

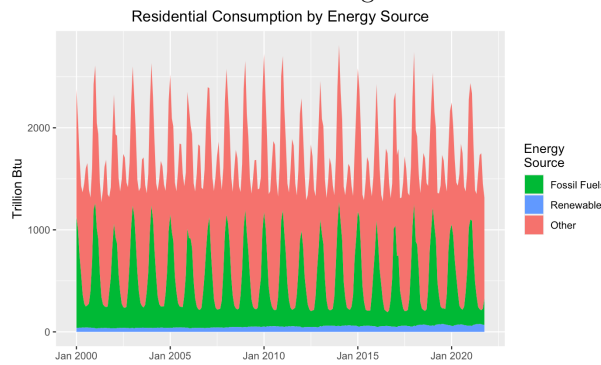
Energy Consumption in the Residential Sector

March 8, 2022

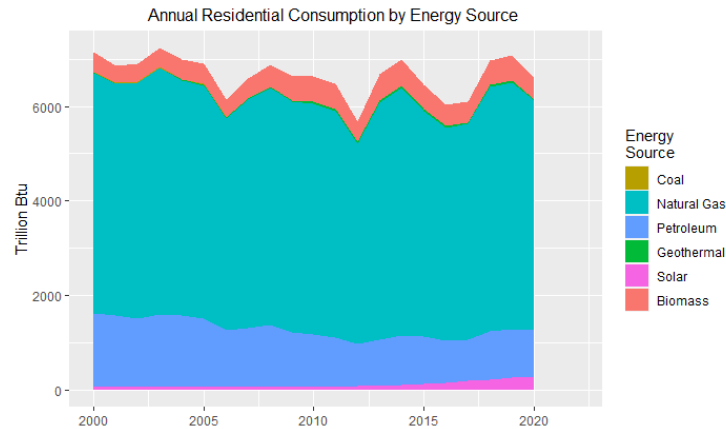
Background

Fossil fuels severely damage our environment with lasting effects on our climate. It is imperative to develop alternative energy sources, like solar, geothermal and wind power. In this report, we are going to explore how the total energy consumed in the residential sector changed in the last two decades.

Renewable energy has grown at a rapid rate over the past decade. However, the bulk of energy source consumed in the residential sector is still dominated by fossil fuels. From the stacked graph below, the total energy consumed in the residential sector has remained constant despite population growth. It is possibly due to a very effective campaign to upgrade homes with energy-efficient appliances.



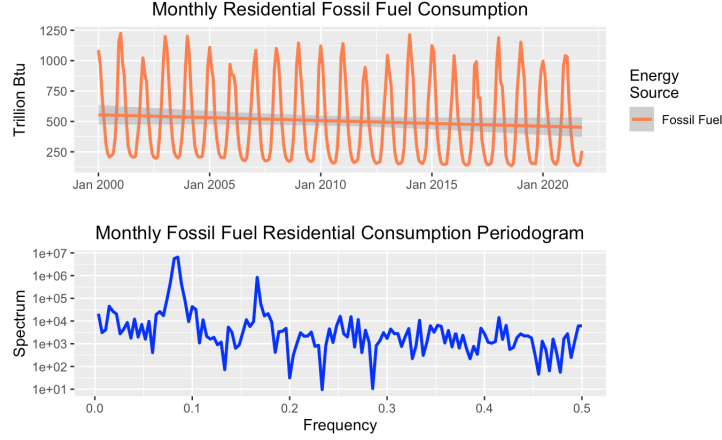
As we break down Fossil Fuels into Coal, Natural Gas and Petroleum and divide Renewable into Geothermal, Solar and Biomass, we can see that solar energy use has grown rapidly (eight fold) over the past twenty years, whereas there is a visible descending trend in Petroleum usage. With seasonality in monthly residential energy use, it is difficult to detect a trend. The graph below shows annual consumption of the six major energy sources.



Fossil fuels, a univariate time series analysis

The main energy source in the residential sector is fossil fuels, we are going to take a closer look at fossil fuels and analyse it as a univariate time series. From the plots below, we cannot detect a linear trend that is statistically significant in the data, hence we will not de-trend the series and proceed with dependence structure analysis. In the ACF and PACF plots, some values lie outside of the 95%

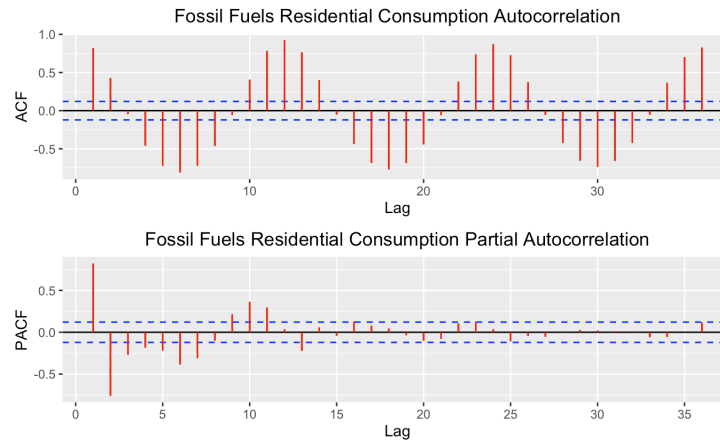
confidence interval blue dotted lines, therefore, we can conclude that there is a seasonal component in the data.



Moreover, the PACF plot illustrates that even though the values are diminished, we observe significant coefficients laying outside the 95% confidence interval past lag 12.

To estimate the frequency of the seasonal component, we look for the the highest spectral density as an indicative of the most significant frequency in explaining the oscillation in the time series. This shows there is a spike in the periodogram at frequency $f_{max} = 0.08519$ corresponding to a seasonal component with a period of approximately 12 months ($1/f_{max}$) which represents an annual cycle in the series.

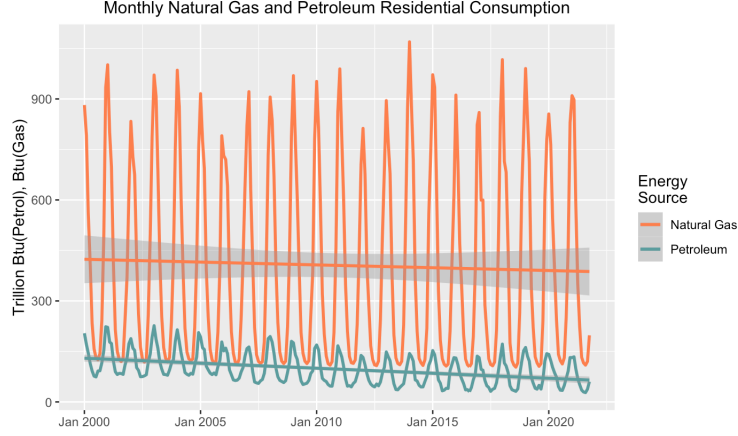
After differencing the data, spectral analysis on the new residual data reveals that it no longer exhibits seasonality. By examining the internal dependence of the residual time series using the ACF and PACF plots below, we see significant presence of positive and negative coefficients outside the 95% confidence interval even at distant lags, so the model is not strictly an $AR(1)$. We can use the augmented Dickey-Fuller test which shows strong evidence at 99% confidence interval against non-stationarity. However, since the series is not strictly an $AR(1)$ model, this stationarity test may not have produced a reliable result. But these preliminary analyses provide a starting point for future investigation.



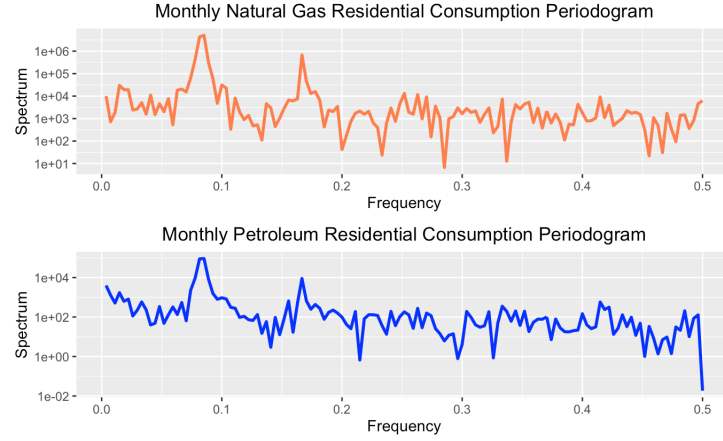
Natural Gas and Petroleum, a multivariate time series analysis

We consider $z_t = (z_{1t}, z_{2t})$, a two-dimensional time series with monthly observations from January 2000 until October 2021, where z_{1t} represents the Natural Gas (expressed in Trillion British thermal units (Btu)) and z_{2t} represents Petroleum consumed in the residential sector. There are no missing data in these series. The objective is to investigate the dynamic relationships between the two components

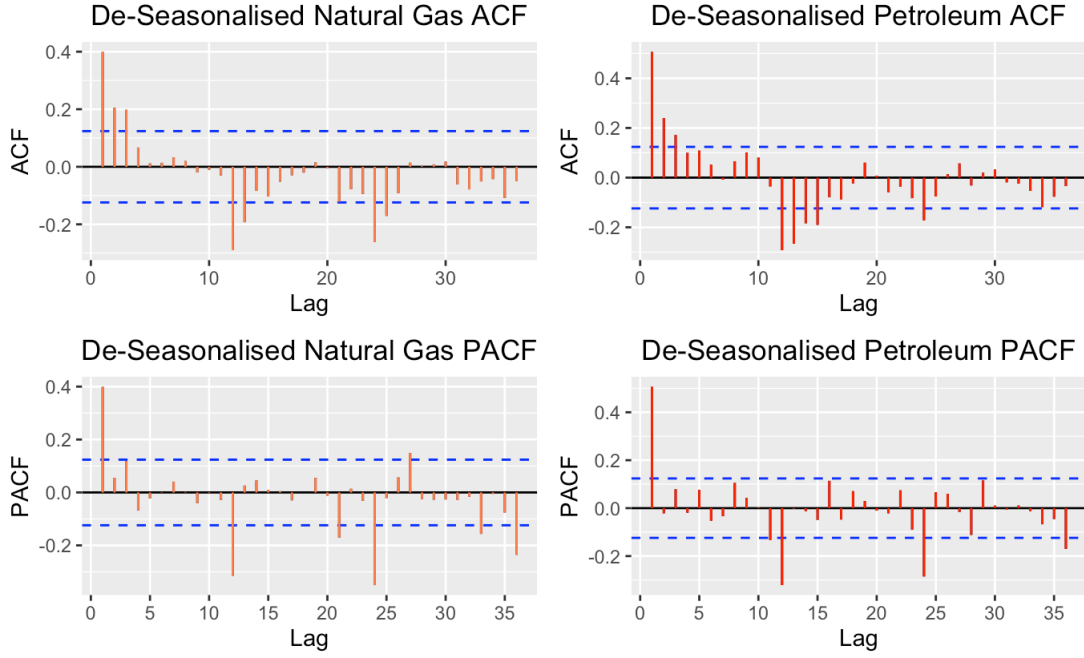
of z_t . We will follow for each individual time series the steps we performed above, eliminating the trend and seasonality by differencing and seasonal decomposition. After fitting linear models to both Natural Gas and Petroleum series, and we observe that Petroleum has a downtrend which is removed before moving on to the analysis of seasonality.



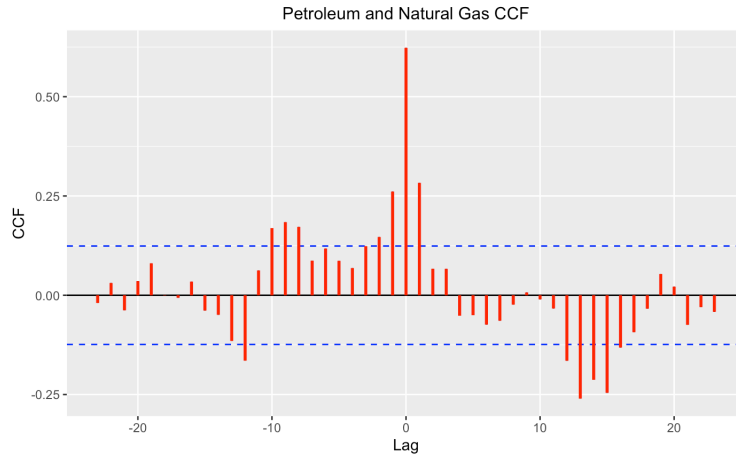
We performed the spectral analysis as we did for Fossil Fuel consumption. We also use the periodogram that detects the spike at frequency $f_{max} = 0.08519$ which corresponds to a seasonal component with a period of approximately 12 months ($1/f_{max}$) for both time series. This tells us that the two series also have a dominant annual cycle as the Total Fossil Fuel series.



After differencing both series and analyse the internal dependence of the residuals using the ACF and PACF plots below, we observe positive and negative coefficients with high significance outside the 95% confidence interval. We perform as a final step the augmented Dickey-Fuller test which outputs a very low $pvalue = 0.01$ which allows us to conclude that there is sufficient evidence against non-stationarity. But again, we suspect that both series are not from an $AR(1)$ process as the PACF have significant lags outside of the first lag, so the results of this test may be questioned.



Next we plot the cross-correlation function for the two series, and we observe a stronger correlation at smaller lags above the dashed line representing the 5% significance tests, which means the two series have more internal dynamics with each other in adjacent months.



We can also use the Portmanteau test to detect the existence of linear dynamic dependence in the data. The null hypothesis H_0 assumes all cross-correlation matrices ρ_i up to lag m are zero, versus the alternative hypothesis H_a that exists $i \in [1, m]$ where ρ_i is non zero. The test outputs very small p-values for the first 12 lags, which shows strong evidence against the null hypothesis of no cross-correlations. This also matches our previous findings.

In general, we discovered that the monthly Petroleum is pointing downward disregarding seasonality, and this could have different reasoning behind such as the increasing popularity of electric cars. Both Natural Gas and Petroleum series have a similar seasonal structure as the total Fossil Fuel consumption whose dominant period is 12 months. We also detect that there are significant linear dependency between these two series especially in adjacent months.