Homework 18

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[A] Suppose X is a discrete random variable with the following PMF: $f(x) = \frac{2+\theta(2-x)}{6}$, for $x = 1, 2, 3$, where the unknown parameter $\theta \in -1, 0, 1$. Suppose a random sample is observed in this distribution: $X_1 = 3$, $X_2 = 2$, $X_3 = 3$, $X_4 = 1$. Find the maximum likelihood estimate of θ based on these observations.

```
f <- function(theta, x){
  (2 + theta *(2-x))/6
}

theta = -1
f(theta,3)*f(theta,2)*f(theta,3)*f(theta,1)
```

```
[1] 0.01388889
```

```
theta = 0
f(theta,3)*f(theta,2)*f(theta,3)*f(theta,1)
```

```
[1] 0.01234568
```

```
theta = 1
f(theta,3)*f(theta,2)*f(theta,3)*f(theta,1)
```

```
[1] 0.00462963
```

```r
pmf <- function(x, theta) {
  return((2 + theta * (2 - x)) / 6)
}

likelihood <- function(theta) {
  return(pmf(3, theta) * pmf(2, theta) * pmf(3, theta) * pmf(1, theta))
}
theta_values <- c(-1, 0, 1)
likelihood_values <- sapply(theta_values, likelihood)

likelihood_values
```

```
[1] 0.01388889 0.01234568 0.00462963
```

```r
theta_mle <- theta_values[which.max(likelihood_values)]
theta_mle
```

```
[1] -1
```

[A] Let independent random samples $X_{1j}, \ldots, X_{nj}$, each of size n, be taken from the k normal distributions with means $\mu_j = c + d[j - (k+1)/2]$, $j = 1, \ldots, k$, respectively, and common known variance σ^2. Find the maximum likelihood estimators of c and d, where both c and d are unconstrained unknown constants.