Code & Outputs

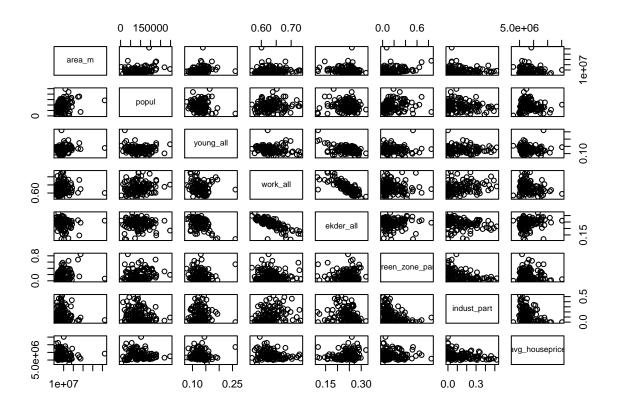
Liziyao, 1500017776 2018.1.15

```
library(stats)
library(forecast)
library(lars)

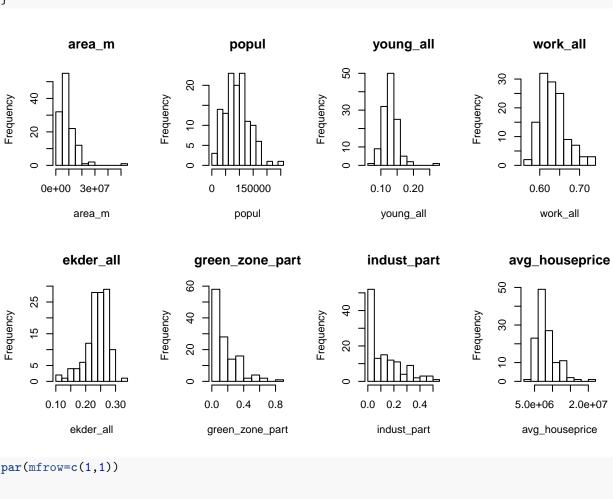
## Loaded lars 1.2

data_original=read.csv("moscow_districts.csv")
data=data_original
n=nrow(data);p=ncol(data)

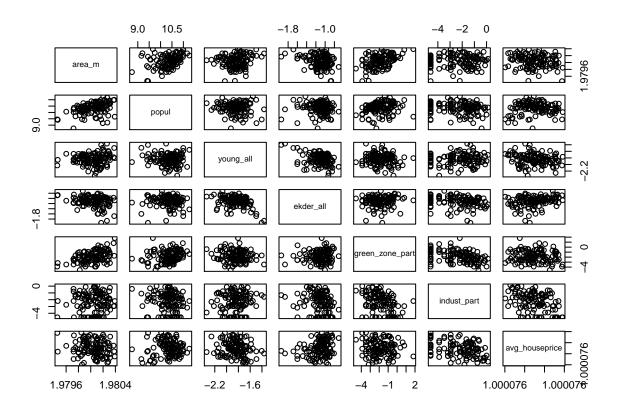
### first exploration ###
# pairwise scatter plot
numeric_cols=c(2,3,4,5,6,7,8,18)
data_numeric=data[,numeric_cols]
pairs(data_numeric)
```

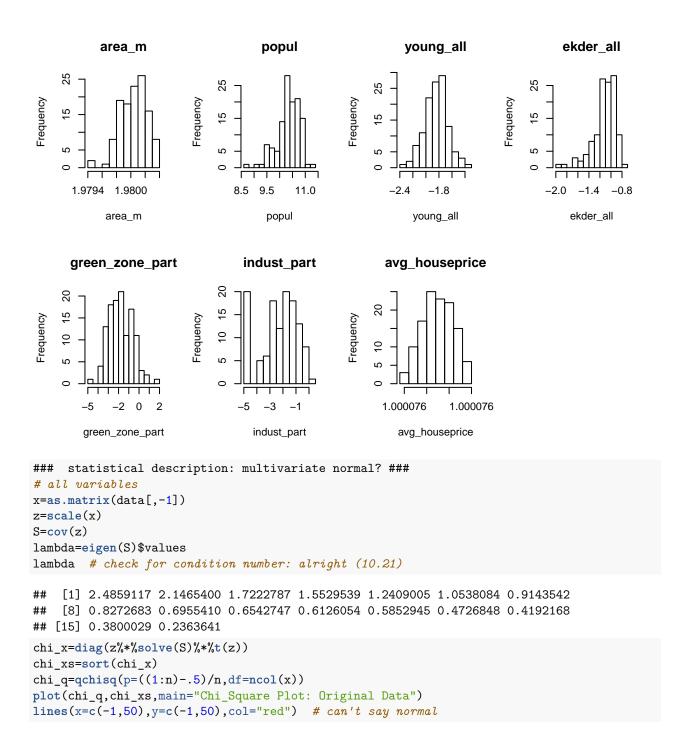


```
# marginal distr.
par(mfrow=c(2,4))
```

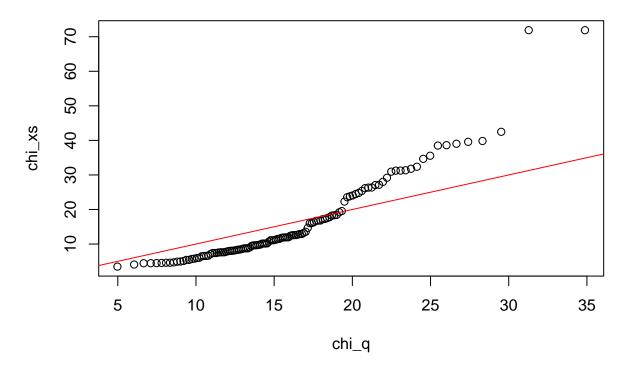


```
data[,i]=logit(data[,i])
}
# boxcox trans. for area,popul and avg_houseprice
boxcox_features=c("area_m","popul","avg_houseprice")
boxcox_lambdas=c(0,0,0)
names(boxcox_lambdas)=boxcox_features
for(i in boxcox_features){
    i
    boxcox_lambdas[i]=BoxCox.lambda(data[,i])
    data[,i]=BoxCox(data[,i],boxcox_lambdas[i])
}
# transformed scatter & hist plots
numeric_cols=c(2,3,4,5,6,7,17)
data_numeric=data[,numeric_cols]
pairs(data_numeric)
```



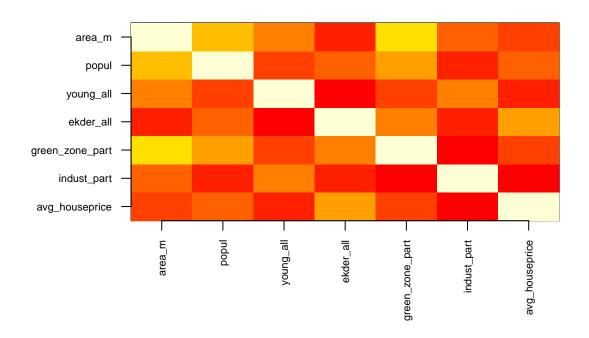


Chi_Square Plot: Original Data



```
# numerical variables
x=as.matrix(data_numeric)
n=nrow(x);p=ncol(x)
z=scale(x)
S=cov(z)
# covariance plot
par(mar=c(7,6,5,4)+.1)
Splot=S[,7:1]
image(Splot,xaxt = 'n', yaxt='n', main="Covariance Plot")
axis(2,labels=colnames(Splot),at=(0:6)/6,las=1,cex.axis=.7)
axis(1,labels=rownames(Splot),at=(0:6)/6,las=3,cex.axis=.7)
```

Covariance Plot



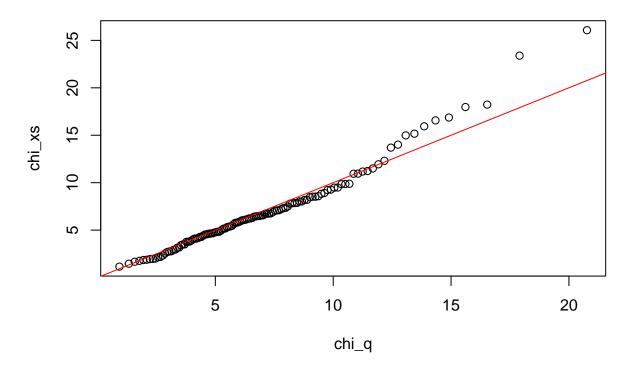
```
par(mar=c(5,4,4,2)+.1)

lambda=eigen(S)$values
lambda # check for condition number: better (7.11)

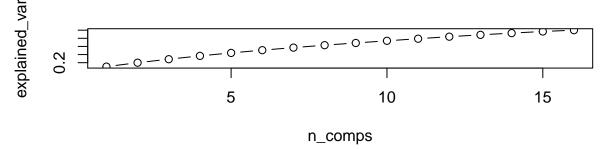
## [1] 1.9220102 1.8152041 1.0019369 0.8742839 0.5618169 0.5536257 0.2711223

chi_x=diag(z%*%solve(S)%*%t(z))
 chi_xs=sort(chi_x)
 chi_q=qchisq(p=((1:n)-.5)/n,df=ncol(x))
 plot(chi_q,chi_xs,main="Chi_Square Plot: Outliers Removed")
 lines(x=c(-1,50),y=c(-1,50),col="red") # can say normal now
```

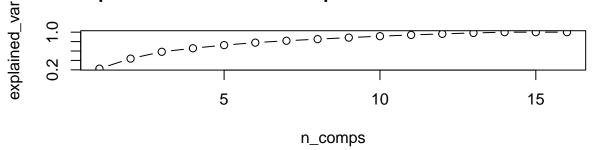
Chi_Square Plot: Outliers Removed



Explained Variance vs. Components numbers: Correlation



Explained Variance vs. Components numbers: Covariance



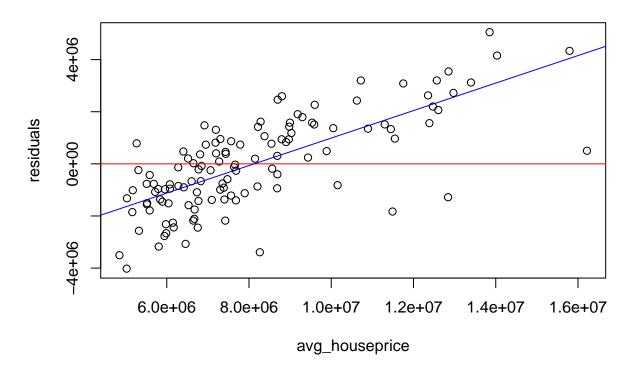
```
# no dominant components; more compressable than correl
par(mfrow=c(1,1))
x.fact=factanal(x,5,scores="Bartlett",rotation="varimax")
x.fact$loadings
```

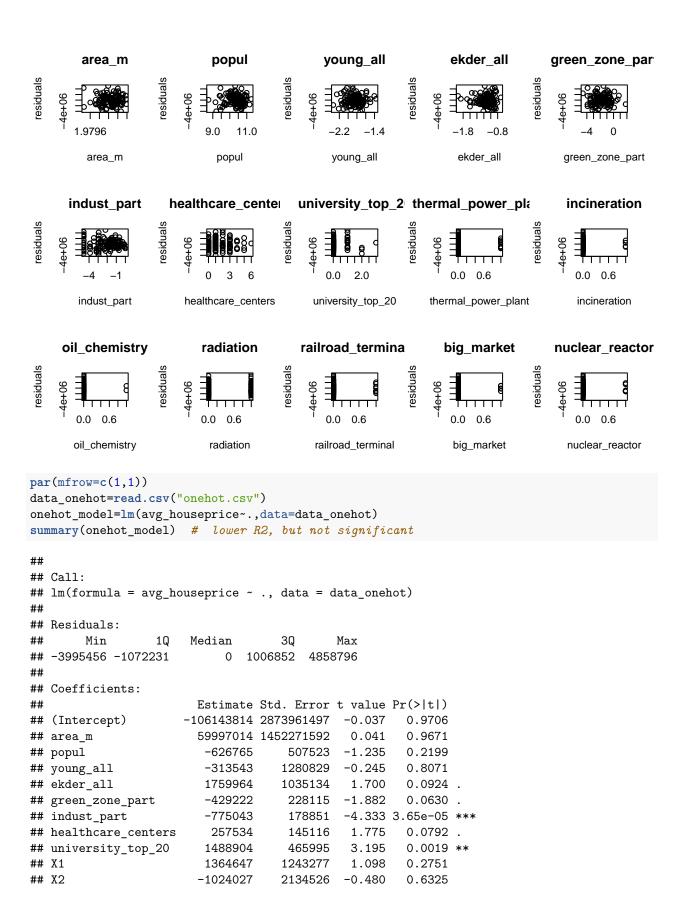
```
##
## Loadings:
##
                       Factor1 Factor2 Factor3 Factor4 Factor5
                                        0.355
                                                0.138
## area_m
                        0.891
## popul
                        0.494
## young_all
                                        0.659
                                              -0.177
                       -0.101
## ekder_all
                                        -0.586
## green_zone_part
                                       -0.180 -0.326
                        0.585
                              -0.374
## indust_part
                       -0.238
                              -0.175
                                        0.332
                                               0.821
## healthcare_centers
                        0.261
                                       -0.129
                                                0.234
                                0.317
## university_top_20
                                0.699
                                                -0.166
## thermal_power_plant 0.139
                                                0.353
                                                         0.154
                                        0.427
## incineration
## oil_chemistry
                                                0.151
                                                         0.984
## radiation
                        0.339
                                0.157
                                                0.124
## railroad_terminal
                                0.541
                                                         0.282
## big_market
## nuclear_reactor
                                                0.317
                                       -0.129
## avg_houseprice
                                0.556 -0.326 -0.200
                                                       -0.154
##
##
                  Factor1 Factor2 Factor3 Factor4 Factor5
```

```
## SS loadings
                   1.668
                            1.416
                                   1.383
                                            1.229
                                                    1.132
## Proportion Var
                            0.088
                                           0.077
                                                    0.071
                   0.104
                                   0.086
## Cumulative Var
                   0.104
                            0.193
                                   0.279
                                            0.356
                                                    0.427
### OLS Regression
                      ###
data reg=data[,-1]
boxcox_model=lm(avg_houseprice~.,data=data_reg)
summary(boxcox_model)
##
## Call:
## lm(formula = avg_houseprice ~ ., data = data_reg)
##
## Residuals:
##
                            Median
                                            3Q
         Min
                      1Q
                                                      Max
## -6.845e-08 -1.894e-08 1.480e-10 2.282e-08 5.225e-08
##
## Coefficients:
##
                         Estimate Std. Error
                                              t value Pr(>|t|)
## (Intercept)
                        1.000e+00 4.028e-05 24830.893 < 2e-16 ***
## area_m
                       -1.116e-05 2.035e-05
                                               -0.549 0.584455
## popul
                       -2.163e-11 7.044e-09
                                               -0.003 0.997556
                       -1.163e-08 1.803e-08
## young all
                                                -0.645 0.520329
## ekder_all
                       1.984e-08 1.483e-08
                                                1.338 0.183901
## green_zone_part
                       -4.376e-09 3.258e-09
                                               -1.343 0.182153
                       -8.994e-09 2.527e-09
## indust_part
                                               -3.560 0.000559 ***
## healthcare_centers
                      4.535e-09 2.064e-09
                                                 2.197 0.030209 *
## university_top_20
                       1.422e-08 6.463e-09
                                                2.200 0.029979 *
## thermal_power_plant 1.311e-08 1.076e-08
                                                1.218 0.225908
                       -7.638e-09 1.632e-08
                                               -0.468 0.640681
## incineration
## oil_chemistry
                       -3.323e-08 2.233e-08
                                                -1.488 0.139815
## radiation
                      -2.320e-09 5.979e-09
                                               -0.388 0.698762
## railroad_terminal
                       1.712e-08 1.278e-08
                                                1.339 0.183404
## big market
                      -9.587e-10 1.277e-08
                                                -0.075 0.940291
## nuclear reactor
                       8.304e-10 1.421e-08
                                                 0.058 0.953508
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.86e-08 on 105 degrees of freedom
## Multiple R-squared: 0.3931, Adjusted R-squared: 0.3064
## F-statistic: 4.535 on 15 and 105 DF, p-value: 1.465e-06
data_reg[,16] = data_original[-outliers,18]
original_model=lm(avg_houseprice~.,data=data_reg)
summary(original_model)
##
## Call:
## lm(formula = avg_houseprice ~ ., data = data_reg)
##
## Residuals:
##
                  1Q
                       Median
## -4020441 -1279666 -134517 1307693
                                       5052714
##
## Coefficients:
```

```
##
                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                        60271364 2685111487
                                            0.022 0.98213
## area m
                       -24261414 1356708098 -0.018 0.98577
## popul
                                    469610 -1.436 0.15401
                         -674298
## young_all
                         -756948
                                    1202096 -0.630 0.53027
## ekder all
                                            1.676 0.09674 .
                         1657048
                                    988779
## green_zone_part
                         -429736
                                     217207 -1.978 0.05050 .
## indust_part
                         -778316
                                    168436 -4.621 1.09e-05 ***
                                            1.888 0.06185 .
## healthcare_centers
                          259744
                                    137611
                                    430838 2.889 0.00469 **
## university_top_20
                         1244742
## thermal_power_plant
                         1121356
                                    717653
                                            1.563 0.12117
## incineration
                                    1087778 -0.201 0.84103
                         -218729
                                    1488967 -1.093 0.27673
## oil_chemistry
                        -1628006
                                     398619 -0.248 0.80430
## radiation
                          -99023
## railroad_terminal
                         1350374
                                     852266
                                            1.584 0.11610
## big_market
                         -139867
                                     851214 -0.164 0.86980
## nuclear_reactor
                          129519
                                     947243
                                             0.137 0.89150
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1907000 on 105 degrees of freedom
## Multiple R-squared: 0.4724, Adjusted R-squared: 0.3971
## F-statistic: 6.268 on 15 and 105 DF, p-value: 2.929e-09
    # R2 of original model is a lot better than a boxcox model
    # use original house price below.
residuals=original_model$residuals
plot(data_reg[,16],residuals,
    xlab="avg_houseprice",ylab="residuals",
    main="Residual Plot")
lines(x=c(0,2e7),y=c(0,0),col="red",type='c')
abline(lm(residuals~data_reg[,16]),col='blue')
```

Residual Plot





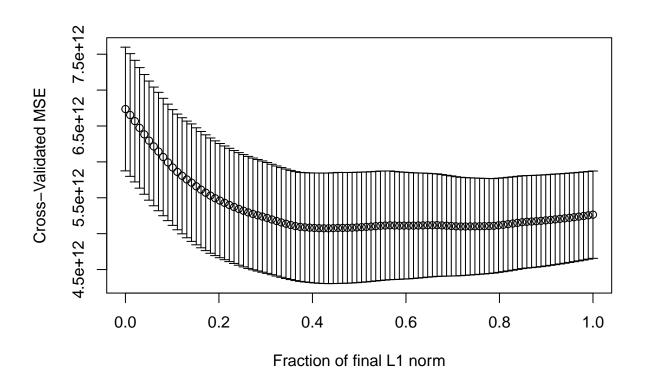
```
## X8
                         -78028
                                   456786 -0.171
                                                    0.8647
                         355166
                                   1455298 0.244
                                                    0.8077
## X9
## X10
                                                    0.7296
                         515279
                                   1486226 0.347
## X11
                        -243558
                                   2030520 -0.120
                                                    0.9048
## X13
                       -1280542
                                   2043488 -0.627
                                                    0.5324
## X16
                        1492146
                                   1087727 1.372 0.1734
## X24
                                   1676084 -0.479 0.6330
                        -802875
## X25
                                   2049578 2.261
                        4635103
                                                    0.0260 *
## X32
                         273396
                                   1447711 0.189
                                                    0.8506
## X36
                        -875518
                                   2052712 -0.427
                                                    0.6707
## X40
                        -702115
                                   1195202 -0.587
                                                    0.5583
                                   2028044 -0.307
## X64
                        -622982
                                                    0.7594
## X65
                         291100
                                   2050951
                                           0.142
                                                    0.8874
## X72
                         264446
                                                    0.8571
                                   1465007
                                            0.181
## X73
                        2582041
                                   2021311 1.277
                                                    0.2046
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 1948000 on 95 degrees of freedom
## Multiple R-squared: 0.5019, Adjusted R-squared: 0.3709
## F-statistic: 3.83 on 25 and 95 DF, p-value: 1.108e-06
### different regression models ###
rmse=function(true,pred){
 mean((true-pred)^2)^.5
}
train=sample(1:n,size=0.8*n)
test_y=data_reg$avg_houseprice[-train]
ols=lm(avg_houseprice~.,data=data_reg[train,])
ols_pred=predict(ols,data_reg[-train,])
ols_rmse=rmse(test_y,ols_pred)
step_ols=step(ols)
## Start: AIC=2804.45
## avg_houseprice ~ area_m + popul + young_all + ekder_all + green_zone_part +
      indust_part + healthcare_centers + university_top_20 + thermal_power_plant +
##
      incineration + oil_chemistry + radiation + railroad_terminal +
##
      big_market + nuclear_reactor
##
##
                        Df Sum of Sq
                                            RSS
                                                   AIC
## - radiation
                         1 7.7244e+09 3.3463e+14 2802.4
                        1 2.2087e+11 3.3484e+14 2802.5
## - big_market
## - area_m
                        1 3.7925e+11 3.3500e+14 2802.6
## - incineration
                        1 4.8956e+11 3.3511e+14 2802.6
                         1 5.0845e+11 3.3513e+14 2802.6
## - young_all
                      1 2.8452e+12 3.3746e+14 2803.3
## - nuclear_reactor
## - healthcare_centers 1 3.7666e+12 3.3839e+14 2803.5
## - oil_chemistry
                        1 4.5661e+12 3.3919e+14 2803.8
## - popul
                        1 6.1388e+12 3.4076e+14 2804.2
## <none>
                                     3.3462e+14 2804.4
## - ekder all
                       1 7.2008e+12 3.4182e+14 2804.5
## - thermal_power_plant 1 8.9841e+12 3.4360e+14 2805.0
## - green_zone_part
                         1 1.1323e+13 3.4594e+14 2805.6
## - railroad_terminal
                         1 1.1930e+13 3.4655e+14 2805.8
```

```
1 2.5220e+13 3.5984e+14 2809.4
## - university top 20
                          1 7.1056e+13 4.0567e+14 2820.9
## - indust_part
##
## Step: AIC=2802.45
## avg_houseprice ~ area_m + popul + young_all + ekder_all + green_zone_part +
       indust part + healthcare centers + university top 20 + thermal power plant +
##
       incineration + oil chemistry + railroad terminal + big market +
##
##
       nuclear reactor
##
##
                         Df Sum of Sq
                                              RSS
                                                      AIC
## - big_market
                          1 2.1881e+11 3.3485e+14 2800.5
                          1 3.9416e+11 3.3502e+14 2800.6
## - area_m
## - young_all
                          1 5.0502e+11 3.3513e+14 2800.6
## - incineration
                          1 5.1119e+11 3.3514e+14 2800.6
                          1 2.8424e+12 3.3747e+14 2801.3
## - nuclear_reactor
## - healthcare_centers
                          1 3.7720e+12 3.3840e+14 2801.5
## - oil_chemistry
                          1 4.5645e+12 3.3919e+14 2801.8
## - popul
                          1 6.7284e+12 3.4136e+14 2802.4
## <none>
                                       3.3463e+14 2802.4
## - ekder all
                          1 7.2911e+12 3.4192e+14 2802.5
## - thermal_power_plant 1 9.0202e+12 3.4365e+14 2803.0
## - green_zone_part
                          1 1.1318e+13 3.4594e+14 2803.6
## - railroad terminal
                          1 1.2066e+13 3.4669e+14 2803.8
## - university_top_20
                          1 2.5758e+13 3.6038e+14 2807.6
## - indust_part
                          1 7.1422e+13 4.0605e+14 2819.0
## Step: AIC=2800.51
## avg_houseprice ~ area_m + popul + young_all + ekder_all + green_zone_part +
       indust_part + healthcare_centers + university_top_20 + thermal_power_plant +
##
##
       incineration + oil_chemistry + railroad_terminal + nuclear_reactor
##
##
                         Df Sum of Sq
                                              RSS
                                                      AIC
## - area_m
                          1 3.7803e+11 3.3522e+14 2798.6
                          1 4.5768e+11 3.3530e+14 2798.6
## - young_all
                          1 5.1657e+11 3.3536e+14 2798.7
## - incineration
                          1 2.8160e+12 3.3766e+14 2799.3
## - nuclear_reactor
## - healthcare centers
                        1 3.8717e+12 3.3872e+14 2799.6
## - oil_chemistry
                          1 4.4460e+12 3.3929e+14 2799.8
## - popul
                          1 6.7221e+12 3.4157e+14 2800.4
## <none>
                                       3.3485e+14 2800.5
                          1 7.4847e+12 3.4233e+14 2800.6
## - ekder all
## - thermal_power_plant 1 8.8017e+12 3.4365e+14 2801.0
## - green_zone_part
                          1 1.1175e+13 3.4602e+14 2801.7
## - railroad_terminal
                          1 1.2140e+13 3.4699e+14 2801.9
## - university_top_20
                          1 2.5561e+13 3.6041e+14 2805.6
                          1 7.1241e+13 4.0609e+14 2817.0
## - indust_part
##
## Step: AIC=2798.62
  avg_houseprice ~ popul + young_all + ekder_all + green_zone_part +
##
       indust_part + healthcare_centers + university_top_20 + thermal_power_plant +
##
       incineration + oil_chemistry + railroad_terminal + nuclear_reactor
##
##
                         Df Sum of Sq
                                              RSS
                                                     ATC
## - young_all
                          1 6.0972e+11 3.3583e+14 2796.8
```

```
## - incineration
                          1 6.2920e+11 3.3585e+14 2796.8
## - nuclear_reactor
                          1 2.8604e+12 3.3808e+14 2797.4
## - healthcare centers
                        1 3.5712e+12 3.3879e+14 2797.6
## - oil_chemistry
                          1 4.6888e+12 3.3991e+14 2797.9
## <none>
                                       3.3522e+14 2798.6
## - thermal power plant 1 8.4431e+12 3.4367e+14 2799.0
## - ekder all
                          1 8.4580e+12 3.4368e+14 2799.0
                          1 8.8819e+12 3.4411e+14 2799.1
## - popul
## - railroad_terminal
                          1 1.2021e+13 3.4724e+14 2800.0
## - green_zone_part
                          1 1.9147e+13 3.5437e+14 2801.9
## - university_top_20
                          1 2.5185e+13 3.6041e+14 2803.6
## - indust_part
                          1 7.5837e+13 4.1106e+14 2816.2
##
## Step: AIC=2796.8
## avg_houseprice ~ popul + ekder_all + green_zone_part + indust_part +
##
       healthcare_centers + university_top_20 + thermal_power_plant +
##
       incineration + oil_chemistry + railroad_terminal + nuclear_reactor
##
##
                         Df Sum of Sq
                                              RSS
                                                     ATC
## - incineration
                          1 7.1521e+11 3.3655e+14 2795.0
## - nuclear_reactor
                          1 3.0460e+12 3.3888e+14 2795.7
## - healthcare centers
                          1 3.6083e+12 3.3944e+14 2795.8
## - oil_chemistry
                          1 4.2092e+12 3.4004e+14 2796.0
                                       3.3583e+14 2796.8
## <none>
                          1 8.5217e+12 3.4436e+14 2797.2
## - popul
## - thermal_power_plant 1 8.7115e+12 3.4454e+14 2797.2
## - railroad_terminal
                          1 1.2639e+13 3.4847e+14 2798.3
## - ekder_all
                          1 1.2770e+13 3.4860e+14 2798.4
                          1 1.9119e+13 3.5495e+14 2800.1
## - green_zone_part
## - university_top_20
                          1 2.4956e+13 3.6079e+14 2801.7
## - indust_part
                          1 7.7229e+13 4.1306e+14 2814.7
##
## Step: AIC=2795
## avg_houseprice ~ popul + ekder_all + green_zone_part + indust_part +
##
       healthcare_centers + university_top_20 + thermal_power_plant +
##
       oil_chemistry + railroad_terminal + nuclear_reactor
##
##
                         Df Sum of Sq
                                                     AIC
                                              RSS
## - nuclear reactor
                          1 3.1774e+12 3.3973e+14 2793.9
## - oil_chemistry
                          1 3.9097e+12 3.4046e+14 2794.1
## - healthcare centers
                          1 4.3677e+12 3.4092e+14 2794.2
                                       3.3655e+14 2795.0
## <none>
## - thermal_power_plant 1 8.2730e+12 3.4482e+14 2795.3
## - popul
                          1 8.7109e+12 3.4526e+14 2795.4
## - railroad_terminal
                          1 1.3069e+13 3.4962e+14 2796.7
## - ekder_all
                          1 1.3889e+13 3.5044e+14 2796.9
## - green_zone_part
                          1 1.8994e+13 3.5554e+14 2798.3
## - university_top_20
                          1 2.4527e+13 3.6108e+14 2799.8
## - indust_part
                          1 8.2683e+13 4.1923e+14 2814.1
## Step: AIC=2793.9
## avg_houseprice ~ popul + ekder_all + green_zone_part + indust_part +
##
      healthcare_centers + university_top_20 + thermal_power_plant +
##
       oil_chemistry + railroad_terminal
```

```
##
                         Df Sum of Sq
##
                                                     ATC
                                              RSS
## - oil chemistry
                          1 4.4260e+12 3.4415e+14 2793.1
## - healthcare_centers
                          1 5.6955e+12 3.4542e+14 2793.5
## <none>
                                       3.3973e+14 2793.9
## - popul
                          1 8.3785e+12 3.4810e+14 2794.2
## - thermal_power_plant 1 9.9759e+12 3.4970e+14 2794.7
## - railroad terminal
                          1 1.2306e+13 3.5203e+14 2795.3
## - ekder_all
                          1 1.5248e+13 3.5497e+14 2796.1
## - green_zone_part
                          1 1.9542e+13 3.5927e+14 2797.3
## - university_top_20
                          1 2.3968e+13 3.6369e+14 2798.4
                          1 8.0691e+13 4.2042e+14 2812.4
## - indust_part
##
## Step: AIC=2793.14
  avg_houseprice ~ popul + ekder_all + green_zone_part + indust_part +
##
       healthcare_centers + university_top_20 + thermal_power_plant +
##
       railroad_terminal
##
##
                         Df Sum of Sq
                                              RSS
                                                     AIC
## - healthcare centers
                          1 5.7233e+12 3.4988e+14 2792.7
## <none>
                                       3.4415e+14 2793.1
## - popul
                          1 7.9105e+12 3.5206e+14 2793.3
## - thermal_power_plant 1 8.0500e+12 3.5220e+14 2793.4
## - railroad terminal
                          1 1.3248e+13 3.5740e+14 2794.8
## - ekder all
                          1 1.5620e+13 3.5977e+14 2795.4
## - green_zone_part
                          1 1.9586e+13 3.6374e+14 2796.5
## - university_top_20
                          1 2.3461e+13 3.6761e+14 2797.5
                          1 8.5210e+13 4.2936e+14 2812.4
## - indust_part
##
## Step: AIC=2792.73
## avg_houseprice ~ popul + ekder_all + green_zone_part + indust_part +
##
       university_top_20 + thermal_power_plant + railroad_terminal
##
                         Df Sum of Sq
##
                                              RSS
                                                     AIC
                          1 4.2889e+12 3.5416e+14 2791.9
## - popul
## - thermal_power_plant 1 6.7260e+12 3.5660e+14 2792.6
## <none>
                                       3.4988e+14 2792.7
## - railroad_terminal
                          1 1.3740e+13 3.6362e+14 2794.4
## - ekder all
                          1 1.6400e+13 3.6628e+14 2795.1
## - green_zone_part
                          1 2.0943e+13 3.7082e+14 2796.3
## - university top 20
                          1 2.8615e+13 3.7849e+14 2798.3
## - indust_part
                          1 8.0253e+13 4.3013e+14 2810.6
##
## Step: AIC=2791.9
## avg_houseprice ~ ekder_all + green_zone_part + indust_part +
       university_top_20 + thermal_power_plant + railroad_terminal
##
##
##
                         Df
                            Sum of Sq
                                              RSS
                                                      AIC
## - thermal_power_plant 1 6.0659e+12 3.6023e+14 2791.5
## <none>
                                       3.5416e+14 2791.9
                          1 1.4350e+13 3.6851e+14 2793.7
## - railroad_terminal
## - ekder all
                          1 1.6208e+13 3.7037e+14 2794.2
## - green_zone_part
                          1 2.4959e+13 3.7912e+14 2796.4
## - university top 20
                          1 2.8460e+13 3.8262e+14 2797.3
```

```
1 7.7790e+13 4.3195e+14 2809.0
## - indust_part
##
## Step: AIC=2791.53
## avg_houseprice ~ ekder_all + green_zone_part + indust_part +
      university_top_20 + railroad_terminal
##
                     Df Sum of Sq
##
                                         RSS
                                                AIC
## <none>
                                   3.6023e+14 2791.5
## - railroad_terminal 1 1.6954e+13 3.7718e+14 2793.9
## - ekder_all
                      1 1.7205e+13 3.7744e+14 2794.0
## - green_zone_part
                      1 2.2557e+13 3.8279e+14 2795.4
## - indust_part
                      1 7.1758e+13 4.3199e+14 2807.0
step_ols_pred=predict(step_ols,data_reg[-train,])
step_ols_rmse=rmse(test_y,step_ols_pred)
trainx=as.matrix(data_reg[train,-16])
trainy=data_reg[train,16]
testx=as.matrix(data_reg[-train,-16])
ridge=lm.ridge(avg_houseprice~.,data=data_reg[train,],
              lambda=seq(0,100,length=10001))
ridge_k=which.min(ridge$GCV)
ridge_coef=coef(ridge)[ridge_k,]
ridge_pred=cbind(1,testx)%*%ridge_coef
ridge_rmse=rmse(test_y,ridge_pred)
lasso=lars(trainx,trainy)
lasso_cv=cv.lars(trainx,trainy,K=5)
```



[1] 1511196 1524160 1417529 1413124 1364878