附录I: 无约束优化检验题目

下面给出无约束优化问题的检验题目，每一题给出的内容包括：

- n,m. 若n或m未给定，则为正整数变量. 变量的维数或规模可以自选.
- $r_i(x)$, $i = 1, ...m$,
- x_0, x^*, $f(x^*)$.

从而优化问题为

$$\min \sum_{i=1}^{m} r_i^2(x).$$

上机习题

1. Penalty I 函数

    $m = n + 1,$

    $r_i(x) = \sqrt{\gamma}(x_i - 1),\ 1 \geq i \geq n,$

    $r_{n+1}(x) = \sum_{j=1} n x_j^2 - \frac{1}{4};\ \gamma = 10^{-5}$

    $x_0 = (\xi_j),\ \xi_j = j,$

    $f = 2.24997...10^{-5}, if\ n = 4; f = 7.08765...10^{-5}, if\ n = 10.$

2. Rosenbrock 函数

$$n = 2, m = 2,$$
$$r_1(x) = 10(x_2 - x_1^2), \ r_2(x) = 1 - x_1,$$
$$x_0 = (-1.2, 1), \quad x^* = (1, 1), \quad f(x^*) = 0.$$

3. Powell badly scaled 函数

$$n = 2, m = 2,$$
$$r_1(x) = 10^4 x_1 x_2 - 1, \ r_2(x) = e^{-x_1} + e^{-x_2} - 1.0001,$$
$$x_0 = (0, 1), \quad x^* = (1.098...10^{-5}, 9.106...), \quad f(x^*) = 0.$$

4. Box three-dimensional function

$$n = 3, m \geq n,$$
$$r_i(x) = e^{-t_i x_1} - e^{-t_i x_2} - x_3(e^{-t_i} - e^{-10t_i}),$$
$$where \ t_i = 0.1i$$
$$x_0 = (0, 10, 20)^T, \ f = 0 \ at \ (1, 10, 10)^T, \ (10, 1, -1)^T$$
$$and \ wherever \ (x_1 = x_2, x_3 = 0).$$

5. Wood 函数

$$n = 4, m = 6,$$
$$r_1(x) = 10(x_2 - x_1^2), \ r_2(x) = 1 - x_1, \ r_3(x) = (90)^{1/2}(x_4 - x_3^2),$$
$$r_4(x) = 1 - x_3, \ r_5(x) = (10)^{1/2}(x_2 + x_4 - 2), \ r_6(x) = (10)^{-1/2}(x_2 - x_4),$$
$$x_0 = (-3, -1, -3, -1)^T, \ x^* = (1, 1, 1, 1), \ f(x^*) = 0.$$

6. Watson 函数

$$2 \le n \le 31,\ m = 31,$$

$$r_i(x) = \sum_{j=2}^{n} (j-1) x_j t_i^{j-2} - (\sum_{j=1}^{n} x_j t_i^{j-1})^2 - 1,$$

$$\text{其中} t_i = i/29, 1 \le i \le 29,$$

$$f_{30}(x) = x_1, \quad f_{31} = x_2 - x_1^2 - 1,$$

$$x_0 = (0, ...0),$$

$$\text{若} n = 6, f^* = 2.28767...10^{-3},$$

$$\text{若} n = 9, f^* = 1.39976...10^{-6},$$

$$\text{若} n = 12, f^* = 4.72238...10^{-10}.$$

7. Trigonometric 函数

$$m = n,$$

$$r_i(x) = n - \sum_{j=1}^{n} cos\ x_j + i(1 - cos\ x_i) - sin\ x_i,$$

$$x_0 = (1/n, ..., 1/n),\ f^* = 0.$$

8. Discrete boundary value 函数

$$m = n,$$
$$r_i(x) = 2x_i - x_{i-1} - x_{i+1} + h^2(x_i + t_i + 1)^3/2,$$
$$\text{其中}h = 1/(n+1),\ t_i = ih,\ x_o = x_{n+1} = 0.$$
$$x_0 = (\xi_j),\text{其中}\xi_j = t_j(t_j - 1),\ f^* = 0.$$

9. Beale 函数

$$n = 2, m = 3,$$
$$r_i(x) = y_i - x_1(1 - x_2^i),\ \text{其中}\ y_1 = 1.5,\ y_2 = 2.25,\ y_3 = 2.625$$
$$x_0 = (1, 1),\quad x^* = (3, 0.5),\quad f(x^*) = 0.$$

10. Biggs EXP6 函数

$$n = 6, m \geq n,$$
$$r_i(x) = x_3 e^{-t_i x_1} - x_4 e^{-t_i x_2} + x_6 e^{-t_i x_5} - y_i,$$
$$\text{其中}t_i = 0.1i,\ y_i = e^{-t_i} - 5e^{-10t_i} + 3e^{-4t_i}$$
$$x_0 = (1, 2, 1, 1, 1, 1).\text{若}\ m = 13,\quad f(x^*) = 5.65565...10^{-3},$$
$$x^* = (1, 10, 1, 5, 4, 3), f(x^*) = 0.$$

11. Extended Rosenbrock 函数

n为偶数$, m = n,$

$r_{2i-1}(x) = 10(x_{2i} - x_{2i-1}^2), \quad r_{2i}(x) = 1 - x_{2i-1},$

$x_0 = (\xi_j),$ 其中 $\xi_{2j-1} = -1.2, \xi_{2j} = 1$

$x^* = (1, ..., 1), \quad f(x^*) = 0.$

12. Extended Powell singular 函数

n为4的整数倍$, m = n,$

$r_{4i-3}(x) = x_{4i-3} + 10x_{4i-2},$

$r_{4i-2}(x) = 5^{1/2}(x_{4i-1} - x_{4i}),$

$r_{4i-1}(x) = (x_{4i-2} - 2x_{4i-1})^2,$

$r_{4i}(x) = 10^{1/2}(x_{4i-3} - x_{4i})^2.$

$x_0 = (\xi_j),$ 其中 $\xi_{4j-3} = 3, \xi_{4j-2} = -1, \xi_{4j-1} = 0, \xi_{4j} = 1,$

$x^* = (0, ..., 0), \quad f(x^*) = 0.$

13. Penalty II 函数

$$m = 2n,$$
$$r_1(x) = x_1 - 0.2;$$
$$r_i(x) = \sqrt{a}(e^{\frac{x_i}{10}} + e^{\frac{x_{i-1}}{10}} - y_i),\ 2 \le i \le n,$$
$$r_i(x) = \sqrt{a}(e^{\frac{x_{i-n+1}}{10}} + e^{\frac{-1}{10}}),\ n < i < 2n,$$
$$r_{2n}(x) = \Big(\sum_{j=1}^{n}(n-j+1)x_j^2\Big) - 1,$$
$$where\ a = 10^{-5}\ and\ y_i = e^{\frac{i}{10}} + e^{\frac{i-1}{10}},$$
$$x_0 = \Big(\frac{1}{2}, ..., \frac{1}{2}\Big),$$
$$f = 9.37629...10^{-6}, if\ n = 4;$$
$$f = 2.93660...10^{-4},\ if\ n = 10.$$

约束优化检验问题

14.

$$\min \quad f(x) = -1,$$
$$s.t. \quad x_1^2 + x_2^2 - 25 = 0,$$
$$x_1 x_2 - 9 = 0.$$

$x_0 = (2, 1)^T$为非可行点,

$x^* = (a, 9/a),\ (-a, -9/a),\ (b, 9/b),\ (-b, -9/b),$

$$\text{其中} a = \sqrt{\frac{25 + \sqrt{301}}{2}},\ b = \sqrt{\frac{25 - \sqrt{301}}{2}},$$

$f(x^*) = -1.$

15.

$$\min \quad f(x) = ln(1 + x_1^2) - x_2,$$
$$s.t. \quad (1 + x_1^2)^2 + x_2^2 - 4 = 0.$$

$$x_0 = (2, 2)\text{为非可行点},$$
$$x^* = (0, \sqrt{3}),\ f(x^*) = -\sqrt{3}.$$

16.

$$\min \quad f(x) = .5x_1^2 + x_2^2 - x_1 x_2 - 7x_1 - 7x_2,$$
$$s.t. \quad 25 - 4x_1^2 - x_2^2 \geq 0.$$
$$x_0 = (0, 0)\text{为可行点}, \quad x^* = (2, 3),\ f(x^*) = -30.$$

17.

$$\min \quad f(x) = \frac{1}{27\sqrt{3}}((x_1 - 3)^2 - 9)x_2^3,$$

$$s.t. \quad x_1/\sqrt{3} - x_2 \geq 0,$$

$$x_1 + \sqrt{3}x_2 \geq 0,$$

$$-x_1 - \sqrt{3}x_2 + 6 \geq 0,$$

$$x_1 \geq 0,$$

$$x_2 \geq 0.$$

$x_0 = (1, 0.5)$为可行点, $x^* = (3, \sqrt{3}), f(x^*) = -1.$

18.

$$\min \quad f(x) = \sum_{i=1}^{99} r_i^2(x),$$

$$r_i(x) = -0.01i + e^{-\frac{1}{x_1}(u_i - x_2)^{x_3}},$$

$$\text{其中} u_i = 25 + (-50ln(0.01i))^{\frac{2}{3}}, \quad i = 1, ..., 99,$$

$$s.t. \quad 0.1 \leq x_1 \leq 100,$$

$$0 \leq x_2 \leq 25.6,$$

$$0 \leq x_3 \leq 5.$$

$x_0 = (100, 12.5, 3)$为可行点, $x^* = (50, 25, 1.5), f^* = 0.$

19.

$$\min \quad f(x) = -x_1 x_2 x_3,$$
$$s.t. \quad -x_1^2 - 2x_2^2 - 4x_3^2 + 48 \geq 0.$$

$$x_0 = (1, 1, 1) \text{为可行点}$$
$$x^* = (a, b, c,), (a, -b, -c), (-a, b, -c), (-a, -b, c)$$
其中 $a = 4, b = 2\sqrt{2}, c = 2,$
$$f(x^*) = -16\sqrt{2}.$$

20.

$$\min \quad f(x) = r_1(x) + r_2(x),$$
$$s.t. \quad 300 - x_1 - \frac{1}{a} x_3 x_4 \cos(b - x_6) + \frac{c}{a} d x_3^2 = 0,$$
$$-x_2 - \frac{1}{a} x_3 x_4 \cos(b + x_6) + \frac{c}{a} d x_4^2 = 0,$$
$$-x_5 + \frac{1}{a} x_3 x_4 \sin(b + x_6) + \frac{c}{a} x_4^2 = 0,$$
$$200 - \frac{1}{a} x_3 x_4 \sin(b - x_6) + \frac{c}{a} e x_3^2 = 0,$$
$$0 \leq x_1 \leq 400, \quad 340 \leq x_3 \leq 420, \quad -1000 \leq x_5 \leq 10000,$$
$$0 \leq x_2 \leq 1000, \quad 340 \leq x_4 \leq 420, \quad 0 \leq x_6 \leq 0.5236,$$
$$a = 131.078, \quad b = 1.48577, \quad c = 0.90798,$$
$$d = \cos 1.47588, \quad e = \sin 1.47588,$$

其中

$$r_1(x) = \begin{cases} 30x_1, & 0 \le x_1 < 300, \\ 31x_1, & 300 \le x_1 \le 400. \end{cases}$$

$$r_2(x) = \begin{cases} 28x_2, & 0 \le x_2 < 100, \\ 29x_2, & 100 \le x_2 < 200, \\ 30x_2, & 200 \le x_2 \le 1000. \end{cases}$$

$x_0 = (390, 1000, 419.5, 340.5, 198.175, 0.5)$为非可行点

$x^* = (107.8119, 196.3186, 373.3807, 420, 213.0713, 0.1532920),$

$f(x^*) = 8927.5977.$

21.

$$\min \quad f(x) =,$$

初始点选作$x_0 = (0,0)^T.$