1 2

3

4

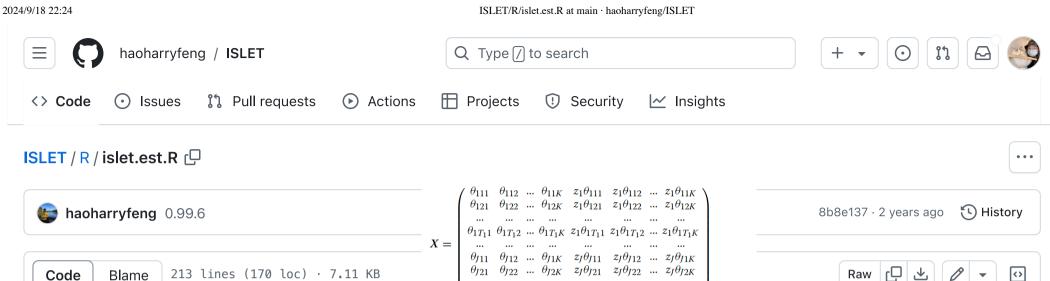
16 17

18

19

20

21



 θ_{IT_11} θ_{IT_12} ... θ_{IT_1K} $z_I\theta_{IT_11}$ $z_I\theta_{IT_12}$... $z_I\theta_{IT_1K}$

###function to implement EM algorithm by block of genes in ISLET algorithm #function here for unix and windows, using lapply, no parallel computing #Y is a GxN matrix for gene expression

dim(u) = (NU×k)×1

islet.solve.block<-function(Y, datuse){</pre> 6 #exp case = as.matrix(datuse@exp case) #exp ctrl = as.matrix(datuse@exp ctrl) X <- datuse@X A <- datuse@A 10 K <- datuse@K 11 NU <- datuse@NU 12 NS <- datuse@NS 13 para<-datuse@para dim=(NUxK)x(NUxK) 14 15

mization (EM) algorithm, although other viable approaches exist. Here, to facilitate the setup of the EM algorithm, we first define the "observed" and the "missing" data: $w = (y, u) := (w_{obs}, w_{mis})$, where $w_{obs} := y$ is the observed data of admixed gene expression, and $w_{mis} := u$ is the missing data of individual-level deviance from the group mean. Then, we have the conditional distribution $w_{obs}|w_{mis} = y|u \sim N(X\beta + Au, \sigma_0^2 I)$ and marginal distribution $w_{mis} = u \sim N(\mathbf{0}, \Sigma_u)$. Here, Σ_u is a block-diagonal matrix $\Sigma_u = diag(\sigma_1^2 I_I, \sigma_2^2 I_I, \cdots, \sigma_k^2 I_I)$. By calculating the variance-covariance matrix of w_{mis} and w_{obs} , we have the following multivariate normal distribution:

$$\begin{pmatrix} \mathbf{w}_{obs} \\ \mathbf{w}_{mis} \end{pmatrix} = N \begin{bmatrix} \begin{pmatrix} X\boldsymbol{\beta} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} A\boldsymbol{\Sigma}_{u}A' + \sigma_{0}^{2}I & A\boldsymbol{\Sigma}_{u} \\ \boldsymbol{\Sigma}'_{u}A' & \boldsymbol{\Sigma}_{u} \end{pmatrix}$$
(6)

The EM algorithm calculation will then follow naturally.

E-step:

$$E[u|w_{obs} = y] = \Sigma_u A' V^{-1} \underbrace{(y - X\beta)}_{Sig}$$

$$E[s's|w_{obs} = y] = tr(A\Sigma_p A') + (A\mu_p + X\beta - y)'(A\mu_p + X\beta - y)$$

$$E[\mathbf{u}_{k}'\mathbf{u}_{k}|\mathbf{w}_{obs}=\mathbf{y}]=tr(\mathbf{\Sigma}_{p_{k}})+\boldsymbol{\mu}_{p_{k}}'\boldsymbol{\mu}_{p_{k}}$$

Here, $s = Au + X\beta - y$, $V := A\Sigma_u A' + \sigma_0^2 I$, Σ_{p_k} is the kth diagonal block of matrix Σ_{p_k} and μ_{p_k} is the *k*th sub-vector in μ_p .

```
Y<-t(Y) after this step, dim(Y) = N×G
22
            G < -ncol(Y)
23
24
            \# Y=log2(Y+1)
25
26
            ####1. Initialization of parameters
27
            #1.1 cell type profiles AND csDE B parameters
28
            \#B \ 0 = solve(X,Y)
            B_0 <- Matrix::tcrossprod( Matrix::tcrossprod(solve( Matrix::crossprod(X)), X), t(Y))
29
                                                                                              NXQ (Q=JF)
30
31
            #1.2 error terms
32
            \# sig <- mean((Y-X%*%B 0)^2)
33
            sig \leftarrow colMeans((Y-X%*%B 0)^2)
34
            \#sig <- 20
35
            #1.3 missing values
36
                                     50 x 6 = 300
            U 0 <- rep(0, NU*K)
37
38
            B t <- B 0
39
            \#sig t = rep(sig, 7)
40
            U t <- U 0
41
42
            #sig0_t <- rep(sig, G)</pre>
43
            #sigK_t <- rep(sig, K)</pre>
                                                                                                                  UJK
            sig0 t <- sig #rep(sig, G)
44
            sigK_t <- matrix(rep(sig, each=K), nrow=K)</pre>
45
46
47
                                   Sig p
                                                             list [500]
                                                                                        List of length 500
            iem <- 1
48
                                     [[1]]
                                                            list [2]
                                                                                        List of length 2
            diff1 <- 100
49
                                                            S4 [300 x 300] (Matrix::dsCM S4 object of class dsCMatrix
                                      Sig
50
            diff2 <- 100
                                      U
                                                             S4 [300 x 1] (Matrix::dgeMatr S4 object of class dgeMatrix
51
            pp <- 1
                                                                                   NSXSK
52
           カノケにst,有G个element.
                                                                       X E [OII)
53
                                                                                                   for each gene g of subject j:

U \in IP [EXI]
            #Sig_U = diag(rep(sigK_t, each = NU))
54
            Sig_p \leftarrow lapply(seq_len(G), function(x, A, sig0_t, sigK_t, NU, Y, X, B_t){
55
                invSig_U<-Matrix::bdiag(diag(rep(1/sigK_t[, x], each=NU)))</pre>
56
                                       \Sigma_{u} = diag(\sigma_{1}^{2} \mathbf{I}_{I}, \sigma_{2}^{2} \mathbf{I}_{I}, \cdots, \sigma_{K}^{2} \mathbf{I}_{I}).
```

```
crossprod (x,y) = XTY
tcrossprod (x,y) = XYT
                                               Matrix::crossprod(A)/sig0_t[x]+invSig_U)
VE(01)
                                      Matrix::tcrossprod( Matrix::tcrossprod(Sig, A),
          ΣωΑT (AΣωΑT+66' IN) (y-xβ)
                                                                                                                                                      1-50: CT1 for subject 1-50
                                                               BiocGenerics::t(Y[, x] - Matrix::tcrossprod(X,
                                                              BiocGenerics::t(B_t[, x])))
                                                                                                                           JK
                                                                                                                        S4 [300 x 500] (Matrix::dgeM S4 object of class dgeMatrix
                                                                                                 C E Up
                                    turn(list(Sig=Sig, U=U))
                                                                                                                                                300 500
                                                                                                     Dim
                                                                                                                        integer [2]
                            }, A, sig0_t, sigK_t, NU, Y, X, B_t)
              63
                                                                                                                        list [2]
                                                                                                                                                List of length 2
                                                                                                   Dimnames
              64
                            E Up<-do.call(cbind, lapply(Sig p, function(x)x$U))</pre>
                                                                                                                        double [150000]
                                                                                                                                                -97.90 -303.88 -79.14 243.73 -2.02 -79.58 ...
              65
                                                                                                     factors
                                                                                                                        list [0]
                                                                                                                                                List of length 0
              66
                            while(iem<15){</pre>
              67
                                  cat("iteration=", iem, "\n")
              68
                                                                                                          E-step:
              69
                                 iem <- iem + 1
                                                                                                            E[u|w_{obs} = v] = \Sigma_u A' V^{-1}(v - X\beta)
              70
                                 ####2. E-step
              71
                                 \#observed data COV(Y) = V
                                                                                                            E[s's|w_{obs} = y] = tr(A\Sigma_{p}A') + (A\mu_{p} + X\beta - y)'(A\mu_{p} + X\beta - y)
              72
              73
                                 \#V = A\% *\% Sig U\% *\% t(A) + diag(rep(sig0 t, 5*600))
                                                                                                            E[\mathbf{u}_{k}'\mathbf{u}_{k}|\mathbf{w}_{obs}=\mathbf{y}]=tr(\mathbf{\Sigma}_{n_{k}})+\boldsymbol{\mu}_{n_{k}}'\boldsymbol{\mu}_{n_{k}}
              74
                                                                                                         Here, s = Au + X\beta - y, V := A\Sigma_u A' + \sigma_0^2 I, \Sigma_{p_k} is the kth diagonal block of matrix \Sigma_p,
              75
                                 #2.1 E[U|Y]: missing data [U|Y] given observed data
                                                                                                        and \mu_{ni} is the kth sub-vector in \mu_{ni}.
                                 \#invV = solve(V)
              76
                                 # E U = mu_p = t(Sig_U) %*% t(A) %*% invV %*% (Y - X %*% B_t)
              77
              78
                                 # Sig p = Sig U - crossprod(Sig U,t(A)) %*% invV %*% A %*% Sig U
              79
                                 # Estimate from last iteration
              80
              81
                                 E U <- E Up
              82
              83
                                 mu_p <- E_Up
                                 E_U_frame <- as.data.frame(as.matrix(E_U))</pre>
              84
              85
                                 #2.2 E[t(S)S|Y]
                                 E_StS <- lapply(seq_len(G), function(x, A, Sig_p, mu_p, X, B_t, Y){</pre>
              86
                                     sum( Matrix::diag(Matrix::tcrossprod( Matrix::tcrossprod(A, Sig_p[[x]]$Sig), A))) +
              87
                                           sum(( Matrix::tcrossprod(A, BiocGenerics::t(mu p[, x])) +
              88
                                                       Matrix::tcrossprod(X, BiocGenerics::t(B_t[, x])) - Y[, x])^2)},
              89
                                     A, Sig_p, mu_p, X, B_t, Y)
              90
                                 E_StS <-unlist(E_StS)</pre>
              91
```

```
92
 93
                  #2.3 E[U k^T U k|Y]
 94
                  mutra split <- lapply(Sig p, function(x){</pre>
                      sig_p<-split(diag(x$Sig), ceiling(seq_len(NU*K)/NU))</pre>
 95
                      tra<-unlist(lapply(sig p, sum))</pre>
 96
                       return(tra)
 97
                  })
 98
                  mu_split <- split(E_U_frame, ceiling(seq_along(E_U_frame[, 1])/NU))</pre>
 99
100
                  E UkTUk <- do.call('cbind', mutra split) + do.call('rbind', lapply(mu split, colss))</pre>
101
102
                                                                                      (\chi^{\tau}\chi)^{-1}
103
                  ####3. M-step
104
                  #3.1 B
                  B_tp <- Matrix::tcrossprod( Matrix::tcrossprod(solve( Matrix::crossprod(X)), X),</pre>
105
                                                   BiocGenerics::t(Y- Matrix::tcrossprod(A, BiocGenerics::t(E U))) )
106
107
                                                                                          M-step:
                  #make correction in case B[1:K]<0 or B_tp[(K+1):2K]<0</pre>
108
                                                                                          For the (t+1)^{th} iteration given the t^{th} iteration:
                  #important to bound the estimation to positive values
109
110
                                                                                             \hat{\boldsymbol{\beta}}^{(t+1)} = (X'X)^{-1}X'(y - AE_{n(t)}(\boldsymbol{u}^{(t)}))
                  # B tp[1:K,]=ifelse(B tp[1:K,]<0,0,B tp[1:K,])
111
                  # B_tp[-(1:K),]=ifelse(B_tp[1:K,]+B_tp[-(1:K),]<0,-B_tp[1:K,],B_tp[-(1:K),])
112
113
114
115
                  #3.2 sigma 0^2
                  sig0_tp <- E_StS/(NS)</pre>
116
117
118
                  #3.3 sigma_k^2
                  sigK_tp <- E_UkTUk/(NU)</pre>
119
120
121
                  ####4. Stopping criteria
122
                  diff1 \leftarrow sum(abs(B_tp - B_t)) + abs(sig0_tp - sig0_t) + sum(abs(sigK_tp - sigK_t))
123
                  n1 <- sum(abs(B_tp - B_t))/length(B_tp)</pre>
124
                  n2 <- sum(abs(B_tp))/length(B_tp)</pre>
125
126
                  pp \leftarrow n1/n2
```

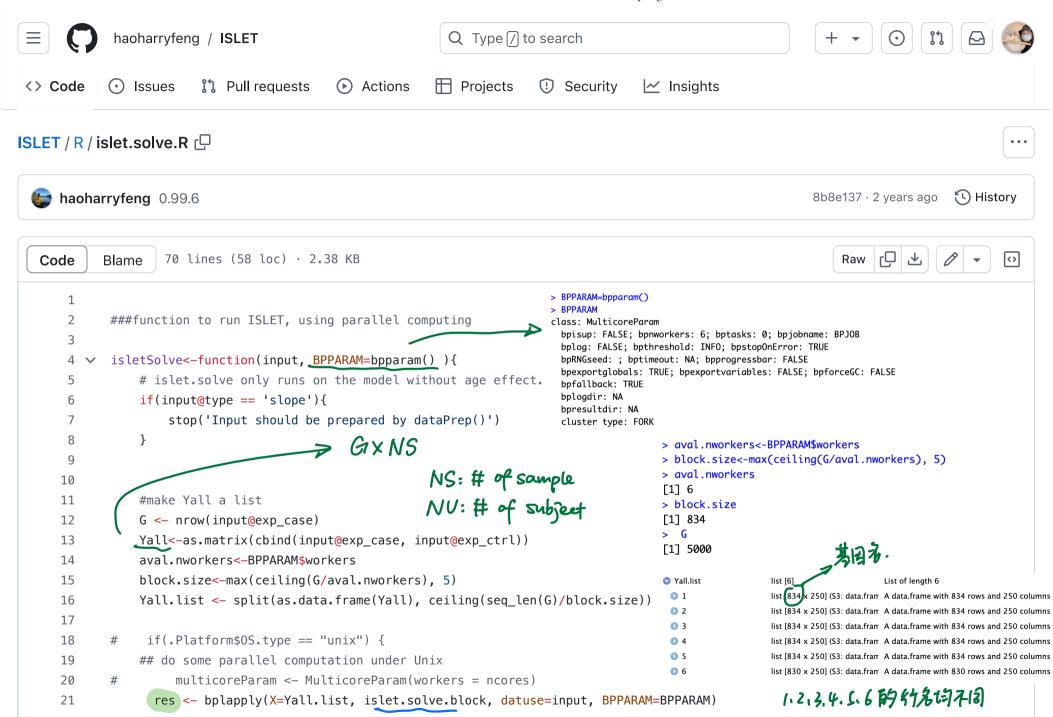
```
cat("B sum val=", n2, "\n")
127
         #
                   cat("B change val=", n1, "\n")
128
                   cat("B change prop=", pp*100,"% \n")
129
                                                                              \hat{\sigma}_{k}^{2(t+1)} = \frac{E_{\eta^{(t)}} \left[ \mathbf{u}_{k}' \mathbf{u}_{k} | \mathbf{w}_{obs} = \mathbf{y} \right]}{\mathbf{v}_{obs}}
130
131
                  ####5. Update params
132
                  B t<-B tp
133
                  sig0_t <- sig0_tp
                                                                            The E-step and M-step above are repeated until convergence. The details of modeling
134
                  sigK_t <- sigK_tp</pre>
                                                                          and algorithm is available in Additional file 5.
135
                  Sig_p<-lapply(seq_len(G), function(x, A, sig0_t, sigK_t, NU, Y, X, B_t){</pre>
136
                       Sig U<- Matrix::bdiag(diag(rep(sigK_t[, x], each=NU)))</pre>
137
                       invSig_U<-Matrix::bdiag(diag(rep(1/sigK_t[, x], each=NU)))</pre>
138
                       Sig<-solve( Matrix::crossprod(A)/sig0_t[x]+invSig_U)</pre>
139
                       U<- Matrix::tcrossprod(Matrix::tcrossprod(Sig, A),
140
                                                 t(as.matrix(Y[, x] -
141
                                                   Matrix::tcrossprod(X, BiocGenerics::t(B_t[, x])))
142
                                                  )/sig0_t[x]
143
144
                       return(list(Sig_U=Sig_U, Sig=Sig, U=U))
                  }, A, sig0_t, sigK_t, NU, Y, X, B_t)
145
                  Sig_p_all<-NULL
146
                  E_Up<-do.call(cbind, lapply(Sig_p, function(x)x$U))</pre>
147
148
                  diff2 <- sum(abs(E Up - E U))/(length(E U)*mean(colMeans(Y))^2)</pre>
149
                   cat("Random effect diff2=", diff2, "\n")
150
         #
151
152
             # Estimate of fixed effect
153
              B est<-cbind(B est, B t)</pre>
154
155
             # Estimate of random effect
             E_U_est<-cbind(E_U_est, E_Up)</pre>
156
157
             # Estimate of variance Sigma_U, Sigma_0
158
             Sig0_est <- cbind(Sig0_est, sig0_t)</pre>
             SigU_est <- cbind(SigU_est, sigK_t)</pre>
159
160
             #calculate LLK Llog-Litelahood)
161
```

```
162
            llk<-lapply(seg len(G), function(x){</pre>
163
                Sig<- Matrix::tcrossprod( Matrix::tcrossprod(A, Sig p[[x]]$Sig U), A)+
164
                    Matrix::bdiag(diag(sig0 t[x], nrow = nrow(A)))
165
                l<- Matrix::determinant(Sig)$modulus+</pre>
                    Matrix::tcrossprod(Matrix::crossprod(Y[, x]-
166
167
                               Matrix::tcrossprod(X, BiocGenerics::t(B t[, x])), solve(Sig)),
                               BiocGenerics::t(Y[, x]-
168
                                   Matrix::tcrossprod(X, BiocGenerics::t(B_t[, x]))) )
169
170
                return(-as.numeric(l))
            })
171
172
173
            llk<-unlist(llk)</pre>
174
           #compile return list
                                                Bk: fixed group effect
175
176
            case.m <- B_est[seq_len(K), ]+B_est[K+seq_len(K), ]</pre>
177
            ctrl.m <- B est[seg len(K).]
178
179
            #(2) the individual value for case and control, for all cell types. 2 matrices of NU by K.
180
            rel <- split(as.data.frame(as.matrix(E U est)), ceiling(seg along(E U est[, 1])/NU))</pre>
181
182
183
            case.indv <- lapply(seg len(K), function(k){rel[[k]][seg len(datuse@case num), ] +</pre>
184

  matrix(rep(case.m[k, ], each=datuse@case_num), nrow=datuse@case_num)})

                                                                                                  case-num:
185
                                                                                                         # of case subject
            ctrl.indv <- lapply(seq_len(K), function(k){rel[[k]][-seq_len(datuse@case_num), ] +</pre>
186
                    matrix(rep(ctrl.m[k, ], each=datuse@ctrl_num), nrow=datuse@ctrl_num)})
187
            names(case.indv) <- names(rel)</pre>
188
            names(ctrl.indv) <- names(rel)</pre>
189
190
            #(3) Variance for K cell types. 1 vector of length K.
191
            #'SigU_est' is already to be rendered.
192
193
            #(4) Variance for grand residuals. 1 scalar.
194
195
            #'Sig0_est' is already to be rendered.
196
```

```
#(5) the model likelihood. 1 scalar.
197
198
            #'llk' is already to be rendered.
199
200
            #compile return list
201
            rval <- list(</pre>
202
                case.m=case.m,
203
                ctrl.m=ctrl.m,
204
                case.indv=case.indv,
205
                ctrl.indv=ctrl.indv,
206
                var.k=SigU_est,
207
                var.0=Sig0_est,
                LLK=llk)
208
209
            return(rval)
210
            message("Complete: parameter estimation from ISLET is complete.")
211
212
        }
```



Rpage 1-7

```
22
       # }
23
       # else {
24
           ## This will be windows
25
           ## Use serial param or do not use any parallel functions, just use 'lapply'
26
           ## result should be of the same "type" from both the if and else statements.
27
            nworkers<-length(Yall.list)</pre>
            cl <- makeCluster(nworkers)</pre>
28
29
           ## Remove clusterExport(), clusterEvalQ() if use devtools::install() to build package
30
            clusterExport(cl,list('colss'))
31
            clusterEvalO(cl,{
32
                library(Matrix)
33
34
                library(BiocGenerics)})
35
            res <-parLapply(cl, X=Yall.list, islet.solve.block, datuse=input)
36
            stopCluster(cl)
37
                                                                       #compile return list
38
       #
                                                                       rval <- list(
39
                                                                           case.m=case.m,
                                                                           ctrl.m=ctrl.m,
40
                                                                           case.indv=case.indv,
         # Organize estimated individual reference
41
                                                                           ctrl.indv=ctrl.indv.
                          C(1,2,...,50)
42
         K<-input@K
                                                                           var.k=SigU_est,
         SubjectID<-unique(input@SubjectID)</pre>
                                                                           var.0=Sig0_est,
43
                                                                          LLK=llk)
         case num<-input@case num
44
                                                                       return(rval)
         case.indv.merge<-lapply(seq_len(K), function(k){</pre>
45
                                                     烟取 res 的第分流。细乡个[[3]]
              case.indv.all<-lapply(res, '[[', 3]</pre>
46
             case.indv.ctk<-t(do.call(cbind, lapply(case.indv.all, '[[', k)))</pre>
47
              case.indv.ctk<-ifelse(as.matrix(case.indv.ctk)<0, 0, case.indv.ctk)</pre>
48
              dimnames(case.indv.ctk)<-list(rownames(input@exp case), SubjectID[seq len(case num)])</pre>
49
             return(case.indv.ctk)
50
                                                        gene_names
51
         })
52
         ctrl.indv.merge<-lapply(seq_len(K), function(k){</pre>
53
              ctrl.indv.all<-lapply(res, '[[', 4)
             ctrl.indv.ctk<-t(do.call(cbind, lapply(ctrl.indv.all, '[[', k)))</pre>
54
             ctrl.indv.ctk<-ifelse(as.matrix(ctrl.indv.ctk)<0, 0, ctrl.indv.ctk)</pre>
55
             dimnames(ctrl.indv.ctk)<-list(rownames(input@exp_ctrl), SubjectID[-seq_len(case_num)])</pre>
56
```

```
57
                    return(ctrl.indv.ctk)
             })
58
59
             names(case.indv.merge)<-input@CT</pre>
60
             names(ctrl.indv.merge)<-input@CT</pre>
61
             llk<-unlist(lapply(res, '[[', 7))</pre>
62
63
              rval<-outputSol(case.ind.ref=case.indv.merge,
64
                                                                                          N25_age.ref <- readRDS("N25_ref_0_0.rds")
                             ctrl.ind.ref=ctrl.indv.merge,
65
                                                                                          caseVal <- caseEst(N25_age.ref)</pre>
66
                             mLLK=11k)
                                                                                          ctrlVal <- ctrlEst(N25_age.ref)</pre>
67
              return(rval)
68
69
                                                                       N25_age.ref
                                                                                           S4 (ISLET::outputSol)
                                                                                                                 S4 object of class outputSol
                                                                        case.ind.ref
                                                                                           list [6]
                                                                                                                 List of length 6
                                                                           B-cells
                                                                                           double [5000 x 25]
                                                                                                                 1968 0 566 3473 6020 0 2035 0 547 3406 5788 0 1612 0 585 3411 ...
      caseEst<-function(res.sol){</pre>
                                                                           CD4
                                                                                           double [5000 x 25]
                                                                                                                 0.0 0.0 5242.0 4223.7 0.0 0.0 0.0 5125.7 4053.3 0.0 ...
                                                                           CD8
                                                                                           double [5000 x 25]
                                                                                                                 5071 0 1535 2855 4692 522 4788 0 1264 2724 4171 534 4297 0 1299 27...
            est <- res.sol@case.ind.ref
                                                                                           double [5000 x 25]
                                                                                                                 9488.65 0.00 1376.65 0.00 1590.27 76.19 9418.68 0.00 1017.64 0.00 ...
                                                                                           double [5000 x 25]
                                                                                                                 10538 254 0 0 3234 256 10612 256 0 0 2459 279 10897 ...
            return(est)
                                                                           Neutrophils
                                                                                           double [5000 x 25]
                                                                                                                 3648.872 282.819 324.877 0.000 432.233 235.798 4116.368 300.163 50...
                                                                           Monocytes
      }
                                                                        ctrl.ind.ref
                                                                                           list [6]
                                                                                                                 List of length 6
                                                                                           double [5000 x 25]
                                                                                                                 8611.35 0.00 0.00 950.40 1633.46 136.96 8275.63 0.00 0.00 ...
                                                                           B-cells
                                                                                           double [5000 x 25]
                                                                                                                 13037 180 0 776 0 0 13201 218 0 742 0 0 12907 ...
                                                                           CD4
                                                                           CD8
                                                                                           double [5000 x 25]
                                                                                                                 12630 0 1057 0 3289 0 11783 0 1161 0 2578 0 11525 ...
                                                                                           double [5000 x 25]
                                                                                                                 0.00 22.66 2097.96 0.00 2401.11 1136.82 0.00 8.09 2160.48 0.00 ...
      ctrlEst<-function(res.sol){
                                                                           Neutrophils
                                                                                           double [5000 x 25]
                                                                                                                 0 892 590 1607 1167 204 0 883 669 1553 923 220 0 876 633 1601 ...
            est <- res.sol@ctrl.ind.ref
                                                                           Monocytes
                                                                                           double [5000 x 25]
                                                                                                                 2510 0 3814 816 9475 377 2473 0 3850 781 9207 383 2378 0 3855 810 ...
                                                                        mLLK
                                                                                           double [5000]
                                                                                                                 -4368 -3070 -3680 -3682 -4044 -3212 ...
            return(est)
      }
              caseVal
                                            list [6]
                                                                          List of length 6
                                            double [5000 x 25]
                    B-cells
                                                                          2920 0 5053 7317 10854 0 3024 0 5033 7161 10605 0 2554 ...
                    CD4
                                            double [5000 x 25]
                                                                          7040.09 0.00 8077.24 9397.88 280.51 0.00 7038.60 0.00 7879.73 8997.4...
                    CD8
                                            double [5000 x 25]
                                                                          11345 0 3885 8104 9141 2365 11024 0 3407 7821 8488 2423 10471 ...
                    NK
                                            double [5000 x 25]
                                                                          7972.8 0.0 1139.8 262.1 2998.0 674.7 8079.7 0.0 677.1 0.0 2514.4 77 ...
                                            double [5000 x 25]
                                                                          13584 1177 1780 786 3981 702 14249 1170 1279 170 3164 797 14306 1 ..
                   Neutrophils
                                            double [5000 x 25]
                                                                          10521.9 1303.8 276.1 0.0 2517.8 891.9 10954.2 1459.2 53.2 0.0 ...
                    Monocytes
```