*Homework 5*



(a) since , that means

Therefore, the gamma distribution is

(b) for exponential distribution X, the mean and variance are

(c) for maximum likelihood estimation for exponential distribution

Then let derivative of be 0

1. The proof is as follows

Based on above inequality, we show that is convex function.

(a) for uniform distribution, we have

Then we have

(b)

From part(a), it might be sometimes negative, but for discrete entropy, it’s impossible to be negative.

And we know that and for every such that . Therefore, .

(c)

From the lecture, the distribution that satisfies these features is normal distribution.

So the maximum entropy distribution meeting these constraints is normal distribution with mean equals to 2 and variance is 10. The distribution can be denoted as .

1. (a)

We use Lagrange for this question, and we have following constraints

Therefore,

Compute derivative then,

Let , we can get

This explains the symmetry we should use.

Then we have constraints, using them will get

(b)

Similar to part(a), we still use Lagrange, with following constraints

Then we can have

Similar to what we have done in part(a), we can obtain

Then using above symmetry, we have two methods to solve

Method (1):

Let , since

Then and

Therefore, the entropy is

Let

Therefore,

Method (2):

Similar to method (1), we get

Then we can find ,

Denote

then

then

Solving this equation set, we can get

So we can get the same answer as method (1)