Untangling the Security of Kilian's Protocol: Upper and Lower Bounds

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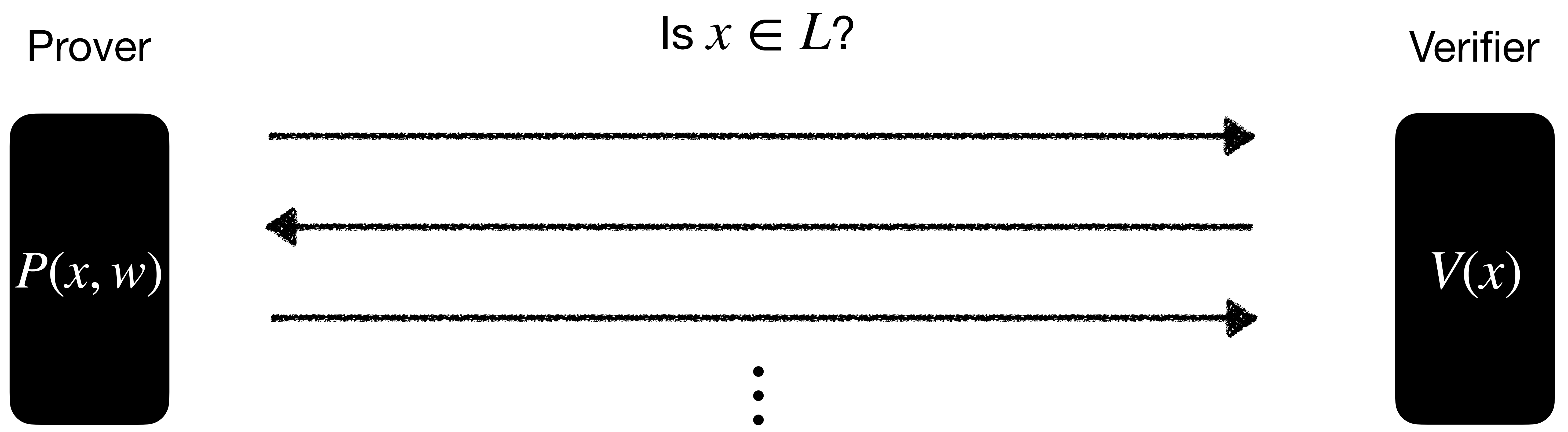
Interactive proofs



Perfect completeness: For every instance $x \in L$,

$$\Pr\left[\langle P(x, w), V(x)\rangle = 1\right] = 1.$$

Soundness: For every instance $x \notin L$ and adversary $\tilde{P}$,

$$\Pr\left[\langle \tilde{P}, V(x)\rangle = 1\right] \leq \epsilon(x).$$

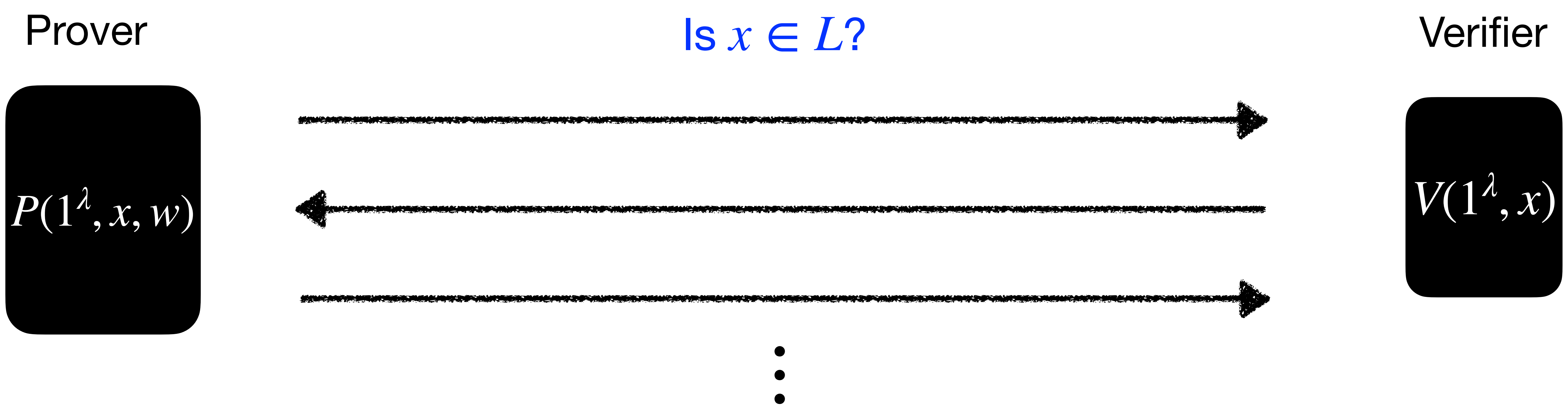
Basic efficiency metric: **COMMUNICATION COMPLEXITY** (number of bits exchanged during the interaction).

Limitation: NP-complete languages do not have IPs with cc $\ll |w|$ (or else the language would be easy).

(Indeed, [GH97] proved that, in general, $\mathrm{IP}[\mathrm{cc}] \subseteq \mathrm{BPTIME}[2^{\mathrm{cc}}]$.)

Interactive arguments

Interactive proofs with computational soundness



relaxes the soundness guarantee of interactive proofs

Computational soundness: For every $x \notin L$, security parameter $\lambda \in \mathbb{N}$, and t_{ARG}-bounded adversary $\tilde{P}$,

$$\Pr\left[\langle \tilde{P}, V(1^\lambda, x)\rangle = 1\right] \leq \epsilon_{\mathsf{ARG}}(\lambda, x, t_{\mathsf{ARG}}).$$

Further relaxation: Expected-time computational soundness $\epsilon^\star_{\mathsf{ARG}}$ against adversaries with bounded expected running time $t^\star_{\mathsf{ARG}}$.

Limitations on the communication complexity of interactive proofs no longer hold.

AMAZING: there exist interactive arguments for NP with cc $\ll |w|$ (given basic cryptography)

These are known as **Succinct Interactive Arguments**.

Why study succinct interactive arguments?

A **fundamental primitive** known to exist assuming only simple cryptography (e.g. collision-resistant hash functions).

The savings in communication (cc $\ll |w|$) or even verification ($\mathsf{time}(V) \ll |w|$) are remarkably useful.

Succinct arguments play a key role in notable applications
(e.g., zero-knowledge with non-black-box simulation, malicious MPC, ...).

They also serve as a stepping stone towards succinct **non-interactive** arguments (SNARGs).

Recall: SNARGs for NP cannot be realized via a black-box reduction to a falsifiable assumption [GW11].

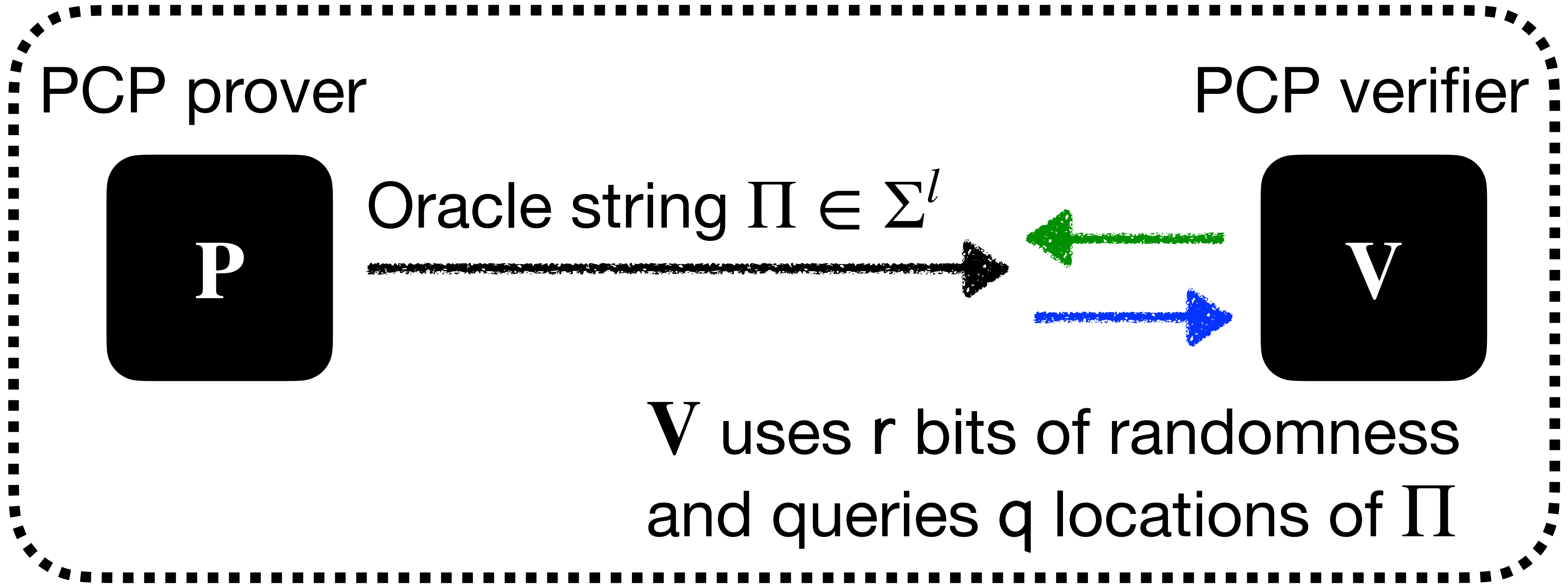
Often (though not always): SNARG = succinct interactive argument + non-falsifiable assumption / idealized model

Kilian's protocol, the first and simplest succinct argument

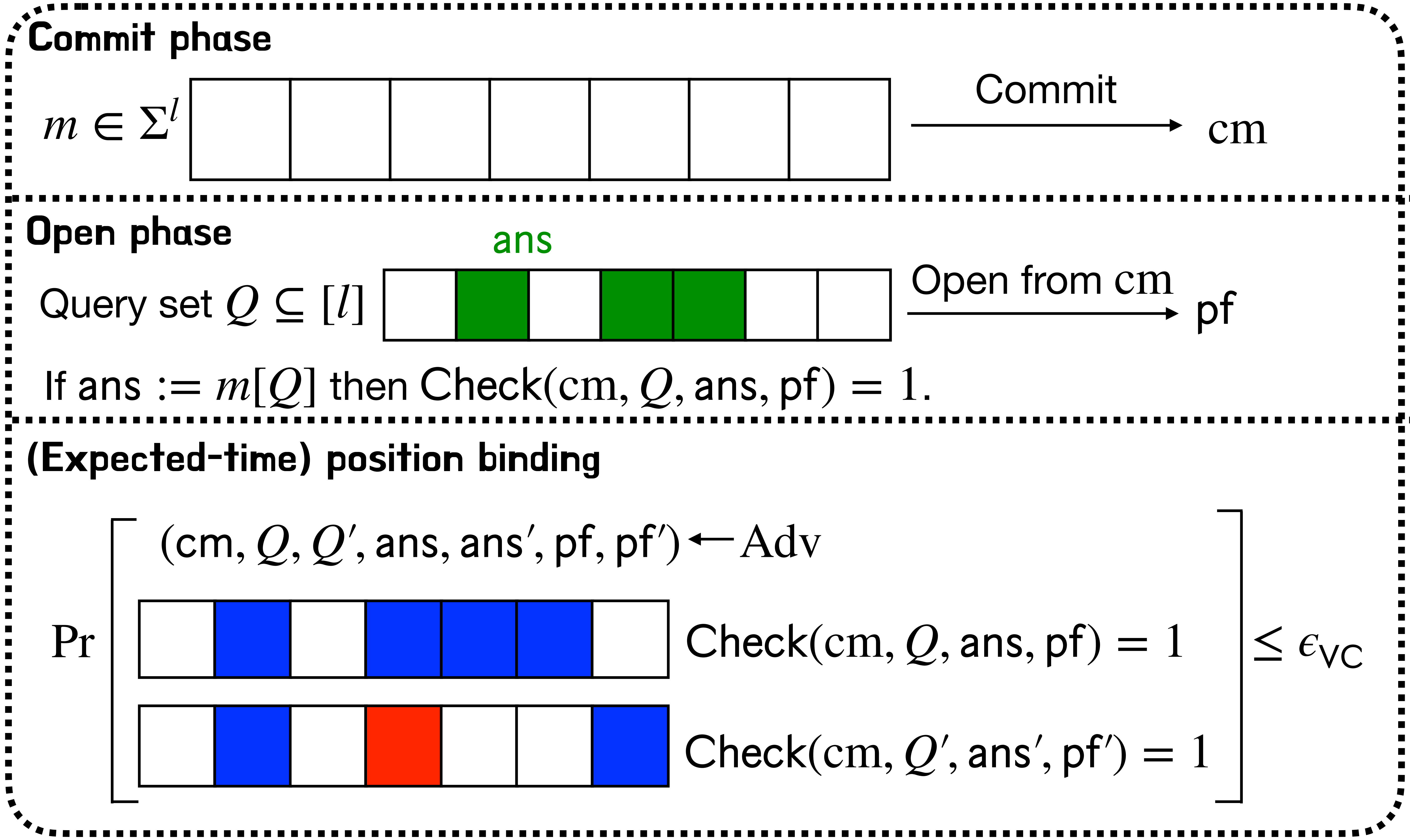
Kilian's protocol

abstraction for a succinct commitment
with local openings (e.g. Merkle tree)

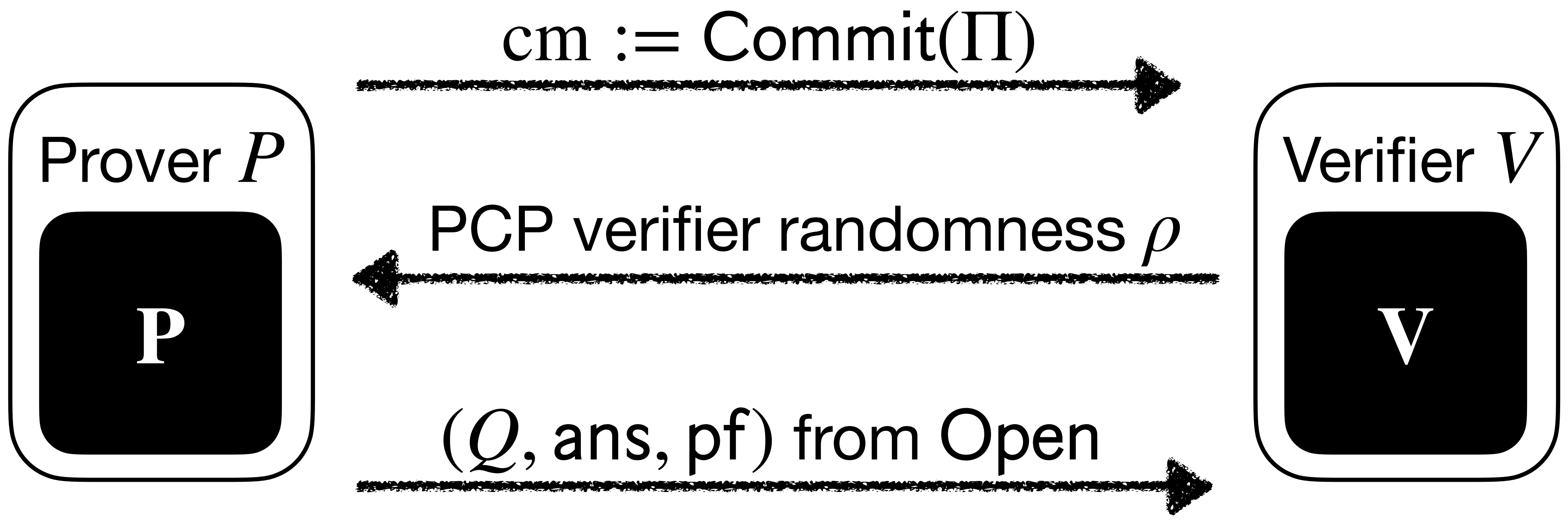
Building block #1: probabilistically checkable proof (PCP)



Building block #2: vector commitment scheme (VC)



The protocol:



Fundamental question:
What is the security of Kilian’s protocol?

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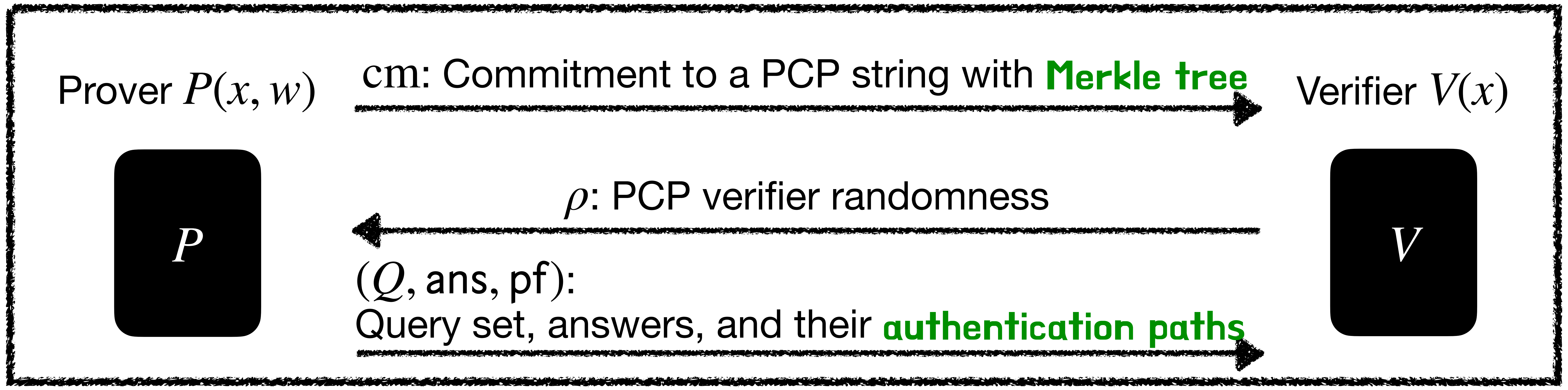


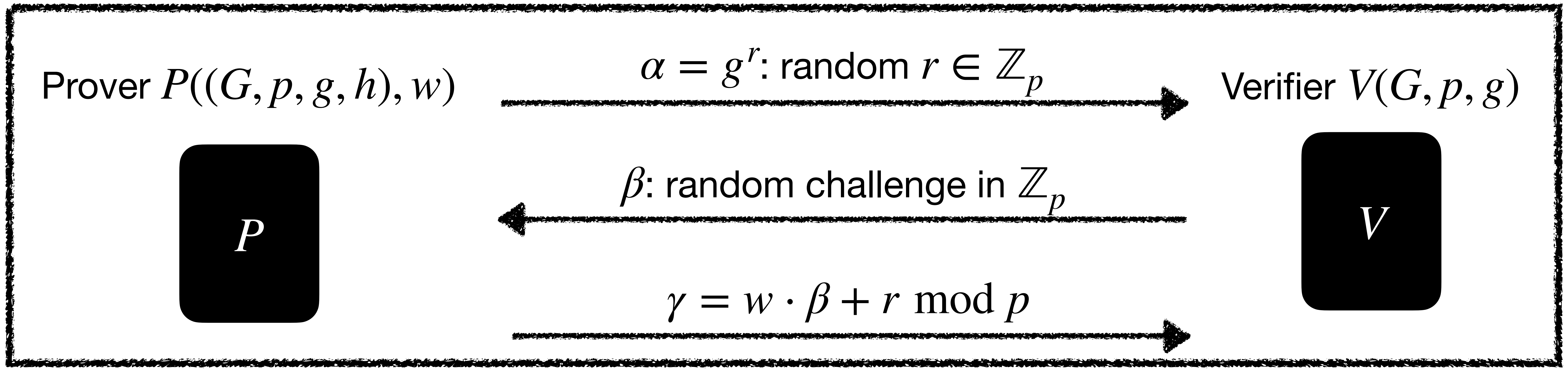
Previously:

- Folklore: well-understood, if ϵ_{PCP} and ϵ_{VC} if negligible, then ϵ_{ARG} is negligible.
- [Kilian92] gives an informal analysis.

non-trivial restrictions on the PCP.

- [BG08] proves security of Kilian's protocol **assuming** the underlying PCP is non-adaptive and reverse-samplable.
  Their analysis is NOT tight: roughly $\epsilon_{\mathrm{ARG}} \leq 8 \cdot \epsilon_{\mathrm{PCP}} + \sqrt[3]{\epsilon_{\mathrm{VC}}}$ (multiplicative constant overhead).
- Kilian's protocol is widely used across cryptography but lacks a security proof in the general case.

A similar protocol: Schnorr identification scheme



Numerous works study the security of Schnorr identification and its variants in different settings [Sho97,PS00,BP02,FPS20,BD20,RS21,SSY23]

Yet, there are gaps in our understanding of Schnorr's protocol - challenging open questions

Our contribution:

- Proving the security of Kilian's protocol is as hard as that of Schnorr's protocol.
    - Is Kilian's protocol really "well-understood"?
- A general and tightest known security analysis of Kilian's protocol.
    - Gaps and barriers remain.

Our results

Upper Bounds.

PCP for language L with
- proof length l
- query complexity q
- soundness error ϵ_{PCP}

ARG := Kilian[PCP, VC]

Vector commitment scheme with
position binding error ϵ_{VC}
(or expected-time position binding error $\epsilon^{\star}_{\mathsf{VC}}$)

For every $x \notin L$ and $\epsilon > 0$,

$$\epsilon_{\mathsf{ARG}}(\lambda, x, t_{\mathsf{ARG}}) \leq \epsilon_{\mathsf{PCP}}(x) + \epsilon_{\mathsf{VC}}(\lambda, t_{\mathsf{VC}}) + \epsilon, \text{ where } t_{\mathsf{VC}} = O\left(t_{\mathsf{ARG}} \cdot l/\epsilon\right);$$
$$\epsilon^{\star}_{\mathsf{ARG}}(\lambda, x, t^{\star}_{\mathsf{ARG}}) \leq \epsilon_{\mathsf{PCP}}(x) + \epsilon^{\star}_{\mathsf{VC}}(\lambda, t^{\star}_{\mathsf{VC}}) + \epsilon, \text{ where } t^{\star}_{\mathsf{VC}} = O\left(t^{\star}_{\mathsf{ARG}} \cdot \log(\mathsf{q}/\epsilon)\right).$$

Lower Bounds. Bounding the soundness error of Kilian's protocol is as hard as that of the *Schnorr identification scheme*.

There exists PCP and VC such that, for every $x \notin L$,

$$\epsilon_{\mathsf{Schnorr}}(\lambda, t_{\mathsf{Schnorr}}) \leq \epsilon_{\mathsf{ARG}}(\lambda, x, t_{\mathsf{ARG}}), \text{ where } t_{\mathsf{ARG}} = O(t_{\mathsf{Schnorr}});$$
$$\epsilon^{\star}_{\mathsf{Schnorr}}(\lambda, t^{\star}_{\mathsf{Schnorr}}) \leq \epsilon^{\star}_{\mathsf{ARG}}(\lambda, x, t^{\star}_{\mathsf{ARG}}), \text{ where } t^{\star}_{\mathsf{ARG}} = O(t^{\star}_{\mathsf{Schnorr}}).$$

How tight are the bounds?

Strict-time setting.

- Setting $\epsilon_{\mathsf{DLOG}}(\lambda, t) \leq O(t^2/2^\lambda)$.
- Best known analysis of the Schnorr identification scheme:

$$\epsilon_{\mathsf{Schnorr}}(\lambda, t_{\mathsf{Schnorr}}) \leq \sqrt{\epsilon_{\mathsf{DLOG}}(\lambda, O(t_{\mathsf{Schnorr}}))} \leq O\left(\sqrt{t_{\mathsf{Schnorr}}^2/2^\lambda}\right).$$

Polynomial gap

- Our bound:

$$\epsilon_{\mathsf{ARG}}(\lambda, x, t_{\mathsf{ARG}}) \leq 2^{-\lambda} + \epsilon_{\mathsf{DLOG}}(\lambda, t_{\mathsf{ARG}} \cdot l/\epsilon) + \epsilon \leq 2^{-\lambda} + l^{2/3} \cdot O\left(\sqrt[3]{t_{\mathsf{ARG}}^2/2^\lambda}\right).$$

Expected-time setting.

- Best known analysis of the Schnorr identification scheme:

$$\epsilon^\star_{\mathsf{Schnorr}}(\lambda, t^\star_{\mathsf{Schnorr}}) \leq \epsilon^\star_{\mathsf{DLOG}}(\lambda, O(t^\star_{\mathsf{Schnorr}})).$$

- Our bound:

$$\epsilon^\star_{\mathsf{ARG}}(\lambda, x, t_{\mathsf{ARG}}) \leq 2^{-\lambda} + \epsilon^\star_{\mathsf{DLOG}}(\lambda, t^\star_{\mathsf{ARG}} \cdot \log(\mathsf{q}/\epsilon)) + \epsilon.$$

Polylogarithmic gap
Almost tight

On the price of rewinding

Goal: achieve $\epsilon_{\mathsf{ARG}} = 2^{-40}$ against adversaries of size 2^{60} for Kilian's protocol.

Standard model

$t_{\mathsf{VC}} = O\left(\frac{l}{\epsilon} \cdot t_{\mathsf{ARG}}\right)$

For every $x \notin L$ and $\epsilon > 0$,
$\epsilon_{\mathsf{ARG}}(\lambda, x, t_{\mathsf{ARG}}) \leq \epsilon_{\mathsf{PCP}}(x) + \epsilon_{\mathsf{VC}}(\lambda, l(x), \mathsf{q}(x), t_{\mathsf{VC}}) + \epsilon.$

- Suppose $\epsilon_{\mathsf{PCP}} = 2^{-42}$ with $l = 2^{30}$.
- Suppose $\epsilon_{\mathsf{VC}} = (\lambda, l, \mathsf{q}, t_{\mathsf{VC}}) \leq \frac{t_{\mathsf{VC}}^2}{2^\lambda}$ (achieved by ideal Merkle trees).
- Setting $\epsilon := 2^{-42}$:
  - $t_{\mathsf{VC}} \leq 4 \cdot \frac{2^{30}}{2^{-42}} \cdot t_{\mathsf{ARG}} < 2^{80} \cdot t_{\mathsf{ARG}}$
  - $\epsilon_{\mathsf{VC}} \leq \frac{(2^{80} \cdot t_{\mathsf{ARG}})^2}{2^\lambda} = 2^{160-\lambda} \cdot t_{\mathsf{ARG}}^2 = 2^{280-\lambda}$
- Set $\lambda = 322$ to achieve the desired bound.

Random oracle model

For every $x \notin L$, [CY24]
$\epsilon_{\mathsf{ARG}}(\lambda, x, t_{\mathsf{ARG}}) \leq \epsilon_{\mathsf{PCP}}(x) + \frac{t_{\mathsf{ARG}}^2}{2^\lambda}.$

- Suppose $\epsilon_{\mathsf{PCP}} = 2^{-42}$
- $\epsilon_{\mathsf{VC}} \leq \frac{t_{\mathsf{ARG}}^2}{2^\lambda} = 2^{120-\lambda}$
- Set $\lambda = 162$ to achieve the desired bound.

- If the hash function is assumed ideal then extraction is straightline.
- If the hash function is merely collision-resistant then extraction is rewinding.
These computations illustrate the **PRICE OF REWINDING**.

Thank you!

https://eprint.iacr.org/2024/1434