

Quantum Rewinding for IOP-Based Succinct Arguments

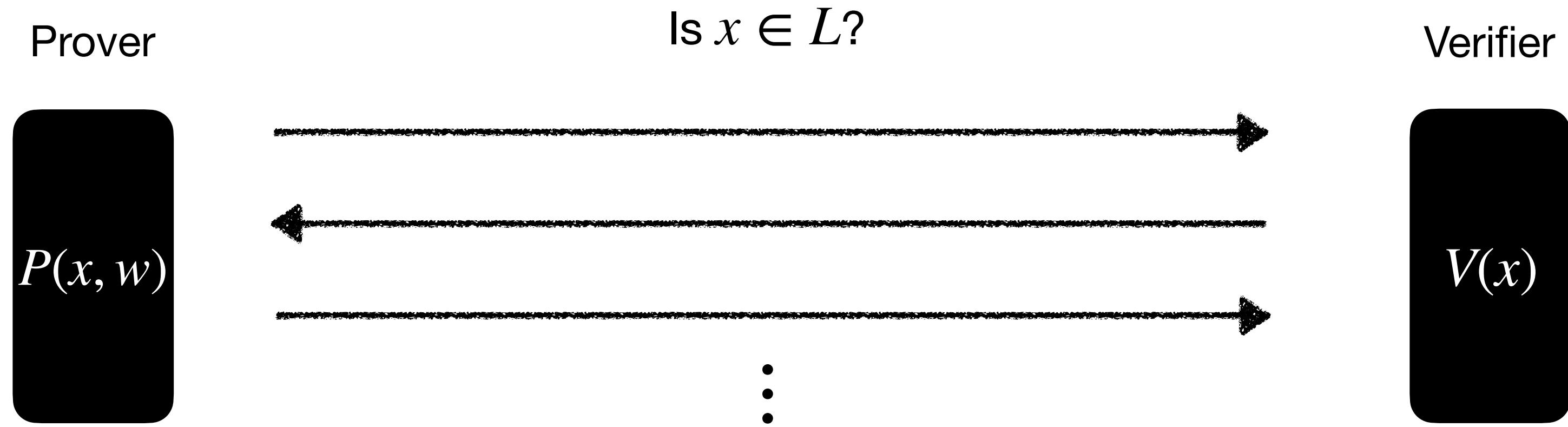
Ziyi Guan

Joint work with Alessandro Chiesa, Marcel Dall'Agnol, Zijing Di, and Nick Spooner



What are succinct arguments?

Interactive proofs



Completeness: $\forall x \in L, \Pr [\langle P(x, w), V(x) \rangle = 1] = 1$

Soundness: $\forall x \notin L$ and adversary $\tilde{P}, \Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon$

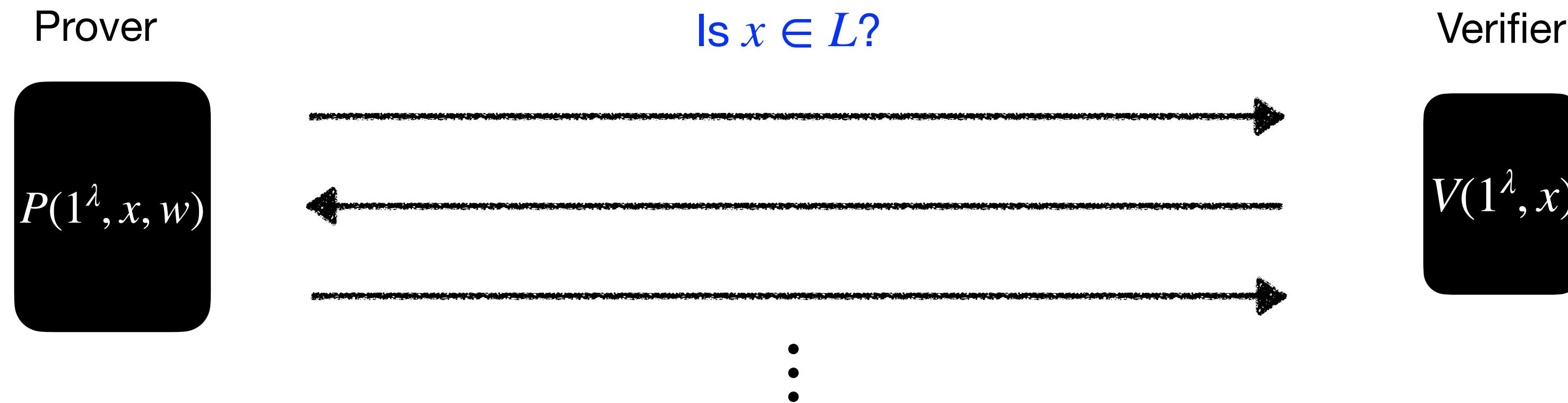
Target metric: COMMUNICATION COMPLEXITY

Limitation: NP-complete languages do not have IPs with $\text{CC} \ll |w|$

[GH97]: $\text{IP}[\text{CC}] \subseteq \text{BPTIME}[2^{\text{CC}}]$

Interactive arguments

Interactive proofs with computational soundness



Computational soundness: $\forall x \notin L$ and t_{ARG} -time adversary \tilde{P} , $\Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon_{\text{ARG}}(t_{\text{ARG}})$

AMAZING: \exists interactive arguments for NP with $\mathbf{CC} \ll |w|$ (given basic cryptography)

Today's protagonist:
Succinct Interactive Arguments

$\text{cc} \ll |w|$

Why study **succinct** interactive arguments?

$\text{time}(V) \ll |w|$

They exist based on simple crypto assumptions...

... so they play a role in numerous cryptotheory results.

zero-knowledge with
non-black-box simulation

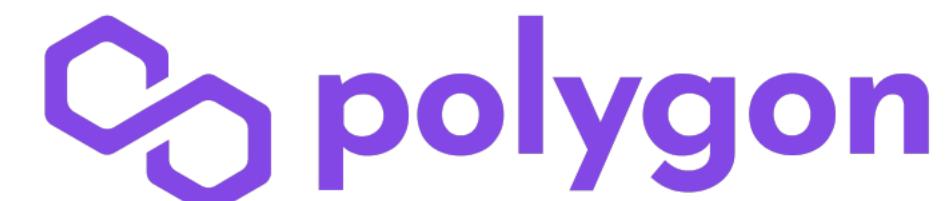
malicious MPC

...

They are a stepping stone for SNARGs, which have numerous real-world applications.



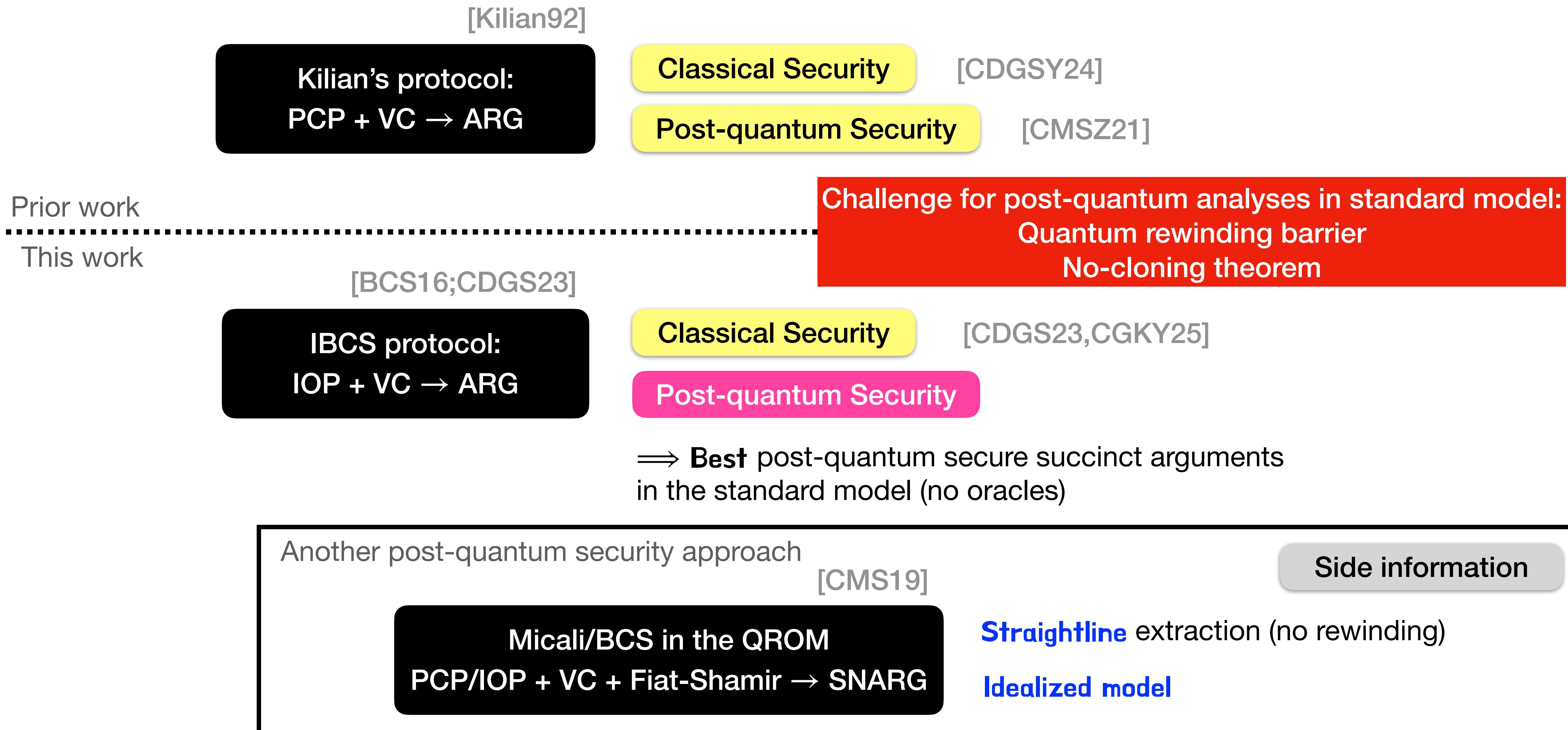
Irrreducible



...

Roadmap

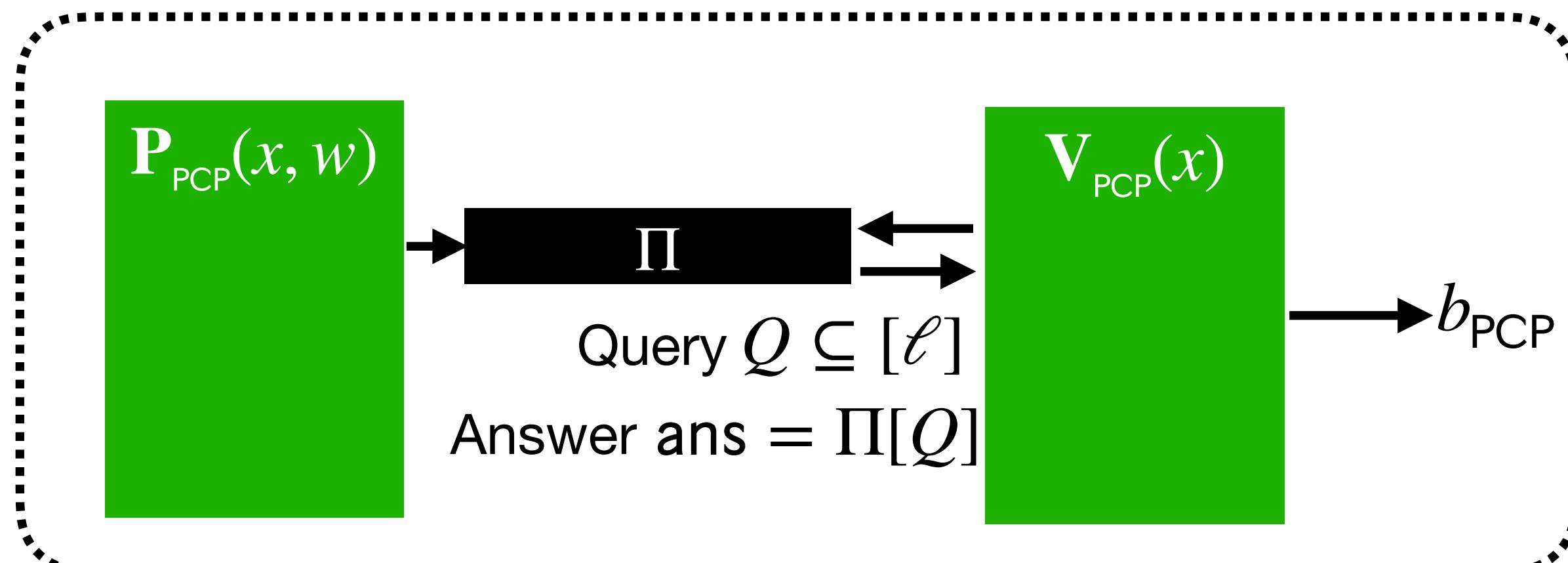
Today: only soundness
Knowledge soundness similar



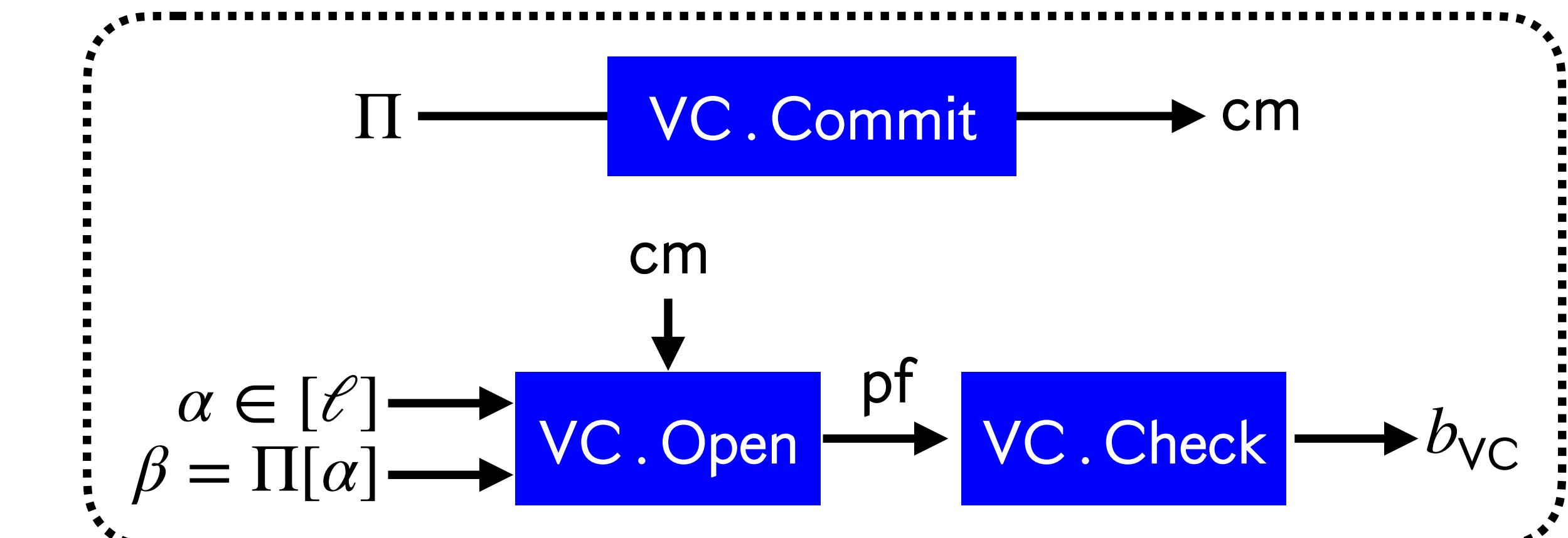
Warm-up: Kilian's protocol
The first and simplest succinct argument

How to construct succinct arguments?

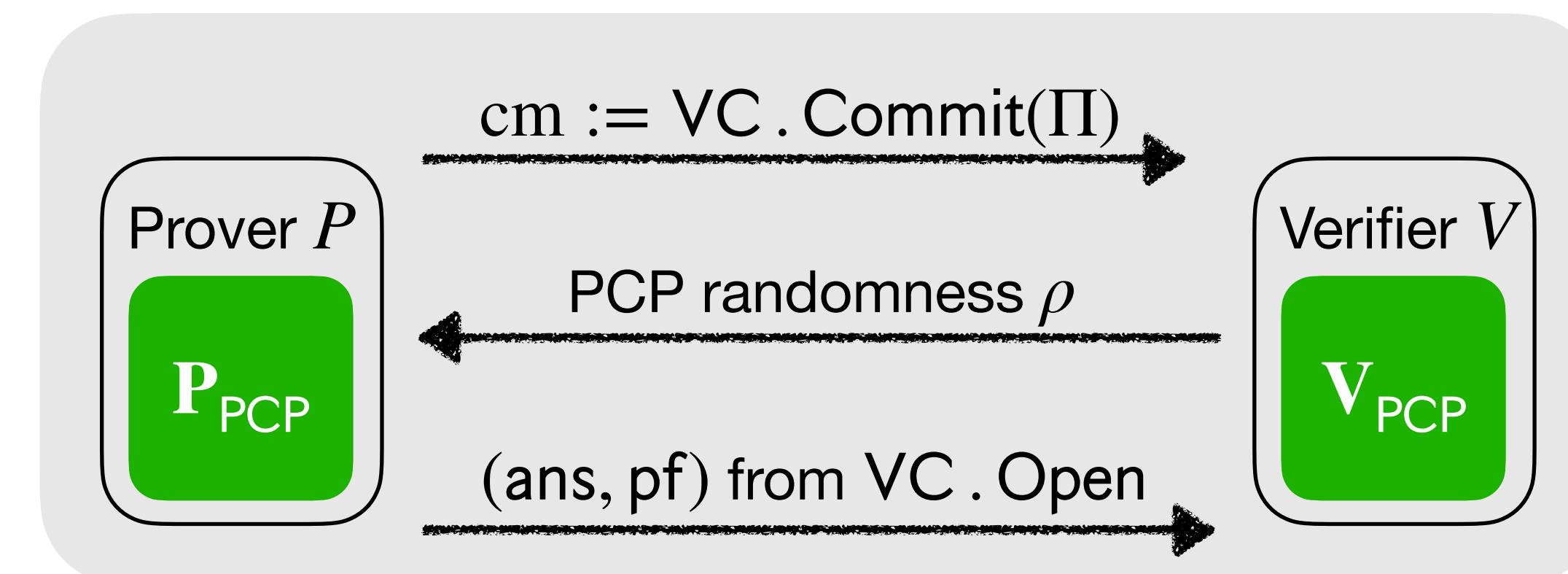
Building block #1: probabilistically checkable proof (PCP)



Building block #2: vector commitment scheme (VC)



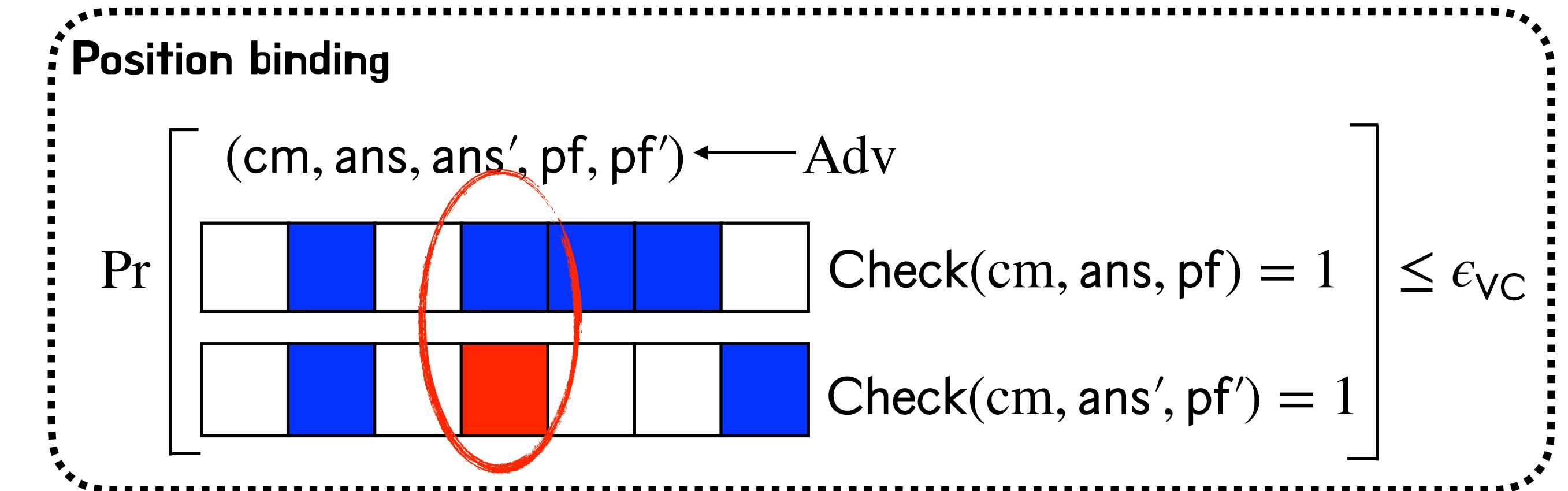
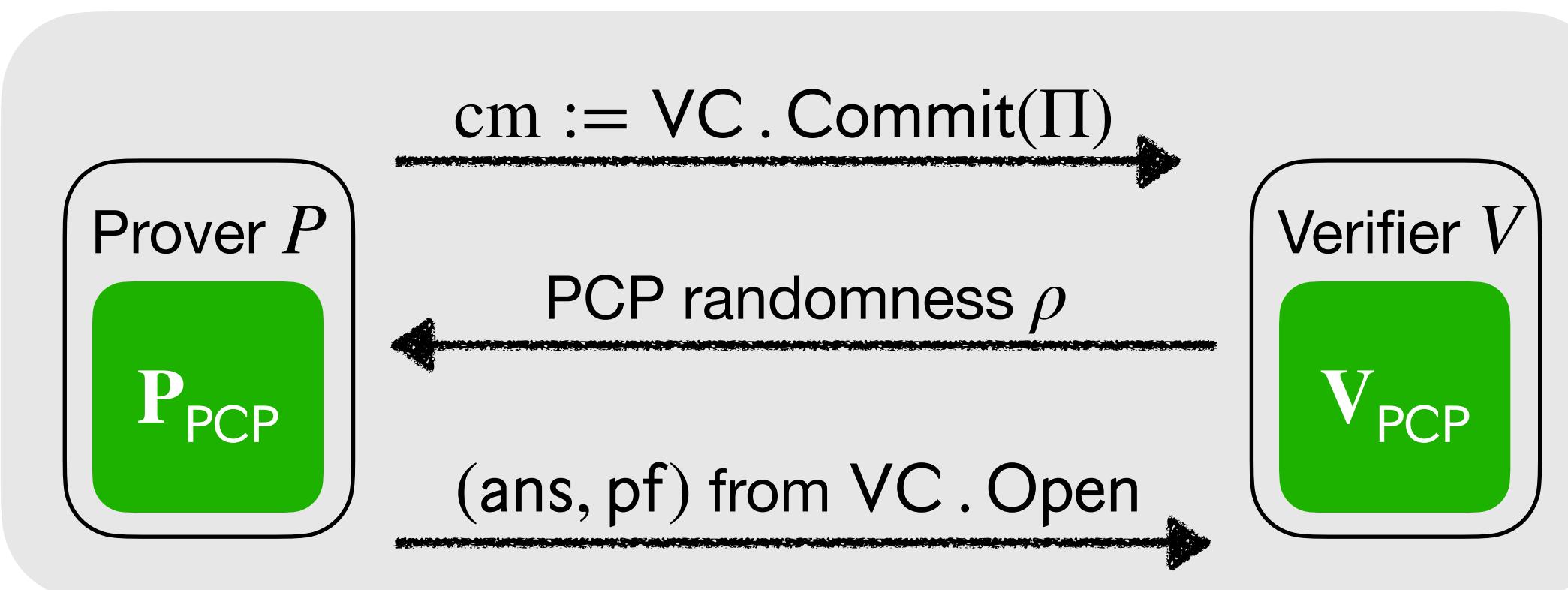
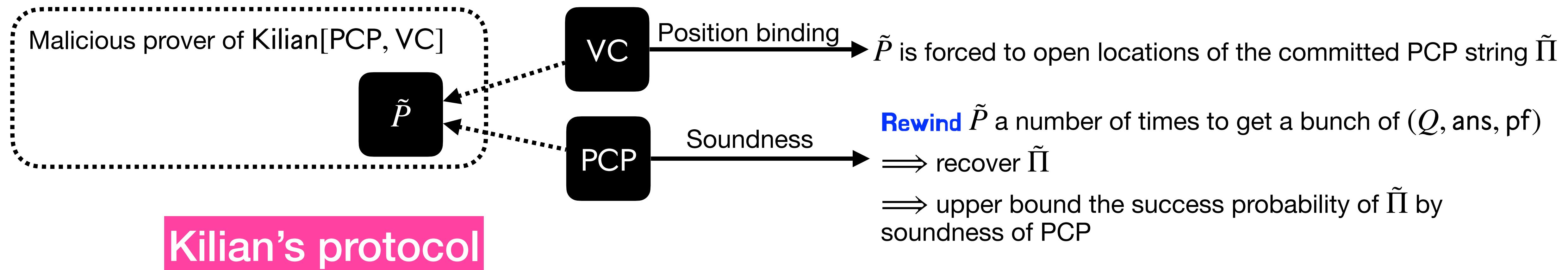
Kilian's protocol



Classical security analysis

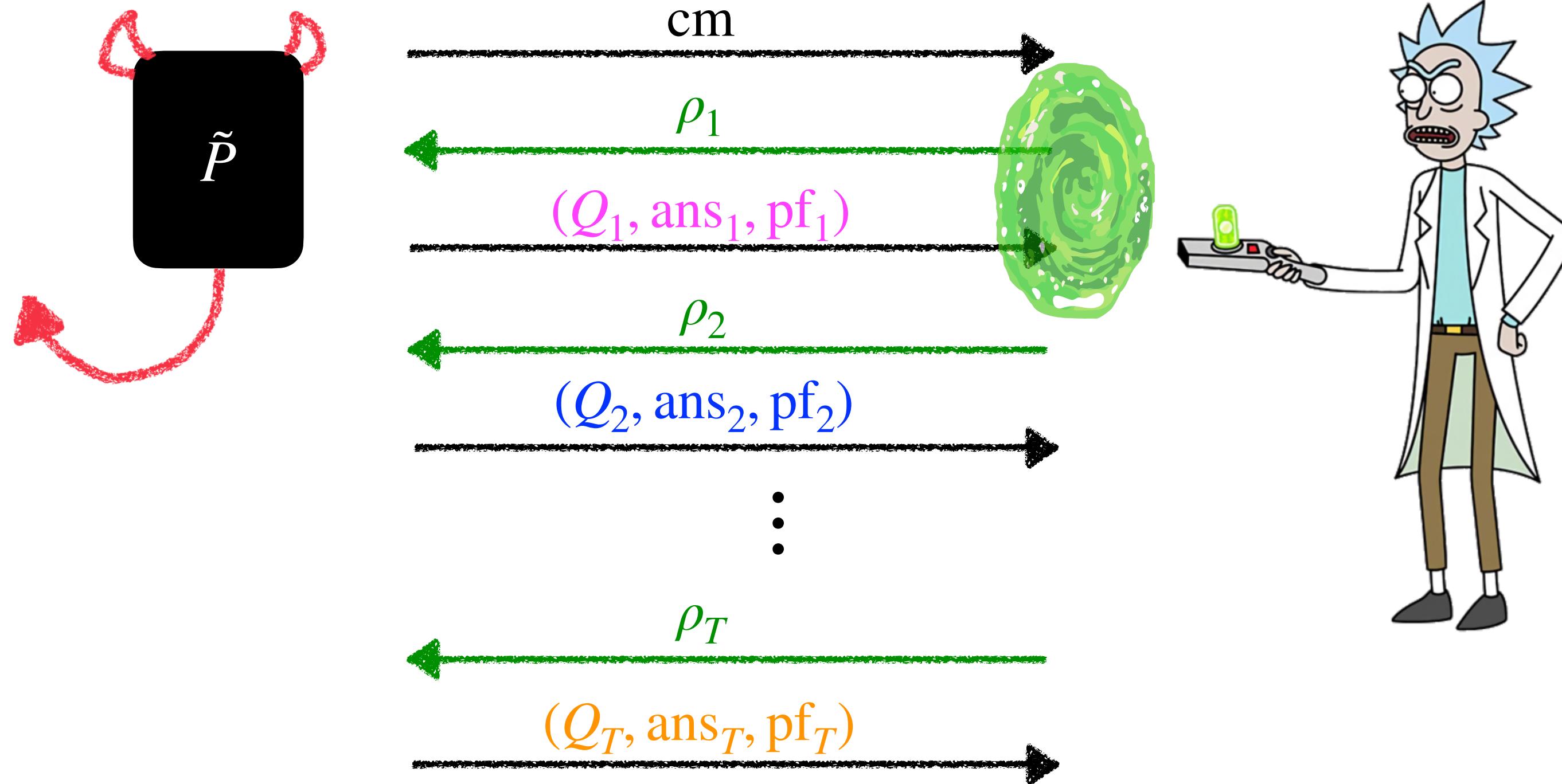
Goal: relate the soundness error of Kilian[PCP, VC]

to the soundness error of PCP and the position binding error of VC.



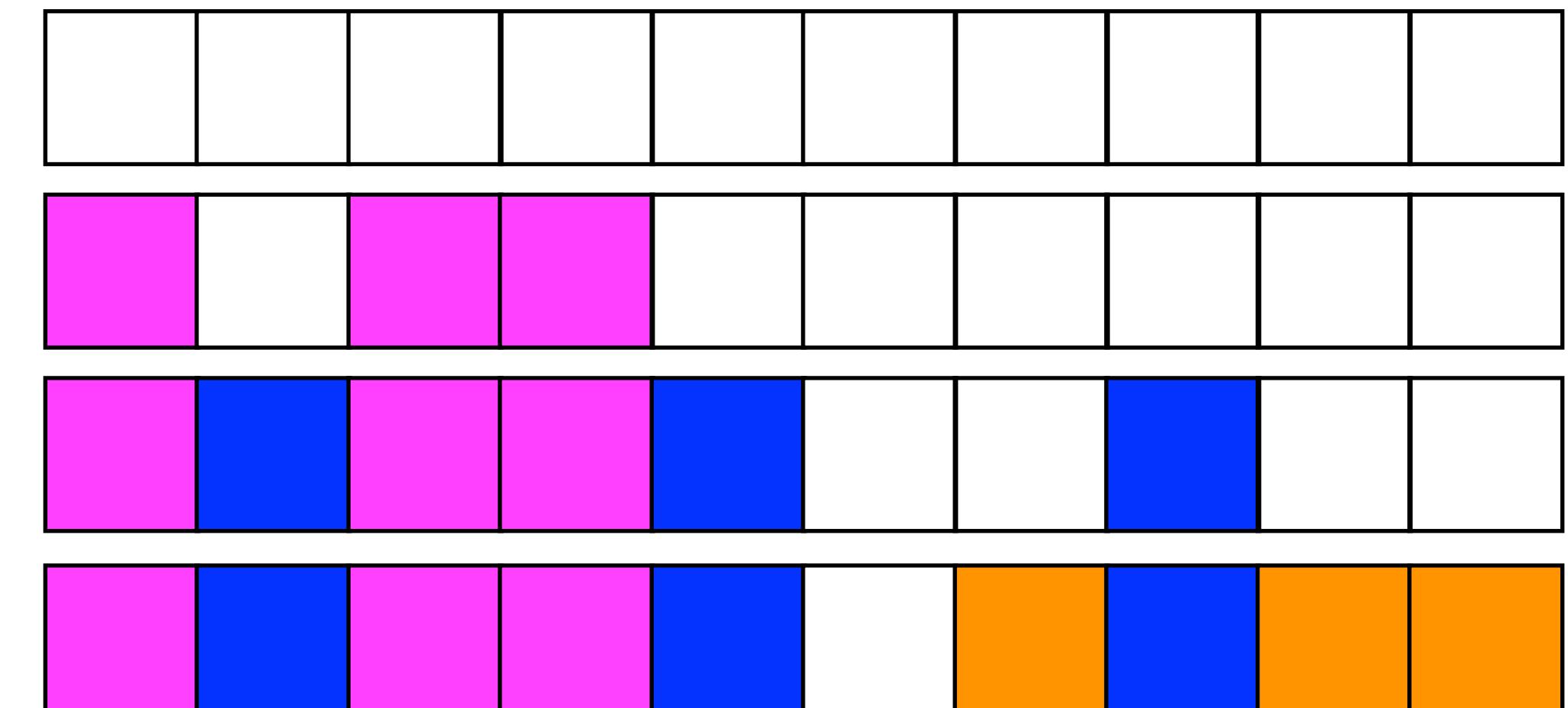
Approach: rewind the prover

Malicious Prover \tilde{P}



Reductor $\mathcal{R}^{\tilde{P}}(cm, \epsilon)$

Recover $\tilde{\Pi}$



Theorem [CDGSY24]. \forall PCP $(\epsilon_{\text{PCP}}, \ell_{\text{PCP}})$, VC (ϵ_{VC}) , $\epsilon > 0$,

$$\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{PCP}} + \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = O(t_{\text{ARG}} \cdot \ell_{\text{PCP}} \cdot \frac{1}{\epsilon}).$$



Overhead from rewinding.
Possibly inherent [CDGSY24]

How about post-quantum security?

Post-quantum soundness: same as classical soundness but adversary is quantum:

$$\forall t_{\text{ARG}}\text{-time QUANTUM adversary } \tilde{P}^{\star}, \Pr [\langle \tilde{P}^{\star}, V \rangle = 1] \leq \epsilon_{\text{ARG}}^{\star}(t_{\text{ARG}})$$

Ethereum Unlocks Millions To Prepare For The Post-quantum Era

Sun 11 May 2025 • 4 min read • by Mikaia A.

The building blocks need to be post-quantum secure at the minimum

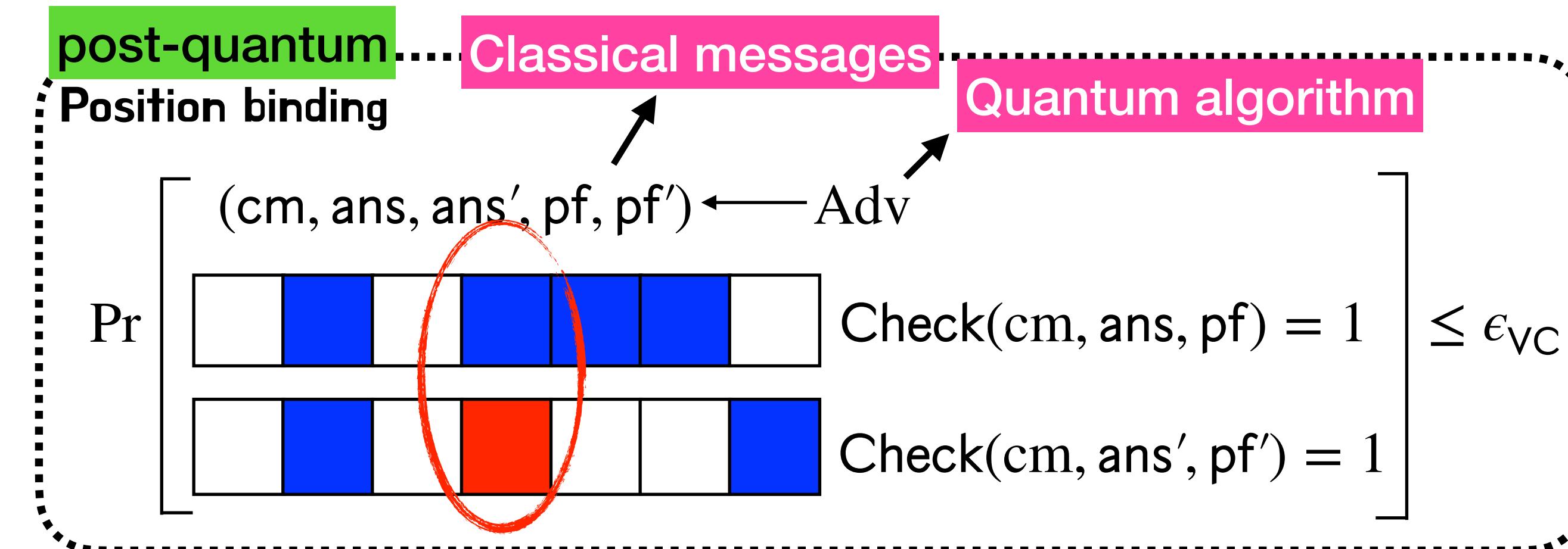
PCP for language L with soundness error ϵ_{PCP}

PCP has statistical soundness!

$$\implies \forall x \notin L \text{ and quantum adversary } \tilde{P}, \Pr [\langle \tilde{P}, V(x) \rangle = 1] \leq \epsilon$$

Vector commitment scheme with position binding error ϵ_{VC}

post-quantum



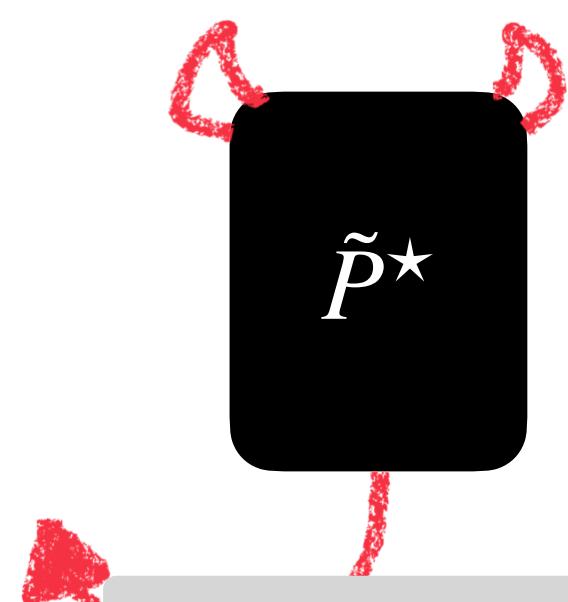
Is this sufficient?

Not with current rewinding techniques...

**Key property for rewinding:
Collapsing**

Quantum reductor

Malicious Prover \tilde{P}^*



Quantum algorithms, but output classical messages

$$\tilde{P}^* = (U_{\text{cm}}, U_{\text{open}})$$

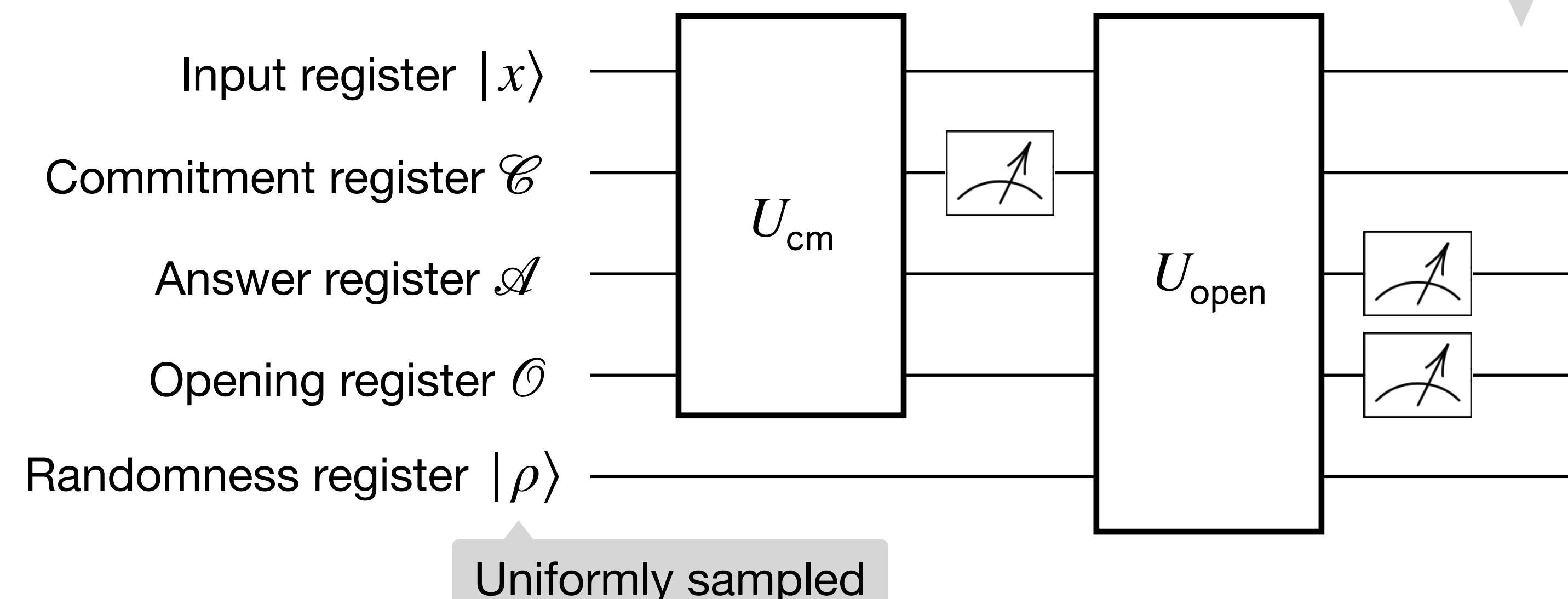
What does it mean to have
black-box access to \tilde{P}^* ?

Reductor $\mathcal{R}^{\tilde{P}^*}(\text{cm}, \epsilon)$

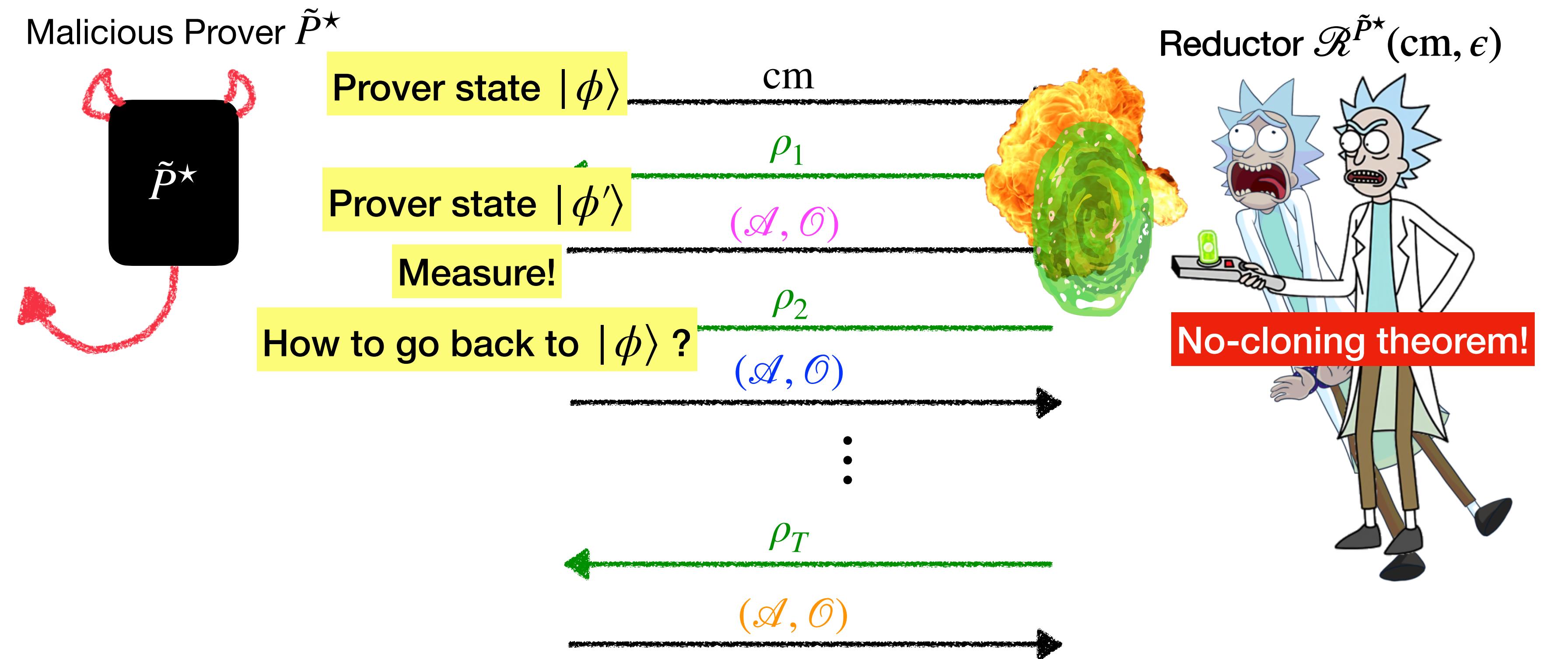


Black-box simulation of $\langle \tilde{P}^*, V \rangle$

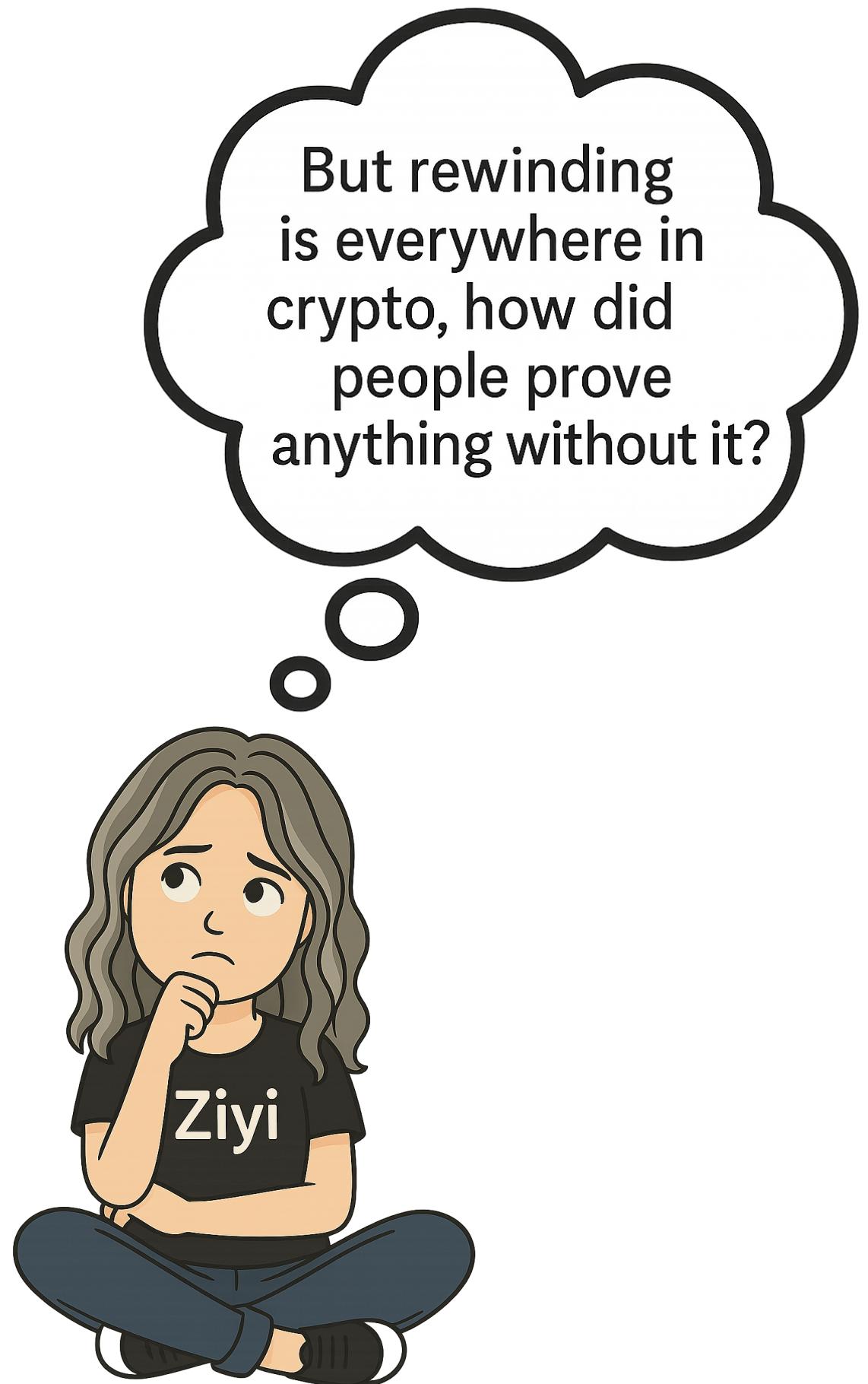
Send measured outcome to V



On quantum rewinding

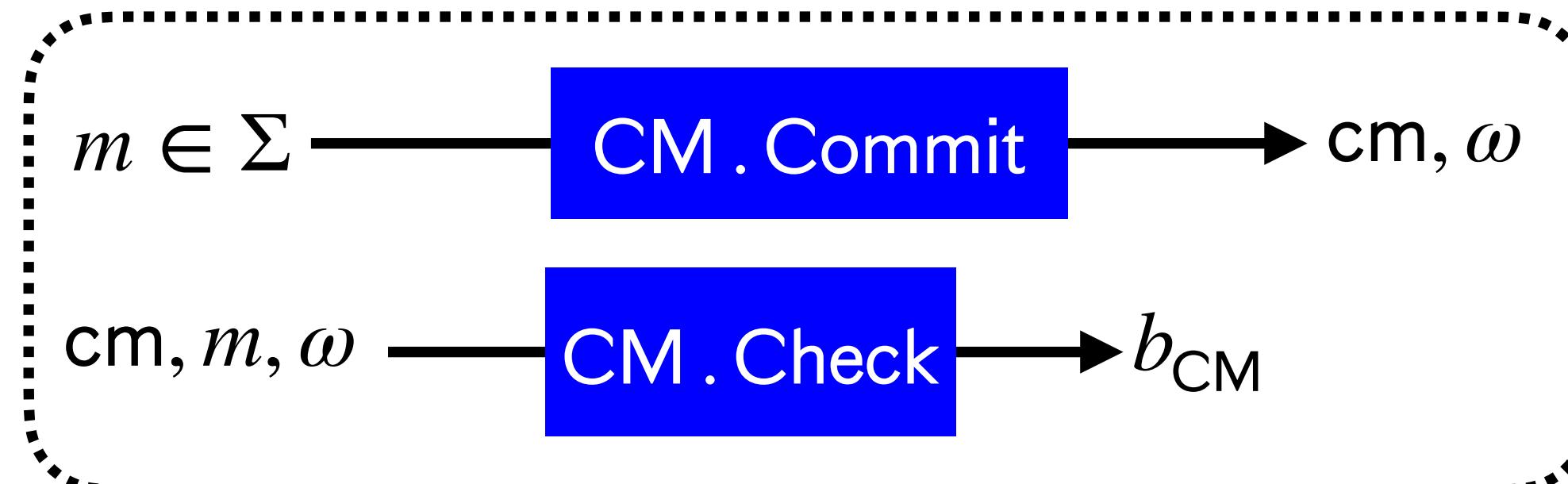


[Unruh16]: standard commitment scheme gives quantum proofs of knowledge
Collapse binding commitment scheme \implies quantum rewinding for $O(1)$ times



Quantum rewinding with commitment schemes

Commitment scheme (CM)



Collapse binding

$cm, (\mathcal{M}, \mathcal{W}) \leftarrow Adv$

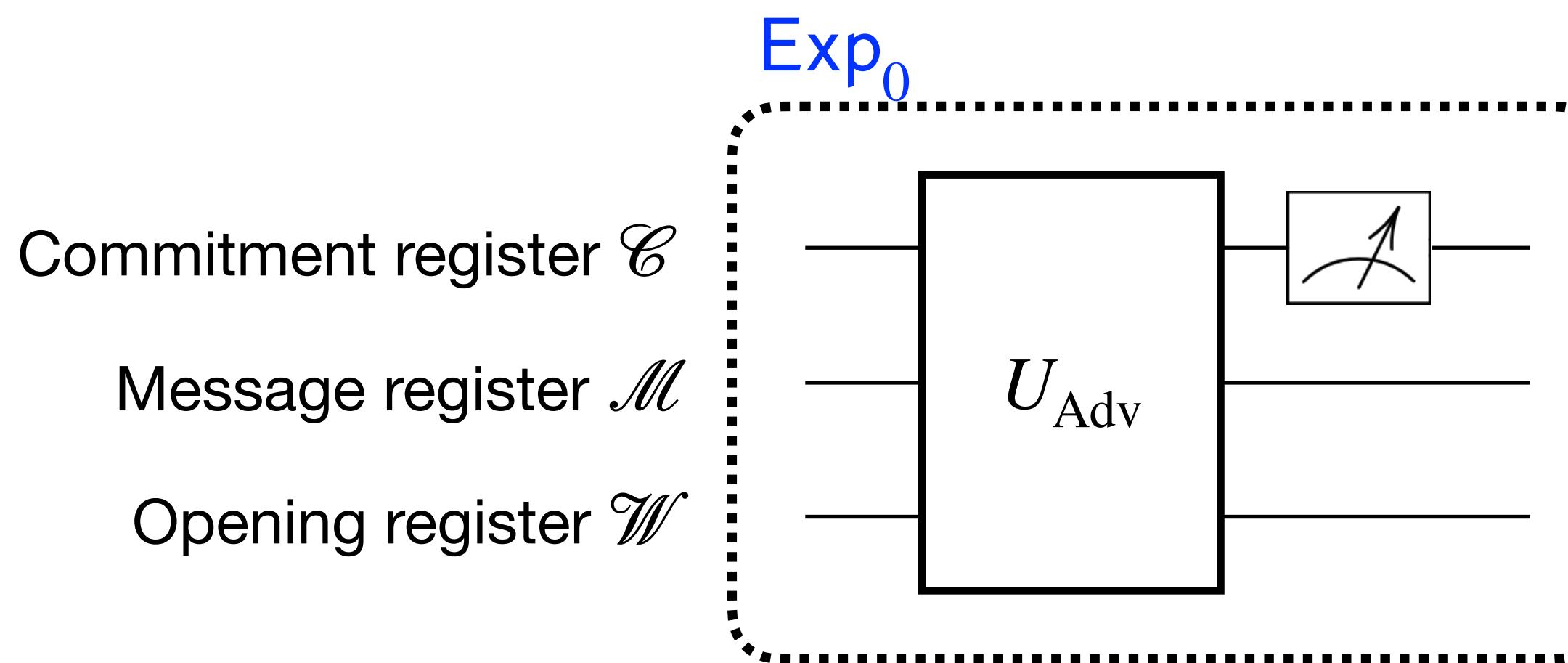
Exp_0 : does nothing

Exp_1 : measure \mathcal{M}

$(\mathcal{M}, \mathcal{W}) \rightarrow Adv$

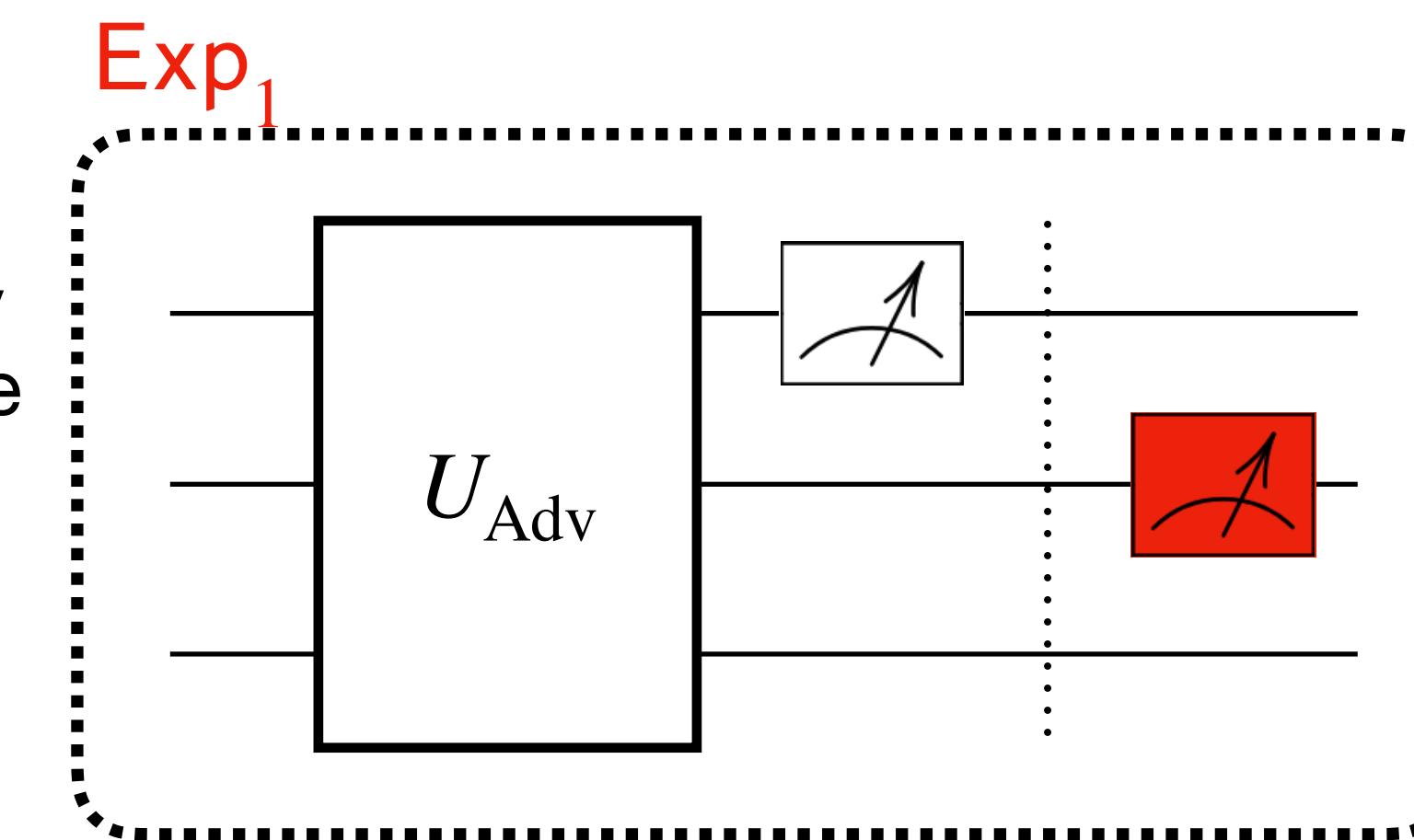
Undetectable measurement

$\Pr[\text{Adv distinguishes } \text{Exp}_0 \text{ and } \text{Exp}_1] \leq \epsilon_{\text{CM Collapse}}^*$



Computationally
Indistinguishable

\approx



Collapse binding \implies binding
(Adv cannot give different openings for one cm)

How about vector commitments?

CMSZ collapsing

$(\text{cm}, Q), (\mathcal{A}, \mathcal{O}) \leftarrow \text{Adv}$

Exp_0 : does nothing

Exp_1 : measure $(\mathcal{A}, \mathcal{O})$

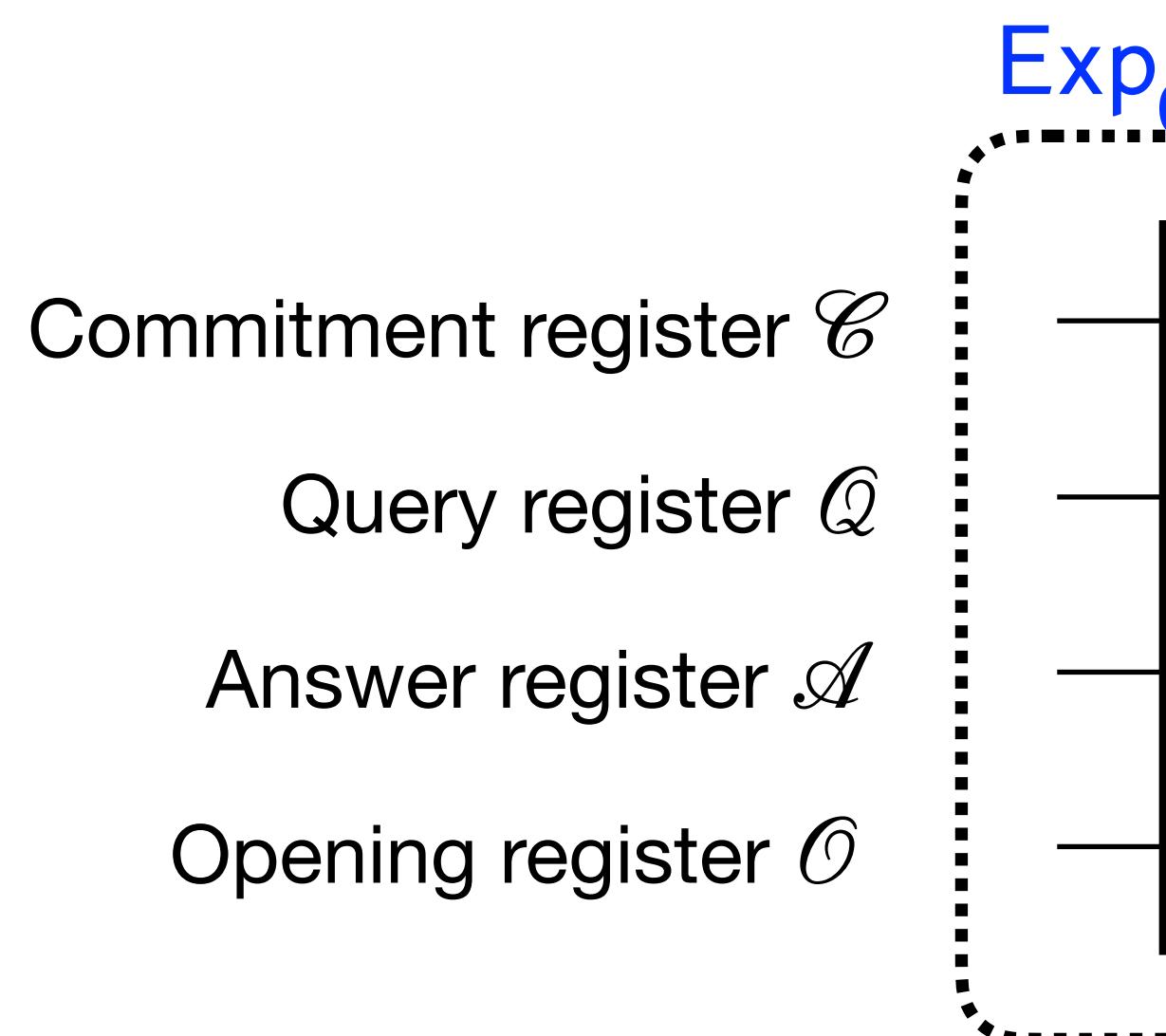
$(\mathcal{A}, \mathcal{O}) \rightarrow \text{Adv}$

Why measure \mathcal{O} ? CM only measure \mathcal{M}
[CMSZ21] security analysis needs \mathcal{O}

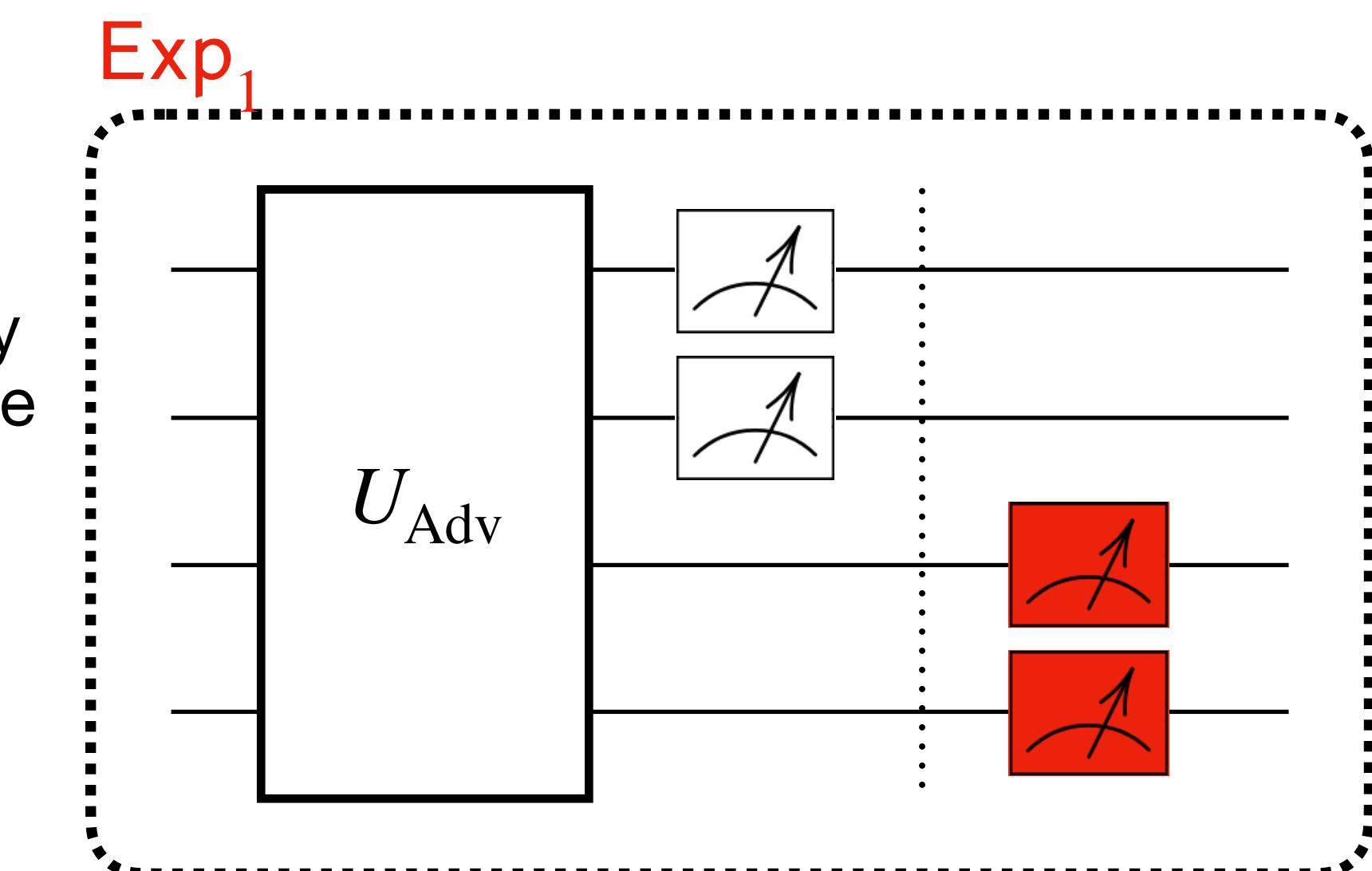
Issue: does not imply position binding

- CMSZ collapsing - one single query set
- Position binding - two query sets Q, Q'

$\Pr[\text{Adv distinguishes } \text{Exp}_0 \text{ and } \text{Exp}_1] \leq \epsilon_{\text{CMSZCollapse}}^*$



Computationally
Indistinguishable
 \approx



Post-quantum security of Kilian's protocol

Queries only depend on randomness

Theorem [CMSZ21]. \forall non-adaptive PCP, VC (negligible ϵ_{PQPB}^* , negligible $\epsilon_{\text{CMSZCollapse}}^*$),
 $\epsilon_{\text{ARG}}^* \leq \epsilon_{\text{PCP}} + \text{negl}$

Can we get a more robust VC collapsing def?
(VC collapsing that implies position binding)

PCP is not concretely efficient - Can we use IOPs?

Can we get concrete bound as classical case?

Can we handle adaptive PCPs?

A new collapsing definition for VC: Collapse position binding

Naive attempt: openings to different subsets

Naive-attempt collapsing

$\text{cm}, (\mathcal{Q}, \mathcal{A}, \mathcal{O}) \leftarrow \text{Adv}$

Impossible to achieve!

Exp_0 : does nothing

Exp_1 : measure $(\mathcal{Q}, \mathcal{A}, \mathcal{O})$

$(\mathcal{Q}, \mathcal{A}, \mathcal{O}) \rightarrow \text{Adv}$

$\Pr[\text{Adv distinguishes } \text{Exp}_0 \text{ and } \text{Exp}_1] \leq \epsilon_{\text{NaiveCollapse}}^*$

Recall:

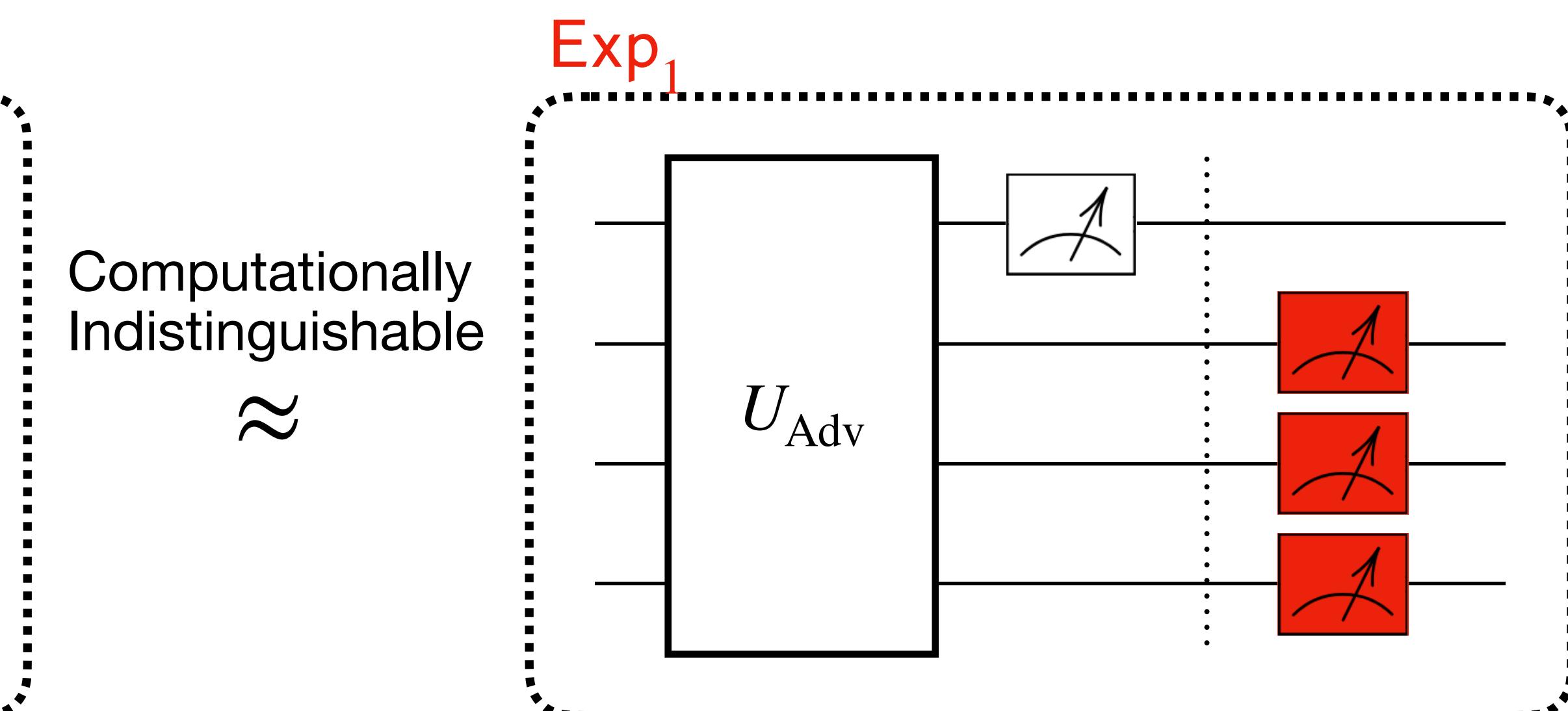
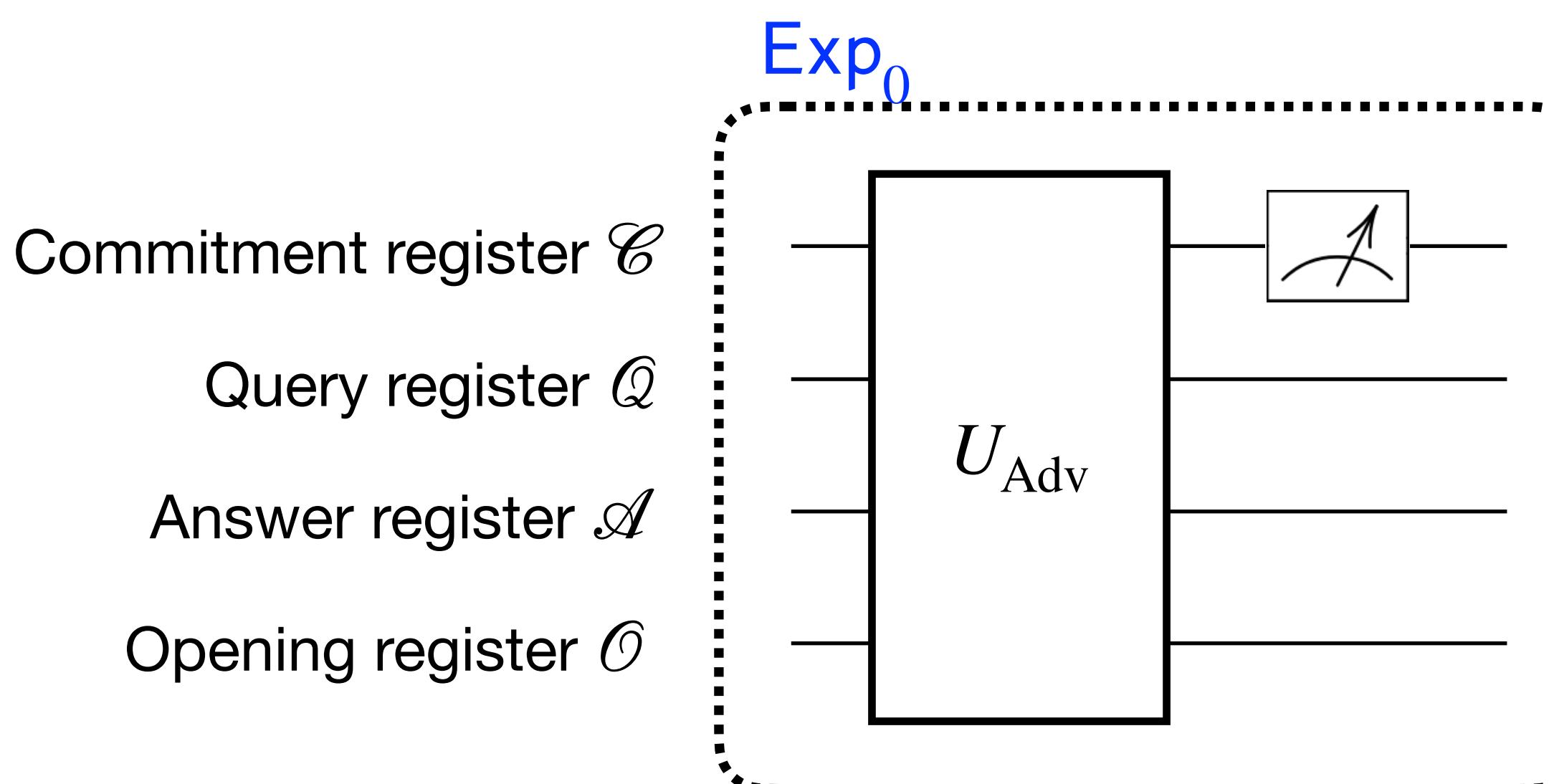
- CMSZ collapsing - one single query set
- Position binding - two query sets $\mathcal{Q}, \mathcal{Q}'$

Assume cm has two valid openings $(\mathcal{Q}, \text{ans}, \text{pf}), (\mathcal{Q}', \text{ans}', \text{pf}')$

$\text{Adv} \rightarrow \text{cm}, |\mathcal{Q}, \text{ans}, \text{pf}\rangle + |\mathcal{Q}', \text{ans}', \text{pf}'\rangle$

Measuring $(\mathcal{Q}, \mathcal{A}, \mathcal{O}) \Rightarrow (\mathcal{Q}, \text{ans}, \text{pf}) \text{ or } (\mathcal{Q}', \text{ans}', \text{pf}')$

\Rightarrow Easily distinguishable from uniform superposition



Collapse position binding

Lifting from commitment schemes

Collapse position binding

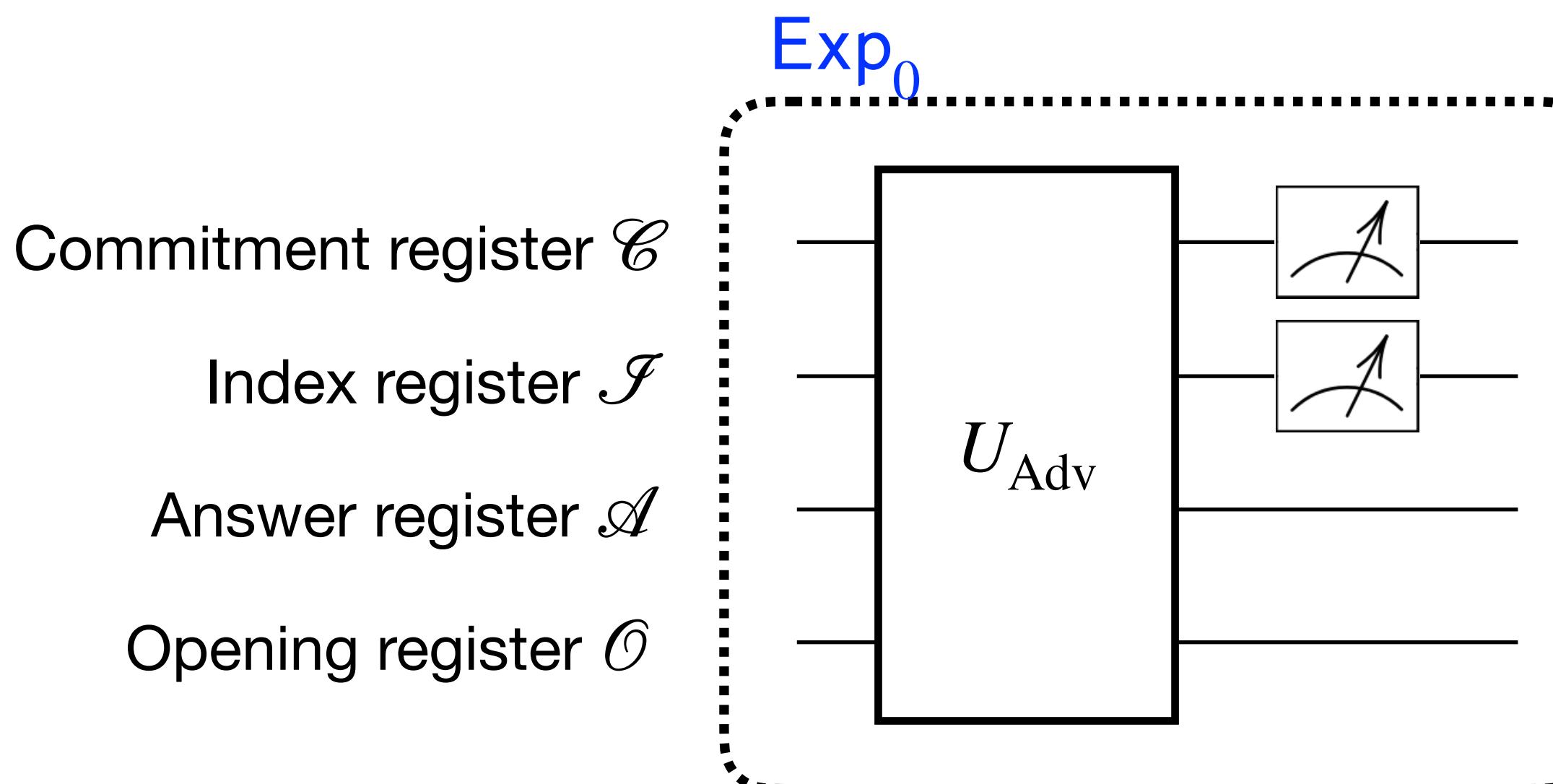
$(\text{cm}, \text{idx}), (\mathcal{A}, \mathcal{O}) \leftarrow \text{Adv}$

Exp_0 : does nothing

Exp_1 : measure \mathcal{A} at location idx

$(\mathcal{A}, \mathcal{O}) \rightarrow \text{Adv}$

$\Pr[\text{Adv distinguishes } \text{Exp}_0 \text{ and } \text{Exp}_1] \leq \epsilon_{\text{VCCollapsePB}}^*$

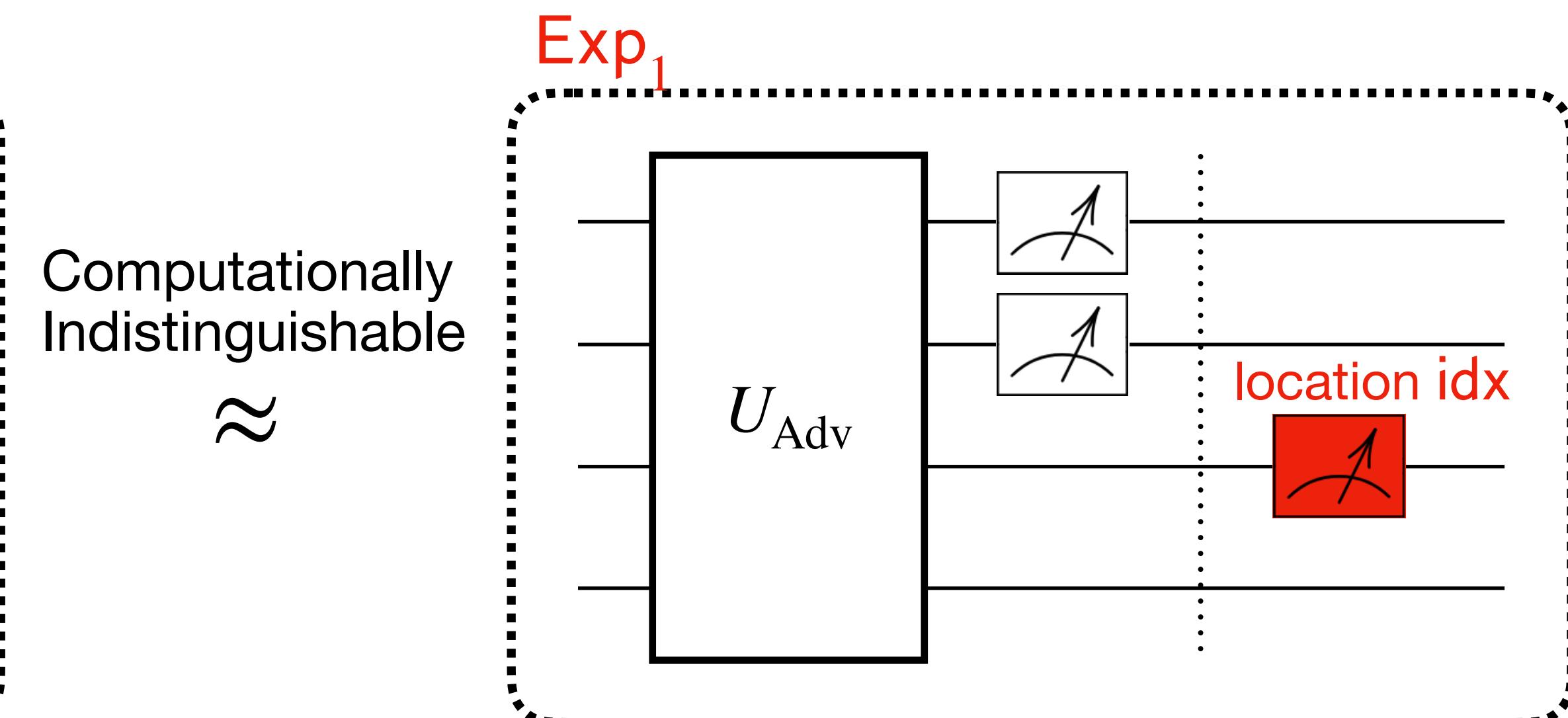


Known:

- VC position binding $\iff \forall i, \text{CM}_i$ binding
- CM collapse binding \implies CM binding

Goal: VC collapse position binding $\iff \forall i, \text{CM}_i$ collapse binding

VC collapse position binding \implies VC position binding



Improved post-quantum security of Kilian's protocol

Queries only depend on randomness

Theorem [CMSZ21]. \forall non-adaptive PCP, VC (negligible ϵ_{PQPB}^* , negligible $\epsilon_{\text{VCCollapsePB}}^*$)

$$\epsilon_{\text{ARG}}^* \leq \epsilon_{\text{PCP}} + \text{negl}$$

Can we get a more robust VC collapsing def?
(VC collapsing that implies position binding)

YES!

PCP is not concretely efficient - Can we use IOPs?

Can we get concrete bound as classical case?

Can we handle adaptive PCPs?

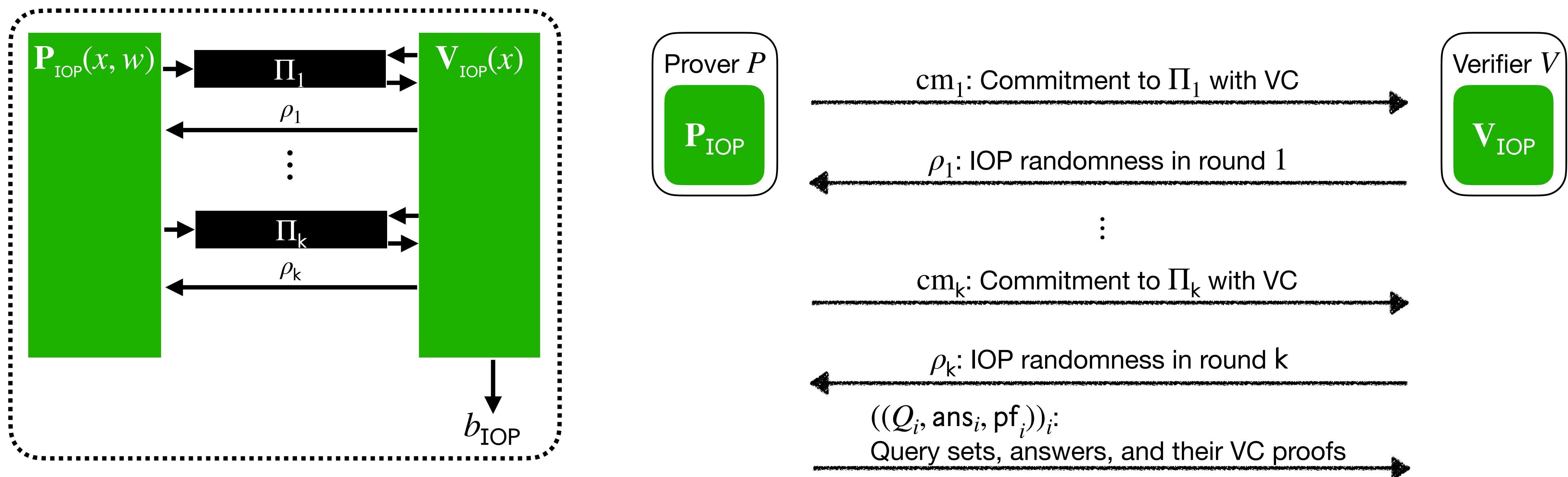
**IBCS protocol:
Using IOPs instead of PCPs**

IBCS protocol

Existing PCPs are not concretely efficient: prover time too big

People use IOPs

Public-coin interactive oracle proof (IOP)



Our result

Queries depend on randomness and answers to queries to previous proofs

Theorem. \forall semi-adaptive IOP, VC, $\epsilon > 0$,

$$\epsilon_{\text{ARG}}^*(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + k \cdot \ell_{\max} \cdot q_{\max} \cdot \epsilon_{\text{VCCollapsePB}}^*(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = \text{poly}(\ell/\epsilon) \cdot t_{\text{ARG}}.$$

Extra $\ell_{\max} \cdot q_{\max}$ factor: cost of quantum rewinding

Quantum rewinding can fail

$\text{poly}(\ell/\epsilon)$ attempts $\Rightarrow \ell/\epsilon$ valid rewinds

IBCS soundness [CDGS23,CGKY25]: $\epsilon_{\text{ARG}}(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + k \cdot \epsilon_{\text{VC}}(t_{\text{VC}}) + \epsilon$, where $t_{\text{VC}} = O(t_{\text{ARG}} \cdot \ell/\epsilon)$.

Corollary: post-quantum secure succinct arguments in the standard model (no oracles),
with **the best asymptotic complexity known**.

Corollary for Kilian's protocol. \forall adaptive PCP, VC, $\epsilon > 0$,

$$\epsilon_{\text{ARG}}^*(t_{\text{ARG}}) \leq \epsilon_{\text{IOP}} + \ell \cdot q \cdot \epsilon_{\text{VCCollapsePB}}^*(t_{\text{VC}}) + \epsilon, \text{ where } t_{\text{VC}} = \text{poly}(\ell/\epsilon) \cdot t_{\text{ARG}}.$$

Theorem [CMSZ21]. \forall non-adaptive PCP, VC (negligible ϵ_{PQPB}^* , negligible $\epsilon_{\text{CMSZCollapse}}^*$),

$$\epsilon_{\text{ARG}}^* \leq \epsilon_{\text{PCP}} + \text{negl}$$

YES!

Can we get a more robust VC collapsing def?

PCP is not concretely efficient - Can we use IOPs?

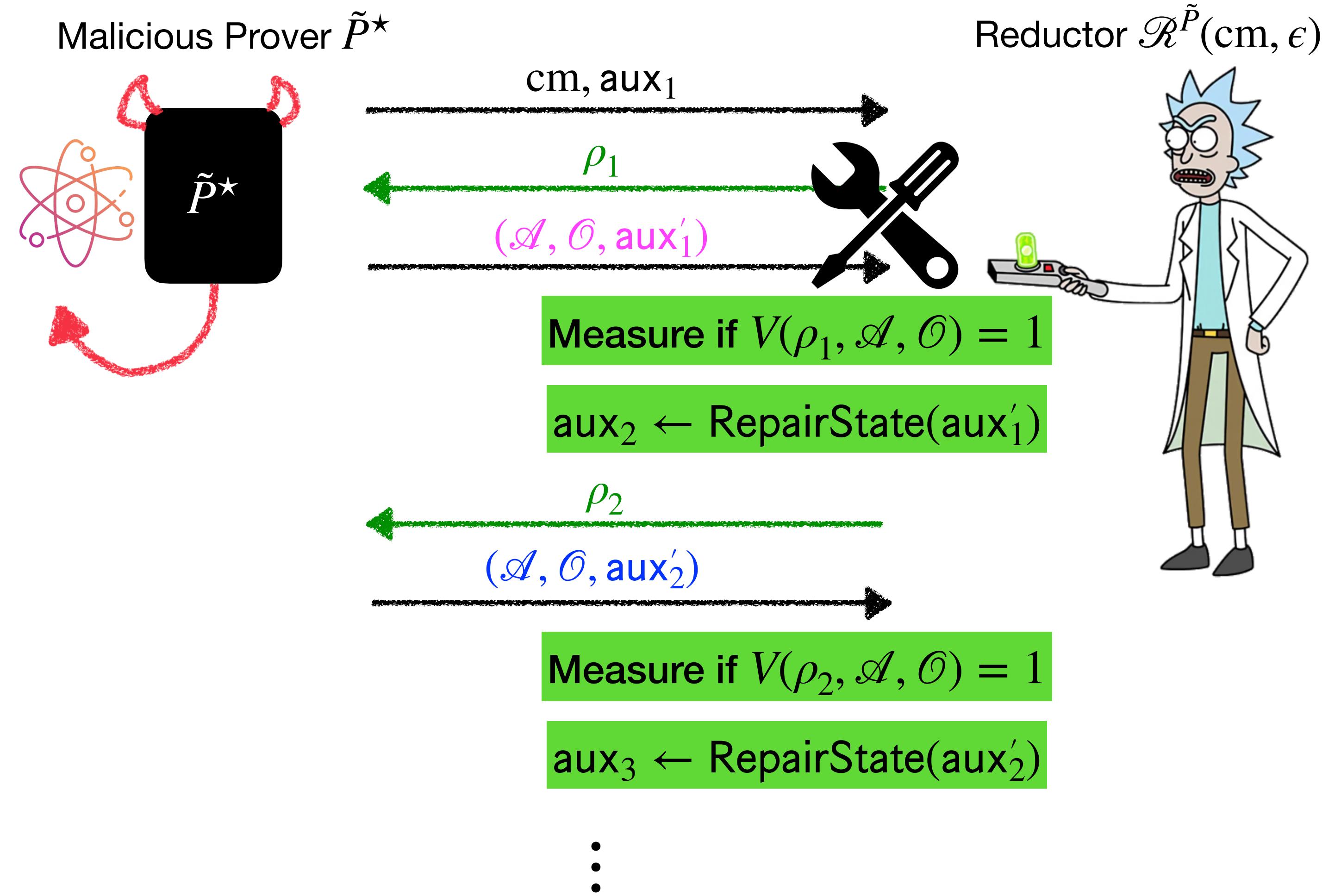
Can we get concrete bound as classical case?

Can we handle adaptive PCPs?

Post-quantum security for Kilian

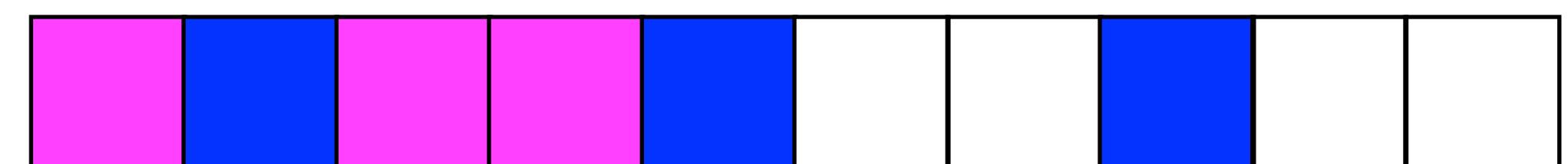
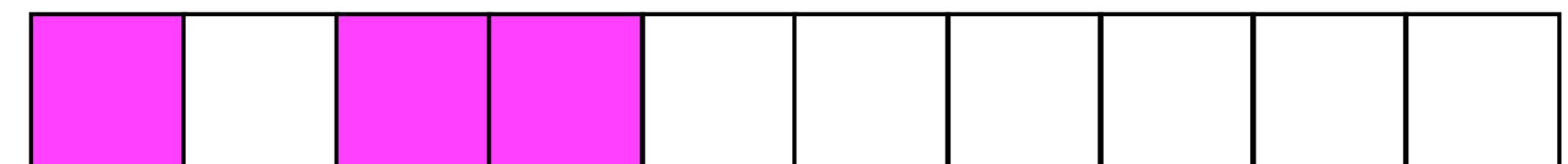
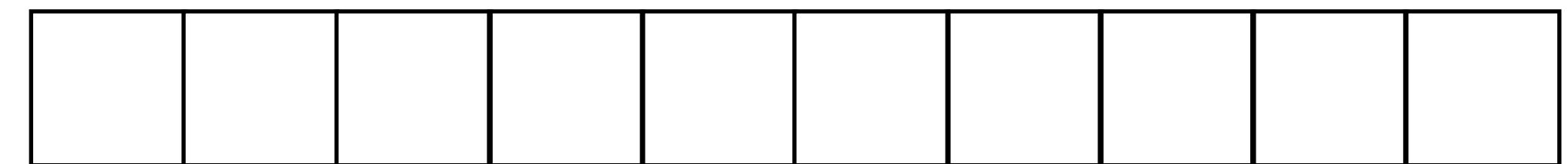
Starting point: [CMSZ21]

[CMSZ21] reductor



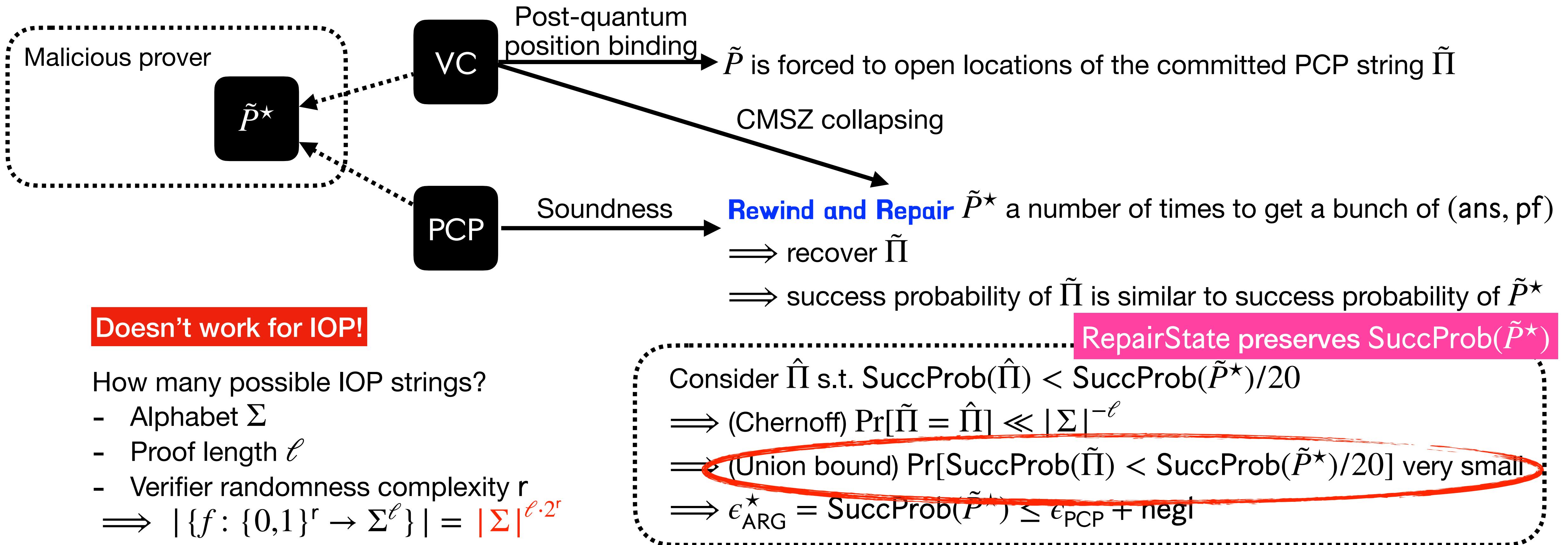
RepairState preserves SuccProb(\tilde{P}^*)

Recover $\tilde{\Pi}$



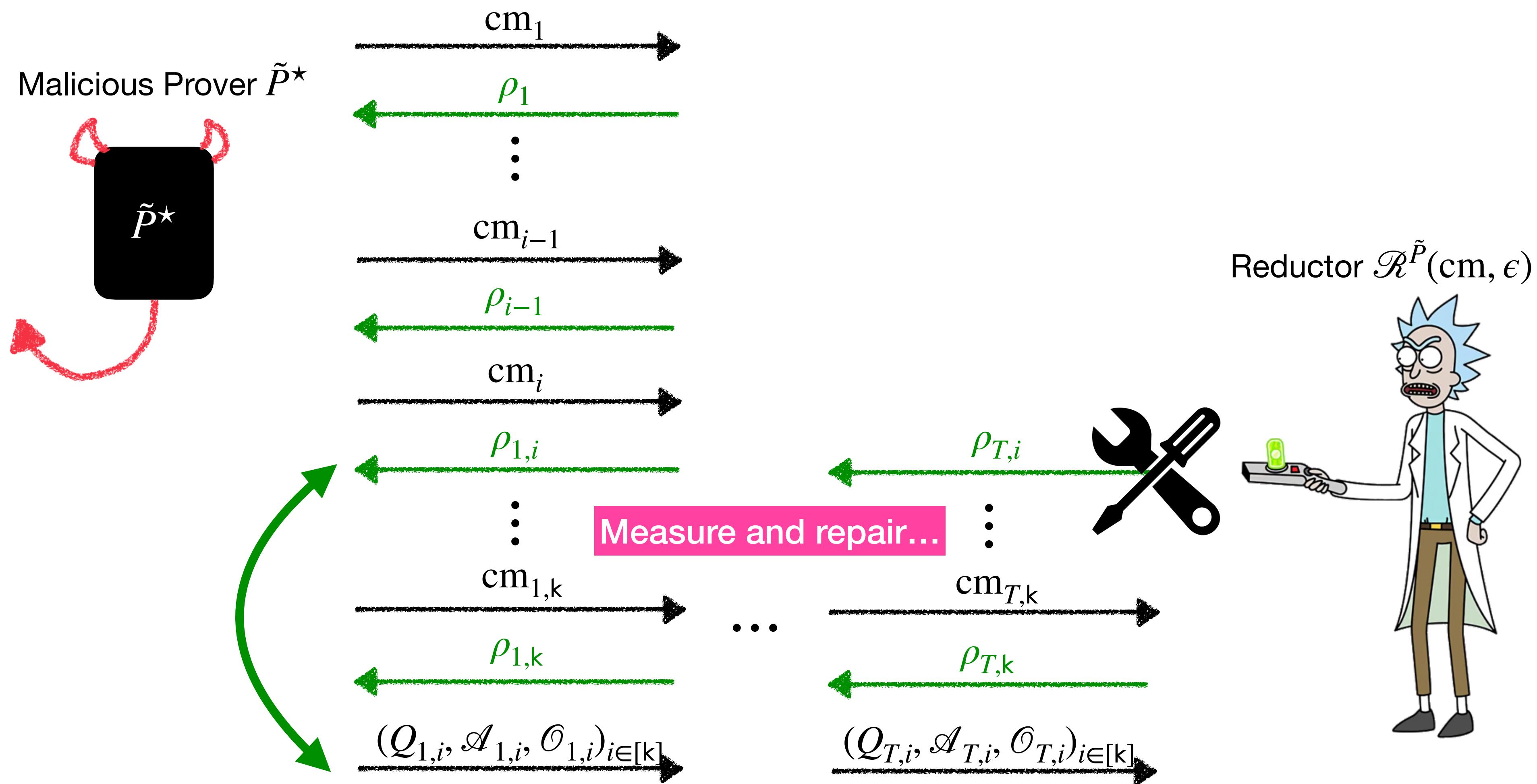
[CMSZ21] security reduction

Goal: relate the soundness error of Kilian[PCP, VC]
to the soundness error of PCP and the post-quantum security of VC.

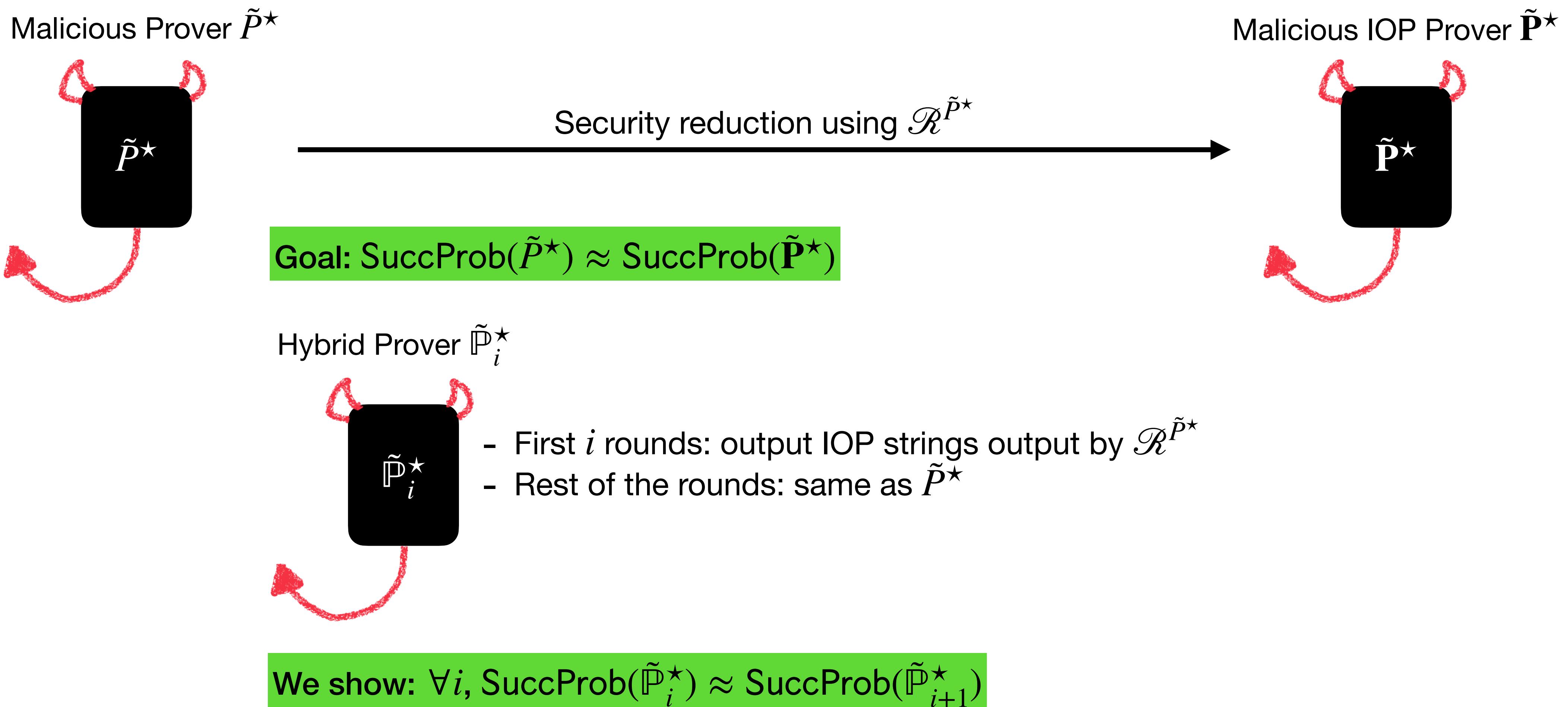


Our security reduction

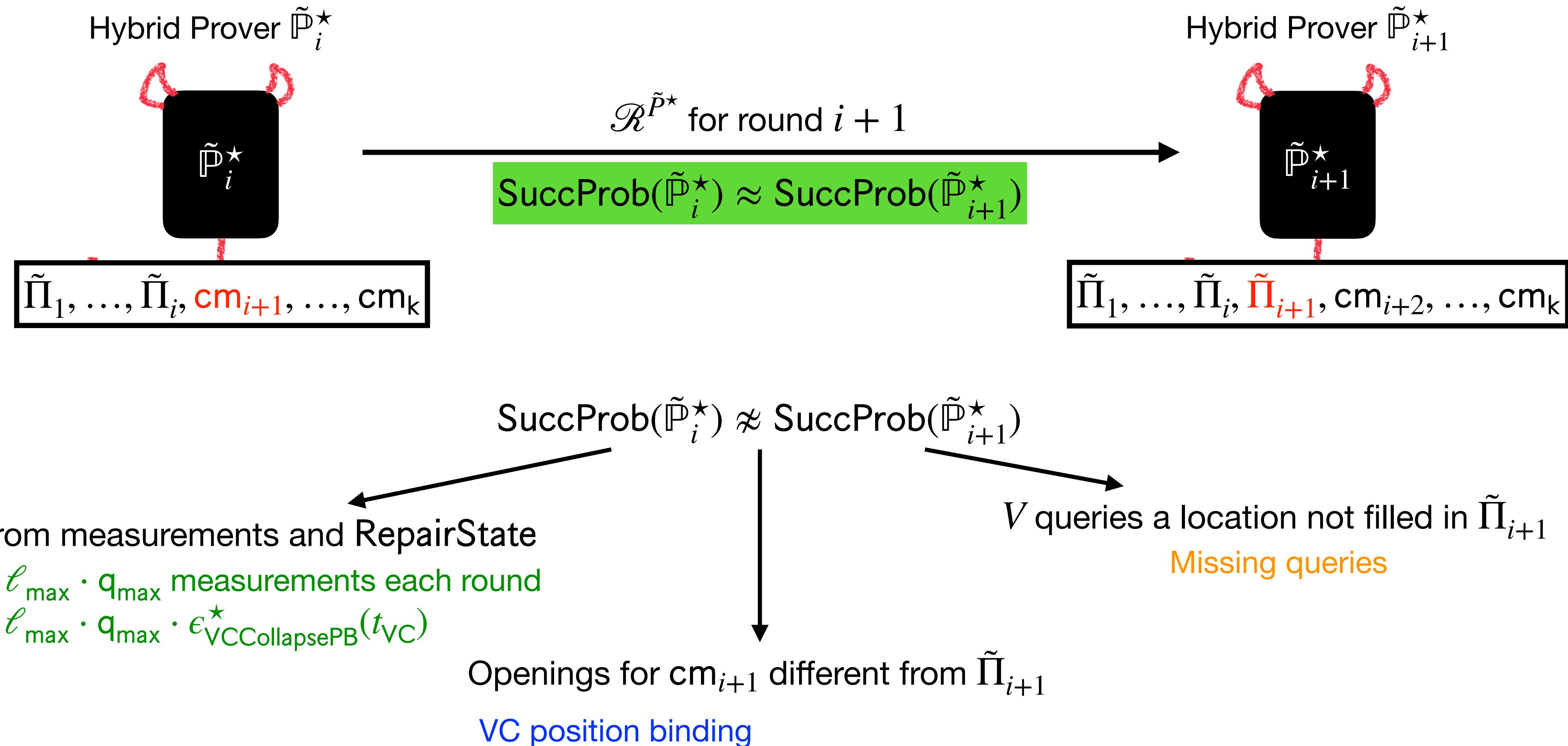
Our quantum reductor



Hybrid argument

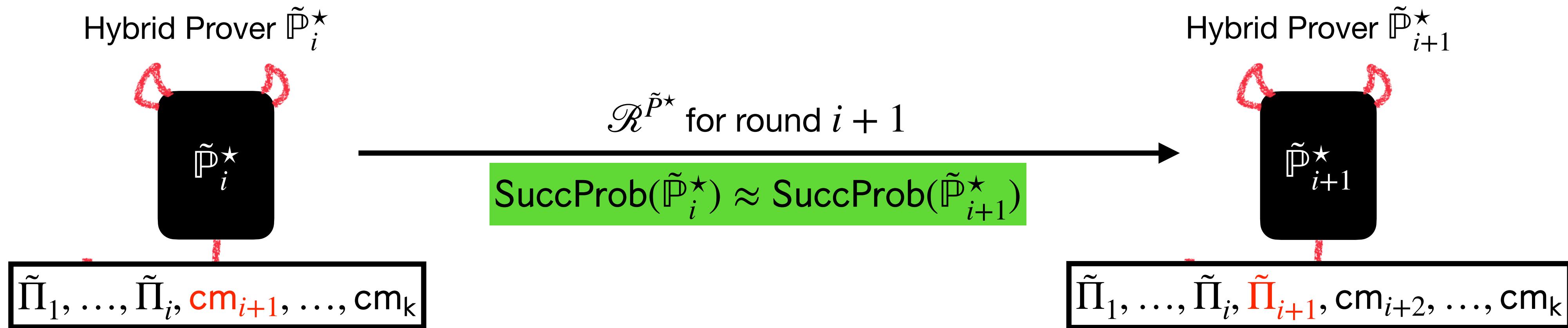


Our security reduction



Missing queries

V queries a location not filled in $\tilde{\Pi}_{i+1}$



Run \tilde{P}_i^* one more time, get openings for \mathbf{cm}_{i+1}
 \implies The $(T + 1)$ -th rewind

Rewind \tilde{P}_i^* T times to get $\tilde{\Pi}_{i+1}$

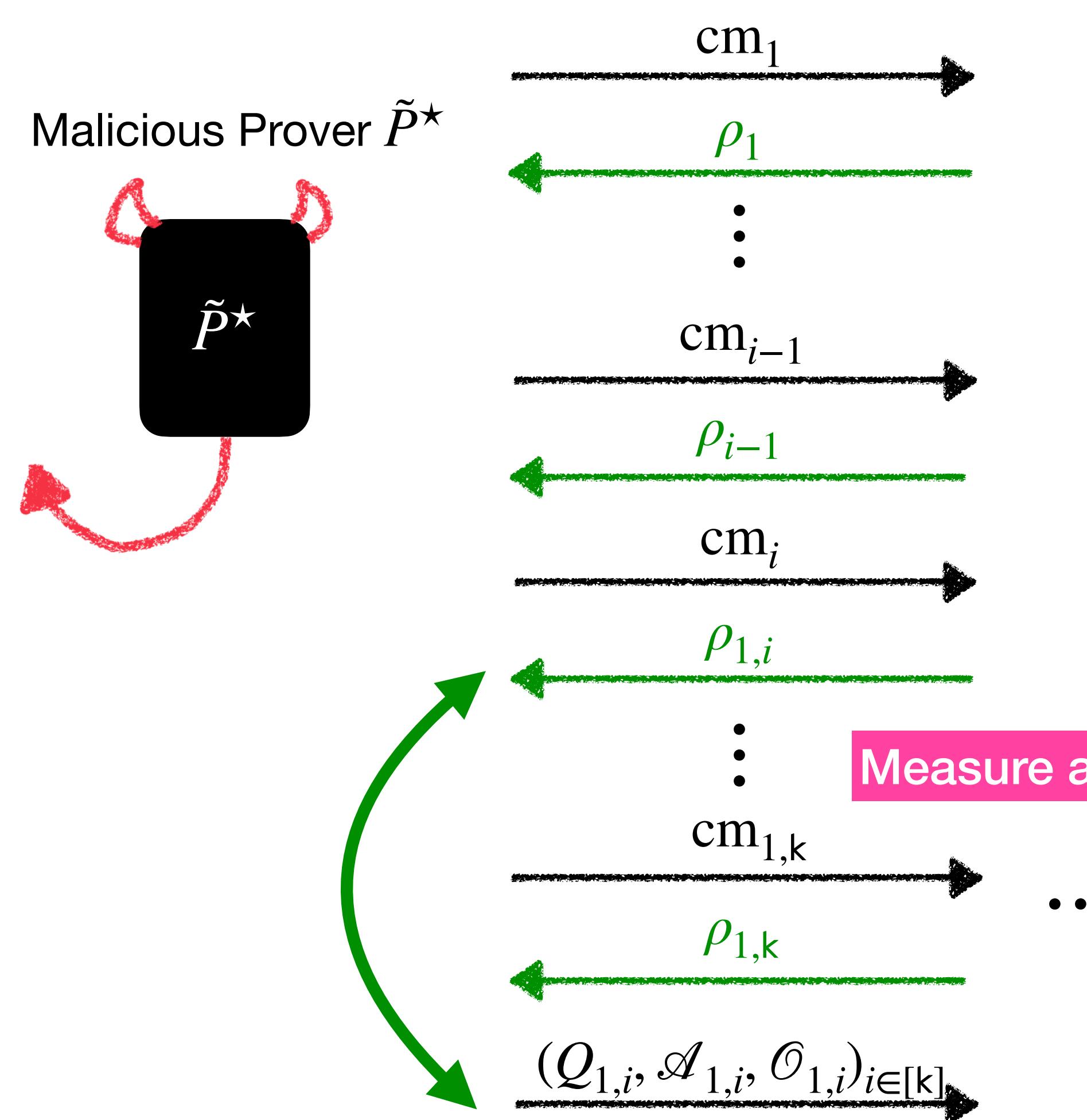
Classical approach

- δ_q : prob $q \in [\ell_i]$ queried by V and correctly opened by \tilde{P}^*
- $\Pr[\exists q, \tilde{\Pi}_{i+1}[q] \text{ unfilled}, q \text{ queried with valid opening}] \leq \ell_i \cdot \delta_q (1 - \delta_q)^T \leq \ell_i / T$
- Setting T to get desired bound

Doesn't work for quantum!

RepairState only preserves $\text{SuccProb}(\tilde{P}^*)$
 \implies does not preserve δ_q

Random stopping time



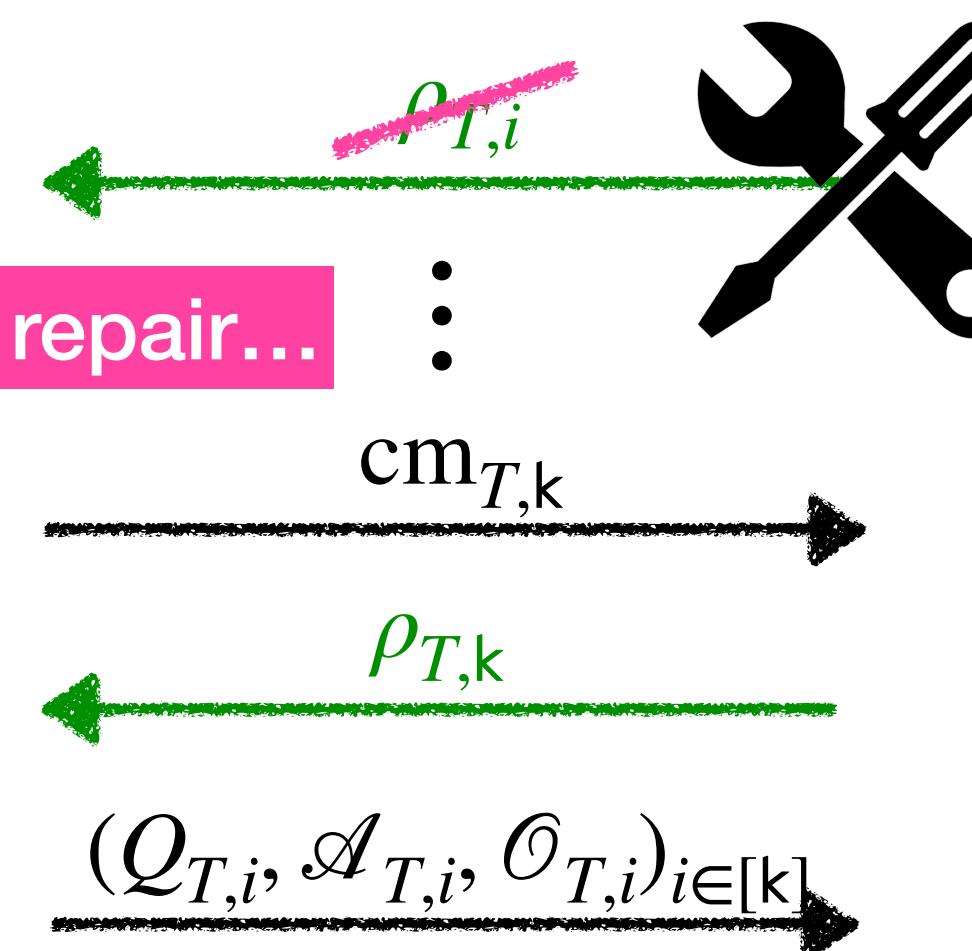
Key: total number of locations $q \in [\ell_i]$ filled in by \mathcal{R} is ℓ_i

$$\zeta_j: \mathbb{I}[Q_{j+1,i} \neq \cup_{c \leq j} Q_{c,i}]$$

$$\implies \Pr[\text{missing queries}] = \mathbb{E}[\zeta_t]$$

$$(\text{Observation}) \mathbb{E}[\zeta_t] = 1/T \cdot \sum_{j \leq T} \mathbb{E}[\zeta_j] \leq \ell_i/T$$

Sample $t \leftarrow [T]$
Rewind for t times



Reductor $\mathcal{R}^{\tilde{P}^*}(cm, \epsilon)$



Why not adaptive IOP?

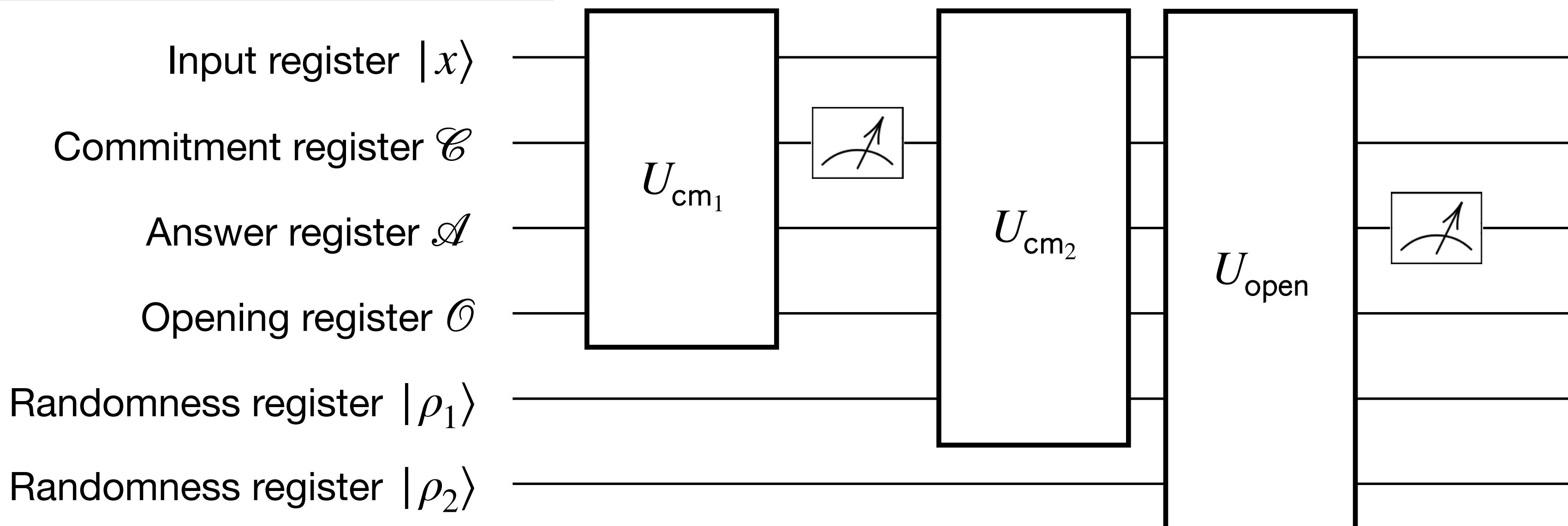
Queries to Π_1 depends on answers from Π_2

$(Q_1, \mathcal{A}_1, \mathcal{O}_1)$ and $(Q_2, \mathcal{A}_2, \mathcal{O}_2)$ entangled

\implies Measuring $(Q_1, \mathcal{A}_1, \mathcal{O}_1)$ collapses $(Q_2, \mathcal{A}_2, \mathcal{O}_2)$

Collapse position binding **does not allow** measurement of $(Q_2, \mathcal{A}_2, \mathcal{O}_2)$

When $\mathcal{R}^{\tilde{P}^*}$ rewinds to get $\tilde{\Pi}_1$...



Collapse position binding

$(cm, idx), (\mathcal{A}, \mathcal{O}) \leftarrow Adv$

Exp_0 : does nothing

Exp_1 : measure \mathcal{A} at location idx

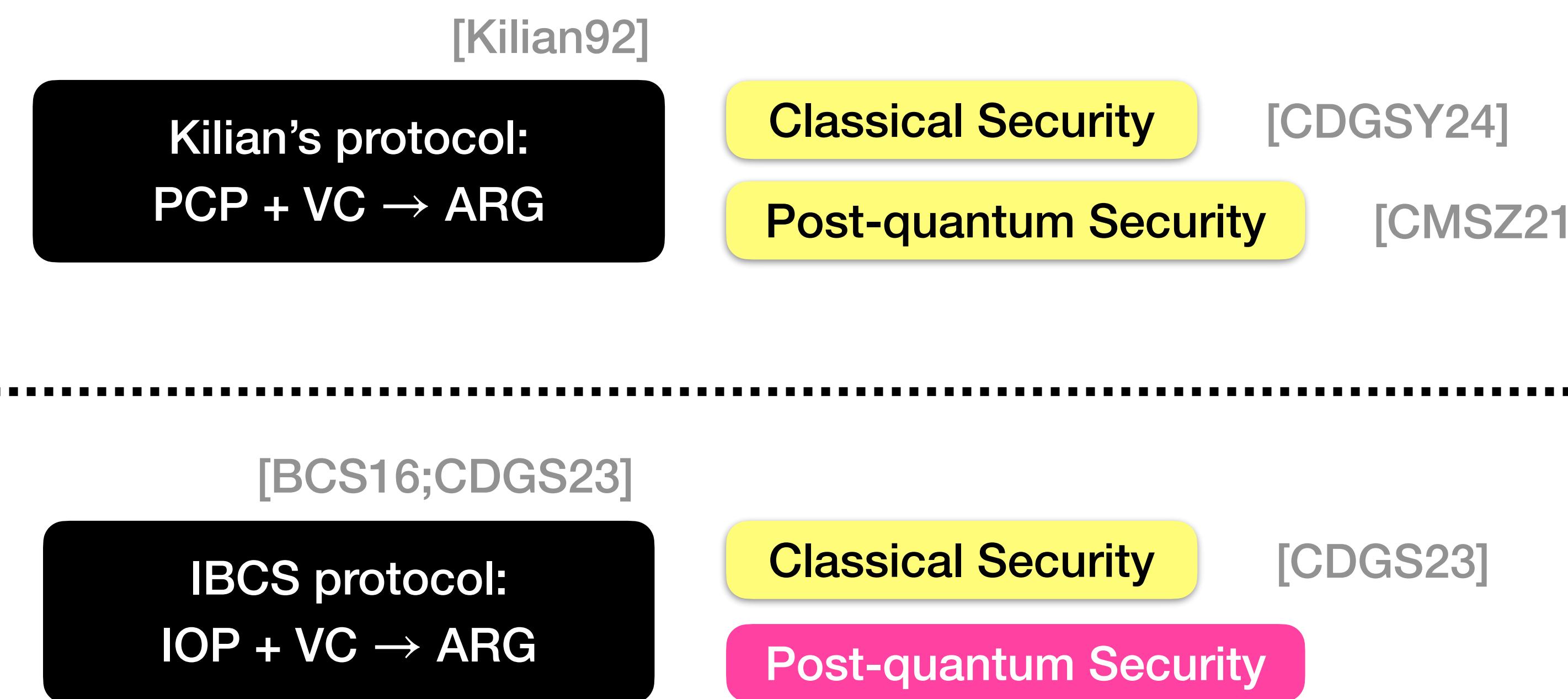
$(\mathcal{A}, \mathcal{O}) \rightarrow Adv$

$\Pr[\text{Adv distinguishes } Exp_0 \text{ and } Exp_1] \leq \epsilon_{VCCollapse}^*$

Recap

Prior work

This work



⇒ **Best** post-quantum secure succinct arguments
in the standard model (no oracles)



<https://eprint.iacr.org/2025/947>



Thank you!

References

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