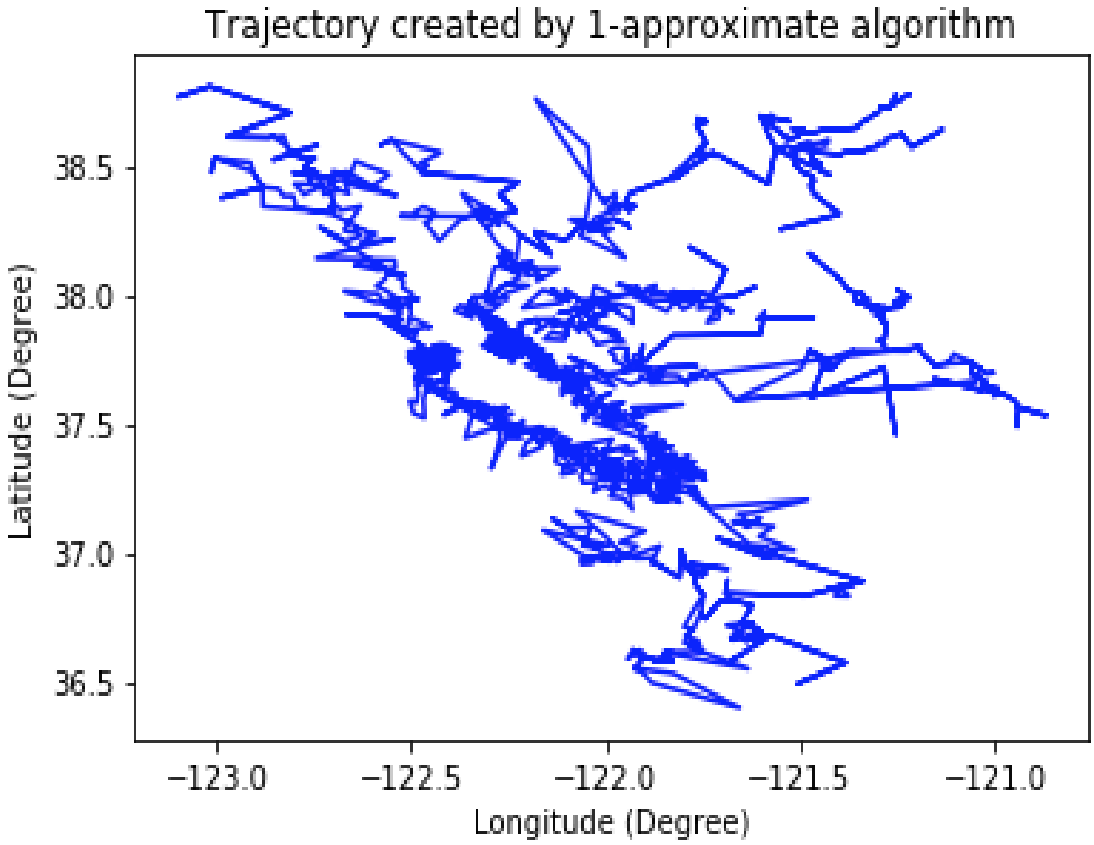
Question 9 (continued)

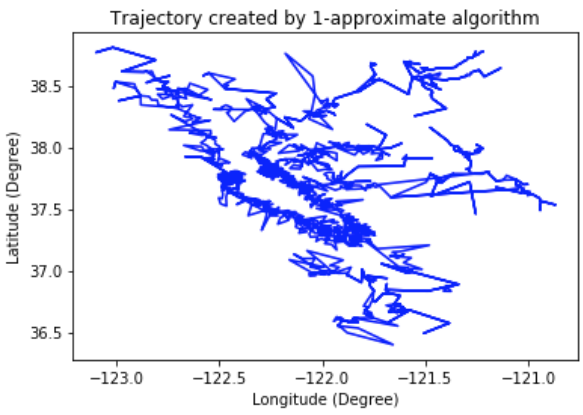
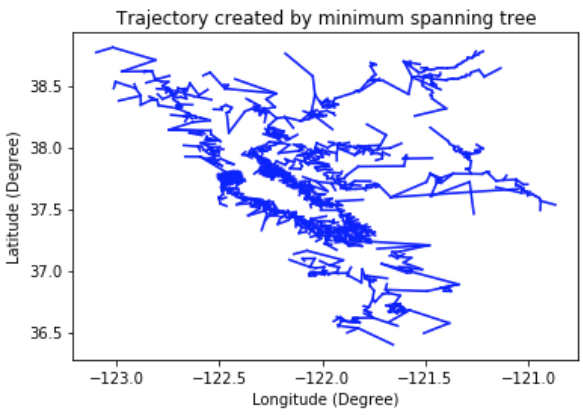
The 1-approximate algorithm's main steps are duplicate edges, find the Eulerian path, shortcutting. All the edges are duplicated, and then find the Eulerian path, to get the shortest Eulerian spanning tree, try to extract the embedding path (to remove duplicate vertices in the Eulerian path). For example, 1-3-2-3-4-5-4-3-1, will first become 1-3-2- -4-5-4-3-1, if edge (2,4) exists in the graph (it is real edge), if not find the minimum path between these two nodes.

This is based on the results from Question 8, from Question 8 we can see that 92.6% satisfy the triangle inequality, which means about there is about 92.6% this triangle relation in triangle (2,3,4) $weight(2,3) + weight(3,4) > weight(2,4)$ satisfies. We could just assume the triangle inequality satisfies during the shortcutting. Through shortcutting we could find the shortest Eulerian path.

Because the Eulerian path is duplicated from the **duplicated** minimum spanning tree, the worst cost of the Eulerian path is the **twice** of the cost of the minimum spanning tree $cost(mst)$, after the shortcutting the cost will be lower than that so $cost(1 - approximate) \leq 2 \cdot cost(mst)$, therefore, the calculated upper bound about **1.74** makes sense. And because $1.74 - 1 \leq 1$it is a 1-approximate method.

Question 10



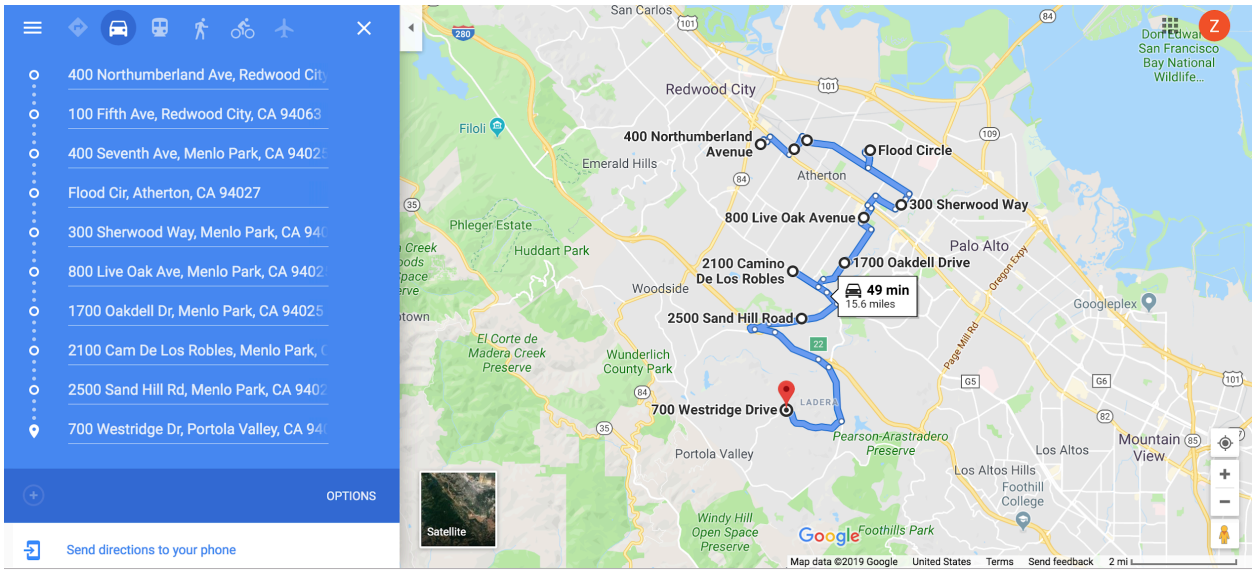
1-Approximate Algorithm	Minimum Spanning Tree
	

The figure above compare the trajectory plot of Santa has to travel generated by the 1-approximate algorithm and the minimum spanning tree in igraph package. They look very similar, which means that the 1-approximate algorithm works very well.

The minimum spanning tree generated by the igraph package uses the Prim's algorithm, which is a kind of greedy algorithm. That is "We start from one vertex and keep adding edges with the lowest weight until we reach our goal".

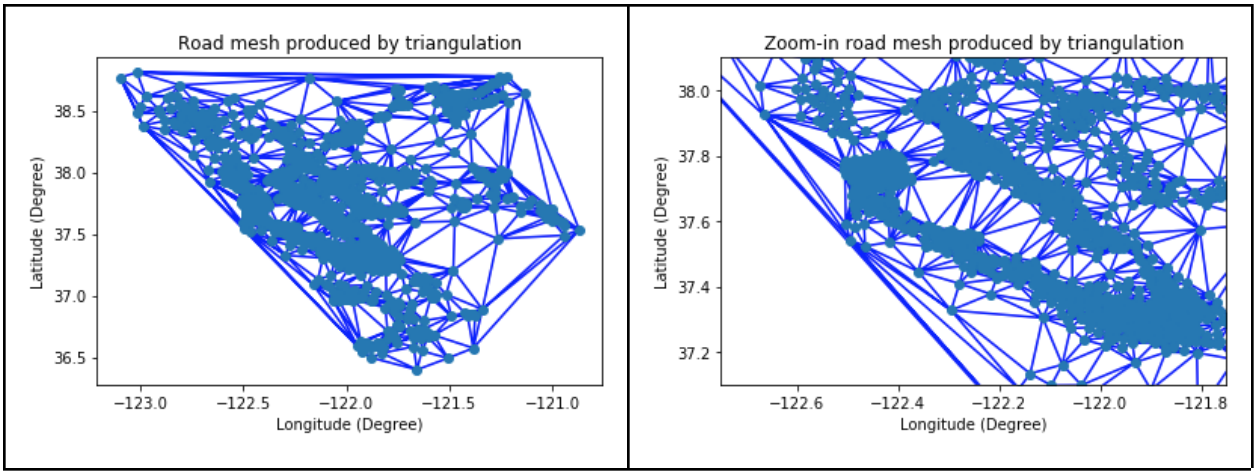
The dataframe below is the initial part of the address sequence produced by 1-approximate algorithm. And the following is the first 10 street addresses in the sequence showed onto the real map. The results make sense, because the adjacent nodes in the sequence are close to each other.

	Step	Trajectory node address
0	1	400 Northumberland Avenue, Redwood Oaks, Redwo...
1	2	100 Fifth Avenue, South Fair Oaks, Redwood City
2	3	400 Seventh Avenue, Fair Oaks, Menlo Park
3	4	0 Flood Circle, Lindenwood, Atherton
4	5	300 Sherwood Way, Linfield Oaks, Menlo Park
5	6	800 Live Oak Avenue, Downtown Menlo Park, Menl...
6	7	1700 Oakdell Drive, Menlo Park
7	8	1700 Oakdell Drive, Menlo Park
8	9	2100 Camino De Los Robles, University Heights,...
9	10	2500 Sand Hill Road, Sharon Heights, Menlo Park
10	11	700 Westridge Drive, Central Portola Valley, P...
11	12	Reservoir Road, Stanford
12	13	700 Welch Road, Palo Alto
13	14	100 Campus Drive, Stanford
14	15	600 Campus Drive, Stanford
15	16	0 Ryan Court, Stanford
16	17	700 Bay Road, Staumbaugh Heller, Redwood City



Question 11

Road mesh	Zoom-in road mesh



The figures above are the road mesh created by Delaunay triangulation. After Delaunay triangulation all the nodes are connected, and the Delaunay algorithm maximize the minimum angles of all the triangles. For each triangle its circumcircle contains no other nodes, and for each edge a circle exists through its endpoints contains no other nodes. The Euclidean minimum spanning tree (the Euclidean minimum spanning tree is the minimum spanning tree based on the Euclidean distance of the edge in space instead of the weight of the edge) is a subset of the Delaunay triangulation of the same set of nodes.

Question 12

$Flow = Density \cdot Speed$, $Flow$ is cars/(road.hour), $Density$is cars/mile, $Speed$ is mile/hour

Safety distance: $d_{safety} = Speed \cdot \frac{2}{3600} = \frac{Speed}{1800}\ (mile)$

Distance between two cars: $d = 0.003 + d_{safety} = 0.003 + \frac{Speed}{1800}\ (mile)$

Density: $Density = \frac{1}{d} = \frac{1}{0.003 + \frac{Speed}{1800}}\ (cars/mile)$

Flow: $Flow = \frac{1}{0.003 + \frac{Speed}{1800}} \cdot Speed = \frac{Speed}{0.003 + \frac{Speed}{1800}}\ (cars/hour)$

Each road has 2 lanes, the flow is:

$$Flow = \frac{2 \cdot Speed}{0.003 + \frac{Speed}{1800}}\ (cars/(road \cdot hour))$$

To calculate the flow of each road the edge of the graph, the edge distance is calculated by the Euclidean distance between the coordinates and then convert the degree to mile, using average 69 mile per degree. The time is the mean travel time of the edge. So $Speed = \frac{distance\ of\ the\ edge}{mean\ travel\ time}$

Then use the equation derived above $Flow = \frac{2 \cdot Speed}{0.003 + \frac{Speed}{1800}}\ (cars/(road \cdot hour))$to calculate the

flow of each edge. The average flow of the graph is about **2996 cars/(road.hour).** Below is part of the flow values of the edges:

3281	2706	3157	3043	3312	2969	3040	3327	2858	3243	3266	2494
3024	3042	3107	3030	3202	3229	2846	3160	2447	3031	3234	2765
3277	3163	2646	3169	3270	3262	2951	3069	3193	2951	2887	2920
3137	3179	3234	3074	3148	3147	2692	3210	3317	2950	3169	3296
2949	2913	3163	2717	3254	3302	2499	3172	2889	2744	3082	3231
3236	3005	2771	3066	2757	2806	3141	2980	3316	3238	3151	3183
3286	2816	3054	3294	3171	3242	2753	2671	3240	3081	2827	2554
2581	3299	3226	2922	3098	3131	3119	2497	2957	3256	3156	2822
2677	3272	2878	2674	2785	3309	3126	3031	3099	3137	3160	3018
2902	2969	3286	3159	3148	2941	2700	3165	3096	2827	2500	3276
2914	3117	2927	3206	3078	3169	3208	2791	3017	3186	3230	3301
2167	2892	2788	3184	3158	2521	2918	3140	2985	3196	3254	3223
2860	2600	2749	3148	2199	2617	2418	3189	3055	2924	2946	3195
3330	2886	3016	2754	3321	3127	3067	3052	3107	3107	3085	3034
2715	2754	2715	2757	2851	3112	2674	2699	2641	3088	2770	2658
3242	3024	3150	3406	3189	3101	3167	2990	3313	3075	2707	2621
2440	3263	2981	3060	2751	3100	3144	2898	3046	3100	3191	2869
3036	2919	3250	3046	3257	3281	2974	2364	2327	2750	3152	3155
2772	3023	2936	2660	3093	3310	2733	2765	3095	3076	3145	2881
… ...											

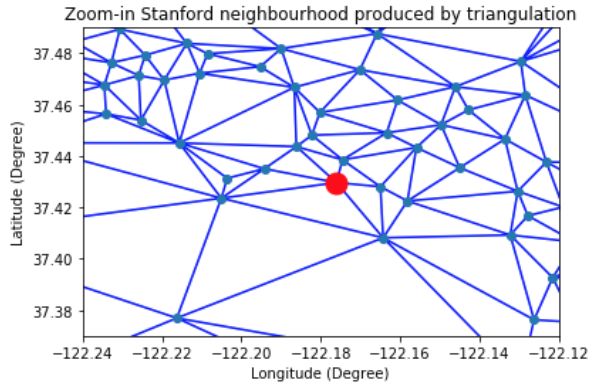
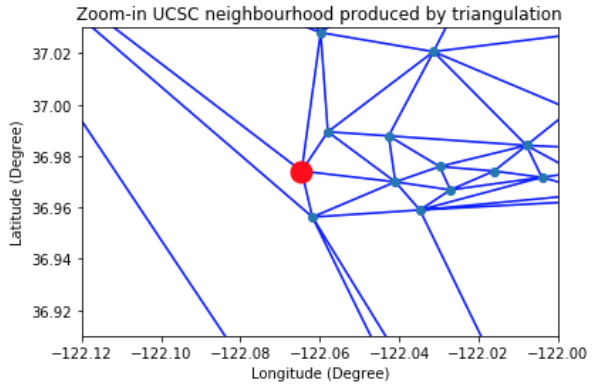
Question 13

Use the maxflow algorithm Ford-Fulkerson algorithm to calculate the maxflow of the graph based on the flow of each edge calculated in Question 12. The maxflow value is **470908 cars/(road.hour)**. It means that in the ideal case (no traffic jam), the maximum total number of cars that can commute on the road travelling from “100 Campus Drive, Stanford” to “700 Meder Street, Santa Cruz” is 470908 per hour.

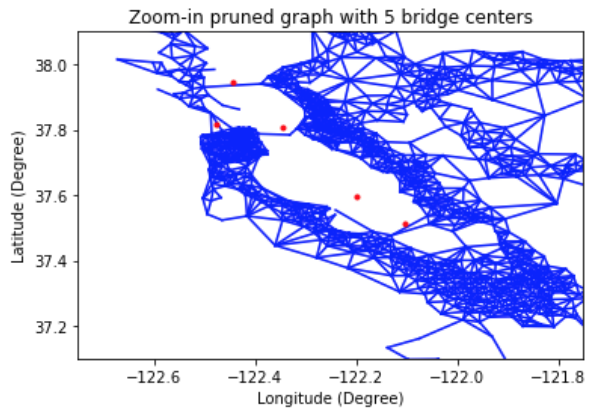
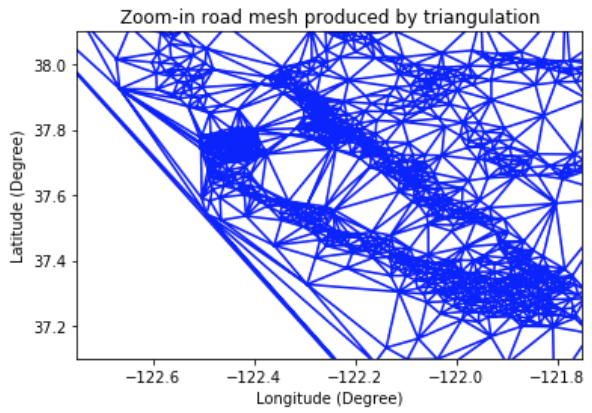
The number of edge-disjoint paths equals the number of edges that have to be removed in order to disconnect the two vertices. The number of edge-disjoint paths between the source and the destination is **6**.

Edge-disjoint paths are the maximum number of paths that don’t have common edges. The number of edge-disjoint paths problem is one of the variants of the maxflow problem. In the edge-disjoint paths problem, each edge has a capacity of 1. And the maxflow between the source and the target nodes in this graph is k if and only if the number of edge-disjoint paths is k.

The results make sense, because from the zoom-in road mesh of source and the destination we can see that there are 6 neighbor nodes directly connected to Stanford node, but there are only 5 neighbor nodes directly connected to UCSC node. So the maximum number of paths that don’t have common edges should be a number that is <= 6.

Stanford neighbourhood (-122.18, 37.43) ID in Geo data = 2607	UCSC neighbourhood (-122.06, 36.97) ID in Geo data = 1968
	

Question 14

Pruned graph	Original Delaunay graph
	

Delaunay triangulation might generate some edges that are not in the graph. To prune the graph:
(1) Check all the edges of all the triangles generated by Delaunay to see if the edge is in the edgelist of the graph;
(2) If the edge is in the edgelist of the graph, then check if the edge mean_travel_time is lower than the set threshold 600;
(3) Remove the edge if it is not in the edgelist of the graph, or if the edge mean_travel_time is lower than the set threshold.

For the pruned graph now, all the edges are “real road” that is they are in the edgelist of the graph, also all of their mean_travel_time is within 600 seconds (10 min).

As for the bridges, from the pruned graph, we can see that all the bridges except for the “San Mateo Bridge” are treated as fake bridge and removed, because it is the longest bridge of the five bridges listed in the question leading to longer mean travel time.

Question 15

We repeat Question 13 with the pruned graph produced in Question 14, that is (1) calculate the flow of each edge of the pruned graph, (2) get the maxflow and the number of edge-disjoint paths between Stanford to UCSC.

The maxflow value of the pruned graph is **21363 cars/(road.hour)**. It means that in the ideal case (no traffic jam), the maximum total number of cars that can commute on the road travelling from “100 Campus Drive, Stanford” to “700 Meder Street, Santa Cruz” is 21363 per hour. And the number of edge-disjoint path remains **6**.

The maxflow value of the pruned graph is lower because the threshold is relatively low, leading to a lot of road are treated as “fake”, according to the Ford-Fulkerson algorithm for the calculation of the maxflow, since there are less road that could be used to travel from Stanford to UCSC, the number of cars that can commute between these two nodes will decrease.

From the zoom-in road mesh of source and the destination we can see that there are 6 neighbor nodes directly connected to Stanford node, but there are only 4 neighbor nodes directly connected to UCSC node. The decreased number of neighbor nodes is due to the edge trimming process. Edge-disjoint paths are the maximum number of paths that don’t have common edges. The results make sense, because from the zoom-in road mesh of source and the destination we can see that there are 6 neighbor nodes directly connected to Stanford node, but there are only 4 neighbor nodes directly connected to UCSC node. So the maximum number of paths that don’t have common edges should be a number that is <= 6.

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