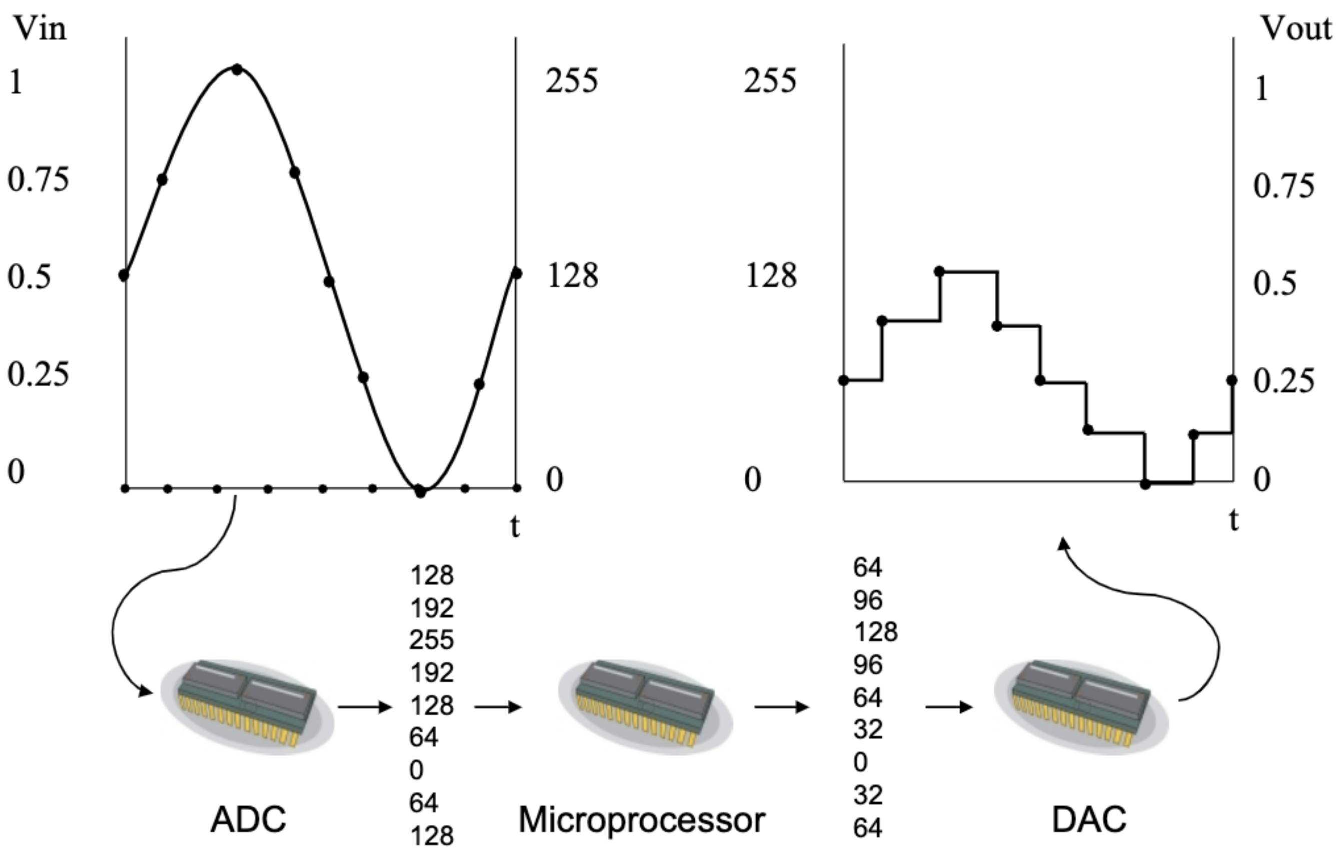
I: Digital Processing and Binary Arithmetic

1: Digital Processing

1.1: Analogue and Digital Signal

- 

- The microprocessor can convert the **Analogue Signal** to **Digital Signal**.

1.1.1: The Analogue Signal

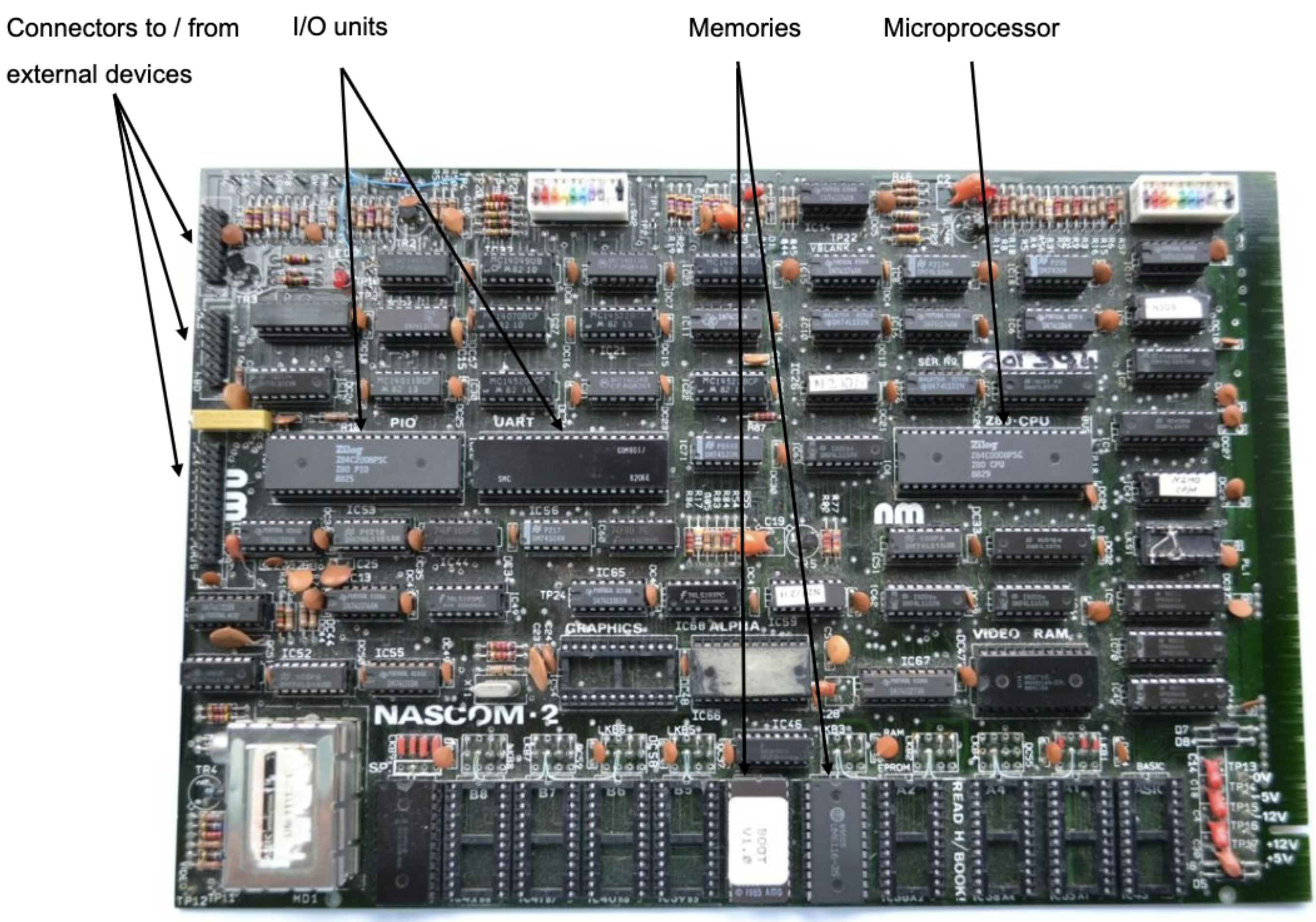
- **Analogue signal** means the signal which can form **continuous function** of **time**.
- The voltage, current,displacement and such physical quantities are **Analogue Signal**.

1.1.2: The Digital Signal

- It is the digital from of a sequence of of **discrete** values.

- **Discrete** means not continuous, which only have values at samples.
- In most digital circuits, the signal only have two values which is called $binarysignal$ or $logicsignal$.

1.2: The construction of processor

- 

2: Binary Arithmetic

2.1: Binary Addition

- $1 + 1 = 10$
- $1 + 1 + 1 = 11$

2.2: Binary-Decimal Conversion

- Binary to Decimal:
    -

1	0	1	1	0	1	0	1
1×2^7	0×2^6	1×2^5	1×2^4	0×2^3	1×2^2	0×2^1	1×2^0
1x128	0x64	1x32	1x16	0x8	1x4	0x2	1x1

which added together give 181 (decimal)

- Decimal to Binary:
  - Use the short-division.

3: Hexadecimal Arithmetic

3.1: The reason why we use Hexadecimal

- The expression of binary numbers is too long to use.
- 4 digits of Binary = 1 digit of Hexadecimal

3.2: Binary-Hexadecimal conversion

Binary	Hexadecimal
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

4: Negative numbers and Subtraction

4.1: The Expression of Subtraction

- It is difficult to do the subtraction, so we use the way of **complementing** to do the subtraction and minus number.
- For the binary, we use the **2's complement arithmetic**.
- The substation **a-b** can be expressed as **a+(the complement of b)+1**.

- For the example of decimal, such as $215 - 145$, we can use the compliment of 145, which is $999 - 145 + 1 = 855$ (No carried number). Then the result will be $215 + 855 = 1070$, then if we **omit the carry**, it will be 070.
- For the example of binary, we can simply **invert** the number then **plus 1**, and we should **ignore the carry out of the highest digit**.
- Such as $01101100 - 00101101$, we can first change 00101101 to 11010011. Then the result will be $01101100 + 11010011 = 001111111$, which ignore the carry out of the highest digit.

4.2: The way processing minus numbers in computers

- In computer, we use the **2's complement** to express a number 's minus value.
- The minus number will be marked as **signed number** while the positive number will be **unsigned**.
- For example, in binary, if the number is marked as unsigned, 10010001 will means 145.
- However, if the value is marked as signed, 10010001 will means -111.

II: Combinational Logic: Introduction

1: The Boolean Operation: AND

- If we A+B in binary and 'C' means carry out, 'S' means sum in the digit.
-

A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

- From the 'C' result of the binary truth table, if '0' means 'false' and '1' means 'true', we can define the operator **'AND'**.
- **AND** can be expressed as $C = A.B$.
- Only if both input A and B are '1', the output C = 1.
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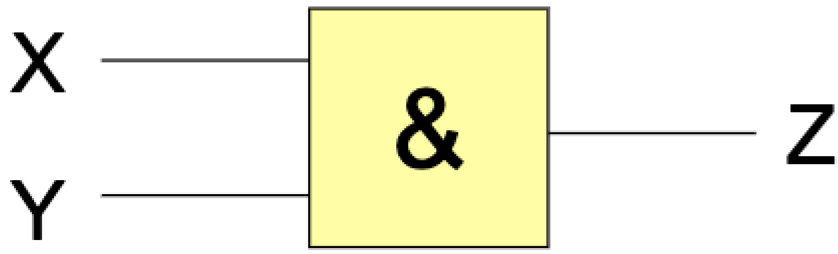

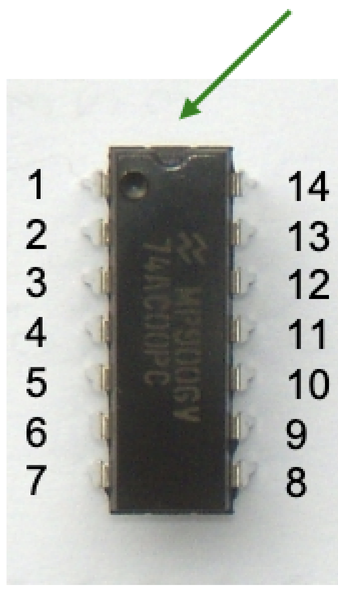

- Logic 1 can also means VCC and logic 0 means 0 V (the ground).

2: The Boolean Operation: OR

- The operation **OR** is expressed as Z=X+Y, if there is one or two '1' in the input, the input will be '1'.
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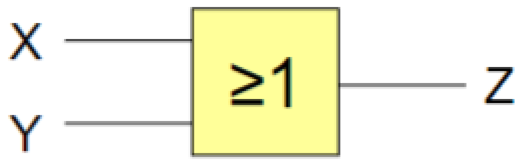
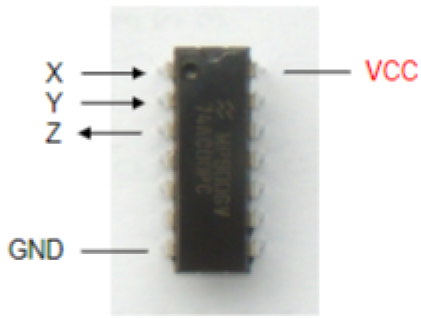
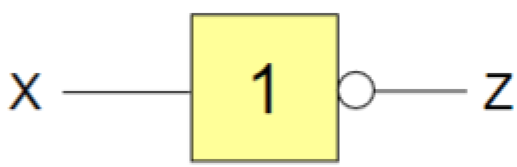
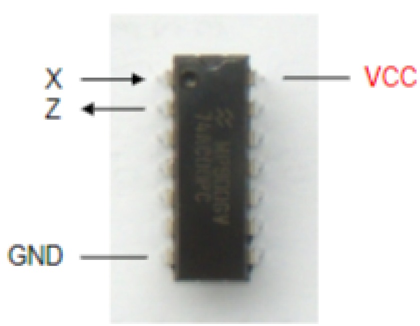




3: The Boolean Operation: NOT

- It can invert the input '1' to '0' and '0' to '1'.
- 



4: The Boolean Operation: OR-EXCLUSIVE

- Only if the input is '1' and '0', not all '1', then the output will be '1'.
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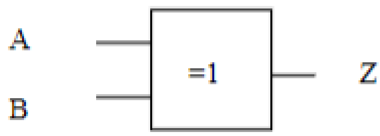
$Z = A \oplus B$



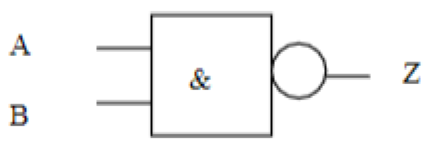
A	B	Z
0	0	0
0	1	1
1	0	1
1	1	0

- This operator can give the one-digit sum of the input.
- The only difference between EXCLUSIVE-OR and AND is the '1' '1' condition.

5: The Boolean Operation: NAND

- NAND

$Z = \overline{A . B}$

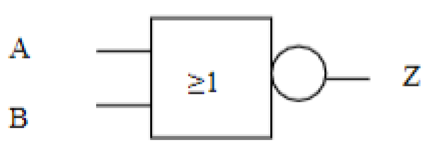


A	B	Z
0	0	1
0	1	1
1	0	1
1	1	0

- It is the invert of the AND.

6: The Boolean Operation: NOR

- $Z = \overline{A + B}$

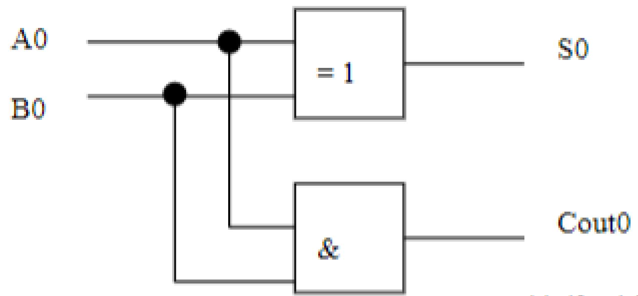


A	B	Z
0	0	1
0	1	0
1	0	0
1	1	0

- The inverse of OR.
- Only one of the input is '1' then the output will be '0'.

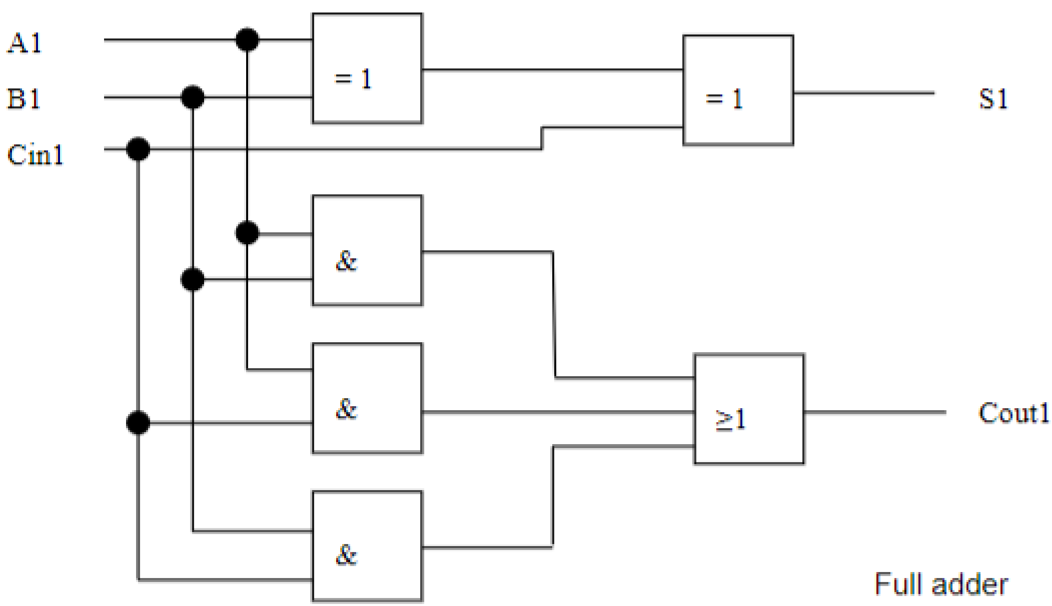
7: The half adder and full adder

7.1: The Half adder

- The half adder use EX-OR to produce sum and another AND to produce the carry out.
- 

Half adder

7.2: The Full adder

- In addition of the half adder, the full adder can accept the carry out from the previous digit.
- The EX-OR can be used to process the sum in one digit.
- Three AND gates are used to justify whether there is a **carry out** during the calculation.
- The final OR gates can be used to analysis the result of the AND gates. IF one of them is '1', it will produce a '1' as the carry out to $C_{out}1$.
- 

Full adder

III: Combinational Boolean Algebra

1: The way to simplify the gates

- Truth tables can be used to simplify the gates.
- The second method to simplify is to write the **Boolean Expression**. Such as $Z = XY + \bar{W}$

2: The Boolean Algebra

2.1: Distribution Theorem

- $A.(B+C) = A.B + A.C$
- $A + (B.C) = (A+B).(A+C)$

2.2: Complement Theorem

- $A + \bar{A} = 1$
- $A.\bar{A} = 0$

2.3: Redundancy Theorem

- $A.B + A = A$
- $A.(A+B) = A$

2.4: De Morgan's Law

- $A \bar{+} B = \bar{A}.\bar{B}$
- $A\bar{.}B = \bar{A} + \bar{B}$
- It is noted that the operation $A \bar{+} B$ means the invert of both **A,B** and **the OR operation**.
- For example, the expression $X = (C.D\bar{)} + E$, can be simplified to $X = (C\bar{.}D).\bar{E}$, then will be $X = (\bar{C} + \bar{D}).\bar{E}$

2.5: Commutation Law

- $A + B = B + A$
- $A.B = B.A$

2.6: Association Law

- $(A+B) + C = A + B + C = A + B + C$
- $(A.B).C = A.(B.C) = A.B.C$
- **Noted** that it does not apply when a expression contain both AND and OR, such as $(A.B) + C \not= A.(B+C)$

2.7: Idempotency Law

- $A + A = A$
- $A.A = A$
- Idempotency means the multiple manipulations have same effect as the first manipulation.