IV: Strain transformation, Principal strain and strain measurement

1: Plane Strain

- Same as the plane stress, ε_{xx},ε_{yy}, γ_{xy} may not ne non-zero, while other strain are zero.

2: Plane strain transformation equation

- For normal strain:

$$\varepsilon_\theta = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\cos(2\theta) + \frac{\gamma_{xy}}{2}\sin(2\theta)$$

- For shear strain:

$$\frac{\gamma_\theta}{2} = -\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\sin(2\theta) + \frac{\gamma_{xy}}{2}\cos(2\theta)$$

- The angle θ is positive in the A/C direction from the horizontal.

3: Principal strain

3.1: The max and min strain

- ε_1 and ε_2 act in the same direction as the principal stresses.
- ε_1 is oriented 90 degrees from ε_2.

3.2: Th formula of the principal strain

- The max and min of the principal strain can be find using the general formula.
- Direction of the strain:

$$\theta = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}$$

- The solution will be θ and $\theta + 90$.

- The max of the shear strain can also be defined as:

$$\gamma_{max} = \sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + (\gamma_{xy})^2}$$

or

$$\gamma_{max} = \varepsilon_2 - \varepsilon_2$$

4: Strain measurement

4.1: Using strain measurement to get the stress and strain

- It is common to get the value of stress using the value of strain cause the stress is hard to measure.
- It is easy to calculate the value of ε using the change of the resistance:

$$R_0 = \frac{\rho L}{A}$$

while the change of the resistance can be expressed as:

$$R_n = (\frac{L + \Delta L}{A - \Delta A})\rho$$

And the volume of the metal is constant:

$$V = L \times A = (L + \Delta L)(A - \Delta A)$$

If we treat $\Delta R = R_n - R_0$:
$\Delta R \approx 2R_0 \frac{\Delta L}{L} = 2R_0\varepsilon$

- A more general form is :
$\Delta R \approx cR_0\varepsilon$,c is a gauge factor that varied with the resistivity and Passion's ratio.(2 for metal and 10 fro carbon)

4.2: The way to detect the tiny change in strain

- In order to account for the tiny change in resistance due to the change of strain, we apply the **Wheatstone bridge** (a kind of electrical bridge):

$$R_U = (\frac{R_2}{R_1} R_v)$$

4.3: The Strain gauge rosettes

- The strain gauge rosettes is the multiple gauges which could measure the strain in different directions, which have 45,60 and 120 degrees in common.
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- For a 45 degrees gauge, $\varepsilon_{xx} = \varepsilon_A$, $\varepsilon_{yy} = \varepsilon_C$ and $\varepsilon_{45} = \varepsilon_B$, the τ_{xy} can also be got using these.