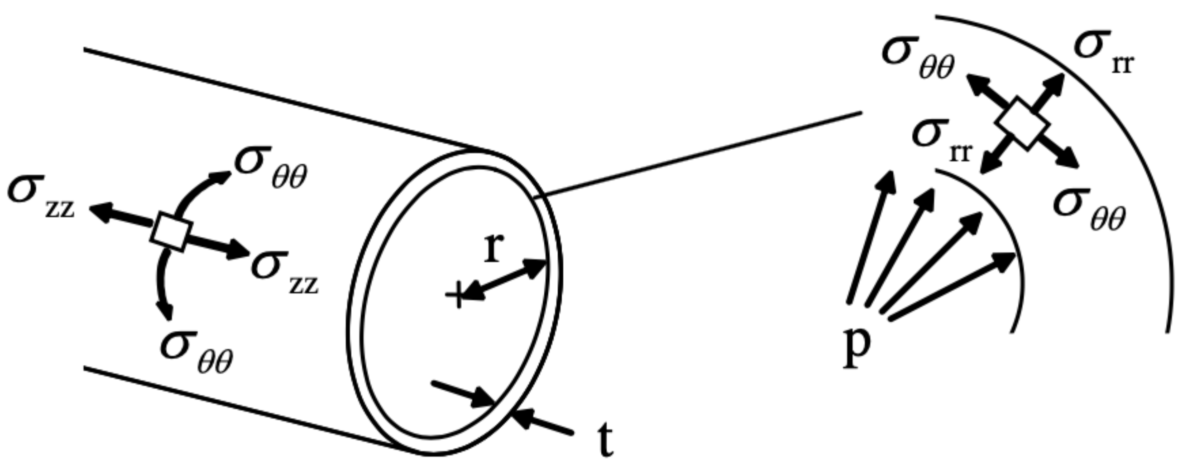
V: Thin-walled pressure vessels

1: Introduction

- Thin-walled pressure vessels can be defined as closed structures containing fluids.
- The term thin-walled refers to the radius being greater than ten times that of the wall thickness.

2: Thin-walled cylindrical pressure vessels under internal pressure

- For a thin-walled cylindrical pressure vessel of radius r and wall thickness t subjected to a uniform internal pressure p.
- Three normal stresses arise:
  - Hoop stress, $\sigma_{\theta\theta}$
  - Axial stress, σ_{zz}
  - Radial Stress, σ_{rr}
  - 

- Hoop Stress:
  - Consider the half od the vessel with unit length in z direction, we only consider the horizontal component of the internal pressure force (have effect to the wall) and p also have vertical components.
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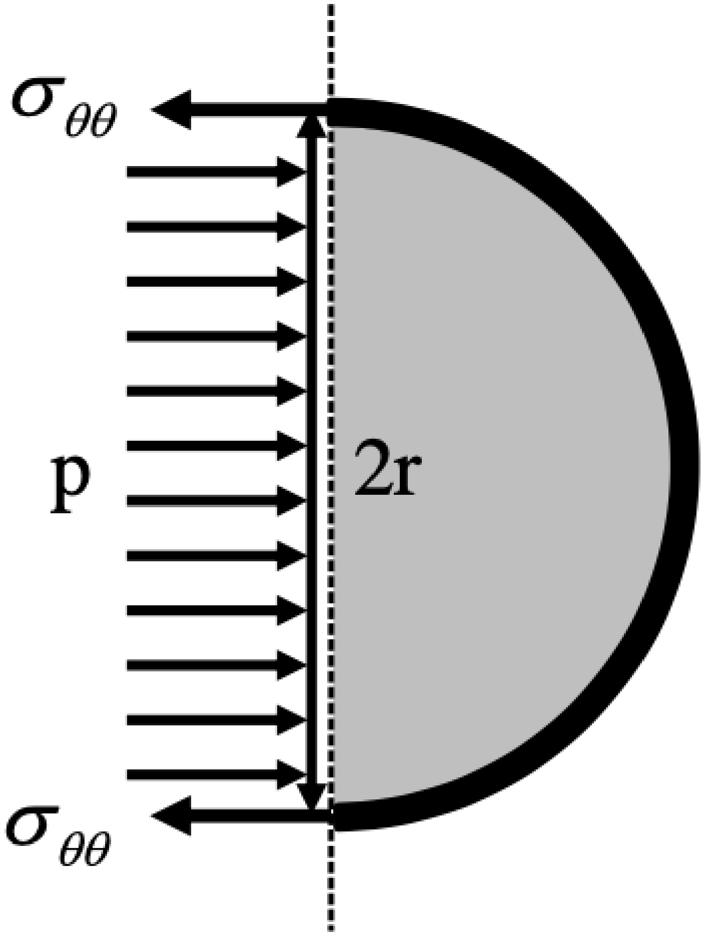

- $(\sigma_{\theta\theta} \times (t \times 1)) + (\sigma_{\theta\theta} \times (t \times 1) = p \times (2r \times 1)$

$$\sigma_{\theta\theta} = \frac{pr}{t}$$

- Axial stress:
  - Now consider the equilibrium in the horizontal direction.
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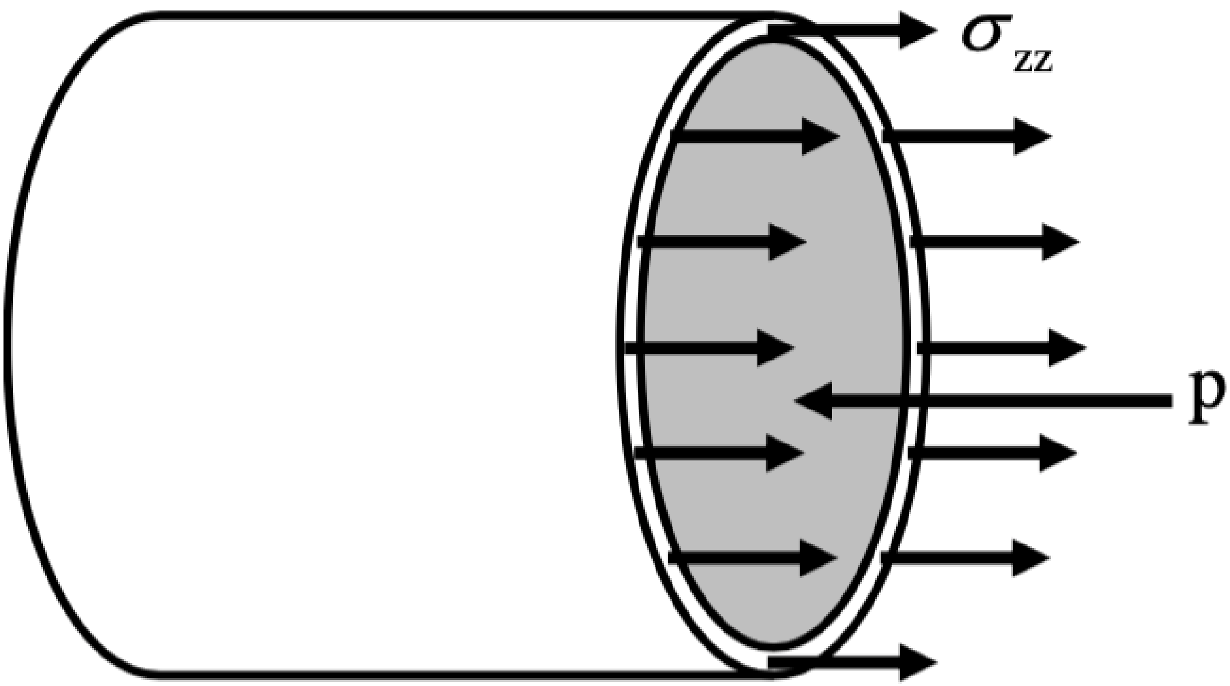


- $\sigma_{zz} = (2\pi r \times t) = p \times \pi r^2$

$$\sigma_{zz} = \frac{pr}{2t}$$

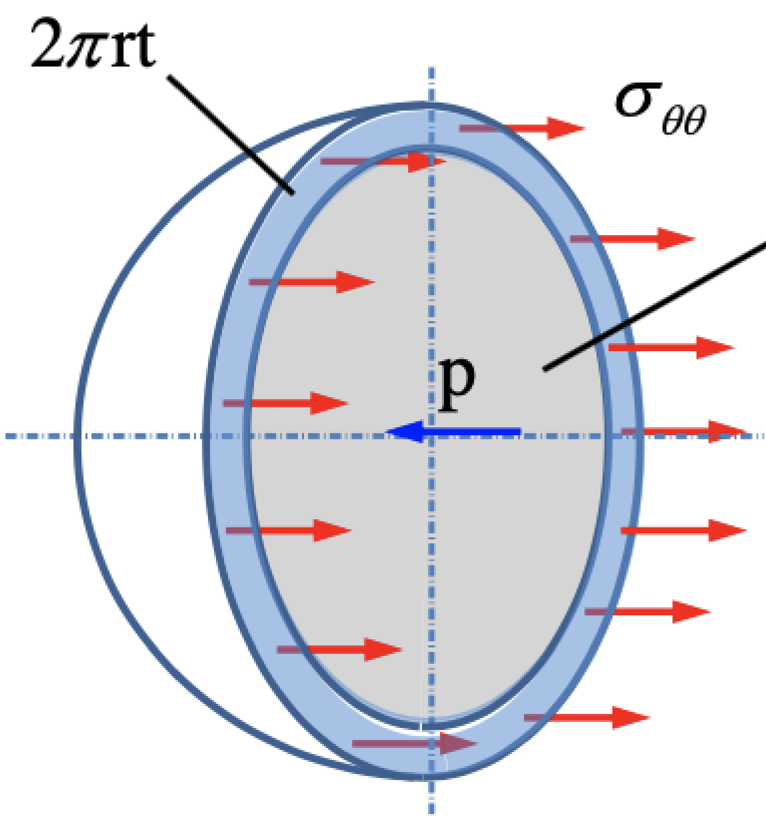
- $\sigma_{\theta\theta} = 2\sigma_{zz}$
- Radial stress:
  - The stress in radial direction σ_{rr}.
  - Varies from $-p$ in the inner surface to 0 in the outer surface.
- Comparison of stresses:
  $\sigma_{\theta\theta} > \sigma_{zz} > \sigma_{rr} \approx 0$
- Principal stresses:
  - The hoop stress is the max principal stress: $\sigma_{\theta\theta}\sigma_1$
  - The axial stress is the min principal stress: $\sigma_{zz} = \sigma_2$
  - The radial stress is the third stress: $\sigma_{rr} = \sigma_3 = 0$

3: Stress-strain relations

- As we know: $\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$
- $\varepsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1$
- Substitute $\sigma_{\theta\theta} = \sigma_1$ and $\sigma_{zz} = \sigma_2$, we can find the relation of thin-wall condition.

- The change in diameter of cylinder, with original diameter D is: $\Delta D = D\varepsilon_{\theta\theta}$
- The change in length of the cylinder, with original length L, is $\Delta L = L\varepsilon_{zz}$
- Use the **partial differential** : $\delta z \approx \frac{\partial f}{\partial x}\delta x + \frac{\partial f}{\partial y}\delta y$
- We can find $\Delta V = \frac{\pi D^2 L}{4}(2\varepsilon_{\theta\theta} + \varepsilon_{zz})$

4: Thin-walled spherical pressure vessels under internal pressure

- 

- The equilibrium in the horizontal :
$\sigma_{\theta\theta} \times 2\pi r t = p \times \pi r^2$, so $\sigma_{\theta\theta} = \frac{pr}{2t}$
- In thin-walled spherical pressure vessels, the stress is same in all directions.
- On the outer surface, every plane and direction are principal, thus, $\sigma_1 = \sigma_2 = \frac{pr}{2t}$ and $\sigma_3 = 0$
- Since the σ_1 and σ_2 are same sign, we can find that :
$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{pr}{4t}$
- Note that all the shear stress are zero in-plane.
- On the inner surface of the spherical shell, the principal stresses are:
$\sigma_1 = \sigma_2 = \frac{pr}{2t}$ and $\sigma_3 = -p$

- The max shear stress which is out of plane:
$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{pr}{4t} + \frac{p}{2}$$
- Since r/t is large, we can consider that the max shear stress inner is same as outer.