V The parallel plate capacitor

1.Capacitance

- $$C(V) = \frac{dQ}{dV}$$
- The units is Farad(F), which is equal to C/V
- $V = 0, C = 0$
- It has a linear relationship
- Other systems have non-linear reactions

2.The E field of the capacitor

2.1 Pairs of planes of charge

$$\vec{D} = \frac{\sigma}{2} \uparrow + \frac{\sigma}{2} \downarrow = 0$$

$$\vec{D} = \frac{\sigma}{2} \downarrow + \frac{\sigma}{2} \downarrow = \sigma \downarrow$$

2.2 $\vec{E}, \vec{D}$ for a paralell plate capacitor

$\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow$

The uniform fields between the planes from the positive to negative :
$|\vec{D}| = \sigma$ and
$|\vec{E}| = \sigma / \varepsilon$

2.3 Approximation

- Real parallel plate capacitor are not infinite
- The distance between the plates is small compared to te diameter.
- $|\vec{D}| \approx \sigma$ inside the capacitor.
  *$|\vec{D}|$is dipolar at the edges.

3. The Potential

3.1 Potential difference for a PPC

- Charged parallel plates
- Approximate the $\vec{E}$ as uniform
- As we have defined before,

$$\Delta\phi = \vec{E}\vec{r}$$

- Here the $\vec{r}$ is the displacement vector between the two plates, length w

$$V = \Delta\phi = |\vec{E}|w$$

  ($\vec{r}$ can both denote the direction and express the magnitude)

3.2 Capacitance of a PPC

- We can find a formula to determine the capacitance :

$$V = \Delta\phi = Ew$$

$$\Delta\phi = \frac{\sigma}{\epsilon}w = \frac{Qw}{\epsilon A}$$

$$C = \frac{Q}{V} = \frac{Q}{\Delta\phi}$$

$$C = \frac{\epsilon A}{w}$$

- So the C only depends on geometry and what is between the plates: $\epsilon = \epsilon_r\epsilon_0$

3.3 The stored energy

3.3.1 Moving electrons from one electrode to the other

- No charge at first
- The $\vec{E}$ field then occurs.
- The next electron moves in the field.
- $\vec{F} = -e\vec{E}$
- The cost energy :

$$|\Delta U| = e\Delta\phi = eV = e\frac{Q}{C}$$

- The cost depends on Q !
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$$\Delta U = \Delta\phi dQ = VdQ = \frac{Q}{C}dQ$$

- The total energy to charge is:

$$\int_0^{Q_{max}} \frac{Q}{C}dQ = \frac{Q_{max}^2}{2C} = Stored \quad Energy$$

$$Q = CV$$