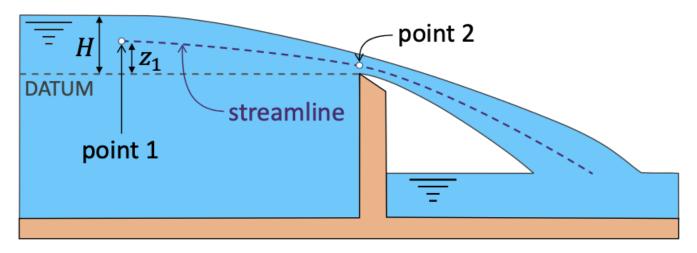
XI: Flow Measurement: Weirs and Orifices

1: Weirs

1.1: Sharp-Crested Weir



- Pressure head at point 1 = $H z_1$, and point 2 is 0.
- According to the Bernoulli equation: $z_1+(H-z_1)+rac{u_1^2}{2a}=z_2+0+rac{u_2^2}{2a}$
- $egin{aligned} ullet u_2 &= \sqrt{2g(H-z_2) + u_1^2} \ ullet \delta Q &= b \delta z \sqrt{2g(H-z) + u_1^2} \end{aligned}$
- Then we can do the integration to find the Q:

$$Q = b\sqrt{2g}\int_0^H{(H-z+rac{u_1^2}{2g})^{1/2}dz}$$

$$Q=rac{2}{3}c_db\sqrt{2g}((H+rac{u_1^2}{2g})^{3/2}-(rac{u_1^2}{2g})^{3/2})$$

- c_d is the coefficient of discharge, without c_d the Q is called the Q_{ideal} .
- If u_1 is very small, $Q=rac{2}{3}c_db\sqrt{2g}H^{3/2}$

1.2: The Example of Sharp-Crested Weirs

• Step 1:

Neglect u_1 term using $Q=rac{2}{3}c_db\sqrt{2g}H^{3/2}$, which could find Q_1 .

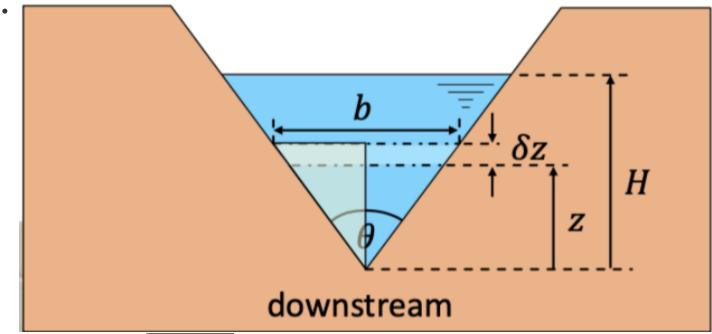
Step 2:

Using $u_1=rac{Q_1}{A}$ to find u_1 , then using the full formula to find Q_2 .

Step 3:

Using $u_2=rac{Q_2}{A}$ to find u_2 . then using the full formula to find Q_3 .

1.3: V North Weir



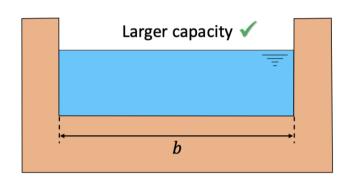
•
$$\delta Q_{ideal} = b \delta z \sqrt{2g(H-z)}$$

$$Q = \frac{8}{15}c_d\tan(\frac{\theta}{2})\sqrt{2g}H^{5/2} \text{, for triangle.}$$

$$Q = \frac{2}{3}c_db\sqrt{2g}H^{3/2} \text{, for rectangle.}$$

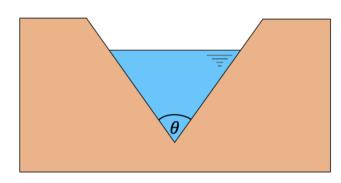
•
$$Q=rac{2}{3}c_db\sqrt{2g}H^{3/2}$$
 , for rectangle.

· Advantages:



$$Q = \frac{2}{3}c_d b \sqrt{2g} H^{3/2}$$

Nappe shape varies with $h \times c_d$ varies with $h \times c_d$

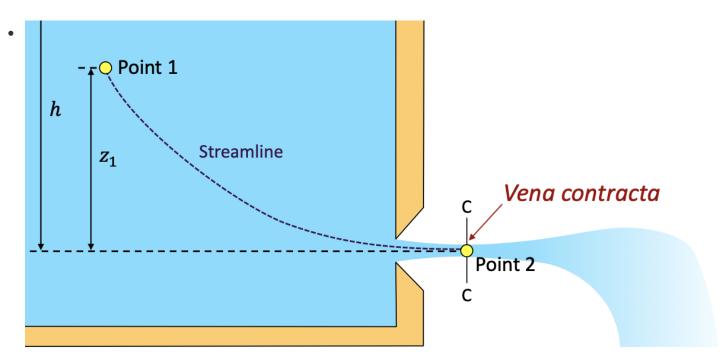


$$Q = \frac{8}{15} c_d \tan\left(\frac{\theta}{2}\right) \sqrt{2g} H^{5/2}$$

Nappe shape is constant \checkmark c_d is more constant with h \checkmark More accurate at low Q

2: Orifices

2.1: Small Orifice



Bernoulli Equation:

$$z_1 + rac{p_1}{
ho g} + rac{u_1^2}{2g} = 0 + rac{p_2}{
ho g} + rac{u_2^2}{2g}$$

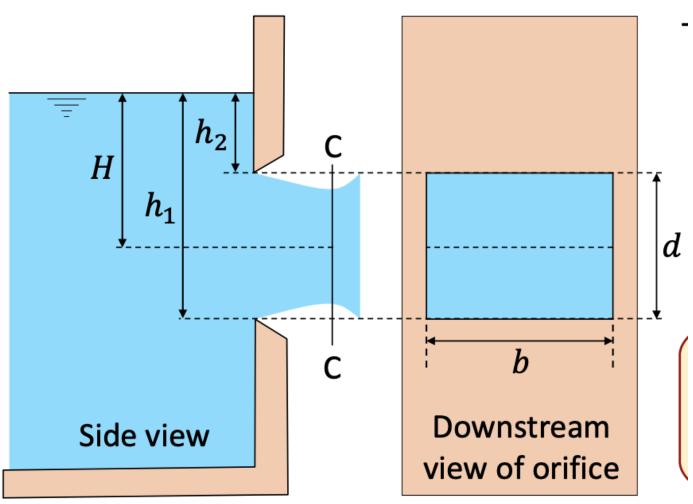
• Cause $\frac{u_1^2}{2g}$ =0, $\frac{p_2}{\rho g}$ =0, we can find:

$$h=rac{u_2^2}{2q}$$

- I.e, $u_2^2=\sqrt{2gh}$
- $Q=c_cAu_2=c_cA\sqrt{2gh},\,c_c=$ area of vena contracta/ area of orifice.

2.2: Large orifice

•



- $\delta Q_{ideal} = b \delta z \sqrt{2g(H-z)}$
- Then we can find that:

$$Q=rac{2}{3}c_db\sqrt{2g}((H+rac{d}{2})^{3/2}-(H-rac{d}{2})^{3/2})$$