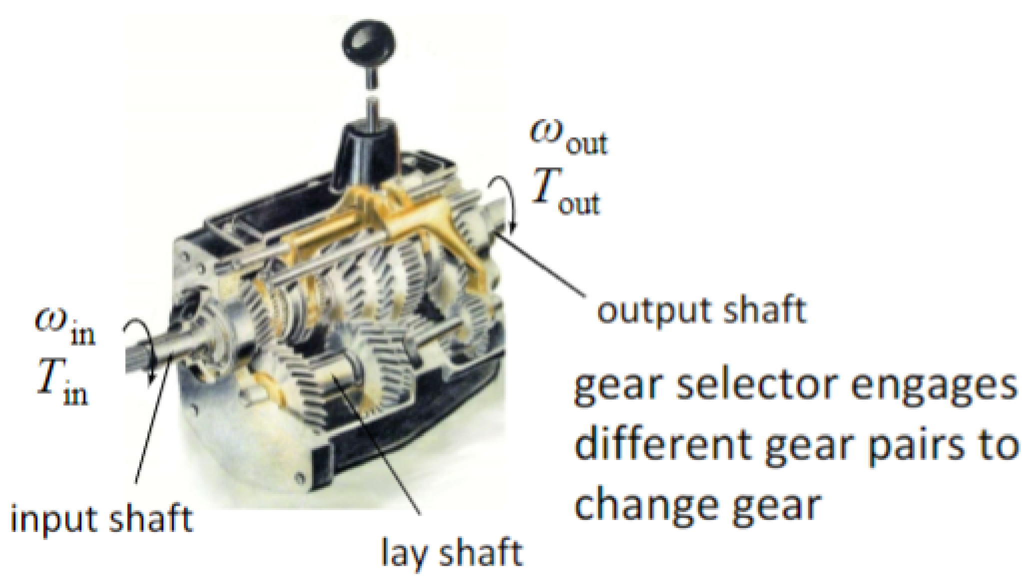
VII: Power and Speeds

1: Introduction

- Power: rate of energy flow , rate of doing work
- SI units: J/S or W
- Various forms:
  - Mechanical_linear: Fv or $F_x \frac{dx}{dt} = F_x \ddot{x}$
  - Mechanical_rotary: ωT or $T \frac{d\theta}{dt} = T\ddot{\theta}$
  - Fluid: $\dot{Q}\Delta P$
  - Electrical:VI

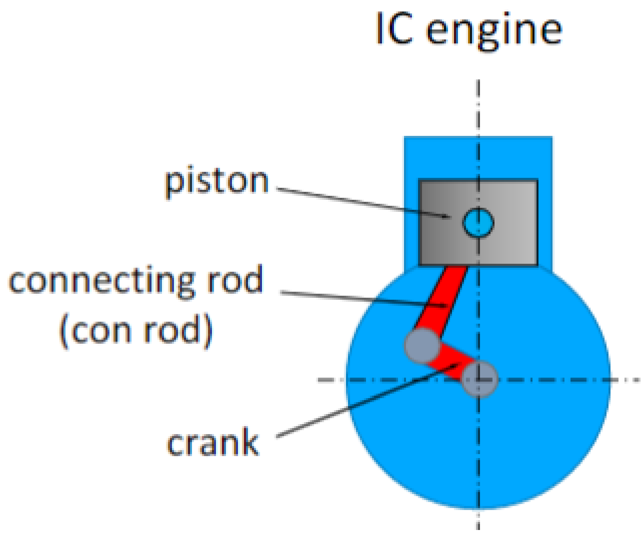
2: The use of gearbox



- According to conservation of energy, $\omega_{in} T_i n = \omega_{out} T_{out}$, if neglecting the losses.
- I.C engines cannot run at very high speed and cannot gain high torque at low speed.

3: Design machines to deliver high power

3.1: IC engine



- In order to deliver high power, we need higher speed or higher torque, so we need large forces and vibration.
- We use big pistons and big crank.

3.2: Electrical Motor

- We use small diameter rotor, which is compact and can be run at high speed.
- The pancake motor, large diameter and low speed.

VIII: Mechanical drives: Gears

1: Gear Kinematic

1.1: Introduction

- Modern gears evolved from ancient cogs, such as the cog in the windmill.
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windmill

- The problem is the rapid wear limits load and uneven speed.
- The modern way to solve these problems: rapid wear limits load, uneven running speed (involute tooth forms)

1.2: Speed ratio

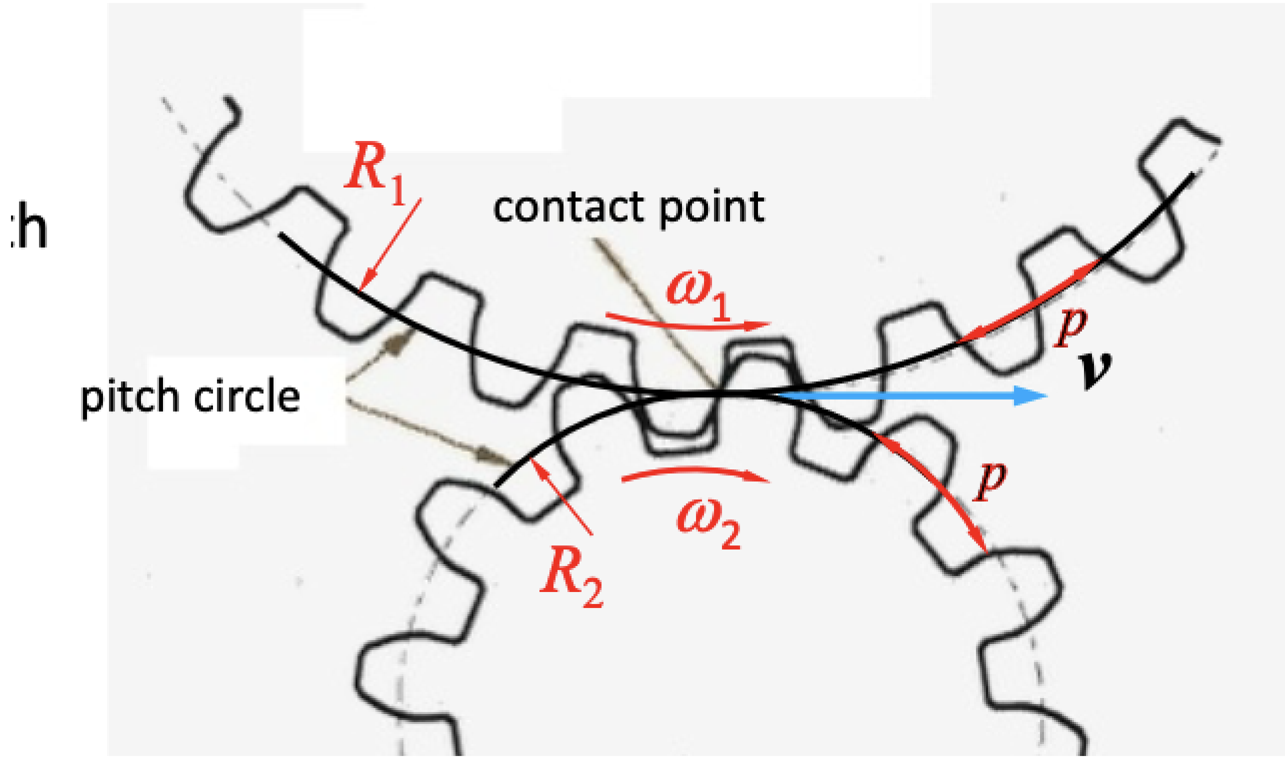
- Gears behave as perfect friction wheels.
- Tangential velocity is same for both gears, i.e.:(minus means different directions)

$$v = R_1\omega_1 = R_2(-\omega_2)$$

- So we can find the gear or speed ratio:

$$\frac{\omega_2}{\omega_1} = -\frac{R_1}{R_2}$$

- How to measure pitch circle radius R (or pitch diameter):



$$N_1 p = 2\pi R_1$$
$$N_2 p = 2\pi R_2$$
$$\frac{R_1}{R_2} = \frac{N_1}{N_2}$$

- So we can find the speed ratio just by the **teeth N**.

2: Types of Gears

2.1: Spur Gears

- Teeth cut parallel to the axis.
- Cheap and easy to make.
- Noisy.

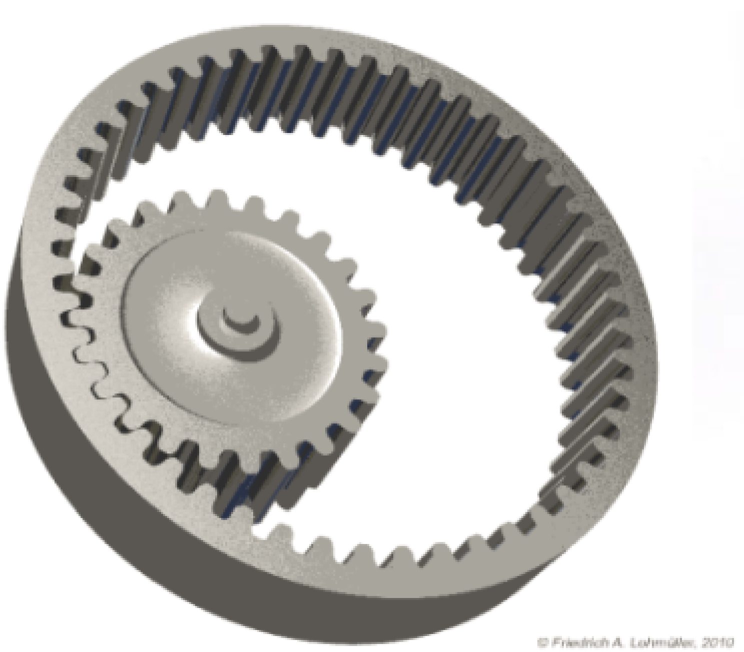
2.2: Helical Gears

- Teeth cut at an angle to the axis.
- Can still be made by hobbing.
- Quiet.
- Introduce to an axial force.

2.3: Double helical gears

- Very quiet.
- No axial force.
- Expansive.

2.4: Internal gears



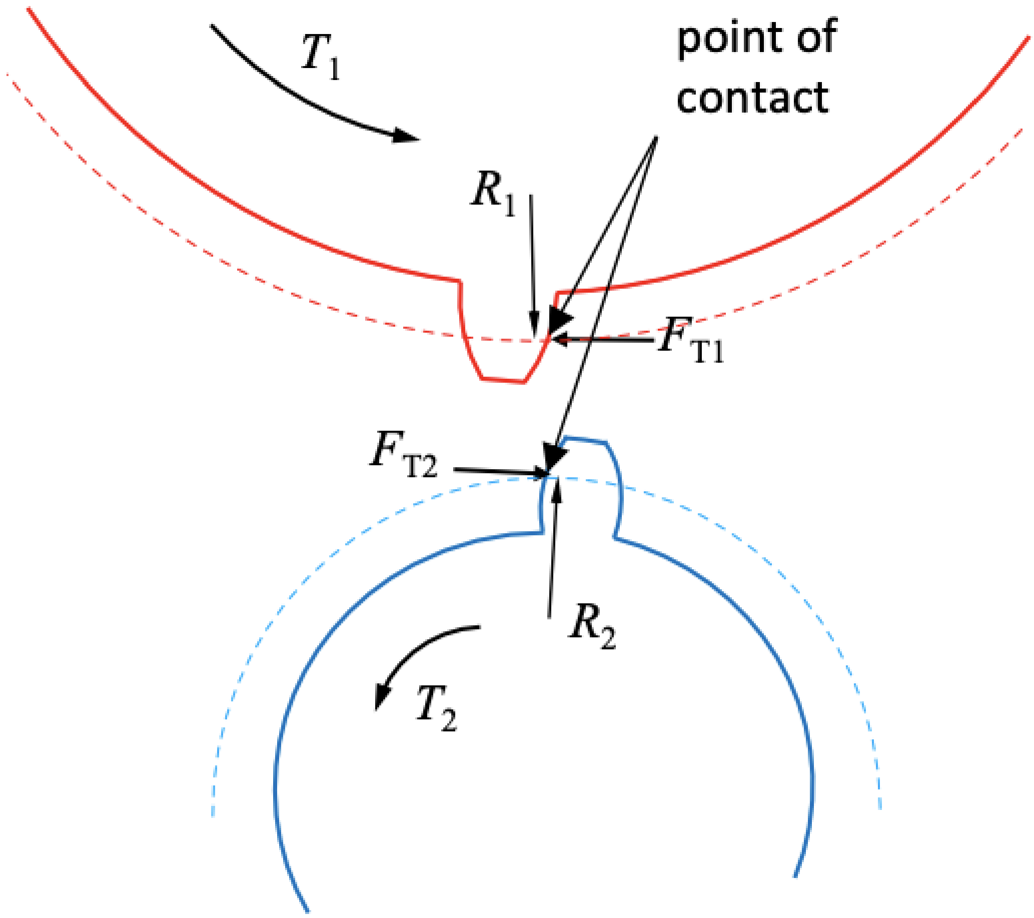
- Straight or helical.
- Difficult to make.

2.5: Bevel Gears

- Rotating transmission axis.
- Straight or helical.

3: Torque Transmission through gears

3.1: Consider only the pair of teeth in contact



- Tangential force F_{T1} and F_{T2} are equal but opposite.
- Taking moments about the gear centres:

$$
\begin{aligned}
T_1 - F_{T1}R_1 = 0 \\
T_2 - F_{T2}R_2 = 0
\end{aligned}
$$

- Then we get the Torque ratio:

$$\frac{T_2}{T_1} = \frac{R_2}{R_2} = \frac{N_2}{N_1}$$

- According to former equation, we can find that:

$$\omega_1 T_1 = \omega_2 T_2$$

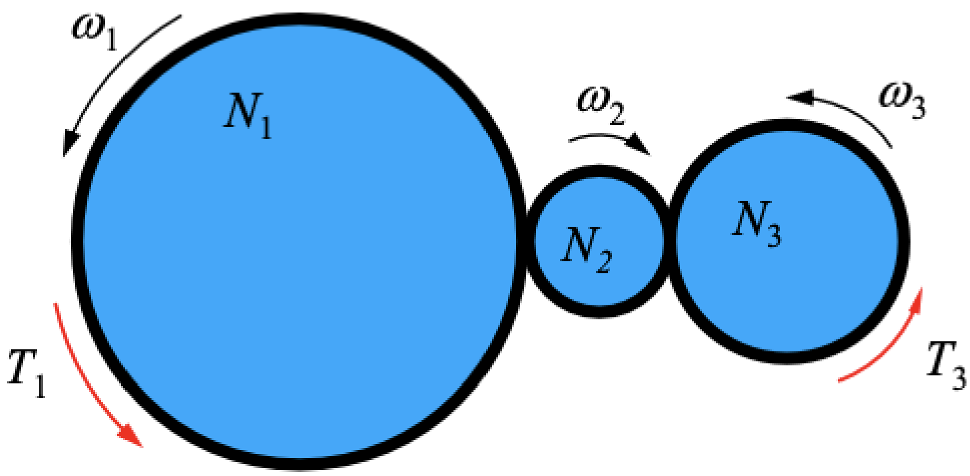
- Which means power in = power out.
- In practice, there is a small loss of power due to friction:

$$\omega_{in} T_{in} > \omega_{out} T_{out}$$

$$\frac{\omega_{in}}{\omega_{out}} = \frac{N_{out}}{N_{in}}$$
$$\frac{T_{out}}{T_{in}} = \eta \frac{N_{out}}{N_{in}}$$

- η is the efficiency.

3.2: Muti-stage Gears: Idler gears

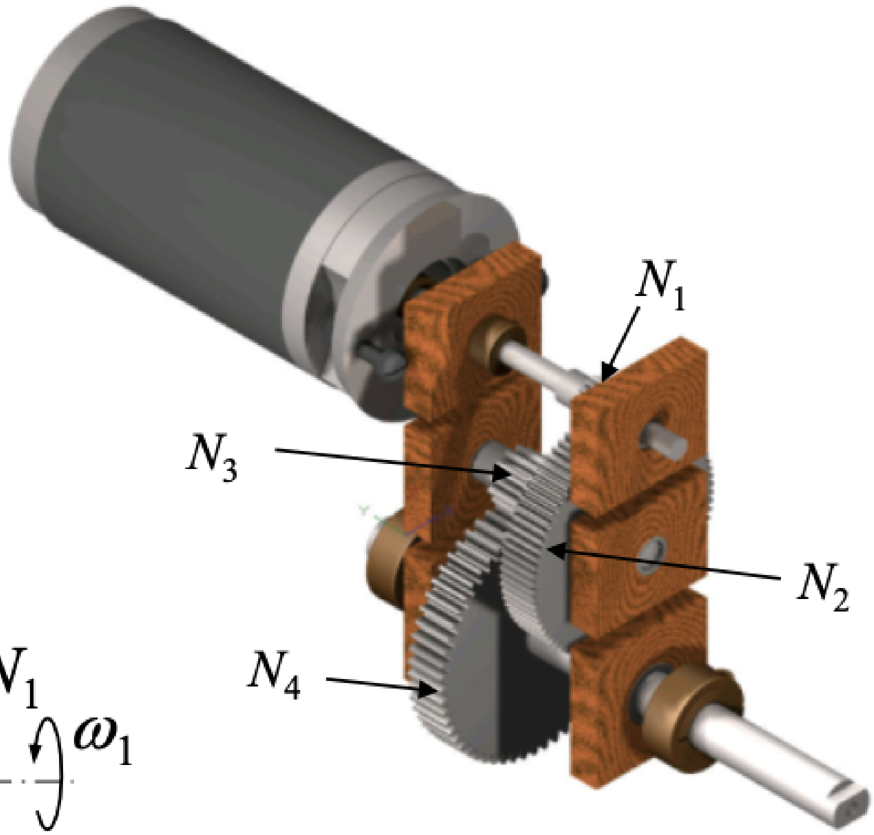


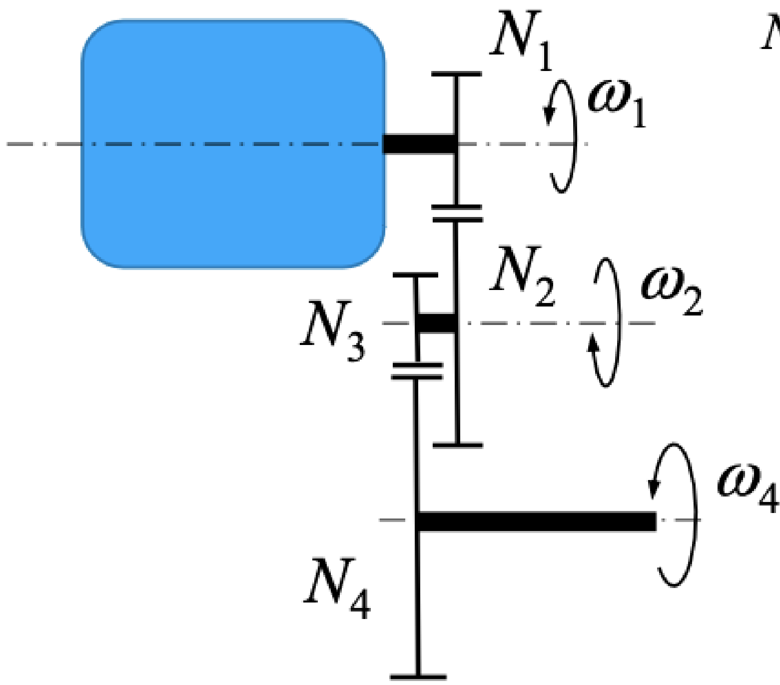
- It is found that:

$$\frac{\omega_3}{\omega_1} = \frac{N_1}{N_3}$$
$$\frac{T_3}{T_1} = \frac{N_3}{N_1}$$

- So the idler gear has no effect on speed or torque, **but is used to increase the distance between gear axes and to change direction of rotation.**
- In fact, it will increase the losses.

3.3: Multi-stage Gears: Double reduction gears





$$\frac{\omega_2}{\omega_1} = -\frac{N_1}{N_2}$$
$$\frac{\omega_3}{\omega_4} = \frac{\omega_4}{\omega_3} = \frac{\omega_4}{\omega_2}$$

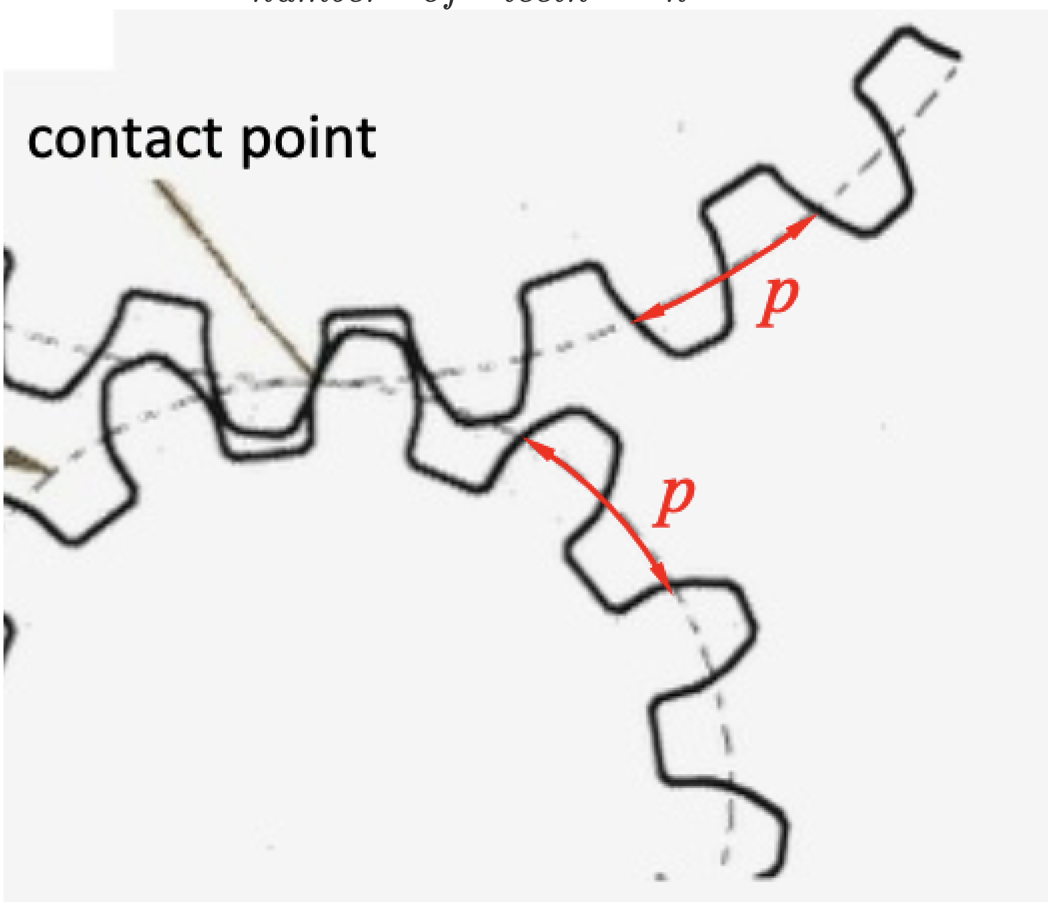
- So we can get:

$$\frac{\omega_4}{\omega_1} = \frac{N_1 N_3}{N_2 N_4}$$
$$\frac{T_4}{T_1} = \frac{N_2 N_4}{N_1 N_3}$$

- Space/weight saving
- Extra complexity
- Extra friction losses (two meshings, so two losses)

4: Gear dimensions

- Tooth length is important.
- Tooth size is defined by **pitch length p**, but it is difficult to measure.
- Define **module**= $\frac{pitch \quad diameter}{number \quad of \quad teeth} = \frac{D_p}{n}$.



- Pitch circle circumference = $\pi D_p = pn$
- So module=$\frac{p}{\pi}$
- By gears by module and number of teeth.
- Gears of the same module will mesh together.