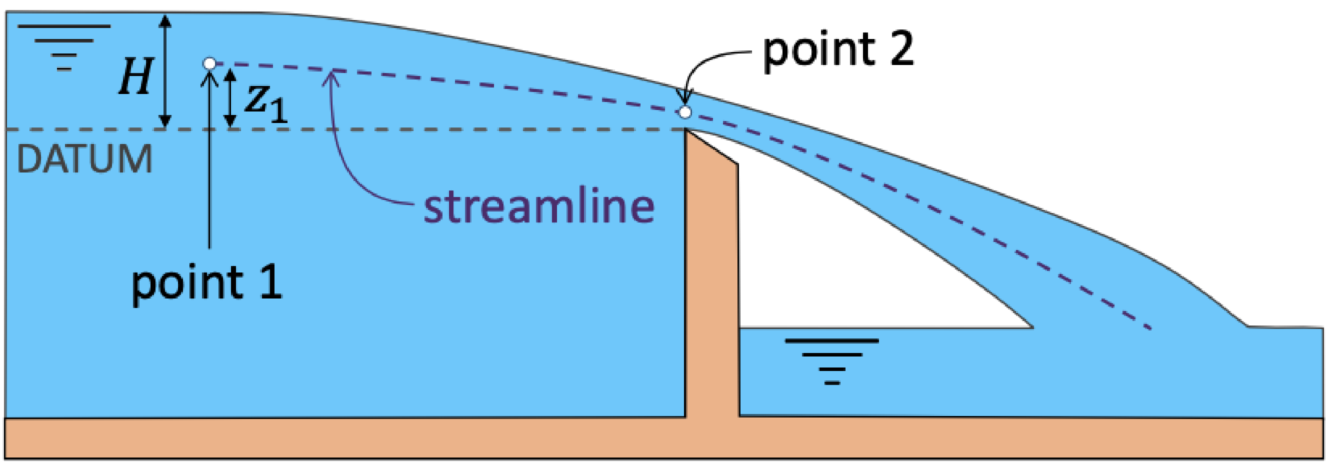
XI: Flow Measurement: Weirs and Orifices

1: Weirs

1.1: Sharp-Crested Weir

- 

- Pressure head at point 1 = $H - z_1$, and point 2 is 0.
- According to the Bernoulli equation: $z_1 + (H - z_1) + \frac{u_1^2}{2g} = z_2 + 0 + \frac{u_2^2}{2g}$
- $u_2 = \sqrt{2g(H - z_2) + u_1^2}$
- $\delta Q = b\delta z\sqrt{2g(H - z) + u_1^2}$
- Then we can do the integration to find the Q:

$$Q = b\sqrt{2g}\int_0^H (H - z + \frac{u_1^2}{2g})^{1/2}dz$$

$$Q = \frac{2}{3}c_d b\sqrt{2g}((H + \frac{u_1^2}{2g})^{3/2} - (\frac{u_1^2}{2g})^{3/2})$$

- c_d is the coefficient of discharge, without c_d the Q is called the Q_{ideal}.
- If u_1 is very small, $Q = \frac{2}{3}c_d b\sqrt{2g}H^{3/2}$

1.2: The Example of Sharp-Crested Weirs

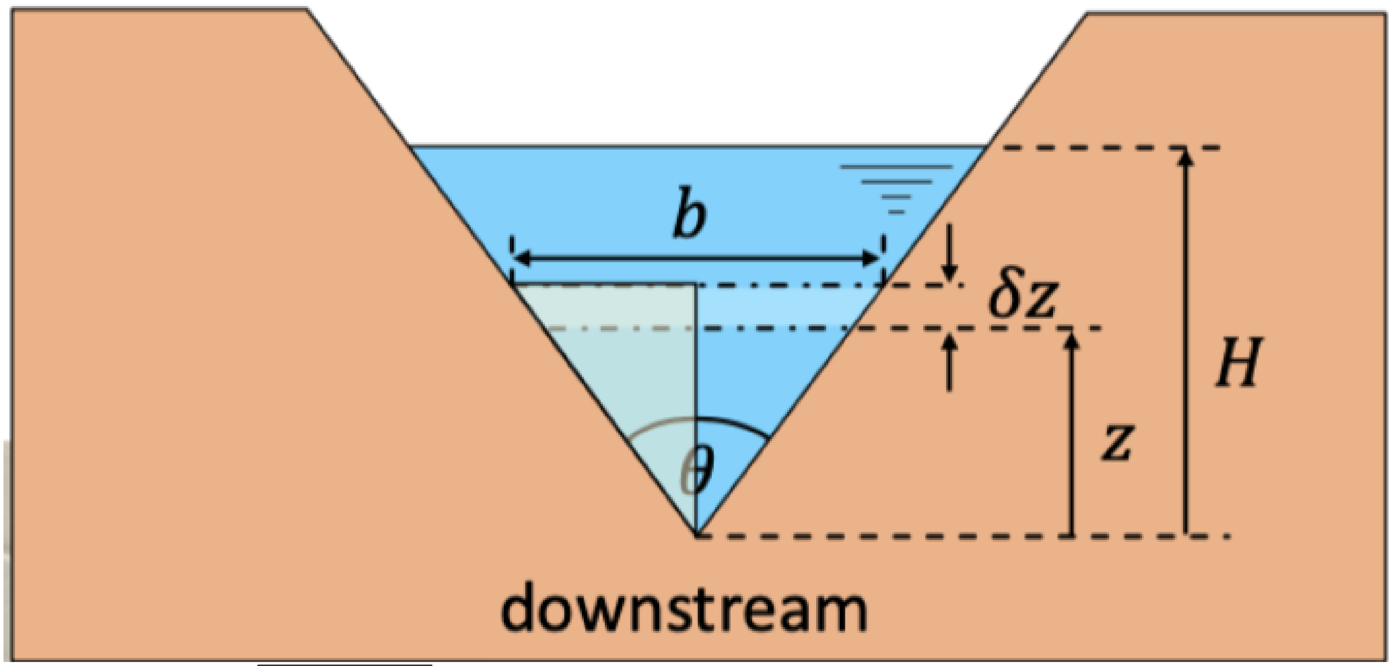
- **Step 1**:

  Neglect u_1 term using $Q = \frac{2}{3} c_d b \sqrt{2g} H^{3/2}$, which could find Q_1.
- **Step 2**:

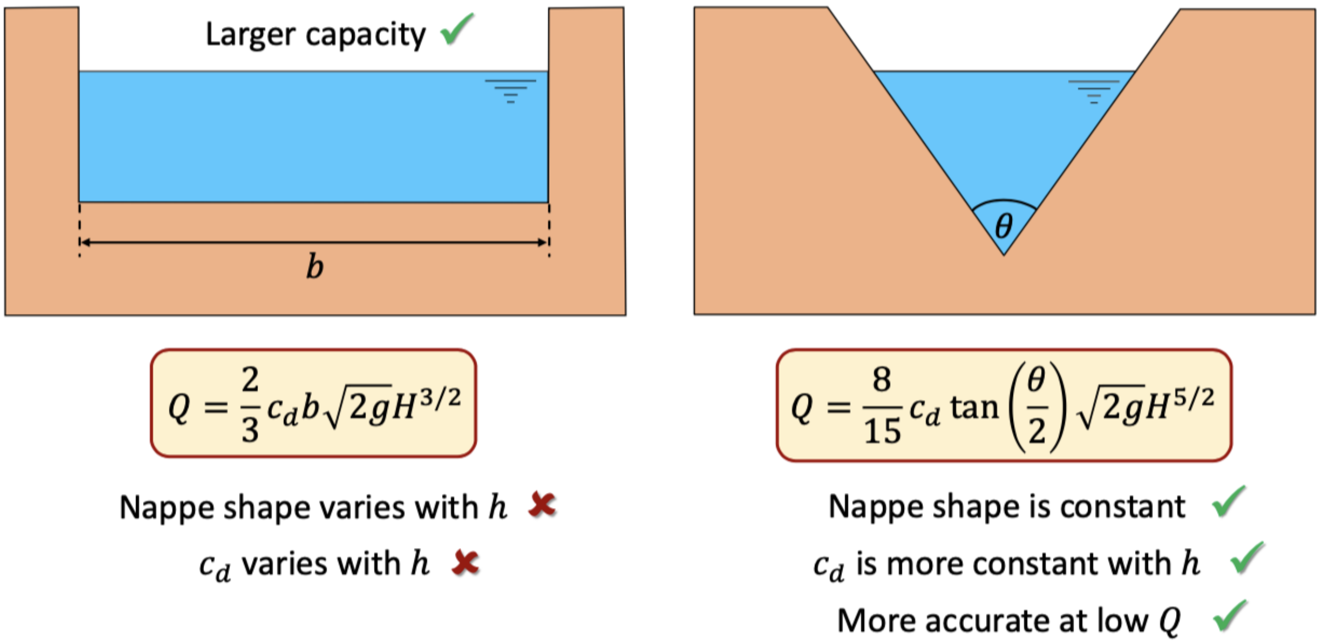
  Using $u_1 = \frac{Q_1}{A}$ to find u_1, then using the full formula to find Q_2.
- **Step 3**:

  Using $u_2 = \frac{Q_2}{A}$ to find u_2. then using the full formula to find Q_3.

1.3: V North Weir

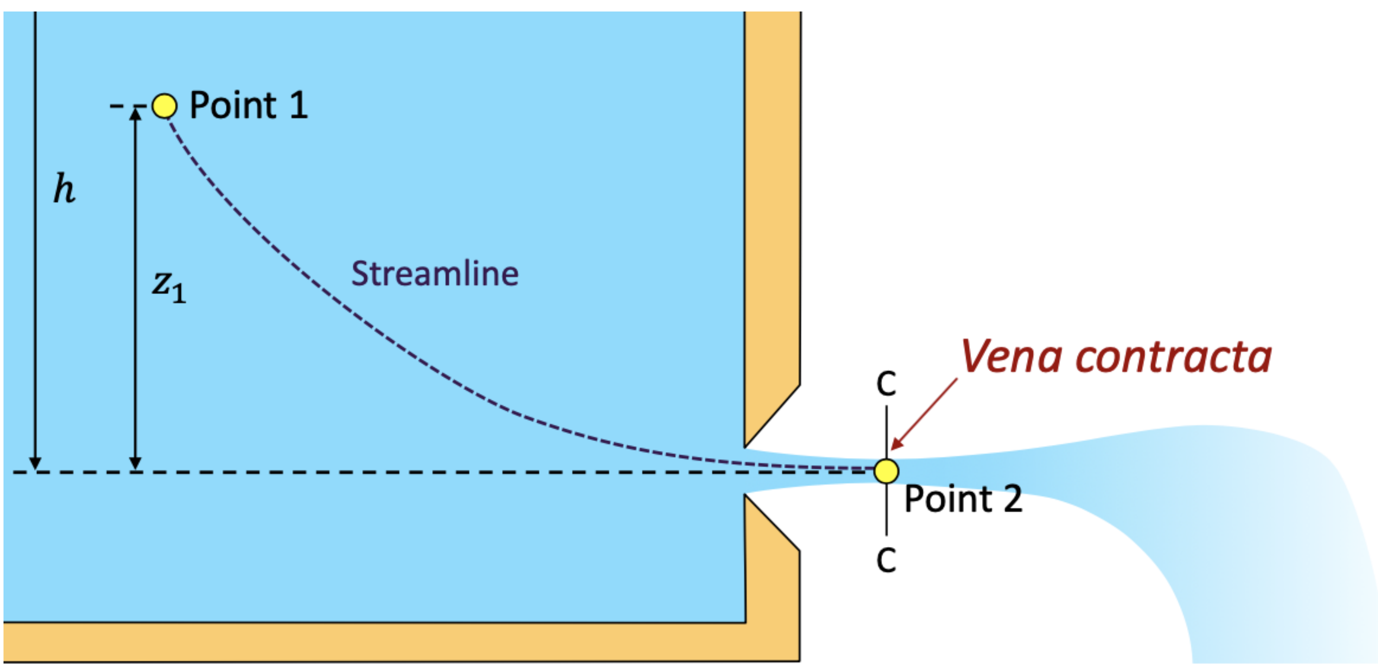
- 

- $\delta Q_{ideal} = b \delta z \sqrt{2g(H-z)}$
- $Q = \frac{8}{15} c_d \tan(\frac{\theta}{2}) \sqrt{2g} H^{5/2}$, for triangle.
- $Q = \frac{2}{3} c_d b \sqrt{2g} H^{3/2}$, for rectangle.
- Advantages:
  -



2: Orifices

2.1: Small Orifice

- 

- Bernoulli Equation:

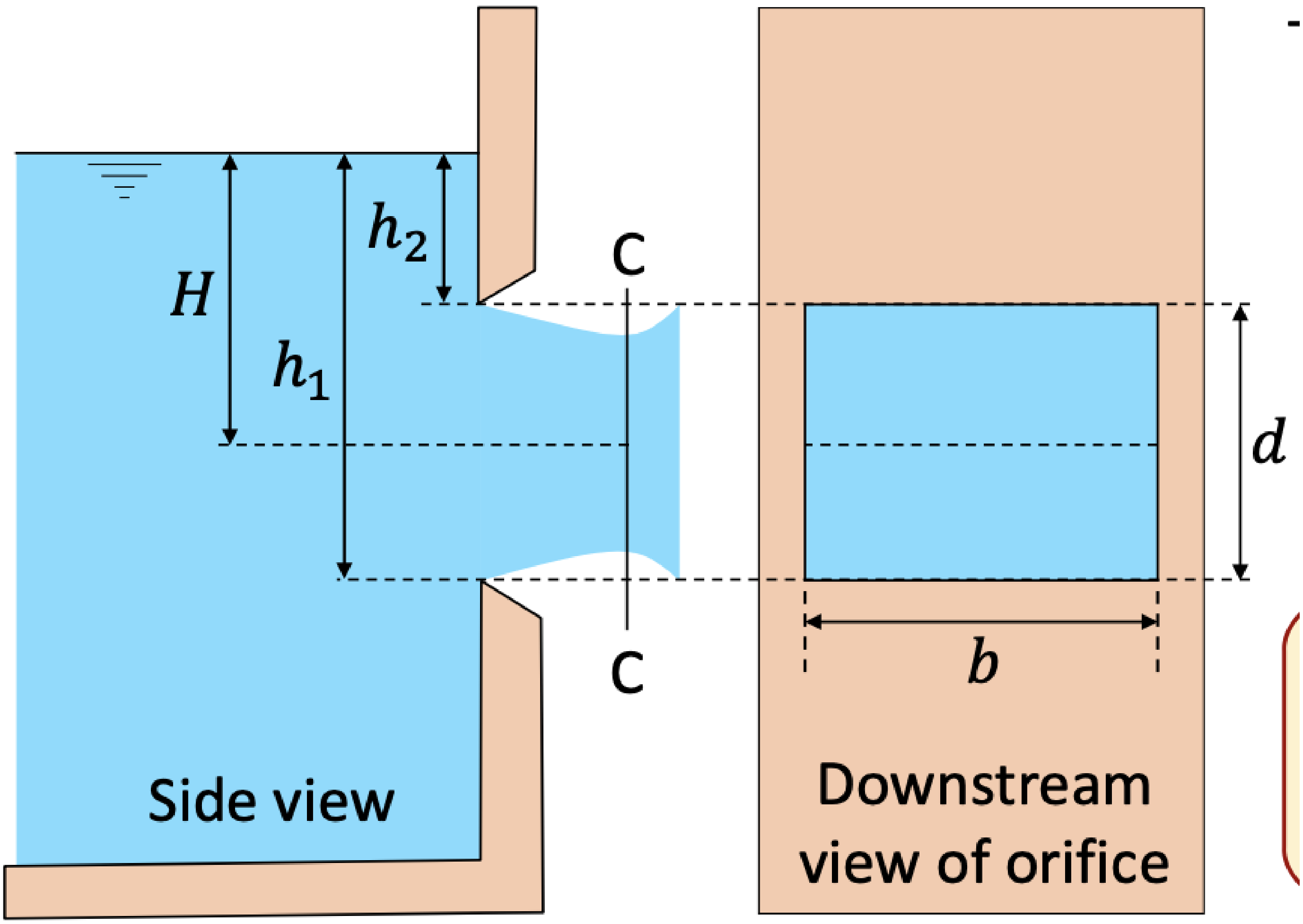
$$z_1 + \frac{p_1}{\rho g} + \frac{u_1^2}{2g} = 0 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g}$$

- Cause $\frac{u_1^2}{2g}$=0, $\frac{p_2}{\rho g}$=0, we can find:

$$h = \frac{u_2^2}{2g}$$

- I.e, $u_2^2 = \sqrt{2gh}$
- $Q = c_c A u_2 = c_c A\sqrt{2gh}$, c_c =area of vena contracta/ area of orifice.

2.2: Large orifice

- 

- $\delta Q_{ideal} = b\delta z\sqrt{2g(H-z)}$
- Then we can find that:

$$Q = \frac{2}{3} c_d b\sqrt{2g}((H+\frac{d}{2})^{3/2} - (H-\frac{d}{2})^{3/2})$$