I Magnetism

1: Introduction

1.1 Gauss's Law of Magnetism

- The law related to the passage of the magnetic flux through a closed surface.
- No point source of M-fields.
- Flux must be circulate.
- For a fixed surface, the flux out is equal to that in.
- The M-pole cannot be enclosed.

1.2 The order of the magnitude for magnetic flux density, $\vec{B}$, in Tesla

- Surface of earth: 10^{-4}
- Interstellar space:10^{-8}
- Iron magnet: 10^{-2}
- Strong magnet: 1
- Superconducting solenoid:1^{1}
- Neutron:10^{8}

2: Magnetic fields

2.1: Empirical observations:Force in wires

- The same direction I affect each other.
- The different direction I reject each other.

2.2: Caparison with the electrical fields

- In electrical fields, $\vec{D} = \epsilon\vec{E}$,while the $\vec{D}$ is material independent.
- In magnetic fields,$\vec{H} = \dfrac{1}{\mu}\vec{B}$, while $\vec{H}$ is materials independent.

2.3: Gauss's Laws

- In E-fields, $\int_S \vec{D}.d\vec{A} = Q_{enclosed}$, and $\nabla.\vec{D} = \rho$
- In magnetic,
  $\int_S .d\vec{A} = 0$, and $\nabla.\vec{B} = 0$

2.4: The Biot-Savart Law

- Biot-Savart Law -analogous to the principle of superposition
- $\vec{H} = \int \frac{id\vec{l} \times \vec{r}}{4\pi r^3}$
- An example:

  a is the distance from a point to the wire with a current. The Biot-Savart shows that: $|\vec{H}| = \frac{i}{2\pi a}$,
  And $|\vec{B}| = \frac{\mu_r \mu_0 i}{2\pi a}$
- Direction of the magnetic fields can be defined by the right hand screw rule.
- The geometric interpretation:
  i can be treated as the current passing the loop, and the $2\pi r$ is the length of the loop.

2.5: Ampere's Law

- $\oint_C \vec{H}.d\vec{l} = i$
- Just as the geometry interpretation above.
- The comparison of Gauss's Law and Ampere's Laws:
  - Gauss's Law: $\oint_S \vec{D}.d\vec{A} = q$
  - Ampere's Law: $\oint_C \vec{H}.d\vec{l} = i$

2.6: Magnetic field from a current loop

- The field at the centre is :
  $\int \frac{id\vec{l} \times \vec{r}}{4\pi r^3}$
- r is the constant, and $d\vec{l} \times \vec{r} = rdl\hat{z}$
- $|\vec{H}| = \frac{i}{2r}$

2.7: An important application of Ampere's Law

- N is the total length of the turn of the solenoid, n is the turns density.
- $\oint_C \vec{H}.d\vec{l} = \int_B^C \vec{H}.d\vec{l} = HL_{AB} = N_{AB}i$

- $H = \frac{N_{AB}}{L_{AB}} i = ni$

2.7: Faraday's Law

2.7.1 Faraday's idea

- $i \to H$
- Faraday surmised $H \to i$
- What we need is to change the magnetic fields

2.7.2 Induced e.m.f(voltage)

- The e.m.f is electromotive force.
- $\epsilon = -\frac{d\Phi}{dt}$, where the φ is the flux passing the loop.
- The sign of the voltage can be inferred by Lenz's Law.
- For coils,

$$\epsilon = -N\frac{d\Phi}{dt} = -\frac{d\Phi}{dt}$$

2.7.3 Application of Faraday 's Law

- Voltage and induction
  - As we know above: $\Phi = Li$
  - Apply Faraday's Law: $|V| = \frac{d\Phi}{dt}$
  - i.e: $|V| = \frac{dL}{dt} i + \frac{di}{dt} L$
  - L is a constant in time so $\frac{dL}{dt} = 0$
  - $|V| = L\frac{di}{dt}$
  - Add the resistance: $|V| = L\frac{di}{dt} + iR$
  - Now for the power: $P = i|V| = iL\frac{di}{dt} + i^2 R$
- Energy in the field
  - $U_{ind} = \int_{t1}^{t2} P(t)dt$, i.e. $U_{ind} = \frac{1}{2}Li_2^2 - \frac{1}{2}Li_1^2$
  - For an alternating current, the stored energy alternated.
- Inductive coupling: a two coil system
  - A ring of metal forms a path for magnetic flux.

- A coil of wire is wrapped around the ring.
- The coil-1 is connected to a power supply while the coil-2 is not.
- The changing flux from the first coil passes and interacts with the second coil,which generate the voltage.
- The flux from coil-1 is: $\Phi = N_1 \Phi_1 = i_1 L_1$
- The flux passes through the coil-2: $\Phi_2 = N_2 \Phi_1 = \frac{N_2}{N_1} L_1 i_1 = M_{21} i_1$
- The quantity M_{21} is the mutual induction.
- $U = \frac{1}{2} L_1 i_1^2$
- $\Phi_1 = i_1 L_1 + M_{12} i_2$
- The total energy is : $U = \frac{1}{2} L_1 i_1^2 + M_{12} i_1 i_2 + \frac{1}{2} L_2 i_2^2$

- Coupled coils and voltage
  - The alternating voltage in coil-1 generate a flux φ_1
  - Then in coil-2:

$$|V_2| = \frac{d\Phi_2}{dt} = \frac{N_2}{N_1} V_1$$