2.Second- order differential equations

2.1 Mechanical System with spring and damping

$$\frac{md^2y}{dt^2} = -ky - C\frac{dy}{dt}$$

$$i.e.ma = -F_{restoring} - F_{damp}$$

It show the relationship between the y and t in the damping system.

2.1.1 Another physical example-an electrical circuit

In a circuit with R,L and C,we can find the voltage relationship:

$$IR + L\frac{dI}{dt} + \frac{Q}{C} = V(t)$$

$$R\frac{dQ}{dT} + L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$$

This a second order differential equation.

2.2

Solve the equation

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dt} + 2y = 0$$

First, use the $y = e^{mt}$ as a trial solution.

$$m^2e^{mt} + 3me^{mt} + 2e^{mt} = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1$$

$$m = -2$$

So we have two solutions.

In fact, the GS is

$$y = Ae^{-t} + Be^{-2t}$$

While A, B are the constants.(when A or B=0,ensure containing all the answer)

Then we can use the

$$y(0) = 1$$

$$\frac{dy}{dt}(0) = 0$$

Then we can find the A=2,B=-1.
So the P.S is

$$y = 2e^{-t}e^{-2t}$$

2.3 Quadratic(repeated) roots the same

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 0$$

This gives only one solution:
$y_1 = Ae^{-t}$

This case is called the **critical damping**.
However, all second-order d.e.s have two independent solutions.

The other one is

$$y_2 = Bte^{-t}$$

So the GS is

$$y = (A + Bt)e^{-t}$$

2.4 Example of simple harmonic motion

If we choose the simplest case of no damping (R=0), we can find this equations:

$$\frac{d^2Q}{dt^2} = -\frac{Q}{LC}$$

This is the simple harmonic motion.

2.5

Oscillations- complex roots of the auxiliary equations

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 0$$

Try $y = e^{mt}$

We can find that m=$-1 \pm i$

So the GS is

$$y = Ce^{(-1+i)t} + De^{(-1-i)t}$$

Next, we use the result:

$$e^{i\theta} = cos\theta + isin\theta$$

$$y = e^{-t}[Acos\theta + Bsin\theta]$$

We use the final real number to do solve the physical problems.

2.6 General Solution for the oscillation equations

If we find $m = \pm Ci + D$;
the GS for real number is:

$$y = e^{Dx}[Acos(Cx) + Bsin(Cx)]$$