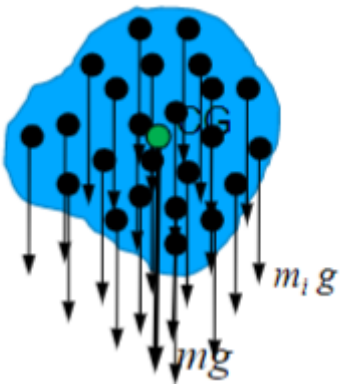
I: Introduction to Dynamics

- Draw the free body diagram
- Newton's second Law in each direction.
- Non-rotational: $\Sigma F = m\ddot{x}$ and $\Sigma M = 0$
- Rotational: $\Sigma M \neq 0$. Note that the moment is about the **centre of mass**.

II: Body Forces

1: Introduction of Body force

- A **external force** acts throughout the volume of a body.
- Force due to gravity, electrical fields and magnetic fields are examples.
- Note that **Normal forces and surface forces instead of body forces**.
- Body forces act on all particles making up a body.
- 

- For a rigid body, these behave as a single force acting through a point.
- Centre of mass = Centre of Gravity

2: Centre of mass

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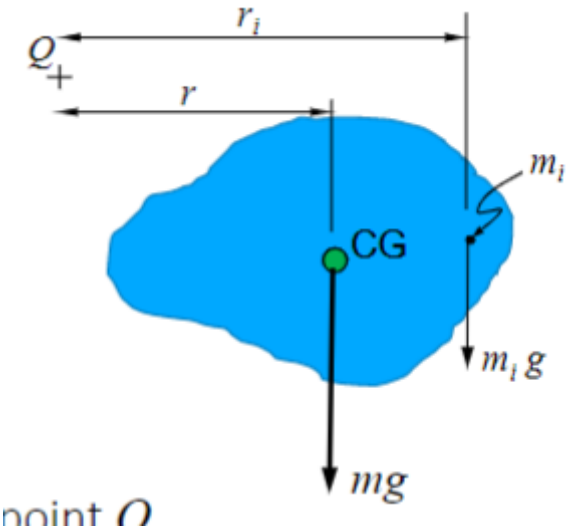

point Q

- Each part feels a force $F_i = m_i g$
- Each part feels a moment $M_i = m_i g r_i$ about any point Q.
- $\Sigma M_i = g \Sigma m_i r_i = mgr$
- So $mr = \Sigma m_i r_i$, this is the 1^{st} **moment of mass**, which can be used to located the centre of mass.

3: Examples of finding the centre of mass

3.1: Example_1

- A uniform, triangular plate, with thickness t and density ρ.
- Define an element of width δx, a distance x from an axis XX.
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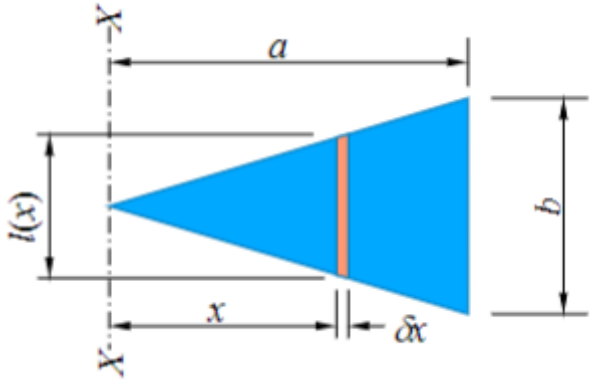

- The length of element can be written as $l(x) = \frac{b}{a}x$
- The mass can be expressed as $\delta x = \rho t \frac{b}{a} x \delta x$
- So the first moment of the element about the axis $XX = \frac{1}{3}$
- And for the whole body integration $= \int_{x=0}^{x=a} \rho t \frac{b}{a} x \delta x = \frac{1}{3} \rho t b a^2$

- $mr = \frac{1}{3}\rho tba^2$, so $r = \frac{1}{3}a$

3.2: Centre of gravity, Centre of mass and Centroid

- Centre of mass, centre of gravity and centroid are not the same thing.
- Assume uniform gravitational field, so **centre of gravity =centre of mass**.
- Centroid is obtained from **first moment of area**: $\Sigma \delta A.x = A\bar{x}$
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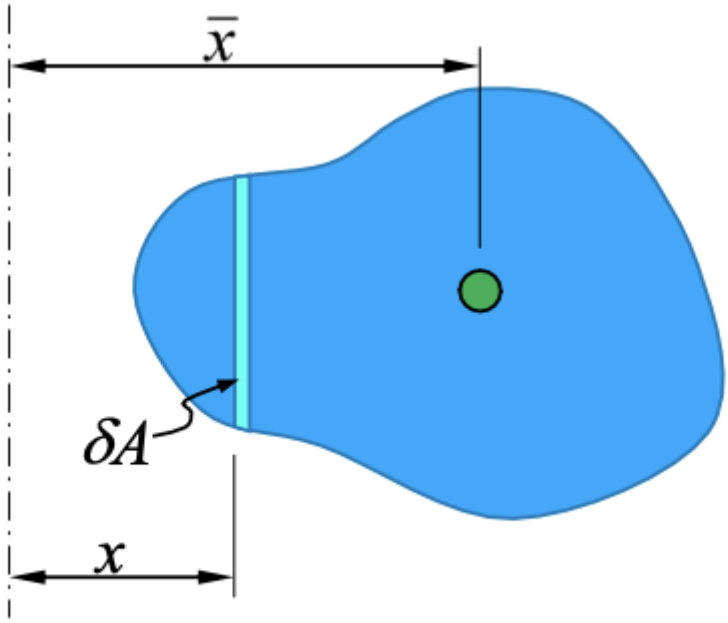

- centroid = centre of mass if t and ρ are uniform.