IV Functions and Curves

1: Functions- general ideas

1.1 What is a function

1.2 Combining of functions

1.3 Domain and Range

- Domain is all the possible input values.
- Range is all the possible output values.

1.4 Inverse Functions

- We use f^{-1} to mean inverse.
- Examples:
  - If $f(x) = 1 - 2x$, we write $f = 1 - 2x$
  - And we can write it as a function of x : $f = 1 - 2x$
    $x = \dfrac{1-f}{2}$
  - $f^{-1}(x) = \dfrac{1-x}{2}$
  - The graph of the functions and its inverse are reflections of each other in the line $y = x$.
- Domain of $f(x)$ is equal to the Range of f^{-1} and vice versa.
- Other important functions and their inverses:
  - e^x and $\ln x$
  - $\sin x$ and $sin^{-1}x$

2: Exponential and Logarithmic functions

2.1 Exponential function e^x or exp(x)

2.1.1 The application of e^x

- e^{-x}-the exponential decay
- e^{-x^2}-the normal distribution
- xe^{-x}-the Poisson distribution

2.1.2 The hyperbolic cosine and hyperbolic sine

- $\cosh x = \dfrac{(e^x + e^{-x})}{2}$
- $\sinh x = \dfrac{(e^x - e^{-x})}{2}$
- Their properties:
  - $(\cosh x)^2 - (\sinh x)^2 = 1$
  - $$\frac{d}{dx}\cosh x = \sinh x$$

    $$\frac{d}{dx}\sinh x = \cosh x$$
  - $\cosh(ix) = \cos(x)$
  - $\sinh(ix) = i\sin(x)$

2.2 Logarithm and powers

3: Sine and Cosine functions

3.1 Combination of sine and cosine functions

The combination of the sine and cosine produce more complicated shapes, such as Fourier Series.

4: Transformation

4.1 Translation and Magnifications

4.2 Plotting a quadratic graph

4.4 Odd,even and periodic functions

- Even function: $f(-x) = f(-x)$
- Odd function: $f(-x) = -f(-x)$
- Periodic function: like $\sin x$

5: Curve sketching

5.1 Aims and strategy

- Cross or touch the axes.
- Max, min and inflection points

5.2 Stationary points- First derivation

Using the first derivation to find the gradient each side.

5.3 Stationary points- Secondary derivation

- The inflection points means the $\frac{dy}{dx} \neq 0$ and $\frac{d^2y}{dx^2} = 0$.
- The gradient of the function reach a max or min at the infection points.
- It is useful only if the $\frac{dy}{dx} \neq 0$

6: Asymptotes and Rational Functions

6.1 Definitions

- The function that is a quotient of two polynomial functions
- As the denominator of the fraction takes the value zero, the function becomes infinite, we get a vertical line called vertical asymptote. The function may have horizontal , sloping and vertical asymptote.
- Theses lines may cross.

6.2 Rewriting the functions by long division

- This is a way to separate the function to make to curves graphing easier.
- Examples:

- $\frac{x^2}{x+1} = \frac{(x-1)(x+1)+1}{x+1} = x - 1 + \frac{1}{x+1}$
- $\frac{x+1}{x-3} = \frac{(x-3)+5}{x-3} = 1 + \frac{5}{x-3}$

7: Curve Sketching Examples

Example 1

$$y = \frac{2x+1}{(x-1)(x+2)}$$

- Finding the roots of the denominator, which is the vertical asymptotes.
- Finding the monotony of each parts of the function.
- Finding the infinite of the function.

Example 2

$$y = \frac{x^3 - 2x^2 + x - 2}{1 - x^2}$$

- Separating the factors as $(x-2)(x^2+1)$.
- Following the example 1 to get the vertical asymptotes and the monopoly.
- Using the long division to separate the constant to find the slope asymptotes:

$$y = \frac{(1-x^2)(2-x)}{1-x^2}$$

i.e.:

$$y = -x + 2 + \frac{2x-4}{1-x^2}$$

As the last part of the term is really small, the slope asymptote is the $y = -x + 2$.

Example 4 (modulus function)

- $y = |x+3| + |x-1|$
- The graph can be drawn by apart the functions.