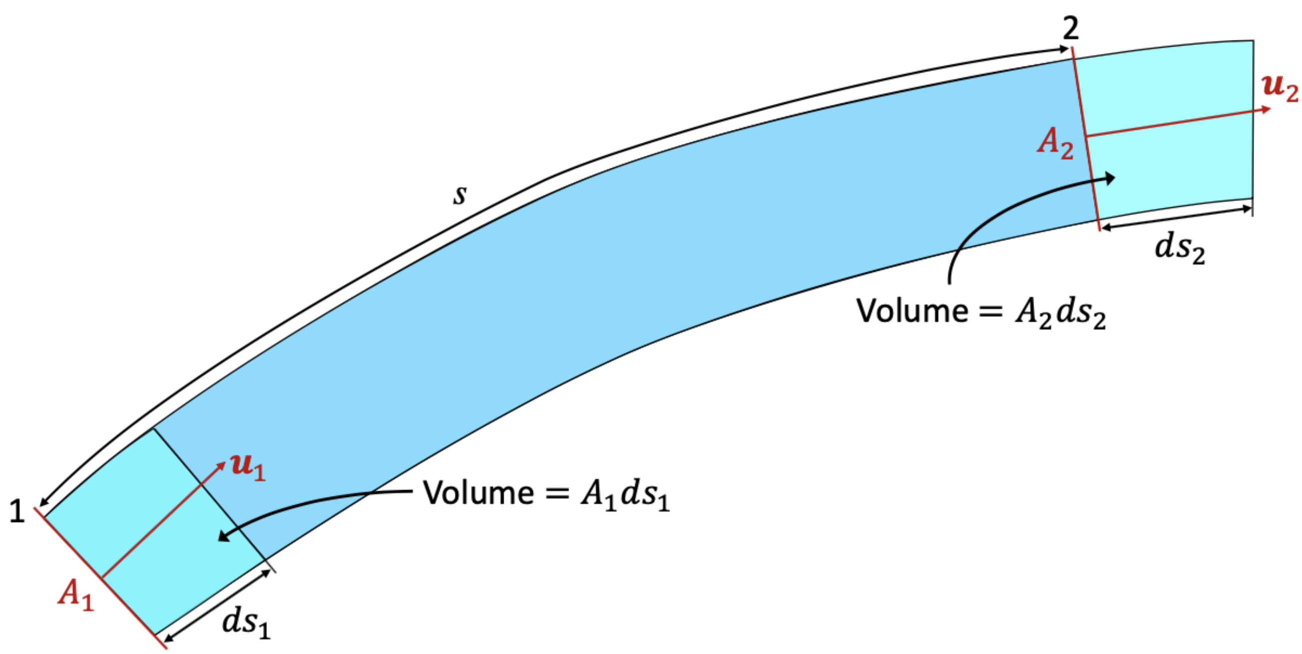
X: Conservation of Momentum

1: Definition

- Momentum, $M = mv$
- Newton's second law of motion: $\Sigma F = \frac{dM}{dt}$
- Impulse-momentum equation: $\Sigma F dt = dM$

2: Impulse-momentum equation of Control Volume

- $dM = \rho A_2 ds_2 u_2 - \rho A_1 ds_1 u_1$



- $\Sigma F dt = dM = \rho A_2 ds_2 u_2 - \rho A_1 ds_1 u_1$
- For ds can be expressed as udt and $Q = Au = constant$, the equation can be simplified as :

$$\Sigma \vec{F} = \rho Q(\vec{u_2} - \vec{u_1})$$

- What's more, we can get this equation in 3D:

$$\Sigma F_x = \rho Q(u_{2x} - u_{1x})$$

$$\Sigma F_y = \rho Q(u_{2y} - u_{1y})$$

$$\Sigma F_z = \rho Q(u_{2z} - u_{1z})$$

3: Bousinesque coefficient

- The Bousinesque coefficient:

$$\beta = \frac{1}{A}\int_A (u/\bar{u})^2 dA$$

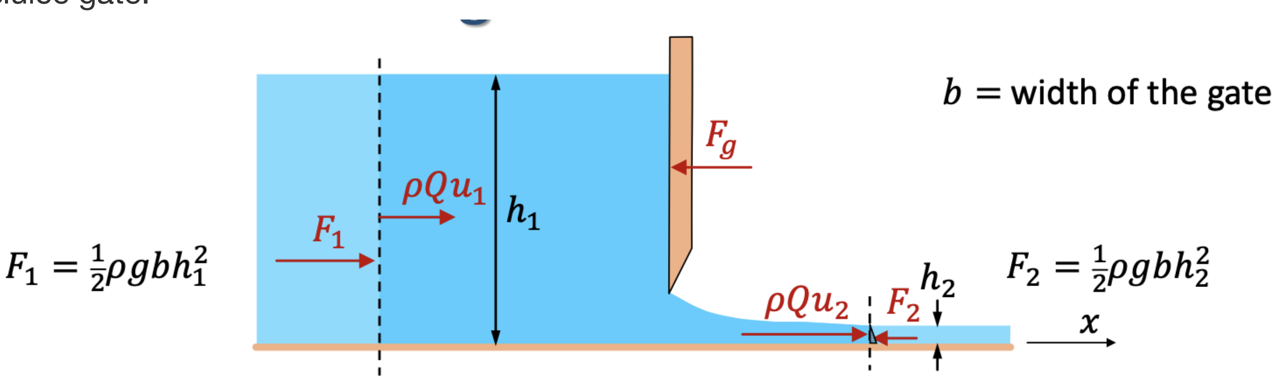
- With the coefficient, we can find:

$$\Sigma \vec{F} = \rho Q \beta(\vec{u_2} - \vec{u_1})$$

4: Application of Conservation of Momentum

4.1: Force in a Sluice Gate

- The sluice gate:

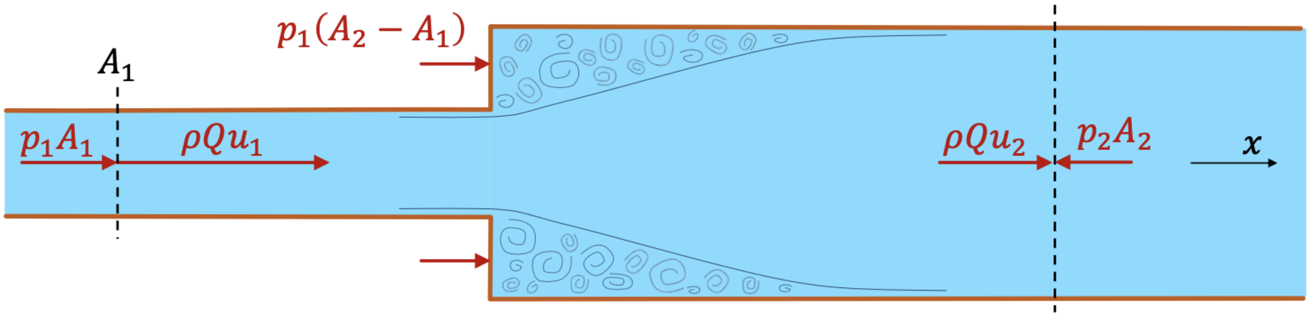


- According to the Conservation of Momentum, $\rho Q u_1 + \Sigma F = \rho Q u_2$ in x-direction.
- It can be found that $\Sigma F = F_1 - F_2 - F_g$, where the F_1 and F_2 are Hydraulic thrust of the beginning and end of the Control Volume and F_g is the Compressive force of the wall.
- So if b is the width of the sluice gate, we can simplify it by:

$$\rho g Q u_1 + \frac{1}{2}\rho g b h_1^2 - F_g - \frac{1}{2}\rho g h_2^2 = \rho Q u_2$$

4.2: Sudden expansion

- The sudden expansion:



- $\rho Q u_1 + \Sigma F = \rho Q u_2$
- $\rho Q u_1 + p_1 A_1 + p_1(A_2 - A_1) - p_2 A_2 = \rho Q u_2$
- $z_1 + \frac{p_1}{\rho g} + \frac{u_1^2}{2g} = z_1 + \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + losses$
- $losses = \frac{(u_1 - u_2)^2}{2g}$