II: Stress transformation

1: Stress at a point

1.1: Point stress at a body

- There should be 9 states of stress in a 3D body:
  - Normal stress: σ_{xx},σ_{yy} and σ_{zz}
  - Shear stress: τ_{yx}, τ_{yz}, τ_{xz}, τ_{xy}, τ_{zy} and τ_{zx}.
- The second subscript means the direction while the first and second means the plane it apply.

1.2: Plane Stress

- For convenience, the plane stress is only 2D and is redrawn in x-y plane.

2: Uniaxial tension

2.1: Definition

- Stresses on an inclined plane, i.e.$\sigma_{yy} = \tau_{xy} = \tau_{yx} = 0$

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2.2: The Transformation equations for uniaxial tension cases

- Cut PQ rotated anti-clockwise through a angle θ from the vertical
- Noted that the tension stress is '+' and press stress is '-', for the angle θ, anti-clockwise is '+' and clockwise is '-'.
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- Then we can epitomize the stress transformation equations for uniaxial case:

$$\sigma_\theta = \frac{\sigma_{xx}}{2} + \frac{\sigma_{xx}}{2}\cos(2\theta)$$

  and

$$\tau_\theta = -\frac{\sigma_{xx}}{2}\sin(2\theta)$$

2.3: Failure of ductile and brittle materials

2.3.1: The ductile materials

- Ductile materials tend fail on planes to the max shear stress.
- According to the sheAR stress equations: $\tau_\theta = -\frac{\sigma_{xx}}{2}\sin(2\theta)$, we can fin that the max shear stress occurs at approximately 45 degrees.
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2.3.2 The brittle materials

- Brittle material fail due to normal stresses and rupture occurs along a surface perpendicular to the surface.
- To find the transformation of the brittle materials, we should use the formula of σ, so the max occurs at $\theta = 0$ degrees.
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2.4: General stress transformation equations

- For a more general case with $\sigma_{xx} \neq 0$, $\sigma_{yy} \neq 0$, $\tau_{xy} \neq 0$ and $\tau + yx \neq 0$.
- To keep the equilibrium, the shear force and normal stress acting in opposite direction must be equal in magnitude, while the shear stress acting on perpendicular direction must be equal.($\tau_{xy} = \tau_{yx}$)
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- Then cut a plane rotated in A/C through an angle θ from the vertical, there will be τ_θ and σ_θ in the interface to keep the balance.
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- The we can get the general stress transformation equations:

$$\sigma_\theta = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} \cos(2\theta) + \tau_{xy} \sin(2\theta)$$

and

$$\tau_\theta = -\frac{\sigma_{xx} = \sigma_{yy}}{2} \sin(2\theta) + \tau_{xy} \cos(2\theta)$$

3: Stress transformation: special cases of plane stress

3.1: Case: uniaxial tension

- For the uniaxial tension case, $\sigma_{xx} \neq 0$ and $\sigma_{yy} = \tau_{xy} = \tau_{yx} = 0$
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3.2: Case: Pure shear

- For the pure shear case, $\sigma_{xx} = \sigma_{yy} = 0$ and $\tau_{xy} = \tau_{yx} \neq 0$.
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3.2: Case: Biaxial tension

- For a biaxial tension case, $\sigma_{xx} \neq 0$, $\sigma_{yy} \neq 0$ and $\tau_{xy} = \tau_{yx} = 0$.
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4: Summary of Equations

- For uniaxial tension:

$$\sigma_\theta = \frac{\sigma_{xx}}{2} + \frac{\sigma_{xx}}{2} \cos 2\theta$$

$$\tau_\theta = -\frac{\sigma_{xx}}{2} \sin 2\theta$$

- General stress transformation equations:

$$\tau_\theta = -\frac{\sigma_{xx} - \sigma_{yy}}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$