VI Gauss' s Law

1: Flux

1.1 An intuitive example

The flux will be the same through the ends as through same imaginary cross section in the middle.

1.2 Definition

Some imagined aera perpendicular to the field direction. This is the flux, φ.
(like several arrows across the aera).

2. Gauss's Law

2.1 Formula

The amount of the **displacement field** flux passing outward through a closed surface is equal to the amount of the **charge inside** the closed surface.

i.e.

$$\varphi_D = \int_s \vec{D} \quad d\vec{A} = \int_V \rho \quad dV = Q$$

2.2 Example for the point charge

Imagine a Gaussian surface "S" (sphere centred on the charge)
By the symmetry, $\vec{D}$ and $d\vec{A}$ must be parallel.

$$\int_S \vec{D} d\vec{A} = \int_S D dA$$

So

$$\varphi_D = D.4\pi r^2 = Q$$

$$D = \frac{Q}{4\pi r^2}$$

2.3 Important considerations

- Applied to any surface and distribution.
- No charge does not mean no field ,just means leaves is equal to the enters.

2.4 Process to follow

- Identified symmetry
- Choose suitable Gauss surface(make it so that the field is parallel or perpendicular to it)

2.5 Application to a plane of charge

2.5.1

Imagine an infinite plane and we add a proton on the plane.
We can now define a uniform positive areal charge density, σ

2.5.2

A cylinder of radius r
and total height $2h$
bisected by the plane of charge is a **suitable Gauss surface**.

2.5.3

Divide the surface integral into regions:

$$\int_S \vec{D} d\vec{A} = \int_{top} \vec{D} d\vec{A} + \int_{side} \vec{D} d\vec{A} + \int_{bottom} \vec{D} d\vec{A}$$

$$\int_{side} \vec{D} d\vec{A} = 0$$

$$\int_{top} \vec{D}(h) d\vec{A} = \int_{bottom} \vec{D}(h) d\vec{A} = D(h)\pi r^2$$

$$\int_V \rho dV = \int_S dA = \sigma A$$

So

$$\rightarrow Q = \sigma \pi r^2$$

To summarize,

$$2\pi r^2 D(h) = \pi r^2 \sigma$$

$$D(h) = \frac{\sigma}{2}$$

$$i.e. D = \frac{\sigma}{2}$$

We can find D is independent of distance from the plane.

2.5.4

We can also find the E in this way.

$$E = \frac{\sigma}{2\varepsilon}$$