VI Finding Hydraulic Thrust and COP by Equations

1. Finding the Hydraulic Thrust by equations

If we let each piece of area of the surface is dA and a small force is dF, and x_0 be the distance from the 0-0 axis to the COP, we can find:

$$dF = pdA = \rho ghdA$$

while

$$h = xsin\theta$$

If we use the moments balancing:

$$Fx_0 = \sum_A xdF = \int_A xdF = \int_A x(\rho gxsin\theta dA) = \rho gsin\theta \int_A x^2 dA$$

2. Second moment of area

- The second moment of area can be defined as $I_0 = \int_A x^2 dA$, x is the distance from dA to the axis x.
- If we use a Cartesian axes, the I about x axis can be expressed as :

$$I_{xx} = \iint_R y^2 dxdy$$

(As for a small area A, the dA=dx*dy)
So:

$$Fx_0 = \rho gsin\theta I_0$$

$$x_0 = \frac{\rho g I_0}{F}$$

- Second moments of area of common shapes
  - Rectangular: $I_g = \frac{bd^3}{12}$
  - Triangle: $I_g = \frac{bh^3}{36}$
  - Circle:
    $I_g = \frac{\pi d^4}{64}$
  - Parallel Axis Theorem:
    $I_o = I_g + \bar{x}^2 A$
- Finding COP with second moments of area:

$$y_0 = \bar{y} + \frac{I_g sin^2\theta}{A\bar{y}^2}$$