II: Electro-magnetism

1: Inductance

1.1 Definition of capacitance

- The electrostatic potential:$\Delta\phi = V = |\vec{E}|w$
- The capacitance is:$C = \frac{\varepsilon A}{w} = \frac{Q}{V}$
- w(or d) is for wide and A (or S)is for area above.
- Definition of inductance
  - Self-inductance of a circuit : $\phi = Li$
  - For multiple turns: $\Psi = N\phi$, and $\Psi = Li$
- The unit is Wb/A or Henry,H.

1.2 Voltage and inductance

- Inductance definition: $\Psi = Li$
- Faraday law:$|V| = \frac{d\Phi}{dt}$
- Product rules: $|V| = \frac{dL}{dt}i + L\frac{di}{dt}$
- Add the resistance:$|V| = L\frac{di}{dt} + iR$
- So the power: $P = i|V| = iL\frac{di}{dt} + i^2 r$
- Energy in the field: $U_{ind} = \int_{t_1}^{t_1} U(t)dt$, i.e:$U_{ind} = \frac{Li_2^2}{2} - \frac{Li_1^2}{2}$

1.3:Inductive coupling: a two coils system

- The metal ring produce a path for the magnetic flux.
- One side is wrapped with coils and a power supply while the other side only has the coils.
- The changing flux in the first coils interact the second side: This is the Faraday's Law apply.
- Th flux in the first coils: $\Psi_1 = N_1\phi_1 = i_1 L_1$
- The flux in the second coils: $\Psi_2 = N_2\phi_1 = \frac{N_2}{N_1}L_1 i_1 = M_{21} i_1$
- The quantity M_{21} is called mutual inductance.

- $M_{21} = M_{12}$

1.4: Inductive coupling : Energy

- L_1 is the self-inductance of coil 1, is a measure of how hard to change the current. i.e: $U = \frac{L_1 i_1^2}{2}$, and U for energy.
- Now ϕ_2 through coil 1 comes from coil 2: $\Psi_1 = i_1 L_1 + M_{12} i_2$
- $U = \frac{L_1 i_1^2}{2} + M_{12} i_1 i_2 + \frac{L_2 i_2^2}{2}$

2: Circuits equivalence

2.1 Magnetomotive force and the reluctance

- The amount of source of flux(MMF)
- MMF is equivalent to the battery(emf).
- The resistance to the flux is called the reluctance.

2.2 The analogy to the Ohm's Law

- emf $\rightarrow$ MMF
- $i \rightarrow \phi$
- emf=V=IR $\rightarrow$ MMF= F =ϕS
- $R = \frac{l}{\sigma A}, S = \frac{l}{\mu A}$
- Materials dependance: σ and μ
- $S = \frac{N^2}{l}$
- S for reluctance.

2.3 Combining reluctance

- Same as the resistance.

3: The Lorentz Force

- $\vec{F} = q\vec{E} + q\vec{u} \times \vec{B}$

- The second part is non-zero only if the charge q is moving in $\vec{B}$ and not in the same direction as $\vec{B}$.
- Use the right hand rule can find the direction of the second part.
- Fore on a moving charge in a uniform magnetic field
  - $\vec{B} = \mu\vec{H}$
  - A electron moving through it at right angles.
  - As it enter the field it experiences the Lorentz force $\vec{F} = q\vec{u} \times \vec{B}$
  - It is circle motion.
- Force on a current in a uniform magnetic fields:
  - $\vec{F} = i\vec{l} \times \vec{B}$
  - For the current and the field at the right angles:$F = Bil$ or $F = Bil\sin\theta$
- Force between two currents:
  - Approximate as vacuum.
  - Current 1 gives a circulating magnetic field.
  - Magnitude of field at wire 2 can be found by Biot-Savart: $\vec{B_1} = \dfrac{\mu_0 i_1}{2\pi d}$

4: Motional emf

4.1 Motional induced emf

- $\vec{B} = \mu\vec{H}$
- $\vec{F} = q\vec{u} \times B$
- Which generate the $\vec{E}$ from separated charges and an opposite force.
- $E = uB$.
- $V = BLu$.(same as BLv)
- u is same as v.

5: Force from the energy

5.1 Force from the filed energy

- For permanent magnets, there is not current so that we can't use the Lorentz's law.
- Energy stored in a magnets : $Energy = V\int_V \vec{H}\vec{B} = \dfrac{B^2}{2\mu_0}V$
- The force is a change that reduce the energy.

- $F = -\dfrac{B^2 A}{2\mu_0}$, which is independence of distant.
- It is not valid for constant current.