Crops Production Optimization Analysis

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Introduction

To investigate the optimum time point to harvest the crops (or replanted the machine), which could make the average yield maximum during the crops production or machine working lifetime, we set the initial model to describe the *Output - Age* relationship shown in *Fig.1.1*.

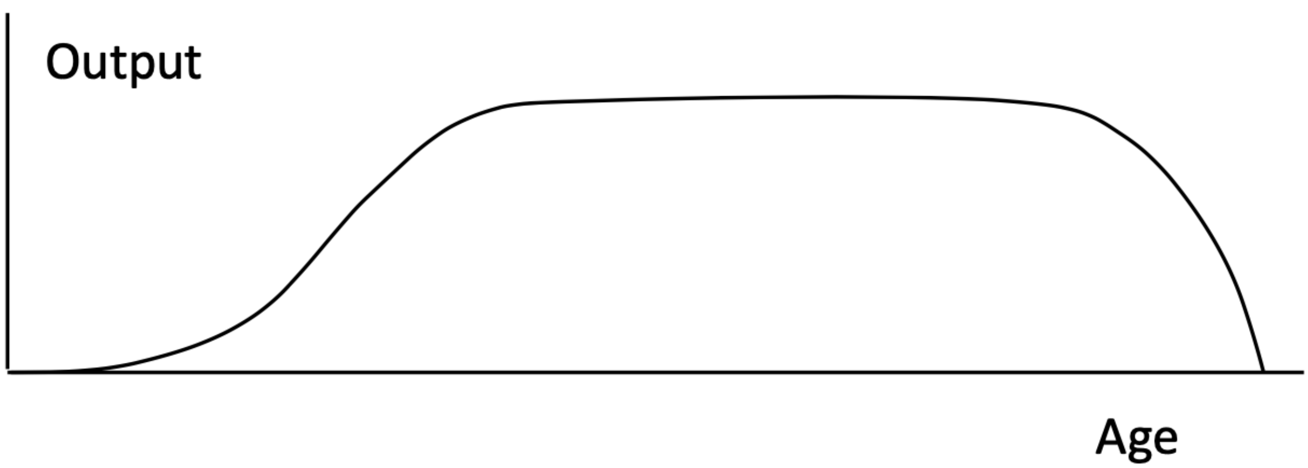

Fig.1.1 Initial Modelling of *Output - Age* Relationship

For this model, an initial period of zero output is to be followed by a gradually increase to max output, which is maintained for some time, before a period of decline. The target is to find the expression of the optimum value of age (time length 'L') to maximum the average output.

Assumptions and Simplifications

To simplify the conditions, we assume that the crops or machines working normally without any interruption and the internal properties keep unchanged during the producing lifetime.

As the graph shown in *Fig.1.2*, we use the maximum value of '1' refer to 100% output efficiency. From 0 to *a* is the zero region, *a* to *b*, *c* to *d* is treated as linear relationship for the purpose of simplification, and *b* to *c* is the flat region of the curve.

We fix the value of *d* is 25 in this condition.

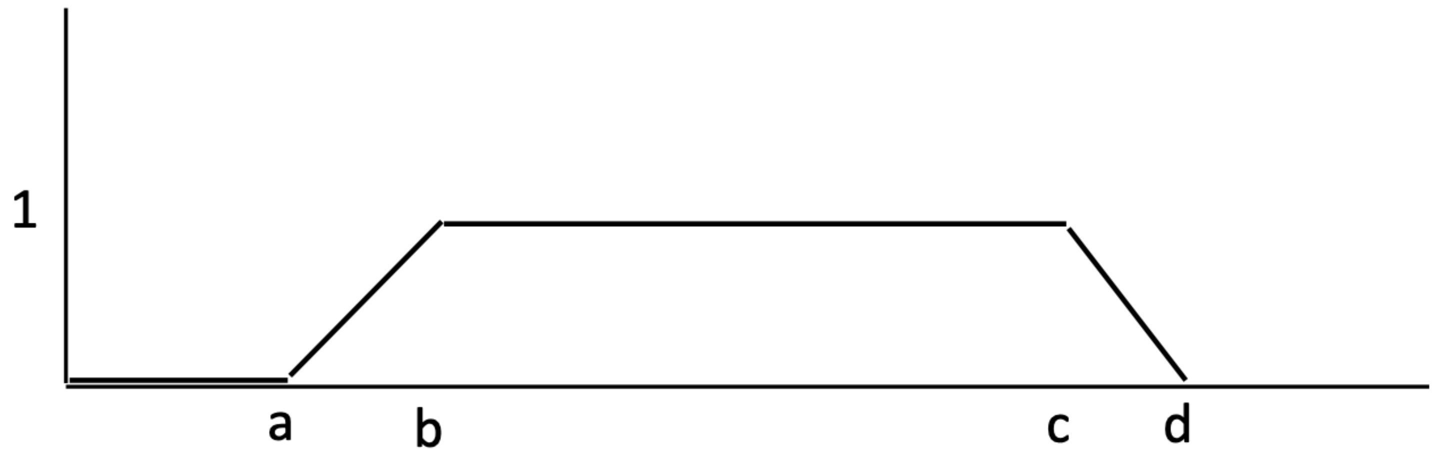

Fig.1.2 Simplified Model of *Output* - *Age* Relationship

First-stage Solutions

Model Construction

If we note the output in the Y-axis is *y* ($y \leq 1$) and L stand for the length of the continuous production ages. The average yield $\bar{Y}$ can be expressed by the integration of output divided by the stopped age L':

$$\bar{Y} = \frac{\int_0^{L'} y \mathrm{d}L}{L'} \tag{1.1}$$

After the simplification of the formula, we can get the expression below:

$$\bar{Y} = -\frac{1}{2(d-c)}L + \frac{d}{d-c} + \frac{1}{L}[\frac{1}{d-c}(\frac{1}{2}c^2 - dc) + c - \frac{a+b}{2}] \tag{1.2}$$

Using the formula 1.2 above, the optimum length L' can be found:

$$L' = \sqrt{(d-c)(a+b-2c) + 2cd - c^2} \tag{2}$$

Testing and Feedback

Now that we got the expression of the optimum length, we have done several simple test of this model with different values of *a*, *b* and *c* shown in *Table.1*:

Optimum Length (L)	a	b	c	d	Output	Average_Y
21.21320344	0	10	20	25	0.75735931	0.7573593
21.27204739	0.5	10	20	25	0.74559052	0.7455905
21.33072901	1	10	20	25	0.7338542	0.7338542
21.38924964	1.5	10	20	25	0.72215007	0.7221501
21.44761059	2	10	20	25	0.71047788	0.7104779
21.50581317	2.5	10	20	25	0.69883737	0.6988374
21.56385865	3	10	20	25	0.68722827	0.6872283
21.62174831	3.5	10	20	25	0.67565034	0.6756503

Table.1 Optimum Length, Output and Average_Y with different *a*,*b*,and *c*

The values of the optimum length with the output and average_Y at the optimum points are about 0.6 to 0.7, which below the max output 100% we assumed previously.

Parameters Analysis

As we assumed before, the parameter *d* is fixed to 25, so we researched the influence to the optimum length of *a, b* and *c* independently by fixing two parameters and changing the other one:

Changing value of 'a'

Optimum Length (L)	a	b	c	d	Output	Average_Y	
21.21320344	0	10	20	25	0.75735931	0.75735931	a < b < c < 25
21.27204739	0.5	10	20	25	0.74559052	0.74559052	we fix the c to 20
21.33072901	1	10	20	25	0.7338542	0.7338542	fix the b to 10
21.38924964	1.5	10	20	25	0.72215007	0.72215007	
21.44761059	2	10	20	25	0.71047788	0.71047788	
21.50581317	2.5	10	20	25	0.69883737	0.69883737	
21.56385865	3	10	20	25	0.68722827	0.68722827	
21.62174831	3.5	10	20	25	0.67565034	0.67565034	
21.67948339	4	10	20	25	0.66410332	0.66410332	
21.73706512	4.5	10	20	25	0.65258698	0.65258698	
21.79449472	5	10	20	25	0.64110106	0.64110106	
21.85177338	5.5	10	20	25	0.62964532	0.62964532	
21.9089023	6	10	20	25	0.61821954	0.61821954	
21.96588264	6.5	10	20	25	0.60682347	0.60682347	
22.02271555	7	10	20	25	0.59545689	0.59545689	
22.07940217	7.5	10	20	25	0.58411957	0.58411957	
22.13594362	8	10	20	25	0.57281128	0.57281128	
22.19234102	8.5	10	20	25	0.5615318	0.5615318	
22.24859546	9	10	20	25	0.55028091	0.55028091	
22.30470802	9.5	10	20	25	0.5390584	0.5390584	
22.36067977	10	10	20	25	0.52786405	0.52786405	

Changing value of 'b'

Optimum Length (L)	a	b	c	d	Output	Average_Y	
21.21320344	5	5	20	25	0.75735931	0.75735931	a < b < c < 25
21.27204739	5	5.5	20	25	0.74559052	0.74559052	we fix the c to 20
21.33072901	5	6	20	25	0.7338542	0.7338542	fix the a to 5
21.38924964	5	6.5	20	25	0.72215007	0.72215007	
21.44761059	5	7	20	25	0.71047788	0.71047788	
21.50581317	5	7.5	20	25	0.69883737	0.69883737	
21.56385865	5	8	20	25	0.68722827	0.68722827	
21.62174831	5	8.5	20	25	0.67565034	0.67565034	
21.67948339	5	9	20	25	0.66410332	0.66410332	
21.73706512	5	9.5	20	25	0.65258698	0.65258698	
21.79449472	5	10	20	25	0.64110106	0.64110106	
21.85177338	5	10.5	20	25	0.62964532	0.62964532	
21.9089023	5	11	20	25	0.61821954	0.61821954	
21.96588264	5	11.5	20	25	0.60682347	0.60682347	
22.02271555	5	12	20	25	0.59545689	0.59545689	
22.07940217	5	12.5	20	25	0.58411957	0.58411957	
22.13594362	5	13	20	25	0.57281128	0.57281128	
22.19234102	5	13.5	20	25	0.5615318	0.5615318	
22.24859546	5	14	20	25	0.55028091	0.55028091	
22.30470802	5	14.5	20	25	0.5390584	0.5390584	
22.36067977	5	15	20	25	0.52786405	0.52786405	
22.47220505	5	16	20	25	0.50555899	0.50555899	

Changing value of 'c'

Optimum Length (L)	a	b	c	d	Output	Average_Y	
18.02775638	5	10	10	25	0.46481624	0.46481624	a < b < c < 25
18.10386699	5	10	10.5	25	0.47559538	0.47559538	we fix the a to 5
18.1934054	5	10	11	25	0.48618533	0.48618533	fix the b to 10
18.29617446	5	10	11.5	25	0.49657967	0.49657967	
18.41195264	5	10	12	25	0.50677287	0.50677287	
18.54049622	5	10	12.5	25	0.5167603	0.5167603	
18.68154169	5	10	13	25	0.52653819	0.52653819	
18.8348082	5	10	13.5	25	0.53610363	0.53610363	
19	5	10	14	25	0.54545455	0.54545455	
19.17680891	5	10	14.5	25	0.55458963	0.55458963	
19.36491673	5	10	15	25	0.56350833	0.56350833	
19.56399755	5	10	15.5	25	0.57221078	0.57221078	
19.77371993	5	10	16	25	0.58069779	0.58069779	
19.99374902	5	10	16.5	25	0.5889707	0.5889707	
20.22374842	5	10	17	25	0.59703145	0.59703145	
20.46338193	5	10	17.5	25	0.60488241	0.60488241	
20.71231518	5	10	18	25	0.6125264	0.6125264	
20.97021698	5	10	18.5	25	0.61996662	0.61996662	
21.23676058	5	10	19	25	0.62720657	0.62720657	
21.51162476	5	10	19.5	25	0.63425004	0.63425004	
21.79449472	5	10	20	25	0.64110106	0.64110106	
22.08506283	5	10	20.5	25	0.64776382	0.64776382	

Table.2 Spreadsheet when changing *a*, *b* or *c* and fixing the other two parameters

To show the different increasing ot decreasing influence trend of different parameters, the scatter-point graph of the *Table.2* can be plotted:

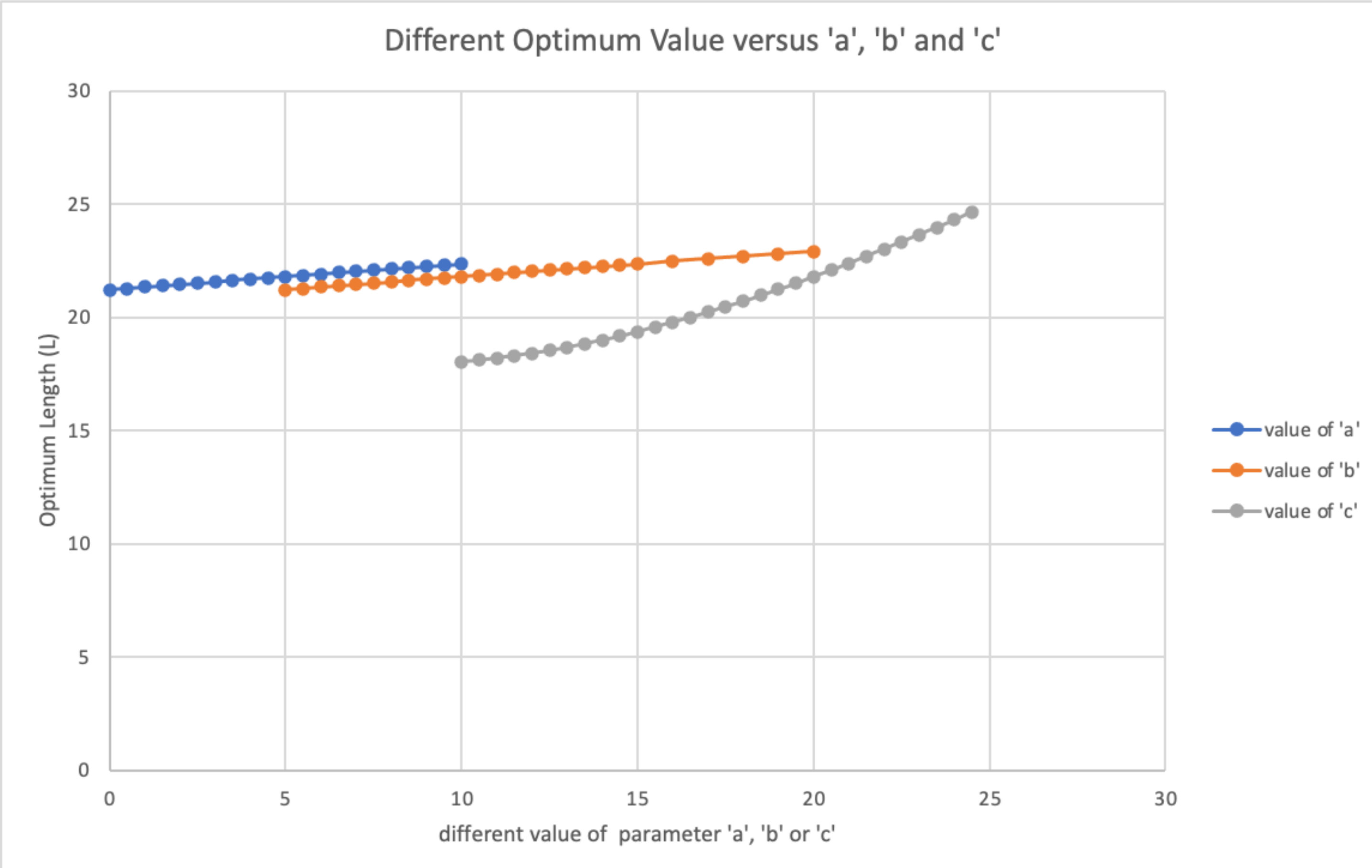

Figure.2 Scatter-point graph for different *a, b, c*

According to the results shown in *Figure.2*, when we changing one of the parameters and fixing the others, parameter *a* and *b* have a linear-like relation with the optimum length with a small positive gradient, while the increasing of parameter *c* have a larger positive gradient than *a* and *b*.

In conclusion, the parameter *c* have a larger effect on the value of optimum length, which is more critical than others.

Model Application

Practical Conclusions

For the application of the model we proposed above, there are several key-points need to be considered before the real-world deployment.

- The first perquisite of applying this model is collecting related data of the crops/machine production output in previous years. It is essential to find the value of parameter *a, b*, and *c*.

- According to the result shown in *Figure.2*, the parameter *c* have more influence on the optimum length *L'*, so the estimation of *c*, which is the beginning of the output declining, is extremely important in the application.
- The second step of the application is choosing the calculated value of optimum stopped using the Formula.2. Due to the calculated result, the client or manager is suggested to harvest or replant the crops (machines) at the optimum length to have the most efficient average yield.

Assumptions and Simplifications Reconsideration

- During the assumption and simplification stage, we assumed that the internal properties and continuous condition remain constant during the whole production process.
- If the real process situation does not fit the assumption and simplification, the conclusion of optimum length may be effected. For example, if there is any interruption during the production process, the *output-age* curve would not be the shape assumed in *Figure 1.1* and *Figure 1.2*, so that the result may not correct.
- In another aspect, the quantity of the researching target also have an effect on the final conclusion:
    - For a single unit or a small number of units, the output can be correctly described using the formula above.
    - For a large area or large numbers of unit, the accuracy of this model might be affected as the different machine may have different parameter *a*, *b* and *c* so that different optimum length.

Second modelling using different curves

To try different fitting the final stage of the curve, we changed the curve between *c* and *d* as a quadratic function:

$$y(L) = -\frac{1}{(d-c)^2}(L^2 - 2cL + 2cd - d^2) \tag{3}$$

The whole model will be like *Figure.3*:

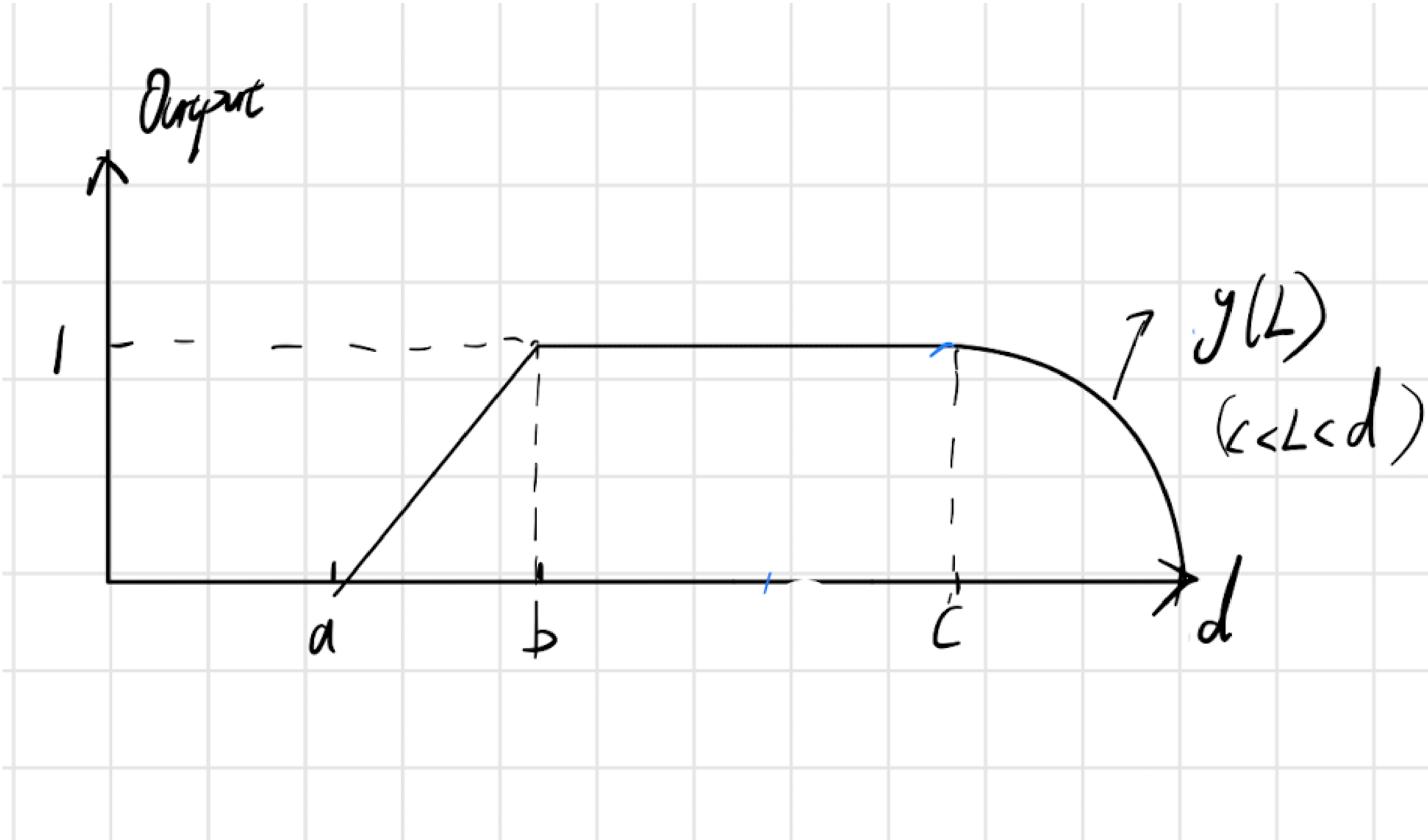

Figure.3 Another fitting model with quadratic function

As the analytic solution of optimum length is too complicated to solve, we got the numerical solution using MATLAB:

```
>> solve(S,L)

ans =

root(z^3 - (3*c*z^2)/2 + (3*b*c*d)/2 + (3*a*c*d)/2 - (3*b*d^2)/4 - (3*b*c^2)/4 - (3*a*d^2)/4 - (3*a*c^2)/4 + (3*c^3)/2 - c^2, z, 1)
root(z^3 - (3*c*z^2)/2 + (3*b*c*d)/2 + (3*a*c*d)/2 - (3*b*d^2)/4 - (3*b*c^2)/4 - (3*a*d^2)/4 - (3*a*c^2)/4 + (3*c^3)/2 - c^2, z, 2)
root(z^3 - (3*c*z^2)/2 + (3*b*c*d)/2 + (3*a*c*d)/2 - (3*b*d^2)/4 - (3*b*c^2)/4 - (3*a*d^2)/4 - (3*a*c^2)/4 + (3*c^3)/2 - c^2, z, 3)
```

Figure.4.1 Numerical Solution in MATLAB

Using the subs() function in MATLAB, the zero points of $\bar{Y}'$ can be expressed as:

```
S_1 =

(3*c^3)/2 - (3*a*c^2)/4 - (3*a*d^2)/4 - (3*b*c^2)/4 - (3*b*d^2)/4 - c^2 - (3*c)/2 + (3*a*c*d)/2 + (3*b*c*d)/2 + 1

>> S_2

S_2 =

(3*c^3)/2 - (3*a*c^2)/4 - (3*a*d^2)/4 - (3*b*c^2)/4 - (3*b*d^2)/4 - c^2 - 6*c + (3*a*c*d)/2 + (3*b*c*d)/2 + 8

>> S_3

S_3 =

(3*c^3)/2 - (3*a*c^2)/4 - (3*a*d^2)/4 - (3*b*c^2)/4 - (3*b*d^2)/4 - c^2 - (27*c)/2 + (3*a*c*d)/2 + (3*b*c*d)/2 + 27
```

Figure.4.2 Zero points of $\bar{Y}'$in MATLAB

However, due to the properties of $\bar{Y}$ and the various position of a, b, c, the optimum choice of L to maximize $\bar{Y}$ is too complicated to find using the given information.

In conclusion, the client or manager is suggested to collect data of a, b, c carefully, so that this model can lead to a more accurate solution of L.

Final Conclusion and Application advice

- According to the solutions above, we can find that different model would lead to different choice of the optimum length:
  - For the first model using linear shown in formula.2, the key-point of the application is to define the parameter *a*, *b* especially *c*, which affects the L' most. The client is suggested to choose the stopped ages as L' calculated above.
  - For the second model, the formula above shown the optimum length is too complicated to solve using limited data of a, b, c. So that it is suggested to collect more data.
- The holistic conclusion above is based on the limited condition shown in the content under title 'Assumptions and Simplifications'. To make the model work more accurately, the following advice could be applied in real conditions:
  - For large quantity of targets (crops area or machine quantity), some type of the crops or machines may not suitable to this single formula. For further work, it is suggested to classify the type of different targets, and construct different model for each types.
  - For the continuity of the production, it is advised that another model with interruption is needed in further research to improve the efficiency of the model.