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THE UNIVERSITY OF HONG KONG

STAT4601 Time Series Analysis

Group Project Report

Forecasting the CNY/HKD Exchange Rate:
A Time Series Analysis of Five Months of Closing Rates

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Contents

1. Introduction

This study focuses on forecasting the CNY/HKD exchange rate through a comprehensive time series analysis of five months of closing rates, where the data was collected from Yahoo Finance. The period under examination spans from February 13, 2023, to July 14, 2023, and holds immense significance in understanding the intricate economic dynamics between Mainland China and Hong Kong. These two regions share a substantial level of financial interdependence, with the Chinese Yuan serving as the official currency of the People's Republic of China and playing a pivotal role in Asian financial markets. Simultaneously, the Hong Kong Dollar, as the legal tender in Hong Kong, holds its own as a global financial hub currency.

The primary objective of this study is to leverage the closing exchange rate data from this five-month timeframe to accurately predict the CNY/HKD exchange rates for the subsequent five days, specifically from July 17 to July 21, 2023. Through time series analysis including data stationarity, model identification, and model fitting and checking, this research aims to provide valuable insights into the future trends of the CNY/HKD exchange rate.

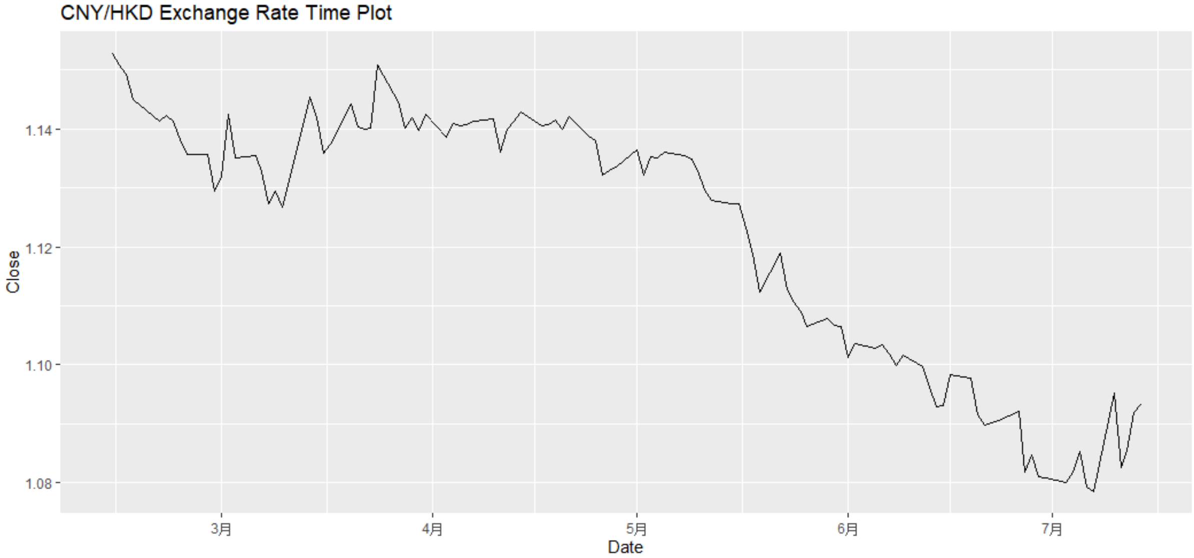
In addition, this study will further explore two specific areas of interest: Holt's Linear Exponential Smoothing and the GARCH Model. These discussions will provide insights into how effective and applicable these forecasting techniques are in predicting the CNY/HKD exchange rate.

2. Data Stationarity

In this section, we examined the stationarity of the data and transformed the original non-stationary time series into a stationary sequence. The time plot of the exchange rate is as follows.

Figure 1

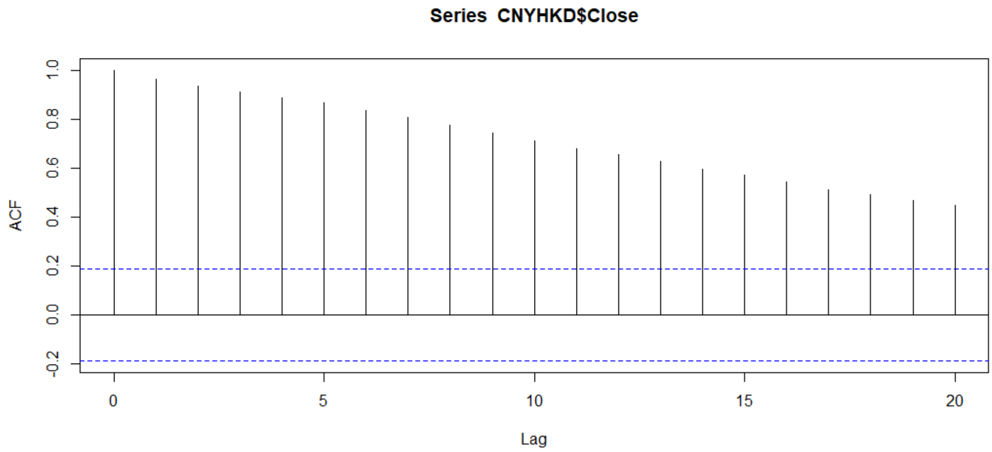
CNY/HKD exchange rate time plot



After observing the original time series in Figure 1, it is clear that it is not stationary. This conclusion is supported by two key observations. Firstly, the sample autocorrelation function (ACF) decays slowly as shown in Figure 2 below. Secondly, we conducted the Dickey-Fuller test and obtained a p-value of 0.8235. Since this p-value is greater than the significance level of 0.05, we cannot reject the null hypothesis. This means that there is not enough evidence to support the claim that the time series is stationary.

Figure 2

ACF plot of the exchange rate

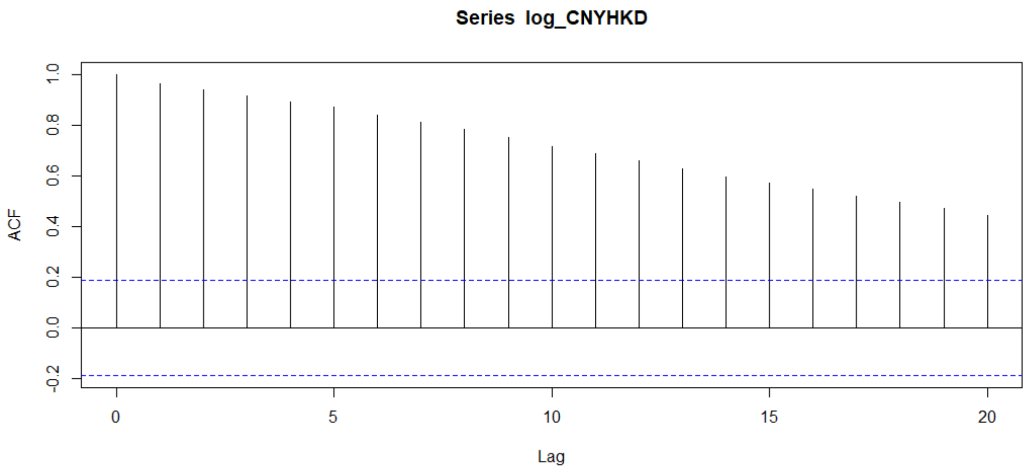


To address the issue of non-stationarity in the time series, we first apply a log transformation. This is necessary because the variance of the time series is not independent of time and the exchange rate data tends to fluctuate due to human activities.

By applying a log transformation, we aim to make the time series closer to a stationary series. To visualize the effect of the log transformation, we plot the ACF of the time series data after the transformation in the below figure.

Figure 3

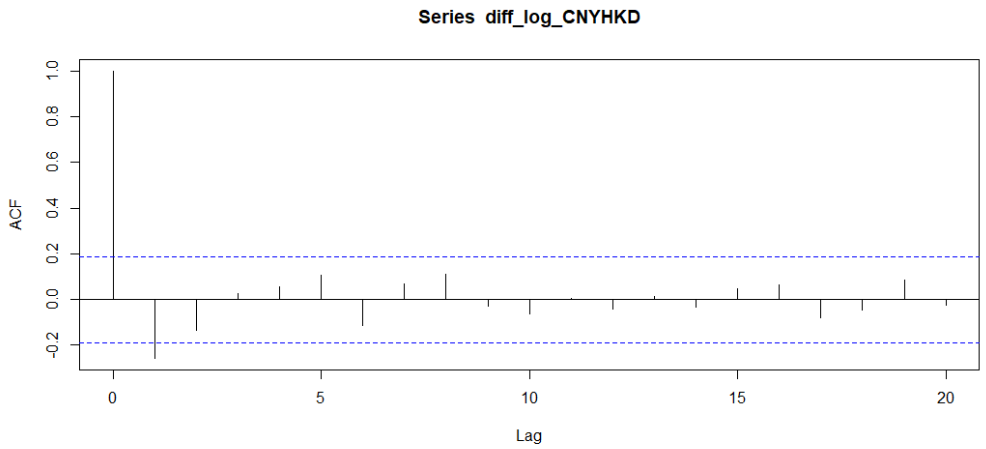
ACF plot of the exchange rate after log transformation



Despite applying the log transformation to the time series, the sample ACF continues to show slow decay. Meanwhile, the p-value of the Dickey-Fuller test is 0.7933, which is greater than 0.05. These suggest that the time series remains non-stationary.

We further noticed that it is non-stationary in the mean of the exchange rate data. Therefore, we further applied differencing on the log-transformed data in order to reach a stationary time series. We plotted the ACF of the time series data after differencing the log-transformed time series as follows.

Figure 4
ACF plot of the exchange rate after log transformation and differencing



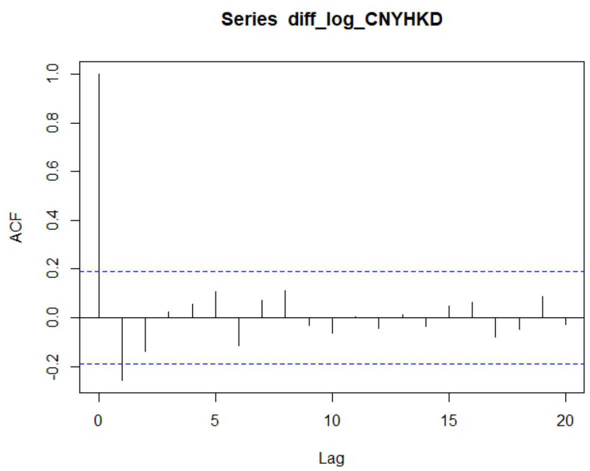
Notably, the ACF exhibits a rapid decay after the log transformation and differencing. At the same time, the p-value of the Dickey-Fuller test is 0.01, which is smaller than 0.05. Therefore, we now reach a stationary time series.

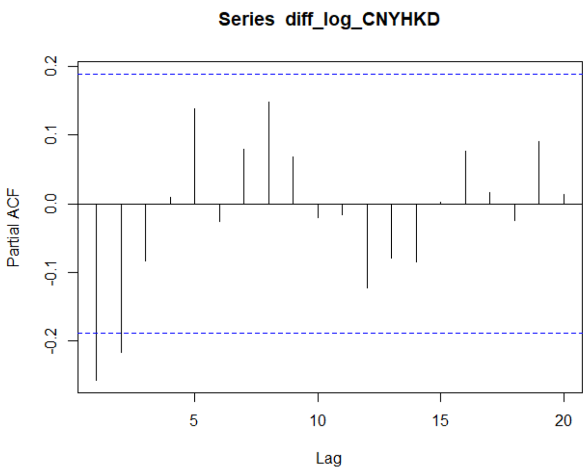
After ensuring the data stationarity, we then identified some preliminary models for further analysis.

3. Model Identification

In the preliminary model identification process, we consider the sample ACF and the sample partial autocorrelation function (PACF) to determine the appropriate model for our data. However, in this specific case, we do not consider a seasonal model due to the absence of seasonality in both the time plot and the ACF plot.

Figure 5
ACF (left) and PACF (right) after log transformation and differencing (exchange rate)





In Figure 5, the sample ACF cut off at lag 2, indicating that there might be a significant correlation up to lag 2. The sample PACF, on the other hand, also cut off at lag 2, suggesting a significant direct relationship up to lag 2. Based on these observations, we specify two models for our data, which are MA(2) and AR(2) for the differenced log-transformed time series.

4. AR Model

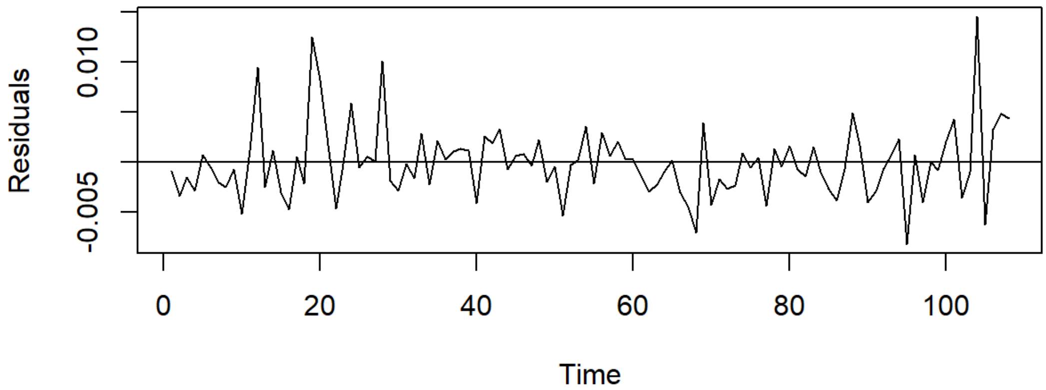
4.1. Model Diagnostics

4.1.1. Residual Analysis

Moving on to the Autoregressive (AR) model, we first conduct model diagnostics, beginning with the residual analysis. Figure 6 below shows the time plot of the residuals of the AR(2) model, which we consider to have the pattern of a white noise sequence.

Figure 6

Time Plot of Residual of the AR(2) Model



Next, we perform the Anderson-Darling normality test for the residuals of the AR(1), AR(2), AR(3), and AR(4) models. The results are presented in Table 1.

Table 1

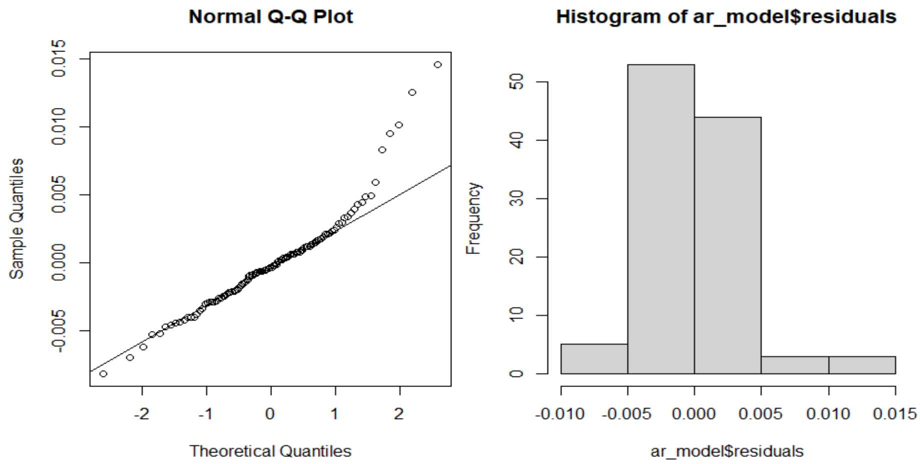
Normality Test Results for AR Models

Model	Test Statistic	p-value
AR(1)	2.4565	3.00e-06
AR(2)	1.8042	0.000121
AR(3)	1.8805	7.86e-05
AR(4)	1.8803	7.87e-05

In all cases, the p-values are significantly below the standard significance level of 0.05, indicating that we can reject the null hypothesis of normal distribution for the residuals in each model. This suggests that the residuals of these models do not follow a normal distribution. Similarly, we observe the distribution of the AR(2) residuals by Q-Q Plot and histogram, which also appear to be not normal. We then considered using the Conditional least squares (CLS) method, instead of the Maximum likelihood (ML) or unconditional least square (ULS) method for better parameter estimation.

Figure 7

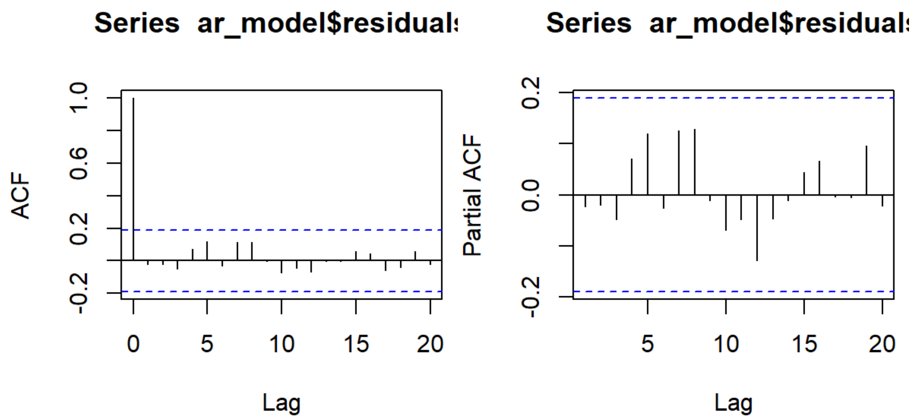
Normal Q-Q Plot and Histogram for AR(2) Residuals



Moreover, we observed that the AR(2) residuals do not cut off at ACF or PACF, indicating the current model is adequate.

Figure 8

ACF and PACF for AR(2) Residuals



Meanwhile, the p-values in the Ljung-Box test results for the residuals of the AR(2) model for each case in the table below are all above 0.05, indicating that the residuals are independently distributed. This suggests that the AR(2) model is adequate in terms of capturing the autocorrelation in the data.

Table 2

Ljung-Box Test Results for AR(2) Model

Lag	Test Statistic	p-value
10	2.5334	0.6387
10	6.2998	0.7096
15	7.4834	0.9145
20	9.0073	0.9734

4.1.2. Analysis of over-parameterized models

We then analyzed the coefficients for the AR(1) to AR(4) models, as shown in Table 3.

Table 3

Coefficients for AR Models

Model	Coefficient
AR(1)	-0.345
AR(2)	-0.259
AR(3)	-0.092
AR(4)	0.024

While the AR1 and AR2 coefficients remain significant, the AR3 and AR4 coefficients are relatively small, suggesting that the AR(2) model might indeed be sufficient.

4.2. Parameter estimation on AR(2)

Since we have concluded that the results from MLE are not normal, we decided to adopt the CLS approach to estimate the parameters. The estimated coefficients for the AR(2) model are presented in Table 4.

Table 4

Coefficients for AR(2) Models

Coefficient	Estimation	p-value
Constant	-0.0005	0.066
AR1	-0.3152	0.000
AR2	-0.2247	0.020

Then we looked at the AIC and BIC for the AR(2) model, which are -909.392 and -898.627, respectively. Both AR1 and AR2 coefficients are statistically significant (p-values < 0.05), indicating that the AR(2) model is a reasonable fit for the data.

4.3 AR Models Selection and Forecast

Based on the AIC comparison for AR models (Table 5), the AR(2) model has a lower AIC compared to the AR(1), AR(3), and AR(4) models, indicating it is the preferred model. We use the AR(2) model to forecast the next 5 values and then compare them with the actual values (Table 6).

Table 5

AIC Comparison for AR Models

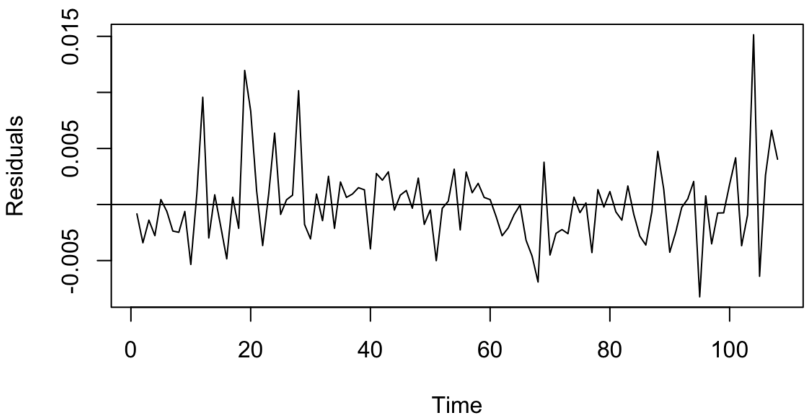
Model	AIC
AR(1)	-906.099
AR(2)	-909.392
AR(3)	-908.343
AR(4)	-906.384

Table 6
Forecast using AR(2) model

Period	Point Forecast	Actual Value
111	1.090562916	1.094058307
112	1.090279407	1.089245223
113	1.090130049	1.088175021
114	1.089402085	1.080250545
115	1.088827033	1.088490637

5. MA Model
5.1 Model Diagnostics
5.1.1. Residual Analysis
After the analysis of the AR model, we then shifted the focus to the Moving average (MA) model. Figure 9 shows a time plot of the residuals of the MA(2) model. Similar to the AR model, the residual seems to have the pattern of a white noise sequence.

Figure 9
Time plot of the residuals of the MA(2) model:



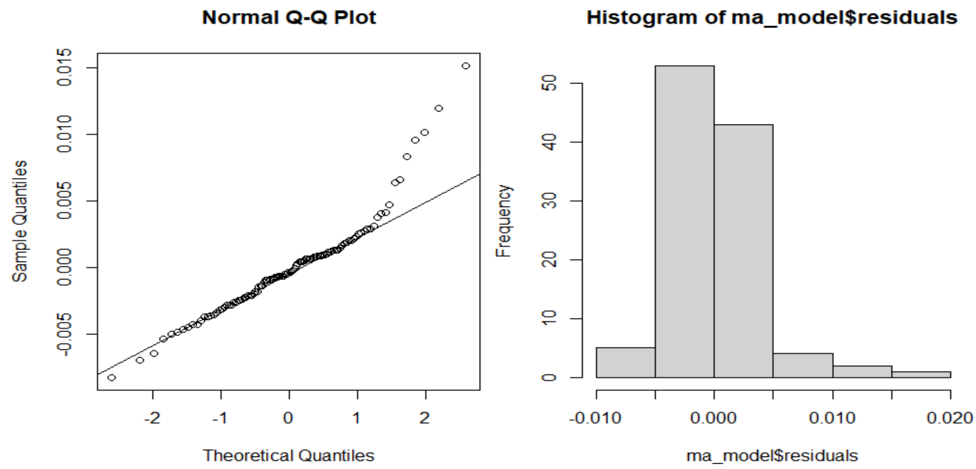
The Anderson-Darling normality test was conducted for the residuals of the MA(1), MA(2), MA(3), and MA(4) models. The results are presented in Table 7, while Figure 10 presents the normal Q-Q plot and histogram for the MA(2) residuals. In all cases, the p-values are significantly below the standard significance level of 0.05, indicating that we can reject the null hypothesis of normal distribution for the residuals in each model. This suggests that the residuals of these models do not follow a normal distribution. Similar to the testing result, the Q-Q Plot and histogram also do not show a normal distribution pattern. The CLS method should then be considered for better parameter estimation as it does not assume the distribution of the time series.

Table 7
Normality Test Results for MA Models

Model	Statistic	p-value
MA(1)	2.486	2.54e-06
MA(2)	2.0824	2.497e-05
MA(3)	1.8388	9.953e-05
MA(4)	2.0028	3.923e-05
ARMA(1,2)	2.0039	3.899e-05

Figure 10

Normal Q-Q Plot and histogram for MA(2) Residuals



After that, we checked the model adequacy by utilizing the ACF and PACF plots of the residuals as well as conducting the Ljung-Box test. The ACF and plots (Figure 11) do not show additional cut-off situations in the residual. Meanwhile, the p-values in the Ljung-Box test (Table 8) are all above 0.05, indicating that the residuals are independently distributed. This suggests that the MA(2) model is adequate in terms of capturing the autocorrelation in the data.

Figure 11

ACF and PACF for MA(2) Residuals

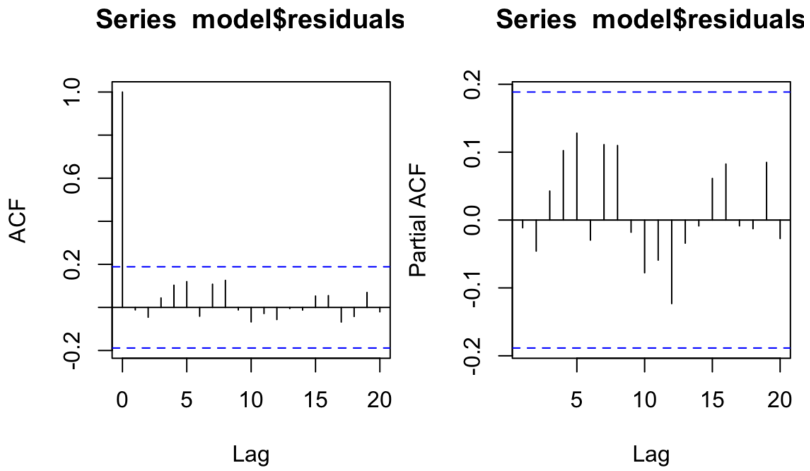


Table 8

Ljung-Box Test Results for MA(2) Model

Lag	Test Statistic	p-value
5	3.3261	0.6499
10	7.3155	0.6954
15	8.1791	0.9164
20	10.096	0.9664

In each case, the p-value is well above 0.05, indicating that the residuals are independently distributed. This suggests that the MA(2) model is adequate in terms of capturing the autocorrelation in the data.

5.1.2. Analysis of over-parameterized models

The analysis of over-parameterized models reveals the coefficients for the MA(1), MA(3), MA(4), and ARMA(1, 2) models as follows:

- MA(1) model: ma1 = -0.3729371
- MA(3) model: ma1 = -0.3514362 ma2 = -0.1365129 ma3 = 0.1193522
- MA(4) model: ma1 = -0.373057 ma2 = -0.1438567 ma3 = 0.05950927 ma4 = 0.1312305
- ARMA(1, 2) model: ar1 = -0.759883 ma1 = 0.4299668 ma2 = -0.3577995

Since MA(1), (3), and (4) models do not show significantly different results for the extra parameters, the MA(2) model seems to be adequate.

5.2. Parameter Estimation

The CLS approach was adopted for parameter estimation. The estimated coefficients for the MA(2) model are as follows:

- Coefficients:

- Constant: -0.000484
- MA1 Coefficient: -0.3172571
- MA2 Coefficient: -0.09325022

At the same time, the model fit is evaluated using the AIC and BIC. For the MA(2) model, the AIC is -899.87 and the BIC is -889.1414.

5.3. Model Selection and Forecast

We then selected the model based on AIC. The table below presents the AIC comparison for the MA models, including the AIC values for each model.

Table 9

AIC Comparison for MA Models

Model	AIC
MA(1)	-900.8653
MA(2)	-899.8699
MA(3)	-899.2088
MA(4)	-898.6051

Based on the AIC values, the MA(2) model has a slightly lower AIC than the MA(3) and MA(4) models, indicating that it could be preferred. However, the ACF does not justify the use of the MA(1) model despite its lower AIC value.

To forecast the next 5 values, the AR(2) model is used and the results are compared with the actual values. The forecasted values and actual values are presented in Table 10.

Table 10

Forecast using MA(2) model

Period	Point Forecast	Actual Value
111	1.090654059	1.094058307
112	1.08971541	1.089245223
113	1.089187843	1.088175021
114	1.088660542	1.080250545
115	1.088133486	1.088490637

6. Model Selection and Forecasting Data

6.1 Model Selection

After selecting and comparing the models and results for AR and MA, we wanted to further compare the fitting and forecasting performance of the selected models from sections 4 and 5. The below compares the metrics and forecasting differences of the AR(2) and MA(2) models by considering their AIC, BIC, and forecasting differences.

Table 11

Metrics comparison of AR(2) and MA(2) model

Metircs	AR(2)	MA(2)
AIC(Akaike Information Criterion)	-909.392	-899.87
BIC (Bayesian Information Criterion)	-898.627	-889.1414
Forecasting difference 2023-07-17	0.003194885	0.003111579
Forecasting difference 2023-07-18	0.00094945	0.000431663
Forecasting difference 2023-07-19	0.001796612	0.000930753
Forecasting difference 2023-07-20	0.008471683	0.007785229
Forecasting difference 2023-07-21	0.000309048	0.000328116

To compare the fitting effectiveness of the AR(2) and MA(2) time series models on our data, we can analyze various criteria and forecasting differences. The AIC and BIC are commonly used criteria for model comparison, with lower values indicating a better model fit. In our dataset, the AR(2) model has an AIC of -909.392 and a BIC of -898.627, while the MA(2) model has an AIC of -899.87 and a BIC of -889.1414. These values suggest that the AR(2) model has a marginally better fit according to these criteria.

Furthermore, examining the forecasting differences for specific dates can provide insights into the models' predictive accuracy. For example, on 2023-07-17, the forecasting difference is 0.0032 for the AR(2) model and 0.003116 for the MA(2) model. A smaller forecasting difference indicates a closer fit to the actual data. Across the provided dates, both models show similar forecasting differences, indicating that both models have good fits.

In conclusion, based on the AIC and BIC values and the comparable forecasting differences, we still select the AR(2) model as the best model for our dataset.

6.2 Forecasting values

In section 6.2, we delve into the forecasting values for both the AR(2) model and the actual values to gain further insights into the performance and accuracy of the models.

Table 12

Metrics comparison of AR(2) and Actual value

Forecast Date	AR(2) forecasting value	Actual value
2023/7/17	1.090562916	1.094058307
2023/7/18	1.090279407	1.089245223
2023/7/19	1.090130049	1.088175021
2023/7/20	1.089402085	1.080250545
2023/7/21	1.088827033	1.088490637

By comparing the forecasting values to the actual values, we can evaluate the models' ability to accurately predict future observations. A smaller difference between the forecasting value and the actual value indicates a closer fit to the real data. Analyzing the values in Table 12, we observe that the forecasting differences between the AR(2) model and the actual values are relatively small across all the listed dates. This suggests that the AR(2) model is performing well in terms of forecasting accuracy. However, it is important to note that the MA(2) model's forecasting differences are also comparable, indicating a good fit for that model as well.

6.3 Insights

Our time series analysis yielded remarkably smooth and successful predictions, a fact we attribute to several key insights. Firstly, the models we hypothesized based on the PACF and ACF analysis proved to be exceptionally well-suited for our data. This strong alignment between our chosen models and the data's characteristics was a crucial factor in the accuracy of our predictions.

Additionally, the results from our model diagnostics were overwhelmingly positive, indicating that our models were well-specified and effectively captured the underlying patterns in the data. This level of model fitness is not always easy to achieve in time series analysis, suggesting that our methodological choices were particularly effective.

A significant factor contributing to this success was the inherent trend within our series. Our analysis revealed a consistent downward pattern in the data, which likely made the series more predictable. In time series analysis, data with clear trends or patterns can often be more accurately forecasted, as models can leverage these inherent structures to make more accurate predictions.

Moreover, it's likely that our initial data transformation process, involving log transformation and differencing, played a pivotal role in smoothing out any irregularities and rendering the series more amenable to analysis. By converting the series into a stationary format, we mitigated the impact of non-stationarity, which often complicates time series analysis and hinders accurate forecasting.

In conclusion, the combination of a well-chosen model, robust diagnostics, and an inherently predictable trend within the data contributed to the smooth and successful predictions in our project. This serves as a testament to the power of rigorous analytical techniques and careful data processing in the realm of time series forecasting.

7. Further Discussion I: Holt's linear exponential smoothing

In this section, we will discuss Holt's Linear Exponential Smoothing as a forecasting method for predicting future values of the CNY/HKD exchange rate. We will provide an introduction to the model, specify the variables used, present the model specification, interpret the results, and provide a forecast using the model.

7.1 Model Introduction

Holt's Linear Exponential Smoothing, also known as double exponential smoothing, is a forecasting method that incorporates trend information to predict future values based on the trend and level of the data. It is an extension of the simple exponential smoothing method. Given that the time plot of the CNY/HKD exchange rate exhibits a clear trend, Holt's linear exponential smoothing can be an appropriate method for forecasting future values.

7.2 Variable Specification

The variable used in this analysis is the CNY/HKD exchange rate. Since the raw exchange rate data has a clear trend but no seasonality, there is no need for any preprocessing.

7.3 Model Specification

The model specification for Holt's Linear Exponential Smoothing involves two main equations: the level equation and the trend equation (Hyndman & Athanasopoulos, 2018). The equations are represented by:

$$Level\ equation:\ L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$Trend\ equation:\ T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1}$$

where L_t is the level at time t, α is the smoothing parameter for the level, Y_t is the time series; T_t is the trend at time t, β is the smoothing parameter for the trend. These equations are used to calculate the smoothed level and trend values for each time period.

For this analysis, Holt's method was employed to forecast the future five values of the CNY/HKD exchange rate using the k-step ahead forecast (Hyndman & Athanasopoulos, 2018):

$$Y_t(k) = L_t + kT_t$$

We fit two models with and without β, the trend smoothing parameter, for comparison. The model parameters are summarized in Table 13 below. In the model with β, the level component (l = 1.1527) represents the estimated average value of the exchange rate, indicating it is expected to be around 1.1527. The trend component (b = -0.0019) suggests a slight downward trend in the exchange rate, with an average decrease of 0.0019 units per time period. In the model without β, only the initial state for the level component (l = 1.1527) is provided, indicating no trend component, which assumes the exchange rate to remain relatively stable. These initial state values, along with the smoothing parameters (alpha and beta), are used in Holt's method to generate forecasts for future time periods.

Table 13

Model Parameters

Model	Smoothing Parameters	Initial States	Sigma	ACF1
With β	alpha = 0.1972, beta = 1	l = 1.1527, b = -0.0019	0.0041	0.313
Without β	alpha = 0.6969	l = 1.1527	0.0044	-0.0179

Based on the error measures in Table 2, it can be observed that both models perform reasonably well in forecasting the CNY/HKD exchange rate. However, the model without β generally exhibits slightly better accuracy, as indicated by its lower values for RMSE, MAE, MPE, MAPE, and MASE. The model with β shows a small positive ME, while the model without β has a slightly negative ME. These findings suggest that the model without β may provide more reliable forecasts for the CNY/HKD exchange rate.

Table 14

Model Error Measures

Model	ME	RMSE	MAE	MPE	MAPE	MASE
With β	0.0002	0.0044	0.0032	0.0183	0.2852	1.044
Without β	-0.0008	0.0041	0.0029	-0.0711	0.2613	0.9545

7.4. Model Forecast and Interpretation

Holt's Linear Exponential Smoothing model provided forecasts for the CNY/HKD exchange rate for the next 5 periods. The point forecasts (model with trend smoothing parameter), as well as the lower and upper bounds for 80% and 95% confidence intervals, are as follows:

Table 15

Forecast result from Holt's Linear Exponential Smoothing model with trend smoothing parameter

Period	Point Forecast	80% CI	95% CI	Actual Value
111	1.095613	[1.089945, 1.095613]	[1.086945, 1.104281]	1.094058
112	1.098211	[1.089370, 1.098211]	[1.084690, 1.111732]	1.089245
113	1.100809	[1.085537, 1.100809]	[1.077453, 1.124166]	1.088175
114	1.103408	[1.079709, 1.103408]	[1.067164, 1.139651]	1.080251
115	1.106006	[1.072428, 1.106006]	[1.054652, 1.157359]	1.088491

The model without trend smoothing parameter (Table 2) also provides point forecasts, 80% and 95% confidence intervals, and actual values for the next 5 periods.

Table 16

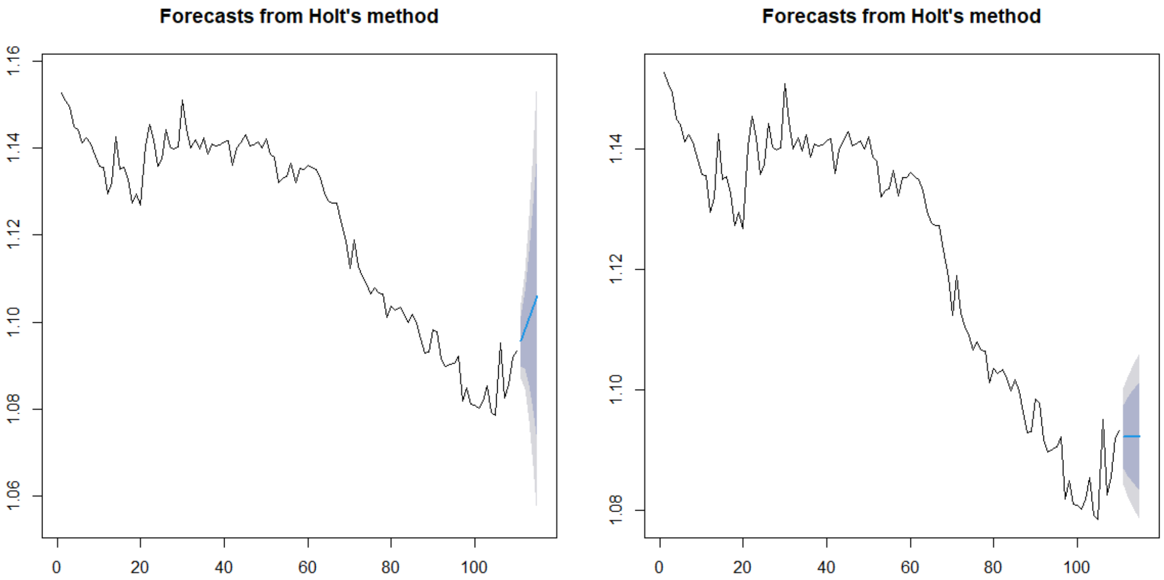
Forecast result from Holt's Linear Exponential Smoothing model without trend smoothing parameter

Period	Point Forecast	80% CI	95% CI	Actual Value
111	1.092248	[1.086979, 1.097516]	[1.084190, 1.100305]	1.094058
112	1.092248	[1.085826, 1.098669]	[1.082427, 1.102069]	1.089245
113	1.092248	[1.084851, 1.099645]	[1.080935, 1.103560]	1.088175
114	1.092248	[1.083990, 1.100506]	[1.079618, 1.104877]	1.080251
115	1.092248	[1.083211, 1.101285]	[1.078427, 1.106069]	1.088491

Comparing the two models, we can see that the point forecasts for the exchange rate are relatively close, with slight variations. The model with the trend smoothing parameter tends to have wider confidence intervals, indicating higher uncertainty in the forecasts as in Figure 10. Additionally, when comparing the actual values to the point forecasts, both models are able to capture the general trend of the exchange rate. However, there are differences in the accuracy of the forecasts for each period.

Figure 12

Comparison of the forecasting result from Holt's Linear Exponential Smoothing model with (left) and without (right) trend smoothing parameter



In some scenarios, the forecast result from Holt's Linear Exponential Smoothing model without the trend smoothing parameter may be better despite the presence of a visible trend in the original data. This could be because the trend component in the data is not strong or consistent, and including a trend smoothing parameter introduces unnecessary complexity and noise into the model. Overall, the inclusion of the trend smoothing parameter provides additional information and flexibility in capturing the trend component of the CNY/HKD exchange rate. However, it also introduces more uncertainty in the forecasts. The choice between the two models depends on the specific needs and preferences of the analysis.

8. Further Discussion II: Garch Model

In previous studies, the ARIMA models were applied to handle temporal dependencies, and they assumed homoscedasticity (i.e., constant variance over time) of the residuals. However, this assumption is often violated in real applications, showing periods of high variability followed by periods of low variability. The ARCH-type models fill this gap by allowing the model's error variance to be a function of past error terms, hence capturing the heteroskedasticity in the data.

ARCH-type models are particularly useful when dealing with financial and economic data. Financial data such as exchange rates can exhibit patterns of volatility, including periods of swings and tranquillity. These patterns are often not captured by simpler models like

ARIMA, which assume constant variance over time. Therefore, to enhance the accuracy and reliability of our predicted confidence intervals, we will be employing ARCH-type models in this section.

8.1 Heteroskedasticity Detection

We first re-examine the residual plots of AR(2) and MA(2) models for the exchange rate after diff_log transformation to check if the variances exhibit certain patterns. The two residual plots are shown below.

From the residual plots, we can observe that the variances of the residuals are not consistent over time intervals, suggesting the presence of volatility clustering in our time series data, where periods of high variance alternate with periods of low variance. Specifically, the variances in the intermediate time period are lower than the time before and after.

To further substantiate this observation, we conduct more formal statistical tests namely the ARCH Lagrange Multiplier test.

- Null hypothesis: there are no ARCH effects in the residuals.
- Alternative hypothesis: The ARCH effects are present.

For AR(2):

```
        ARCH LM-test; Null hypothesis: no ARCH effects

data:  ar2_model1$residuals
Chi-squared = 21.347, df = 12, p-value = 0.04552
```

Since the p-value is less than 0.05, we may reject the null hypothesis at a 5% significance level to conclude that the ARCH effects are present.

For MA(2):

```
        ARCH LM-test; Null hypothesis: no ARCH effects

data:  ma2_model$residuals
Chi-squared = 20.775, df = 12, p-value = 0.05378
```

Since the p-value is greater than 0.05, we cannot reject the null hypothesis at a 5% significance level. But it is very close to 0.05, so there may be mild ARCH effects. Therefore, it is appropriate to apply the ARCH-type model to our time series data for further analysis.

8.2 Model Introduction

The Generalised Autoregressive Conditional Heteroscedasticity-GARCH (p,q) model is an extension of the ARCH model that combines moving average elements (MA) with autoregressive elements (AR). In this case, the GARCH (p,q) model is given by

$$r_t = \sigma_t \varepsilon_t; \sigma_t^2 = \omega + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{p} \alpha_j a_{t-j}^2$$

where r_t is the error term of the ARIMA model with non-constant variance. To guarantee a positive variance in all instances, the restrictions are imposed as $w > 0, \alpha_j, \beta_i \geq 0$.

In general, the returns of financial assets are often characterized more in terms of "bad" news rather than "good" news, known as “leverage effect” (Black, 1976). The term "leverage" comes from the observation that an asset's volatility tends to increase when its returns are negative. In order to capture the asymmetry in return volatility or the "leverage effect," asymmetric GARCH models have been developed. Our study also applies two of them, namely Exponential GARCH (EGARCH) and GJR-GARCH.

The EGARCH is defined as

$$\log \sigma_t^2 = \omega + \sum_{k=1}^{q} \beta_k g(Z_{t-k}) + \sum_{k=1}^{p} \alpha_k \log \sigma_{t-k}^2$$

where $g(Z_t) = \theta Z_t + \lambda(|Z_t| - E(|Z_t|))$. The function uses log transformation to capture the exponential increase or decrease trend regarding variance.
The formulation of GJR-GARCH is

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{p} (\alpha_j a_{t-j}^2 + \gamma_j I_{t-j} a_{t-j}^2)$$

where γ_j represents the leverage term. The indicator function I takes on the value of 1 for $a_{t-j} < 0$ and 0 otherwise.

8.3 Model Fitting and Selection

In this section, the “rugarch” package in R will be used for model fitting as it provides a more flexible and extensive framework for univariate GARCH modeling than the “fGarch” package. We compare the three GARCH models mentioned above with different combinations of parameters and evaluate their performances based on AIC and BIC.
The parameters and possible values are listed below:

Table 17
Parameters and possible values of GARCH models

Parameters	Possible Values
variance model	'sGARCH', 'eGARCH', 'gjrGARCH'
AR order	0,1
MA order	0,1
ARCH order	0,1,2
GARCH order	0,1
distribution model	Normal, Student-t

We then visualized the performances of the models in the following figure.

Figure 13

Performances visualization of the Garch models

Then, we examine the model with the lowest AIC or BIC, denoted by the star in the above image. The best-performed model is shown below

Figure 14

Best-performed Garch Model

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model     : gjrGARCH(1,0)
Mean Model      : ARFIMA(1,0,0)
Distribution    : std

Optimal Parameters
------------------------------------
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000526    0.000001 -437.283        0
ar1    -0.363428    0.000813 -447.064        0
omega   0.000027    0.000000  501.197        0
alpha1  0.409746    0.000817  501.292        0
gamma1 -0.819383    0.001637 -500.539        0
shape   2.409996    0.110126   21.884        0

Robust Standard Errors:
        Estimate  Std. Error  t value Pr(>|t|)
mu     -0.000526    0.000007 -76.6619        0
ar1    -0.363428    0.009952 -36.5176        0
omega   0.000027    0.000002  11.4053        0
alpha1  0.409746    0.049192   8.3295        0
gamma1 -0.819383    0.082209  -9.9670        0
shape   2.409996    0.118163  20.3955        0

LogLikelihood : 470.0456
```

We can observe that the p-values for all parameters are less than 0.05, implying that all parameters are significantly different from zero. In other words, each parameter contributes meaningfully to the model. Therefore, the estimated GJR-GARCH model is

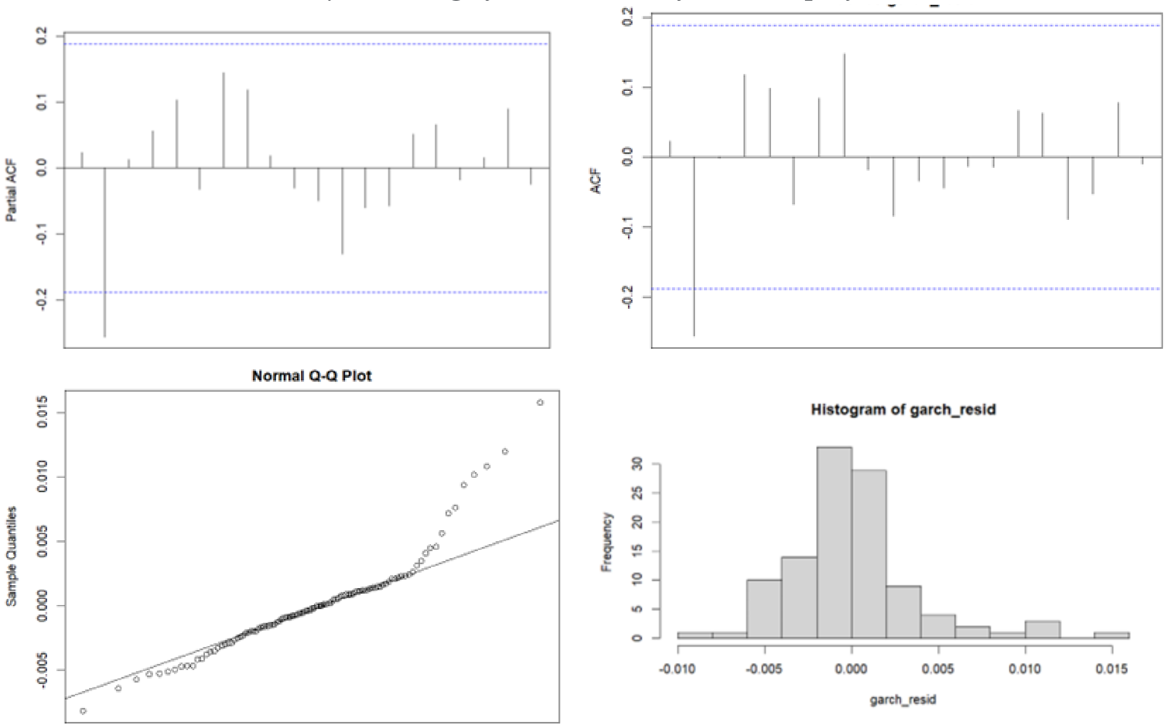
Mean Equation (ARFIMA): $y_t = -0.000526 - 0.3634y_{t-1}$

Variance Equation (GJR-GARCH): $h_t = 0.000027 + 0.409746 \times a_{t-1}^2 - 0.819383I_{t-1}a_{t-1}^2$

By plotting the ACF and PACF of residuals, most lag values fall within the required boundaries, with only a few outliers that can be treated as random occurrences. The Q-Q plot and histogram are also consistent suggesting a normal distribution with constant mean and variance. These all indicate that the model fitting is reasonable.

Figure 15

ACF, PACF, and Normality checking of the residual of the best-performed Garch model



Besides, we also perform the Ljung-Box test on the squared residuals to check for possible autocorrelation structures.

- Lag 10: Test statistic = 7.5061, p-value = 0.677
- Lag 15: Test statistic = 9.133, p-value = 0.8705
- Lag 20: Test statistic = 10.865, p-value = 0.950

For each case, the p-value is significantly greater than 0.05. We cannot reject the null hypothesis, indicating that any correlations in the residuals do not significantly differ from what would be expected by chance, suggesting that your GARCH model could be a good fit.

8.4 Model Forecasting

Eventually, the GJR-GARCH model specified above is used to generate forecasts for the return of the CNY/HKD exchange rate. The forecast is called an "unconditional" forecast because it does not involve any rolling-window analysis. It's a simple "one-shot" forecast based on the fitted model. The forecasted results are as follows.

Table 18

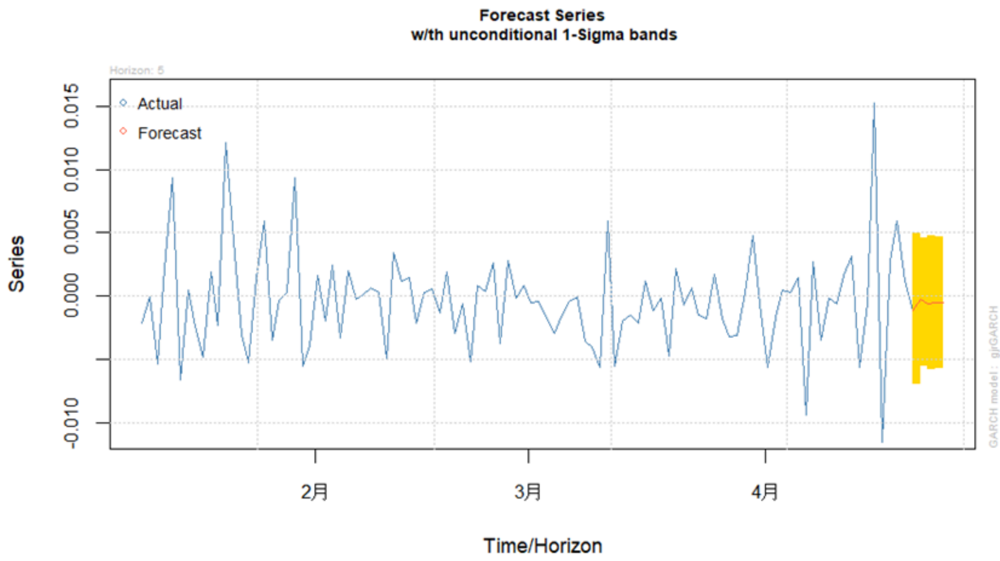
Forecast results of the best-performed GARCH models

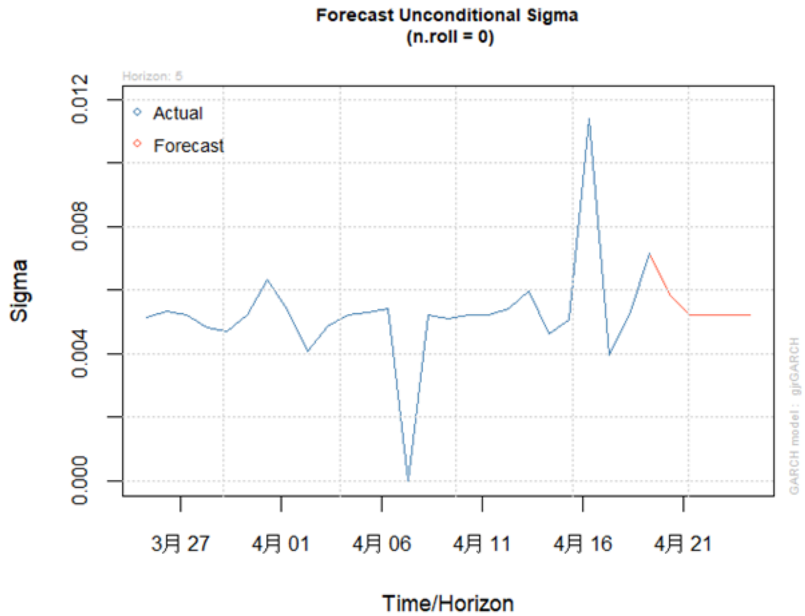
	Mean	SD	95% Upper	95% Lower
1	-0.001157035	0.07643351	0.1486527	-0.1509667
2	-0.000296469	0.07231846	0.1414477	-0.1420407
3	-0.000609222	0.07231823	0.1411345	-0.1423529
4	-0.000495559	0.07231822	0.1412481	-0.1422393
5	-0.000536868	0.07231822	0.1412086	-0.1422806

We can also visualize the results in the graphs below.

Figure 16

Visualization of the Best-performed Garch Model





Based on the graphs, the forecasted return line fluctuates marginally and remains flat, indicating that the return of the exchange rate will stay relatively stable. Besides, the volatility decreases and remains stable. This reduction in volatility may suggest that there is less uncertainty and a lower possibility of significant changes.

9. Conclusion

In concluding our time series report, we focused on predicting the CNY/HKD exchange rate through an extensive analysis of five months of closing rates, sourced from Yahoo Finance. Initially, our assessment revealed that the original data was not stationary. To address this, we applied log transformation and differencing, effectively converting the series into a stationary format.

Our analysis proceeded with the examination of ACF and PACF. This exploration led us to hypothesize that AR(2) and MA(2) models could be appropriate for the differenced and logarithmically transformed stationary series.

Subsequently, we conducted a thorough model diagnosis for both AR(2) and MA(2) models, which included normality tests and Ljung-Box tests. We also analyzed over-parameterized models similar to AR(2) and MA(2) to ensure the robustness of our findings. Despite examining various models, AR(2) and MA(2) consistently emerged as the best fit for our data.

After that, we estimated parameters and forecasted future values using both models. To determine the most effective model, we compared their Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), and forecast differences. This comparison led us to conclude that the AR(2) model yielded the most reliable results for forecasting the CNY/HKD exchange rate.

Considering the descending trend observed from the time plot, we explored Holt's Linear Exponential Smoothing as a forecasting method for the CNY/HKD exchange rate. There were two models fitted, one with the trend smoothing parameter and one without. Both models performed reasonably well, even just by an 80% confidence interval, but the model without the trend smoothing parameter generally exhibited slightly better accuracy in terms of error measure and forecasting results. A possible explanation might be the diminishing or inconsistent trend in the exchange rate.

Finally, our extensive analysis using GARCH models has provided valuable insights into the volatility of the CNY/HKD exchange rate. Utilizing the "rugarch" package in R, we compared the performance of several GARCH models, including sGARCH, eGARCH, and gjrGARCH, by using AIC and BIC criteria. The most appropriate model appeared to be gjrGARCH, which effectively captured the "leverage effect" inherent in the return of the exchange rate, with its parameters showing statistical significance. In forecasting the CNY/HKD exchange rate return, the GJR-GARCH model predicted a relatively stable outlook.

10. Reference

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11. Contribution

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