AMATH 482 Homework 1

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January 2020

Abstract

Locating a submarine and finding its trajectory by using the acoustic signature.

1 Introduction and Overview

By using a broad spectrum recoding of acoustics, the data of a moving submarine is collected over a 24-hour period in half-hour increments. In order to hunt the submarine using the acoustic signature and identify its acoustic admissions, we use the following ways to achieve this goal:

1.1 Determine the Frequency Signature

In order to find the center frequency generated by the submarine, we need to average the spectrum by using 3D Fourier Transform.

1.2 Plot the Path of the Submarine

By filtering the data around center frequency we found above, we can find the path of the submarine.

1.3 Sending Aircraft

According to the trajectory found above, we can provide a prediction of sending the P-8 Poseidon subtracking aircraft with a specific x and y coordinates.

2 Theoretical Background

As we learned from our lecture [1], Fourier introduced the concept of representing a given function $f(x)$ by a trigonometric series of sines and cosines:

$$f(x) = \frac{a_0}{2} + \sum_{i=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad x \in (-\pi, \pi].$$

Then we learned about the fast Fourier Transform to calculate the discrete Fourier Transform with total complexity far quicker than Fourier Transform.

As we learned from lecture [2], filter in math is a function we multiply the signal by, for example, Gaussian:

$$F(k) = e^{\tau(k-k0)^2}$$

where τ determines the width of the filter and constant k_0 determines the center of the filter.

3 Algorithm Implementation and Development

Prepare the data by transform the signals into Fourier domain:

```
L = 10; % spatial domain
n = 64; % Fourier modes
x2 = linspace(-L,L,n+1); x = x2(1:n); y =x; z = x;
```

Find center frequency by building a zero matrix first:

```
Uave = zeros(n,n);
```

Then add all 49 realizations within the loop and use Fourier Transform to shift them to Fourier domain (we use fftn() here because the data is three-dimensional):

```
for j=1:49
   Un(:,:,:)=reshape(subdata(:,j),n,n,n);
   Uave = Uave + fftn(Un(:,:,:));
end
```

To have the average, we need to shift the average to the correct order by using fftshift(), and divide it by length of 49.

```
Uave = fftshift(Uave(:,:,:))/length(Uave);
```

Plug in the average domain into frequency domain, which is [Kx, Ky, Kz], then we get the center frequency, which is the highest point in this domain.

```
[row,ind] = max(Uave(:));
[a,b,c] = ind2sub(size(Uave),ind);
xp = Kx(a,b,c);
yp = Ky(a,b,c);
zp = Kz(a,b,c);
center = [xp, yp, zp];
```

Filtering the data:

1. Filtering the data around the center frequency by denoise it, using Gaussian function
2. Determine the trajectory of the submarine and plot it
3. Making a prediction of the x, y coordinates where the aircraft should be sent.

4 Computational Results

The center frequency is shown in the table as following:

x-coordinate	y-coordinate	z-coordinate
5.3407	-6.9115	2.1991

Table 1

The trajectory of submarine is shown in Figure 1.

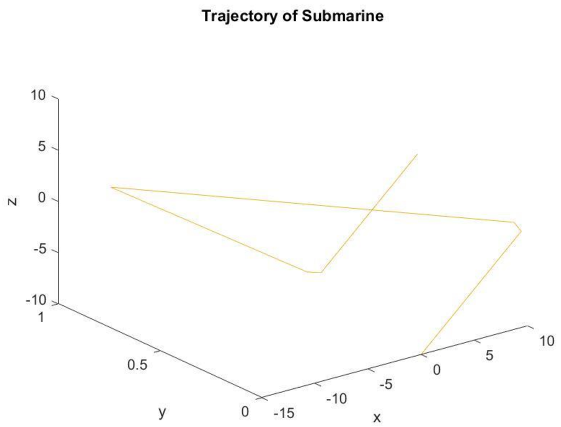

Figure 1

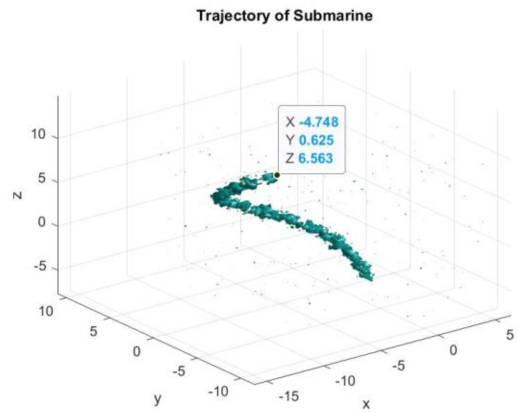

Figure 2

The predicted x,y coordinates we should send the aircraft is shown in Figure 2 above.

Algorithm 1: Fourier Transform

Import data from subdata.mat
for $j = 1 : 49$ **do**
Extract measurement j from subdata
Sum the average after using the Fourier transform
end
Calculate the average by dividing the Shifted Fourier transform by length.
Finally find the center frequency.

Algorithm 2: Filter data using Gaussian function

Import data from subdata.mat
Set up tau and k_0 with Gaussian function
Get the equation for filter ready
Plot the trajectory.

3 Summary and Conclusions

- The center frequency is centered around the point (5.3407, -6.9115, 2.1991)
- The trajectory of the submarine is clear to see in Figure 1 after filtering the data.
- According to Figure 2, the trajectory of the submarine, we may assume that the next step is (-4.748, 0.625), where we should send the aircraft to.

References

[1] Jose Nathan Kutz. *Data-driven modeling & scientific computation: methods for complex systems & big data*. Oxford University Press, 2013.

[2] MathWork. *https://ww2.mathworks.cn/help/matlab/ref/fft.html*

[3] Jason J. Bramburger, Lecture Notes for AMATH482, Winter, 2021

Appendix A MATLAB Functions

- y = linspace(x1,x2,n) returns a row vector of n evenly spaced points between x1 and x2.
- [X,Y] = meshgrid(x,y) returns 2-D grid coordinates based on the coordinates contained in the vectors x and y. X is a matrix where each row is a copy of x, and Y is a matrix where each column is a copy of y. The grid represented by the coordinates X and Y has length(y) rows and length(x) columns.
- fftn(x) returns multidimensional Fourier transform of the array using fast Fourier transform algorithm.
- sech(x) returns the hyperbolic secant of the elements of x.
- plot3(x, y, z) draw coordinates in 3D spaces.
- Y = fftshift(x) rearranges the fourier transform by moving the zero frequency component to the center of the array x
- [row,col] = Ind2sub(size, ind) convert linear index to subscript

Appendix B MATLAB Code

```
clear all; close all; clc

load subdata.mat % Imports the data as the 262144x49 (space by time) matrix called subdata

L = 10; % spatial domain
n = 64; % Fourier modes
x2 = linspace(-L,L,n+1);
x = x2(1:n);
y =x;
z = x;
k = (2*pi/(2*L))*[0:(n/2 - 1) -n/2:-1];
ks = fftshift(k);
Uave = zeros(n,n,n);
[X,Y,Z]=meshgrid(x,y,z);
[Kx,Ky,Kz]=meshgrid(ks,ks,ks);

for j=1:49
   Un(:,:,:)=reshape(subdata(:,j),n,n,n);
   Uave = Uave + fftn(Un(:,:,:));

%     M = max(abs(Un),[],'all');
%     close all,
%     isosurface(X,Y,Z,abs(Un)/M,0.7)
%     axis([-20 20 -20 20 -20 20]), grid on, drawnow
%     pause(1)
end
Uave = fftshift(Uave(:,:,:))/length(Uave);
[row,ind] = max(Uave(:));
[a,b,c] = ind2sub(size(Uave),ind);
xp = Kx(a,b,c);
yp = Ky(a,b,c);
zp = Kz(a,b,c);
center = [xp, yp, zp];

% Filter
tau = 30;
k0 = 0;
filter = exp(-tau*(k-k0).^2);
plot3(fftshift(ks), fftshift(filter),x,fftshift(ks), fftshift(filter),y,fftshift(ks), fftshift(filter),z);
title('Trajectory of Submarine')
xlabel('x')
ylabel('y')
zlabel('z')
set(gca, 'Fontsize',10)
```

Listing 1: Example code from external file.