Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		2220171

A trading strategy for bitcoin and gold based on an original method and the Time Series Method

Summary

We make some assumptions based on economic principles and human nature.

For this question, cases under long-term are our focus. We try to analyze it from two perspectives. One is treat it as stochastic process by using tools like Markov chain and determine constants as its components. Another way is view it from the prospective of probability and statistics. Then compare the difference between two models.

At the end of our report, according to our results , we give the treader some advice.

Content

Notations

B_n	the component of bitcoin in nth state vector
C_n	the component of bitcoin in nth state vector
G_n	the component of bitcoin in nth state vector
B_n'	the component of bitcoin in nth state vector
C_n'	the component of bitcoin in nth state vector
G_n'	the component of bitcoin in nth state vector
B	a notation represents Bitcoin
C	a notation represents
G	a natation represents
A	a matrix to show the transition of state
P	a matrix to show the transition of state
W	the investment profits
$Cov(X_1, X_2)$	Covariance of X_1, X_2
$E(X)$	Expectation
$Var(X_1, X_2)$	Variance of X_1, X_2

Restate of the problem

There are several portfolios. The components of all the portfolios are cash, gold and Bitcoin. Different ways of flow from one component to another is an operation. Existence of commission for each transaction means the loss of principal. So, it is obvious that frequent transaction may cause huge loss in the final profit. Intuitively speaking, the more frequent operations are done, the less the profit is. To take the commission into account, a necessary thing is to divide the transaction into two ways: long-term and short-term.

Operation

(1)Defination and Classification

We have three operations: buying, selling and holding an entity actually. Because the exchanging between Bitcoin and gold must go through the joint, cash. However, if we consider the initial and final state, four possible cases occur: selling, buying, holding and transferring.

Define four processes:

selling: changing Bitcoin or gold into cash

buying: changing cash into Bitcoin or gold (the reverse process of selling)

holding: keeping something rather than changing it into any other things

transferring: exchanging between Bitcoin and gold

Then the four process can be expressed as below:

selling:

buying:

holding:

transferring:

Figure 1: This describes the flowing diagram of money throughout the entire trading. 1 means selling gold or bitcoin, 2 means buying gold or bitcoin, 3 means the percent growth of the assets in each system between each transaction, and 4 means the indirect monetary connections between gold and bitcoin.

(2) Allocation

For each operation, we assign one entity another one or itself by controlling the proportion. For example, initially we had 1000 dollars. In the first operation, we have to determine weights to allocate cash to Bitcoin and gold or only to hold.

~~expression:~~Clarification

~~the~~The measurements of cash, Bitcoin and gold are unified as the unit of cash, dollar.

Assumption

~~–~~We first assume that the person participating the trading is rational, which means he or she will always make decisions according to the situation instead of emotions. We also assumed the trading can be classified as a long-term one and a short-term one. For the long-term trading, we assumed the person operates the trading in only a comparatively small period of time in a half of a year, such as an hour. During the trading on an asset, the more gradually the daily price decreases, the more assets that the person will buy, because, according to the figure 1 and figure 2 from the document of contest's problem of MCM, the daily price will always increase after a smooth decline. In another hand, the more rapid the price rises, the more assets that the person will sell, because, from the figure 1 and figure 2, we can notice that there will always be a drop in the price after a noticeable growth. In another situation, the person will keep the asset. In the end of the long-term trading, the person will just transfer all of the assets from the bitcoin and gold to the currency.

Long term Method

Our first method focused on the long-term trading, and the unit for calculating this case is a month. We regrade three assets as three independent systems, and there are connections between bitcoin and currency and between gold and currency. To be more specific, the person can transfer money form bitcoin to currency through buying or from currency to bitcoin through selling, and it is the same for the gold. Furthermore, there will be an implicit connection between bitcoin and gold made by transferring money from bitcoin to currency then form currency to gold and vice versa.

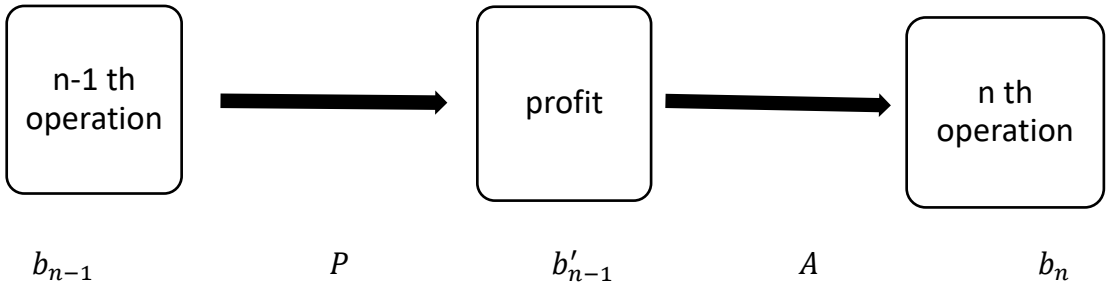
There are four types of transferring money throughout the whole process, which are transferring from bitcoin to currency, from gold to currency, from currency to gold, and from currency to bitcoin. Additionally, transaction between gold and currency and between bitcoin and currency require cost of 1% and 2% respectively. The amount of money transferred from currency to gold can be calculated multiplying the assets in the currency by portion of money transferring to the gold, then by 99% due to the cost, while that from gold to the currency can be calculated by multiplying the assets in the gold by portion of money transferring to the currency, then also by 99% due to the cost.

Also, the amount of money transferred from currency to bitcoin can be calculated multiplying the assets in the currency by portion of money transferring to the bitcoin, then by 98% due to the cost, while that from gold to the currency can be calculated by

multiplying the assets in the bitcoin by portion of money transferring to the currency, then also by 98% due to the cost. The person can also transfer assets between the gold and the bitcoin through the medium of currency, building an indirect relationship between the bitcoin and gold.

We first assume that the person participating the trading is rational, which means he or she will always make decisions according to the situation instead of emotions. We also assumed the trading can be classified as a long-term one and a short-term one. For the long-term trading, we assumed the person operates the trading in only a comparatively small period of time in a half of a year, such as an hour. During the trading on an asset, the more gradually the daily price decreases, the more assets that the person will buy, because, according to the figure 1 and figure 2 from the document of contest's problem of MCM, the daily price will always increase after a smooth decline. In another hand, the more rapid the price rises, the more assets that the person will sell, because, from the figure 1 and figure 2, we can notice that there will always be a drop in the price after a noticeable growth. In another situation, the person will keep the asset. In the end of the long-term trading, the person will just transfer all of the assets from the bitcoin and gold to the currency.

According to our assumptions, we first split a period into two stages: harvesting and allocation. Since the investment is long-term, each time of operation accounts for relatively short time comparing to the tiresomely long period. So, an approximation is to treat each operation as an instantaneous process. Note that the length of each arrow in the diagram below indicates neither duration itself nor the scale to the duration.



Operation matrices $P = \begin{pmatrix} r & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \end{pmatrix}$, $A =$

$$\begin{pmatrix} 1-a & b & bd \\ a & 1-b-c & d \\ ac & c & 1-d \end{pmatrix}$$

Operation matrices

Each operation matrix represents an operation.

In matrix P, the constants r, 1 and s are profit rate of Bitcoin, cash. and gold, respectively.

In matrix A, containing the commission, a, b, c and d are transition rate for each step in the diagram below. Consider that all the transitions are completed in an instant.

[diagram]

State vectors $b_n = \begin{pmatrix} B_n \\ C_n \\ G_n \end{pmatrix}$, $b'_n = \begin{pmatrix} B'_n \\ C'_n \\ G'_n \end{pmatrix}$

Each vector is a state in one specific instant.

Relationship $b'_{n-1} = Pb_{n-1}$ ___ $b_n = Ob'_{n-1}$

So, the problem has transmitted to find the coefficient in each matrix.

In accordance to our assumptions, it is natural to use the slope of the graph to broadcast whether we should choose which one.

Figure 2: In those graph of the polynomials, the x-axis means time with the unit of month, and September 2016 is the origin.

Then, we calculated the average polynomial for the gold and the bitcoin. The polynomials are the following.

$$B(x)=-0.0001x^6+0.0217x^5-1.239x^4+31.45x^3-361.96x^2+2083x-1940$$

$$G(x)=(1.6E-5)x^6-0.002x^5+0.0981x^4-2.17x^3+21.53x^2-78.16x+1304$$

B(x) is for the bitcoin, and G(x) is for the gold. These polynomials will be used to describe the general trend on the price of gold and bit coin.

Based on the assumption, the more gradually the daily price decreases, the more assets that the person will buy, while the more rapid the price rises, the more assets that the person will sell. This can be interpreted as: When the derivative of the polynomial of a system is negative and large, the more money the person will spend on buying it, while, when the derivative is positive and large, the more money the person will sell it. Therefore, when the derivative is negative, it is inversely proportional to the amount of buying, while, when the derivative is positive, it is proportional to the amount of selling. This can be described by the following formula.

$$w = \begin{cases} w_{enter} = \dfrac{1}{\dot{f}(x_i)} & \left(\dot{f}(x_i) < 0\right) \\ w_{out} = \dot{f}(x_i) & \left(\dot{f}(x_i) > 0\right) \end{cases} \qquad (i = 0,1,2,3 \dots)$$

In the formula, we set a value w reflecting the amount of buying and selling, and xi means the date we choose to do the transaction.

$$S(w) = \frac{2}{1 + e^{-w}} - 1$$

As for calculating the portion of buying and selling, the above formula derived from the sigmoid function is introduced, because the absolute value of its outputs is always between 0 and 1, and the bigger the positive or negative w value, the larger portion of buying and selling. We can get the portion from calculating the absolute of the S(x) function. There might be a special case that two portion of transferring assets out of currency are both greater than 0.5. In this case, we just normalize two portions. The above calculation can be generalized as the following formula.

$$\begin{cases} \dot{f}(x_i) > 0: & S(x_i) = \dfrac{2}{1 + e^{-\dot{f}(x_i)}} - 1 \\ \dot{f}(x_i) < 0: & S(x_i) = \dfrac{2}{1 + e^{-\frac{1}{\dot{f}(x_i)}}} - 1 \end{cases} \qquad (i = 0,1,2,3 \dots)$$

From the above formula, we can get the portions, meaning that the Markov's matrices for every transactions. Then we can multiply all calculated matrices to the initial vector (1000, 0, 0) for getting the final vector, which is the final result. The calculation can be shown as the following way. (A means a matrix, and v means a vector.)

From this method, we get the final result of (1033.5, 0, 0), meaning 1033.5 dollars in currency and no asset in both gold and bitcoin.

Long term ---Method 2

Table 1 quarter profit

quarter	gold	bitcoin
2016.9	1314.85	610.19
2017.1	1196.05	895.798875
2017.5	1257.4	1807.48506
2017.9	1283.35	4201.98905
2018.1	1333.4	11223.064
2018.5	1319.85	8652.03833
2018.9	1201.9	6583.45
2019.1	1282.1	3540.02
2019.5	1282.5	7880.29
2019.9	1485.3	8056.74

2020.1	1560.15	8703.36
2020.5	1737.95	9719.37
2020.9	1859.7	10732.43
2021.1	1856.85	32276.84
2021.5	1899.75	38410.5

Since the tendency of profit is not so clear, we just do simple random sampling to take out a datum from evenly spaced months (a quarter) to represent the entire sample.

Table 2 quarter profit rate

quarter	gold	bitcoin
2017.1	0.909647488	1.46806548
2017.5	1.051293842	2.017735354
2017.9	1.020637824	2.32477111
2018.1	1.038999494	2.670893205
2018.5	0.989838008	0.770915886
2018.9	0.910633784	0.76091318
2019.1	1.066727681	0.537715028
2019.5	1.000311988	2.226058045
2019.9	1.158128655	1.022391308
2020.1	1.05039386	1.080258268
2020.5	1.113963401	1.116737674
2020.9	1.070053799	1.104231036
2021.1	0.998467495	3.007412114
2021.5	1.023103643	1.190032853

Each datum in table 2 is obtained by take the ratio of this quarter to previous one.

Suppose that X_1 represent the ratio of annual yield of gold

X_2

The expectation

,

where ,

The decision variables x1, x2 and x3 show the proportion of gold, bitcoin and cash which the trader holds at a particular time respectively.

Then

The investment profits

$$W = x_1E(X_1) + x_2E(X_2) + x_3E(X_3) = x_1E(X_1) + x_2E(X_2)$$

Since $E(X3) = 0$, the last term $x_3E(X_3)$ can be removed

$Cov(X1, X2) = E[(X - E(X))][(Y - E(Y))]$

$Cov(X_1, X_2) = -0.006, \quad Cov(X_1, X_2) < 0$,

Then

The investment profit

Since , the last term can be removed

, ,which means that bitcoin and gold are negatively corelated. More specifically, if bitcoin increases its value, gold teds to decrease, and vice versa. However, the absolute value of $Cov(X_1, X_2)$ is very close to 0. This convinces us that they probably would have no relationship.

The variance of annual harvest

$$R = Var\big(x_1E(X_1) + x_2E(X_2) + x_3E(X_3)\big)$$

$$= x_1^2Var(X_1) + x_2^2Var(X_2) + x_3^2Var(X_3) +$$

$$2x_1x_2cov(X_1, X_2) + 2x_1x_3cov(X_1, X_3) + 2x_2x_3cov(X_2, X_3)$$

$$= \sum_{i=1}^{3}\sum_{j=1}^{3} x_i x_j cov(X_i, X_j)$$

$$= x_1^2 Var(X_1) + x_2^2 Var(X_2) + 2x_1 x_2 cov(X_1, X_2)$$

Under the above restrictions, R should be as little as possible and the expectation should be as large as possible.

We can consider the function

$$f(x) = \frac{x_1 E(X_1) + x_2 E(X_2)}{\sqrt{x_1^2 Var(X_1) + x_2^2 Var(X_2) + 2x_1 x_2 cov(X_1, X_2)}},$$

so this converts question into finding the maximum of this function. The reason why we do this is we can transform two variables in this function into one variable

$$f(t) = \frac{tE(X_1) + E(X_2)}{\sqrt{t^2 Var(X_1) + Var(X_2) + 2tcov(X_1, X_2)}}, \text{ where } t = \frac{x_1}{x_2}$$

$$f(s) = \frac{E(X_1) + sE(X_2)}{\sqrt{Var(X_1) + s^2 Var(X_2) + 2scov(X_1, X_2)}}, \text{ where } s = \frac{x_2}{x_1}$$

However, it does not have the maximum under the restrictions but has limit which is.

$$\lim_{t\to\infty} f(t) = \frac{E(X_1)}{\sqrt{Var(X_1)}} \quad , \ t \to \infty \ \ as \ \ x_2 \to 0$$

$$\lim_{s\to\infty} f(s) = \frac{E(X_2)}{\sqrt{Var(X_2)}} \quad , \ s \to \infty \ \ as \ \ x_1 \to 0$$

That is to say, the way to minimize the risk and enlarge the profit at the same time is invest all the money to one sector: gold or Bitcoin.

Strength and weaknesses

1. In method 1

We find the relationship between each step and its previous one. This relationship relates to concrete matrices. It is similar to the Markov chain but each component is not constant such that it is hard to determine. Even though we can represent it, we are unable to precisely manipulate it. Adopting a polynomial to fit the line, as far as we think, is not so appropriate.

2. In method 2

In another method, our focus is the uncertainty and we try to find how it corresponds to concepts in probability. There are indeed some threads pointing to the expectation and variance. Actually, it was concluded as a nonlinear programming problem. However, limited by our knowledge, we cannot go further and draw more useful conclusions. We get vague notion about the premise and cannot solve such a seemingly easy but complex programming problem. In the end, an intuition-dependent solution is posted which seems to be a compromise.

3. Optimization

ARIMA model seems to be a much better way to do that. It can reveal serial correlation and intrinsic tendency of that curve.

4. Other

Limited by time and ability, we did not compare the effect of long-term and short-term.

Memorandum

------A suggestion letter to

Long- term or short-term?

From our model, the optimal way is to do long-term investment. Because over frequent transaction reduces the capital. Both of them are significantly profitable. The sensitivity of strategy is significantly related to the frequency of transaction.

Strategy?

To invest all the money to one sector. Our advice is to choose bitcoin. From short-term perspective, you may have to undertake a higher risk, but on the other hand, you will get a considerable income from long-term perspective, since the amplitude of

increasing is also much higher than gold. This is always true---Risks always coexists with the opportunity. You may just as well have a try!

Conclusion

In conclusion, we use an original method and Time Series Method to find the best strategy for the trading on bitcoin and gold, and finally find out that the best strategy is to invest all the money to bitcoin.

References

[1]Meade, J. (1978). The Meaning of "Internal Balance." *The Economic Journal*, *88*(351), 423–435. https://doi.org/10.2307/2232044

[2] Tobin, James (1955). *A Dynamic Aggregative Model. Journal of Political Economy, 63(2), 103–115.*doi:10.1086/257652

[3] Wassily W. Leontief (1936). *Quantitative Input and Output Relations in the Economic Systems of the United States. The Review of Economics and Statistics, 18(3), 105–125.* doi:10.2307/1927837