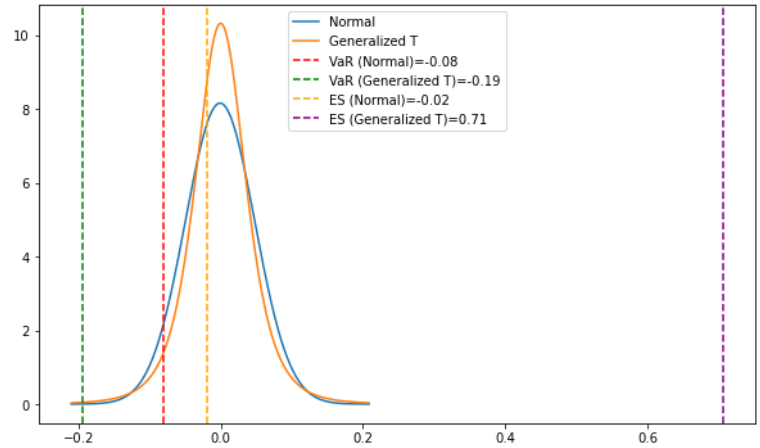
Problem 1

Based on the results, we can see that the VaR and ES values differ between the two fitted distributions. The Normal distribution has a VaR of -0.08 and ES of -0.02, while the Generalized T distribution has a VaR of -0.19 and ES of 0.71.

The Generalized T distribution has a larger VaR than the Normal distribution, indicating that there is a higher probability of larger losses with this distribution. The ES value for the Generalized T distribution is also positive, indicating that the expected loss given the VaR is greater than zero, which is in contrast to the negative ES value for the Normal distribution.

These differences can be attributed to the different shapes of the two distributions. The Generalized T distribution has heavier tails than the Normal distribution, meaning that extreme events are more likely to occur. This results in a larger VaR value for the Generalized T distribution. The positive ES value for the Generalized T distribution may be due to skewness in the data, which the Normal distribution does not account for.

```
Normal Distribution:
  VaR(0.05): -0.08
  ES(0.05): -0.02
Generalized T Distribution:
  VaR(0.05): -0.19
  ES(0.05): 0.71
```



Problem 2

Module 1: covariance_estimators
This module will contain functions for estimating the covariance matrix of asset returns.

- sample_covariance_matrix(): This function takes in a matrix of asset returns and returns the sample covariance matrix.
- ledoit_wolf_shrinkage(): This function implements the Ledoit-Wolf shrinkage estimator to estimate the covariance matrix. It takes in a matrix of asset returns and returns the estimated covariance matrix.
- fit_generalized_t: This function is a Generalized t-distribution to a vector of returns using maximum likelihood estimation.

Module 2: correlation_matrix_fixes
This module will contain functions for handling non-positive definite covariance matrices.

- nearest_correlation_matrix(): This function takes in a non-positive definite covariance matrix and returns the nearest positive definite matrix.
- higham_algorithm(): This function implements the Higham algorithm for finding the nearest positive definite matrix to a non-positive definite matrix.

Module 3: simulation_methods
This module will contain functions for simulating asset returns.

- monte_carlo_simulation(): This function takes in a covariance matrix, a vector of mean returns, and the number of simulations, and returns simulated asset returns.

Module 4: risk_metrics
This module will contain functions for calculating Value at Risk (VaR) and Expected Shortfall (ES).

- historical_var(): This function calculates the historical VaR given a vector of returns and a confidence level.
- parametric_var(): This function calculates the parametric VaR given a vector of returns, a confidence level, and a normal distribution assumption.
- monte_carlo_var(): This function calculates the VaR using Monte Carlo simulation given a vector of returns, a confidence level, and the number of simulations.
- historical_es(): This function calculates the historical ES given a vector of returns and a confidence level.
- parametric_es(): This function calculates the parametric ES given a vector of returns, a confidence level, and a normal distribution assumption.
- monte_carlo_es(): This function calculates the ES using Monte Carlo simulation given a vector of returns, a confidence level, and the number of simulations.

Module 5: test_suite
This module will contain tests for all the functions in the previous modules.

Problem 3

Analysis of Result:

The results show that Portfolio A has the highest VaR at 2,078,005.04 USD, followed by Portfolio B at 1,609,582.87 USD, and then Portfolio C at 1,315,425.30 USD. This suggests that Portfolio A is the riskiest of the three portfolios.

Similarly, when looking at the ES at a 95% confidence level, Portfolio A has the highest ES at 9,436.09 USD, followed by Portfolio C at 8,630.39 USD, and then Portfolio B at 1,801.94 USD. This suggests that

Portfolio A has the highest expected losses in the worst-case scenario.

The total portfolio VaR at a 95% confidence level is 4,834,000.64 USD, which is the sum of the VaRs of the three portfolios. The total portfolio ES at a 95% confidence level is 19,243.20 USD, which is the weighted sum of the ESs of the three portfolios.

Overall, these results suggest that Portfolio A is the riskiest of the three portfolios and has the highest expected losses in the worst-case scenario. Investors may want to consider rebalancing their portfolio to reduce their exposure to Portfolio A and increase their exposure to the less risky portfolios, B and C.

```
Portfolio A VaR (95%): 2078005.038445803
Portfolio A ES (95%): 9436.094195227448
Portfolio B VaR (95%): 1609582.8690287422
Portfolio B ES (95%): 1801.9390276677134
Portfolio C VaR (95%): 1315425.3035742636
Portfolio C ES (95%): 8630.393503268093
Total Portfolio VaR (95%): 4834000.635738503
Total Portfolio ES (95%): 19243.202553278297
```

Compared with the Result of Week04:

Using a multivariate normal distribution assumes that the individual asset returns are normally distributed, and the covariance matrix is constant over time. While using a generalized T distribution allows for more flexibility in the shape of the distribution, allowing for skewness and heavier tails. It appears that the two methods have led to different VaR estimates for the three portfolios and the total portfolio. The multivariate normal method has resulted in higher VaR estimates for all portfolios and the total portfolio compared to the generalized T method. This suggests that the multivariate normal method may be more conservative in its estimates.