Problem 1

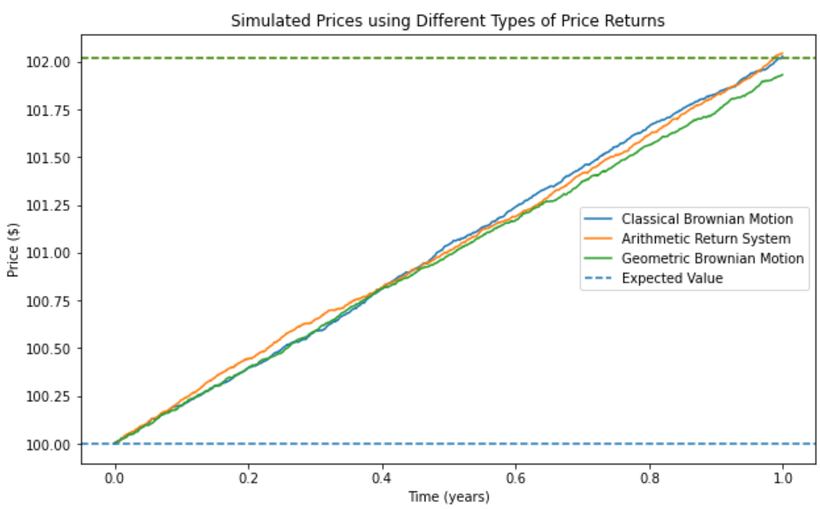
- **Classical Brownian motion**: It has an expected value of 100.00 and a standard deviation of 20.61. This means that the expected final price is equal to the initial price, and the standard deviation indicates the level of volatility in the stock price.

- **Arithmetic Return System**: It has an expected value of 102.02 and a standard deviation of 21.88. This method takes into account the arithmetic returns, which are assumed to be normally distributed with mean zero and a constant standard deviation. The expected value is higher than the initial price, indicating that the stock price is expected to grow over time.

- **Log Return or Geometric Brownian Motion**: It also has an expected value of 102.02 but a lower standard deviation of 21.03. This method takes into account the logarithmic returns, which are assumed to be normally distributed with mean equal to the drift term (r - 0.5*sigma^2) and a constant standard deviation. The expected value is the same as the arithmetic return system, but the lower standard deviation indicates that the volatility is lower in this case.

```
Classical Brownian Motion:
Expected value of P_T: 100.00
Standard deviation of P_T: 20.61
Arithmetic Return System:
Expected value of P_T: 102.02
Standard deviation of P_T: 21.88
Geometric Brownian Motion:
Expected value of P_T: 102.02
Standard deviation of P_T: 21.03
```

In terms of performance, the Classical Brownian Motion method is the simplest and fastest to simulate, while the Arithmetic Return System and Geometric Brownian Motion methods require additional calculations and are slightly slower. However, the added complexity of these methods allows for a more accurate representation of stock prices, especially over long time periods. If a quick and simple simulation is sufficient, Classical Brownian Motion may be adequate. However, for more accurate and realistic simulations, Arithmetic Return System or Geometric Brownian Motion may be more appropriate.



Problem 2

- **VaR using a normal distribution**: This method assumes that the returns follow a normal distribution. It is simple and widely used but may not be appropriate for portfolios with non-normal returns or during extreme market conditions. It estimate is -0.0546, which means that there is a 5% chance that the loss on the portfolio will exceed this amount. The VaR estimate obtained using the normal distribution with exponentially weighted variance is much lower at -0.0003, which indicates that the variance is decreasing over time.

- **VaR using a normal distribution with an EWMA variance**: This method is similar to the first method, but instead of using a constant variance, it uses an exponentially weighted moving average (EWMA) of the past variances to estimate the current variance. This can be useful for capturing changes in volatility over time. It estimate is -0.0576, which is close to the normal VaR estimate.

- **VaR using MLE fitted T distribution**: This method assumes that the returns follow a T distribution, which allows for fatter tails than the normal distribution. The parameters of the T distribution are estimated using maximum likelihood estimation (MLE) based on the historical data. This can be useful for capturing extreme events but may be sensitive to the choice of the distribution and the estimation method.

- **VaR using a fitted AR(1) model**: This method assumes that the returns follow an autoregressive (AR) process of order 1, which means that each return depends on the previous return. This can be useful for capturing the dependence structure of the returns but may not be appropriate for portfolios with complex dynamics. It estimate is -0.0658, which is the highest among the five methods

- **VaR using Historic Simulation**: This method estimates VaR based on the empirical distribution of historical returns. It can be useful for capturing the actual distribution of the returns but may be sensitive to the length and quality of the historical data. It estimate is positive at 0.0546, which means that the loss on the portfolio is not likely to exceed this amount.

```
VaR (normal): -0.0546200790823787
VaR (normal with EW variance): -0.0003419529681526706
VaR (T distribution): -0.057579649751533055
VaR (AR(1)):  -0.06581442019119715
VaR (Historic Simulation): 0.0546200790823787
```

Problem 3

- The first method assumes that the returns of the portfolio are normally distributed (using ewnormal covariance with lambda=0.94) and calculates the VaR as the product of the portfolio value, the portfolio volatility, and the inverse of the standard normal cumulative distribution function at a specified confidence level (). This method is simpler and faster, but it has limitations because it assumes normality and ignores tail risk.

```
Portfolio A VaR: $1,074.38
Portfolio B VaR: $1,620.58
Portfolio C VaR: $-1,881.20
Total VaR: $813.76
```

- The second method simulates possible returns of the portfolio using a Monte Carlo simulation and calculates the VaR as the difference between the initial portfolio value and the specified percentile of the simulated returns distribution. This method takes into account tail risk and non-normality of returns, but it is computationally more intensive.

```
Portfolio A VaR: $726.57
Portfolio B VaR: $1,431.09
Portfolio C VaR: $-2,407.40
Total VaR: $-249.74
```

Comparing the results, we can see that the VaR estimates from the two methods are different, with some portfolios having significantly different VaR estimates. The total VaR estimate from the first method is positive, indicating that the portfolios have a positive expected return, while the total VaR estimate from the second method is negative, suggesting that the portfolios have a negative expected return. For choosing the method, if the portfolio has non-normal returns and tail risk, then the Monte Carlo simulation approach may be more appropriate; while if the portfolio has relatively simple characteristics and the investor wants a quick estimate of VaR, then the normal distribution assumption may be more appropriate.