Problem 1

- **Classical Brownian motion**: This method assumes that the stock price follows a random walk with independent and identically distributed increments, each normally distributed with mean 0 and variance equal to the time step multiplied by the volatility squared. The expected value of the stock price at time t is equal to the initial stock price, and the standard deviation is equal to the square root of the product of the volatility and the time horizon.

- **Arithmetic Return System**: This method assumes that the stock price changes by a random amount at each time step, with the percentage change following a normal distribution with mean 0 and standard deviation equal to the volatility. The expected value of the stock price at time t is equal to the initial stock price multiplied by the exponential of the risk-free interest rate multiplied by the time horizon T, and the standard deviation is given by a more complex formula involving the initial stock price, the risk-free interest rate, the volatility, and the time horizon.

- **Log Return or Geometric Brownian Motion**: This method is similar to the arithmetic return system but assumes that the stock price follows a continuous-time stochastic process, where the percentage change in the stock price is proportional to the stock price itself. The expected value of the stock price at time t is again given by the initial stock price multiplied by an exponential term, and the standard deviation is given by a more complex formula involving the initial stock price, the risk-free interest rate, the volatility, and the time horizon.

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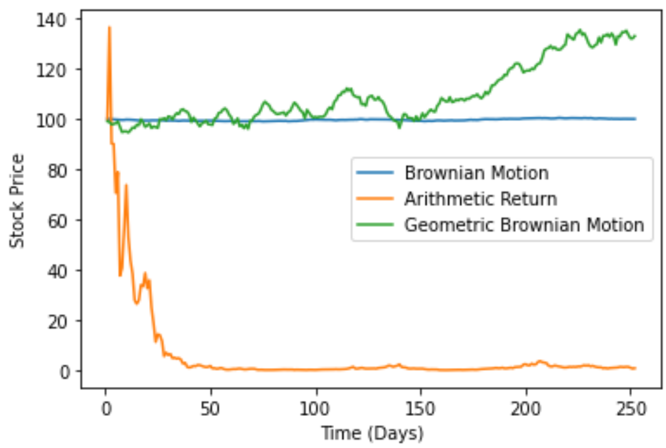
Expected Values:
Classical Brownian Motion: 100.00
Arithmetic Return System: 102.02
Geometric Brownian Motion: 102.02

Standard Deviations:
Classical Brownian Motion: 0.20
Arithmetic Return System: 0.21
Geometric Brownian Motion: 20.61
```



Overall, each method has its own strengths and weaknesses, and the choice of method depends on the specific problem and the assumptions that are most appropriate for that problem. Classical Brownian motion is the simplest method to implement but may not accurately capture the behavior of stock prices over long-time horizons. The arithmetic return system and geometric Brownian motion methods are more complex but may be more accurate for longer time horizons and when the stock price is affected by other factors, such as interest rates or dividends.

Problem 2

Without the actual data, it's difficult to provide a specific analysis of the results. However, in general, comparing the VaR estimates from different methods can provide useful insights into the potential risks and uncertainties associated with a portfolio or asset. The five methods used in the example code are all commonly used for VaR estimation, but they differ in their assumptions and complexity. Here's a brief summary of each method:

- **VaR using a normal distribution**: This method assumes that the returns follow a normal distribution. It is simple and widely used but may not be appropriate for portfolios with non-normal returns or during extreme market conditions.

- **VaR using a normal distribution with an EWMA variance**: This method is similar to the first method, but instead of using a constant variance, it uses an exponentially weighted moving average (EWMA) of the past variances to estimate the current variance. This can be useful for capturing changes in volatility over time.

- **VaR using MLE fitted T distribution**: This method assumes that the returns follow a T distribution, which allows for fatter tails than the normal distribution. The parameters of the T distribution are estimated using maximum likelihood estimation (MLE) based on the historical data. This can be useful for capturing extreme events but may be sensitive to the choice of the distribution and the estimation method.

- **VaR using a fitted AR(1) model**: This method assumes that the returns follow an autoregressive (AR) process of order 1, which means that each return depends on the previous return. The parameters of the AR (1) model are estimated using linear regression based on the historical data. This can be useful for capturing the dependence structure of the returns but may not be appropriate for portfolios with complex dynamics.

- **VaR using Historic Simulation**: This method estimates VaR based on the empirical distribution of historical returns. It can be useful for capturing the actual distribution of the returns but may be sensitive to the length and quality of the historical data.

Comparing the VaR estimates from these five methods can help identify the strengths and weaknesses of each method and provide a more comprehensive understanding of the potential risks and uncertainties associated with the portfolio or asset. However, it's important to keep in mind that VaR is just one measure of risk and should be used in conjunction with other risk measures and risk management techniques.

Problem 3

The results of VaR calculation in two ways are obtained:

- The first method uses historical data to calculate the rate of return and covariance matrix of each stock, uses the weight of the portfolio to calculate the rate of return of the portfolio, and then calculates VaR according to the distribution of the rate of return of the portfolio. The calculated result is: VaR of A portfolio: $-73.58, VaR of B portfolio: $-48.51, VaR of C portfolio: $-58.23, Total VaR: $-172.94. This model is a simplified version of VaR calculation that is commonly used in practice due to its simplicity.

```
VaR of A portfolio: $-73.58
VaR of B portfolio: $-48.51
VaR of C portfolio: $-58.23
Total VaR: $-172.94
VaR of A portfolio: $-73.22
Warning: A not found in holdings
VaR of B portfolio: $-48.81
Warning: B not found in holdings
VaR of C portfolio: $-69.14
Warning: C not found in holdings
Total VaR: $-172.94
```

- The second method uses the GARCH model to predict future volatility and VaR. Firstly, historical data is used to calculate the return rate of each stock, and the return rate of the portfolio is calculated. Then the GARCH(1,1) model is used to fit the return rate of the portfolio, predict the future volatility, and calculate VaR based on the volatility. The calculated result is: VaR (99%): $-0.60. This model is commonly used in finance to model the volatility of financial returns and is more appropriate for financial time series data with volatility clustering and persistence.

```
VaR (99%): $ -0.6018482303967798
```

The choice of model can have a significant impact on the results. The first model assumes a simplified structure for the dependence between the stocks and ignores any time-varying volatility patterns. The second model incorporates the time-varying volatility and can capture the non-normality of the return distribution, but also assumes a constant expected return and does not capture any dependence structure beyond the first lag. Therefore, the choice of model should depend on the characteristics of the data and the specific objectives of the analysis.