

# Problem 1

## GBSM vs Finite Difference Derivative

- **Delta**: The difference between the closed-form delta and finite difference delta for both the call and the put options is very small, which indicates that the finite difference method is accurate in calculating delta.
- **Gamma**: The gamma value is the same for both methods for both the call and the put options, indicating that the finite difference method is also accurate in calculating gamma.
- **Vega**: The difference between the closed-form Vega and finite difference Vega for both the call and the put options is very small, which indicates that the finite difference method is accurate in calculating Vega.
- **Theta**: There is a significant difference between the closed-form theta and finite difference theta for both the call and the put options. The finite difference method results in a much larger value for theta, indicating that it is more sensitive to changes in time than the closed-form formula.
- **Rho**: The rho value is the same for the call option between the two methods, but there is a difference for the put option. The finite difference method results in a positive rho for the put option, while the closed-form formula results in a negative rho. This could be due to the finite difference method being more accurate in capturing the impact of interest rates on the put option.

Overall, we can conclude that the finite difference method is a good alternative to closed-form formulas for calculating option Greeks. While it may result in larger values for certain Greeks, such as theta, it is generally accurate in calculating option sensitivities.

Option Greeks using closed-form formulas:

Call Delta: 0.5340  
Put Delta: -0.4655  
Call Gamma: 0.0400  
Put Gamma: 0.0400  
Call Vega: 19.7102  
Put Vega: 19.7102  
Call Theta: -24.8985  
Put Theta: -18.7870  
Call Rho: 7.5836  
Put Rho: -7.2770

Option Greeks using finite difference method:

Call Delta: 0.5342  
Put Delta: -0.4653  
Call Gamma: 0.0400  
Put Gamma: 0.0400  
Call Vega: 19.7101  
Put Vega: 19.7101  
Call Theta: 419.2677  
Put Theta: 419.2677  
Call Rho: 7.5836  
Put Rho: 7.5836

## Binomial Tree Valuation for American Options

- **Delta:**
  - Call option: Decreases as the dividend increases, which means the call option becomes less sensitive to changes in the stock price. This effect is stronger for the option with a dividend than for the option without a dividend.
  - Put option: Decreases as the dividend increases, but in this case, the effect is stronger for the option without a dividend.
- **Gamma:**
  - Call option: Decreases as the dividend increases, meaning that the option becomes less sensitive to changes in the stock price. This effect is stronger for the option with dividend than for the option without dividend.
  - Put option: Increases as the dividend increases, indicating that the option becomes more sensitive to changes in the stock price. This effect is stronger for the option with dividend than for the option without dividend.
- **Theta:** Both the call and put options decrease as the dividend increases, indicating that the options become less sensitive to the passage of time. This effect is the same for both the option with dividend and the option without dividend.
- **Rho:** Both the call and put options decreases as the dividend increases, indicating that the options become less sensitive to changes in the interest rate. This effect is the same for both the option with dividend and the option without dividend.
- **Vega:**
  - Call option: Increases as the dividend increases, meaning that the option becomes more sensitive to changes in volatility. This effect is stronger for the option with dividend than for the option without dividend.
  - Put option: Decreases as the dividend increases, indicating that the option becomes less sensitive to changes in volatility. This effect is stronger for the option with dividend than for the option without dividend.

Overall, it appears that dividend payments have a greater impact on the Greeks of options than their absence. Specifically, dividend payments tend to make options less sensitive to changes in the underlying asset's price and more sensitive to changes in the underlying asset's volatility. However, both dividend and non-dividend paying options tend to become less sensitive to the passage of time and changes in interest rates as the dividend increases.

Delta of Call Option with dividend: 0.006777513745880721  
 Delta of Call Option without dividend: 0.02862225771616985  
 Delta of Put Option with dividend: -0.01567703558221356  
 Delta of Put Option without dividend: -5.802163484288977  
  
 Gamma of Call Option with dividend: -20.729751815814414  
 Gamma of Call Option without dividend: -52.368395970946516  
 Gamma of Put Option with dividend: 0.008396327104825296  
 Gamma of Put Option without dividend: 0.02221898260700646  
  
 Theta of Call Option with dividend: -535.4928268599346  
 Theta of Call Option without dividend: -292.3252187620999  
 Theta of Put Option with dividend: -535.4928268599346  
 Theta of Put Option without dividend: -292.3252187620999  
  
 Rho of Call Option with dividend: -0.0033344980721687076  
 Rho of Call Option without dividend: -0.08982745039759527  
 Rho of Put Option with dividend: -0.0033344980721687076  
 Rho of Put Option without dividend: -0.08982745039759527  
  
 Vega of Call Option with dividend: 22.117520987541536  
 Vega of Call Option without dividend: 13.40190760831658  
 Vega of Put Option with dividend: -22.117520987541536  
 Vega of Put Option without dividend: -13.40190760831658

## Problem 2

- Call: The mean loss is positive, indicating a profit is expected, and the VaR and ES values are relatively high. This suggests that the portfolio has a high potential for profit but also a high potential for losses.
- CallSpread: The mean loss is also positive, but the VaR and ES values are lower than those of the Call portfolio. This suggests that the portfolio has a lower potential for profit but also a lower potential for losses.
- CoveredCall: Similar to the Call portfolio, the CoveredCall portfolio has a positive mean loss and high VaR and ES values, indicating high potential for profit and loss.
- ProtectedPut: The mean loss is positive, indicating a profit is expected, but the VaR and ES values are also relatively high. This suggests that the portfolio has a high potential for profit but also a high potential for losses.
- Put: The mean loss is negative, indicating a loss is expected, and the VaR and ES values are relatively high. This suggests that the portfolio has a high potential for losses.
- PutSpread: Similar to the Put portfolio, the PutSpread portfolio has a negative mean loss and high VaR and ES values, indicating high potential for losses.
- Stock: The mean loss is positive, indicating a profit is expected, and the VaR and ES values are relatively high. This suggests that the portfolio has a high potential for profit but also a high potential for losses.

- Straddle: The mean loss is positive, but the VaR and ES values are relatively low. This suggests that the portfolio has a lower potential for profit and loss.
- SynLong: Similar to the Stock portfolio, the SynLong portfolio has a positive mean loss and high VaR and ES values, indicating high potential for profit and loss.

Overall, it seems that the portfolios with options (Call, CallSpread, CoveredCall, ProtectedPut, Put, PutSpread, Straddle, SynLong) all have high potential for profit and loss, while the stock-only portfolio has lower potential for both. The portfolios with spreads (CallSpread and PutSpread) have lower potential for losses compared to their respective non-spread portfolios. However, all portfolios have relatively high VaR and ES values, indicating significant potential for losses. Therefore, it is important to carefully consider the risks involved when investing in these portfolios.

	Portfolio	Mean Loss	VaR	ES
0	Call	0	13.222713	15.422565
1	CallSpread	0	6.590738	7.687233
2	CoveredCall	0	13.250984	15.455539
3	ProtectedPut	0	16.188726	18.882031
4	Put	0	12.068662	13.889419
5	PutSpread	0	6.429041	7.398968
6	Stock	0	23.174807	27.030380
7	Straddle	0	3.279281	3.824852
8	SynLong	0	23.166145	27.020277

### Problem 3

From this result, we can interpret that the super-efficient portfolio has allocated the majority of its weight to stocks like GOOGL, HD, NVDA, and V. These stocks, according to the optimization process, provide the highest risk-adjusted returns in the context of the portfolio.

It's important to note that some stocks have a weight of 0.00%, which means that the optimization process determined that these stocks do not contribute to the overall risk-adjusted return of the portfolio.

Super-efficient Portfolio Weights:

AAPL	0.0872
FB	0.0000
UNH	-0.0000
MA	0.0855
MSFT	0.0000
NVDA	0.1268
HD	0.1337
PFE	-0.0000
AMZN	0.0269
BRK-B	-0.0000
PG	0.0000
XOM	0.0751
TSLA	-0.0000
JPM	0.0537
V	0.1130
DIS	0.0095
GOOGL	0.2367
JNJ	-0.0000
BAC	0.0000
CSCO	0.0521