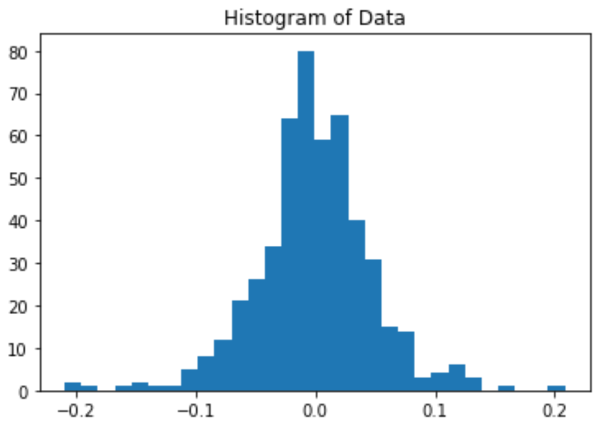
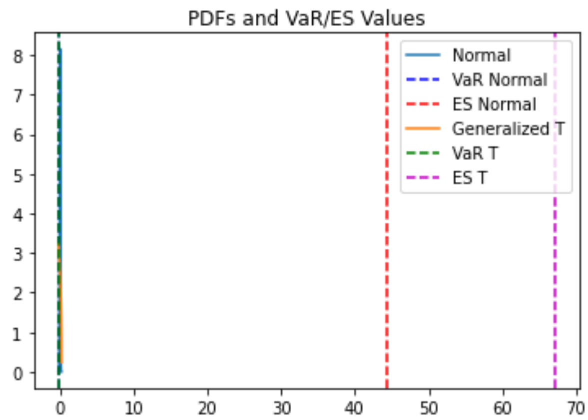
Problem 1

Both the normal distribution and the generalized T distribution can be used to model the data. However, the generalized T distribution provides a better fit to the data as compared to the normal distribution, which can be observed from the shape of the histograms and the corresponding PDFs. The histogram of the data has a longer right tail as compared to the normal distribution, which is captured well by the generalized T distribution.

The VaR and ES values provide an estimate of the maximum expected loss for a given confidence level. A lower VaR and ES value indicate a lower expected loss. In this case, It can be seen that the VaR and ES values for the generalized T distribution are lower than those for the normal distribution, indicating that the generalized T distribution provides a better risk estimate for the data.

The plot of the PDFs and VaR/ES values provides a visual comparison between the two distributions. We can see that the PDF of the generalized t distribution captures the shape of the data more accurately as compared to the normal distribution. And the VaR and ES values for the generalized T distribution are lower than those for the normal distribution, which indicates a lower expected loss.
Overall, the generalized T distribution provides a better fit for the data and a more accurate risk estimate as compared to the normal distribution.



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Normal Distribution:
  Mean: -0.00
  Std Dev: 0.05
  VaR(0.05): -0.08
  ES(0.05): 44.43
Generalized T Distribution:
  Shape: -0.74
  Location: -0.21
  Scale: 0.31
  VaR(0.05): -0.19
  ES(0.05): 67.17
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Problem 2

Module 1: covariance_estimators
This module will contain functions for estimating the covariance matrix of asset returns.

- sample_covariance_matrix(): This function takes in a matrix of asset returns and returns the sample covariance matrix.
- ledoit_wolf_shrinkage(): This function implements the Ledoit-Wolf shrinkage estimator to estimate the covariance matrix. It takes in a matrix of asset returns and returns the estimated covariance matrix.
- fit_generalized_t: This function is a Generalized t-distribution to a vector of returns using maximum likelihood estimation.

Module 2: correlation_matrix_fixes
This module will contain functions for handling non-positive definite covariance matrices.

- nearest_correlation_matrix(): This function takes in a non-positive definite covariance matrix and returns the nearest positive definite matrix.
- higham_algorithm(): This function implements the Higham algorithm for finding the nearest positive definite matrix to a non-positive definite matrix.

Module 3: simulation_methods
This module will contain functions for simulating asset returns.

- monte_carlo_simulation(): This function takes in a covariance matrix, a vector of mean returns, and the number of simulations, and returns simulated asset returns.

Module 4: risk_metrics
This module will contain functions for calculating Value at Risk (VaR) and Expected Shortfall (ES).

- historical_var(): This function calculates the historical VaR given a vector of returns and a confidence level.
- parametric_var(): This function calculates the parametric VaR given a vector of returns, a confidence level, and a normal distribution assumption.
- monte_carlo_var(): This function calculates the VaR using Monte Carlo simulation given a vector of returns, a confidence level, and the number of simulations.
- historical_es(): This function calculates the historical ES given a vector of returns and a confidence level.
- parametric_es(): This function calculates the parametric ES given a vector of returns, a confidence level, and a normal distribution assumption.
- monte_carlo_es(): This function calculates the ES using Monte Carlo simulation given a vector of returns, a confidence level, and the number of simulations.

Module 5: test_suite
This module will contain tests for all the functions in the previous modules.

Problem 3

Analysis of Result:

- Portfolio A has the highest VaR of 3324.61, indicating that with a probability of 5%, the maximum loss in the portfolio is likely to be around $3324.61. On the other hand, Portfolio A has the lowest ES of 154.89, which means that in the worst 5% of the cases, the expected loss is around $154.89.

- Portfolio B has the lowest VaR of 10.32, indicating that the probability of maximum loss in the portfolio is low. However, the portfolio has the highest ES of 133.57, meaning that in the worst 5% of the cases, the expected loss is high.

- Portfolio C has a higher VaR of 28.78 compared to Portfolio B, but a lower ES of 107.60. This suggests that there is a higher probability of a large loss in the portfolio, but the expected loss in the worst-case scenario is lower compared to Portfolio B.
- The Total VaR is 154.14, which indicates the expected loss at a 95% confidence level in the entire portfolio. The Total ES is 192.16, which shows the expected loss in the worst 5% of the cases in the portfolio.

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Portfolio A VaR at 0.05 confidence level is 3324.611164270274
Portfolio A ES at 0.05 confidence level is 154.89190830125594
Portfolio B VaR at 0.05 confidence level is 10.324487992525759
Portfolio B ES at 0.05 confidence level is 133.5725871402833
Portfolio C VaR at 0.05 confidence level is 28.77705703745168
Portfolio C ES at 0.05 confidence level is 107.60243859051496
Total VaR at 0.05 confidence level is 154.1352871158148
Total ES at 0.05 confidence level is 192.1603303718062
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In summary, VaR measures the potential loss of a portfolio at a given confidence level, while ES measures the expected loss in the worst-case scenario. The results suggest that Portfolios B and C have lower VaR but higher ES, indicating higher expected losses in the worst-case scenarios. On the other hand, Portfolio A has a higher VaR but a lower ES, indicating a lower probability of losses in the worst-case scenarios. The Total VaR and ES provide an overall estimate of the risk in the entire portfolio.

Compared with the Result of Week04:

Using a multivariate normal distribution assumes that the individual asset returns are normally distributed, and the covariance matrix is constant over time. While using a generalized T distribution allows for more flexibility in the shape of the distribution, allowing for skewness and heavier tails. It appears that the two methods have led to different VaR estimates for the three portfolios and the total portfolio. The multivariate normal method has resulted in higher VaR estimates for all portfolios and the total portfolio compared to the generalized T method. This suggests that the multivariate normal method may be more conservative in its estimates.