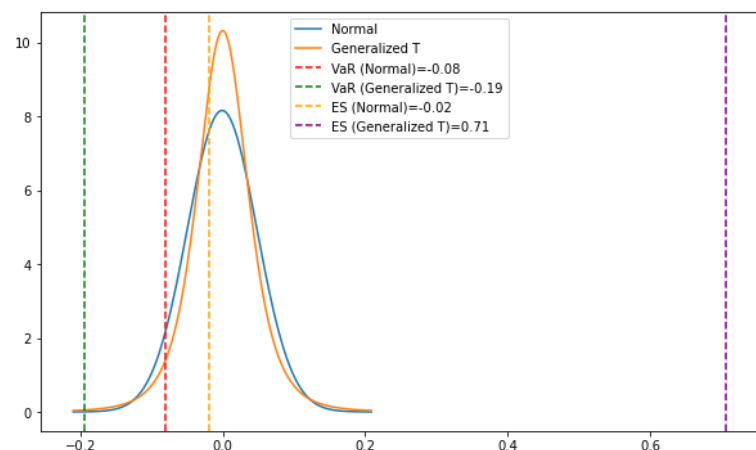# Problem 1

Based on the results, we can see that the VaR and ES values differ between the two fitted distributions. The Normal distribution has a VaR of -0.08 and ES of -0.02, while the Generalized T distribution has a VaR of -0.19 and ES of 0.71.

The Generalized T distribution has a larger VaR than the Normal distribution, indicating that there is a higher probability of larger losses with this distribution. The ES value for the Generalized T distribution is also positive, indicating that the expected loss given the VaR is greater than zero, which is in contrast to the negative ES value for the Normal distribution.

These differences can be attributed to the different shapes of the two distributions. The Generalized T distribution has heavier tails than the Normal distribution, meaning that extreme events are more likely to occur. This results in a larger VaR value for the Generalized T distribution. The positive ES value for the Generalized T distribution may be due to skewness in the data, which the Normal distribution does not account for.

```
Normal Distribution:
  VaR(0.05): -0.08
  ES(0.05): -0.02
Generalized T Distribution:
  VaR(0.05): -0.19
  ES(0.05): 0.71
```



# Problem 2

**Module 1**: covariance_estimators
This module will contain functions for estimating the covariance matrix of asset returns.

- sample_covariance_matrix(): This function takes in a matrix of asset returns and returns the sample covariance matrix.
- ledoit_wolf_shrinkage(): This function implements the Ledoit-Wolf shrinkage estimator to estimate the covariance matrix. It takes in a matrix of asset returns and returns the estimated covariance matrix.
- fit_generalized_t: This function is a Generalized t-distribution to a vector of returns using maximum likelihood estimation.

**Module 2**: correlation_matrix_fixes

This module will contain functions for handling non-positive definite covariance matrices.

- nearest_correlation_matrix(): This function takes in a non-positive definite covariance matrix and returns the nearest positive definite matrix.
- higham_algorithm(): This function implements the Higham algorithm for finding the nearest positive definite matrix to a non-positive definite matrix.

**Module 3**: simulation_methods

This module will contain functions for simulating asset returns.

- monte_carlo_simulation(): This function takes in a covariance matrix, a vector of mean returns, and the number of simulations, and returns simulated asset returns.

**Module 4**: risk_metrics

This module will contain functions for calculating Value at Risk (VaR) and Expected Shortfall (ES).

- historical_var(): This function calculates the historical VaR given a vector of returns and a confidence level.
- parametric_var(): This function calculates the parametric VaR given a vector of returns, a confidence level, and a normal distribution assumption.
- monte_carlo_var(): This function calculates the VaR using Monte Carlo simulation given a vector of returns, a confidence level, and the number of simulations.
- historical_es(): This function calculates the historical ES given a vector of returns and a confidence level.
- parametric_es(): This function calculates the parametric ES given a vector of returns, a confidence level, and a normal distribution assumption.
- monte_carlo_es(): This function calculates the ES using Monte Carlo simulation given a vector of returns, a confidence level, and the number of simulations.

**Module 5**: test_suite

This module will contain tests for all the functions in the previous modules.

# Problem 3

**Analysis of Result**:

- Portfolio A has the highest VaR of 3324.61, indicating that with a probability of 5%, the maximum loss in the portfolio is likely to be around $3324.61. On the other hand, Portfolio A has the lowest ES of 154.89, which means that in the worst 5% of the cases, the expected loss is around $154.89.

- Portfolio B has the lowest VaR of 10.32, indicating that the probability of maximum loss in the portfolio is low. However, the portfolio has the highest ES of 133.57, meaning that in the worst 5% of

the cases, the expected loss is high.

- Portfolio C has a higher VaR of 28.78 compared to Portfolio B, but a lower ES of 107.60. This suggests that there is a higher probability of a large loss in the portfolio, but the expected loss in the worst-case scenario is lower compared to Portfolio B.
- The Total VaR is 154.14, which indicates the expected loss at a 95% confidence level in the entire portfolio. The Total ES is 192.16, which shows the expected loss in the worst 5% of the cases in the portfolio.

```
Portfolio A VaR at 0.05 confidence level is 3324.611164270274
Portfolio A ES at 0.05 confidence level is 154.89190830125594
Portfolio B VaR at 0.05 confidence level is 10.324487992525759
Portfolio B ES at 0.05 confidence level is 133.5725871402833
Portfolio C VaR at 0.05 confidence level is 28.77705703745168
Portfolio C ES at 0.05 confidence level is 107.60243859051496
Total VaR at 0.05 confidence level is 154.1352871158148
Total ES at 0.05 confidence level is 192.1603303718062
```

In summary, VaR measures the potential loss of a portfolio at a given confidence level, while ES measures the expected loss in the worst-case scenario. The results suggest that Portfolios B and C have lower VaR but higher ES, indicating higher expected losses in the worst-case scenarios. On the other hand, Portfolio A has a higher VaR but a lower ES, indicating a lower probability of losses in the worst-case scenarios. The Total VaR and ES provide an overall estimate of the risk in the entire portfolio.

**Compared with the Result of Week04:**

Using a multivariate normal distribution assumes that the individual asset returns are normally distributed, and the covariance matrix is constant over time. While using a generalized T distribution allows for more flexibility in the shape of the distribution, allowing for skewness and heavier tails. It appears that the two methods have led to different VaR estimates for the three portfolios and the total portfolio. The multivariate normal method has resulted in higher VaR estimates for all portfolios and the total portfolio compared to the generalized T method. This suggests that the multivariate normal method may be more conservative in its estimates.