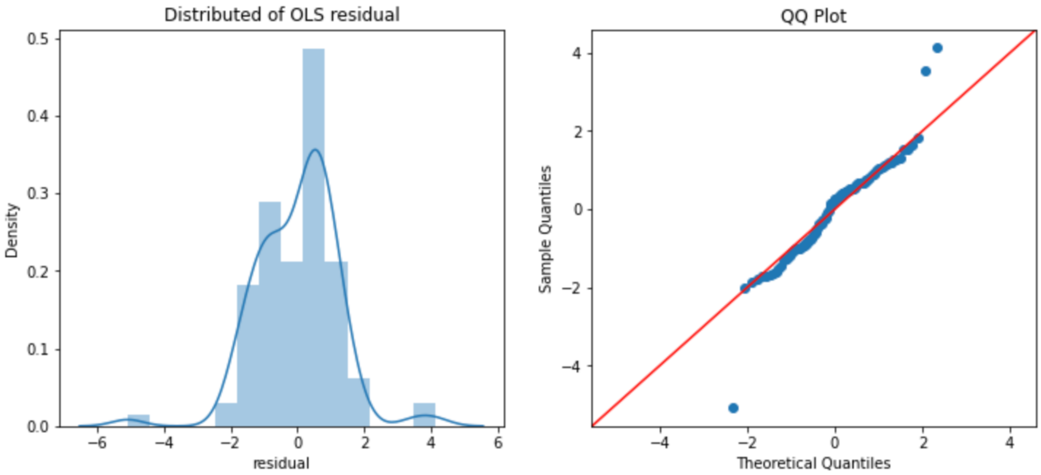
Problem 1

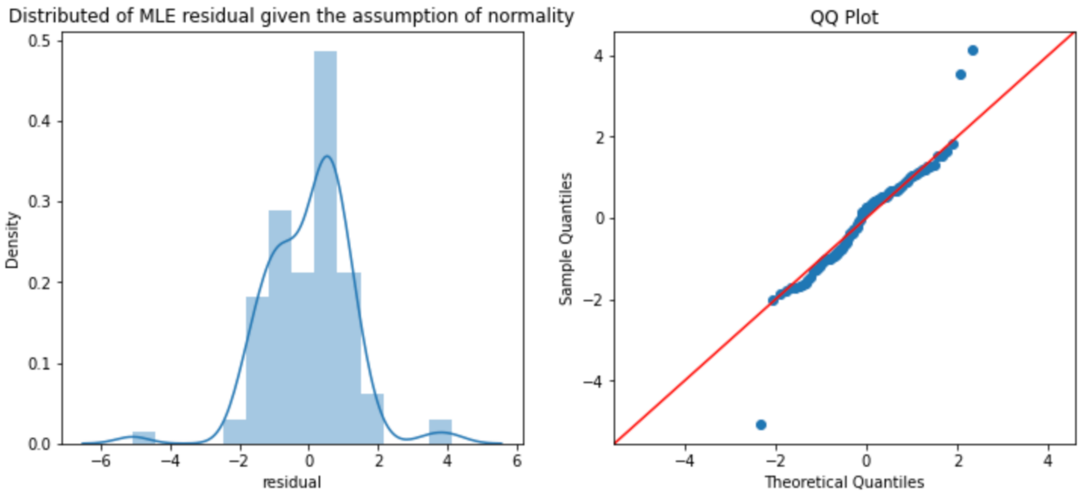
First, generate 100 random numbers from a normal distribution with a mean of 100 and a standard deviation of 10. We compute skewness and kurtosis on these 100 random numbers. If you use the scipy.stats.skew and scipy.stats.kurtosis functions in python's scipy statistical packages to calculate, in the case of default parameters, the calculation results are biased. But as long as the bias parameter in the scipy function is set to False, an unbiased estimate can be obtained, and our function result is unbiased.

Problem 2

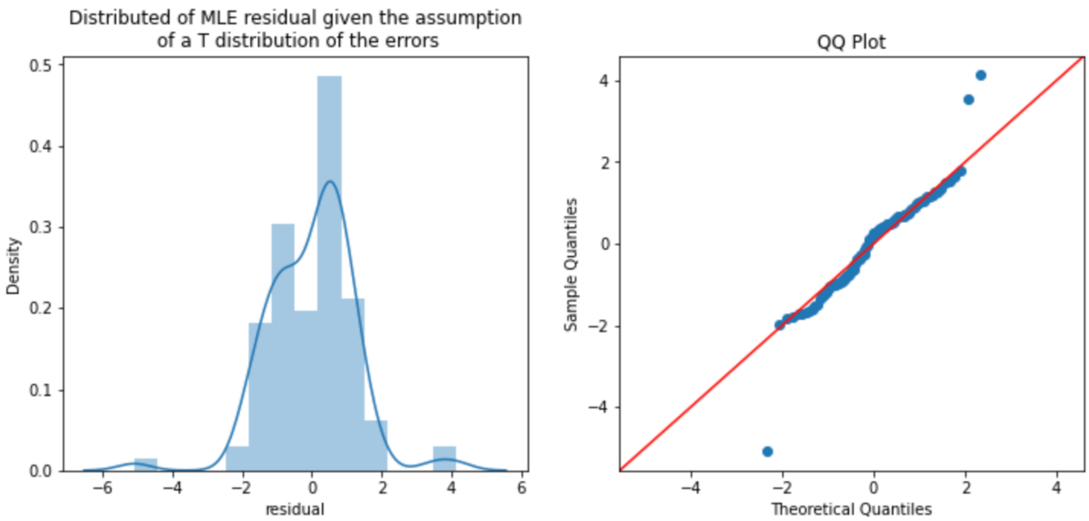
Fit the data in problem2.csv using OLS, calculate the regression residuals, and observe their distribution. Both the distribution histogram and the QQ plot reveal that the regression residuals violate the assumption of normally distributed errors.



Fit the data using MLE given the assumption of normality, compute regression residuals, and observe their distribution. Both the distribution histogram and the QQ plot reveal that the regression residuals violate the assumption of normally distributed errors.



Fit the MLE using the assumption of a T distribution of the errors, compute the regression residuals, and observe their distribution. Both the distribution histogram and the QQ plot reveal that the regression residuals violate the assumption of normally distributed errors.

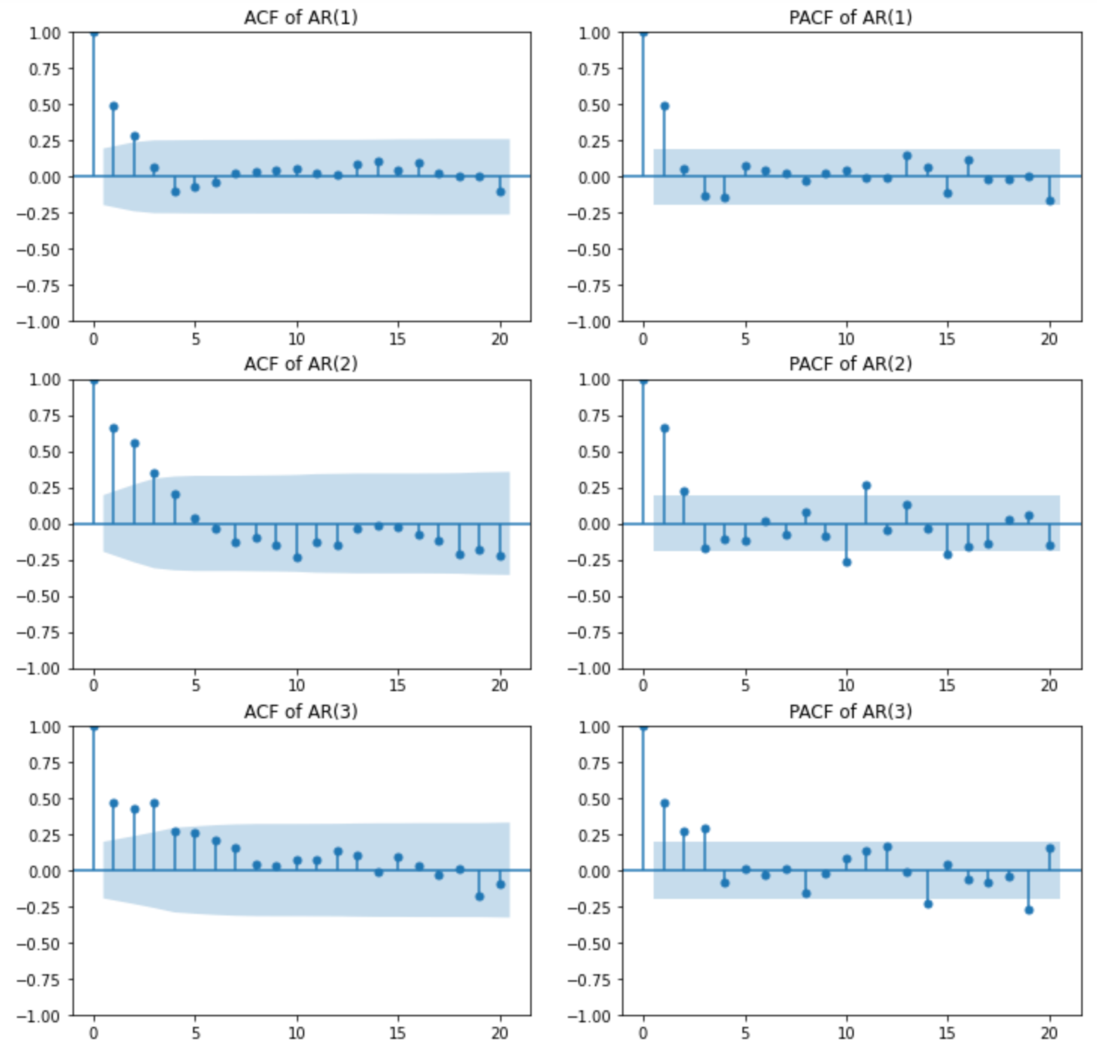


We use MSE to evaluate the fitting effect of the three methods. The following table shows the regression coefficients of the three models and MSE, OLS and MLE given the assumption of normality. The regression parameters and MSE are the same, and fit the MLE using the assumption of a T distribution of the errors has the highest MSE and is the worst fitting method. At the same time, all three methods are breaking of the normality assumption in regards to expected values in this case, which means that the prediction results of the model are not accurate enough, there may be potential model bias, or the assumption of the model may be incorrect.

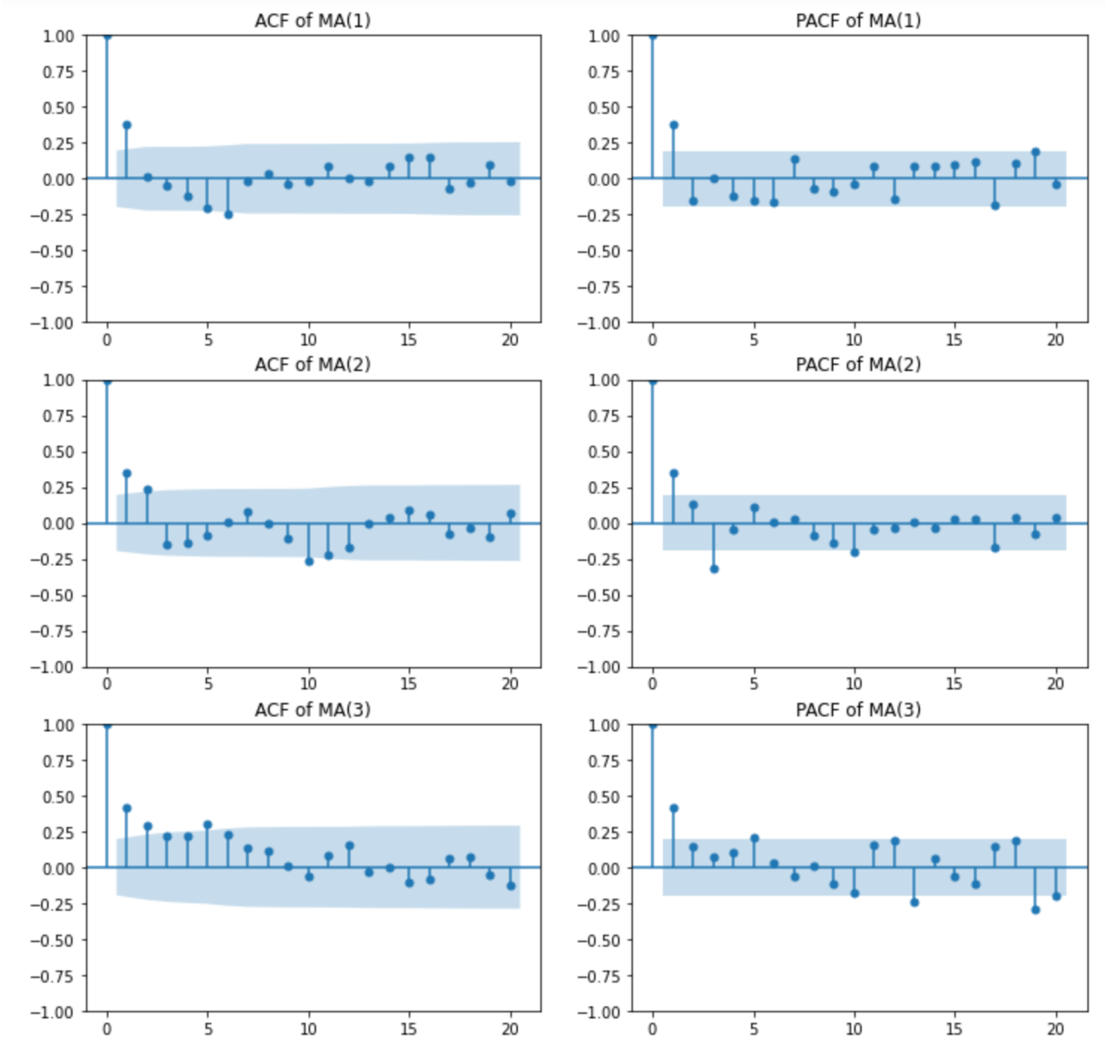
	regression intercept	regression slope	MSE
OLS	0.119836	0.605205	1.436148
MLE given the assumption of normality	0.119836	0.605205	1.436148
MLE using the assumption of a T distribution of the errors	0.123685	0.595034	1.436261

Problem 3

Simulate AR(1) through AR(3) processes， compare their ACF and PACF graphs.



Simulate MA(1) through MA(3) processes， compare their ACF and PACF graphs.



The ACF and PACF graphs help us to identify the type and order of each process by showing the pattern of autocorrelations at different lags. For AR processes, the ACF graph will show an exponential decay while the PACF graph will show a sharp cut-off after the order of the process. For example, the ACF graph for an AR(2) process will show an exponential decay, with the lag-1 and lag-2 autocorrelation coefficients being the highest. The PACF graph will show a sharp cut-off after lag-2, indicating that only the lag-1 and lag-2 autocorrelations are significant.

For MA processes, the ACF graph will show a sharp cut-off after the order of the process while the PACF graph will show an exponential decay. For example, The ACF graph for an MA(2) process will show a sharp cut-off after lag-2, indicating that only the lag-1 and lag-2 autocorrelations are significant. The PACF graph will show an exponential decay, with the lag-1 autocorrelation coefficient being the highest.