Exercise 8 : MFKTIT004

Backward equation

$$U(x-\Delta x) = U(x) - \frac{\partial u}{\partial x}\Delta t + \frac{1}{2!}\frac{\partial^2 u}{\partial x^2}\Delta x^2 - \frac{1}{3!}\frac{\partial^3 u}{\partial x^3} + O(\Delta x^4)$$

Solving for $\frac{\partial u}{\partial x}$ and divide by Δx.

$$\frac{\partial U}{\partial x}\frac{\cancel{\Delta x}}{\cancel{\Delta x}} = \frac{U(x) - U(x-\Delta x)}{\Delta x} + \frac{1}{2!}\frac{\partial^2 u}{\partial x^2}\frac{\Delta x^2}{\Delta x} - \frac{1}{3!}\frac{\partial^3 u}{\partial x^3}\frac{\Delta x^3}{\Delta x} + \frac{O(\Delta x^4)}{\Delta x}$$

Below is velocity gradient !

$$\frac{\partial U}{\partial x} = \frac{U(x) - U(x-\Delta x)}{\Delta x} + \frac{1}{2!}\frac{\partial^2 U}{\partial x^2}\Delta x - \frac{1}{3!}\frac{\partial^3 U}{\partial x^3}\Delta x^2 + O(\Delta x^3)$$

Backward in space aproximation : (First-order)

-> The higher order polynomials x^2/x^3 are removed:

$$\frac{\partial U}{\partial x} = \frac{U(x) - U(x-\Delta x)}{\Delta x}$$

Now using discre indices :

$$\frac{\partial U}{\partial x} \approx \frac{U_i - U_{i-1}}{\Delta x} + O(\Delta x)$$