Differences between Arithmetic, Geometric, and Harmonic Means

Oct 1st, 2010 | By **Andrew Matuszak** | Category: **Applied Finance**, **Lead Story**

Although the concept of an average seems simple, we continue to see many people using an incorrect method to calculate averages. We would be surprised if most people were familiar with the three different types of averages mentioned in the title. In fact, many people (including journalists) routinely use the wrong type of average–leading to incorrect results and often poor decisions.

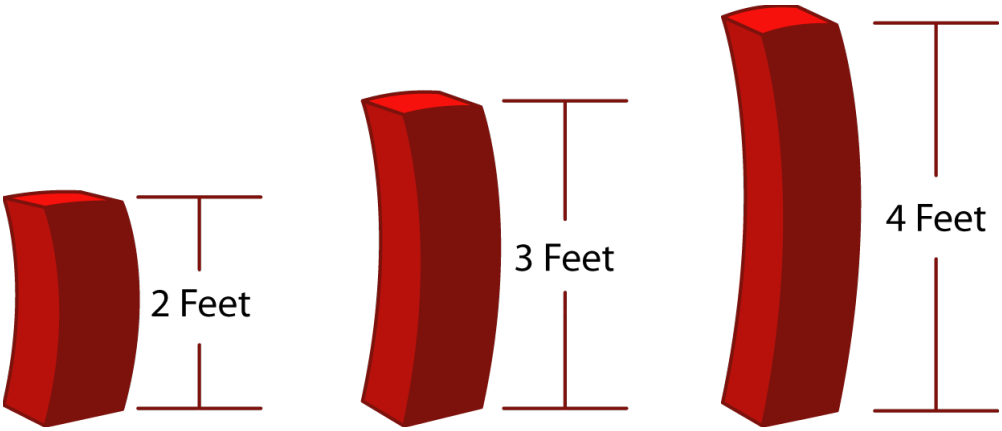
In this article, we will explore each of these averages and discuss when to use each. We will avoid mathematical derivations since most people will be using a spreadsheet.

For this article, we use Microsoft Excel to solve for the arithmetic, harmonic, and geometric means. However, we've also create an **article** that shows how to use the R software to calculate the various averages as well as compare them using graphs. We encourage you to read this **article** as well as it provides additional context that may be helpful in understanding the key differences between these averages.

Arithmetic Mean

The arithmetic mean is by far the most common average. It is the simplest to compute and the easiest to understand. In fact, most people are only familiar with the arithmetic mean; however, this is often inaccurate and misleading.

To illustrate the arithmetic mean, consider the three bars below.



The arithmetic mean is simply computed by adding the three heights together and dividing by three; the result is an average height of 3 feet.

In many instances, this is the correct type of average to use. The arithmetic mean is best used in situations where:

- the data are not skewed (no extreme outliers)
- the individual data points are not dependent on each other (see the section below for examples of where data are interrelated, e.g., financial analysis)

Harmonic Mean

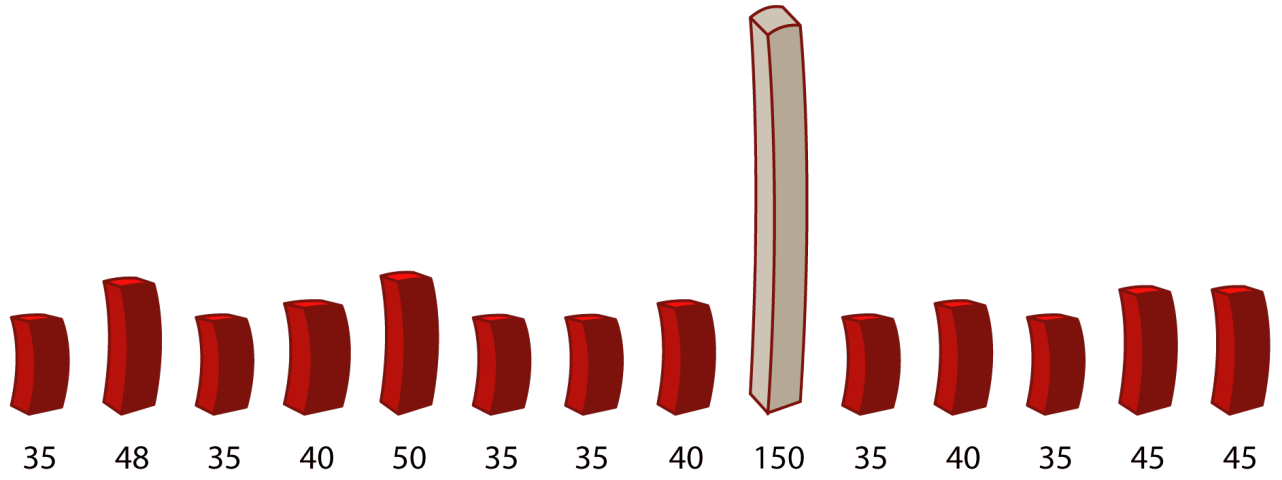
Probably the least understood, the harmonic mean is best used in situations where extreme outliers exist in the population. The harmonic mean can be manually calculated; however, most people will find it much easier to

just use Excel. In Excel, the harmonic mean can be calculated by using the HARMEAN() function. If you interested in using R to calculate the harmonic mean, please visit **here**.

There are plenty of online resources (see Wikipedia) that cover the mathematical derivation of the harmonic mean; we are going to focus on when one should use it.

If the population (or sample) has a few data points that are much higher than the rest (outliers), the harmonic mean is the appropriate average to use. Unlike the arithmetic mean, the harmonic mean gives less significance to high-value outliers–providing a truer picture of the average.

For example, let's consider how we could calculate the average height of the following bars.



In this population, there are 13 bars that are fairly similar in height and 1 bar that is more than 3 times larger. This outlier has a large effect on the arithmetic average, resulting in an average that is biased due to that one bar. If the objective is to calculate the average that most representative of the population, than the harmonic average is the best approach.

The harmonic average is less biased due to a small number of outliers. In the example above, the arithmetic average is approximately 47.7 while the harmonic average is approximately 41. In this case, the arithmetic average overstates the average by over 15%. The harmonic average provides a better picture of the true average of this population, discounting the large outlier.

The harmonic average is best to use when there is:

- A large population where the majority of the values are distributed uniformly but where there are a few outliers with significantly higher values

Geometric Average

The geometric average should be used much more frequently than we suspect it is, especially when discussing average investment returns or interest rates.

Use the geometric average should be used whenever the data are inter-related–for example, when discussing returns on investment or interest rates. Using the arithmetic average will result in the wrong answer in these cases. Let's consider an example.

An investment has the following returns over the next five years:

2010	2011	2012	2013	2014
5%	20%	25%	-10%	20%

The simple arithmetic average is 12%. However, this is not correct. To understand why it is not correct, consider an initial investment of $10,000.

If one were to use the arithmetic average, they would arrive at an amount at the end of 2014 of $17,623.42. To confirm this, type the following formula into Excel:

```
=FV(.12,5,0,-10000)
```

This is incorrect. The table below illustrates the actual cash flows from the investment above.

2010	2011	2012	2013	2014
5%	20%	25%	-10%	20%
10,500	12,600	15,750	14,175	17,010

The investment is actually worth $17,010 at the end of the five years. The arithmetic mean overstated the return by almost 4%. The correct mean to use in these situations is the geometric mean. To calculate the geometric mean in Excel, simply enter the annual returns and use the `GEOMEAN` function.

1	Column A	Column B
2	2010 Percentage	1.05
3	2011	1.20
4	2012	1.25
5	2013	0.90
6	2014	1.20
7	Geometric Mean Function	=GEOMEAN(B2:B6)

The geometric mean is approximately 11.20924. Confirm that this is the correct answer by typing the following formula into Excel:

```
=FV(.1120924,5,0,-10000)
```

The result is approximately $17,010 (slightly off due to rounding).

NOTE: When entering percentages in Excel for use in calculating the geometric mean, they must be entered as (1+percentage). For example, enter 40% as 1.40 and not .40. Similarly, a percentage of -20% would be .80.

If you are interested in using R to calculate the geometric mean of a sample of observations, please find **here**.

Summary

Choosing the correct mean is essential to correctly estimating the central tendency of a population or calculating investment returns. Although the concept of an average may seem simple, it is imperative for a user to carefully consider which mean to use–and to communicate to reader or intended audience the method of deriving the average and the rationale for doing so.

Please read the article posted here for additional information on the differences between these three averages as well as videos and helpful R code.

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Tags: **average**, **Economics**, **geometric mean**, **harmonic mean**, **mean**

30 Comments to “Differences between Arithmetic, Geometric, and Harmonic Means”

1. ***Javed Iqbal*** says:
   April 20, 2011 at 10:27 am

   Very well explained.
   Useful hints
   

2. *mona* says:
   May 5, 2011 at 12:04 pm

   wonderful and amazing explanation

3. *kartikeya pandey* says:
   June 13, 2011 at 4:57 am

   It is wonderful discussion and extremly helpful .

4. *Jenny Falcon* says:
   December 17, 2011 at 4:04 pm

   well explained. i was just looking for such a detailed explanation.

5. *Tatyana* says:
   February 7, 2012 at 2:55 am

   Simple and interesting examples with reasonable investment application.
   Thank you.

6. *grace* says:
   February 8, 2012 at 5:03 pm

   perfect

7. *Cathrine* says:
   March 20, 2012 at 2:59 pm

   Thanks for the help
   

8. *sumiya* says:
   July 25, 2012 at 4:58 am

   it is understandable and very helpful

9. *Kelly* says:
   August 8, 2012 at 1:41 am

   It is vivid and very helpful. Perfect explanation.!!!!!!

10. ***Steve Roberts*** says:
    September 8, 2012 at 2:49 pm

    Our Glossary Term "Harmonic Mean" at Mr. X, Mentor of Math leaves a lot to be desired. We make math videos. May I take your descriptions from **http://economistatlarge.com/finance/applied-finance/differences-arithmetic-geometric-harmonic-means** and build a better video, giving you good folks due credit?
    More succinctly, I want to use your words in our video to explain mean, especially harmonic and geometric mean.

Thank you.
In your service,
Steve

11. *Kamster* says:
October 23, 2012 at 6:51 am

Above you stated you said it is appropriate to use harmonic mean when outliers are lower than the rest, this would overstate to these values since the reciprocals give more wait to lower values.

12. ***Andrew Matuszak*** says:
October 23, 2012 at 1:09 pm

Kamster, you are absolutely correct; the harmonic mean would overstate the significance of extremely lower values. For example, if the population contains {5, 3, 2, -10000} the harmonic mean be overly biased towards the lower number. The harmonic mean works best when used on data sets where there are higher outliers. I've amended the post to more accurately reflect this. Thank you for pointing this out!

13. *lale* says:
November 5, 2012 at 10:13 am

geometric mean is about multiplicative quantities
just like investment returns
arithmetic means is about additive ones..

14. *Geoff Patton* says:
November 6, 2012 at 2:31 pm

For deciding when to use the Harmonic Mean, how big (or small) is too much? That is, by what criteria is a value an outlier?

15. ***Andrew Matuszak*** says:
November 6, 2012 at 3:18 pm

Geoff,

That is a great question–and one subject to one's own subjective interpretation. The answer you are probably not looking for is that 'it depends'. By visualizing the data, you may quickly find that one or two points lie far outside what you'd consider an expected distribution. That isn't always feasible though and so I'd recommend that calculate both the arithmetic and harmonic mean and compare the results. If you find that the difference is large (also subjective), you should investigate further to understand whether only a small number of points are influencing your results. Remember to look for data points significantly larger that the rest of the set; the harmonic mean isn't appropriate to compensate for data significantly lower than the rest. I hope this helps.

16. ***A Case of Two Means:Geometric & Arithmetic « Sharad Sinha*** says:
November 7, 2012 at 3:55 pm

[…] cited in the beginning, GM should be used. Some nice references to read are : ref1, ref2, ref3, ref4. Similary, understanding when to use Harmonic Mean (HM) is also important. Whichever mean you […]

17. ***Samuel*** says:
November 16, 2012 at 9:25 am

I find this article to be interesting and quite educative. Please maintain the standard.

18. ***Compounded Average Growth Rate - Economist at Large*** says:
December 3, 2012 at 6:12 pm

[…] useful view to remember is that CAGR is the geometric mean of the returns. Please visit here for an in-depth discussion on how to calculate geometric means and a comparison with other […]

19. ***Compound Average Growth Rate - Economist at Large*** says:
December 3, 2012 at 6:14 pm

[…] useful view to remember is that CAGR is the geometric mean of the returns. Please visit here for an in-depth discussion on how to calculate geometric means and a comparison with other […]

20. *Pawan Bhatt* says:
December 4, 2012 at 6:45 am

very satisfactory answer ….
Thanks


21. *Rohit* says:
December 6, 2012 at 8:15 am

Very well explained.

22. ***rs gp*** says:
April 21, 2013 at 9:12 am

Hello to all, it担 actually a pleasant for me to go to see this web page, it consists of useful Information.

23. ***Gender Development Index – Systematic Biases | Stand up for a Cause*** says:
June 9, 2013 at 11:12 am

[…] The index is combined to get a single number signifying how worse of women are (as a ratio of 1 which was discussed in the first para). The combination is based on the % of population and a 'Harmonic Mean' is found. Now harmonic mean gives undue weightage to the lower values. The justification of harmonic mean was not adequate and seemed like biased. It stated that 'index is combined in such a way which penalizes differences in achievement'. An example on the difference of harmonic v/s arithmetic mean is given here. […]

24. *Muhammad Rafi* says:
July 29, 2013 at 6:09 pm

The discussion above about the comparision and uses of average is quite nice and fruitful. But if you allow me I wish to add some comments.
Firstly for the selection of an appropriate average one should keep three points in mind
i. type of data ii. shape of distribution iii. and purpose for which it would be used
Secondly you if please consider the mode and median then discussion would be more useful.
Regards
Muhammad Rafi
Lecturer and Researcher in Statistics
Govt. P/G Islamia College, Gujranwala

25. *desmond* says:
January 17, 2014 at 6:30 am

good but didnt give the manual formulae to use.

26. *vipin* says:
July 17, 2014 at 2:46 am

Fabulous sir…

27. *Jahanzeb Ahsan* says:
October 10, 2014 at 6:41 am

Splendid, very simple and direct approach for layman

28. *shanza Nadeem* says:
    October 27, 2014 at 10:23 am

    great help (y) thanks
    

29. *JAMES* says:
    November 6, 2014 at 2:44 pm

    Am very thankful, the site is very helpful to me; and am real enjoying to get more and favorable concepts! which I didn't understand before.

30. ***pushap ratn*** says:
    December 6, 2014 at 3:03 am

    Fantastic ellabration

Leave a Comment