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$$16. (1) \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{\sqrt{2}}{2}$$

$$\frac{1}{2}c^2 = b^2 - a^2$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\sin B - \sin^2 A = \pm \sin^2 C$$

$$\sin^2 A \cos^2 C + \pm \sin 2A \cdot \pm \sin 2C + \pm \sin 2C \cdot \pm \sin 2A + \sin^2 C \cos^2 A$$

$$-\sin^2 A = \pm \sin^2 C$$

$$\frac{1}{2} \cos^2 C + \frac{1}{2} \sin 2C + \frac{1}{2} \cos^2 C \pm \sin^2 C - \frac{1}{2} = \pm \sin^2 C$$

$$\cos^2 C + \sin 2C + \sin^2 C - 1 = \sin^2 C$$

$$\cos^2 C + 1 - 2\sin^2 C + \sin^2 C - 1 = \sin^2 C$$

$$\cos^2 C = 2\sin^2 C$$

$$\therefore \frac{\sin^2 C}{\cos^2 C} = \frac{1}{2} \quad \therefore \tan C = \pm \frac{1}{\sqrt{2}} \quad \therefore \tan C = \frac{\sqrt{2}}{2}$$

$$12) \sin C = \frac{\sqrt{2}}{3} \quad \cos C = \frac{\sqrt{7}}{3}$$

$$\sin B = \sin(A+C) = \sin A \cos C + \cos A \sin C$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{7}}{3} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{3} = \frac{2\sqrt{6} + \sqrt{3}}{6}$$

$$S_{\triangle ABC} = \frac{1}{2} bc \sin A = 3$$

$$bc = 6\sqrt{2}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\therefore b = 2\sqrt{2}$$

AI Response:

根据提供的学生答案，按照标准数学公式格式要求进行批改和规范，具体如下：

16. (I)

学生答案分析与批改：

原始答案中有一些不规范的地方，如乘号未使用“ \times ”，部分公式缺少适当的括号等。批改并规范后的答案如下：

1. **计算 $\cos A$ ：**

$$\begin{aligned} & \backslash[\\ & \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & \frac{1}{2}c^2 = b^2 - a^2 \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & c^2 = a^2 + b^2 - 2ab \cos C \end{aligned}$$

2. **计算 $\sin B - \sin A$ ：**

$$\begin{aligned} & \backslash[\\ & \sin B - \sin A = \frac{1}{2} \sin C \\ & \backslash[\\ & \sin^2 A \cos C + \frac{1}{2} \sin 2A \times \frac{1}{2} \sin 2C + \frac{1}{2} \sin 2A \times \sin C \cos A - \sin^2 A = \frac{1}{2} \sin C \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & \frac{1}{2} \cos C + \frac{1}{4} \sin 2C + \frac{1}{4} \cos \frac{1}{2} \sin C - \frac{1}{2} = \frac{1}{2} \sin C \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & \cos^2 C + \sin 2C + \sin^2 C - 1 = \sin C \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & \cos^2 C + 1 - 2 \sin^2 C + \sin^2 C - 1 = \sin C \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & \cos^2 C = 2 \sin^2 C \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & \end{aligned}$$

$\therefore \frac{\sin^2 C}{\cos^2 C} = \frac{1}{2} \quad \therefore \tan C = \frac{\sqrt{2}}{2}$

16. (II)

学生答案分析与批改：

原始答案中也存在一些不规范的地方，如乘号未使用“ \times ”，部分公式缺少适当的括号等。批改并规范后的答案如下：

1. **计算 $\sin C$ 和 $\cos C$ ：**

$$\sin C = \frac{\sqrt{3}}{2}, \quad \cos C = \frac{\sqrt{2}}{2}$$

$$\sin B = \sin(A + C) = \sin A \cos C + \cos A \sin C$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{2}}{6}$$

2. **计算 $\triangle ABC$ 的面积：**

$$S_{\triangle ABC} = \frac{1}{2} bc \sin A = 3$$

$$bc = 6\sqrt{2}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\therefore b = 2\sqrt{2}$$

通过以上批改和规范，使得数学公式更加清晰、规范和专业。