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Level 5 Paper 1
Exemplar 1

2022-DSE
MATH CP

PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2022

Please stick the barcode label here.

Candidate Number									

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

8:30 am – 10:45 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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SECTION A(1) (35 marks)

1. Simplify $\dfrac{(a^3b^{-2})^4}{a^{-5}b^6}$ and express your answer with positive indices. (3 marks)

$\dfrac{(a^3b^{-2})^4}{a^{-5}b^6} = \dfrac{a^{12}b^{-8}}{a^{-5}b^6}$

$= \dfrac{a^{17}}{b^{14}}$

2. Let x and y be two numbers. The sum of x and y is 456 while the product of 7 and x is y. Find x. (3 marks)

$\begin{cases} x+y=456 & —(1) \\ 7x=y & —(2) \end{cases}$

Put (2) into (1),

$x+7x=456$

$8x=456$

$x=57$

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3. Simplify $\dfrac{3}{k-9}+\dfrac{2}{5k+6}$. (3 marks)

$$\frac{3}{k-9}+\frac{2}{5k+6}=\frac{3(5k+6)+2(k-9)}{(k-9)(5k+6)}$$

$$=\frac{(15k+18)+(2k-18)}{(k-9)(5k+6)}$$

$$=\frac{17k}{(k-9)(5k+6)}$$

4. Factorize

(a) $9c^2-6c+1$,

(b) $(4c+d)^2-9c^2+6c-1$.

(4 marks)

(a) $(3c-1)^2$

(b) $(4c+d)^2-9c^2+6c-1$

$=(4c+d)^2-(3c-1)^2$

$=[(4c+d)+(3c-1)][(4c+d)-(3c-1)]$

$=(7c+d-1)(c+d+1)$

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5. A fan is sold at a discount of 30% on its marked price. After selling the fan, the profit is \$78 and the percentage profit is 26%. Find the marked price of the fan. (4 marks)

let s, m and c be the selling price, marked price and cost respectively

$\begin{cases} s = (1-30\%)m & —① \\ s = c+78 & —② \\ s = c(1+26\%) & —③ \end{cases}$

Put ② into ③

$c+78 = c(1+26\%)$

$c+78 = 1.26c$

$c = 300$

Put $c=300$ into ②, $s=300+78=378$

Put $s=378$ into ①, $378=(1-30\%)m$

$m=540$

$\therefore$ marked price = \$540

6. Consider the compound inequality

$-2(3x+2) > x+10$ or $2x \le -8$ .... (*).

(a) Solve (*).

(b) Write down the greatest integer satisfying (*).

(4 marks)

(a) $-2(3x+2) > x+10$ or $2x \le -8$

$-6x-4 > x+10$ or $x \le -4$

$-7x > 14$ or $x \le -4$

$x < -2$ or $x \le -4$

$\therefore x < -2$

(b) -3

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7. The coordinates of the points S and T are $(12, -5)$ and $(-3, -7)$ respectively. S is rotated anticlockwise about O through $90°$ to S', where O is the origin. T' is the reflection image of T with respect to the x-axis.

(a) Write down the coordinates of S' and T'.

(b) Find the slope of $S'T'$.

(4 marks)

(a) coordinates of S' are $(5, 12)$
coordinates of T' are $(-3, 7)$

(b) slope of $S'T' = \frac{12-7}{5-(-3)} = \frac{5}{8}$

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8. In Figure 1, A is a point lying inside the quadrilateral $BCDE$ such that $AC // ED$ and $AD // BC$. It is given that $\angle ABC = \angle AED$ and $AB = AE$.

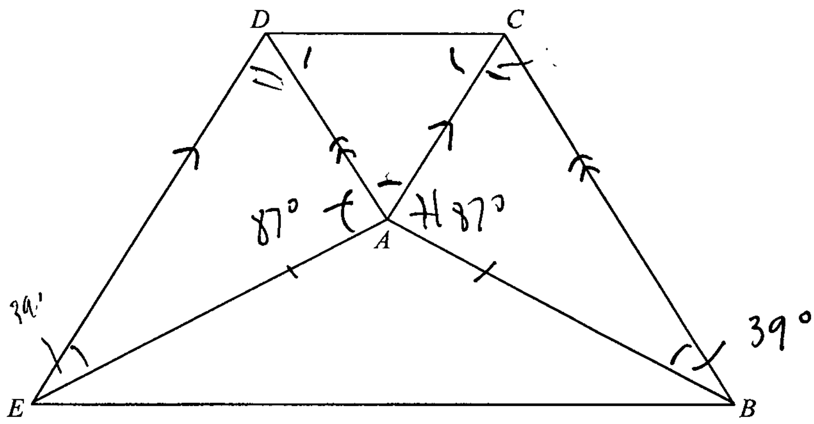

Figure 1

(a) Prove that $\Delta ABC \cong \Delta AED$.

(b) If $\angle ABC = 39°$ and $\angle DAE = 87°$, find $\angle ACD$.

(5 marks)

(a) $\angle ABC = \angle AED$ (given)

$\angle BCA = \angle CAD$ (alt $\angle$s $BC // AD$)

$\angle CAD = \angle ADE$ (alt $\angle$s $DE // AC$)

$\therefore \angle BCA = \angle ADE$

$\therefore \angle BAC = \angle DAE$ ($\angle$ sum of Δ)

$\therefore \Delta ABC \cong \Delta AED$ (AAA)

(b) $\angle DEA = \angle ABC = 39°$

$\angle EDA = 180° - \angle DAE - \angle DEA = 180° - 87° - 39° = 54°$

$\angle CAD = \angle EDA = 54°$

Note that $AC = AD$

$\therefore \angle ADC = \angle ACD$

$2\angle ACD + 54° = 180°$

$\angle ACD = 63°$

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9. The frequency distribution table and the cumulative frequency distribution table below show the distribution of the times taken to complete a 3 km race by a group of students.

Time taken (minutes)	Frequency
10 – 14	a
15 – 19	9
20 – 24	b
25 – 29	3

Time taken less than (minutes)	Cumulative frequency
14.5	3
19.5	x
24.5	y
29.5	20

(a) Write down the value of x .

(b) Find the mean of the distribution.

(c) Find the probability that the time taken to complete the 3 km race by a randomly selected student from the group is less than 19.5 minutes.

(5 marks)

(a) $x = 12$

(b) mean $= \frac{(12)(3)+(17)(9)+(22)(5)+(27)(3)}{3+9+5+3}$

$= 17.5$

(c) The required probability $= \frac{12}{20} = \frac{3}{5}$

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SECTION A(2) (35 marks)

10. It is given that $f(x)$ partly varies as x^2 and partly varies as x. Suppose that $f(4)=96$ and $f(-5)=15$.

(a) Find $f(x)$. (3 marks)

(b) Write down the x-intercept(s) of the graph of $y=8f(x)$. (1 mark)

(c) Let k be a real constant. Find the range of values of k such that the equation $f(x)=k$ has two distinct real roots. (2 marks)

(a) let $f(x)=ax^2+bx$, where a and b are non-zero constant

Put $f(4)=96$, $16a+4b=96$ —— ①

Put $f(-5)=15$, $25a-5b=15$ —— ②

By ①×5 + ②×4 : $180a=540$

$a=3$

Put $a=3$ into ①, $16(3)+4b=96$

$b=12$

$\therefore f(x)=3x^2+12x$

(b) $y=8f(x)=8(3x^2+12x)=24x^2+96x$

Put $y=0$, $24x^2+96x=0$

$24x(x+4)=0$

$x=0$ or $x=-4$

$\therefore$ x-intercepts are 0 or -4

(c) $f(x)=k$

$3x^2+12x=k$

$3x^2+12x-k=0$

$\Delta=(12)^2-4(3)(-k)>0$

$144+12k>0$

$12k>-144$ $\therefore k>-12$

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11. The stem-and-leaf diagram below shows the distribution of the ages of the players of a football team.

Stem (tens)	Leaf (units)
1	7 8 9
2	0 a a 8 8 9
3	b b 5 5 6 6 6 6 7 8
4	3

The inter-quartile range and the median of the distribution are 14 and 31 respectively.

(a) Find a and b. (3 marks)

(b) A player now leaves the football team.

(i) Is there any change in the mode of the distribution due to the leaving of the player? Explain your answer.

(ii) If the range of the distribution is decreased, find the greatest possible standard deviation of the distribution.

(4 marks)

(a) inter-quartile range $= 36 - (20 + a) = 14$

$a = 2$

median $= (30 + b) = 31$

$b = 1$

(b)(i) The original mode is 36

Even through a player of 36 ages leaves, it still has the highest number, so the mode remain unchanged

(ii) When a player of 43 ages leaves

standard deviation $= 7.13$

When a player of 17 ages leaves

standard deviation $= 7.16$

$\therefore$ greatest possible standard deviation $= 7.16$

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12. The equation of the circle C is $x^2 + y^2 - 154x - 128y + 224 = 0$. Denote the centre of C by G . The coordinates of the point H are $(65, 48)$.

(a) Find the distance between G and H . (3 marks)

(b) Let P be a moving point on C . When the area of ΔGHP is the greatest,

(i) describe the geometric relationship between GH and GP ;

(ii) find the perimeter of ΔGHP .

(4 marks)

(a) $x^2+y^2-154x-128y+224=0$

$(x-77)^2+(y-64)^2 = 9801$

$\therefore$ coordinates of G are $(77, 64)$

distance between G and $H = \sqrt{(77-65)^2+(64-48)^2}$

$= 20$

(b) (i) GH and GP are perpendicular to each other

(ii) Note that $GP =$ radius of $C = \sqrt{9801} = 99$

$\cancel{\tan \angle PHG = \frac{99}{20}}$

$\cancel{\angle PHG \approx 78.57881373^\circ}$

$\cancel{\sin \angle PHG = \frac{PG}{PH}}$

$\sin \cancel{78.57881373} = \frac{99}{PH}$

$\frac{99}{101}$ $\cancel{PH = 99}$

$\cancel{PH = 101}$

$PH^2 = HG^2 + PG^2$

$PH = \sqrt{20^2+99^2}$

$= 101$

$\therefore$ perimeter of $\Delta GHP = 101 + 20 + 99$

$= 220$

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13. There are two solid metal spheres. The ratio of the surface area of the smaller sphere to the surface area of the larger sphere is $4:9$. The radius of the larger sphere is 9 cm.

(a) Express, in terms of π, the volume of the smaller sphere. (3 marks)

(b) The two spheres are melted and recast into two solid right circular cones. Denote these two circular cones by A and B. It is given that the height and the base radius of A are 10 cm and 6 cm respectively. A student finds that the base radius of B is 12 cm. The student claims that A and B are similar. Is the claim correct? Explain your answer. (4 marks)

(a) let r_s be the radius of smaller sphere

$\left(\frac{r_s}{9}\right)^2 = \frac{4}{9}$

$\frac{r_s}{9} = \frac{2}{3}$

$r_s = 6$

$\therefore$ volume of smaller sphere $= \frac{4}{3}\pi(6)^3 = 288\pi \text{ cm}^3$

(b) volume of larger sphere $= \frac{4}{3}\pi(9)^3 = 972\pi \text{ cm}^3$

volume of cone A $= \frac{1}{3}\pi(6)^2(10) = 120\pi \text{ cm}^3$

volume of cone B $= (288\pi + 972\pi) - 120\pi$

$= 1140\pi \text{ cm}^3$

let h_B be ~~the~~ height of cone B

$1140\pi = \frac{1}{3}\pi(12)^2(h)$

$3420 = 144h$

$h = 23.75$

$\frac{\text{radius of A}}{\text{radius of B}} = \frac{6}{12} = \frac{1}{2}$

Answers written in the margins will not be marked.

(b) $\dfrac{\text{height of } A}{\text{height of } B} = \dfrac{6}{23.75} = \dfrac{8}{19} \neq \dfrac{1}{2} = \dfrac{\text{radius of } A}{\text{radius of } B}$

$\therefore$ cones A and B are not similar

$\therefore$ The claim is incorrect

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14. Let $\mathrm{p}(x)=2x^3+ax^2+bx-20$, where a and b are constants. When $\mathrm{p}(x)$ is divided by x^2-2x+3, the remainder is $x+13$.

(a) Find a and b. (3 marks)

(b) Is $x-5$ a factor of $\mathrm{p}(x)$? Explain your answer. (2 marks)

(c) Someone claims that the equation $\mathrm{p}(x)=0$ has two irrational roots. Do you agree? Explain your answer. (3 marks)

(a) let $p(x)=(x^2-2x+3)(px+q)+x+13$

$=px^3+(q-2p)x^2+(3p-2q+1)x+(3q+13)$

$=2x^3+ax^2+bx-20$

Therefore, we have:

$$\begin{cases} 2=p & \text{—(1)} \\ a=q-2p & \text{—(2)} \\ b=3p-2q+1 & \text{—(3)} \\ -20=3q+13 & \text{—(4)} \end{cases}$$

By (4), $-33=3q$

$-11=q$

put $p=2, q=-11$ into (2), $a=(-11)-2(2)=-15$

put $p=2, q=-11$ into (3), $b=3(2)-2(-11)+1=29$

(b) $p(x)=2x^3-15x^2+29x-20$

$p(5)=2(5)^3-15(5)^2+29(5)-20=0$

$\therefore$ $x-5$ is a factor of $p(x)$

Answers written in the margins will not be marked.

(c) When $p(x)=0$,

$2x^3-15x^2+29x-20=0$

$(x-5)(2x^2-5x+4)=0$

$\therefore x=5$, or $2x^2-5x+4=0$

$\Delta=(-5)^2-4(2)(4)$

$=-7<0$

$\therefore$ no real roots

$\therefore$ There is no irrational roots for $p(x)=0$

$\therefore$ I ~~does~~ disagree

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SECTION B (35 marks)

15. There are 10 boys and 12 girls in a class. If 4 students are randomly selected from the class to form a committee,

(a) find the probability that there are 2 boys and 2 girls in the committee; (2 marks)

(b) find the probability that the number of boys and the number of girls in the committee are different. (2 marks)

(a) The required probability $= \frac{C_2^{10} \times C_2^{12}}{C_4^{22}} = \frac{54}{133}$

(b) The required probability $= 1 - \frac{54}{133} = \frac{79}{133}$

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16. Let $g(x)=3x^2+12kx+16k^2+8$, where k is a non-zero real constant.

(a) Using the method of completing the square, express, in terms of k, the coordinates of the vertex of the graph of $y=g(x)$. (2 marks)

(b) On the same rectangular coordinate system, denote the vertex of the graph of $y=g(x)$ and the vertex of the graph of $y=2g(-x)$ by A and B respectively. Let M be a point lying on AB such that the area of ΔOBM is the triple of the area of ΔOAM , where O is the origin. Express, in terms of k, the coordinates of M. (3 marks)

(a) $g(x)=3x^2+12kx+16k^2+8$

$=3(x^2+4kx)+16k^2+8$

$=3(x^2+4kx+4k^2)-12k^2+16k^2+8$

$=3(x+2k)^2+4k^2+8$

$\therefore$ coordinates of the vertex are $(-2k, 4k^2+8)$

(b) The vertex of $y=2g(-x)$ is $(2k, 8k^2+16)$

Note that when the area of ΔOBM : the area of $\Delta OAM=3:1$. Then $BM:AM=3:1$

$\therefore$ Coordinates of $M=\left(\frac{(2k)(1)+(-2k)(3)}{1+3}, \frac{(8k^2+16)(1)+(4k^2+8)(3)}{1+3}\right)$

$=(-k, 5k^2+10)$

Answers written in the margins will not be marked.

17. Let c be a real constant. The roots of the equation $x^2+cx-9=0$ are α and β.

(a) Express $\alpha^2+\beta^2$ in terms of c. (3 marks)

(b) The 1st term, the 2nd term and the 3rd term of an arithmetic sequence are c^2, $\alpha^2+\beta^2$ and 85 respectively. Find the least value of n such that the sum of the first n terms of the sequence is greater than 2×10^6. (4 marks)

(a) $\alpha+\beta=-c$

$\alpha\beta=-9$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=(-c)^2-2(-9)$

$=c^2+18$

(b) common difference $=(\alpha^2+\beta^2)-c^2=85-(\alpha^2+\beta^2)$

$(c^2+18)-c^2=85-(c^2+18)$

$18=67-c^2$

$c^2=49$

$\therefore$ 1st term $=c^2=49$

common difference $=(\alpha^2+\beta^2)-c^2$

$=(49+18)-49$

$=18$

sum of first n terms

$=\frac{n[2\times49+(n-1)18]}{2}>2\times10^6$

$n(9n+40)>2\times10^6$

$9n^2+40n-2\times10^6>0$

$n<-474$, $n>469$

$\therefore$ least value of $n=470$

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18. In Figure 2, the triangular paper card PQR is held such that PQ lies on the horizontal ground. It is given that $PQ = 30$ cm , $PR = 25$ cm and $\angle QPR = 95°$.

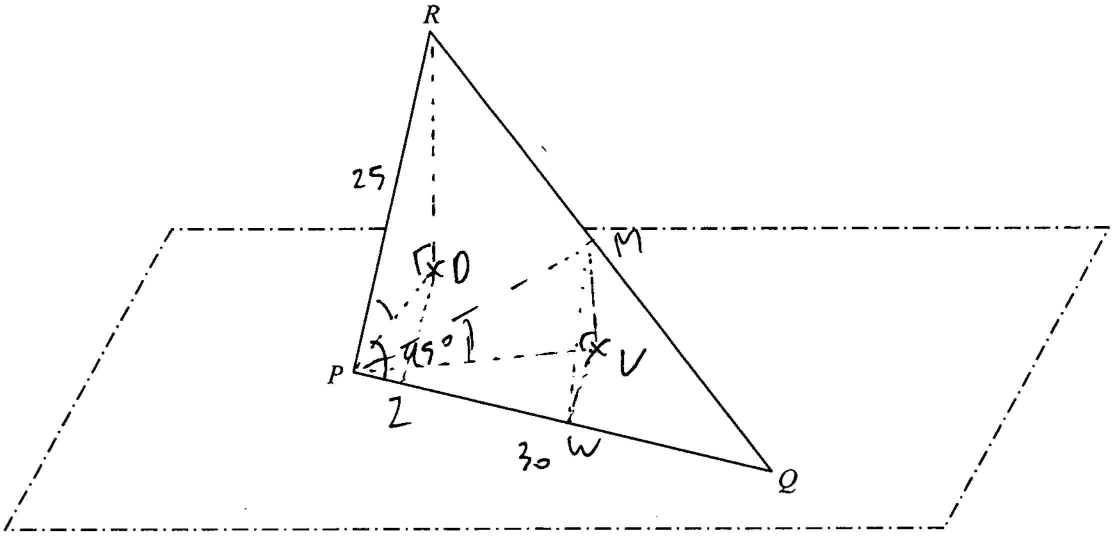

Figure 2

(a) Find

(i) the length of QR,

(ii) $\angle PQR$.

(4 marks)

(b) Let M be the mid-point of QR . A craftsman finds that the angle between PR and the horizontal ground is $70°$. The craftsman claims that the angle between PM and the horizontal ground exceeds $40°$. Is the claim correct? Explain your answer. (3 marks)

(a)(i) By cosine formula, we have

$QR^2 = PR^2 + PQ^2 - 2(PR)(PQ)\cos\angle QPR$

$QR^2 = 25^2 + 30^2 - 2(25)(30)\cos 95°$

$QR \approx 40.69070673$

≈ 40.7 cm

(ii) By sine formula, we have

$\frac{PR}{\sin\angle PQR} = \frac{QR}{\sin\angle QPR}$

$\frac{25}{\sin\angle PQR} \approx \frac{40.69070673}{\sin 95°}$

$\angle PQR \approx 37.73809375° \approx 37.7°$

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(b) area of $\Delta PQR = \frac{1}{2}(30)[25\sin(180-95^\circ)]$

≈ 373.5730118

let D be the point of projection of R on the horizontal ground

$RD = 25\sin\angle RPD = 25\sin 70^\circ \approx 23.49231552$ cm

let Z on PQ such that $RZ \perp PQ$

$\frac{1}{2}(30)(RZ) \approx 373.5730118$

$RZ \approx 24.90486745$ cm

$\sin\angle RZD \approx \frac{23.49231552}{24.90486745}$

$\angle RZD \approx 70.61535843^\circ$

let W on PQ such that $MW \perp PQ$

$MQ = \frac{RQ}{2} \approx 20.34535336$ cm

By cosine formula

$\cos\angle PQR = \frac{RQ^2 + PQ^2 - PR^2}{2(RQ)(PQ)}$

$\angle PQR \approx 37.73809375^\circ$

$MW = MQ\sin\angle PQR \approx 12.45243372$ cm

let V be the point of projection of M on the horizontal ground

Note that $\angle RZD = \angle MWV$

$MV = MW\sin\angle MWV \approx 11.74652598$ cm

By cosine formula,

$PM^2 = MQ^2 + PQ^2 - 2(MQ)(PQ)\cos\angle PQR$

$PM \approx 18.66993831$ cm

$\sin\angle MPV = \frac{MV}{PM}$

$\angle MPV = 38.98875875$

$< 40^\circ$

(Note that the angle between PM and the horizontal ground $= \angle MPV$)

$\therefore$ The claim is incorrect

Answers written in the margins will not be marked.

19. The centre of the circle C is the point $G(83, 112)$. It is found that the point $A(158, 12)$ lies outside C. AP and AQ are the tangents to C at the points P and Q respectively. It is given that C passes through the point $(23, 67)$.

(a) Find the equation of the straight line passing through A and G. (2 marks)

(b) Find the coordinates of the point of intersection of AG and PQ. (3 marks)

(c) Find the equation of the inscribed circle of ΔAPQ. (4 marks)

(d) Someone claims that the ratio of the area of the inscribed circle to the area of the circumcircle of ΔAPQ is $1:4$. Do you agree? Explain your answer. (3 marks)

(a) slope of stright line passing through A and G $= \frac{112-12}{83-158}$

$= -\frac{4}{3}$

Equation : $y - 12 = -\frac{4}{3}(x - 158)$

$4x + 3y - 668 = 0$

(b) radius of circle $= \sqrt{(83-23)^2 + (112-67)^2} = 75$

$\therefore$ equation of C : $(x-83)^2 + (y-112)^2 = 75^2$

$x^2 + y^2 - 166x - 224y + 13808 = 0$

let the slope of tangent be m

$\therefore$ equation of tangent : $y - 12 = m(x - 158)$

$y = mx + (12 - 158m)$

Put ~~y~~ = mx + (12 − 158m) into ~~equation of C~~

$(x-83)^2 + [mx + (12-158m) - 112]^2 = 75^2$

$(x^2 - 166x + 6889) + [mx - (100 + 158m)]^2 = \cancel{5625}$

$(x^2 - 166x + 6889) + [m^2x^2 - (200m + 316m^2)x + (31600m + 34964)]$

$= 5625$

$(1+m^2)x^2 - (316m^2 + 200m + 166)x + (31600m + 36228) = 0$

$\Delta = [(316m^2 + 200m + 166)]^2 - 4(1+m^2)(31600m + 36228) = 0$

(b) Distance between $mx-y+(12-158m)=0$ and $(83,112)$

$$=\left|\frac{83m-112+(12-158m)}{\sqrt{m^2+1}}\right|=75$$

$$\left|\frac{-75m-100}{\sqrt{m^2+1}}\right|=75$$

$$\frac{-75m-100}{\sqrt{m^2+1}}=75 \quad \text{or} \quad \frac{-75m-100}{\sqrt{m^2+1}}=-75$$

$$-3m-4=3\sqrt{m^2+1} \qquad 75m+100=75\sqrt{m^2+1}$$

$$9m^2+24m+16=9(m^2+1) \qquad 3m+4=3\sqrt{m^2+1}$$

$$24m=-7 \qquad 9m^2+24m+16=9(m^2+1)$$

$$m=-\frac{7}{24} \qquad m=-\frac{7}{24}$$

$\therefore$ equation of tangent: $-\frac{7}{24}x-y+\frac{697}{12}=0$

$\frac{7}{24}x-y-\frac{409}{12}=0$

By solving $-\frac{7}{24}x-y+\frac{697}{12}=0$ and $x^2+y^2-166x-224y+138\ =0$

we have, $(62,40)$

By solving $\frac{7}{24}x-y-\frac{409}{12}=0$ and $x^2+y^2-166x-224y+13808=0$

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Answers written in the margins will not be marked.

(d) Disagree

END OF PAPER

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Level 5 Paper 1
Exemplar 2

2022-DSE
MATH CP

PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2022

Please stick the barcode label here.

Candidate Number									

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

8:30 am – 10:45 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

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SECTION A(1) (35 marks)

1. Simplify $\dfrac{(a^3b^{-2})^4}{a^{-5}b^6}$ and express your answer with positive indices. (3 marks)

$\dfrac{(a^3b^{-2})^4}{a^{-5}b^6}$

$= \dfrac{a^{12}b^{-8}}{a^{-5}b^6}$

$= \dfrac{a^{17}}{b^{14}}$

2. Let x and y be two numbers. The sum of x and y is 456 while the product of 7 and x is y. Find x. (3 marks)

$x+y=450$ —— (1)

$7x=y$ —— (2)

Sub (2) into (1)

$x+7x=450$

$x=56.25$ //

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3. Simplify $\frac{3}{k-9}+\frac{2}{5k+6}$. (3 marks)

$$\frac{3}{k-9}+\frac{2}{5k+6}$$

$$=\frac{3(5k+6)+2(k-9)}{(k-9)(5k+6)}$$

$$=\frac{15k+18+2k-18}{(k-9)(5k+6)}$$

$$=\frac{17k}{(k-9)(5k+6)}$$

4. Factorize

(a) $9c^2-6c+1$,

(b) $(4c+d)^2-9c^2+6c-1$.

(4 marks)

a) $9c^2-6c+1$

$=(3c-1)^2$ //

b) $(4c+d)^2-9c^2+6c-1$

$=(4c+d)^2-(3c-1)^2$

$=(4c+d-3c+1)(4c+d+3c-1)$

$=(c+d+1)(7c+d-1)$ //

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5. A fan is sold at a discount of 30% on its marked price. After selling the fan, the profit is \$78 and the percentage profit is 26%. Find the marked price of the fan. (4 marks)

Let \$$x$ be the marked price. and \$$c$ be the cost.

$(1-30\%)x - c = 78$ —— ①

$\frac{(1-30\%)x - c}{c} \times 100\% = 26\%$

$0.7x = 1.26c$ —— ②

$\therefore x = 540,\ c = 300$

$\therefore$ The marked price is \$540.

6. Consider the compound inequality

$-2(3x+2) > x+10$ or $2x \le -8$ .... (*).

(a) Solve (*).

(b) Write down the greatest integer satisfying (*).

(4 marks)

a) $-2(3x+2) > x+10$ or $2x \le -8$

$-6x-4 > x+10$ $\quad$ $x \le -4$

$-14 > 7x$

$-2 > x$

$\therefore x < -2$

b) -3.

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7. The coordinates of the points S and T are $(12, -5)$ and $(-3, -7)$ respectively. S is rotated anticlockwise about O through $90°$ to S', where O is the origin. T' is the reflection image of T with respect to the x-axis.

(a) Write down the coordinates of S' and T'.

(b) Find the slope of $S'T'$.

(4 marks)

a) $S' = (5, 12)$
$T' = (-3, 7)$

b) Slope of $S'T' = \frac{12-7}{5+3}$

$= \frac{5}{8}$ //

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8. In Figure 1, A is a point lying inside the quadrilateral $BCDE$ such that $AC // ED$ and $AD // BC$. It is given that $\angle ABC = \angle AED$ and $AB = AE$.

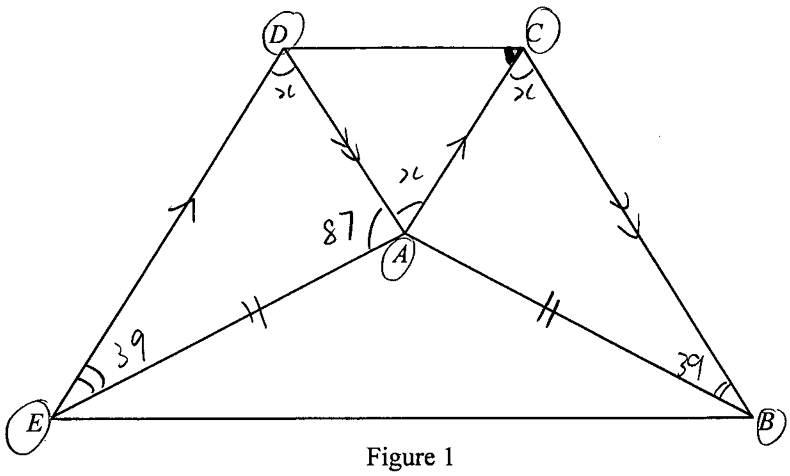

Figure 1

(a) Prove that $\Delta ABC \cong \Delta AED$.

(b) If $\angle ABC = 39°$ and $\angle DAE = 87°$, find $\angle ACD$.

(5 marks)

a) $\angle EDA = \angle DAC$ (alt $\angle$s, $ED // AC$)

$= x$

$\angle BCA = \angle DAC$ (alt $\angle$s, $AD // BC$)

$= x$

$\therefore \angle EDA = \angle BCA$

$\angle ABC = \angle AED$ (given)

$AB = AE$ (given)

$\therefore \Delta ABC \cong \Delta AED$ (AAS)

b) In ΔADE

$x = 180° - 39° - 87°$ ($\angle$ sum of Δ)

$= 54°$

$DA = AC$ (corr sides, $\cong \Delta$s)

$\therefore \angle CDA = \angle DCA$ (base $\angle$s, isos Δ)

$\angle ACD = (180° - x) \div 2$

$= (180° - 54) \div 2$

$= 63°$ //

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9. The frequency distribution table and the cumulative frequency distribution table below show the distribution of the times taken to complete a 3 km race by a group of students.

Time taken (minutes)	Frequency
10 – 14	a 3
15 – 19	9
20 – 24	b 5
25 – 29	3

Time taken less than (minutes)	Cumulative frequency
14.5	3
19.5	x 12
24.5	y
29.5	20

(a) Write down the value of x.

(b) Find the mean of the distribution.

(c) Find the probability that the time taken to complete the 3 km race by a randomly selected student from the group is less than 19.5 minutes.

(5 marks)

a) $x = 12$

b) Mean $= \dfrac{12 \times 3 + 17 \times 9 + 22 \times 5 + 27 \times 3}{20}$

$= 19$ mins.

c) The probability $= \dfrac{3+9}{20}$

$= \dfrac{3}{5}$ //

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SECTION A(2) (35 marks)

10. It is given that $f(x)$ partly varies as x^2 and partly varies as x. Suppose that $f(4)=96$ and $f(-5)=15$.

(a) Find $f(x)$. (3 marks)

(b) Write down the x-intercept(s) of the graph of $y=8f(x)$. (1 mark)

(c) Let k be a real constant. Find the range of values of k such that the equation $f(x)=k$ has two distinct real roots. (2 marks)

a) $f(x) = k_1 x^2 + k_2 x$

$f(4) = k_1(4^2) + k_2(4) = 96$

$f(-5) = k_1(-5)^2 + k_2(-5) = 15$

$\therefore k_1 = 3, \quad k_2 = 12$

$f(x) = 3x^2 + 12x$ //

b) $y = 8f(x)$

$0 = 8(3x^2+12x)$

$0 = 24x^2+96x$

$x=0$ // or $x=-4$ //

c) $f(x) = k$

$3x^2 + 12x = k$

$3x^2+12x-k = 0$

$\Delta = 12^2 - 4(3)(-k) > 0$

$144 + 12k > 0$

$k > -12$ //

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11. The stem-and-leaf diagram below shows the distribution of the ages of the players of a football team.

Stem (tens)	Leaf (units)
1	7 8 9 2
2	0 a a 8 8 9
3	b b 5 5 6 6 6 6 7 8
4	3

The inter-quartile range and the median of the distribution are 14 and 31 respectively.

(a) Find a and b. (3 marks)

(b) A player now leaves the football team.

(i) Is there any change in the mode of the distribution due to the leaving of the player? Explain your answer.

(ii) If the range of the distribution is decreased, find the greatest possible standard deviation of the distribution.

(4 marks)

a) Median $= \dfrac{30+b+30+b}{2} = 31$

$b = 1$ //

Interquartile range $= 36-(20+a) = 14$

$36-20-a = 14$

$a = 2$ //

b i) No, The original mode is 36, which have 4 members, even a member aged 36 leave, there are still 3 members left, the new mode is still 36. If member with other age leave, the mode will not be changed.

ii) The greatest possible standard deviation

$= 7.1625$

≈ 7.16.

12. The equation of the circle C is $x^2+y^2-154x-128y+224=0$. Denote the centre of C by G. The coordinates of the point H are $(65, 48)$.

(a) Find the distance between G and H. (3 marks)

(b) Let P be a moving point on C. When the area of ΔGHP is the greatest,

(i) describe the geometric relationship between GH and GP;

(ii) find the perimeter of ΔGHP.

(4 marks)

a) $G=(77, 64)$

$GH = \sqrt{(77-65)^2+(64-48)^2}$

$= 20$ units.

b) i) GH is prependicular to GP.

ii) Radius of C $= \sqrt{77^2+64^2-224}$

$= 99$ units.

$PH^2 = 20^2+99^2$ (Pyth. Thm)

$PH = 101$ units

The perimeter of $\Delta GHP = 101+99+20$

$= 220$ units.

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13. There are two solid metal spheres. The ratio of the surface area of the smaller sphere to the surface area of the larger sphere is 4 : 9 . The radius of the larger sphere is 9 cm .

(a) Express, in terms of π, the volume of the smaller sphere. (3 marks)

(b) The two spheres are melted and recast into two solid right circular cones. Denote these two circular cones by A and B. It is given that the height and the base radius of A are 10 cm and 6 cm respectively. A student finds that the base radius of B is 12 cm . The student claims that A and B are similar. Is the claim correct? Explain your answer. (4 marks)

a) Volume of the smaller sphere

$= \frac{4}{3}\pi(9)^3 \times \frac{8}{27}$

$= 288\pi \text{ cm}^3$.

b) Total volume of spheres

$= \frac{4}{3}\pi(9)^3 + 288\pi$

$= 1260\pi \text{ cm}^3$

Let h cm be the height of B.

$\pi(6)^2 \times 10 \times \frac{1}{3} + \pi(12)^2 \times h \times \frac{1}{3} = 1260\pi$

$120\pi + 48h\pi = 1260\pi$

$h = 23.75$ cm.

$\therefore$ The height is 23.75 cm.

The ratio of radius to height of B

$= 12 : 23.75$

$= 48 : 95$

The ratio of radius to height of A

$= 6 : 10$

$= 3 : 5 \neq 48 : 95$

$\therefore$ They are not similar.

$\therefore$ The claim is incorrect.

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14. Let $p(x) = 2x^3 + ax^2 + bx - 20$, where a and b are constants. When $p(x)$ is divided by $x^2 - 2x + 3$, the remainder is $x + 13$.

(a) Find a and b. (3 marks)

(b) Is $x - 5$ a factor of $p(x)$? Explain your answer. (2 marks)

(c) Someone claims that the equation $p(x) = 0$ has two irrational roots. Do you agree? Explain your answer. (3 marks)

a)

$$\begin{array}{r} 2x + a+4 \\ x^2-2x+3 \overline{\big) \, 2x^3 + ax^2 + bx - 20} \\ 2x^3 - 4x^2 + 6x \\ \hline (a+4)x^2 + (b-6)x - 20 \\ (a+4)x^2 - 2(a+4)x + 3(a+4) \\ \hline [(b-6)+2(a+4)]x \\ -20-3(a+4) \end{array}$$

$[(b-6)+2(a+4)]x - 20 - 3(a+4)$

$= x + 13$

$\therefore -20 - 3(a+4) = 13$

$-20 - 3a - 12 = 13$

$a = -15$ //

$\therefore (b-6) + 2(a+4) = 1$

$b - 6 + 2(-15+4) = 1$

$b = 29$ //

b) $p(x) = 2x^3 - 15a^2 + 29x - 20$

$p(5) = 2(5)^3 - 15(5)^2 + 29(5) - 20$

$= 0$

$\therefore x - 5$ is a factor.

$p(x) = 2x^3 - 15a^2 + 29x - 20$

$= (x-5)(2x^2 - 5x + 4)$

$\therefore x = 5$ or $x = \dfrac{5 \pm \sqrt{(-5)^2 - 4(2)(4)}}{2(2)}$

$= \dfrac{5 \pm \sqrt{-7}}{4}$

$\therefore p(x)$ has two irrational roots.

$\therefore$ The claim is agreed.

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SECTION B (35 marks)

15. There are 10 boys and 12 girls in a class. If 4 students are randomly selected from the class to form a committee,

(a) find the probability that there are 2 boys and 2 girls in the committee; (2 marks)

(b) find the probability that the number of boys and the number of girls in the committee are different. (2 marks)

a) The probability $= \dfrac{C_2^{10} \times C_2^{12}}{C_4^{22}}$

$= \dfrac{54}{133}$

b) The probability $= 1 - \dfrac{54}{133}$

$= \dfrac{79}{133}$.

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16. Let $g(x) = 3x^2 + 12kx + 16k^2 + 8$, where k is a non-zero real constant.

(a) Using the method of completing the square, express, in terms of k, the coordinates of the vertex of the graph of $y = g(x)$. (2 marks)

(b) On the same rectangular coordinate system, denote the vertex of the graph of $y = g(x)$ and the vertex of the graph of $y = 2g(-x)$ by A and B respectively. Let M be a point lying on AB such that the area of ΔOBM is the triple of the area of ΔOAM, where O is the origin. Express, in terms of k, the coordinates of M. (3 marks)

a) $g(x) = 3x^2 + 12kx + 16k^2 + 8$

$= 3(x^2 + 4kx + (2k)^2 - (2k)^2) + 16k^2 + 8$

$= 3(x+2k)^2 - 3(2k)^2 + 16k^2 + 8$

$= 3(x+2k)^2 + 4k^2 + 8$

$\therefore$ Vertex $= (-2k, 4k^2+8)$

b) $A = (-2k, 4k^2+8)$

$B = (2k, 8k^2+16)$

$\because$ ΔOBM and ΔOAM have the same height.

$\therefore$ $BM : AM = 3 : 1$

The coordinate of M

$= \left(\dfrac{2k + (3(-2k))}{4}, \dfrac{8k^2+16+3(4k^2+8)}{4}\right)$

$= \left(k, \dfrac{8k^2+16+12k^2+24}{4}\right)$

$= (k, 5k^2+10)$ //

Answers written in the margins will not be marked.

17. Let c be a real constant. The roots of the equation $x^2+cx-9=0$ are α and β.

(a) Express $\alpha^2+\beta^2$ in terms of c. (3 marks)

(b) The 1st term, the 2nd term and the 3rd term of an arithmetic sequence are c^2, $\alpha^2+\beta^2$ and 85 respectively. Find the least value of n such that the sum of the first n terms of the sequence is greater than 2×10^6. (4 marks)

a) $\alpha^2+\beta^2$

$=\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta$

$=(\alpha+\beta)^2-2\alpha\beta$

$=(\frac{-c}{1})^2-2(\frac{-9}{1})$

$=c^2+18$ //

b) The common different $=c^2+18-c^2$

$=18.$

1st term $=85-18-18$

$=49.$

Sum of first n term

$=[49+49+(n-1)(18)]\frac{n}{2}>2\times10^6$

$49n+9n^2-9n>2\times10^6$

$9n^2+40n-2\times10^6>0$

$n<-473.63$ (rej) or $n>469.19$

$\therefore$ The least value of n is 470.

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18. In Figure 2, the triangular paper card PQR is held such that PQ lies on the horizontal ground. It is given that $PQ = 30$ cm , $PR = 25$ cm and $\angle QPR = 95°$.

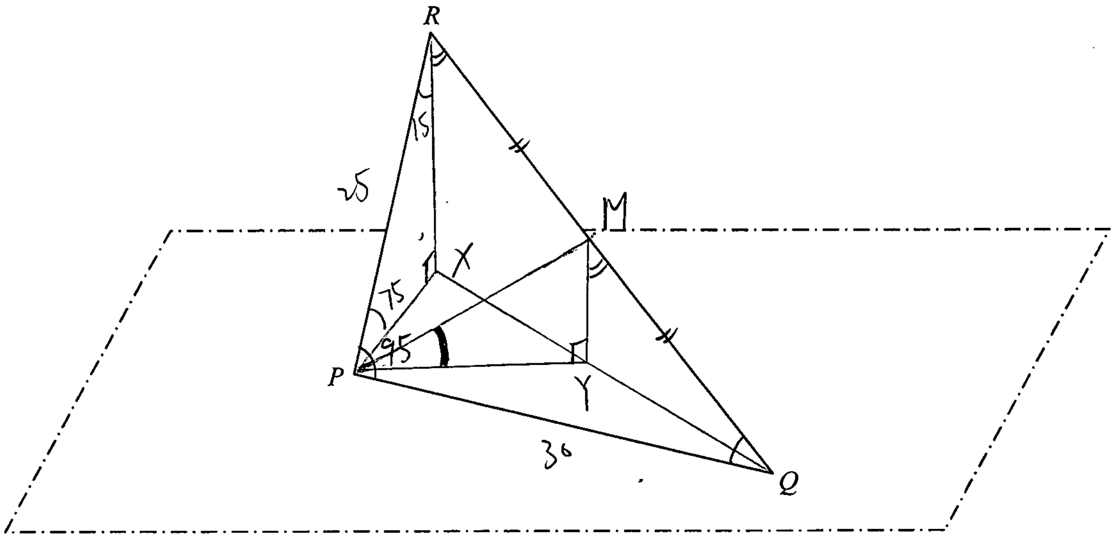

Figure 2

(a) Find

(i) the length of QR,

(ii) $\angle PQR$.

(4 marks)

(b) Let M be the mid-point of QR . A craftsman finds that the angle between PR and the horizontal ground is $70°$. The craftsman claims that the angle between PM and the horizontal ground exceeds $40°$. Is the claim correct? Explain your answer. (3 marks)

a i) $QR^2 = 25^2 + 30^2 - 2(25)(30)(\cos 95°)$

$QR = 40.6907$ cm.

≈ 40.7 cm

ii) $\frac{25}{\sin \angle PQR} = \frac{QR}{\sin 95°}$

$\angle PQR = 37.738°$

$\approx 37.7°$

b) $PM^2 = (\frac{QR}{2})^2 + 30^2 - 2(\frac{QR}{2})(30)(\cos \angle PQR)$

$PM = 18.6699$ cm

Draw $RX \perp$ horizontal ground. and $RX \perp PX$.

$\frac{PX}{25} = \cos 75°$

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$PX = 6.4705$ cm.

Draw $MY \parallel RX$ and $MY \perp$ horizontal ground.

$\frac{RX}{25} = \sin 75^\circ$

$RX = 24.1481$ cm.

$\angle RQX = \angle MQY$ (common $\angle$s)

$\angle XRQ = \angle YMQ$ (corr. $\angle$s, $MY \parallel RX$)

$\therefore \Delta RXQ \sim \Delta MYQ$ (AA)

$\frac{MQ}{RQ} = \frac{MY}{RX}$

$\frac{1}{2} = \frac{MY}{24.1481}$

$MY = 12.07407$ cm.

$\frac{MY}{PM} = \sin \angle MPY$

$\frac{12.07407}{18.6699} = \sin \angle MPY$

$\angle MPY = 40.2943^\circ > 40^\circ$

$\therefore$ The claim is correct.

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19. The centre of the circle C is the point $G(83, 112)$. It is found that the point $A(158, 12)$ lies outside C. AP and AQ are the tangents to C at the points P and Q respectively. It is given that C passes through the point $(23, 67)$.

(a) Find the equation of the straight line passing through A and G. (2 marks)

(b) Find the coordinates of the point of intersection of AG and PQ. (3 marks)

(c) Find the equation of the inscribed circle of ΔAPQ. (4 marks)

(d) Someone claims that the ratio of the area of the inscribed circle to the area of the circumcircle of ΔAPQ is $1:4$. Do you agree? Explain your answer. (3 marks)

a) Equation = $\frac{y-12}{x-158} = \frac{112-12}{83-158}$

$\frac{y-12}{x-158} = \frac{-4}{3}$

$3y-36 = -4x+632$

$4x+3y-635=0$

b) Radius of C $= \sqrt{(83-23)^2+(112-67)^2}$

$= 75$ units.

$AG = \sqrt{(83-158)^2+(112-12)^2}$

$= 125$ units

$\because GP \perp PA$ (radius $\perp$ tangent)

$PA^2 = AG^2 - GP^2$ (Pyth. Thm)

$PA = 100$

$\tan \angle PGA = \frac{100}{75}$

$\angle PGA = 53.1301°$

$\because GA \perp QP$

Let the point of intersection be X.

$\frac{GX}{GP} = \cos 53.1301°$

$GX = 45$.

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~~$(x-83)^2+(y-112)^2=45^2$ — ①~~

~~$\frac{y-83}{x-112}=-\frac{4}{3}$~~

~~$3y-249=-4x+448$~~

~~$4x+3y-697=0$ — ②~~

~~From ①~~

~~$x^2-166x+6889+y^2-224x+12544-2025=0$~~

~~$x^2+y^2-166x-224y+17408=0$~~

$GX:XA=45:80$

$=9:16$

Coordinate of $X=\left(\frac{16(83)+9(158)}{9+16},\frac{16(112)+9(12)}{9+16}\right)$

$=(110,76)$

$\sin\angle XPA=\frac{80}{100}$

$\angle XPA=53.1301^\circ$.

Let L be the angle bisector of $\angle XPA$.

$\frac{\text{Radius of }\triangle APQ}{60}=\tan 26.565^\circ$.

Radius of $\triangle APQ=30$ units.

Let M be the centre of the circle.

$GM:MA=75:50$

$=3:2$

$M=\left(\frac{2(83)+3(158)}{5},\frac{2(112)+3(12)}{5}\right)$

$=(128,52)$

Equation $=(x-128)^2+(y-52)^2=900$

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