Transfer Function Modeling of DC Motors

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DC Motors

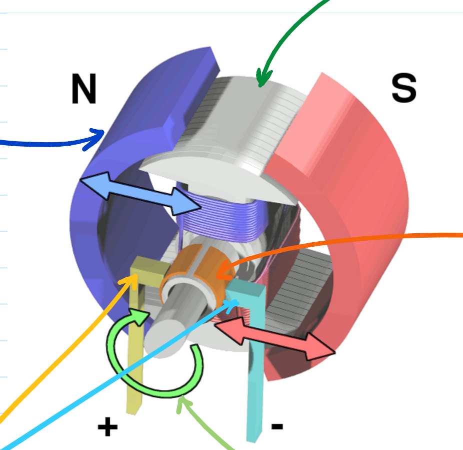
→ INexpensive
Reliable
High performance

→ Electromechanical actuator for control of rotational motion

→ Aplications:
automation/Robotics
automobiles, etc.

Rotor/armature
(Rotating windings or magnet)

Stator/field
(stationary windings or magnet)



commutator ring
(wired to armature windings)

commutator brushes

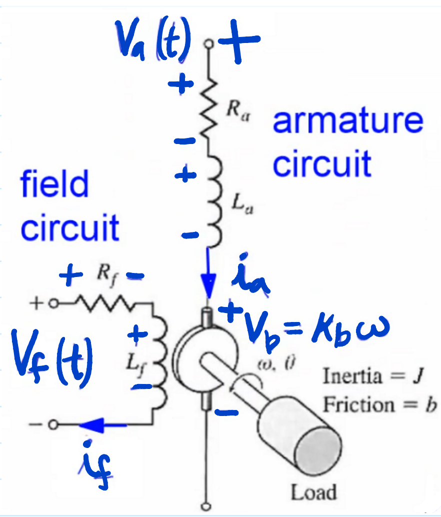
opposite magnetic poles attrack creating a torque

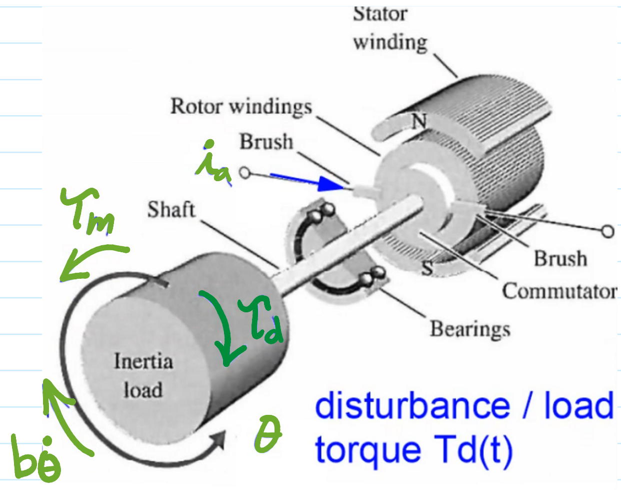
Note: To model an electro-mechanical system, both the mechanical & electrical systems must be modelled

→ Equations for both models are then combined to get the electro-mechanical system model.

Electrical Circuit

Mechanical System





∴ Motor Torque

<u>Case 1:</u>

- Assume armature current $i_a(t)$ is constant (using current source amplifier) or armature uses a permanent magnet.
- Field current $i_f(t)$ is controlled using voltage $V_f(t)$

Non Linear so simplify by arranging with previous assumptions

$\tau_m = K i_a i_f$

$$\tau_m = K i_a i_f = K_m i_f \, , \quad K_m = K i_a = \text{const.}$$

∴ <u>KVL</u>

u_1

$$L_f \frac{di_f}{dt} + i_f R_f = V_f(t) \qquad (1)$$

<u>N2L</u>

u_2

$$J\ddot{\theta} = -b\dot{\theta} + K_m i_f - \tau_d(t) \qquad (2)$$

Say $y = \theta$

∴ Doing Laplace for Eqs ① & ②

① $$L s I_f + I_f R = V_f(s) \qquad ③$$

② $$J s^2 \Theta(s) = -b s \Theta(s) + K_m I_f(s) - \tau_d(s) \qquad ④$$

∴ solve for $I_f(s) \rightarrow (Ls + R) I_f(s) = V_s(s)$

$$\boxed{I_f(s) = \frac{V_f(s)}{Ls + R}} \rightarrow \text{substitute in eq. ④}$$

∴ $$J s^2 \Theta(s) = -b s \Theta(s) + K_m \left(\frac{V_f(s)}{Ls + R} \right) - \tau_d(s)$$

- solve for $\Theta(s)$

∴ $$(J s^2 + b s)\Theta(s) = K_m \left(\frac{V_f(s)}{LS + R} \right) - \tau_d(s)$$

$$\overset{Y}{\Theta(s)} = \underbrace{\frac{K_m \overset{U_1}{V_f}}{(LS + R)(JS^2 + bs)}}_{G_1} - \underbrace{\frac{\overset{U_2}{\tau_d(s)}}{(JS^2 + bs)}}_{G_2}$$

∴ $$\boxed{Y(s) = G_1 U_1 + G_2 U_2}$$ ← Proves the linear system property for this case.