To be completed individually or in groups of two people (for groups, please be sure both names and matriculation numbers are clearly included at the top of your submission). Submissions can be handwritten or in LaTeX formatting, but hard-to-read handwritten submissions will not be graded.

Please submit via Ilias. Submissions should be a single PDF document (note that Jupyter notebooks can and should also be downloaded as PDFs, and not submitted as .ipynb files).

Each question will be graded "pass" (full points) or "fail" (no points). We award .5 bonus points for the exam for each theory and practical question solved. You must complete 50% of all exercises to enter the final exam.

1. **EXAMple Question** Consider the bivariate distribution on (X, Y) defined by

$$p(x,y) = \begin{cases} 0.5 & \text{for } 0 < x < 1, 0 < y < 1 \\ 0.5 & \text{for } -1 < x < 0, -1 < y < 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Draw the region on which $p(x, y) > 0$ (i.e. the 'support' of the distribution).
   (b) Are X and Y independent, or not?
   (c) Verify that $\int \int p(x, y) dx dy = 1$.
   (d) Calculate the marginal distributions $p(x)$ and $p(y)$.
   (e) Calculate the conditionals $p(y|x)$ and $p(x|y)$.
   (f) Calculate the mean of X the conditional mean $E(X|Y)$.
   (g) Calculate the variance of X.
   (h) Calculate the covariance of X and Y.
   (i) Let $Z = e^{-X}$. Compute the pdf of Z.

2. **Theory Question** We are attempting to estimate the outside temperature using two thermometers, one of which is miscalibrated. For the first thermometer A, the measurement t_A can be approximated well by a univariate normal distribution: $t_A \sim \mathcal{N}(t_A|t, 1)$. For the second, miscalibrated, thermometer B, the measurement t_B can be approximated well by a univariate normal distribution: $t_B \sim \mathcal{N}(t_B|2t + 3, 2^2)$, where t is the outside temperature in Celsius.

   Our prior belief is that the outside temperature is normally distributed: $t \sim \mathcal{N}(t|10, 3^2)$. Given the measurements $t_A = 10$, $t_B = 18$ and our prior belief, calculate the posterior distribution on the outside temperature t.

3. **Practical Question** In previous lecture courses you have become accustomed with various models of *Deep Learning.* In popular texts it can sometimes sound as if deep learning has made all other concepts of machine learning obsolete. The point of this exercise is to reflect on this sentiment. On Ilias you can find a jupyter notebook that loads the famous "Keeling curve", a time series of atmospheric CO_2 concentrations collected at the Mauna Loa observatory in Hawaii. Your task is, using only a deep learning framework (like TensorFlow, pyTorch, etc.) to produce an *extrapolation* of this dataset from today 40 years into the future, until 2060. You are free to use whichever architecture you like, but try to use only 'deep neural networks' (that's a vague term, of course). For more, refer to `Exercise_02.ipynb`

   Note: this question has been adapted from ProbML 2021 by Prof. Dr. Philipp Hennig, Emilia Magnani and Lukas Tatzel.