

To be completed individually or in groups of two people (for groups, please be sure both names and matriculation numbers are clearly included at the top of your submission). Submissions can be handwritten or in LaTeX formatting, but hard-to-read handwritten submissions will not be graded.

Please submit via Ilias. Submissions should be a single PDF document (note that Jupyter notebooks can and should also be downloaded as PDFs, and not submitted as .ipynb files).

Each question will be graded "pass" (full points) or "fail" (no points). We award .5 bonus points for the exam for each theory and practical question solved. You must complete 50% of all exercises to enter the final exam.

1. EXAMple Question

- (a) Show that conditional probabilities satisfy the axioms of probability (nonnegativity, normalization and countable additivity).
- (b) We have two biased coins, a gold and a bronze. In any given toss, the probability of heads for the gold coin is 0.99, whereas for the bronze coin is 0.01. We choose one of the two at random, with equal probability, and proceed with two independent tosses. Let B be the event that the bronze coin was selected and H_i be the event that the i th toss resulted in heads.
 - i. Are H_1 and H_2 conditionally independent given B ?
 - ii. Are H_1 and H_2 independent?
- (c) You and your friend are candidates in a TV show, and you are told that there is a cash-prize behind two of three doors (with equal probabilities for each). Each of you, in turn, chooses a single door and the show-master immediately opens it (thus the second person cannot choose the same door).
 - i. If you choose before your friend, what is the probability that you get a prize?
 - ii. If you choose first and get a prize, what is the probability that your friend gets a prize?
 - iii. If you choose first and don't get a prize, what is the probability that your friend gets a prize?
 - iv. Is it in your best interest if your friend chooses first?
 - v. If you choose first, what is the probability that you get a prize, given that your friend gets a prize?
2. **Theory Question** On your desk, there is a very special die with a prime number p of faces, and you throw this die once. Show that no two events A and B can be independent unless either A or B is the whole sample space or the empty set.
3. **Practical Question** In class, you were shown how to calculate the probability of someone having COVID given a positive COVID test. In this exercise, we ask you to code up a method that performs this calculation, given the sensitivity and specificity of a COVID test, as well as the prevalence of COVID. More details can be found in the .ipynb notebook.

(Note: Some of the questions are adapted from a textbook on probability theory, which will be credited and revealed after the hand-in deadline.)