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Sprinkle Magic on the Dance: Enriching a Verified Choreographic Language with a Simply Typed Lambda Calculus

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October, Xin Lu

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Abstract

Distributed systems are ubiquitous but writing endpoint programs can be error-prone since mismatched message sending and receiving can lead to errors such as deadlock, where the system indefinitely awaits a message. Choreography offers a solution by providing a global description of how messages are exchanged among endpoints, where message mismatches are disallowed — a property called "deadlock-free by design." The global choreography is then projected into process models for each endpoint via EndPoint Projection (EPP), preserving the deadlock-free property.

While many choreography languages focus on message exchange behaviors, few address the local computations occurring within endpoints. Most current languages assume local computation results or delegate them to external languages. While this offers a reasoning ground for studying message exchange behaviours of choreography, when it comes to writing a concrete choreography program, the former can only exchange literal values and the latter leads to cumbersome code due to the addition of an external computation program which typically involves conversions between choreography values and external data types.

Hence in this thesis, we extend Kalas, a state-of-art choreography language with verified end-to-end compilation, with a local language Sprinkles, such that local computations are handled gracefully within a few lines of codes. Moreover, it also allows us to formally analyse the message exchange behaviours of choregraphy when local computations are considered.

We design Sprinkles as a simply typed lambda calculus with function closure. We use a functional bis-step semantics with clocks to ensure the evaluation function for Sprinkles is total. We prove type soundness for the proposed semantics and typing rules. We also provide a strong normalisation proof for Sprinkles.

We extend the Kalas' *let* transition with Sprinkles expressions. Besides common data types such as integer, string, and boolean, we also add function, pair, and sum types to the local computation in choregraphy. Common operators for our data types are included, such as addition, modulo, and negation. An integer-string converter is implemented as well to handle message strings in Kalas. Last but not least, we prove the enriched Kalas enjoys progress. We also show type preservation holds for non-recursive, synchronised transitions.

Table of Contents

Chapter 1

Introduction

Distributed systems consist of multiple endpoints that communicate by exchanging messages, operating with asynchrony and parallelism as these messages are sent and received between the various endpoints. But programming distributed system is notoriously error-prone as programmer has to implement the communication protocol by developing individual endpoint programs. Mismatched message sending and receiving can lead to errors such as *deadlock*, where the system is waiting forever for a message.

Choreography arises as a programming diagram to address this issue by providing a concrete global description of how the messages are exchanged between endpoints in a distributed system. A choreography program is written in a similar style to the "Alice and Bob" notation by Needham and Schroeder (1978):

$$\begin{array}{ll} \texttt{1.} & \texttt{Alice} \to \texttt{Bob} : \textit{key} \\ \texttt{2.} & \texttt{Bob} \to \texttt{Alice} : \textit{message} \end{array}$$

Thus message mismatches are disallowed from the choreographic perspective. A property we refer to as *deadlock free by design*. The global choreography is then projected into process models for each endpoint via EndPoint Projection (EPP), with properties such as deadlock free by design preserved (Hallal et al., 2018).

While most choreography languages focus on the message exchange behaviours, few pay attention to the local computation happening in the individual endpoint. It is shown by Hirsch and Garg (2022) that as long as the local language exhibits type preservation and progress, the choreography inherits these properties as well. Thus, when analyzing message exchange behaviors in choreography—such as multiparty sessions, asynchrony, and parallelism—one can safely assume good behaviours of local computation (Montesi and Yoshida, 2013; Cruz-Filipe and Montesi, 2017; Carbone and Montesi, 2013).

But when it comes to writing the choreography program that implements concrete system behaviours, if local computation is ever required by the system, one must describe the inputs and outputs of local computations. Current work either delegate this part to an assumed well-behaved external implementation, for example, Kalas Pohjola et al. (2022) and Pirouette Hirsch and Garg (2022), or provide a basic framework where only Church numerals are considered, as in Core Choreography (CC) Cruz-Filipe and Montesi (2017). This makes writing choreography program with local computation implementation an unpleasant experience. For instance, a choreography program in Kalas, where the client computes modulo locally using input from the server and then sends the result back, will look like:

Example 1.0.1 (Local computation — Modulo)**.**

Kalas	External computation
`server`.var → `client`.x;	**fun** $mod\ x\ =$
let v@`client` = `mod`(x) **in**	case Option.map (fn s ⇒ valOf (Int.fromString s)) (hd x) of
`client`.v → `server`.$result$;	None ⇒ None
	| Some n ⇒ Some [Int.toString (n MOD y)]

In Kalas, processes communicate exclusively via strings. Therefore, in the external implementation of `mod`(x), the string value of x must first be converted to a number. Then client computes the modulo on the converted input. Afterward, the result is converted back to a string before updating the client process variable v. We can easily see from this example that how data type conversions between choregraphy and external language lead to cumbersome code.

Thus this thesis extends Kalas, a verified choregraphy language with machine-checked end-to-end compilation, by introducing Sprinkles, a simple language of expressions over types such as integers, strings and booleans. Using the extended Kalas, local computations can be handled gracefully within a few lines of codes. The previous choregraphy where client computes the modulo of input integer can be written as in Example 1.0.2.

By providing a concrete local language syntax and semantics, we are able to formally anlayse the message exchange behaviours of choregraphy in terms of progress and type preservation when local computations are considered. We show that type soundness and strong normalisation properties of Sprinkles lead to progress for the enriched Kalas. Our semantics and typing system also allow us to show type preservation for non-recursive and synchronised transitions in the enriched Kalas.

Example 1.0.2 (Local computation with Sprinkles — Modulo)**.**

1. `server`.var → `client`.x;
2. **let** v@`client` = StrOf ((NumOf (Var x)) Mod (Var y)) **in**
3. `client`.v → `server`.$result$;

To summarise, this thesis provides three main contributions.

- The first contribution is Sprinkles, a simply typed lambda calculus with function closure. According to the approach taken by Owens et al. (2016), we use a functional big-step semantics with clocks to ensure the evaluation function for Sprinkles' expressions is total. Our evaluation strategy is call-by-value. We also provide a typing system and give type soundness proof for the proposed semantics and typing rules. Sprinkles is implemented using the higher-order logic proof assistant HOL4 (Slind and Norrish, 2008) and all the proofs are conducted within HOL4.
- The second contribution is the strong normalization proof for Sprinkles. We follow the standard practice for proving strong normalization in simply typed lambda calculus, with a specific case for function closure in the definition of the strong normalization relation.

- The third contribution is Kalas enriched with Sprinkles. Besides common data types such as integer, string, and boolean, we also add function, pair, and sum types to the local computation in Kalas. Common operators for our data types are included, such as addition, modulo, and negation. Integer-string convertor is implemented as well for the integration with Kalas. Last but not least, we prove the enriched Kalas enjoys progress. We also show type preservation holds for non-recursive, synchronised transitions in the enriched Kalas.

Chapter 2

Background

2.1 Choreographic Programming

A distributed system can be defined as a collection of autonomous computing elements that behaves to its users' expectations (Mullender, 1990). Message exchanges are heart of the distributed system designs cause it allows nodes to collaborate and share resources with each other. Otherwise there is no need to put different nodes inside one connected network. Using a traditional way to describe a distributed system, one will have to give a detailed description of operations at each node in the system. For example, a communication between node A and node B is achieved by A sending the message and B receiving the message. But mismatched message sending and receiving can happen and these endpoint programs may fail to prevent the system from deadlocks or race among messages.

Choreography raises as an effort to eliminate incorrect system implementation by only providing a global description on how messages should be exchanged within the system. This approach is analogous to dance choreography, which outlines steps and movements for an entire performance without focusing on individual dancers' control points.

While it is fascinating to have no mismatched messages in choregraphy perspective, choregraphy cannot be run directly on individual nodes in a system and thus endpoint programs are still required by implementation. Thus the idea of EndPoint Projection (EPP) is proposed in which a choreography is projected into endpoint programs such that each endpoint correctly implements the behaviours described by its role in the choreography. This idea is first described by the design document of Web Services Choreography Description Language (WS-CDL) (W3C WS-CDL Working Group, 2005), and Carbone et al. (2007) further formalise it into the theory of EPP, namely the *soundness* and *completeness* properties for a given EPP. Soundness means that all projected endpoint communications adhere to the choregraphy description and completeness means that all communications described by the choregraphy are projected into endpoint codes.
- maybe two important results but not namely

- idea of CC

- common aspected in choreography: asynchrony ...

2.2 Interactive Theorem Proving

- what is higher-order logic - how proofs are done using HOL - recInduct in HOL - tactices - finite maps?

2.3 Kalas

- overview including intro to its compiler - semantics - implementation in HOL
Definition 1. - *kalas syntax*

Chapter 3

Related Work

some intro ..

3.1 Choreography Models

some intro ..

3.1.1 Typing System for Choreography

Choreography language model can ensure deadlock freedom without using a typing system. The property is typically proven through structural induction on the choreography's semantics, where an applicable rule always enables reduction, or reduction follows by inductive hypothesis (Carbone and Montesi, 2013; Cruz-Filipe and Montesi, 2017; Pohjola et al., 2022). Typing system in this case might still be desired since it can discipline choreography in other ways such as correct protocol implementation.

Channel Choreography (ChC) by Carbone and Montesi (2013) is a rich choregraphy language where a choreography program consists of where a program consists of roles, threads, and sessions that implement communication protocols. Deadlock freedom is guaranteed by its semantics, while the typing system ensures correct protocol implementation by sessions. The typing context includes three components: a service environment Γ, which stores global types for public channels specifying session execution and local expressions (annotated with threads); a thread environment Θ, which tracks the roles of threads in each session; and a session environment Δ, which stores the types of active sessions.

It ensures that a well-typed choreography, with public channels specified by Γ and threads assuming roles in Θ, maintains disciplined sessions governed by Δ. Additionally, runtime typing introduces a delegation environment to handle changes in typing context due to asynchrony or parallelism, ensuring the program adheres to the protocol during execution.

When local computation is involved in the choreography, since we cannot solve the halting problem in semantics by deciding whether the local computation will terminate, a typing system is desired to reassert deadlock freedom. To the best of our knowledge, the closet work to this discussion is Pirouette by Hirsch and Garg (2022). They assume a substitution model with small-step semantics for the local language. Based on a set of admissible typing rules for the sake of providing a reasoning ground, their results show that type preservation and progress in the local language ensure the same for the choreography language. Their progress result aligns with our results, but our local language adopts big-step semantics and thus preservation for choreography requires strong normalisation from local language rather than type preservation.

Pirouette is a higher-order functional choreography language with three value types: local values, local functions (mapping local values to choreographies), and global functions (mapping choreographies to choreographies). This structure enables choreographies to return values, forming the basis of its typing system. On the other hand, since neither Kalas or the enriched Kalas has return values, our typing system mainly checks for the well-formedness of a choreography within the typing environment and if any local computation is involved, we discipline it with its own typing system in a localised typing environment. Pirouette types its local values in a similar projected typing environment, binding variables to types at a specific location.

Another state-of-art functional choregraphy language Chorλ by Cruz-Filipe et al. (2022) uses a different approach than Pirouette, where choreographies are interpreted as terms in λ-calculus. Chorλ assumes local values for communications, without focusing on how they are computed. This results in a distinct typing system: local value types are annotated with roles and are part of the global types, rather than being projected from a global environment. Type preservation and progress follow from Chorλ's typing and semantic rules.

3.1.2 Handling Exceptions

- Choral: an object-oriented choregraphy implemented as a java library (Giallorenzo et al., 2024). It treats choregraphy as class and EPP as generating role classes from the choregraphy implementation. It also offloads the local computation to Java. Higher-order functional choreography models such as Pirouette and Chorλ can be viewed as formal model candidates for it. Its type checker is similar to the typing system in ChC where it has types for public channels where roles for sender and receiver are specified as well as the type of messages being communicated an it also has local types annotated with roles.

In terms of communication failure which may raise an exception when a role is trying to perform operation based on reading from the place where a received message has not arrived yet, Choral implements the failure model in RC by Montesi and Peressotti (2017), using recover strategies such as capped attempts or timer within java try-catch block.

- RC, a model where communication failure is considered, recover strategy for senders and receivers, either while loop until received (exactly once delivery for setting 1), or with a timer or capped tries (best efforts); typing system to ensure almost once delivery (and exactly one delivery where message won't be lost); they use configuration where sender and receiver have stacks and it is initially false, with payload value, or ticked meaning no longer in the stack (sent/received) to implement send/received and the failure rules for semantics and typings; we do not consider message sending failure, but we do have exception caused by local computation failure (e.g. division by zero) or bad message value type (not a string), and we record the exception using transition labels. But we do not consider any recover strategy and always transits the choregraphy into a termination state (nil)

3.2 Some Title

- functional big-step semantics Owens et al. (2016)

- STLC; language with environment semantics - our typing is not unique

- SN for STLC; Weak and strong SN in terms of non-deterministic; my language is deterministic

Chapter 4

Sprinkles: A Simply Typed Lambda Calculus

- we introduce Sprinkles, a language over expressions; Adopting a functional big-step semantics (owens), we define an interpreter that evaluate expressions to values; we evaluate expressions with clocks to ensure termination, thus our evaluation function is total. Expressions in Sprinkles can also be evaluated into exceptions. We include common arithmetic errors such as division by zero, and integer-string conversion errors such as bad formatted strings. The latter is included for the integration with Kalas, since a process may want to perform integer arithmetic based on the received string which does not encode a valid integer.

Sprinkles is deterministic in a sense that if we increase the clocks an expression will be evaluated to the same values or exceptions.

- Sprinkles closely resembles a STLC, but uses a dynamic environment to store values of free variables, so we always evaluate the given expression inside an environment E. This leads to typing expressions in a typing environment G that stores types for free variables. This setup, further requires the typed expression being evaluated in a *correct* environment E in the type soundness proof, given the typing environment G where the expression was typed.

4.1 Syntax

- syntax definition (+ exceptions) with brief explanation
Definition 2 (Sprinkles syntax)**.** Values *and* expressions *in Sprinkles, ranged over by v, e, are inductively defined by the grammar*

$$
\begin{array}{rcl}
v & ::= & \mathsf{IntV}\ n \mid \mathsf{StrV}\ s \mid \mathsf{BoolV}\ b \mid \mathsf{Clos}\ s\ e\ E \mid \mathsf{PairV}\ v_1\ v_2 \mid \mathsf{SumLV}\ v \mid \mathsf{SumRV}\ v \\
\mathit{bop} & ::= & \mathsf{Add} \mid \mathsf{Concat} \mid \mathsf{Mult} \mid \mathsf{Div} \mid \mathsf{Mod} \mid \mathsf{Less} \mid \mathsf{And} \mid \mathsf{Or} \mid \mathsf{Eq} \mid \mathsf{Sub} \mid \mathsf{Pair} \\
\mathit{uop} & ::= & \mathsf{Not} \mid \mathsf{NumOf} \mid \mathsf{StrOf} \mid \mathsf{Fst} \mid \mathsf{Snd} \mid \mathsf{SumL} \mid \mathsf{SumR} \\
e & ::= & \begin{array}{llll}
\mathsf{Var}\ x & \mathit{(var)} & \mathsf{StrLit}\ s & \mathit{(str)} \\
\mathsf{IntLit}\ n & \mathit{(int)} & \mathsf{BoolLit}\ b & \mathit{(bool)} \\
\mathsf{BinOp}\ \mathit{bop}\ e_1\ e_2 & \mathit{(bop)} & \mathsf{Uop}\ \mathit{uop}\ e & \mathit{(uop)} \\
\mathsf{If}\ \mathit{bg}\ e_1\ e_2 & \mathit{(if)} & \mathsf{Let}\ x\ e_1\ e_2 & \mathit{(let)} \\
\mathsf{Fn}\ x\ e & \mathit{(fn)} & \mathsf{App}\ e_1\ e_2 & \mathit{(app)} \\
\mathsf{Case}\ e\ x\ e_1\ y\ e_2 & \mathit{(case)} & &
\end{array}
\end{array}
$$

Syntax for function and application syntax are standard. Same as if and let. Case here is for sum types, where *s1*, *e1* represent the left branch and *s2*, *e2* represent the right branch. Var x represents a variable where x is the string that represents variable's name. We include literals for string, integer and boolean in our syntax to allow expressions such as BinOp Add (Var x) (IntLit 1). For the sake of readability, we will write the addition example as Add (Var x) (IntLit 1), and same for any binary or unary operators examples.

We abstract binary and unary operators to allow shorter syntax definitions. This makes extending binary and unary operators easy since we only need to register the new operator in *bop* or *uop* definition and provide the corresponding evaluation rule. It also greatly reduces the number of cases generated in proofs that rely on syntax forms of an expression since we only have one case for all binary operators and one case for all unary operators. Though those two cases typically rely on lemmas that establish the desired properties for all operators, as discussed in Section ?, most operators cases can be proven using the same tactics and where some operator case in the lemma proof differs from the others is usually the essential subproof that is required to re-establish the desired property after introducing this operator to our language. Thus separating concrete operator syntax definitions from language syntax definitions also enabling a neat proof maintenance process when new operators are introduced.

We include exceptions to handle division by zero and bad formatted integer strings. They are thrown by the corresponding operators. We define three observable evaluation results: as either values, or we encounter a type error (for example, Add (IntLit 1) (BoolLit T)), or we reach an exception, or the evaluation times out for the given clock.

4.2 Statics

Our typing rules for expressions are similar to those for a simply typed lambda calculus. We define a typing relation for values as we want the evaluated values in the type soundness statement to be typed as well.

Our expressions are typed in a typing environment G which stores mappings from variable names to types. Similar to a variable expression is evaluated by looking up variable name in the given environment E, a variable expression is typed by looking up type stored under the variable name in the given typing environment G. This setup, requires us to constrain the typing environment G and dynamic environment E in the type soundness statement to make sure we are evaluating the typed expression using a correct dynamic environment that aligns with the typing constrain specified in G.

Similar to how we provide only one general case for binary/unary operation evaluation and delegate the evaluation of concrete operators to another function, we provide only one general rule for typing binary/unary operations and delegate type checking concrete input/output types against a concrete operator to external relations, boptype and uoptype. As we will see in Section ?, this leads to only two cases in the type soundness proof for operators and a separate soundness lemma regarding the operator typing relation is required by the general case.

4.2.1 Syntax and Typing Rules

Definition 3. - *types*
Definition 4. - *typing relations: typecheck*
Definition 5. - *typing relations: value_type*

- typing relations: typecheck, value_type, uop_type, bop_type

A typing relation for expression is written as $G \vdash_s e : ty$ where G is the typing environment. For typing a binary operation expression, we require the expressions as arguments to the operator to have some type *t1*, *t2*, and those types and the output type satisfy the typing relation specified in boptype for the concrete binary operator involved. Typing for a unary operation is the same. Other rules are standard.

Before we explain value_type, we define a typing constrain between a typing environment G and a dynamic environment E.
Definition 6. - *envtype definition.*

Intuitively, envtype G E means that values of variables stored in E satisfy the typing constrains specified in G.

value_type relates a value v to a type t. Integer, string and boolean literal cases are obvious. For typing a closure value, we require the function body e to have the output type *t2* in a typing environment G that extends with the type *t1* for function argument s, where such G is a correct descriptions of types of variable values stored in E. This constrain is equivalent as requiring envtype G E. [1]

We require envtype G E is because we will evaluate the function body e in an extended environment E(+s,v) and we want the extended typing environment G(+s,v) in which e is typed to correctly describes the types of values stored in E(+s,v), i.e. envtype $E(+s, v)$ $G(+s, v)$. We will see in Section ? that envtype $E(+s, v)$ $G(+s, v)$ is necessary to be able to use the inductive hypothesis in the application case of type soundness proof as our type soundness statement requires a typed expression to be evaluated in an environment that aligns with the typing environment that types the expression.

value_type for pair and sum cases are also standard. A SumLV v value does not have a unique type since *t2* is arbitrary and so as a SumRV v value.

4.2.2 Typing Examples

Example ? discussed in Section ? to show our function closure supports currying can be typed using the rules in Definition ?. A derivation tree is provided in Appendix ?.

However, the recursion program (Example ?) discussed earlier cannot be typed. Since it is a let expression, the last rule applied must be T-let. We can type f 1 in the extended typing environment (f:int-¿int). But we cannot type the definition (fn ... some complex expression) in

[1]We do not use envtype G E directly in the closure case of value_type because definition of envtype depends on value_type and we want to avoid mutually recursive definition between a definition and a relation in HOL4.

the empty typing environment since f is free in the definition expression. A stuck derivation tree is provided in Appendix ?.

4.3 Dynamics

As discussed above, we defined our semantics using evaluation function rather than relation. The size of expressions are decreasing except for the application case, so we decrease the clock to ensure termination. Semantics for if, let, case are standard. When evaluating a variable, we try to look up for the value corresponds to the variable name in the given environment. If we try to look up an unknown variable name, the evaluation function will return with type error. Environments are implemented as finite maps in HOL4, as discussed in Section ?.
Definition 7. *- semantics rules*

Our evaluation strategy is call-by-value. For evaluating binary or unary operations, we first evaluate the argument expressions into values, and then use another function eval_bop or eval_uop to apply the operators by the specified rules on the evaluated values and return the result. eval_bop and eval_uop are where concrete evaluation rules for each operator is defined.

For example, eval_exp c E (NumOf (Var x)) will first compute eval_exp c E (Var x) by looking up name x in E. If x is unknown to E, the function will return TypeError and propagate it to the top. Otherwise it will return the value v stored under the name x in E and we move to compute eval_uop NumOf v. If v is not a correctly formatted integer string (for example, if it contains a non-numeric character), eval_uop NumOf v will return $ExnBadStr$. Otherwise, it will return the converted integer value IntV n. Again, the format checking and conversion algorithm are implemented in eval_uop's definition. The same applies to evaluating any binary operations. Definitions of eval_uop and eval_bop are included in Appendix ?.

- typed example: eval_exp c $\emptyset$

4.3.1 Function Closure

Definition 8. *- function semantics rule (unrestricted version)*

For a function Fn x e, closure is a triple (x, e, E). x is the function parameter. e is the function body. E is the environment that stores bindings of free variables when the function definition is evaluated.

We stores the bindings of free variables (i.e. the environment) into the closure value when evaluate a function definition so we never lose track of values for bound variables when evaluating a function definition that appears nested in another function definition. If we don't keep track of values for bound variables, when we finishing evaluating a function definition that contains nested function definitions, only the inner most bound variable will appear bound and the inner most function body will be the expression to be evaluated when we apply this nested function definition to a value. Since we now lose bindings for all the other bound variables from outer scopes, we are unable to finish the evaluation of application.

Example 4.3.1. - example: failure without closure

This is illustrated by Example 4.3.1. When we are evaluating Fn y (Add (Var x) (Var y 1)) in line 5 we know x is bound to 1. But we lose this information when we choose $(y, \text{Add (Var } x) \text{ (Var } y))$

to be the evaluation result. So we only know y is bound to IntLit 2 when we try to evaluate Add (Var x) (Var y 1) in line 7.

By using function closure to record values of all bound variables when a function definition is being evaluated, we are able to evaluate Add (Var x) (Var y 1), as illustrated in line 7 of Example 4.3.2. It is possible that a function body contains unbound free variables, but such expression cannot be typed in an empty environment and it is reasonable to make its evaluation fails. Discussions on typing for function closure are in Section ?.

Example 4.3.2. - another example

Restricted Function Closure

The Kalas repository states an interesting property: Given a bigger state Γ_1 that contains Γ, the choregraphy c still transits into c' and Γ_1' contains Γ'.
Theorem 1. - *Property theorem.*

However, our S-Fn invalidates this property since a bigger state may result in a closure value that contains a bigger environment, so we will not be able to transit into the same c'. In order to re-establish this property, we modify S-Fn to store a restricted environment instead: (DRESTRICT E (free$_\mathsf{v}$ars e) \\ s), where DRESTRICT is defined as:
Definition 9. - *definition of* DRESTRICT

And the modified S-Fn is defined as:
Definition 10. - *restricted eval_exp fn*

The restricted closure should at least contain bindings of variables that appear free from the perspective of the current function definition scope, as much as provided by the environment in which the current function definition is evaluated. Such that if we apply the evaluated restricted function closure to a value, if we were able to evaluate the application without restriction, we will be able to evaluate to the same result with restriction.

But we further require the restricted environment to exclude storing the value of the variable bound by current function definition in the current environment if any. This still allows us to evaluate an application to the same result because if we ever apply this function to a value we will always bind s with the provided value, leaving the previous value of s stored in the environment irrelevant.

Excluding storing s is also necessary. If we only restrict E with free$_\mathsf{v}$ars e, since e can have s as free variables and Γ does not need to contain binding for s for evaluating Fn s e, (DRESTRICT (localise Γ p) (free$_\mathsf{v}$ars e)) may not contain binding for s. But since Γ' contains Γ, Γ' may contain binding for s and thus (DRESTRICT (localise Γ' p) (free$_\mathsf{v}$ars e)) contains binding for s, making the closure evaluated in a bigger state larger than the closure evaluated in the smaller state.

The restricted S-Fn still evaluates Example 4.3.2 to the same result, as illustrated in Example 4.3.3. When the inner application is evaluated, the parameter x of the current function is assigned with the evaluated argument value 1 in the environment. Since x, which is bound by the outer function definition, is a free variable from the perspective of the inner function definition, the binding of x is stored into closure evaluated in line 5. So we are able to evaluate $x + y$ to 3 in line 8.

Example 4.3.3. - restricted currying evaluation example

Excluding the parameter and storing only bindings of free variables into the closure environment also makes sense in a way that bindings of free variables are necessary to evaluate the function body, and we will always have an evaluated value for the function parameter when the application is evaluated regardless of whether the closure environment contains a previous binding of the function parameter or not.

Another possible approach to re-establish Theorem 1 is to define an equivalence relation on function closures where closures with the same parameter, function body are equivalent if their environments all contain the same bindings of free variables in the function body, excluding by not considering the binding of function parameter if any. This leads to a more complex approach since one has to deal with choreographies that contains different but equivalent function closures. Though our approach modifies the S-FN rule and requires extra lemmas to prove the function case in the type soundness proof (discussed in Section 4.4.1), it is simpler since given a bigger state the function definition will still be evaluated to the same closure and no discussion on equivalence relation is required. If not specified, the restricted S-FN will be used for the rest of the thesis.

Recursion

Sprinkles does not support recursion, as we see in the Example ?, the name f is unbound in the definition thus we don't have a value for the recursively invoked name f in line 6.

- example

We argue that recursion can be added to Sprinkles by introducing Letrec syntax to our expressions, and Letrec f s $e1; e2$ will be evaluated into a quadruple (f, x, e, E). The quadruple is similar to the one described by Hardin et al. (2021).

4.3.2 Main Properties

We first show that the evaluation function for Sprinkles is total. That is, for any expression e, environment E, and clock value c, eval_exp always returns a *result.*
Lemma 1. *- eval_exp is total*

Proof: By Definition 7, $(c,$ exp_size $e)$ always becomes smaller in the lexicographic order over $c \times$ (exp_size e) in every recursive call to eval_exp. Box

The total evaluation function obtained by using a functional big-step semantics makes Sprinkles's evaluation deterministic. We say that the evaluation for an expression *terminates* if there exists a clock with which the evaluation function return a result that is not Timeout. Otherwise if for all clocks the evaluation for an expression returns Timeout, we say the evaluation for this expression *diverges.* Then the evaluation of Sprinklesis deterministic in a sense that if the evaluation for an expression terminates for an initial clock, then if we increase the clock we will always evaluate to the same result. We show this result as a corollary follows from Lemmas 2, 3, and 4.

The first lemma says that if expression e is evaluated to a value v in environment E given some clock $c0$, then if we evaluate the same expression in the same environment with a lager clock value $c1$, we will have the same value v as our result. The statements for Lemmas 3 and 4 are similar.
Lemma 2. *- clock_value_increment*

Proof: Using the well-founded lexicographic induction principle by Definition 7, results for each

case directly follows the inductive hypotheses. The induction principle can be found in Appendix ?. Box.
Lemma 3. - *clock_exn_increment*

Proof: We use the same induction principle in Lemma 2 proof. Base cases are trivial since by Definition 7 variables $\mathsf{Var}\ x$ or literals such as $\mathsf{IntLit}\ 1$ cannot be evaluated into an exception. For the inductive case, when any of the sub-expressions of the current expression evaluates into an exception under the initial clock *c0*, by the inductive hypothesis we know this sub-expression also evaluates into an exception under the bigger clock *c1*, which then by the use of monadic binds in Definition 7, the evaluation of the current expression returns an exception. When all the sub-expressions of the current expression evaluates into values under the initial clock *c0*, we know from the premise of the goal that it is the final evaluation step that uses the evaluated values under the initial clock *c0* that throws an exception. For example, in the $\mathsf{App}\ e_1\ e_2$ case, the premise of the goal gives us $\mathsf{eval\_exp}\ c_0\ E\ e_1\ =\ \mathsf{Value}\ (\mathsf{Clos}\ s\ e\ E')$, $\mathsf{eval\_exp}\ c_0\ E\ e_2\ =\ \mathsf{Value}\ v$, and $\mathsf{eval\_exp}\ (c_0\ -\ 1)\ E'[s\ :=\ v]\ e\ =\ \mathsf{Exn}\ exn$. We have $\mathsf{eval\_exp}\ c_1\ E\ e_1\ =\ \mathsf{Value}\ (\mathsf{Clos}\ s\ e\ E')$ and $\mathsf{eval\_exp}\ c_1\ E\ e_2\ =\ \mathsf{Value}\ v$ by Lemma 3, and the last evaluation corresponds to the inductive hypotheses for $(c0-1, e)$. Then by applying the inductive hypothesis we have $\mathsf{eval\_exp}\ (c_1\ -\ 1)\ E'[s\ :=\ v']\ e\ =\ \mathsf{Exn}\ exn$. Similar analyses apply to other inductive cases. Box
Lemma 4. - *clock_typeerr_increment*

Proof: Similar to the proof for Lemma 3. Box
Corollary 1. - *clock_increment*

Proof: Cases analyses based on the syntax form of r. Each case follows either Lemma 2, 3, or 4. Box

Corollary 1 is important in the strong normalisation proof. For example, in the if case of the proof, we know evaluation for boolean guard and two branches terminates by the inductive hypothesis on their separate clock, we want to ensure that there exists a common clock on which the evaluation of all three expressions also terminate and return the same results. More discussion on the use of Corollary 1 in strong normalisation proof can be found in Section ?.

4.4 Type Soundness

some intro ...

4.4.1 Lemmas

We discuss several lemmas and propositions for the proposed semantics and typing rules before attempting the type soundness proof.

Our type soundness proof is similar to the type soundness proof for a simply typed lambda calculus by Pierce (2002) in a sense that we require, or at least require an equivalent version of, the generation lemma, the lemma of canonical forms of types, and the type preservation under substitution lemma used in Pierce (2002)'s proof.
Lemma 5. - *typecheck_*_thm: the generation lemma*

Proof: Immediately follows from the typecheck definition. Box

The generation lemma allows us to move one step forward in the type derivation tree in the bottom to top direction because it tells us how the current type was derived. We can obtain types or typing information of subterms in a well-typed term.

Lemma 6. - *valuetype_EQ_*: the lemma of canonical forms of types (only useful cases)*

Proof: Immediately follows from the value_type definition. Box.

This gives us the concrete syntax of a value v and typing information for terms involved in the concrete syntax if we have value_type $v\ t$. This is useful for inductive cases in the type soundness proof, since we will have expressions evaluated to a value v and value_type $v\ t$ by inductive hypothesis. And the concrete syntax of v matches terms in our goal.

The type preservation under substitution lemma used by Pierce (2002) says substituting variables with term of appropriate types preserves the original type. Since we evaluate variables into values by looking up in an environment E, envtype $G\ E$ (Definition ?) is analogous to the type preservation under substitution lemma.

As discussed in Section ? and Section ?, we evaluate the argument expressions into values first and delegate the evaluation of binary/unary operation to eval_bop and eval_uop. We also type the argument expressions first and then delegate checking input/output types against the concrete operator to a separate typing relation boptype and uoptype. This leads to two important properties: if argument values are typed and the operator types with the argument types, eval_bop and eval_uop will always output a value of correctly type or we reach an exception. We refer those properties as type soundness for eval_bop and eval_uop.
Proposition 1. - *bop soundness*

Proof: boptype is non-recursive. We prove by performing cases analyses based on boptype rules and each case follows from Lemma 6. Box
Proposition 2. - *uop soundness*

Proof: Same as the above. Box

Proposition 1 and Proposition 2 are needed for the binary and unary operator cases in the type soundness proof. As we will have a well-typed evaluated argument values by the inductive hypothesis and we need to show eval_bop and eval_uop output a well-typed value or an exception.

There is also a lemma that allows us to update G and E with a typed value and re-establish the envtype property:
Lemma 7. - *envtype_lemma*

Proof: Since value_type (Definition ?) is inductively defined, we prove by rule induction over value_type. Integer, string, boolean literals are base cases and directly follows the definition of envtype (Definition ?). Pair and sum values cases follow directly from the inductive hypothesis. The closure value case does not require using inductive hypothesis, but rather follows the closure case of the lemma of canonical forms of types (Lemma 6). Box

This lemma is useful in the function, application and sum case cases of the type soundness proof, where we have subterms inside the goal expression typed in an extended typing environment and we have to re-establish the envtype property for the extended typing environment and dynamic environment so we can use the results from corresponding inductive hypothesis.

Restricted Function Closure

If we adopt the restricted S-Fn semantics (discussed in Section 4.3.1), we will need the following lemmas so we can type the closure value that contains a more restricted dynamic environment Clos $x\ e$ (DRESTRICT$Efv(e)deletes$) given that the expression of function body e is typed in an un-restricted extended typing environment $G(+x, t)$. However, the following lemmas hold regardless of whether we adopt a restricted function closure. They are simply unnecessary for the type soundness proof when using an unrestricted function closure.

The first lemma says that if en expression e is typed in an environment G then G contains type information for all free variables in e.
Lemma 8. - *typecheck_env_fv*

Proof: By rule induction on typecheck and rewriting the definition of subset relation and the commutativity property of subset relation. Box.

The second lemma says that if en expression e is typed in an environment $G1$, then e has the same type in a smaller typing environment $G2$, given $G2$ contains type information for all free variables in e.
Lemma 9. - *typecheck_env_submap*

Proof: By rule induction on typecheck and results in the inductive hypothesis match the goal. We define a typing environment G in HOL4 as a finite map that maps a finite set of name strings to a finite set of types. Thus proof for most cases is re-establishing the subset relation between free variables of an expression and the domain of a finite map, and the submap relation between finite maps, in order to use results in the inductive hypothesis. Box.

The third lemma says that we can always type the expression e in a smaller typing environment as long as the current typing environment still contains the bindings for free variables of e.
Lemma 10. - *typecheck_drestrict*

Proof: Follows directly from Lemma 8 and Lemma 9. Box

Lemma 10 and Lemma 8 gives us the existence of a minimal typing environment for an expression e, which contains exactly bindings for all free variables of e.
Corollary 2. - *minimal typing env*

Proof: By Lemma 10, we know that if e is typable, i.e. there exists some G such that $G \vdash_s e : ty$, then we have $\mathsf{DRESTRICT}\ G\ (\mathsf{free_vars}\ e) \vdash_s e : ty$. We know that G contains bindings for all free variables in e by Lemma 8, thus $\mathsf{DRESTRICT}\ G\ (\mathsf{free_vars}\ e)$ contains exactly bindings for all free variables in e, by Definition 9. For the other direction, if there exists a typing environment G' that contains less bindings than $\mathsf{DRESTRICT}\ G\ (\mathsf{free_vars}\ e)$ and we have $G' \vdash_s e : ty$, by Lemma 8 G' contains bindings for all free variables in e. Contradiction. Box

4.4.2 Type Soundness

We are now ready to prove the type soundness property for Sprinkles.
Theorem 2.

$$\begin{array}{l} \vdash \mathsf{envtype}\ G\ E\ \wedge\ G \vdash_s e : ty \Rightarrow \\ \quad (\exists v.\ \mathsf{eval\_exp}\ c\ E\ e = \mathsf{Value}\ v\ \wedge\ \mathsf{value\_type}\ v\ ty)\ \vee \\ \quad (\exists exn.\ \mathsf{eval\_exp}\ c\ E\ e = \mathsf{Exn}\ exn)\ \vee \\ \quad \mathsf{eval\_exp}\ c\ E\ e = \mathsf{Timeout} \end{array}$$

Proof: We prove via rule induction on the definition of eval_exp. Var case follows directly from Definition 6. For $\mathsf{Let}\ s\ e_1\ e_2$ case, by Lemma 5, we have $G \vdash_s e_1 : t_0$ and $G[s := t_0] \vdash_s e_2 : t$ given $G \vdash_s \mathsf{Let}\ s\ e_1\ e_2 : t$. For all the following cases, we assume Lemma 5 is applied first. Then by the inductive hypothesis for $e1$ we know $e1$ is evaluated to some value v and $\mathsf{value\_type}\ v\ t_0$. Lemma 7 then gives us $\mathsf{envtype}\ G[s := t_0]\ E[s := v]$. This allows us to use the results in the induction hypothesis for $e2$ which prove the goal.

For Fn s e case, given envtype G E and $G[s := t_1] \vdash_s e : t_2$, we need to show value_type (Clos s e (DRESTRICT
By the definition of value_type (Definition 5) and envtype (Definition 6), we need to show there exists some typing environment *G0* such that $G_0[s := t_1] \vdash_s e : t_2$ and envtype G_0 (DRESTRICT E (free$_\mathsf{v}$ars e) \\
By Corollary 2 we know there exists a minimal G' such that $G'[s := t_1] \vdash_s e : t_2$ and it contains bindings for all free variables of e, excluding binding for s if $s \in$ free$_\mathsf{v}$ars e. Then by the definition of DRESTRICT (Definition 9) and Definition 6 we have found such *G0* which is G'.

if we use the unrestricted S-Fn, we need to show value_type (Clos s e E) (fnT t_1 t_2) instead, which is straight forward given Definition 5.

For BinOp and Uop cases, results follow directly from Proposition 2 and Proposition 1, as discussed in Section 4.4.1.

For If e e_1 e_2 case, from the inductive hypothesis for the guard expression e we know e is evaluated to value v and value_type v boolT. By Lemma 6 we know v is either true or false. And we have true and false branches all covered by the inductive hypotheses.

For App e_1 e_2 case, by the inductive hypothesis for *e1* and *e2* we know they are evaluated to *v1* and *v2* separately. We also have value_type v_1 (fnT t_1 t_2) and value_type v_2 t_1. By Lemma 6 we know *v1* is a function closure Clos s e E which matches S-App in the goal. Lemma 6 also gives us $G[s := t_1] \vdash_s e : t_2$ in which envtype G E. By Lemma 7, we have envtype $G[s := t_1]$ $E[s := v_2]$. This allows us to apply the inductive hypothesis for e which solves the goal.

For Case e s_1 e_1 s_2 e_2 case, we have e evaluated to value v and value_type v (sumT t_1 t_2) by the inductive hypothesis. By Lemma 6 we know v is either SumLV v' or SumRV v' for some value v'. In both cases we either have value_type v' t_1 or value_type v' t_2. Then by Lemma 7 we have either envtype $G[s_1 := t_1]$ $E[s_1 := v']$ or envtype $G[s_2 := t_2]$ $E[s_2 := v']$. This allows us to apply the inductive hypothesis for *e1* or *e2* which solves the goal. Box

Chapter 5

Strong Normalisation

We find that the strong normalisation property of Sprinkles is required to re-establish the progress property for the enriched choregraphy. Otherwise since we cannot solve the halting problem, the choregraphy will have no knowledge of whether the local computation terminates or not, and thus it cannot make an informed transition into the next state, $(s, c)arrow(s', c')$.

Since Sprinkles is an extension of simply typed lambda calculus (STLC), the strong normalisation proof is similar to that of a simply typed lambda calculus: If we try to directly prove that a well-typed expression always evaluates to a value or an exception by rule induction over typecheck, in the App e_1 e_2 case we cannot prove that given that *e1* and *e2* terminate, their application also terminates. Hence a stronger inductive hypothesis is required.

However, since we use a functional big-step semantics, we don't need to define the strong normalisation as an accessibility relation as discussed in (Abel et al., 2019)'s proof, and analyse the behaviours of reduction sequence step by step as in a standard strong normalisation proof for STLC with small step semantics. We define the strong normalisation of an expression in Sprinkles as there exists a clock in which the expression evaluates to a value or an exception. And we adapt the logical relations required by the Fundamental Lemma according to Sprinkles's values, especially the function closure, and its use of environments to store free variables' bindings. Strong normalisation proof for Sprinkles is similar to the proof for a simpler call-by-value STLC in HOL's repository by Owens and Laurent (2016), in a sense that they both adopt a functional big-step semantics and use an environment to store bindings for free variables, but Sprinkles has richer data types compared to the STLC in HOL example.

Sprinkles's evaluation function is deterministic given Lemma **??**, so the strong normalisation proof for Sprinkles is the same as the weak normalisation proof.

5.1 Defining Strong Normalisation

We define the strong normalisation property and the related logical relations in a standard way, with some adaptions for functional big-step semantics and environments.

Evaluation of Expressions

Since Sprinkles's evaluation function is total (Lemma 1), we don't define the strong normalisation of evaluation of an expression as it has no infinite reduction sequences, but rather require the evaluation returns a value or an exception for some initial clock c.
Definition 11. - *sn_exec*

Environments

Standard approach also defines an inductive notion of normalisation for terms and then prove the definition of normalisation notion is sound by showing the notion entails the definition of strong normalisation. In the proof by Abel et al. (2019) this inductive normalisation notion handles the so called strong head reduction case, in which the substituting term N is required to have the strong normalisation notion in order to conclude that $\lambda x : A.MN|[N/x]M$ is a strong head reduction. The intuition is that a term is strongly normalising if it is in a normal form or it is obtained by expanding from a normal form. But during the expansion we may have $[N/x]M$ being strongly normalising but $(\lambda x : A.M)N$ or N are not due to there are no occurrences of x in N.

Since we look up values of free variables in an environment rather than using substitution, we introduce a constrain between typing environment and dynamic environment similar to Definition 6 to replace the notion of strong head reduction:
Definition 12. - *envsn*

sn_v is to replace the standard inductive normalisation notion and will be introduced in Section 5.1. We require that the dynamic environment E contains values for all free variables covered by G and for every such value v of some free variable we have good evaluation behaviours according to the type of that free variable t stored in G defined by $\mathsf{sn_v}\ t\ v$.

Values

In the recursive cases of our definition of evaluation function (Definition 7), we first evaluate the sub-expressions of current expression into values, and then invoke the recursive evaluation function based on the values of sub-expressions. Values are not part of the expression syntax of Sprinkles but they are used in the recursively invoked evaluation functions. This is different to a standard simply typed lambda calculus where terms themselves can be values. Thus, different than defining an inductive strong normalisation notion for terms to describe how an expansion of reduction sequence at certain terms can maintain the established strong normalisation notion so far (Abel et al., 2019), we use $\mathsf{sn_v}\ t\ v$ to describe whether the value v of type t is a good value to use in the future evaluation function so using the value v in the next evaluation function won't break the strong normalisation behaviours established so far.

For example, we define a function closure $\mathsf{Clos}\ s\ e\ E$ of type $\mathsf{fnT}\ t_1\ t_2$ to be a good value to use as stating that if we evaluate the function body expression e by binding the function parameter s to any good value v defined by $\mathsf{sn_v}\ t_1\ v$ in the closure environment E, i.e. $E[s := v]$ with $\mathsf{sn_v}\ t_1\ v$, the evaluation of e always return a good value of type *t2* or an exception.
Definition 13. - *sn_v*

We don't need to address the issue of strong head reduction in sn_v cause this is handled by envsn (Definition 12), as a standard inductive strong normalisation notion of terms would have to. For example, when extending STLC with disjoint sums, the strong normalisation notion used by Abel et al. (2019) adapts by adding strong head reduction rules for the case term to discipline the expansion. Using our approach, we don't need a separate rule to describe the evaluation of a sum case expression. Rather, when evaluating $\mathsf{Case}\ e\ s_1\ e_1\ s_2\ e_2$, we will have e evaluates to some v and $\mathsf{sn_v}\ t\ v$ by re-establishing the envsn property for the corresponding (left or right branch of the sum case expression) inductive hypothesis. This is essentially what the strong head reduction rules for the case term do (Abel et al., 2019). We only need to add a left sum value case and a right sum value case in sn_v to describe what will make the sum value a good value to use in the future evaluation.

5.2 Auxillary Lemmas

In this section we show several lemmas for the logical relations proposed in Section 5.1. Those lemmas are used in the proof of the Fundamental Lemma.

Similar to the lemma of canonical forms for typed values (Lemma 6), we have the lemma of canonical forms for sn_v values. This lemma allows us to write an sn_v value v in its concrete syntax, which is useful in proving Lemma 13 and 14. Lemma 11 is also useful for proving if, app, and sum case of the Fundamental Lemma, since we want to write the boolean guard value, the function closure value, and the sum value in its concrete syntax rather then referring it using the name v.
Lemma 11. *- sn_v_**

Proof: Immediately follows from Definition 13. Box

The following lemmas about envsn $G\ E$ turn out quite useful in the Fundamental Lemma proof when we want to weaken the constrains for E specified in G, or pairing G with a larger or smaller E while still keeping the envsn property.
Lemma 12. *envsn lemmas*

1. *envsn_g_submap*
2. *envsn_update*
3. *envsn_e_submap2*
4. *envsn_e_submap*

Proof: Follows directly from Definition 12. Box

Property 1 allows use to establish envsn $G'\ E$ for some G' smaller than G. This is useful when we have envsn (DRESTRICT G ($\mathsf{free_vars}\ e$)) E and we want to deduce envsn (DRESTRICT G ($\mathsf{free_vars}\ e_1$)) E and envsn (DRESTRICT G ($\mathsf{free_vars}\ e_2$)) E for applying inductive hypotheses for e's sub-expressions $e1$ and $e2$. Because free variables of an expression is usually defined as a union of free variables of each of its sub-expression. Property 2 is used in the let and sum case cases of the Fundamental Lemma since we evaluate the function body or the expression in sum case branch using the function argument value or the sum value.

Property 3 allows to establish envsn property between G and a larger E. This lemma is used to re-stablish the envsn property required by inductive hypotheses when the variable x bound by Fn, Let, or Case is not used in some later expression e. Thus variable x is not included in the typing environment for e since it is not a free variable of e. But our dynamics rules do not know this information and still evaluate e with the previous environment updated with value for x. In this case Property 1 is not directly applicable since we do not update the typing environment, but with Property 3 we can extend the dynamic environment to contain extra information of values of unused variable.

Property 4 is required to handle the restricted function closure in the Fn case of the Fundamental Lemma proof. The premise of current goal gives us Don't expect to find a ¡end of input¿ in this position after a in compiler-generated text and at line 165, character 118.)) E, but we need to establish the envsn property for the restricted environment in function closure, so we can apply the inductive hypothesis for the function body expression stored in closure. Property 4 allows us to shrink the dynamic environment as long as the reduced environment still covers all variables covered by the typing environment.

Finally, we need the last two lemmas to handle the BinOp and Uop cases. For BinOp $bop\ e_1\ e_2$ case, if by inductive hypotheses *e1*, *e2* all evaluate to values *v1*, *v2*, we need Lemma 13 to ensure that given $\mathsf{sn}_\mathsf{v}\ t_1\ v_1$ and $\mathsf{sn}_\mathsf{v}\ t_2\ v_2$, $\mathsf{eval}_\mathsf{b}\mathsf{op}\ bop\ v_1\ v_2$ always returns a value or an exception. The same with Uop case.
Lemma 13. - *bop_sn*

Proof: boptype is non-recursive. We prove by performing cases analyses based on boptype rules and each case follows from Lemma 11. Box
Lemma 14. - *uop_sn*

Proof: Same as the above. Box

5.3 The Fundamental Lemma

5.3.1 Formalisation

Based on the discussions in Section 5.1, we state the Fundamental Lemma as:

- normal version without restricting G

And define sn_e as:

- normal version without restricting G

However, if we just require envsn $G\ E$ for any E in the definition of sn_e, the inductive hypothesis for function expression Fn $s\ e$ case is not applicable and we cannot prove the evaluated closure value is related to the type of function expression by sn_v as a result. This is because by Definition 13, the body e in function closure needs to be evaluated in the restricted dynamic environment DRESTRICT E ($\mathsf{free}_\mathsf{v}\mathsf{ars}\ e$) \\ s captured by closure. Hence we need to instantiate the dynamic environment in the inductive hypothesis for e to be DRESTRICT E ($\mathsf{free}_\mathsf{v}\mathsf{ars}\ e$) \\ s. But this requires proving envsn $G[s\ :=\ t]$ (DRESTRICT E ($\mathsf{free}_\mathsf{v}\mathsf{ars}\ e$) \\ s) first in order to be able apply the results of inductive hypothesis for e. But envsn $G[s\ :=\ t]$ (DRESTRICT E ($\mathsf{free}_\mathsf{v}\mathsf{ars}\ e$) \\ s) is not deducible from envsn $G[s\ :=\ t]\ E$, the premise of current goal, since G may contain types for arbitrary variables.

So we modify the definition of sn_e to used a restricted typing environment that matches the restrict dynamic in a function closure. We require the restricted typing environment to contain at most types for all free variables of the expression being evaluated. After introducing restriction for the typing environment, the condition for inductive hypothesis of e becomes envsn (DRESTRICT G ($\mathsf{free}_\mathsf{v}\mathsf{ars}\ e$))$[s\ :=\ t]$ (DRESTRICT E ($\mathsf{free}_\mathsf{v}\mathsf{ars}\ e$))$[s\ :=\ v]$, which is deducible given envsn (DRESTRICT G (free_vars e DIFF s)) E, the new premise of current goal after restricting the typing environment. Hence we give the modified definition of sn_e:

Definition 14. - *sn_e*

We state the Fundamental Lemma as:
Lemma 15. - *sn_lemma*

5.3.2 Strategy

We prove via rule induction over typecheck. The main strategy in each case is to first deduce the envsn property required by the inductive hypotheses from the premise of the current goal. Then we have the sub-expressions of current expression either evaluate to a value or an exception by the corresponding inductive hypotheses. If some sub-expression evaluates to an exception, the monadic binds used in Definition 7 will ensure the evaluation in the goal return the exception. We only need to find a clock value under which the first sub-expression evaluating to an exception and all the sub-expressions evaluating to values before it still evaluate to the same exception and the values. By Lemma 2 and 3, we can always use the summation of clock values used to evaluate those sub-expressions as the clock value used in evaluations in the goal statements to evaluate those sub-expressions to the same results.

If no sub-expression evaluates to an exception, we either have an inductive hypothesis for evaluation of some sub-expression that uses the evaluated values, or an external eval_bop or eval_uop uses the evaluated values. In the first case, we need to deduce the envsn property required by the inductive hypothesis, which usually depends on Lemma 12. In the second case, we need to use Lemma 13 and 14.

The proof strategy itself is not difficult, but evaluation of many expressions requires evaluating sub-expressions first (Definition 7), which generates many cases that are similar but are proved with slightly different tactics, making the HOL proof long. For example, BinOp bop e_1 e_2 requires to evaluate *e1* and *e2* first and calls eval_bop with the evaluated values. This generate 4 cases since both *e1* and *e2* may evaluate into a value or an exception.

Additionally, based on the discussions for Lemma 12 (3), if the bound variable vn is not used in a sub-expression e, the envsn statement in inductive hypothesis of e will be envsn (DRESTRICT G (free$_{\mathsf{v}}$ars e)) $E[vn := v]$ rather than envsn (DRESTRICT G (free$_{\mathsf{v}}$ars e))$[vn := t]$ $E[vn := v]$ if vn is used in e. These two cases require different proofs and thus this extra layer of differences makes the number of generated cases even more for the Fn, Let, and Case cases.

5.3.3 Proof

Base cases. Evaluation for literals always return values and those values are related to the types of literal expressions by sn_v. For variable expressions, the goal follows from Definition 12.

Function. Consider a function expression Fn s e of type fnT t_1 t_2. Since we are doing induction over typecheck we have one inductive hypothesis for evaluating e. By Definition 7 we know a function expression always evaluates to a closure. We then need to show this closure is related to fnT t_1 t_2 by sn_v. By Definition 13 we need to show for any v such that sn$_{\mathsf{v}}$ t_1 v, if we evaluate e in the closure's environment extended with binding s to v, the evaluation either returns a value or an exception. This matches the result of inductive hypothesis for e. From the discussion in Section 5.3.1, we know that the envsn property required for using inductive hypothesis for e is deducible from the premise of current goal.

If expression. Consider an if expression If bg e_1 e_2. The If case follows the strategy discussed in Section 5.3.2. By Definition 14, we have envsn (DRESTRICT G (free$_{\mathsf{v}}$ars (If bg e_1 e_2))) E as the condition for $\mathsf{sn_exec}\ t\ E\ (\mathsf{If}\ bg\ e_1\ e_2)$. Since free variables of $\mathsf{If}\ bg\ e_1\ e_2$ are union of those for bg, $e1$, and $e2$, by Lemma 12 (1) we have $\mathsf{envsn}\ (\mathsf{DRESTRICT}\ G\ (\mathsf{free_vars}\ bg))\ E$, $\mathsf{envsn}\ (\mathsf{DRESTRICT}\ G\ (\mathsf{free_vars}\ e_1))$ and $\mathsf{envsn}\ (\mathsf{DRESTRICT}\ G\ (\mathsf{free_vars}\ e_2))\ E$. Thus we are able to apply the inductive hypotheses for bg, $e1$, and $e2$.

Since each of them can evaluate into either a value or an exception, 2^3 cases are generated. If bg evaluates to an exception, we have $\mathsf{If}\ bg\ e_1\ e_2$ evaluates into an exception regardless of the results of $e1$ and $e2$. If bg evaluates to a value v, we apply the canonical form lemma (Lemma 11) to write v as $\mathsf{BoolV}\ b$. Then based on the value of b and the results of $e1$ and $e2$ (either a value or an exception as implied by the corresponding inductive hypothesis), we instantiate Lemma 2 and 3 with clock value $c_{bg} + c_1 + c_2$, proving the goal.

Binary operation. Consider a binary operation $\mathsf{BinOp}\ bop\ e_1\ e_2$. The BinOp case is similar to If and follows the strategy discussed in Section 5.3.2. Given the premise for current goal we are able to deduce the envsn results required for using inductive hypotheses for $e1$ and $e2$ by Lemma 12 (1). Since both of $e1$ and $e2$ can evaluate into either a value or an exception, 2^2 cases are generated. If any one of them evaluates to an exception, we instantiate Lemma 2 and 3 with clock value $c_1 + c_2$, proving the goal. If $e1$, $e2$ evaluate to $v1$, $v2$ separately, we have $\mathsf{sn_v}\ t_1\ v_1$ and $\mathsf{sn_v}\ t_2\ v_2$ by the inductive hypotheses. We instantiate Lemma 2 with a clock value $c_1 + c_2$ so $e1$, $e2$ evaluate with the same clock value and still reach the previous results $v1$, $v2$. Then Lemma 13 ensures $\mathsf{eval_bop}\ v_1\ v_2$ returns a value or an exception, proving the goal.

Unary operation. Same as the binary operation case.

Let expression. Consider a binary operation $\mathsf{Let}\ s\ e_1\ e_2$. Similar as the above cases, we use Lemma 12 (1) to deduce the envsn results required for using inductive hypothesis for $e1$. If $e1$ evaluates to an exception, we have the goal proved. If $e1$ evaluates into a value v, we need to show evaluation of $e2$ within $E[s := v]$ either returns a value or an exception. This matches the results in the inductive hypothesis for $e2$. By Lemma 12 (2), we have $\mathsf{envsn}\ G[s := t_1]\ E[s := v]$. Thus applying the inductive hypothesis for $e2$ gives us evaluation of $e2$ within $E[s := v]$ returns a value or an exception. Instantiating Lemma 2 and 3 with clock value $c_1 + c_2$ proves the goal.

As the discussion on Lemma 12 (3) in Section 5.2 points out, if s is not used in $e2$, i.e. s is not free in $e2$, $\mathsf{envsn}\ G\ E[s := v]$ is required instead to use the inductive hypothesis for $e2$, and Lemma 12 (3) allows us to deduce $\mathsf{envsn}\ G\ E[s := v]$.

Application. Application case follows the strategy stated in Section 5.3.2 and is similar to the If case.

Case expression. Consider a case expression $\mathsf{Case}\ e\ s_1\ e_1\ s_2\ e_2$. The proof follows the strategy stated in Section 5.3.2 where Lemma 12 (1) is applied for using the inductive hypothesis for e. But extra case analyses on whether $s1$ is free in $e1$, and on $s2$ is free in $e2$ are required. This extra part of analyses are similar to the one in Let case, where one of Lemma 12 (3) and Lemma 12 (2) is applied for using the inductive hypothesis in each case. Box

5.3.4 Strong Normalisation

We now state the strong normalisation property for Sprinkles:
Corollary 3.

$$\begin{aligned}
&\vdash \emptyset \vdash_s e : t \Rightarrow \\
&\quad (\exists\, cl\ v\ E.\ \mathsf{eval\_exp}\ cl\ E\ e = \mathsf{Value}\ v \land \mathsf{sn_v}\ t\ v) \lor \\
&\quad \exists\, cl\ exn\ E.\ \mathsf{eval\_exp}\ cl\ E\ e = \mathsf{Exn}\ exn
\end{aligned}$$

Proof: Follows from Lemma 15 Box

Corollary 3 says that if a closed expression of Sprinkles is typed, then it either evaluates to a value or to an exception.

Chapter 6

The Enriched Choreography

We introduce the enrich Kalas, where local computations are handled by Sprinkles . As a result, the external function $f(v)$ in the previous Let rule of Kalas (Definition 17) is replaced with a Sprinkles expression e. Messages in Kalas used to be a list of strings, but now they are string values in Sprinkles. Since evaluations in Sprinkles may return an exception, we introduce new transition rules in the enriched Kalas to handle local computation failure.

We provide typing rules for the enrich Kalas. Our choregraphy does not return values and thus our rules only specify whether a choregraphy type checks. Kalas uses *states* to store values for variables located at a certain process, so we introduce a similar *typing state* that stores types for variables located at a certain process.

Since a local expression e neither cares neither about values of variables in other processes nor about their types, we introduce the concept of *localised* environment and *localised* typing environment to describe the result of filtering out values or types of variables in other processes. So e can be evaluated in the *localised* environment and typed in the *localised* typing environment.

We prove that our enriched Kalas has an important property: *progress*. The strong normalisation property of Sprinkles is crucial for the progress proof, otherwise we would have had to solve the halting problem. We also prove that the non-recursive, synchronised transitions in the enriched Kalas has type preservation property, where the type soundness and strong normalisation of local computations is required.

6.1 Syntax

Definition 15. - *syntax for the enriched Kalas*

In Let $v\ p\ e\ C$, e represents a Sprinkles expression and (v, p) represents a variable v located at process p. The message values are Sprinkles's results (Definition 4.1). Other syntax are the same as Kalas' (Definition 17).

6.2 Statics

6.2.1 Syntax and Typing Rules

A typing state Γ is a finite map that stores mappings from (v, p) to a Sprinkles's type (Definition 3). A type statement for the enriched kalas is written as $\Gamma\ ,\ \Theta\ \vdash\ C\ \checkmark$, where Γ is a typing state, Θ being a set of processes, and C is the choregraphy being typed.

- chorEnvtype: ensure chortype_no_undefined_vars; and the trans_submap adapted let rule extra constrains in the progress proof

- example of typable choregraphy

6.3 Dynamics

- exceptions

- adaptions for the trans_submap

6.4 Progress

6.4.1 Lemmas

6.4.2 Proof

6.5 Type Preservation

Chapter 7

Concluding Remarks

If you wish, you may also name that section *"Conclusion and Future Work"*, though it might not be a perfect choice to have a section named "A & B" if it has subsections "A" and "B". Also note that you don't necessarily have to use these subsections; that also depends on how much content you have in each. (E.g., having a section header might be odd if it contains just three lines.)

7.1 Conclusion

This section usually summarizes the entire paper including the conclusions drawn, e.g., did the developed techniques work? Maybe add why or why not. Also don't hold back on limitations of your work; it shows that you understood what you have done. And science isn't about claiming how great something is, but about objectively testing hypotheses. Also note that every single scientific paper has such a section, so you can check out many examples, preferably at top-tier venues, e.g., by your supervisor(s).

7.2 Future Work

- communicate with richer data types rather than merely strings

- asynchronus messages; confluence property - Progress for EPP

- what if local language does not have SN property and we may do some special handling to recover the progress for choregraphy 5

Chapter 8

Test

We define what it is for a choreograph to be well-formed with the $G\ ,\ Th\ \vdash\ c\ \checkmark$ relation.

This is a theorem:

$$\begin{array}{l}\vdash \Gamma\ ,\ \Theta\ \vdash\ c\ \checkmark\ \wedge\ \mathsf{chorEnvtype}\ \Gamma\ s\ \wedge\ \mathsf{chorEnvsn}\ \Gamma\ s\ \Rightarrow \\ \quad \exists\,\tau\ l\ s'\ c'.\ s \rhd c\ \underset{l}{\xrightarrow{\tau}}\ s' \rhd c'\ \vee\ \neg\mathsf{not\_finish}\ c\end{array}$$

$$\begin{array}{l}\vdash \mathsf{not\_finish}\ c\ \wedge\ \mathsf{chorEnvsn}\ \Gamma\ s\ \wedge\ \mathsf{chorEnvtype}\ \Gamma\ s\ \wedge \\ \quad \Gamma\ ,\ \Theta\ \vdash\ c\ \checkmark\ \wedge\ s \rhd c\ \underset{l}{\xrightarrow{\tau}}\ s' \rhd c'\ \Rightarrow \\ \quad\ \exists\,\Gamma'. \\ \quad\quad \mathsf{chorEnvsn}\ \Gamma'\ s'\ \wedge\ \mathsf{chorEnvtype}\ \Gamma'\ s'\ \wedge \\ \quad\quad \Gamma'\ ,\ \Theta\ \vdash\ c'\ \checkmark\end{array}$$

$$\begin{array}{l}\vdash \mathsf{envtype}\ G\ E\ \wedge\ G\ \vdash_s\ e\ :\ ty\ \Rightarrow \\ \quad (\exists\,v.\ \mathsf{eval\_exp}\ c\ E\ e\ =\ \mathsf{Value}\ v\ \wedge\ \mathsf{value\_type}\ v\ ty)\ \vee \\ \quad (\exists\,exn.\ \mathsf{eval\_exp}\ c\ E\ e\ =\ \mathsf{Exn}\ exn)\ \vee \\ \quad \mathsf{eval\_exp}\ c\ E\ e\ =\ \mathsf{Timeout}\end{array}$$

$$\begin{array}{l}\vdash \emptyset\ \vdash_s\ e\ :\ t\ \Rightarrow \\ \quad (\exists\,cl\ v\ E.\ \mathsf{eval\_exp}\ cl\ E\ e\ =\ \mathsf{Value}\ v\ \wedge\ \mathsf{sn}_{\mathsf{v}}\ t\ v)\ \vee \\ \quad \exists\,cl\ exn\ E.\ \mathsf{eval\_exp}\ cl\ E\ e\ =\ \mathsf{Exn}\ exn\end{array}$$

BinOp Add (Var x) (IntLit 1)

The transition relation looks like eval_exp $clk\ E\ exp$
Theorem 3. *some text here*

1. *(Operational completeness) If* $G\ ,\ Th\ \vdash\ c\ \checkmark$ *then there exist ...*
2. *(Operational soundness) If* eval_exp $clk\ E\ exp$ *then there exist ...*

Definition 16 (Kalas syntax)**.** Choreographies *in Kalas, ranged over by* C*, are inductively defined by the grammar*

C	$::=$	$p_1.v_1 \rightarrow p_2.v_2; C$	*(com)*	$p_1 \rightarrow p_2[b]; C$	*(sel)*
		if $v@p$ **then** C_1 **else** C_2	*(if)*	Let $v\ p\ e\ C$	*(let)*
		$\mu X.\ C$	*(fix)*	X	*(var)*
		$\mathbf{0}$	*(nil)*		

Definition 17 (Sprinkles syntax)**.** Values *and* expressions *in Sprinkles, ranged over by* v, e, *are inductively defined by the grammar*

v ::= IntV n | StrV s | BoolV b | Clos s e E | PairV v_1 v_2 | SumLV v | SumRV v

bop ::= Add | Concat | Mult | Div | Mod | Less | And | Or | Eq | Sub | Pair

uop ::= Not | NumOf | StrOf | Fst | Snd | SumL | SumR

e	$::=$	Var x	*(var)*	StrLit s	*(str)*
		IntLit n	*(int)*	BoolLit b	*(bool)*
		BinOp bop e_1 e_2	*(bop)*	Uop uop e	*(uop)*
		If bg e_1 e_2	*(if)*	Let x e_1 e_2	*(let)*
		Fn x e	*(fn)*	App e_1 e_2	*(app)*
		Case e x e_1 y e_2	*(case)*		

T ::= intT | strT | boolT | fnT t_1 t_2 | pairT t_1 t_2 | sumT t_1 t_2

sn$_\mathsf{v}$ intT (IntV n) $\stackrel{\text{def}}{=}$ T
sn$_\mathsf{v}$ strT (StrV s) $\stackrel{\text{def}}{=}$ T

```
eval_exp $c$ $E$ (Var $str$) $\stackrel{\text{def}}{=}$
 case $E$ $str$ of None $\Rightarrow$ TypeError | Some $v$ $\Rightarrow$ Value $v$
eval_exp $c$ $E$ (Fn $s$ $e$) $\stackrel{\text{def}}{=}$
 Value (Clos $s$ $e$ (DRESTRICT $E$ (free_vars $e$) \\ $s$))
eval_exp $c$ $E$ (App $e_1$ $e_2$) $\stackrel{\text{def}}{=}$
 if $c$ $>$ 0 then
  do
   $v_1$ $\leftarrow$ eval_exp $c$ $E$ $e_1$;
   $v_2$ $\leftarrow$ eval_exp $c$ $E$ $e_2$;
   case $v_1$ of
    IntV $v_{11}$ $\Rightarrow$ TypeError
   | StrV $v_{12}$ $\Rightarrow$ TypeError
   | BoolV $v_{13}$ $\Rightarrow$ TypeError
   | PairV $v_{14}$ $v_{15}$ $\Rightarrow$ TypeError
   | SumLV $v_{16}$ $\Rightarrow$ TypeError
   | SumRV $v_{17}$ $\Rightarrow$ TypeError
   | Clos $s$ $e$ $E_1$ $\Rightarrow$ eval_exp ($c$ $-$ 1) $E_1$[$s$ := $v_2$] $e$
  od
 else Timeout
```

Choreography	External Computation
1. server.var -> client.x; 2. **let** v@client = mod(x) **in** 3. client.v -> server.$result$;	**fun** mod x = case Option.map (fn s $\Rightarrow$ valOf (Int.fromString s)) (hd x) of None $\Rightarrow$ None | Some n $\Rightarrow$ Some [Int.toString (n MOD y)]

Table 8.1: semantics: communication rules. The function wv(α) returns the variable (if any) that is modified by α.

$$\textsc{Com}\ \frac{s\ (v_1, p_1)\ =\ \mathsf{StrV}\ d \quad p_1\ \neq\ p_2}{s \rhd p_1.v_1 \mathrel{{-}{>}} p_2.v_2;\, C \ \xrightarrow[\epsilon]{p_1.v_1 \gg p_2.v_2}\ s[(v_2, p_2)\ :=\ \mathsf{StrV}\ d] \rhd C}$$

$$\text{T-}\textsc{Var}\ \frac{G\ x\ =\ ty}{G\ \vdash_s\ \mathsf{Var}\ x\ :\ ty}$$

$$\text{T-}\textsc{Fn}\ \frac{G[s\ :=\ t] \vdash_s\ e\ :\ ty}{G\ \vdash_s\ \mathsf{Fn}\ s\ e\ :\ (\mathsf{fnT}\ t\ ty)}$$

$$\text{T-}\textsc{App}\ \frac{G\ \vdash_s\ e_1\ :\ (\mathsf{fnT}\ t\ ty) \quad G\ \vdash_s\ e_2\ :\ t}{G\ \vdash_s\ \mathsf{App}\ e_1\ e_2\ :\ ty}$$

$$\text{CT-}\textsc{Com}\ \frac{\Gamma\ (v_1, p_1)\ =\ \mathsf{strT} \quad \{\ p_1;\ p_2\ \}\ \subseteq\ \Theta \quad p_1\ \neq\ p_2 \quad \Gamma[(v_2, p_2)\ :=\ \mathsf{strT}],\ \Theta\ \vdash\ c \checkmark}{\Gamma\ ,\ \Theta\ \vdash p_1.v_1 \mathrel{{-}{>}} p_2.v_2;\, c \checkmark}$$

$$\text{CT-}\textsc{Let}\ \frac{\mathsf{localise}\ \Gamma\ p\ \vdash_s\ e\ :\ ety \quad \Gamma[(v, p)\ :=\ ety],\ \Theta\ \vdash\ c \checkmark}{\Gamma\ ,\ \Theta\ \vdash\ \mathsf{Let}\ v\ p\ e\ c \checkmark}$$

1. `server`.var -> `client`.x;
2. **let** v@`client` = StrOf ((NumOf (Var x)) Mod (Var y)) **in**
3. `client`.v -> `server`.$result$;

Appendix A

Appendix: Explanation on Appendices

Appendix B

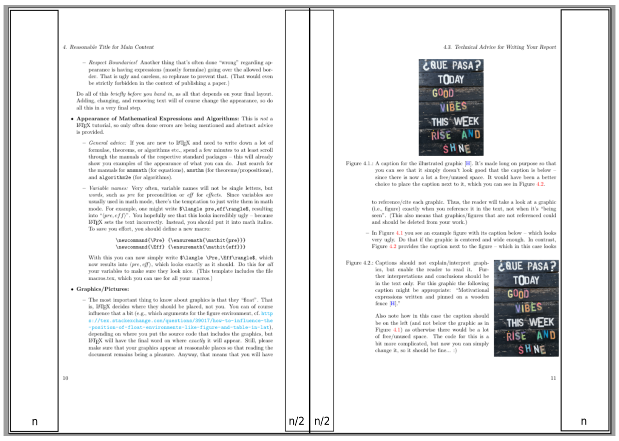
Appendix: Explanation on Page Borders

What you find here is an explanation of why the border width keeps flipping from left to right – which you might have spotted and wondered why that's the case.

Firstly, that is *intended* and thus correct, so there is no reason to worry about this. The reason is that this document is configured as a two-sided book, which means:

- We assume the document will be printed out,
- that this will be done in a two-sided mode (i.e., the document will be printed on both sides of each page), and
- that the bookbinding will be in the middle, just like in every book.

When you open the book, there are three borders of equal size n. This however requires that even pages have a border of n on their left and $\frac{n}{2}$ on their right, and odd pages have a border of $\frac{n}{2}$ on their left and n on their right. This is illustrated in Figure B.1.



4. Reasonable Title for Main Content

- *Respect Boundaries!* Another thing that's often done "wrong" regarding appearance is having expressions (mostly formulae) going over the allowed border. That is ugly and careless, so rephrase to prevent that. (That would even be strictly forbidden in the context of publishing a paper.)

Do all of this *briefly before you hand in*, as all that depends on your final layout. Adding, changing, and removing text will of course change the appearance, so do all this in a very final step.

- **Appearance of Mathematical Expressions and Algorithms:** This is *not* a LaTeX tutorial, so only often done errors are being mentioned and abstract advice is provided.
  - *General advice:* If you are new to LaTeX and need to write down a lot of formulae, theorems, or algorithms etc., spend a few minutes to at least scroll through the manuals of the respective standard packages – this will already show you examples of the appearance of what you can do. Just search for the manuals for `amsmath` (for equations), `amsthm` (for theorems/propositions), and `algorithm2e` (for algorithms).
  - *Variable names:* Very often, variable names will not be single letters, but *words*, such as *pre* for precondition or *eff* for *effects*. Since variables are usually used in math mode, there's the temptation to just write them in math mode. For example, one might write `$\langle pre,eff\rangle$`, resulting into "$\langle pre, eff\rangle$". You hopefully see that this looks incredibly ugly – because LaTeX sets the text incorrectly. Instead, you should put it into math italics. To save you effort, you should define a new macro:

```
\newcommand{\Pre} {\ensuremath{\mathit{pre}}}
\newcommand{\Eff} {\ensuremath{\mathit{eff}}}
```

    With this you can now simply write `$\langle \Pre,\Eff\rangle$`, which now results into $\langle \mathit{pre}, \mathit{eff}\rangle$, which looks exactly as it should. Do this for *all* your variables to make sure they look nice. (This template includes the file macros.tex, which you can use for all your macros.)

- **Graphics/Pictures:**
  - The most important thing to know about graphics is that they "float". That is, LaTeX decides where they should be placed, not you. You can of course influence that a bit (e.g., which arguments for the figure environment, cf. https://tex.stackexchange.com/questions/39017/how-to-influence-the-position-of-float-environments-like-figure-and-table-in-lat), depending on where you put the source code that includes the graphics, but LaTeX will have the final word on where *exactly* it will appear. Still, please make sure that your graphics appear at reasonable places so that reading the document remains being a pleasure. Anyway, that means that you will have

10

4.3. Technical Advice for Writing Your Report

Figure 4.1.: A caption for the illustrated graphic [H]. It's made long on purpose so that you can see that it simply doesn't look good that the caption is below – since there is now a lot a free/unused space. It would have been a better choice to place the caption next to it, which you can see in Figure 4.2.

to reference/cite each graphic. Thus, the reader will take a look at a graphic (i.e., figure) exactly when you reference it in the text, not when it's "being seen". (This also means that graphics/figures that are not referenced could and should be deleted from your work.)

- In Figure 4.1 you see an example figure with its caption below – which looks very ugly. Do that if the graphic is centered and wide enough. In contrast, Figure 4.2 provides the caption next to the figure – which in this case looks

Figure 4.2.: Captions should not explain/interpret graphics, but enable the reader to read it. Further interpretations and conclusions should be in the text only. For this graphic the following caption might be appropriate: "Motivational expressions written and pinned on a wooden fence [H]."

Also note how in this case the caption should be on the left (and not below the graphic as in Figure 4.1) as otherwise there would be a lot of free/unused space. The code for this is a bit more complicated, but now you can simply change it, so it should be fine... :)

11

Figure B.1: Illustration showing why page borders flip.

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