[3]

Design Analysis of a Dirt Jump Mountain Bike Frame

Zach Bebe, Zach Moyse, Alex Allardi

March 9th, 2018

Dirt Jump and Urban Freeride are both a part of the most demanding discipline of extreme cycling. Their discipline involves large drops, big air, flat landings and extreme features. With this aggressive form of cycling comes extreme impacts and grueling crashes. Early dirt jump frames weighed as much as modern downhill bikes however, they had no rear suspension and little to no front suspension. The weight of these bikes coming mostly from their burly overbuilt frames. As the frames developed, through parameter variation and design changes, the bicycles have been able to be reduced to around 25lbs.

This modern dirt jump frame is designed to be ridden on extreme terrain and features. These bikes have to absorb large rider impacts, variable cyclic loading, and their frames have to be able to last at least 3 years of consistent riding. These frames will be exposed to a number of different environments, as the frame is used in both off road dirt jump and urban city environments. The engineering for these frames must be perfect, as a failure in the frame could lead to injury or death of the user.

[6]

When analyzing a dirt jump frame for failure, it was determined that the frame must be able to withstand a rider dropping off of a 10 foot tall ledge and landing flat on a hard surface that does not deform. The assumption made is that the riders weight is fully distributed on the bottom bracket. It should be noted that this is a very extreme case.

Mass of Rider: 220 lb

Mass of Bicycle: 30 lb

Height of Drop: 10 ft

Gravity: 32.2 ft/s^2

Bottom Bracket Length: 0.2395 ft (73mm)

Bottom Bracket Inner Diameter: 0.1345 ft (41mm)

Bottom Bracket Outer Diameter: 0.1427 ft (43.5mm)

Elastic Modulus (Alum. 6061 T6, Chromoly Steel 4130): 10,000 ksi, 11,600 ksi

**First, the static deformation is calculated using Equation 1.*[5]

$$\delta_{st} = \frac{PL}{AE}$$

**The equivalent force was then calculated using Equation 2.*[5]

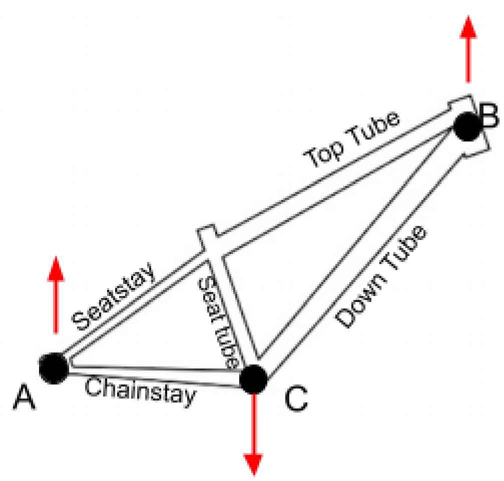
$$Fe = W\sqrt{\frac{V^2}{g\delta_{st}}}$$

**These calculations were executed using Matlab, the code can be seen in Figure 1*

Figure 1: Equivalent Force Calculation Code

```
%Equivelent force for a cyclinder

%prompt user for values
P=input('Load Value P (lbf): ');
L1=input('Length of cylinder(mm): ');
E=input('Elastic Modulus of Material (ksi):  ');
D1=input('Inner diameter of cylinder (mm):  ');
t1=input('Thickness of cylinder wall (mm):  ');
h=input('Height of drop (ft):  ') ;

%Conversions
D=D1*0.0393701;
t=t1*0.0393701;
L=L1*0.0393701;

% Calculate static deformation
A=(pi/4)*((D+2*t)^2-(D^2));
delta=((P*L)/(E*A))/12;

%Calulate velocity from enery equation
v=sqrt(2*32.2*h);

%Calculate Equivalent force
Fe=sqrt(v^2/(32.2*delta));

%print resulting Fe
 fprintf('Fe= %f\n,lbf',Fe)
```

Figure 2: Static Truss Force Diagram



A dirt jump mountain bike can be analyzed in 2d as a truss. At point A and Point B in figure 2 are fixed connections. Point C represents the bottom bracket, where the resultant force is applied. With a resultant Force of 1367 lbf. Reaction forces of 811.418 lbf at A and 556.185 lbf at B occur at the two fixed connections. These external forces result in internal forces throughout the frame. Both the top tube and seatstay are in compression. The seat tube, down tube, and chainstay are in tension. The values of the internal forces of the frame were calculated for 6061 T6 aluminum (Table 1) and 4130 Chromoly Steel (Table 2) using method of sections.

Table 1: Forces for 6061 T6 Aluminum frame

Member	Internal Force (lbf)	Tension/Compression
Top Tube	1466.5	Compression
Seatstay	745.74	Compression
Seat Tube	212.836	Tension
Chainstay	652.38	Tension
Downtube	1741.05	Tension

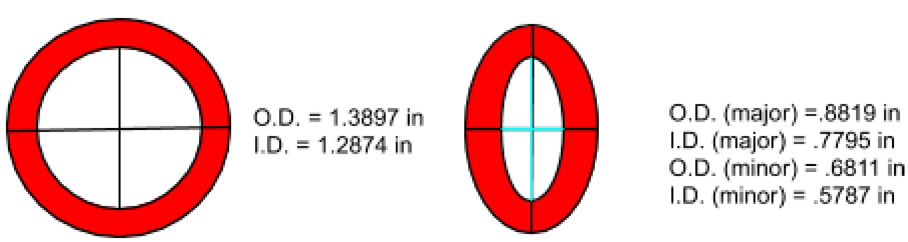
Table 2: Forces for 4130 Chromoly Steel frame

Member	Internal Force (lbf)	Tension/Compression
Top Tube	2527.3	Compression
Seatstay	1285.2	Compression
Seat Tube	366.79	Tension
Chainstay	1124.3	Tension
Downtube	3000.5	Tension

*Notice that in 2d the seatstay and chainstay are represented as a single element. In a 3d representation there are two tubes for both the seatstay and chainstay. The tubing is symmetric and the force is applied at an equidistant point from each set of tubes. For this reason, it was assumed that the internal force seen in 3d is half of the calculated force in 2d for each tube. This is true for both Table 1 & 2.

Both the Top Tube and Seatstay are in compression. Both are long thin cylindrical members. For this reason, it was determined that failure from buckling would have the highest likelihood of occuring in these elements. Buckling analysis was performed. The cross sections of both the top tube and seat stay can be seen in figure 3.

Figure 3: Cross Sections of Top Tube and Seatstay



Below in figure 4 is the code used to evaluate for buckling in the Top Tube and Seatstay components of the bike frame. The code prompts the user for inputs that will generate a factor of safety (F.O.S.), stress of the member (S), weight of the member (W), and a critical buckling stress value (S_{CR}) for a circular or ellipsoidal cylinder of any material. The factor of safety is determined by dividing dividing Euler's Column Buckling stress, S_{CR}, by the stress that the member was evaluated at.

Figure 4: Buckling Code for Calculation of Compressive Members

```
%Equation for Buckling, Pcr: For cylindrical cross scetion
x=input('Is the crosssection a circ, 1 or a elip, 2?');
if x==1
E=input('Mod. Elasticity (ksi) = ');
D1=input('Outer diameter (mm) = ');
d1=input('Inner diameter (mm) = ');
L=input('Length of Member (inches)= ');
F=input('Compressive force in member (kip) = ');
rho=input('Material Density = (lb/ft^3) = ');
D=D1*0.0393701;
d=d1*0.0393701;
S=F/((pi/4)*(D^2-d^2));
%Le equation
Le=0.65*L;
% Moment of inertia equation
I=(pi/32)*(D^4-d^4);
%Pcr and Scr equation
Pcr=pi^2*E*I/(Le^2);
Scr=Pcr/((pi/4)*(D^2-d^2));
%Weight equation
W=rho*pi*((D/2)^2-(d/2)^2)*L*0.000578704;
if Scr>S
    disp('No buckling occurs')
    Fos=Scr/S;
    fprintf('Scr = %f\n ',Scr)
    fprintf('Stress in member = %f\n ',S)
    fprintf('F.O.S. = %f\n',Fos)
    fprintf('Weight= %f\n',W)
else
    disp('Buckling will occur')
end
%end of circular arguement
else
E=input('Mod. Elasticity (ksi) = ');
D1=input('Outer long diameter (mm) = ');
D2=input('Inner long diameter (mm) = ');
d1=input('Outer short diameter (mm) = ');
d2=input('Inner Short diameter (mm) = ');
L=input('Length of Member (inches)= ');
F=input('Compressive force in member (kip) = ');
rho=input('Material Density = (lb/ft^3) = ');
%converststs mm to in
D=D1*0.0393701;
Di=D2*0.0393701;
d=d1*0.0393701;
di=d2*0.0393701;
%Stress equation
S=F/(pi*(0.5*D)*(0.5*d)-pi*(0.5*Di)*(0.5*di))
%Le equation
Le=0.65*L;
% Moment of inertia equation
I=0.25*((pi*(0.5*D)*(0.5*d))*((0.5*D)^2+(0.5*d)^2)-
(pi*(0.5*Di)*(0.5*di))*((0.5*Di)^2+(0.5*di)^2))
%Pcr and Scr equation
Pcr=pi^2*E*I/(Le^2)
Scr=Pcr/(pi*(0.5*D)*(0.5*d)-pi*(0.5*Di)*(0.5*di))
W=rho*pi*(0.5*D-0.5*Di)*(0.5*d-0.5*di)*L*0.000578704;
if Scr>S
    disp('No buckling occurs')
    Fos=Scr/S;
    fprintf('Scr = %f\n ',Scr)
    fprintf('Stress in member = %f\n ',S)
    fprintf('F.O.S. = %f\n',Fos)
    fprintf('Weight= %f\n',W)
else
    disp('Buckling will occur')
end
end
```

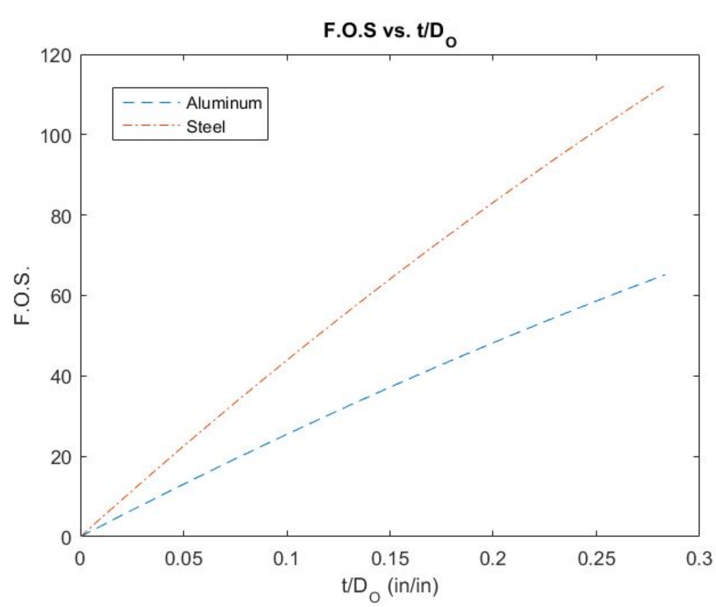
In table 3 below, the values for F.O.S., S, W, and S_{CR} for the Top Tube and the Seatstay are displayed. Both were calculated using Steel and aluminum. Aluminum being the weaker of the two materials still yielded a F.O.S. of 10.794 and 11.847 for the Top Tube and Seatstay, respectively.

Table 3: Factor of Safety for Buckling in Compressive Members.[1,2]

	Top Tube		**Seatstay**	
	4130 Chromoly Steel	**6061 T6 Aluminum**	**4130 Chromoly Steel**	**6061 T6 Aluminum**
S_{CR} (ksi)	439.381	147.939	448.325	150.951
F.O.S.	37.419	21.712	10.945	23.769
S (ksi)	11.742	6.814	40.963	6.351
W (lb)	1.627	0.559	0.480	0.165

In order to optimize the frame to reduce its overall weight and cost without changing the exterior geometry, the inner radius can be reduced to fit certain F.O.S.'s or weight constraints. Below in figure 5 is a plot of the F.O.S. vs the tube thickness over the tube outer diameter for steel and aluminum. This shows the linear relationship of F.O.S. and the non-dimensionalized parameter of cylinder thickness.

Figure 5: Factor of Safety v.s. Thickness/ Outer Diameter



The remaining members of the frame (seat tube, chainstay, and downtube) are all members in tension. These members were not evaluated for fracture toughness due to the manufacturer instructing consumers to not use the product in any way if a frame crack is seen upon purchase or if it develops as a result of use. The F.O.S. of these members can be seen in table 4 for both steel and aluminum. These F.O.S. are extremely large for the seat tube (since the force within the tube is so minimal) which leads to the conclusion that if the frame were to fail it would not be from the seat tube breaking.

Table 4: Factor of Safety for members loaded in tension.[1,2]

F.O.S.		
	4130 Chromoly Steel	**6061 T6 Aluminum**
Seat tube	82.683	38.279
Chainstay	8.351	6.987
Downtube	7.608	6.07

Simple code was written to calculate the F.O.S. of the members in tension. The code used the outer diameter of the tube, the tube thickness, internal tube force and material ultimate strength. This code can be seen in figure 6.

Figure 6: Factor of Safety of Members in Tension

```
D1=input('Outdiameter:  ');
t1=input('thickness:  ');
F=input('Force:  ');
Su=input('Ult strength:  ');

D=D1*0.0393701;
t=t1*0.0393701;
A=(pi*(D/2)^2)-(pi*((D-t)/2)^2);
S=F/A;
fos=Su/S
```

Over the course of this products lifetime it will take an estimate of 10^4 impacts before it starts to show signs of fatigue or failure. An average fatigue stress for the entire frame was taken for both steel and aluminum and was compared to the stresses in each member when the frame was first evaluated at a 10 ft drop. The member that is most at risk to be failing from this cyclic loading would be the steel seatstay (table 3). However, the expectation that this frame will experience 10^4 10 ft impacts through its lifetime is rather unrealistic. Instead it would be safe to say that the seatstay will more than likely remain intact. Below in table 5 is the fatigue stresses for steel and aluminum at 10^4 cycles.

Table 5: Fatigue Stress at 10^4 cycles. [5]

	4130 Chromoly Steel	**6061 T6 Aluminum**
Fatigue Stress (ksi)	32.668	15.858

The dropout of the frame must be analyzed for shear strength. The frame has a thickness of .2657 in and a height of .4921 in. There is an upwards acting force on the dropout with a magnitude of 405.709 lbs. Figure 7a depicts a free body diagram of the dropout. The figure also depicts the thickness and height of the rear dropout at the point where the resulting force is being applied. Figure 7b shows the code used to determine the factor of safety on the rear dropout.

Figure 7a[12]

```
F=405.709
Area= .130782;
Sigma=F/Area;
Tau=Sigma/2;
y=input('Steel, 1 or Aluminum, 2:  ');
if y==1
Taumax=63100;
FOS= Taumax/Tau
else
Taumax=40000;
FOS= Taumax/Tau
end
```

Figure 7b

Table 6: Maximum allowable shear stress with F.O.S.

Material	**6061 T6 Aluminum**	**4130 Chromoly Steel**
Max Shear seen by Frame (PSI)	1550	1550
Factor of Safety	25.7883	40.6811

Conclusion:

After reviewing our analysis, we found that all of the components in the assembly have large safety factors. The frame's components are designed with rider safety in mind. Engineers have designed the tubing of the frame with a larger than necessary wall thickness as well as double or triple butted the joints in order to withstand extreme impact forces. Through complex evaluation and risk assessment the engineers, designing the frame, were able to make a compromise between strength and weight without putting the rider at risk or making the frame to heavy to compete with other frames.

In cases where riders are not as concerned with the longevity of their bike's frame, like professional riders, frame design takes a heavier consideration towards weight savings. Since the members in compression in the frame have very large factors of safety, they could be reduced. The easiest way to do this is by increasing the inner diameter of the top tube, seat tube, and seat stay. This would be reduce wall thickness therefore reducing the weight at the cost of frame strength. The rear dropout also has a very large factor of safety, the cross sectional area could easily be reduced without putting the rider at risk. It should be noted that these parameters should only be varied a small amount as it is crucial to have a high factor of safety for the frame.

In cases where riders are not concerned with weight, such as aggressive street riding, riders look for a high level of rigidity in their frames. It is important that the frame experiences little deformation in order to provide a precise and responsive bicycle. Unlike road riding, in most situations it is ideal for the rider to be able to feel all trail inputs through the frame. For this is a reason, the safety factor should remain very high. The more rigid the members of the frame are, the less the frame will deform when a load is applied.

Dirt jump mountain bikes have been continuously optimized since they were created. Early era dirt jumpers were simple, heavy under designed bicycles. As popularity in the discipline of cycling increased so did the market for the frames. Engineers began analyzing and redesigning these frames. By varying parameters such as the cross sectional area of the tubing (wall thickness), both material costs and frame weight can be reduced. This will not only save money in production, it will allow bicycle companies to provide customers with a higher quality bicycle frame at a more competitive price.

Works Cited

[1] ASM Inc. “AISI 4130 Steel .” *ASM Material Data Sheet*, asm.matweb.com/search/SpecificMaterial.asp?bassnum=m4130r.

[2] ASM Inc. “Aluminum 6061-T6.” *ASM Material Data Sheet*, asm.matweb.com/search/SpecificMaterial.asp?bassnum=ma6061t6.

[3] Davies, Sam. “Matt Macduff at Pure Darkness Step Up - Photo .” *Mountain Bike News, Photos, Videos and Events - Pinkbike*, 2 June 2015, www.pinkbike.com/photo/12299976/.

[4] Deity Company. “DEITY ::: CRYPTKEEPER DJ FRAME.” *DEITY | Premium Race Goods*, 2017, www.deitycomponents.com/cryptkeeper_frame.html.

[5] Juvinall, Robert C., and Kurt M. Marshek. *Fundamentals of Machine Component Design*. 6th ed., John Wiley & Sons, 2017.

[6] Lehner, Laurin. “Looping-Weltrekord: MacDuff Stürzt [Video].” *BIKE-MAGAZIN.de*, 28 Apr. 2016, www.bike-magazin.de/matt-macduff-sturz-beim-looping-weltrekord/a31465.html.

[7] Novotny, John. “Loading Analysis.” Design of Elements . Design of Elements , Feb. 2018, Burlington, VT.

[8] Novotny, John. “Chapter 2-5: Review.” Design of Elements . Design of Elements , Feb. 2018, Burlington, VT.

[9] Novotny, John. “Failure Theories.” Design of Elements . Design of Elements , Feb. 2018, Burlington, VT.

[10] Novotny, John. “Impact.” Design of Elements . Design of Elements , Feb. 2018, Burlington, VT.

[11] Novotny, John. “Fatigue.” Design of Elements . Design of Elements , Feb. 2018, Burlington, VT.

[12] Smith, Anthony. “First Impressions: Deity Cryptkeeper.” *Bike Magazine*, 15 Sept. 2012,