Graphs

10.1 Graphs and Graph Models

DEFINITION 1

A *graph* $G = (V, E)$ consists of V, a nonempty set of *vertices* (or *nodes*) and E, a set of *edges*. Each edge has either one or two vertices associated with it, called its *endpoints*. An edge is said to *connect* its endpoints.

A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a *simple graph*. Note that in a simple graph,

each edge is associated to an unordered pair of vertices, and

no other edge is associated to this same edge. Consequently, when there is an edge of a simple graph associated to {u, v}, we can also say, without possible confusion, that {u, v} is an edge of the graph.

A computer network may contain multiple links between data centers, as shown in Figure 2.

To model such networks we need graphs that have more than one edge connecting the same pair of vertices.

Graphs that may have multiple edges connecting the same vertices are called *multigraphs*.

Sometimes a communications link connects a data center with itself, perhaps a feedback loop for diagnostic purposes. Such a network is illustrated in Figure 3. To model this network we need to include edges that connect a vertex to itself. Such edges are called loops, and sometimes we may even have more than one loop at a vertex. Graphs that may include loops, and possibly multiple edges connecting the same pair of vertices or a vertex to itself, are sometimes called *pseudographs*.

DEFINITION 2

A *directed graph* (or *digraph*) (V, E) consists of a nonempty set of vertices V and a set of *directed edges* (or *arcs*) E. Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair (u, v) is said to *start* at u and *end* at v.

TABLE 1 Graph Terminology.

Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

Because of the relatively modern interest in graph theory, and because it has applications to a wide variety of disciplines, many different terminologies of graph theory have been introduced. The reader should determine how such terms are being used whenever they are encountered. The terminology used by mathematicians to describe graphs has been increasingly standardized, but the terminology used to discuss graphs when they are used in other disciplines is still quite varied. Although the terminology used to describe graphs may vary, three key questions can help us understand the structure of a graph:

- Are the edges of the graph undirected or directed (or both)?
- If the graph is undirected, are multiple edges present that connect the same pair of vertices? If the graph is directed, are multiple directed edges present?
- Are loops present?

Answering such questions helps us understand graphs. It is less important to remember the particular terminology used.

10.2 Graph Terminology and Special Types of Graphs

Basic Terminology (undirected graphs)

Two vertices u and v in an undirected graph G are called *adjacent* (or *neighbors*) in G if u and v are endpoints of an edge e of G. Such an edge e is called *incident with* the vertices u and v and e is said to *connect* u and v.

The set of all neighbors of a vertex v of $G = (V, E)$, denoted by $N(v)$, is called the *neighborhood* of v. If A is a subset of V, we denote by $N(A)$ the set of all vertices in G that are adjacent to at least one vertex in A. So, $N(A) = \bigcup_{v \in A} N(v)$.

The *degree of a vertex in an undirected graph* is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex v is denoted by $\deg(v)$.

THE HANDSHAKING THEOREM Let $G = (V, E)$ be an undirected graph with m edges. Then

$$2m = \sum_{v \in V} \deg(v).$$

(Note that this applies even if multiple edges and loops are present.)

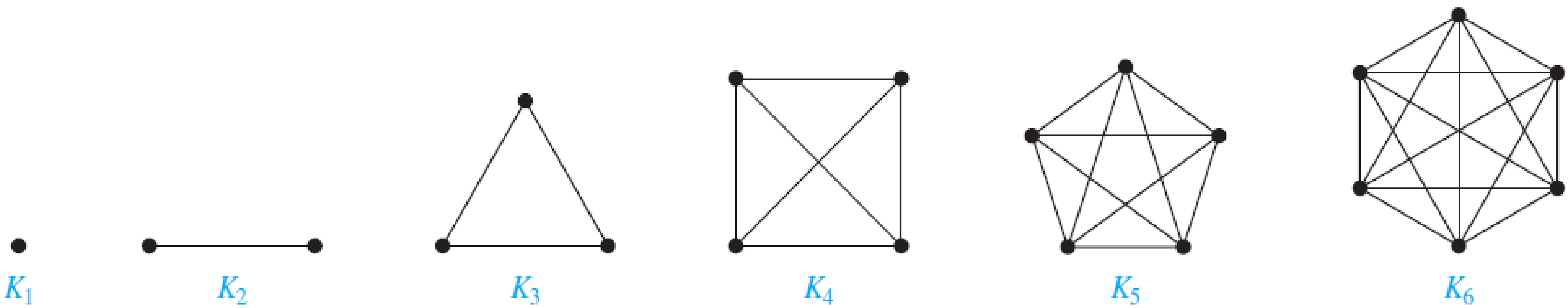
An undirected graph has an even number of vertices of odd degree.

When (u, v) is an edge of the graph G with directed edges, u is said to be *adjacent to* v and v is said to be *adjacent from* u. The vertex u is called the *initial vertex* of (u, v), and v is called the *terminal* or *end vertex* of (u, v). The initial vertex and terminal vertex of a loop are the same.

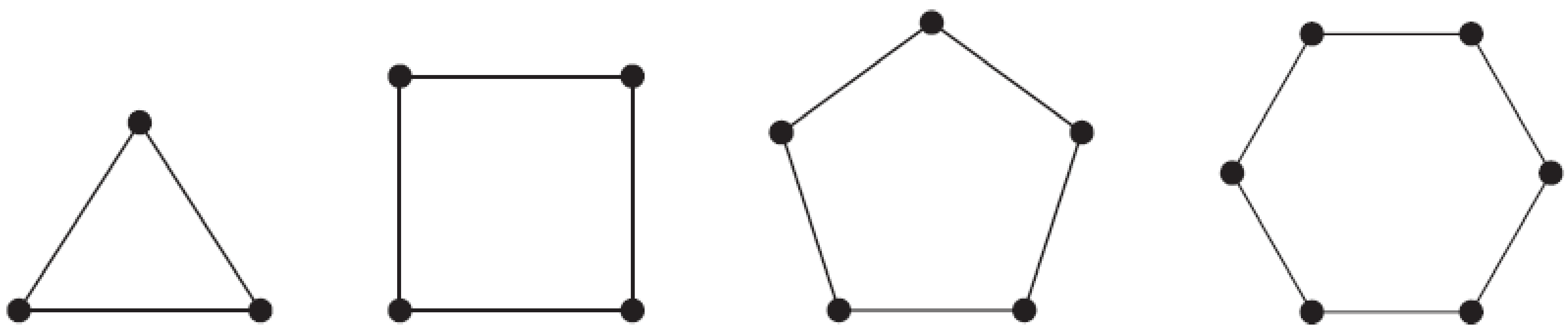
In a graph with directed edges the *in-degree of a vertex* v, denoted by $\deg^-(v)$, is the number of edges with v as their terminal vertex. The *out-degree of* v, denoted by $\deg^+(v)$, is the number of edges with v as their initial vertex. (Note that a loop at a vertex contributes 1 to both the in-degree and the out-degree of this vertex.)

Some Special Simple Graphs

Complete Graphs

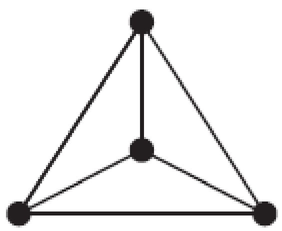


Cycles

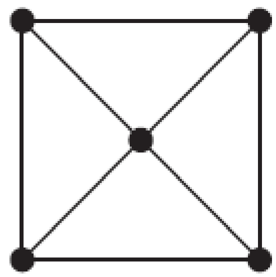


C_3 C_4 C_5 C_6

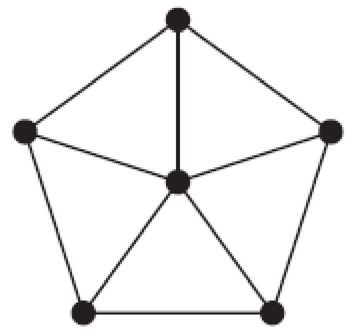
Wheels



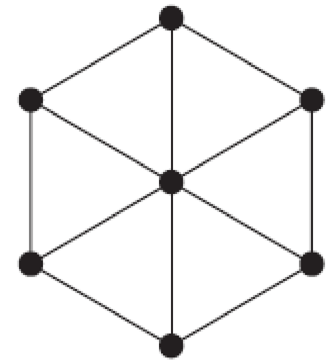
W_3



W_4

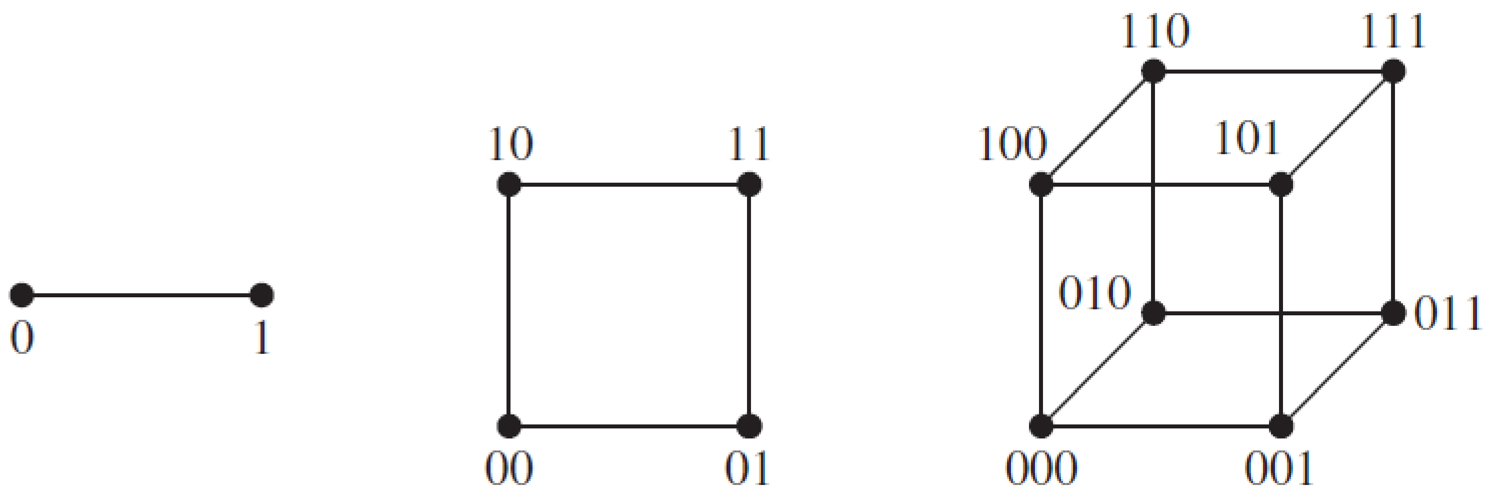


W_5



W_6

n-dimensional hypercube

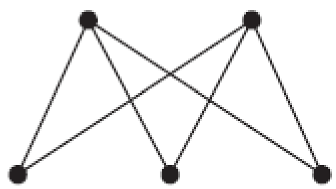


Q_1 Q_2 Q_3

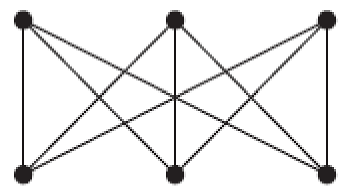
A simple graph G is called *bipartite* if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2). When this condition holds, we call the pair (V_1, V_2) a *bipartition* of the vertex set V of G.

A simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color.

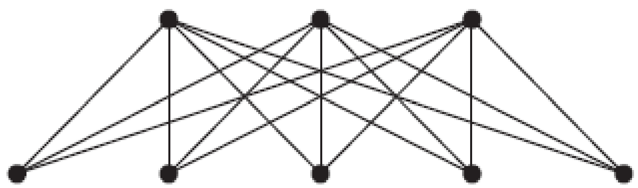
Complete Bipartite Graphs



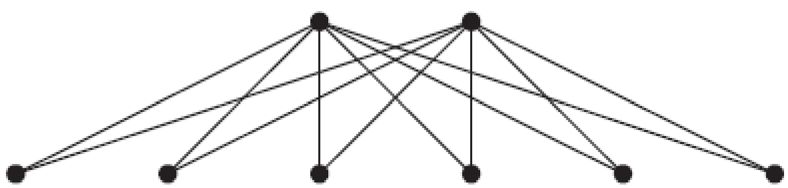
$K_{2,3}$



$K_{3,3}$



$K_{3,5}$



$K_{2,6}$

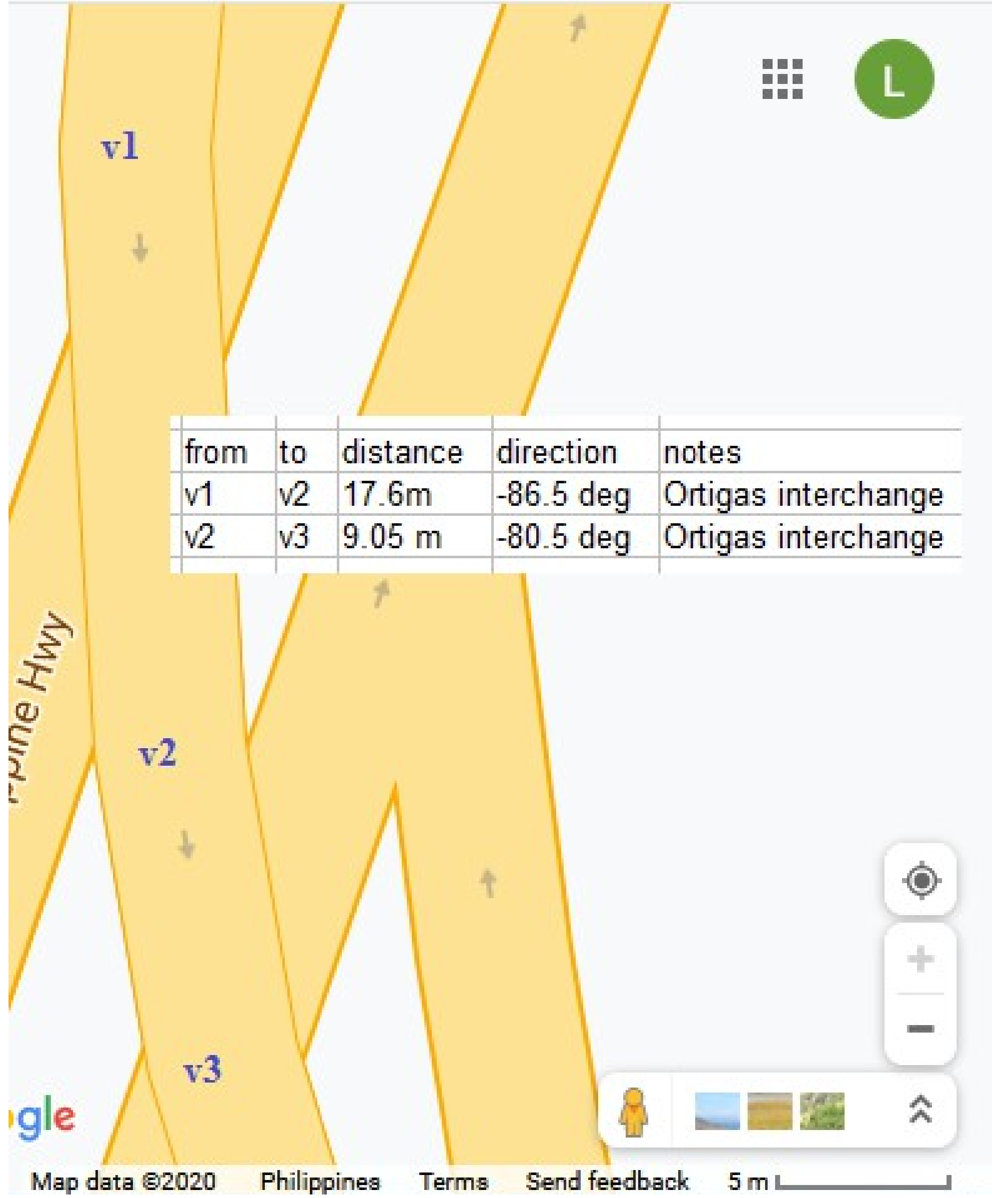
Application: Internal Representation of Maps

curves approximated by straight segments

nodes/vertices marked
v1, v2, v3

distances, direction from node to node must be stored internally so that rendering software could reproduce map

ortigas-interchange.odt/pdf

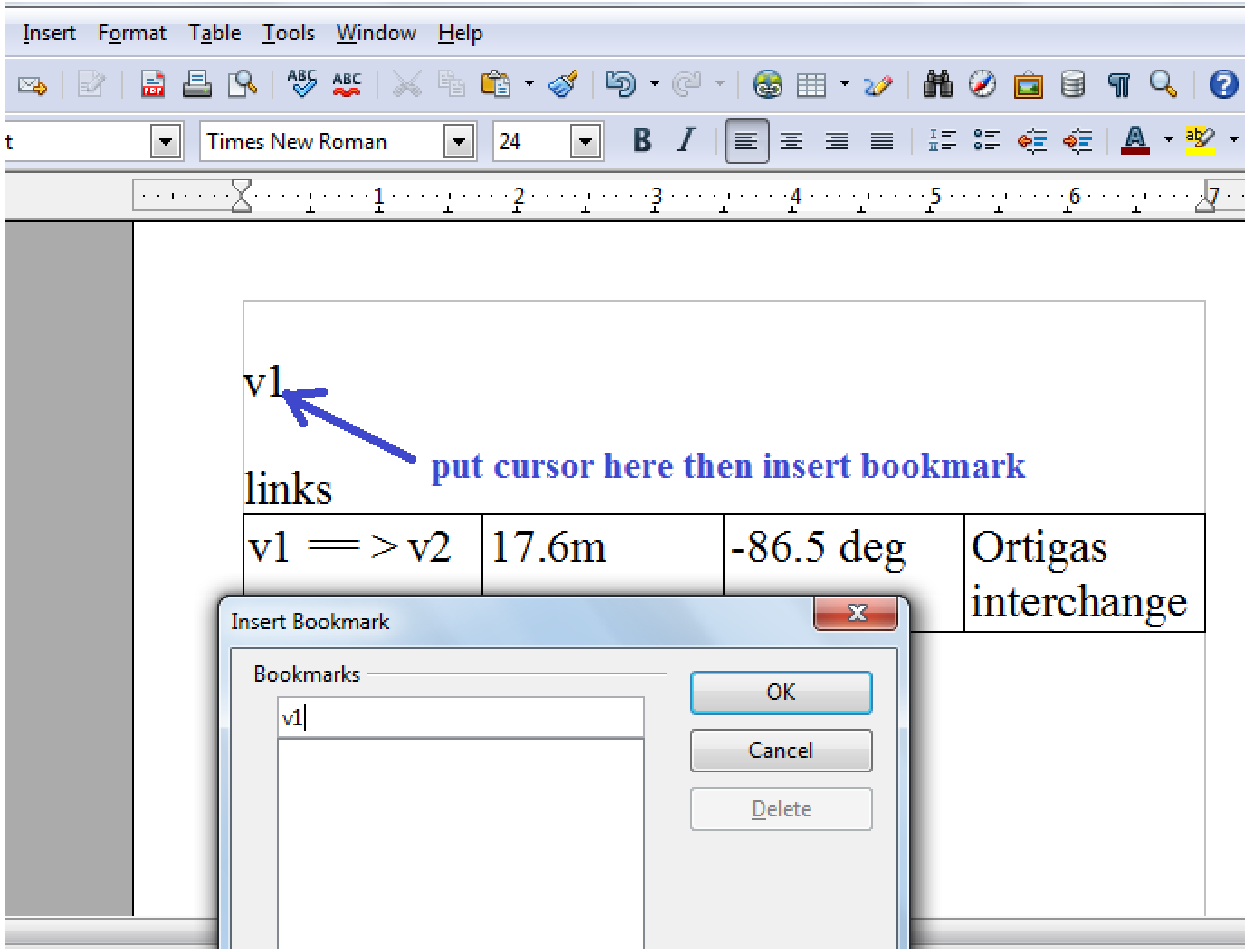


from	to	distance	direction	notes
v1	v2	17.6m	-86.5 deg	Ortigas interchange
v2	v3	9.05 m	-80.5 deg	Ortigas interchange

image from Google Maps

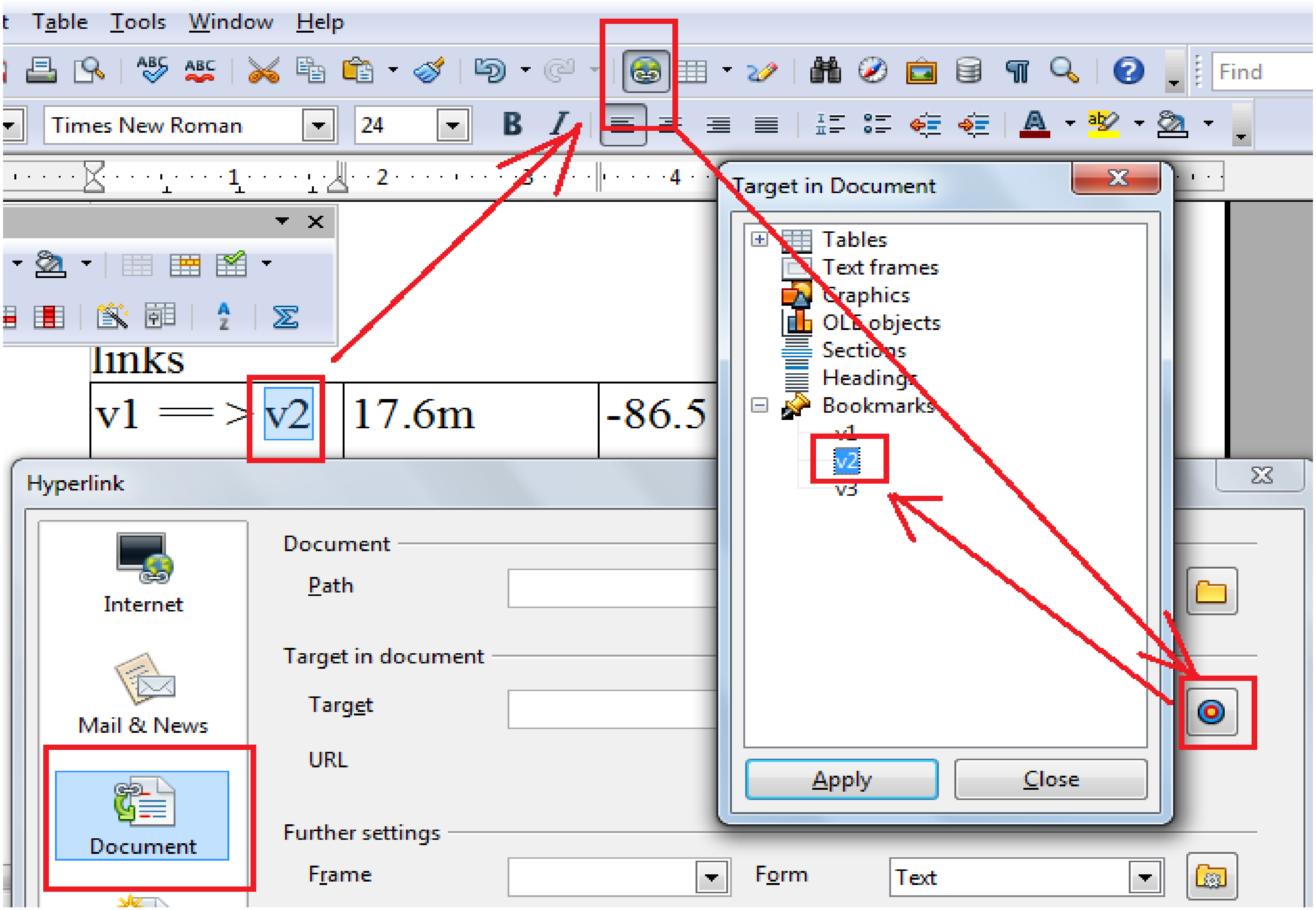
Bookmarks are the nodes.

Bookmarks are what actually identify the nodes.



Insert bookmarks v2, v3 similarly.

To link to a node, insert a hyperlink to its bookmark.



v1

links

v1 == > v2 v1 ==> v4 ? ...	17.6m	-86.5 deg	Ortigas interchange

This table will list all links from v1. There's only one because of the nature of the flyover.

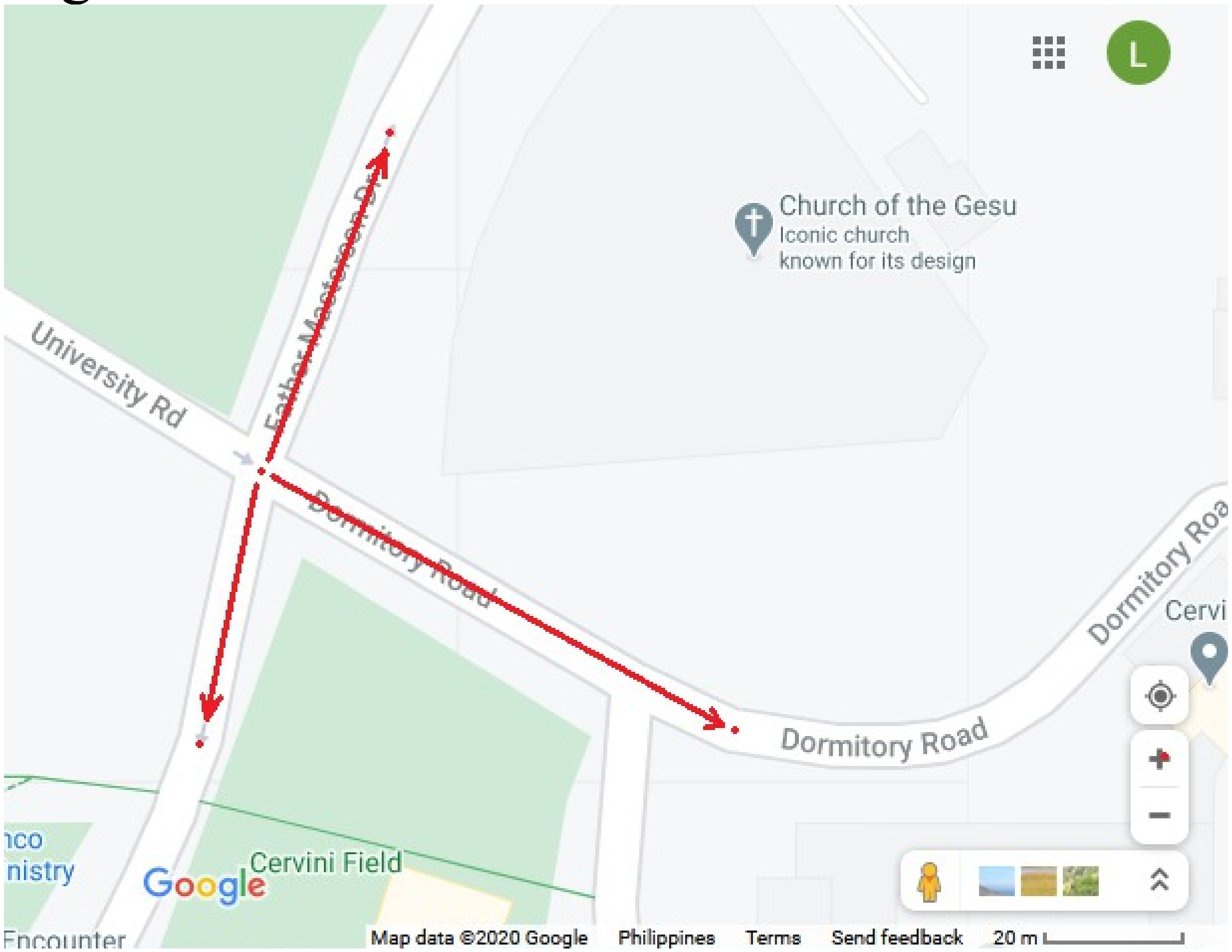
v2

links

v2 == > v3	9.05 m	-80.5 deg	Ortigas interchange

v3

A node linking to 3 nodes



from Google Maps

ENGG 24 A-Q1
2020-1
Project 2, Option 1: **Map Representation**

Due at end of class time, Monday, October 19, 2020.

General Specifications

Construct a hyperlinked document, with at least 20 nodes, representing a map of the Ateneo de Manila University Loyola Heights Campus, or a portion of it.

Detailed Specifications

This project may be worked on by groups.

Construct a hyperlinked document with at least 20 nodes representing a map of the Ateneo de Manila University Loyola Heights Campus, or a portion of it.

Obtain a map of the Ateneo de Manila University Loyola Heights Campus and label at least 20 nodes in the map or a part of it such that the nodes are linked by segments on the map and there is at least one node from which all other nodes can be reached either directly or thru other nodes.

Do not include any nodes on Katipunan Ave.

Use one page in the document for each node on the map. A page representing a node must include hyperlinks to all nodes directly reachable from the node as well as information regarding the distance to the node and the direction to be taken.

Distances are to be specified in meters. Directions are to be specified in degrees with the eastward direction as reference, with positive angles representing directions counterclockwise toward the north and negative angles representing directions clockwise toward the south.

Have a page on the document showing the location of the nodes on a map of the campus (or a portion of it). This page must contain direct hyperlinks to each of the nodes. This page must come before the nodes of the map.

Export your final document to pdf.

Submission

This project is due at end of class time, Monday, October 19, 2020.

Submission procedure:

Your main document should be in pdf format. Include supporting documents as needed.

Collect all files for submission into a single location. Ensure that the files are complete and are exactly what should be submitted. Do not include any other files. Submit a single zip file containing all files for submission.

Do NOT include executables in your submission.

Next synchronous session: Oct. 9

Study Graphs (Chap.10)

Homework #5: Project Proposal

[10 minor points]

Come up with a project proposal involving graphs. This should either be a project that might be considered as an option to Project 2, or might be considered as an option for Project 3. Come up with general specifications. Explain why you think the project is worth doing and how it is consistent with your learning objectives.

Due: end of class time, Oct. 7