

Planck constraints on the tensor-to-scalar ratio

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Abstract

We present constraints on the tensor-to-scalar ratio r using *Planck* data. We use the latest release of *Planck* maps (PR4), processed with the NPIPE code, which produces calibrated frequency maps in temperature and polarization for all *Planck* channels from 30 GHz to 857 GHz using the same pipeline. We compute constraints on r using the *BB* angular power spectrum, and also discuss constraints coming from the *TT* spectrum. Given *Planck*'s noise level, the *TT* spectrum gives constraints on r that are cosmic-variance limited (with $\sigma_r = 0.093$), but we show that the marginalized posterior peaks toward negative values of r at about the 1.2σ level. We derive *Planck* constraints using the *BB* power spectrum at both large angular scales (the “reionization bump”) and intermediate angular scales (the “recombination bump”) from $\ell = 2$ to 150, and find a stronger constraint than that from *TT*, with $\sigma_r = 0.069$. The *Planck BB* spectrum shows no systematic bias, and is compatible with zero, given both the statistical noise and the systematic uncertainties. The likelihood analysis using *B* modes yields the constraint $r < 0.158$ at 95 % confidence using more than 50 % of the sky. This upper limit tightens to $r < 0.069$ when *Planck EE*, *BB*, and *EB* power spectra are combined consistently, and tightens further to $r < 0.056$ when the *Planck TT* power spectrum is included in the combination. Finally, combining *Planck* with BICEP2/Keck 2015 data yields an upper limit of $r < 0.044$.

Key words. cosmology: observations – cosmic background radiation – cosmological parameters – gravitational waves – methods: data analysis

1. Introduction

Gravitational waves entering the horizon between the epoch of recombination and the present day generate a tensor contribution to the large-scale cosmic microwave background (CMB) anisotropy. Hence, primordial tensor fluctuations contribute to the CMB anisotropies, both in temperature (T) and in polarization (E and B modes).

As described in [Planck Collaboration VI \(2020\)](#) and [Planck Collaboration X \(2020\)](#), the comoving wavenumbers of tensor modes probed by the CMB temperature anisotropy power spectrum have $k \lesssim 0.008 \text{ Mpc}^{-1}$, with very little sensitivity to higher wavenumbers because gravitational waves decay on sub-horizon scales. The corresponding multipoles in the harmonic domain are $\ell \lesssim 100$, for which, in temperature, the scalar perturbations dominate with respect to tensor modes. The tensor component can be fitted together with the scalar one, and the precision of the *Planck* constraint is limited by the cosmic variance of the large-scale anisotropies.

In polarization, the *EE* and *TE* spectra also contain a tensor signal coming from the last-scattering and reionization epochs.

However, the addition of *Planck* polarization constraints at $\ell \gtrsim 30$ does not significantly change the results coming from temperature and low- ℓ polarization data (see [Planck Collaboration XIII 2016](#)). *BB* power spectra are treated differently in determining the tensor contribution, since the model does not predict any primordial scalar fluctuations in *BB*. As a consequence, a primordial *B*-mode signal would be a direct signature of tensor modes. However, depending on the amplitude of the tensor-to-scalar ratio, such a signal may be masked by *E*-mode power transformed to *B*-mode power through lensing by gravitational potentials along the line of sight (so-called “*BB* lensing”). *BB* lensing has been measured with high accuracy by *Planck* in both harmonic ([Planck Collaboration VIII 2020](#)) and map ([Planck Collaboration Int. XLI 2016](#)) domains. But a primordial *BB* tensor signal has not yet been detected.

The scalar and tensor CMB angular power spectra are plotted in [Fig. 1](#) for the *Planck* 2018 cosmology and for two values of the tensor-to-scalar ratio, namely $r = 0.1$ and $r = 0.01$. For further discussion of the tensor-to-scalar ratio and its implications for inflationary models, see [Planck Collaboration XXII \(2014\)](#), [Planck Collaboration XX \(2016\)](#), and [Planck Collaboration X](#)

(2020). Note that the signal from tensor modes in EE is similar to that in BB modes, which makes EE (in particular at low multipoles) an important data set for tensor constraints. In this paper, we will make use of a polarized E - B likelihood, which consistently includes the correlated polarization fields E and B , and covers the range of multipoles where tensor modes can be constrained using *Planck* data (i.e., from $\ell = 2$ to $\ell = 150$).

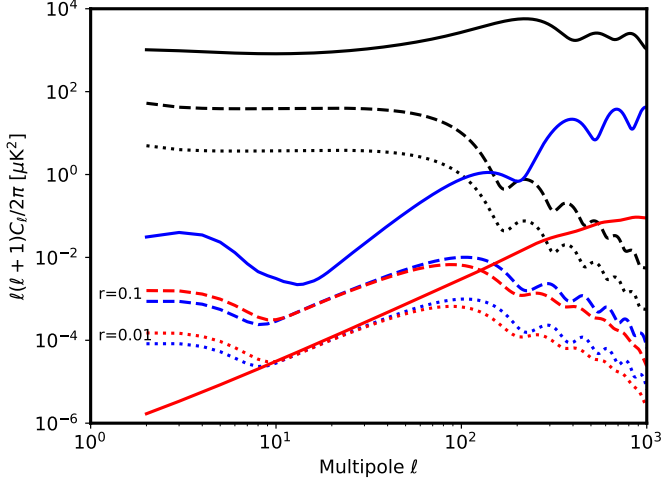


Figure 1: Scalar (thick solid lines) versus tensor spectra for $r = 0.1$ (dashed lines) and $r = 0.01$ (dotted lines). Spectra for TT are in black, EE in blue, and BB in red. The red solid line corresponds to the signal from BB lensing.

At present the tightest B -mode constraints on r come from the BICEP/Keck measurements (BK15; BICEP2 Collaboration 2018), which cover approximately 400 deg^2 centred on RA = 0^h , Dec = -57.5° . These measurements probe the peak of the B -mode power spectrum at around $\ell = 100$, corresponding to gravitational waves with $k \approx 0.01 \text{ Mpc}^{-1}$ that enter the horizon during recombination (i.e., somewhat smaller than the scales contributing to the *Planck* temperature constraints on r). The results of BK15 give a limit of $r < 0.07$ at 95 % confidence, which tightens to $r < 0.06$ in combination with *Planck* temperature and other data sets.

Planck Collaboration V (2020) presented *Planck* B -mode constraints from the 100- and 143-GHz HFI channels with a 95 % upper limit of $r < 0.41$ (at a somewhat larger pivot scale, as described in the next section), using only a limited number of multipoles around the so-called “reionization bump” ($2 \leq \ell \leq 29$). Using *Planck* NPIPE maps (Planck Collaboration Int. LVII 2020), called *Planck* Release 4 (PR4), we are now able to constrain the BB power spectrum for a much larger number of modes, including both the reionization bump at large angular scales ($\ell \lesssim 30$) and the so-called “recombination bump” at intermediate scales ($50 \lesssim \ell \lesssim 150$). In this paper, we first describe, in Sect. 2, the cosmological model used throughout the analysis. We then detail the data and the likelihoods in Sect. 3. Section 4 focuses on constraints from TT and in particular the impact of the low- ℓ data in temperature. Section 5 gives constraints from the BB angular power spectrum using *Planck* data, while results from the full set of polarization power spectra are given in Sect. 6. Finally, in Sect. 7, we combine all data sets to provide the most robust constraints on BB coming from *Planck* and in combination with other CMB data sets, such as the results from the BICEP/Keck Collaboration.

2. Cosmological model

We use the base- Λ CDM model, which has been established over the last couple of decades to be the simplest viable cosmological model, in particular with the *Planck* results (e.g., Planck Collaboration VI 2020). In this model, we assume purely adiabatic, scale-invariant perturbations at very early times, with curvature-mode (scalar) and tensor-mode power spectra parameterized by

$$\mathcal{P}_s(k) = A_s \left(\frac{k}{k_0} \right)^{n_s-1}, \quad (1)$$

$$\mathcal{P}_t(k) = A_t \left(\frac{k}{k_0} \right)^{n_t}, \quad (2)$$

where A_s and A_t are the initial super-horizon amplitudes for curvature and tensor perturbations, respectively. The primordial spectral indexes for scalar (n_s) and tensor (n_t) perturbations are taken to be constant. This means that we assume no “running,” i.e., a pure power-law spectrum with $dn_s/d \ln k = 0$. We set the pivot scale at $k_0 = 0.05 \text{ Mpc}^{-1}$, which roughly corresponds to approximately the middle of the logarithmic range of scales probed by *Planck*; with this choice, n_s is not strongly degenerate with the amplitude parameter A_s . Note that for historical reasons, the definitions of n_s and n_t differ, so that a scale-invariant scalar spectrum corresponds to $n_s = 1$, while a scale-invariant tensor spectrum corresponds to $n_t = 0$.

The late-time parameters, on the other hand, determine the linear evolution of perturbations after they re-enter the Hubble radius. We use the basis ($\Omega_b, \Omega_c, \theta_*, \tau$) following the approach in *Planck* cosmological studies (Planck Collaboration VI 2020), where Ω_b is the density parameter of baryons, Ω_c is the density parameter of cold dark matter, θ_* is the observed angular size of the sound horizon at recombination, and τ is the reionization optical depth.

The amplitude of the small-scale linear CMB power spectrum is proportional to $A_s e^{-2\tau}$. Because *Planck* measures this amplitude very accurately, there is a tight linear constraint between τ and $\ln A_s$. For this reason, we usually adopt $\ln A_s$ as a base parameter with a flat prior; $\ln A_s$ has a significantly more Gaussian posterior than A_s . A linear parameter redefinition then allows the degeneracy between τ and A_s to be explored efficiently. Note that the degeneracy between τ and A_s is broken by the relative amplitudes of large-scale temperature and polarization CMB anisotropies and by the effect of CMB lensing.

We define $r \equiv A_t/A_s$, the primordial tensor-to-scalar ratio defined explicitly at the scale $k_0 = 0.05 \text{ Mpc}^{-1}$. Our constraints are only weakly sensitive to the tensor spectral index, n_t (which is assumed to be close to zero). Note that the *Planck* Collaboration also discussed r constraints for $k_0 = 0.002 \text{ Mpc}^{-1}$ (Planck Collaboration V 2020). Given the definitions in Eqs. (1) and (2), the relation scales like $(0.05/0.002)^{-r/8}$, which means that $r_{0.002}$ is lower by 4 % at $r \approx 0.1$ compared to $r_{0.05}$ and less than 0.4 % lower for $r < 0.01$.

In this work, we will use an effective tensor-to-scalar ratio r_{eff} , which we extend into the negative domain by modifying the Boltzmann-solver code CLASS¹ (Blas et al. 2011). While negative tensor amplitudes are unphysical, this approach will allow us to derive posteriors without boundaries, facilitating detection of potential biases, and enabling us to determine a more accurate statistical definition of the constraints on r . With r_{eff} we will be able to independently discuss both the uncertainty of r (σ_r)

¹ <http://class-code.net>

and corresponding upper limits (depending on the maximum a posteriori probability). In the rest of this paper, we will simply write r as the effective tensor-to-scalar ratio, and report upper-limits for positive tensor amplitudes, for which $r_{\text{eff}} = r$. We use 95 % confidence levels when reporting upper limits, and a 68 % confidence interval with the maximum a posteriori probability.

3. Data and likelihoods

3.1. Data and simulations

The sky measurements used in this analysis are the PR4 maps available from the Planck Legacy Archive² (PLA). They have been produced with the NPIPE processing pipeline, which creates calibrated frequency maps in temperature and polarization from the *Planck* Low Frequency Instrument (LFI) and High Frequency Instrument (HFI) data. As described in Planck Collaboration Int. LVII (2020), NPIPE processing includes several improvements, resulting in lower levels of noise and systematics in both frequency and component-separated maps at essentially all angular scales, as well as notably improved internal consistency between the various frequencies.

NPIPE achieves an overall lower noise level in part by incorporating the data acquired during the 4-minute spacecraft re-pointing manoeuvres that take place between the 30-to-70-min stable science scans. Residual systematics are suppressed using a mitigation strategy that combines aspects of both LFI and HFI processing pipelines. Most importantly, gain fluctuations, bandpass mismatch, and other systematics are formulated into time-domain templates that are fitted and subtracted as a part of the mapmaking process. Degeneracies between sky polarization and systematic templates are broken by constructing a prior of the polarized foreground sky using the extreme polarization-sensitive frequencies (30, 217, and 353 GHz).

Moreover, the PR4 release comes with 400 simulations of signal, noise, and systematics, component-separated into CMB maps, which allow for an accurate characterization of the noise and systematic residuals in the *Planck* maps. This is important because *Planck* polarization data are cosmic-variance-dominated only for a few multipoles at very large scales in *EE* ($\ell < 8$, as shown in Fig. 2). These simulations, even though limited in number, represent a huge effort in terms of CPU time. They are essential in order to compute the following two additional quantities.

1. The end-to-end transfer function from the data reduction (including TOI processing, mapmaking and template fitting for mitigation of systematics, component separation, and power-spectrum estimation). The transfer function is defined as the ratio between the output and the input, averaged over all the simulations (see section 4.3 of Planck Collaboration Int. LVII 2020, for the details).
2. The covariance of the data (here we will use the cross-power spectra), which is the only way to propagate uncertainties when those are dominated by systematics (from the instrument or from foregrounds).

Note that these two quantities estimated from the simulations are directly related to two different characteristics of the final parameter posteriors: the bias of the mean (the transfer function); and the width of the posterior (as propagated into parameter constraints by the covariance matrix in the likelihood). They can be separated from each other, meaning that one systematic effect

can easily produce a significant bias without any strong impact on the variance, while another effect can produce a large increase of the variance with no associated bias.

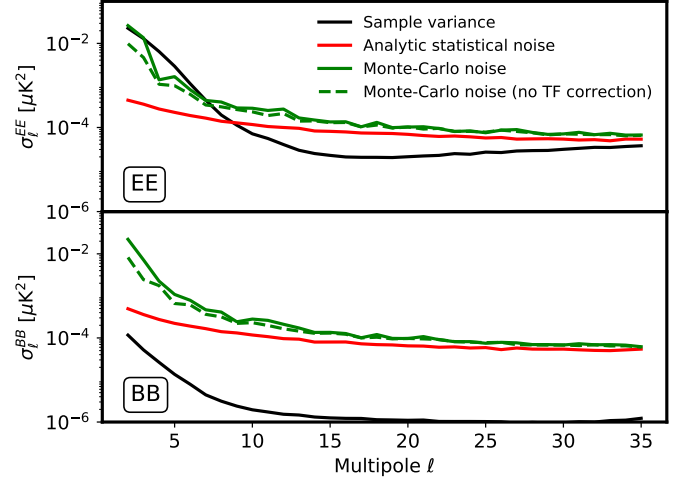


Figure 2: Variances for cross-spectra in *EE* and *BB* based on PR4 simulations, including: cosmic (sample) variance (black); analytic statistical noise (red); and PR4 noise from Monte Carlo simulations (green), including noise and systematics with (solid line) or without (dashed line) correction for the transfer function. The sky fraction used here is 80 %, as it illustrates well the effect of both systematics and transfer-function corrections (Planck Collaboration Int. LVII 2020).

The NPIPE simulations include the systematic effects relevant for polarization studies, specifically analogue-to-digital-converter nonlinearities, gain fluctuations, bandpass mismatch between detectors, correlated noise (including 4-K line residuals), and full-beam convolutions for each detector.

The use of a polarization prior in NPIPE processing causes a suppression of large-scale ($\ell < 20$) CMB polarization, which needs to be corrected. As explained in Planck Collaboration Int. LVII (2020), allowing for a non-trivial transfer function is a compromise between measuring very noisy but unbiased large-scale polarization from all low- ℓ modes, and filtering out the modes that are most affected by the calibration uncertainties left in the data by the *Planck* scan strategy. As detailed in Planck Collaboration Int. LVII (2020), the transfer function to correct for this bias is determined from simulations. It is then used to correct the power spectrum estimates, just as instrumental beam and pixel effects must be deconvolved. Due to the fact that *E* modes dominate the CMB polarization, the simulations do not yield a definitive measurement of the *B*-mode transfer function. We have nevertheless chosen to conservatively deconvolve the *E*-mode transfer function from the *B*-mode spectrum in order to provide a robust upper limit on the true *B*-mode spectrum. Indeed, in the situation where primordial *B*-mode power is not detected, the transfer function correction essentially increases the variance estimate at low multipoles, which propagates the uncertainty induced by the degeneracy between the sky and the systematic templates used in NPIPE. Note that this uncertainty is small compared to the impact of systematics in the error budget (see Fig. 2).

To compute unbiased estimates of the angular power spectra, we perform cross-correlations of two independent splits of the data. As shown in Planck Collaboration Int. LVII (2020), the

² <https://pla.esac.esa.int>

most appropriate split for the *Planck* data is represented by the detector-set (hereafter “detset”) maps, comprising two subsets of maps at each frequency, with nearly independent noise characteristics, made by combining half of the detectors. Note that time-split maps (made from, e.g., “odd-even rings” or “half-mission data”) share the same instrumental detectors, and therefore exhibit noise correlations due to identical spectral band-passes and optical responses. Using time-split maps would result in systematic biases in the cross-power spectra, as well as underestimation of the noise properties when computing the half-differences.

Uncertainties at the power-spectrum level are dominated by noise and systematics, as illustrated in Fig. 2. Thanks to the NPIPE processing, we are now able to show the impact of the systematics at low ℓ . This is illustrated by comparing the PR4 end-to-end noise (based on the Monte Carlo simulations, including instrumental noise, systematics, and foreground uncertainties, and corrected for the transfer function both in *EE* and *BB*) with the propagation of the statistical noise coming from the analytic pixel-pixel covariance matrix. The systematic uncertainties dominate at $\ell \lesssim 15$, then slowly decrease so that the effective uncertainties converge towards the analytic estimate at higher multipoles.

3.2. Polarized sky masks

Foreground residuals in the foreground-cleaned maps dominate the polarized CMB signal near the Galactic plane. To avoid contamination from these residuals in the cosmological analysis, we mask the Galactic plane. We use a series of different retained sky fractions (from 30 % to 70 %) to check the consistency of our results with respect to foreground residuals (Fig. 3).

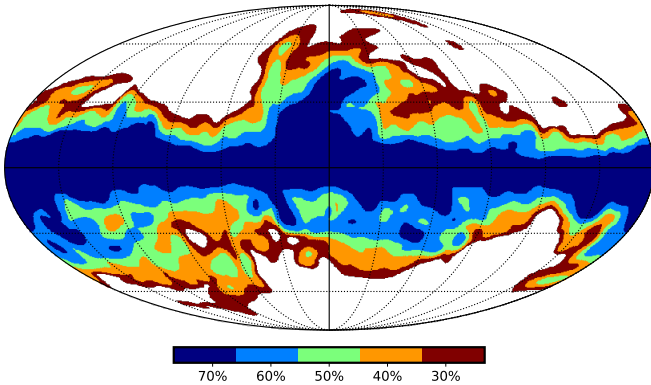


Figure 3: Galactic masks used for the *Planck* likelihoods. The mask shown in dark blue indicates the sky rejected in order to retain a 70 % sky fraction for analysis. The masks shown in light blue, green, orange and red incrementally omit further parts of the sky, corresponding in turn to 60, 50, 40 and 30 % retained sky fractions, the latter shown in white.

The masks used in this analysis are a combination of a mask for polarization intensity (to avoid polarized foreground residuals), a mask for total intensity (to avoid potential temperature-to-polarization leakage residuals), and the confidence mask for component separation provided by the *Planck* Collaboration. The intensity mask is obtained by thresholding the 353-GHz intensity map (which traces dust) scaled to 143 GHz, and the 30-GHz intensity map (which traces synchrotron) scaled to 100 GHz. The polarization map is constructed similarly. Both foreground trac-

ers are smoothed beforehand with a 10° Gaussian window function.

The impact of the emission of extragalactic polarized sources on the power spectra is negligible, given the *Planck* resolution and noise level. The confidence mask for component separation ensures the masking of the strongest sources, which could also produce residuals through temperature-to-polarization leakage.

3.3. Likelihoods

Table 1 summarizes the likelihoods used in this analysis, which are described below.

3.3.1. Low- ℓ temperature likelihood

We use the *Planck* public low- ℓ temperature-only likelihood based on the PR3 CMB map recovered from the component-separation procedure (specifically *Commander*) described in detail in *Planck Collaboration V* (2020). At large angular scales, *Planck* temperature maps are strongly signal-dominated, and there is no expected gain in updating this likelihood with the PR4 data.

As discussed in *Planck Collaboration XX* (2016), the low- ℓ temperature data from *Planck* have a strong impact on the r posterior and the derivation of the corresponding constraints. This is because the deficit of power in the measured C_ℓ s at low- ℓ in temperature (see the discussions in *Planck Collaboration XVI* 2014 and *Planck Collaboration Int. LI* 2017) lowers the probability of tensor models, which *add* power at low multipoles. This shifts the maximum in the posterior of r toward low values (or even negative values when using r_{eff} , as we will show in Sect. 4).

3.3.2. High- ℓ likelihood

At small angular scales ($\ell > 30$), we use the HiLLiPoP likelihood, which can include the TT , TE , and/or EE power spectra. HiLLiPoP has been used as an alternative to the public *Planck* likelihood in the 2013 and 2015 *Planck* releases (*Planck Collaboration XV* 2014; *Planck Collaboration XI* 2016), and is described in detail in *Couchot et al.* (2017a). In this paper, the HiLLiPoP likelihood is applied to the PR4 detset maps at 100, 143, and 217 GHz. We focus on the TT spectra, since there is no additional information at small scales in TE or EE for tensor modes. We only make use of TE in Sect. 7 in order to help constrain the spectral index n_s . The likelihood is a spectrum-based Gaussian approximation, with semi-analytic estimates of the C_ℓ covariance matrix based on the data. The cross-spectra are debiased from the effects of the mask and the beam leakage using *Xpol* (a generalization to polarization of the algorithm presented in *Tristram et al.* 2005³) before being compared to the model, which includes CMB and foreground residuals. The beam window functions are evaluated using *QUICKPOL* (*Hivon et al.* 2017), adapted to the PR4 data. These adaptations include an evaluation of the beam-leakage effect, which couples temperature and polarization modes due to the beam mismatch between individual detectors.

The model consists of a linear combination of the CMB power spectrum and several foregrounds residuals. These are:

- Galactic dust (estimated directly from the 353-GHz channel);

³ <https://gitlab.in2p3.fr/tristram/Xpol>

Table 1: Summary of the likelihoods used in this paper.

| Name | Mode | ℓ range | <i>Planck</i> release | Description |
|-------------------|----------|--------------|-----------------------|--|
| lowT | TT | 2–30 | PR3 | Commander likelihood for Temperature |
| hlpTT | TT | 30–2500 | PR4 | HiLLiPoP likelihood for high- ℓ TT |
| hlpTTTE | TT+TE | 30–2500 | PR4 | HiLLiPoP likelihood for high- ℓ TT+TE |
| lowIE | EE | 2–150 | PR4 | LoLLiPoP likelihood for low- ℓ EE |
| lowIB | BB | 2–150 | PR4 | LoLLiPoP likelihood for low- ℓ BB |
| lowIEB | EE+BB+EB | 2–150 | PR4 | LoLLiPoP likelihood for low- ℓ EE+BB+EB |

- the cosmic infrared background (as measured in [Planck Collaboration XXX 2014](#));
- thermal Sunyaev-Zeldovich emission (based on the *Planck* measurement reported in [Planck Collaboration XXI 2014](#));
- kinetic Sunyaev-Zeldovich emission, including homogeneous and patchy reionization components from [Shaw et al. \(2012\)](#) and [Battaglia et al. \(2013\)](#);
- a tSZ-CIB correlation consistent with both models above; and
- unresolved point sources as a Poisson-like power spectrum with two components (extragalactic radio galaxies and infrared dusty galaxies).

On top of the cosmological parameters associated with the computation of the CMB spectrum, with HiLLiPoP we sample seven foreground amplitudes (one per emission source, the spectral energy density rescaling the amplitude for each cross-frequency being fixed) and six nuisance parameters (one overall calibration factor plus intercalibrations for each map). See Appendix A for more details.

3.3.3. Large-scale polarized likelihood

We construct a polarized *E-B* likelihood based on power spectra, focusing on the large scales where the tensor signal is dominant. Because it carries very little information about the tensor modes, we do not include the *TE* spectrum in this analysis.

In polarization, especially at large angular scales, foregrounds are stronger relative to the CMB than in temperature, and cleaning the *Planck* frequencies using C_ℓ templates in the likelihood (as done in temperature) is not accurate enough. In order to clean sky maps of polarized foregrounds, we use the Commander component-separation code ([Eriksen et al. 2008](#)), with a model that includes three polarized components, namely the CMB, synchrotron, and thermal dust emission. Commander was run on each detset map independently, as well as on each realization from the PR4 Monte Carlo simulations. Maps are available on the PLA in HEALPix⁴ format ([Górski et al. 2005](#)) at a resolution $N_{\text{side}} = 2048$.

To compute unbiased estimates of the angular power spectra, we calculate the cross-correlation of the two detset maps. We make use of two different angular cross-power spectra estimators (described below), which are then concatenated to produce a full-multipole-range power spectrum. There is no information loss in this process, since the covariances are deduced using Monte Carlo simulations including the correlations over the entire multipole range.

- For multipoles $2 \leq \ell \leq 35$, we compute power spectra using an extension of the quadratic maximum likelihood estimator ([Tegmark & de Oliveira-Costa 2001](#)) adapted for

cross-spectra in [Vanneste et al. \(2018\)](#).⁵ At multipoles below 40, it has been shown to produce unbiased polarized power spectra with almost optimal errors. We use downgraded $N_{\text{side}} = 16$ maps after convolution with a cosine apodizing kernel $b_\ell = \frac{1}{2} \left(1 + \cos \pi \frac{\ell-1}{3N_{\text{side}}-1} \right)$. The signal is then corrected with the PR4 transfer function, to compensate for the filtering induced by the degeneracies between the signal and the templates for systematics in the mapmaking procedure (see Sect. 3.1).

- For multipoles $35 < \ell < 300$, we compute power spectra with a classical pseudo- C_ℓ estimator Xpol (Sect. 3.3.2). We used $N_{\text{side}} = 1024$ maps and the native beam of Commander maps (i.e., $5'$). In this case, we apodize the mask (see Sect. 3.2) with a 1° Gaussian taper. Given the low signal-to-noise ratio in polarization, we bin the spectra with $\Delta\ell = 10$.

The *EE*, *BB*, and *EB* power spectra estimates are presented in Fig. 4 for 50 % of the sky, which provides the best combination of sensitivity and freedom from foreground residuals. Power spectra computed on different sky fractions (using masks from Sect. 3.2) are compared in Fig. B.1. A simple χ^2 test on the first 34 multipoles shows no significant departure from the *Planck* 2018 Λ CDM model for any of these spectra. The most extreme multipole in the *BB* spectrum is $\ell = 6$, which, for a Gaussian distribution, would correspond conditionally to a 3.4σ outlier (reducing to 2.3σ after taking into account the look-elsewhere effect, including the first 34 multipoles). However, at such low multipoles, the distribution is not Gaussian and the “probability to exceed” values are certainly higher than the numbers of σ would suggest. In *EE*, the largest deviation from the model is for $\ell = 17$ at 3.1σ and in *EB* it is $\ell = 19$ at 2.7σ .

The C_ℓ covariance matrix is computed from the PR4 Monte Carlo. For each simulation, we compute the power spectra using both estimators. The statistical distribution of the recovered C_ℓ then naturally includes the effect of the components included in the Monte Carlo, namely the CMB signal, instrumental noise, *Planck* systematic effects incorporated in the PR4 simulations (see Sect. 3.1), component-separation uncertainties, and foreground residuals. The residual power spectra (both for the simulations and the data) are shown in Fig. B.2.

Given the *Planck* noise level in polarization, we focus on multipoles below $\ell = 150$, which contain essentially all the information on tensor modes in the *Planck* CMB angular power spectra. At those scales, and given *Planck* noise levels, the likelihood function needs to consistently take into account the two polarization fields *E* and *B*, as well as all correlations between multipoles and modes (*EE*, *BB*, and *EB*).

LoLLiPoP (LOW- ℓ Likelihood on POLarized Power-spectra) is a *Planck* low- ℓ polarization likelihood based on cross-spectra, and was previously applied to *Planck* *EE* data for investigating the reionization history in [Planck Collaboration Int. XLVII](#)

⁴ <http://healpix.sourceforge.net>

⁵ <https://gitlab.in2p3.fr/xQML/xQML>

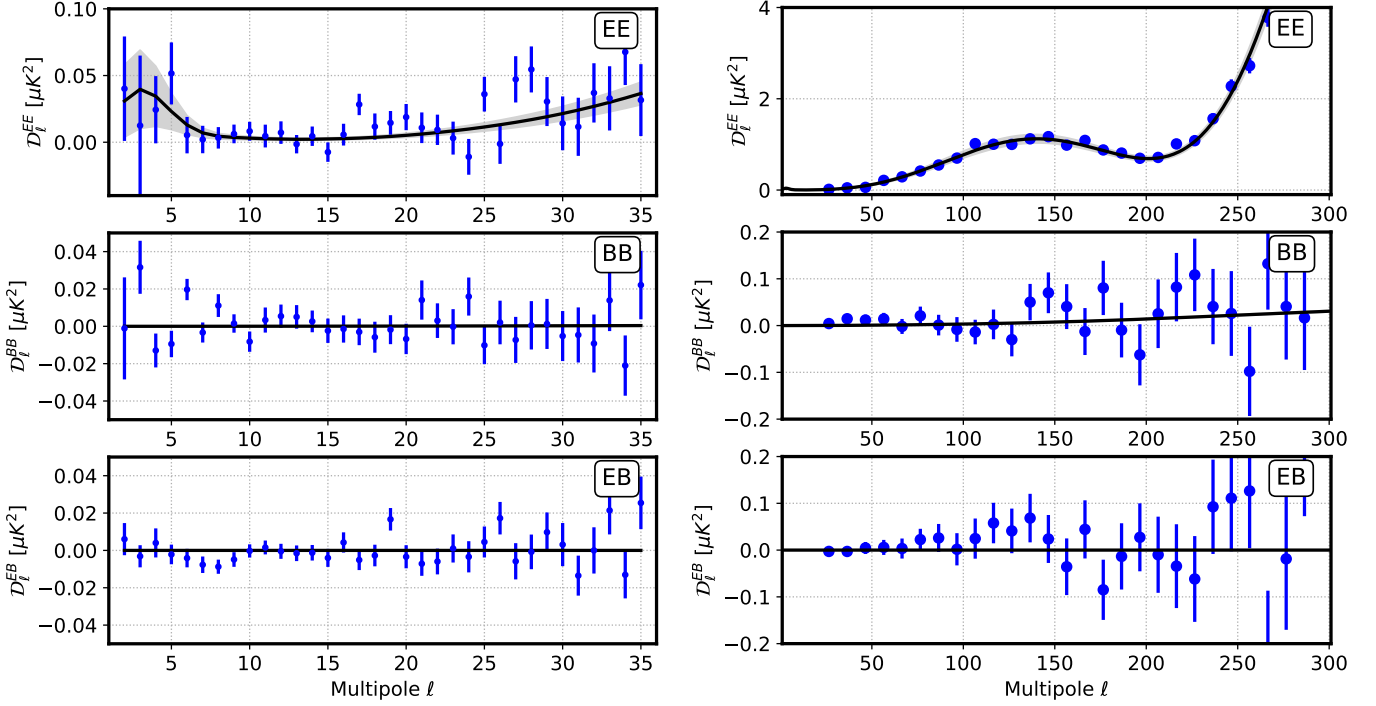


Figure 4: *EE*, *BB*, and *EB* power spectra of the CMB computed on 50 % of the sky with the PR4 maps at low (left panels) and intermediate multipoles (right panels). Grey bands represent the associated cosmic variance. Error bars are deduced from the PR4 Monte Carlo simulations. Correlations between data points are given in Appendix C. A simple χ^2 test shows no significant departure from the model for any of these spectra.

(2016). The version used here is updated to use cross-spectra calculated on component-separated CMB dataset maps processed by `Commander` from the PR4 frequency maps. Systematic effects are considerably reduced in cross-correlation compared to auto-correlation, and LoLLiPoP is based on cross-power spectra for which the bias is zero when the noise is uncorrelated between maps. It uses the approximation presented in Hamimeche & Lewis (2008), modified as described in Mangilli et al. (2015) to apply to cross-power spectra. The idea is to apply a change of variable $C_\ell \rightarrow X_\ell$ so that the new variable X_ℓ is nearly Gaussian-distributed. Similarly to Hamimeche & Lewis (2008), we define

$$X_\ell = \sqrt{C_\ell^f + O_\ell} g\left(\frac{\tilde{C}_\ell + O_\ell}{C_\ell^f + O_\ell}\right) \sqrt{C_\ell^f + O_\ell}, \quad (3)$$

where $g(x) = \sqrt{2(x - \ln(x) - 1)}$, \tilde{C}_ℓ are the measured cross-power spectra, C_ℓ are the power spectra of the model to be evaluated, C_ℓ^f is a fiducial model, and O_ℓ are the offsets needed in the case of cross-spectra. For multi-dimensional CMB modes (here we restrict ourselves to *E* and *B* fields only), the C_ℓ generalise to C_ℓ , a 2×2 matrix of power spectra,

$$C_\ell = \begin{pmatrix} C_\ell^{EE} + O_\ell^{EE} & C_\ell^{EB} \\ C_\ell^{BE} & C_\ell^{BB} + O_\ell^{BB} \end{pmatrix}, \quad (4)$$

and the g function is applied to the eigenvalues of $C_\ell^{-1/2} \tilde{C}_\ell C_\ell^{-1/2}$ (with $C_\ell^{-1/2}$ the square root of the positive-definite matrix C_ℓ). In the case of auto-spectra, the offsets O_ℓ are given by the noise bias effectively present in the measured power spectra. For cross-power spectra, the noise bias is zero, and we use effective offsets

defined from the C_ℓ noise variance:

$$\Delta C_\ell \equiv \sqrt{\frac{2}{2\ell + 1}} O_\ell. \quad (5)$$

The distribution of the new variable $X_\ell \equiv \text{vecp}(X_\ell)$, the vector of distinct elements of X_ℓ , can be approximated as Gaussian, with a covariance given by the covariance of the C_ℓ s. The likelihood function of the C_ℓ given the data \tilde{C}_ℓ is then

$$-2 \ln P(C_\ell | \tilde{C}_\ell) = \sum_{\ell\ell'} X_\ell^T M_{\ell\ell'}^{-1} X_{\ell'}. \quad (6)$$

Uncertainties are incorporated into the C_ℓ -covariance matrix $M_{\ell\ell'}$, which is evaluated after applying the same pipeline (including `Commander` component separation and cross-spectrum estimation on each simulation) to the Monte Carlo simulations provided in PR4. The resulting C_ℓ covariance thus consistently includes CMB sample variance, statistical noise, and systematic residuals, as well as foreground-cleaning uncertainties, together with the correlations induced by masking. These uncertainties are then propagated through the likelihood up to the level of cosmological parameters. Figures of the correlation matrices are given in Appendix C.

Using this approach, we are able to derive three different likelihoods, one using only information from *E* modes (lowIE), one using only information from *B* modes (lowIB), and one using *EE+BB+EB* spectra (lowIEB). We have used these likelihoods from $\ell = 2$ up to $\ell = 300$ with a nominal range of $\ell = [2, 150]$, since multipoles above $\ell \simeq 150$ do not contribute to the result due to the *Planck* noise (see Sect. 5).

The approach used in this paper is different from the one used for the *Planck* 2018 results. Indeed, in [Planck Collaboration V \(2020\)](#), the probability density of the polarized spectra at low multipoles was modelled with a polynomial function adjusted on simulations in which only τ is varied, with all other cosmological parameters in a Λ CDM model fixed to the *Planck* 2018 best-fit values. As a consequence, the probability density is not proportional to the likelihood $\mathcal{L}(\Omega^{\text{model}}|C_\ell^{\text{data}})$ when the model is not Λ CDM (and in particular for our case Λ CDM+ r), and even in the Λ CDM case it neglects correlations with other parameters that affect the posterior on τ . In addition, the simulations used in [Planck Collaboration V \(2020\)](#) were generated with the same CMB realization for the mapmaking solution. Cosmic variance was included afterwards by adding CMB realizations on top of noise-only maps, neglecting correlations between foregrounds or systematic templates and the CMB. The information in polarization at low- ℓ was then extracted using a polynomial function fitted to the distribution from simulations. While this is supposed to empirically take into account the effects of systematics on the likelihood shape, it does not include ℓ -by- ℓ correlations, and is limited in the C_ℓ power that one can test (for example imposing a strong prior on the EE power at $\ell = 3$). As a consequence, the combination of those two effects reduces the covariance, especially at low multipoles, leading to error bars (especially on τ) that are underestimated.

4. Constraints from TT

To derive constraints on the tensor-to-scalar ratio from the temperature power spectrum, we use the high- ℓ HiLLiPoP likelihood for $30 \leq \ell \leq 2500$, and the Commander likelihood (lowT) in temperature for $\ell < 30$, with a prior on the reionization optical depth to break the degeneracy with the scalar amplitude A_s . We use a Gaussian prior $\tau = 0.055 \pm 0.009$. For the base- Λ CDM model, using PR4 data, we obtain the same results as presented in [Planck Collaboration VI \(2020\)](#).

We now describe the results obtained when fitting the tensor-to-scalar ratio r in addition to the six Λ CDM parameters ($\Omega_b h^2$, $\Omega_c h^2$, θ_* , A_s , n_s , τ). In [Planck Collaboration X \(2020\)](#), the constraint from TT is reported as $r_{0.002} < 0.10$ (95 % CL) using PR3 data. This is much lower than the expected 2σ upper bound on r . Indeed, when we calculate r_{eff} as proposed in Sect. 2, we find that the maximum of the posterior is in the negative region by about 1.7σ . That the maximum happens to fall at negative values is the major reason for the apparently strong constraint on r .

With PR4 data, we find that the maximum of the posterior is negative by less than 1.2σ when using HiLLiPoP in temperature (hlpTT) along with lowT. As discussed in [Planck Collaboration X \(2020\)](#), this result is related to the low- ℓ deficit in the temperature power spectrum. Indeed, removing lowT from the likelihood moves the maximum of the posterior closer to zero, as illustrated in Fig. 5. The corresponding posterior maximum and 68 % confidence interval are

$$r_{0.05} = +0.031 \pm 0.120 \quad (\text{hlpTT}+\tau\text{-prior}), \quad (7a)$$

$$r_{0.05} = -0.131 \pm 0.093 \quad (\text{hlpTT}+\text{lowT}), \quad (7b)$$

$$r_{0.05} = -0.101 \pm 0.094 \quad (\text{hlpTT}+\text{lowT}+\tau\text{-prior}). \quad (7c)$$

Using the temperature power spectrum from PR4, we recover the same constraints on other parameters, in particular the scalar spectral tilt n_s , as found using PR3 data (see Appendix D). With the full posterior distribution on r , we get a better idea

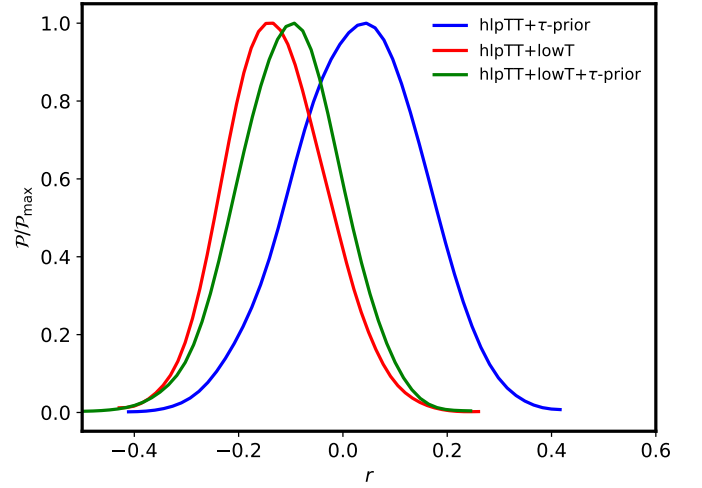


Figure 5: Constraints on the tensor-to-scalar ratio $r_{0.05}$ based on high- ℓ temperature data from *Planck* PR4 (hlpTT), in combination with lowT, and with a prior on τ .

of the intrinsic sensitivity of the *Planck* temperature data to r . Using only high- ℓ data, with a prior on the reionization optical depth τ , we find $\sigma_r = 0.12$ for TT . Note that we find $\sigma_r = 0.43$ for TE , indicating that TE is much less constraining for r than TT . When adding information from low multipoles in temperature, σ_r reduces to 0.094, but at the price of pushing the maximum distribution towards negative values. The fact that the distribution peaks in the non-physical domain can be considered as a statistical fluctuation (with a significance between 1 and 2σ , depending on the data set used), which on its own is not a serious problem. However, the fact that this behaviour is strongly related to the deficit of power at low- ℓ in temperature is worth noting.

Final upper limits from the *Planck* temperature power spectrum using PR4 are then

$$r_{0.05} < 0.13 \quad (95 \text{ \% CL, hlpTT}+\text{lowT}), \quad (8a)$$

$$r_{0.05} < 0.12 \quad (95 \text{ \% CL, hlpTT}+\text{lowT}+\tau\text{-prior}). \quad (8b)$$

5. Constraints from BB

To derive constraints on the tensor-to-scalar ratio from BB using the PR4 maps, we sample the likelihood with a fixed Λ CDM model based on the *Planck* 2018 best fit, to which we add tensor fluctuations with a free amplitude parametrized by the tensor-to-scalar ratio r . We use the LoLLiPoP likelihood described in Sect. 3.3.3, restricted to BB only (referred as “lowIB”). As discussed in Sect. 3.3.3, we construct the C_ℓ covariance matrix using the PR4 Monte Carlo simulations, which include CMB signal, foreground emission, realistic noise, and systematic effects.

Before giving the final constraints coming from the *Planck* BB spectra, we should distinguish between the two different regimes, corresponding to large scales (the reionization bump) and intermediate scales (the recombination bump). Across the reionization bump, uncertainties are dominated by systematic residuals, as discussed in Sect. 3.1, while foreground residuals may bias the results. Across the recombination bump, uncertainties are dominated by statistical noise; however, systematic effects, as well as foreground residuals, can still bias constraints

on r . In order to test the effects of potential foreground residuals, we calculate the posterior distributions of r using various Galactic masks, as described in Sect. 3.2. While large sky fractions ($f_{\text{sky}} > 60\%$) show deviations from $r = 0$, the posteriors for 40, 50, and 60 % of the sky are consistent with zero (Fig. E.1). As a robustness test, we also calculate the posterior distribution when changing the range of multipoles (Fig. E.2) and find consistent results, with posteriors compatible with $r = 0$. Multipoles above $\ell \simeq 150$ do not contribute to the result, since the noise in BB is too high. For the rest of this paper, unless otherwise noted, we use a sky fraction of 50%, and compute the likelihood over the range of multipoles from $\ell = 2$ to $\ell = 150$.

For the reionization and recombination bumps we find

$$r_{0.05} = -0.014^{+0.108}_{-0.111} \quad (\text{lowIB, reionization bump}), \quad (9)$$

$$r_{0.05} = 0.069^{+0.114}_{-0.113} \quad (\text{lowIB, recombination bump}). \quad (10)$$

Both results are obtained over 50 % of the sky, with multipoles in the range $\ell = [2, 35]$ for the former and $\ell = [50, 150]$ for the latter. With these ranges of multipoles, and given the statistics of the PR4 maps, we can see that the reionization bump ($\sigma_r = 0.110$) and the recombination bump ($\sigma_r = 0.113$) contribute equally to the overall *Planck* sensitivity to the tensor-to-scalar ratio.

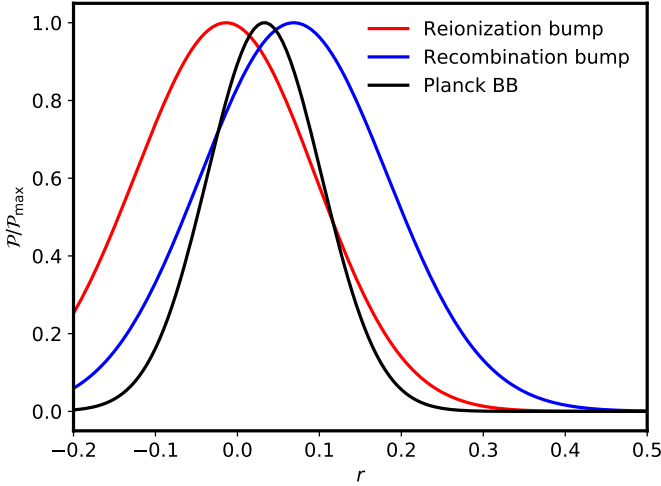


Figure 6: Posterior distribution of r from PR4 data, using LoLLiPoP and the BB spectrum on 50 % of the sky (black). Constraints from the reionization bump and the recombination bump are plotted in red and blue, respectively. Constraints from *Planck* BB with the full multipole range $\ell = [2, 150]$ are in black.

We can combine the results from the two bumps in order to give the overall constraints on the tensor-to-scalar ratio from the *Planck* BB spectrum (Fig. 6). The full constraint on r from the PR4 BB spectrum over 50 % of the sky, including correlations between all multipoles between $\ell = 2$ and $\ell = 150$, is

$$r_{0.05} = 0.033 \pm 0.069 \quad (\text{lowIB}). \quad (11)$$

This is fully compatible with no tensor signal, and we can derive an upper limit by integrating the posterior distribution out to 95 %, after applying the physical prior $r > 0$, which yields

$$r_{0.05} < 0.158 \quad (95\% \text{ CL, lowIB}). \quad (12)$$

This result can be compared with the BICEP2/Keck Array constraints (BICEP2 Collaboration 2018) of

$$r_{0.05} < 0.072 \quad (95\% \text{ CL, BK15}), \quad (13)$$

with $\sigma_r = 0.02$ compared to $\sigma_r = 0.069$ for the *Planck* result presented in this analysis

6. Additional constraints from polarization

As shown in Fig. 1, the EE tensor spectrum is similar in amplitude to the BB tensor spectrum, even though the scalar mode in EE is stronger. Given that noise dominates the tensor signal at all multipoles in both EE and BB , we expect the likelihood for EE to give useful constraints on r . We thus present the constraints from polarized low- ℓ data ($\ell < 150$) using different combinations of the LoLLiPoP likelihood (specifically EE , BB , and $EE+BB+EB$) in Fig. 7. We emphasize that $EE+BB+EB$ is a likelihood of the correlated polarization fields E and B and not the combination of individual likelihoods (see Sect. 3.3.3).

The first thing to notice is that the posterior distribution for EE peaks at $r = 0.098 \pm 0.097$, while the other modes give results compatible with zero within 1σ . Given the lower sensitivity of lowIE to r ($\sigma_r \simeq 0.10$) compared to that of lowIB ($\sigma_r \simeq 0.07$), this is mitigated when adding the information from other modes. The posterior distributions for r give

$$r_{0.05} = 0.033 \pm 0.069 \quad (\text{lowIB}), \quad (14)$$

$$r_{0.05} = -0.031 \pm 0.046 \quad (\text{lowIEB}). \quad (15)$$

As a consistency check, Fig. 7 also shows the constraints when fitting the BB tensor model on the EB data power spectrum, which is compatible with zero ($r = -0.012 \pm 0.068$) as expected.

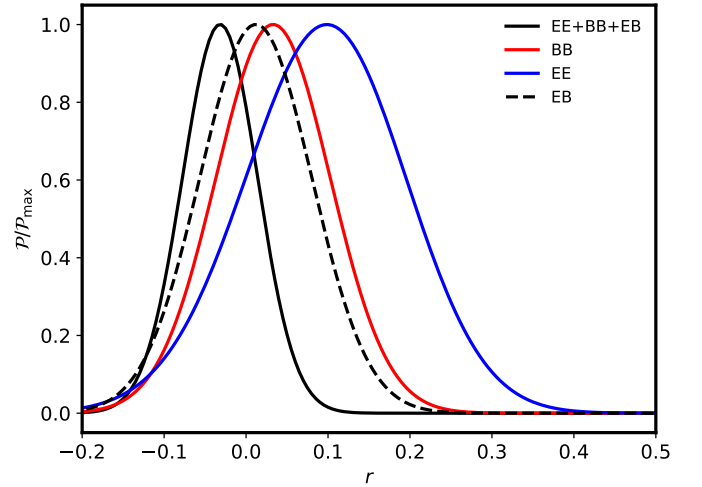


Figure 7: Posterior distributions for r from *Planck* polarized low- ℓ data ($\ell < 150$) using LoLLiPoP and the EE , BB , and $EE+BB+EB$ spectra. The dashed black line is obtained from EB data by fitting a BB tensor model. The sky fraction used here is $f_{\text{sky}} = 50\%$

Using polarization data, *Planck*'s sensitivity to the tensor-to-scalar ratio reaches $\sigma_r = 0.046$. Combining all *Planck* polarization modes (EE , BB , and EB) out to $\ell = 150$ leads to the following upper limit:

$$r_{0.05} < 0.069 \quad (95\% \text{ CL, lowIEB}). \quad (16)$$

Note that this constraint is almost independent of the other Λ CDM parameters, and in particular the reionization optical depth τ . To demonstrate this, using the same data set (lowIB and

lowIEB), we derive 2-dimensional constraints for τ and r and plot them in Fig. 8. The constraint is stable when sampling for τ . Indeed, in this case, we obtain

$$r_{0.05} = 0.025 \pm 0.064 \quad (\text{lowIB}), \quad (17)$$

$$r_{0.05} = -0.015 \pm 0.045 \quad (\text{lowIEB}), \quad (18)$$

and for the reionization optical depth

$$\tau = 0.0577 \pm 0.0056 \quad (\text{lowIEB}), \quad (19)$$

compatible with lowIE results, while lowIB shows no detection of τ , since BB is dominated by noise.

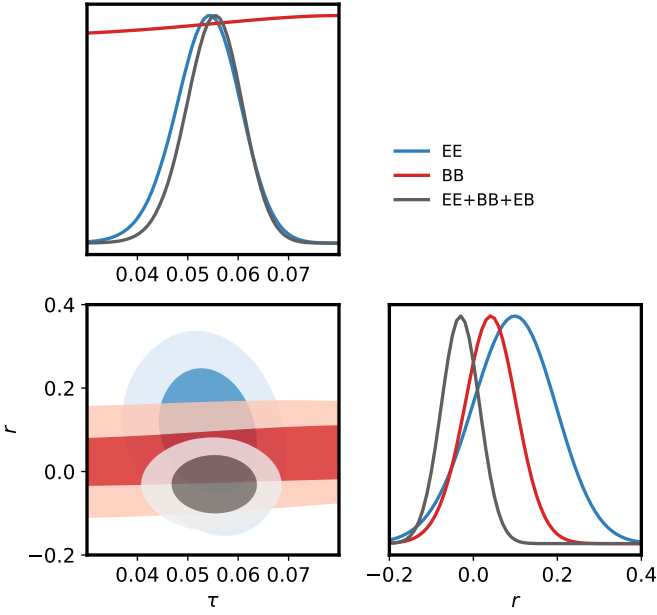


Figure 8: LoLLiPoP posterior distribution in the τ - r plane using lowIE (blue), lowIB (red), and lowIEB (black). The sky fraction here is $f_{\text{sky}} = 50\%$.

7. Combined results

Up to this point, the constraints on r have been derived relative to a fixed fiducial Λ CDM spectrum based on the *Planck* 2018 results. Including the *Planck* temperature likelihoods (both lowT and hlpTT) in a combined analysis of the *Planck* CMB spectra allows us to properly propagate uncertainties from other cosmological parameters to r , as well as to self-consistently derive constraints in the n_s - r plane. In this section, we combine the lowT and hlpTT with the low- ℓ polarized likelihood lowIEB to sample the parameter space of the Λ CDM+ r model. The comparison of contours at 68 % and 95 % confidence levels between PR3 and PR4 data is presented in Fig. F.1 of Appendix F.

We also include the BK15 constraints from [BICEP2 Collaboration \(2018\)](#). When combining *Planck* and BK15, we neglect the correlation between the two data sets and simply multiply the likelihood distributions. This is justified because the BK15 spectra are estimated on 1 % of the sky, while the *Planck* analysis is derived from 50 % of the sky.

Figure 9 gives posteriors on r after marginalization over the nuisance and the other Λ CDM cosmological parameters. We obtain the following 95 % CL upper limits:

$$r_{0.05} < 0.060 \quad (95\% \text{ CL, hlpTT+lowT+BK15}); \quad (20a)$$

$$r_{0.05} < 0.056 \quad (95\% \text{ CL, hlpTT+lowT+lowIEB}); \quad (20b)$$

$$r_{0.05} < 0.044 \quad (95\% \text{ CL, hlpTT+lowT+lowIEB+BK15}). \quad (20c)$$

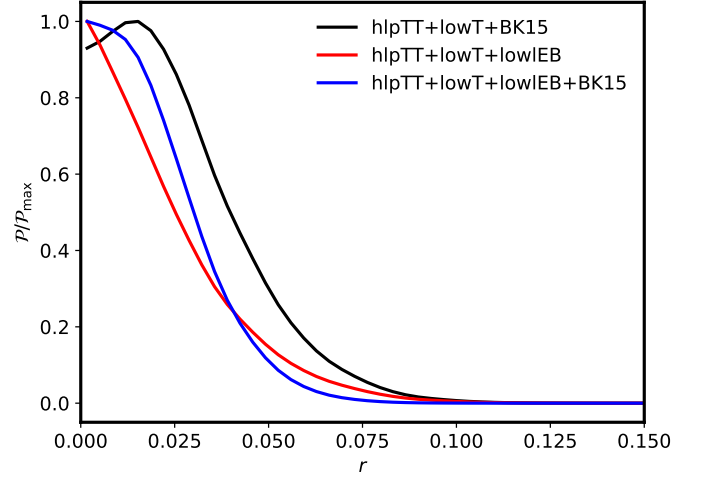


Figure 9: Posterior distributions for r after marginalization over the nuisance parameters and the other Λ CDM parameters, for the *Planck* temperature data (hlpTT+lowT) in combination with BK15 and the large-scale polarized *Planck* likelihood (lowIEB).

Figure 10 shows the constraints in the r - n_s plane for *Planck* data in combination with BK15. The constraints from the full combination of *Planck* data are comparable to those from BK15. The addition of the high- ℓ TE likelihood produces tighter constraints on the spectral index n_s (as already reported in [Planck Collaboration VI 2020](#)).

8. Conclusions

In this paper, we have derived constraints on the amplitude of tensor perturbations using *Planck* PR4 data. We investigated the intrinsic sensitivity of the TT spectrum, which is cosmic-variance limited, and found $\sigma_r = 0.094$ using the full range of multipoles. We noted the impact of the low- ℓ anomaly, which pushes the maximum posterior distribution toward negative values of r_{eff} at roughly the 1σ level.

For the first time, we analyzed the *Planck* BB spectrum for r and obtained $\sigma_r = 0.069$, which is lower than in temperature. The *Planck* B -mode spectrum, being dominated by noise, gives a constraint on r that is fully compatible with zero from both low and intermediate multipoles, in other words from both the reionization and recombination peaks. Multipoles above $\ell \simeq 150$ do not contribute to the result, since the noise in BB is too high.

Using an appropriate likelihood in polarization, we showed that the *Planck* EE spectrum is also sensitive to the amplitude of the tensor-to-scalar ratio r . The combined constraints from *Planck* EE and BB , including EB correlations, lead to a sensitivity on r of $\sigma_r = 0.046$, two times better than in temperature. We also investigated the impact of foreground residuals using

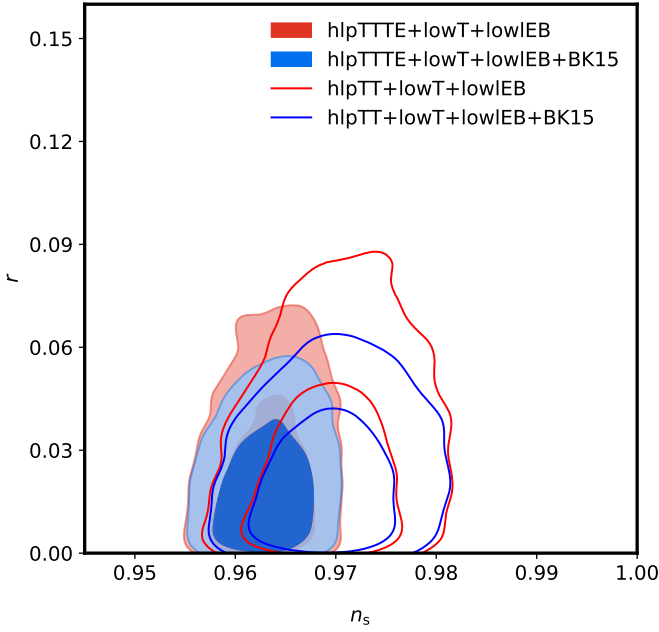


Figure 10: Marginalized joint 68 % and 95 % CL regions for n_s and $r_{0.05}$ from *Planck* alone (hlp+lowT+lowEB) and in combination with BK15. The solid lines correspond to hlpTT+lowT+lowEB, while the filled regions correspond to hlpTTTE+lowT+lowEB.

different Galactic cuts and by varying the range of multipoles used in the polarized likelihood. Finally, by combining temperature and polarization constraints, we derived the posterior distribution on r marginalized over the Λ CDM cosmological parameters as well as the nuisance parameters from the TT high- ℓ likelihood. The result gives an upper-limit of $r < 0.056$ at the 95 % confidence level using *Planck* data only. In combination with the BICEP/Keck measurements from 2015, this constraint is further reduced to $r < 0.044$ (95 % CL), the tightest limit on r to date.

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References

Battaglia, N., Natarajan, A., Trac, H., Cen, R., & Loeb, A., Reionization on Large Scales. III. Predictions for Low- l Cosmic Microwave Background Polarization and High- l Kinetic Sunyaev-Zel'dovich Observables. 2013, *ApJ*, 776, 83, [arXiv:1211.2832](#)

Béthermin, M., Daddi, E., Magdis, G., et al., A Unified Empirical Model for Infrared Galaxy Counts Based on the Observed Physical Evolution of Distant Galaxies. 2012, *ApJ*, 757, L23, [arXiv:1208.6512](#)

BICEP2 Collaboration, Constraints on Primordial Gravitational Waves Using *Planck*, WMAP, and New BICEP2/Keck Observations through the 2015 Season. 2018, *Phys. Rev. Lett.*, 121, 221301, [arXiv:1810.05216](#)

Blas, D., Lesgourgues, J., & Tram, T., The Cosmic Linear Anisotropy Solving System (CLASS). Part II: Approximation schemes. 2011, *Journal of Cosmology and Astroparticle Physics*, 2011, 0347034

Couchot, F., Henrot-Versillé, S., Perdureau, O., et al., Cosmology with the CMB temperature-polarization correlation. 2017a, *ArXiv e-prints*, [arXiv:1609.09730](#)

Couchot, F., Henrot-Versillé, S., Perdureau, O., et al., Relieving tensions related to the lensing of the cosmic microwave background temperature power spectra. 2017b, *A&A*, 597, A126, [arXiv:1510.07600](#)

Eriksen, H. K., Jewell, J. B., Dickinson, C., et al., Joint Bayesian Component Separation and CMB Power Spectrum Estimation. 2008, *ApJ*, 676, 10, [arXiv:0709.1058](#)

Górski, K. M., Hivon, E., Banday, A. J., et al., HEALPix: A Framework for High-Resolution Discretization and Fast Analysis of Data Distributed on the Sphere. 2005, *ApJ*, 622, 759, [arXiv:astro-ph/0409513](#)

Hamimeche, S. & Lewis, A., Likelihood analysis of CMB temperature and polarization power spectra. 2008, *Phys. Rev. D*, 77, 103013, [arXiv:0801.0554](#)

Hivon, E., Mottet, S., & Ponthieu, N., QuickPol: Fast calculation of effective beam matrices for CMB polarization. 2017, *A&A*, 598, A25, [arXiv:1608.08833](#)

Mangilli, A., Plaszczyński, S., & Tristram, M., Large-scale CMB temperature and polarization cross-spectra likelihoods. 2015, *MNRAS submitted*, [arXiv:1503.01347](#)

Planck Collaboration XV, *Planck* 2013 results. XV. CMB power spectra and likelihood. 2014, *A&A*, 571, A15, [arXiv:1303.5075](#)

Planck Collaboration XVI, *Planck* 2013 results. XVI. Cosmological parameters. 2014, *A&A*, 571, A16, [arXiv:1303.5076](#)

Planck Collaboration XXI, *Planck* 2013 results. XXI. Power spectrum and high-order statistics of the *Planck* all-sky Compton parameter map. 2014, *A&A*, 571, A21, [arXiv:1303.5081](#)

Planck Collaboration XXII, *Planck* 2013 results. XXII. Constraints on inflation. 2014, *A&A*, 571, A22, [arXiv:1303.5082](#)

Planck Collaboration XXX, *Planck* 2013 results. XXX. Cosmic infrared background measurements and implications for star formation. 2014, *A&A*, 571, A30, [arXiv:1309.0382](#)

Planck Collaboration XI, *Planck* 2015 results. XI. CMB power spectra, likelihoods, and robustness of parameters. 2016, *A&A*, 594, A11, [arXiv:1507.02704](#)

Planck Collaboration XIII, *Planck* 2015 results. XIII. Cosmological parameters. 2016, *A&A*, 594, A13, [arXiv:1502.01589](#)

Planck Collaboration XX, *Planck* 2015 results. XX. Constraints on inflation. 2016, *A&A*, 594, A20, [arXiv:1502.02114](#)

Planck Collaboration XXII, *Planck* 2015 results. XXII. A map of the thermal Sunyaev-Zeldovich effect. 2016, *A&A*, 594, A22, [arXiv:1502.01596](#)

Planck Collaboration V, *Planck* 2018 results. V. Power spectra and likelihoods. 2020, *A&A*, 641, A5, [arXiv:1907.12875](#)

Planck Collaboration VI, *Planck* 2018 results. VI. Cosmological parameters. 2020, *A&A*, 641, A6, [arXiv:1807.06209](#)

Planck Collaboration VIII, *Planck* 2018 results. VIII. Gravitational lensing. 2020, *A&A*, 641, A8, [arXiv:1807.06210](#)

Planck Collaboration X, *Planck* 2018 results. X. Constraints on inflation. 2020, *A&A*, 641, A10, [arXiv:1807.06211](#)

Planck Collaboration Int. XXX, *Planck* intermediate results. XXX. The angular power spectrum of polarized dust emission at intermediate and high Galactic latitudes. 2016, *A&A*, 586, A133, [arXiv:1409.5738](#)

Planck Collaboration Int. XLI, *Planck* intermediate results. XLI. A map of lensing-induced B -modes. 2016, *A&A*, 596, A102, [arXiv:1512.02882](#)

Planck Collaboration Int. XLVII, *Planck* intermediate results. XLVII. Constraints on reionization history. 2016, *A&A*, 596, A108, [arXiv:1605.03507](#)

Planck Collaboration Int. LI, *Planck* intermediate results. LI. Features in the cosmic microwave background temperature power spectrum and shifts in cosmological parameters. 2017, *A&A*, 607, A95, [arXiv:1608.02487](#)

Planck Collaboration Int. LVII, *Planck* intermediate results. LVII. NPIPE: Joint *Planck* LFI and HFI data processing. 2020, *A&A*, in press, [arXiv:2007.04997](#)

Shaw, L. D., Rudd, D. H., & Nagai, D., Deconstructing the Kinetic SZ Power Spectrum. 2012, *ApJ*, 756, 15, [arXiv:1109.0553](#)

Tegmark, M. & de Oliveira-Costa, A., How to measure CMB polarization power spectra without losing information. 2001, *Phys. Rev. D*, 64, 063001, [arXiv:astro-ph/0012120](#)

Tristram, M., Macías-Pérez, J. F., Renault, C., & Santos, D., XSPECT, estimation of the angular power spectrum by computing cross-power spectra with analytical error bars. 2005, *MNRAS*, 358, 833, [arXiv:astro-ph/0405575](#)

Tucci, M., Toffolatti, L., de Zotti, G., & Martínez-González, E., High-frequency predictions for number counts and spectral properties of extragalactic radio sources. New evidence of a break at mm wavelengths in spectra of bright blazar sources. 2011, *A&A*, 533, 57

Vanneste, S., Henrot-Versillé, S., Louis, T., & Tristram, M., Quadratic estimator for CMB cross-correlation. 2018, *Phys. Rev. D*, 98, 103526, [arXiv:1807.02484](#)

Appendix A: The hillipop likelihood

HiLLiPoP (High- ℓ Likelihood on Polarized Power-spectra) is one of the high- ℓ likelihoods developed for analysis of the *Planck* data. It was used as part of the 2013 (Planck Collaboration XV 2014) and 2015 (Planck Collaboration XI 2016) releases and also described in Couchot et al. (2017b) and Couchot et al. (2017a).

HiLLiPoP is very similar to the *Planck* public likelihood (plik). HiLLiPoP is a Gaussian likelihood based on cross-spectra from the HFI 100-, 143-, and 217-GHz detset split-maps. The cross-spectra are estimated using a pseudo- C_ℓ algorithm with a mask adapted to each frequency to reduce the contamination from Galactic emission and point sources. The C_ℓ model includes foreground residuals on top of the CMB signal. These foreground residuals are both Galactic (dust emission) and extragalactic (CIB, tSZ, kSZ, SZ \times CMB, and point sources). HiLLiPoP also introduced nuisance parameters to take into account map calibration uncertainties.

The most significant differences compared with plik are that HiLLiPoP uses:

- detset-split maps instead of time splits (so that the cross-spectra do not need to account for a noise-correlation correction as in plik);
- point-source masks that were obtained from a procedure that extracts compact Galactic structures;
- a Galactic dust C_ℓ model that (as a result of the mask difference) follows closely and is parametrized by the power law discussed in Planck Collaboration Int. XXX (2016), while plik uses an ad-hoc effective function in ℓ ;
- foreground templates derived from *Planck* measurements (Planck Collaboration Int. XXX 2016 for the dust emission, Planck Collaboration XXX 2014 for the CIB, and Planck Collaboration XXII 2016 for the SZ) with a free amplitude for each emission mechanism, but spectral energy distributions fixed by *Planck* measurements;
- a two-component model for the signal from unresolved point sources, which incorporates the contribution from extragalactic radio (Tucci et al. 2011) and infrared dusty (Béthermin et al. 2012) galaxies, as well as taking into account the variation of the flux cut across the sky and in “incompleteness” of the source catalogue at each frequency;
- all the 15 cross-spectra built from the 100-, 143-, and 217-GHz detset maps (while plik keeps only five of them);
- all multipole values (while plik bins the power spectra).

In the end, we have a total of six instrumental parameters (for map calibration) and nine astrophysical parameters (seven for TT , one for TE , and one for EE) in addition to the cosmological parameters (see Table A.1).

Using HiLLiPoP on the PR3 data, we recover essentially identical constraints on the 6-parameter base- Λ CDM model as the public *Planck* likelihood. The results for the Λ CDM+ r model with PR4 are discussed in Appendix D.

Table A.1: Nuisance parameters for the HiLLiPoP likelihood.

| Name | Definition | Prior (if any) |
|---|---|-------------------|
| Instrumental | | |
| c_0 | map calibration (100-A) | 0.000 ± 0.002 |
| c_1 | map calibration (100-B) | 0.000 ± 0.002 |
| c_2 | map calibration (143-A) | fixed |
| c_3 | map calibration (143-B) | 0.000 ± 0.002 |
| c_4 | map calibration (217-A) | 0.000 ± 0.002 |
| c_5 | map calibration (217-B) | 0.000 ± 0.002 |
| A_{pl} | absolute calibration | 1 ± 0.0025 |
| Foreground modelling | | |
| $A_{\text{PS}}^{\text{radio}}$ | scaling parameter for radio sources in TT | |
| $A_{\text{PS}}^{\text{IR}}$ | scaling parameter for IR sources in TT | |
| A_{SZ} | scaling parameter for the tSZ in TT | |
| $A_{\text{CIB}}^{\text{TT}}$ | scaling parameter for the CIB in TT | 1.00 ± 0.20 |
| $A_{\text{dust}}^{\text{TT}}$ | scaling parameter for the dust in TT | 1.00 ± 0.20 |
| $A_{\text{dust}}^{\text{EE}}$ | scaling parameter for the dust in EE | 1.00 ± 0.20 |
| $A_{\text{dust}}^{\text{TE}}$ | scaling parameter for the dust in TE | 1.00 ± 0.20 |
| A_{kSZ} | scaling parameter for the kSZ effect in TT | |
| $A_{\text{SZ}\times\text{CIB}}$ | scaling parameter for SZ \times CIB in TT | |

Appendix B: Large-scale polarized angular power spectra

We estimate CMB large-scale polarized power spectra by cross-correlating the two independent detset splits, A and B, after Commander component separation. As detailed in Sect. 3.3.3, we use two different power-spectrum estimators for the low ($2 \leq \ell \leq 35$) and the intermediate ($35 < \ell < 300$) multipole ranges. For the lower multipole range we use a quasi-QML estimator (Vanneste et al. 2018), while for the higher multipoles we use a classic pseudo- C_ℓ approach (a generalization to polarization of the method presented in Tristram et al. 2005) with a binning of $\Delta\ell = 10$.

Figure B.1 shows the reconstructed EE , BB , and EB power spectra for various sky fractions, from 30 to 70 %. The 6-parameter Λ CDM model based on the best fit to the *Planck* 2018 data is plotted in black, together with the cosmic variance (computed for

the full sky) in grey. The spectra show a remarkable consistency with the model, and are largely insensitive to sky fraction. The *BB* and *EB* spectra are dominated by noise. At low multipoles, the *BB* spectrum computed on the largest sky fraction (70 %) exhibits an excess at very low multipoles ($\ell \leq 5$), attributable to Galactic residuals. For both *BB* and *EB*, a reduction of the sky fraction corresponds to a larger dispersion.

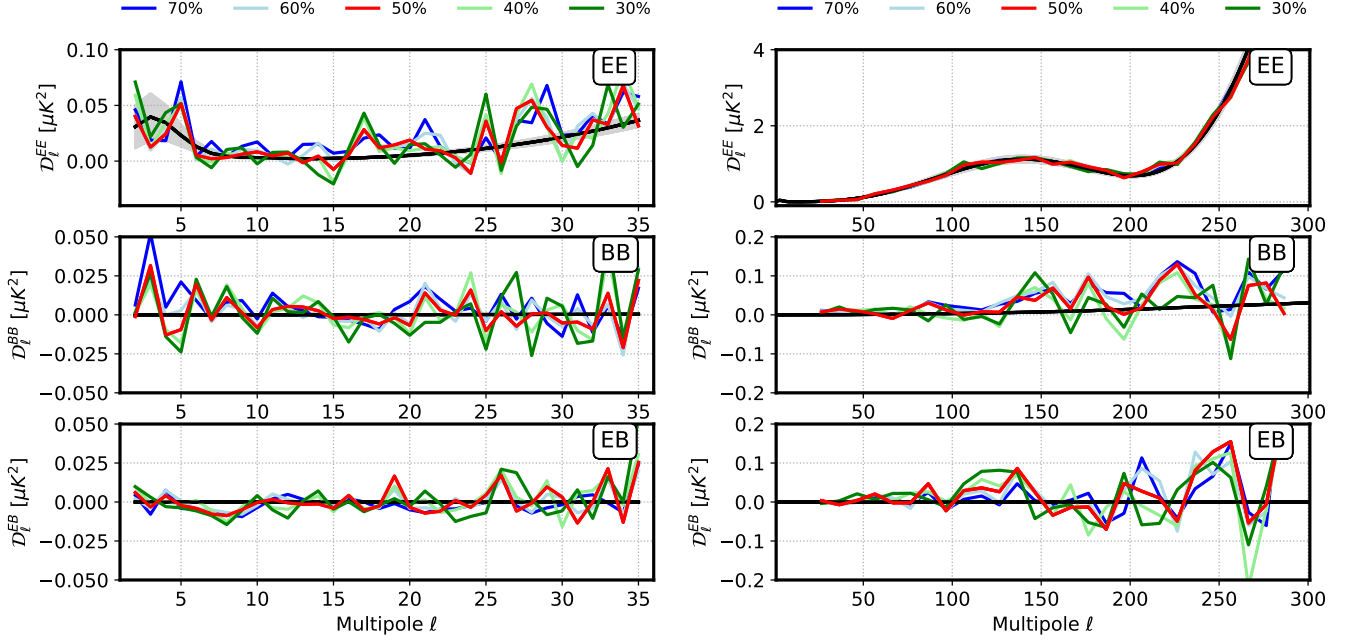


Figure B.1: *EE*, *BB*, and *EB* power spectra computed from the PR4 maps for sky fractions from 30 to 70 %. The black lines represent the Λ CDM model and the grey bands show its associated full-sky cosmic variance.

Figure B.2 shows the residuals for the *EE*, *BB*, and *EB* power spectra compared to the best-fit base- Λ CDM model from the *Planck* 2018 results. The plot on the left shows the residuals from the Monte Carlo simulations computed as the average spectra over the simulations divided by the uncertainty in the mean, $\bar{\sigma}_\ell = \sigma_\ell n_{\text{sim}}^{-1/2}$. The average here is computed independently for each multipole. The plot on the right shows the residuals of the PR4 data compared to the Λ CDM model divided by the spectrum uncertainties (i.e., the dispersion over the simulations).

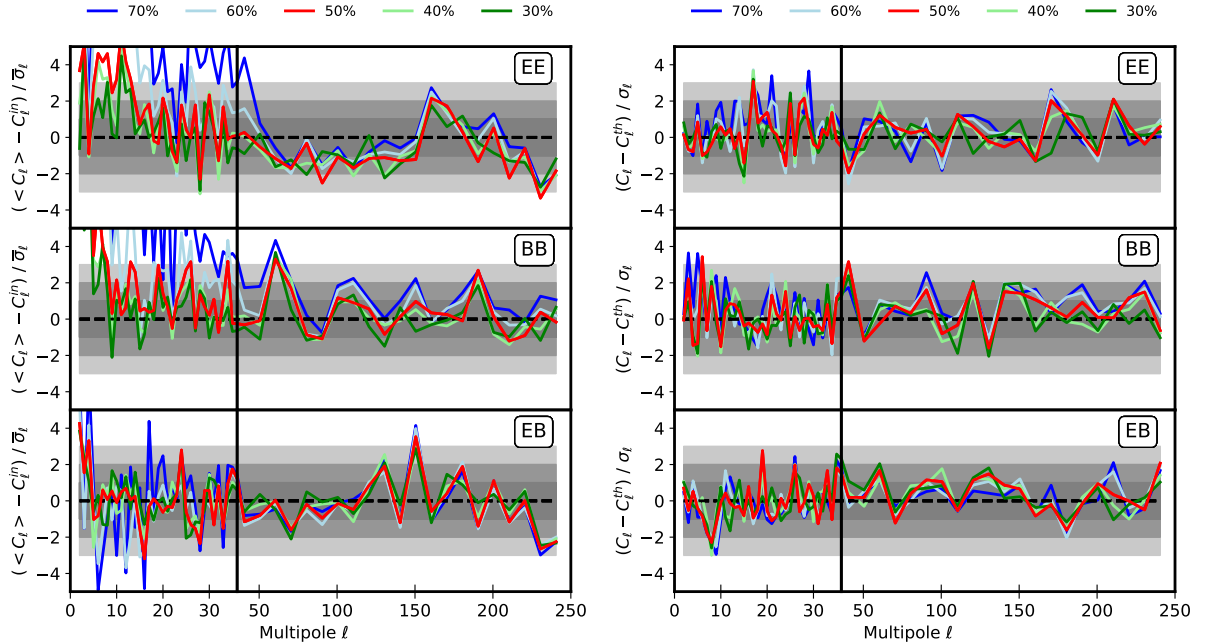


Figure B.2: Residuals of the *EE*, *BB*, and *EB* power spectra. *Left*: Mean value of the Monte Carlo simulations compared to the error on the mean, $\bar{\sigma}_\ell = \sigma_\ell n_{\text{sim}}^{-1/2}$. *Right*: PR4 data compared to the standard deviation σ_ℓ . Grey bands show the 1, 2, and 3 σ levels.

Appendix C: Cross-spectrum correlation matrix

Uncertainties are propagated to the likelihood function through the C_ℓ covariance matrices (see Sect. 3.3.3). We use Monte Carlo simulations to estimate the covariance over the multipoles considered in this analysis ($2 \leq \ell \leq 150$). Four hundred simulations are available, so the covariance matrices are accurate at the 5 % level. This is enough to propagate the uncertainties of the C_ℓ s to r as well as to τ .

Figure C.1 shows the correlations of the C_ℓ^{BB} computed from the covariance matrix. Correlations from bin to bin are below 15 %, except for the next-to-neighbour bins at $\ell < 40$. Figure C.2 shows the correlations computed from the full covariance matrix, including EE , BB , and EB spectra for all multipole bins from 2 to 150.

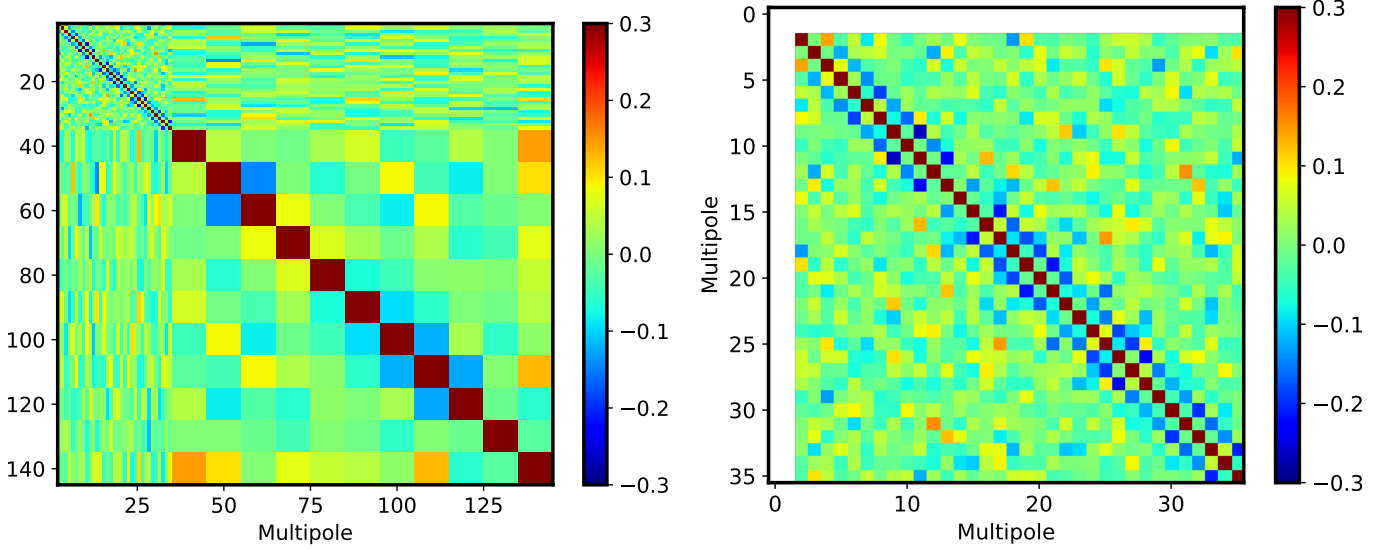


Figure C.1: *Left:* Correlation matrix for C_ℓ^{BB} . *Right:* Enlargement of the upper-left corner. Covariances are estimated from 400 end-to-end simulations, including signal, noise, systematics, and foreground residuals.

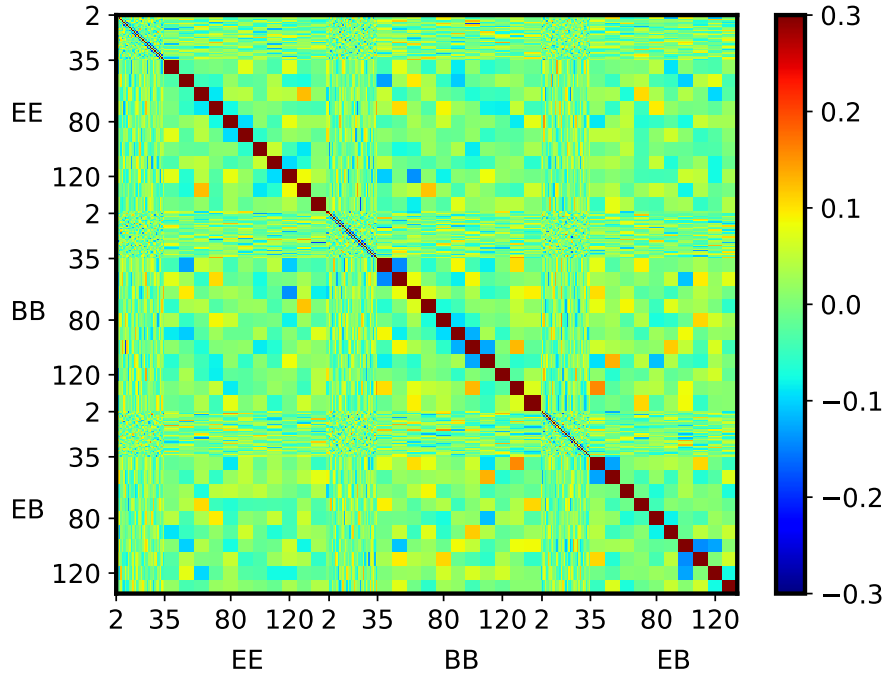


Figure C.2: Correlation matrix for the C_ℓ^{EE} , C_ℓ^{BB} , and C_ℓ^{EB} . Covariances are estimated from 400 end-to-end simulations, including signal, noise, systematics, and foreground residuals.

Appendix D: Λ CDM+ r parameters for PR3 and PR4

In this paper, we use a new data set, PR4, and an alternative likelihood, HiLLiPoP. Figure D.1 shows that we obtain essentially the same parameter values for the Λ CDM+ r model based on the *temperature* power spectrum as are obtained from PR3 and plik (available on the Planck Legacy Archive). The differences between PR3 and PR4 are primarily seen in polarization.

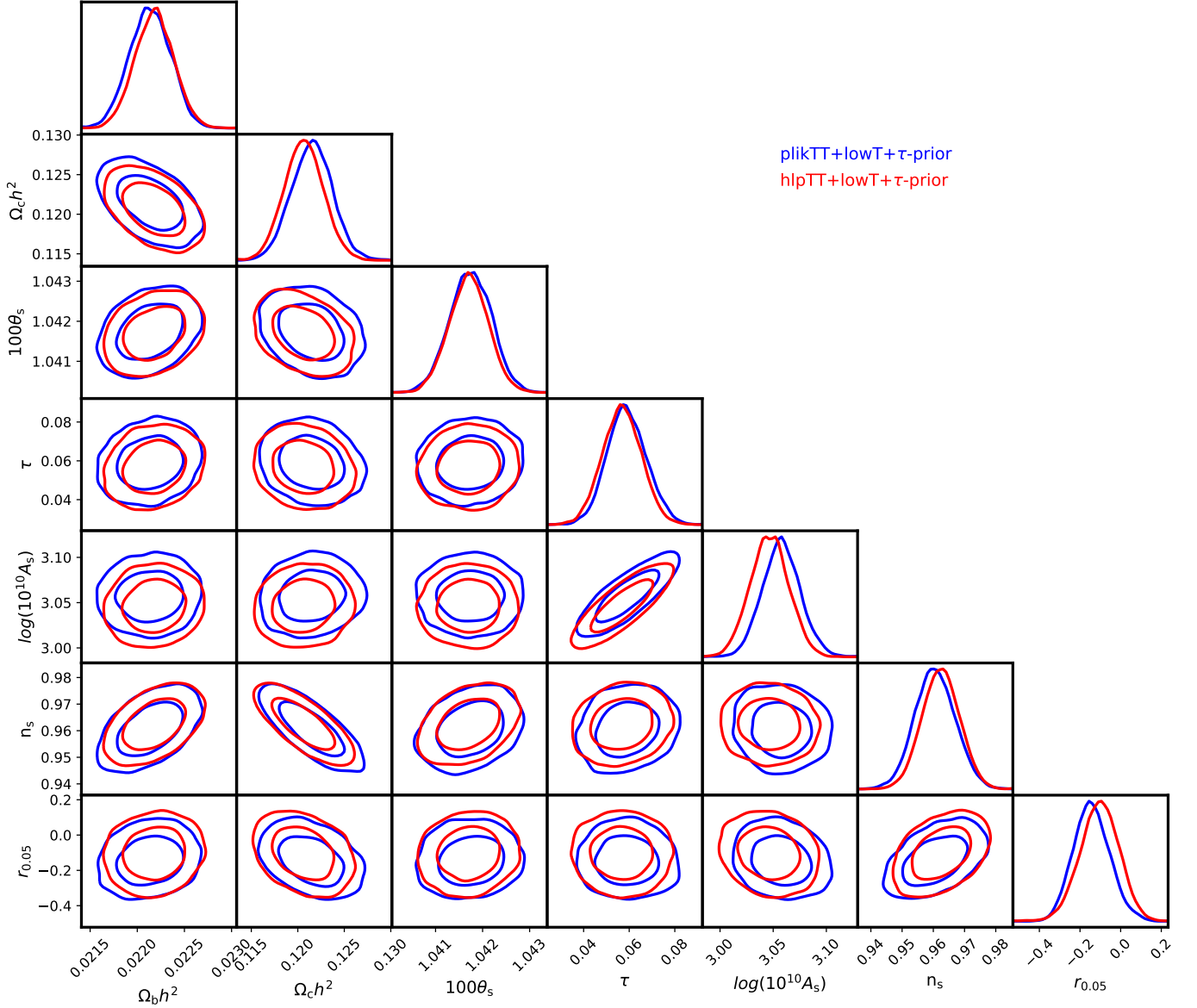


Figure D.1: Constraint contours (at 68 and 95 % confidence) on parameters of a Λ CDM+ r model using PR4 and HiLLiPoP (red), compared to those obtained with PR3 and plik (blue). Both sets of results use the Commander likelihood for temperature multipoles $\ell \leq 30$, and a Gaussian prior to constrain τ .

Appendix E: Robustness tests

As a test of robustness, we computed the posterior distributions for r_{eff} considering spectra estimated on different sky fractions from 30 to 70 % (Sect. 3.2). Figure E.1 shows results for the BB (left) and EE+BB+EB (right) likelihoods. A sky fraction of 50 % provides the best combination of sensitivity (which increases with the sky fraction) and freedom from foreground residuals (which decreases with sky fraction). Figure E.2 shows that the BB posterior is robust with respect to the choice of ℓ_{min} up to $\ell_{\text{min}} \approx 20$, and that the width of the posterior is stable for $\ell_{\text{max}} \gtrsim 150$, a consequence of the increase of signal-to-noise ratio with multipole.

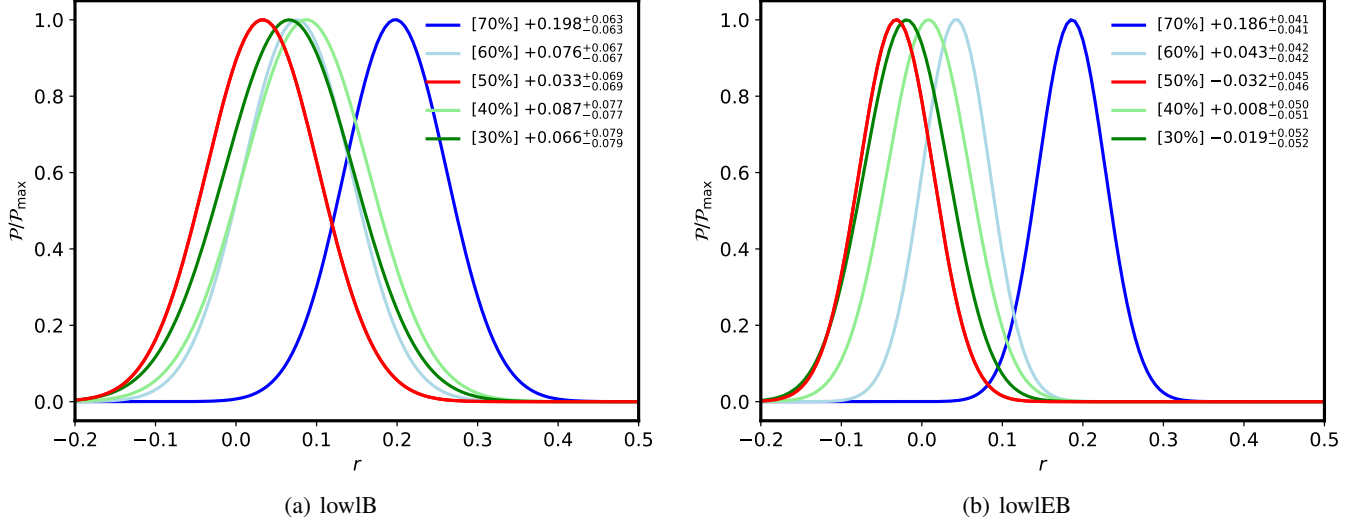


Figure E.1: Posterior distributions for r using LoLLiPoP and BB (panel a) and $EE + BB + EB$ (panel b) for sky fractions from 30 % to 70 %. The multipole range is the same in all cases, $\ell = [2, 145]$. The best combination of sensitivity and freedom from foreground residuals is achieved at 50 %.

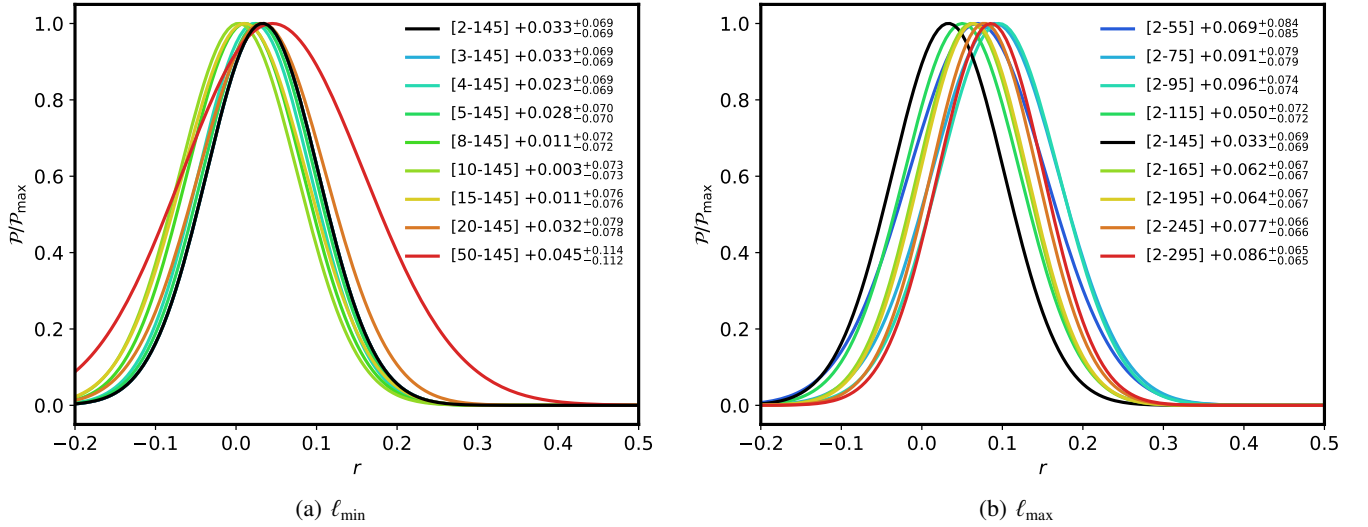


Figure E.2: Posterior distributions for r using LoLLiPoP and the BB power spectrum for various multipole ranges. The sky fraction is fixed at 50 %.

Appendix F: Triangle plot for Λ CDM+ r parameters

Figure F.1 shows the differences in parameter values obtained for PR3 (plikTT+lowT+lowE, Planck Legacy Archive) and PR4 (hlpTT+lowT+lowEB, this analysis) when both temperature and polarization data are included. There are no significant differences. In general, uncertainties are slightly smaller with PR4.

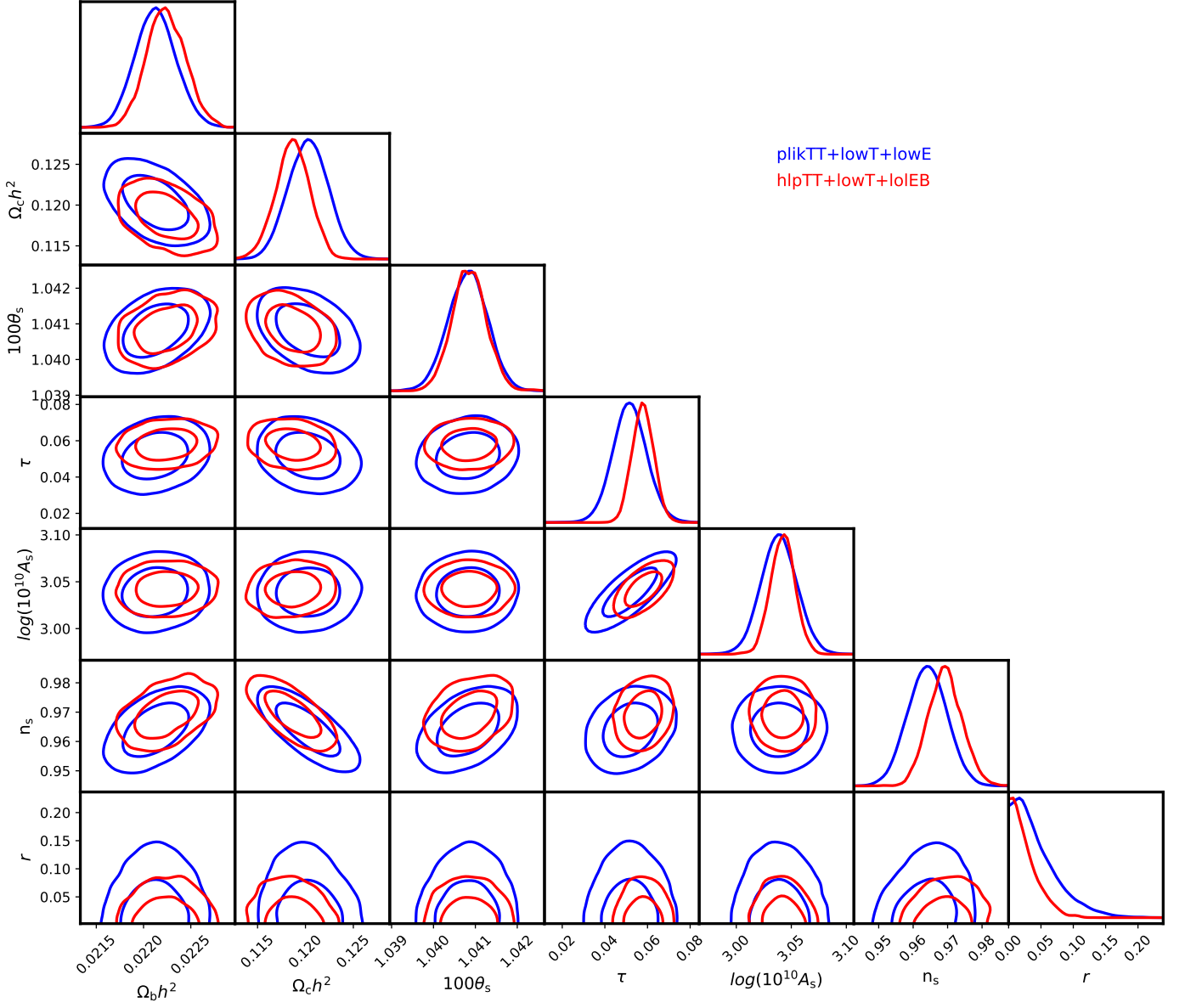


Figure F.1: Contour constraints for Λ CDM+ r parameters for PR3 (plikTT+lowT+lowE) and for PR4 (hlpTT+lowT+lowIEB).