CHAOTIC INFLATION

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A new scenario of the very early stages of the evolution of the universe is suggested. According to this scenario, inflation is a natural (and may be even inevitable) consequence of chaotic initial conditions in the early universe.

At present it seems plausible that the new inflationary universe scenario [1,2] can be completely realized in the context of $N = 1$ supergravity coupled to matter [3,4]. This possibility became rather attractive after a simple solution to the primordial monopole problem in this scenario has been suggested [4]. Still it could seem that inflation of the universe is a rather peculiar phenomenon, which may occur only in some restricted class of theories.

In the present paper we would like to show that under some natural assumptions about initial conditions in the very early universe, inflation may occur in a wide class of theories including the Weinberg–Salam theory, the SU(5) theory etc. Strong constraints on these theories still may appear if one wishes to obtain small density perturbations after inflation. However the original problem of obtaining a sufficiently large inflation in the context of some natural theory of elementary particles does not seem to be a problem any more.

To illustrate the main idea of the new scenario, which we call the chaotic inflation scenario for reasons to be explained soon, let us first consider an extreme and unrealistic example of a theory of a scalar field φ with a degenerate effective potential $V(\varphi) = V_0 = \text{const.} > 0$. It is clear that in such a theory there are no reasons to expect that the classical field φ is equal to any particular value (say, $\varphi = 0$) in the whole universe. On the contrary, one may expect that *all* values of φ may appear in different regions of space, sufficiently far removed from each other, with equal probability. This means that in such a theory the field φ may take absolutely arbitrary values varying from $-\infty$ to $+\infty$ in different regions of the early universe. In particular, the values $\varphi \gg M_p \sim 10^{19}$ GeV are quite legitimate. The only possible constraint on the distribution of the field φ in the very early universe is that $(\partial_\mu\varphi)^2 \lesssim M_p^4$ for $\mu = 0,1,2,3$, since otherwise the corresponding part of the universe would be in the preplanckian era, in which the classical description of space and time is hardly possible. During the expansion of the universe both the energy density connected with the inhomogeneity of the field φ, which is proportional to $(\partial_\mu\varphi)^2$, and the thermal energy $\sim T^4$ rapidly decrease, the energy density of matter reduces to $V(\varphi) = V_0 = \text{const.} > 0$, and the universe becomes exponentially expanding with the scale factor $a(t) \sim \exp(Ht)$, where $H = (\frac{8}{3}\pi V_0/M_p^2)^{1/2}$. As a result of this expansion the universe becomes divided into many exponentially large domains containing an almost homogeneous field φ. This model, of course, is unrealistic, but it is clear that some very similar effects may appear in any theory with an effective potential $V(\varphi)$ which is sufficiently flat.

Indeed, let us consider now a theory $V(\varphi) = \frac{1}{4}\lambda \times \varphi^4$ with $\lambda \ll 1$. A classical description of the evolution of the universe becomes possible only after the Planck time $t \sim t_p \sim M_p^{-1}$, when the energy density becomes smaller than M_p^4. Before that time the universe is usually assumed to be in some chaotic quantum state, see e.g. refs. [5–7]. If λ is sufficiently small, there is no reason to expect that at $t \sim t_p$ the field φ should be equal to zero everywhere. On the contrary, one may expect that the field φ may take any value

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between $-M_p/(\lambda)^{1/4}$ and $M_p/(\lambda)^{1/4}$ in different regions of space, so that $V(\varphi) = \frac{1}{4}\lambda\varphi^4 \lesssim M_p^4$ [Note, that the value of $V(\varphi)$ at $t \sim t_p \sim M_p^{-1}$ can be measured with an accuracy $\sim M_p^4$ only due to the uncertainty principle]. We will discuss later which configurations of the field φ in the very early universe might be most probable, but now we just note that since fields φ as large as $M_p/(\lambda)^{1/4}$ are not forbidden by any known laws of nature [quantum gravity effects become important at $\varphi > M_p/(\lambda)^{1/4}$ only, where $V(\varphi) \gtrsim M_p^4$], in the open (infinite) universe at $t \sim t_p$ there should exist infinitely many locally homogeneous and isotropic domains of size $l \gg M_p^{-1}$, containing a locally homogeneous field φ such that $M_p \lesssim \varphi \lesssim M_p/(\lambda)^{1/4}$. One may wonder what is the reason to discuss such nonequilibrium field configurations, which may occur at some very early stages of the evolution of the universe and which rapidly disappear due to the rolling of the field φ to the minimum of $V(\varphi)$. To answer this question let us consider the evolution of the locally homogeneous field φ in the early universe.

The part of the universe inside a domain filled with a homogeneous field φ expands as de Sitter space with the scale factor $a(t) = a_0 \exp(Ht)$, where

$$H = (\tfrac{8}{3}\pi V(\varphi)/M_p^2)^{1/2} = (\tfrac{2}{3}\pi\lambda)^{1/2}\varphi^2/M_p \,. \quad (1)$$

The equation of motion of the field φ inside this domain is

$$\ddot{\varphi} + 3H\dot{\varphi} = -\lambda\varphi^3 \,, \quad (2)$$

which implies that at $\varphi^2 \gg M_p^2/6\pi$

$$\varphi = \varphi_0 \exp\{-[\sqrt{\lambda}M_p/(6\pi)^{1/2}]t\} \,. \quad (3)$$

This means that at $\lambda \ll 1$ the typical time $\Delta t \sim (6\pi)^{1/2}/\sqrt{\lambda}M_p$, during which the field φ decreases considerably, is much greater than the Planck time $t_p \sim (6M_p)^{-1}$ (see below). During the main part of this period the universe expands exponentially, and the scale factor of the universe after expansion grows as follows:

$$a(\Delta t) \sim a_0 \exp(H\Delta t) \sim a_0 \exp(2\pi\varphi_0^2/M_p^2) \,. \quad (4)$$

From eq. (4) it follows that inflation of the universe is sufficiently large ($\exp(H\Delta t) \gtrsim \exp(65)$ [8]) if

$$\varphi_0 \gtrsim 3M_p \,. \quad (5)$$

Such a value of φ_0 is quite possible if $\frac{1}{4}\lambda\varphi_0^4 \lesssim M_p^4$, which implies that

$$\lambda \lesssim 10^{-2} \,. \quad (6)$$

For a typical initial value $\varphi_0 \sim M_p/(\lambda)^{1/4}$ eq. (4) yields

$$a(\Delta t) \sim a_0 \exp(2\pi/\sqrt{\lambda}) \,. \quad (7)$$

Thus we see that the domains of the field $\varphi_0 \gtrsim 3M_p$ (5), which inevitably exist in a universe with a chaotic initial distribution of the field φ, in the theory $\frac{1}{4}\lambda\varphi^4$ with $\lambda \lesssim 10^{-2}$, grow exponentially, just as the bubbles in the first version of the new inflationary universe scenario [1], and give rise to mini-universes of a size exceeding the size of the observable part of our universe. Conditions necessary for the existence of intelligent life can hardly appear in all other domains (with $\varphi_0 \lesssim M_p$), if the new inflationary universe scenario [1] is not realized at $\varphi_0 \lesssim M_p$. However for our purposes it is sufficient that in the chaotic inflation scenario there should exist infinitely many domains with $\varphi_0 \gtrsim 3M_p$, in which a very large inflation occurs and life becomes possible [2].

In the previous discussion we have neglected high-temperature corrections to $V(\varphi)$, which usually lead to symmetry restoration and to the vanishing of the field φ in the early universe [9]. However, high-temperature symmetry restoration does not actually occur in the regions of the universe in which the field φ was sufficiently large initially, so that $\varphi \gtrsim M_p$ and $V(\varphi) \gtrsim 10^{-5}M_p$. Indeed, according to ref. [9], symmetry restoration in the early universe occurs due to the high-temperature contribution to the effective mass squared of the field φ [to the curvature of $V(\varphi)$]

$$\Delta m^2(T) = cT^2 \,, \quad (8)$$

where c is some constant. In GUTs $c = O(1)$, whereas in the theory of chiral singlet fields coupled to supergravity the value of C may be negligibly small [3,4]. Let us consider the most dangerous case $C \sim 1$. In this case the time necessary for field relaxation near the minimum of $V(\varphi)$ should exceed the time $\tau \sim T^{-1}$, which is necessary for one oscillation of the field φ to occur. Let us estimate the age of the universe at that moment. Since the value of $V(\varphi)$ remains essentially constant during the process, one may expect that initially the energy density of the universe was dominated by relativistic particles. In this case the age of the universe is given by [7]

$$t = \tfrac{1}{4}(3/2\pi)^{1/2}M_p/\sqrt{\rho} \,, \quad (9)$$

where ρ is the energy density of relativistic particles,

$$\rho = \tfrac{1}{30} N \pi^2 T^4 . \qquad (10)$$

Here N is the effective number of degrees of freedom (of particles) in the theory. Typically in GUTs $N \gtrsim 200$. By comparison of $\tau \sim T^{-1}$ and t for $N \gtrsim 200$ one concludes that the field φ can be influenced by high-temperature effects at $T \lesssim \frac{1}{50} M_p$ only. However at such a temperature the energy density of relativistic particles becomes negligibly small as compared with $V(\varphi)$ in the domains with $V(\varphi) \sim M_p^4$. Therefore the expansion of such domains becomes exponential much earlier than the temperature decreases down to $\frac{1}{50} M_p$, the temperature inside the domains becomes exponentially small and all high-temperature effects disappear. This means that the high-temperature effects are irrelevant for the behaviour of the field φ in the domains with $V(\varphi) \sim M_p^4$. {We do not consider here the effects connected with the Hawking "temperature" $T_H = H/2\pi$, which are important in some theories with comparatively large coupling constants, but can be included into the definition of $V(\varphi)$ in curved space [2]}.

Now let us try to understand which initial configurations of the field φ might be most probable. A classical description of the universe expansion becomes possible at $\rho \sim \rho_p = M_p^4$, which, according to (9), occurs at $t_p \sim (6M_p)^{-1}$. The values of the field φ in domains at a distance l from each other are presumably almost uncorrelated when l is much greater than the size of the horizon $\sim t_p$. Some small correlation may still exist if one assumes that $(\partial_\mu \varphi)^2 \lesssim M_p^4$, since we would like to consider the post-planckian era, in which the energy density is smaller than M_p^4. Thus one may assume that in different regions of space of size $\sim t_p$ the field φ takes (almost) random values in the interval $-M_p/(\lambda)^{1/4} \lesssim \varphi \lesssim M_p/(\lambda)^{1/4}$, in such a way that $|\partial_\mu \varphi| \lesssim M_p^2$. In this case a simple statistical analysis indicates that at $t \sim t_p$ the universe is divided into domains filled with fields φ of different sign. The typical amplitude of the field φ inside each domain is $O(M_p/(\lambda)^{1/4}) \gg M_p$, and the typical size of a domain is $O(6/M_p\sqrt{\lambda}) \gg M_p^{-1} \sim H^{-1}$. Each domain becomes a mini-universe after exponential expansion. The only part of the universe which does not expand exponentially is the space between the domains, in which $\varphi \lesssim M_p$. However with decrease of λ most part of the universe becomes contained inside the domains with $\varphi \gtrsim 3M_p$ and exponential expansion occurs for most parts of the universe. Moreover, even if the probability of the existence of domains with $\varphi \gtrsim 3M_p$ of size $l \gg H^{-1}$ is very small [suppose e.g. that our assumption $(\partial_\mu \varphi)^2 \lesssim M_p^4$ is not valid], one may argue that just these domains will cover most parts of the physical volume of the universe, since other parts of the universe do not expand exponentially. In any case, in the infinite (open) universe at $t \sim t_p$ there should exist infinitely many domains of the type desired, which give rise to an infinite number of mini-universes in which life may exist [2].

The main difficulty of our scenario is similar to that of all other versions of the inflationary universe scenario: It is not very easy to obtain density perturbations $\delta\rho/\rho \sim 10^{-4}$ after inflation [10,11]. We have estimated the value of $\delta\rho/\rho$ at the galaxy scale by means of the methods described in ref. [11]. The result is very similar to that obtained in ref. [11] numerically: To get $\delta\rho/\rho$ as small as 10^{-4} one should have $\lambda \sim 10^{-10}$ at $\varphi \sim 3M_p$. This is a rather strong constraint. However in our scenario this constraint is not as dangerous as before. Many theories of some field φ, very weakly interacting with itself and with all other fields, can be easily suggested if there is no need for the effective potential $V(\varphi)$ to satisfy very restrictive and not very natural constraints, both on $m^2(\varphi) = V''(\varphi)$ and on $\lambda(\varphi)$ near $\varphi = 0$ [1,2], to have a zero-temperature minimum at a scale $\varphi \sim 10^{16}$–10^{19} GeV, and to have a high-temperature minimum at $\varphi = 0$. Moreover, the chaotic inflation scenario can be implemented not only in theories in which the evolution of the field φ starts at $\varphi \gtrsim 3M_p$, but in *any* theory in which the effective potential $V(\varphi)$ is sufficiently flat at some φ, at which $V(\varphi) < M_p^4$. In particular, the chaotic inflation scenario can be implemented in the context of $N = 1$ supergravity [3,4], and even in the models considered in ref. [3], in which the standard scenario [1] cannot be realized [4,12].

One should note also, that the calculation of $\delta\rho/\rho$ in ref. [11] was based originally on theories in which reheating of the universe occurs during the time $\Delta t \lesssim H^{-1}$, i.e. immediately after the end of inflation [13]. However in the models based on $N = 1$ supergravity the scalar field φ remains oscillating near the minimum of $V(\varphi)$ during the time Δt, which is many orders larger than H^{-1} [4,14]. The investigation of the generation of density perturbations in such theories is more complicated and may lead to a smaller value of $\delta\rho/\rho$ [15], which may relax the constraint $\lambda \sim 10^{-10}$.

Let us summarize our results. We have suggested a novel version of the inflationary universe scenario based on the assumption of chaotic initial conditions in the very early universe. This assumption seems very natural since presumably there was no (or almost no) correlation between the physical processes at different regions of space which were at a distance $l \gg t$ from each other. Under this assumption, at $t \sim t_p \sim M_p^{-1}$, there should exist some sufficiently isotropic and homogeneous domains of space of a size greater than M_p^{-1} filled with the classical homogeneous field $\varphi \gtrsim M_p$. The amplitude of this field in the early universe decreases very slowly, and during this time the domains filled with the field $\varphi \gtrsim M_p$ expand exponentially. In the theories of the type of $\frac{1}{4}\lambda\varphi^4$, the scale factor of space inside these domains increases $\sim\exp(2\pi\varphi^2/M_p^2)$ times during the period of exponential expansion. This suggests that the main part of the physical volume of our universe appears not due to the expansion of domains of $\varphi < M_p$, but due to the exponential expansion of domains filled with the maximally nonequilibrium field $\varphi \gg M_p$. The only possible constraint on the amplitude of the field φ at $t \sim t_p$ is connected with the condition $V(\varphi) \lesssim M_p^4$, which should be valid in the post-planckian epoch [though in principle the exponential expansion could start earlier, at $V(\varphi) > M_p^4$].

Our scenario may resemble many previous attempts to obtain an isotropic and homogeneous universe after expansion of a universe with chaotic initial conditions, see e.g. refs. [6,7]. However the main aim of the previous works was to obtain a *globally* homogeneous and isotropic universe, which is hardly possible. In our approach, just as in other versions of the new inflationary universe scenario [1,2], it is sufficient to have small homogeneous and isotropic domains, which after exponential expansion become greater than the observable part of the universe.

On the other hand, the chaotic inflation scenario differs considerably from all other versions of the inflationary universe scenario suggested so far [8,1–4], since it is *not* based on the theory of high-temperature phase transitions in the early universe. According to this scenario, a very large inflation occurs in a wide class of theories in which the effective potential is not too curved at $\varphi \gtrsim 3M_p (\lambda \lesssim 10^{-2})$ *at least for one of the scalar fields* φ. As distinct from the previous versions of the new inflationary universe scenario, in this scenario there is no need for $V(\varphi)$ to have a minimum at $\varphi \sim 10^{15}-10^{19}$ GeV: The chaotic inflation scenario can be implemented as well in theories in which there is no symmetry breaking in the equilibrium state at all.

Now let us note, that even in the theories with $\lambda \sim 10^{-1}$ a rather large inflation should occur in our scenario (7). Moreover, it is usually believed that in the pre-planckian epoch there was a singularity, and the energy density was much greater than M_p^4. Therefore it seems natural to assume that large fields φ with $V(\varphi) \gg M_p^4$ are also possible at the very early stages of the evolution of the universe. In this case our constraints on λ (6) disappear completely. Note also that our conclusion about the chaotic inflation in the domains with $\varphi \gtrsim M_p$ is almost model-independent. Indeed, the main condition for large inflation is $H^2 \gg m^2(\varphi)$ [1,2], where $H^2 = 8\pi V(\varphi)/3M_p^2$ and $m^2(\varphi) = d^2V/d\varphi^2$. At large φ the value of $m^2(\varphi)$ can be roughly estimated as $V(\varphi)/\varphi^2$, from which follows that $H^2 \gg m^2(\varphi)$ at $\varphi^2 \gg M_p^2$, and inflation occurs for all reasonable potentials $V(\varphi)$. This suggests that inflation is not a peculiar phenomenon which is desirable for a number of well-known reasons [1,2], but that it is a natural and may be even inevitable consequence of the chaotic initial conditions in the very early universe.

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