CSC373 Homework 3

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Question 1 Auction Optimization

(a) Let g_{ijk} be the amount of good k that supplier i sold to customer j. Then the LP formation is:

$$\begin{aligned}
\text{Maximize } & \sum_{i=1}^{S}\sum_{k=1}^{G} g_{ijk}|B_{jk} - A_{ik}| \\
\text{s.t. } & \sum_{j=1}^{C} g_{ijk} \leq S_{ik}, \forall i \in \{1,2,\cdots,S\}, k \in \{1,2,\cdots,G\} \\
& \sum_{i=1}^{S} g_{ijk} \leq D_{jk}, \forall j \in \{1,2,\cdots,C\}, k \in \{1,2,\cdots,G\} \\
& g_{ijk}, A_{ik}, B_{jk} \geq 0, \forall i \in \{1,2,\cdots,S\}, j \in \{1,2,\cdots,C\}, k \in \{1,2,\cdots,G\}
\end{aligned}$$

Constraint 1 satisfies that each supplier will sell no more units than it can supply. Constraint 2 satisfies that each customer will buy no more units than it demands. The last constraint ensures that the amount of each type of good that each supply sells to each customer is non-negative, and all asking prices and bidding prices are non-negative.

(b) The LP formation is:

$$\begin{aligned}
\text{Maximize } & \sum_{i=1}^{S}\sum_{k=1}^{G} g_{ijk}|B_{jk} - A_{ik} - C_{ij}| \\
\text{s.t. } & \sum_{j=1}^{C} g_{ijk} \leq S_{ik}, \forall i \in \{1,2,\cdots,S\}, k \in \{1,2,\cdots,G\} \\
& \sum_{i=1}^{S} g_{ijk} \leq D_{jk}, \forall j \in \{1,2,\cdots,C\}, k \in \{1,2,\cdots,G\} \\
& \sum_{k=1}^{G} g_{ijk} \leq U_{ij}, \forall i \in \{1,2,\cdots,S\}, j \in \{1,2,\cdots,C\} \\
& g_{ijk}, A_{ik}, B_{jk} \geq 0, \forall i \in \{1,2,\cdots,S\}, j \in \{1,2,\cdots,C\}, k \in \{1,2,\cdots,G\}
\end{aligned}$$

Given the additional conditions, Constraint 3 is added to ensure that the total units of goods transported from supplier i to customer j will not be more than the available fleet capacity U_{ij}.

(c) The LP formation is:

$$\begin{aligned}
\text{Maximize } & \sum_{i=1}^{S}\sum_{k=1}^{G} g_{ijk}|B_{jk} - A_{ik} - C_{ij}| \\
\text{s.t. } & \sum_{j=1}^{C} g_{ijk} \leq S_{ik}, \forall i \in \{1,2,\cdots,S\}, k \in \{1,2,\cdots,G\} \\
& \sum_{i=1}^{S} g_{ijk} \leq D_{jk}, \forall j \in \{1,2,\cdots,C\}, k \in \{1,2,\cdots,G\} \\
& \sum_{k=1}^{G} g_{ijk}W_k \leq U_{ij}, \forall i \in \{1,2,\cdots,S\}, j \in \{1,2,\cdots,C\} \\
& g_{ijk}, A_{ik}, B_{jk} \geq 0, \forall i \in \{1,2,\cdots,S\}, j \in \{1,2,\cdots,C\}, k \in \{1,2,\cdots,G\}
\end{aligned}$$

Constraint 3 is modified by multiplying the unit of good k with its unit weight W_k in kilograms.

Question 2 Minimizing Maximum Deviation

(a) we want to minimize the maximal l_∞ distance, which we denoted here by ϵ_{max}.
For each $i \in \{1, 2, \cdots, n\}$:

$$\epsilon_i \leq \epsilon_{max}$$
$$\Rightarrow |y_i - ax_i - b| \leq \epsilon_{max}$$
$$\Rightarrow y_i - ax_i - b \leq \epsilon_{max} \text{ and } -y_i + ax_i + b \leq \epsilon_{max}, \text{ for each data point } (x_i, y_i)$$
$$\Rightarrow y_i - ax_i - \epsilon_{max} \leq b \text{ and } -y_i + ax_i - \epsilon_{max} \leq -b, \text{ for each data point } (x_i, y_i)$$

Therefore, we can write the linear program that will produce a line of best fit with minimum l_∞ error as:
variables: a, b, ϵ_{max}
maximize $-\epsilon_{max}$
subject to
$y_i - ax_i - \epsilon_{max} \leq b$, for each data point (x_i, y_i)
$-y_i + ax_i - \epsilon_{max} \leq -b$, for each data point (x_i, y_i)
$\epsilon_{max} \geq 0$

(b) My interpretation of the question: the LP will determine if there is a line that separates the points such that all points of type 1 satisfies $y_i < ax_i + b$ and all points of type 2 satisfies $y_i > ax_i + b$. If one exists, maximize the gap δ and $\delta = \min(e_1, e_2)$, where e_i = minimum l_∞ distance between the line and points of type i.

Let data points in type 1 be represented by $(x_i, y_i), i = 1, 2, ..., m$, and data points in type 2 be represented by $(x_j, y_j), j = 1, 2, ..., n$.
If there exists a line separates the points, then $y_i < ax_i + b \Rightarrow y_i - ax_i - b < 0 \Rightarrow l_{\infty 1} = -y_i + ax_i + b$.
Since $e_1 = \min(l_{\infty 1})$, then $e_1 \leq -y_i + ax_i + b, i = 1, 2, ..., m$.
Similarly, from $y_j > ax_j + b$, we can get $e_2 \leq y_j - ax_j - b, j = 1, 2, ..., n$.
Since $\delta = \min(e_1, e_2)$, then if there exists a line separates the points, these two inequalities should be satisfied:
$\delta \leq -y_i + ax_i + b$, for all data points (x_i, y_i) of type 1, $i = 1, 2, ..., m$
$\delta \leq y_j - ax_j - b$, for all data points (x_j, y_j) of type 2, $j = 1, 2, ..., n$.

From the above, we define the LP as follows:
variables: a, b, δ
maximize δ
subject to
$\delta \leq -y_i + ax_i + b$, for all data points (x_i, y_i) of type 1, $i = 1, 2, ..., m$
$\delta \leq y_j - ax_j - b$, for all data points (x_j, y_j) of type 2, $j = 1, 2, ..., n$.
$\delta \geq 0$