Stat 134 lec 15 (MT1 review pt 2)

Warmup

An urn contains 90 marbles, of which there are 20 greens, 20 blacks and 50 red marbles. Tom draw marble with replacement until the 6^{th} green marble. Let $X = \#$ of marbles drawn. Example: **GGG**BRB**GG**BR**G** with $x = 11$. Find $\mathbb{E}[X]$.

How is the answer different from last time if we draw with replacement?

with replacement this is $X \sim \text{NegBin}\left(r=6, p=\frac{2}{9}\right)$

$$E(X) = \frac{r}{p} = \frac{6}{\frac{2}{9}} = \boxed{6 \cdot \frac{9}{2}}$$

without replacement we got $6 \cdot \underbrace{\left(\frac{70}{21} + 1\right)}_{= \frac{91}{21}}$

Before

$$X = I_1 + \cdots + I_{70} + 1$$

$$E(X) = 70 \cdot \frac{1}{21} + 1$$

$$I_2 = \begin{cases} 1 & \text{if 2nd nongreen before 1st green} \\ 0 & \end{cases}$$

$p = 1/21$

Teo Tiburzi

2:27pm

From the book, Pitman, midterm examination #1 pg. 490 problem #5

Let X_2 and X_3 be indicators of independent events with probabilities $\frac{1}{2}$ and $\frac{1}{3}$ respectively.

b) Calculate $E(X_2 - X_3)$

c) Calculate $SD(X_2 - X_3)$

5. Let X_2 and X_3 be indicators of independent events with probabilities $\frac{1}{2}$ and $\frac{1}{3}$, respectively.

a) Display the joint distribution table of $X_2 + X_3$ and $X_2 - X_3$.

b) Calculate $E(X_2 - X_3)$.

c) Calculate $SD(X_2 - X_3)$.

$$\underbrace{E(X_2)}_{=\,1/2} - \underbrace{E(X_3)}_{=\,1/3} = 1/6$$

$$Var(aX) = a^2 Var(X)$$

$$Var(-X) = Var(X)$$

$$Var(X_2 - X_3) = \underbrace{Var(X_2)}_{=\,1/2 \cdot 1/2} + \underbrace{Var(X_3)}_{=\,1/3 \cdot 2/3}$$

$$SD(X_2 \cdot X_3) = \sqrt{1/2 \cdot 1/2 + 1/3 \cdot 2/3}$$

Audrey Su

4:07pm

I would like to go over Properties of Expectation and and Properties of Variance, and whether or not we are allowed to apply the Addition Rule for Variance or not (i.e. how to know when trials are dependent or independent). The source of my question is from Exercise 3.3.8 from *Pitman*. The first sentence of the problem is "Let A1, A2, and A3 be events with probabilities 1/5, 1/4, and 1/3, respectively."

Note if indicators are iid then their sum is Binomial

8. Let A_1, A_2, and A_3 be events with probabilities $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{3}$, respectively. Let N be the number of these events that occur.

a) Write down a formula for N in terms of indicators. — $N = I_1 + I_2 + I_3$

b) Find $E(N)$. — $E(N) = E(I_1) + E(I_2) + E(I_3) = \frac{1}{5} + \frac{1}{4} + \frac{1}{3} = \boxed{\frac{47}{60}}$

In each of the following cases, calculate $Var(N)$:

c) A_1, A_2, A_3 are disjoint; — $N = \begin{cases} 1 & \text{if } A_1 \cup A_2 \cup A_3 \quad P = \frac{47}{60} \\ 0 & \text{else} \quad \frac{13}{60} \end{cases}$

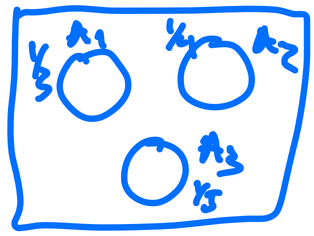
$Var(N) = \boxed{\frac{47}{60} \cdot \frac{13}{60}}$

d) they are independent;

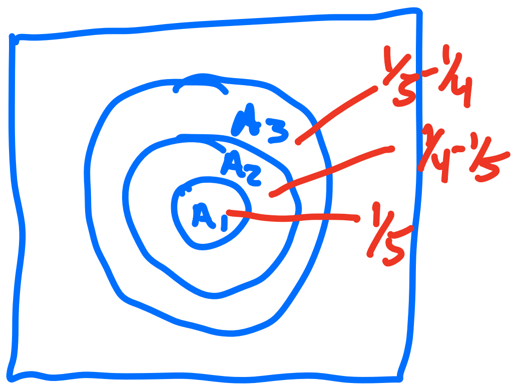
$Var(N) = Var(I_1) + Var(I_2) + Var(I_3)$

$\quad\;\; = \frac{1}{5} \cdot \frac{4}{5} \qquad \frac{1}{4} \cdot \frac{3}{4} \qquad \frac{1}{3} \cdot \frac{2}{3}$

e) $A_1 \subset A_2 \subset A_3$.



N	0	1	2	3
P(N=n)	$1 - \frac{1}{3} = \frac{2}{3}$	$\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$	$\frac{1}{4} - \frac{1}{5} = \frac{1}{20}$	$\frac{1}{5}$



$Var(N) = E(N^2) - (E(N))^2$

$E(N^2) = 0^2 \cdot \frac{1}{3} + 1^2 \cdot \frac{1}{12} + 2^2 \cdot \frac{1}{20} + 3^2 \cdot \frac{1}{5} = \frac{25}{12}$

$E(N) = \frac{47}{60}$

ex

X = # aces in a poker hand from a deck of cards

$X \sim HG(n = 5, N = 52, G = 4)$

X is a sum of dependent indicators.

Sofia Lendahl

7:49pm

Can you go over number 5 of midterm fa18 ("Three couples attend a dinner.")? Thank you!

5. (5 pts) Three couples attend a dinner. Each of the six people chooses a seat randomly from a round table with six seats. What is the probability that no couple sits together? (Hint: use the inclusion-exclusion rule.)

A_i = couple i sit together

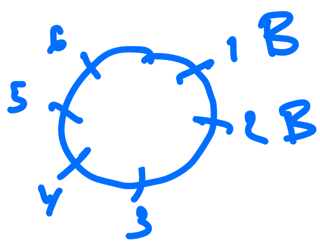
$P(A_1^c A_2^c A_3^c) = 1 - P(A_1 \cup A_2 \cup A_3)$

$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1A_2) - P(A_1A_3) - P(A_2A_3) + P(A_1A_2A_3)$

$P(A_1) = P(A_2) = P(A_3)$

BBXXXX

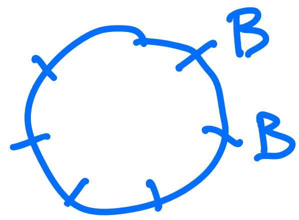
$\dfrac{\binom{6}{1}}{\binom{6}{2}}$

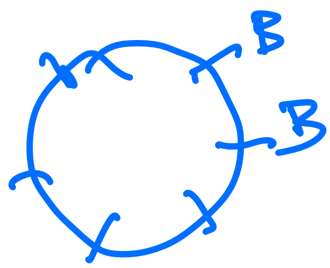


$P(A_1A_2)$

BBCCDD

$\dfrac{\binom{6}{1}}{\binom{6}{2}} \cdot \dfrac{\binom{3}{1}}{\binom{4}{2}}$





$P(A_1A_2A_3)$

$\dfrac{\binom{6}{1}}{\binom{6}{2}} \dfrac{\binom{2}{1}}{\binom{4}{2}}$

ex extra

In a certain card game for a 52 card deck, each card has a point value.

- Numbered cards in the range 2 to 9 are worth five points each.
- The cards numbered 10 and the face cards (jack, queen, king) are worth ten points each.
- Aces are worth fifteen points each.

1. What is the expected number of numbered cards (in range 2 to 9) you get when you draw 3 cards?
2. We pick 3 cards at random. What is the expected total point value of the three cards on the top of the deck after the shuffle?

1. Let X_1 be the the number of numbered cards out of 3. $X_1 = I_1 + I_2 + I_3$ where I_2 is 1 if the second card is a numbered card. This has probability $8/13$. It follows that $E(X_1) = 3(8/13) = 24/13$.
2. Let X_1 be the number of numbered cards in 3 draws, X_2 the number of face cards in 3 draws, and X_3 the number of aces in 3 draws. Let X be the total point value of the three cards. We have $X = 5X_1 + 10X_2 + 15X_3$ and want $E(X)$. Similar to part (a), $E(X_2) = 3(4/13) = 12/13$ and $E(X_3) = 3(1/13) = 3/13$. Then $E(X) = 5(24/13) + 10(12/13) + 15(3/13) = 288/13$.