# Warmup

ex A 5 card poker hand consists of a SRS of 5 cards from a 52 card deck. There are $\binom{52}{5}$ poker hands.

a) Find P(poker hand has 4 aces and a king)

$$\frac{\overset{A}{\binom{4}{4}}\overset{K}{\binom{4}{1}}\binom{44}{0}}{\binom{52}{5}}$$

optional since equal to 1

b) Find P(poker hand has 4 aces)

choose non ace

$$\frac{\overset{A}{\binom{4}{4}}\overset{\text{non aces}}{\binom{48}{1}}}{\binom{52}{5}} = \frac{\binom{12}{1}\binom{4}{4}\binom{4}{1}}{\binom{52}{5}}$$

c) Find P(poker hand has 4 of a kind)

choose 4 of a kind

choose 1 of a kind

$$\frac{\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1}}{\binom{52}{5}}$$

aaaab a ≠ b

Quiz 1 next Tuesday covers 1.1–1.6, 2.1,
Practice Quiz 1 is available on bCourses/pages.

# Last time

Binomial — independent trials
Hypergeometric — dependent trials.

ex 100 person Class with a grade distribution:

A grade : 70 students
B grade : 30 students.

Sample 5 students at random w/o replacement (SRS).

Find P(3A's, 2Bs)

exact
hypergeometric

$$= \frac{\binom{70}{3}\binom{30}{2}}{\binom{100}{5}} = \binom{5}{3}\frac{70}{100}\frac{69}{99}\frac{68}{98}\frac{30}{97}\frac{29}{96} = .316$$

approx
binomial

$$= \binom{5}{3}(.7)^3(.3)^2 = .309$$

When N is large relative to n, $HG(5,100,70) \approx Bin(5,.7)$

why?

$$HG(n,N,G) \approx Bin\left(n,\frac{G}{N}\right)$$

## Summary of approximations

$HG(n,N,G)$

approx by binomial
N large, n small
$p=\frac{G}{N}$

binomial$(n,p)$

approx by Poisson
$p\to 0,\ n\to\infty,\ np\to\mu$

Poisson$(\mu)$

approx by normal
n large
$\mu=np,\ \sigma=\sqrt{npq}$
$0<\mu\pm3\sigma<n$
use continuity correction

Normal$(\mu,\sigma^2)$

# Today

1. sec 2.5 Hypergeometric distribution
2. sec 3.1 — random variables (RV)
   — joint distribution of 2 RVs and independence

① sec 2.5 Hypergeometric distribution

tinyurl.com/feb21-2023

**Stat 134**

1. The probability of being dealt a three of a kind poker hand (ranks $aaabc$ where $a \neq b \neq c$) is:

**a** $\binom{4}{3}\binom{4}{1}\binom{44}{1} / \binom{52}{5}$

**b** $\binom{13}{1}\binom{12}{\underline{2}}\binom{4}{3}\binom{4}{1}\binom{4}{1} / \binom{52}{5}$

→ **c** $\binom{13}{1}\binom{12}{1}\binom{4}{3}\binom{4}{1}\underset{\cancel{\binom{11}{1}\binom{4}{1}}}{\underset{\|}{\binom{44}{1}}} / \binom{52}{5}$

**d** none of the above

No

Is aaabc = bbbac in a poker hand

Is aaabc = aaacb in a poker hand — Yes

set card poker hand

aaabbc

$$\frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}\binom{11}{1}\binom{4}{1}}{\binom{52}{6}}$$

Find the probability that a poker hand has two 2 of a kind

eg K, K, Q, Q, 7

double K Q single 7

$$\frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$$

KKqq7

qqKK7

Find the probability that a 6 card poker hand has two 2 of a kind and 2 singles

eg K, K Q Q 7, 8

$$\frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{6}}$$

## ② Sec 3.1 Intro to Random Variables (RV)

A RV, $X$, is the outcome of an experiment.

What distribution is the following RV?

$X$ = The number of aces in 5 cards drawn from a standard deck?

$$X \sim HG(\overset{52}{N}, \overset{5}{n}, \overset{4}{G})$$

ex flip a prob $p$ coin 2 times

$X$ = # heads

we write $X \sim \text{Bin}(2, p)$

More precisely,

$$X : \overset{\text{outcome space}}{\Omega} \longrightarrow \mathbb{R} \quad \text{is a function}$$

$$\begin{aligned} HH &\longmapsto 2 \\ HT &\longmapsto 1 \\ TH &\longmapsto 1 \\ TT &\longmapsto 0 \end{aligned}$$

so $X=1$ means $\{HT, TH\} \subseteq \Omega$

$X=1$ is an event

$P(X=1) = \binom{2}{1} p^1 (1-p)^1$ binomial formula

# Joint Distribution

Let $(X, Y)$ be the joint outcome of 2 RVs $X, Y$.

ex

$X$: one draw from [ [1] [2] [2] [3] ]

Given $X = x$, $Y$ = number of heads in $x$ coin tosses.

$$P(X=x, Y=y) = P(Y=y \mid X=x) \cdot P(X=x)$$

$$P(X=1, Y=1) = \underbrace{P(Y=1 \mid X=1)}_{= 1/2} \cdot \underbrace{P(X=1)}_{= 1/4} = \boxed{1/8}$$

Find what the range of values of $X$? 1, 2, 3

$Y$? 0, 1, 2, 3

$P(1, 0)$ $\quad \underbrace{P(Y=0 \mid X=1)}_{= 1/2} \underbrace{P(X=1)}_{= 1/4} = 1/8$

marginal prob of X
$P(x) = \sum_{y \in Y} P(x,y)$

| $Y \backslash X$ | 1 | 2 | 3 | |
|---|---|---|---|---|
| | $P(X=1) = 1/4$ | $1/2$ | $1/4$ | |
| 3 | 0 | 0 | $\frac{1}{8} \cdot \frac{1}{4} = 1/32$ | $P(Y=3) = 1/32$ |
| 2 | 0 | $1/8$ | $3/32$ | $P(Y=2) = 7/32$ |
| 1 | $1/8$ | $1/4$ | $3/32$ | $P(Y=1) = 15/32$ |
| 0 | $1/8$ | $1/8$ | $1/32$ | $P(Y=0) = 9/32$ |

marginal prob of Y
$P(y) = \sum_{x \in X} P(x,y)$

Is X, Y dependent? Yes

$$P(X=1, Y=3) \overset{?}{\neq} P(X=1)\,P(Y=3)$$

$$\underset{0}{\shortparallel} \qquad \underset{1/4}{\shortparallel} \qquad \underset{1/32}{\shortparallel}$$

Defⁿ Two RVs are independent if

$P(Y=y \mid X=x) = P(Y=y)$ for all $x \in \mathcal{X}$, $y \in \mathcal{Y}$

By the multiplication rule,

if $X, Y$ are indep,

$$P(X=x, Y=y) = \underbrace{P(Y=y \mid X=x)}_{= P(Y=y)} P(X=x)$$

so $\boxed{P(X=x, Y=y) = P(X=x)P(Y=y)}$.

Equivalently,

$P(X=x \mid Y=y) = P(X=x)$ for all $x \in \mathcal{X}$, $y \in \mathcal{Y}$.

# extra

$(X_1, X_2)$ has joint distribution:

| | $X_2=0$ | $X_2=1$ | |
|---|---|---|---|
| $X_1=0$ | $\frac{5}{36}$ | $\frac{25}{36}$ | $5/6$ |
| $X_1=1$ | $\frac{1}{36}$ | $\frac{5}{36}$ | $1/6$ |
| | $1/6$ | $5/6$ | |

Is $X_1, X_2$ independent?

check

$P(x,y) = P(x)P(y)$ ✓

$\Rightarrow$ $X, Y$ independent

Note the entries in the cells are $P(X=x, Y=y)$ not $P(X=x \mid Y=y)$

To find $P(X=x \mid Y=y)$ use Bayes rule

$$= \frac{P(X=x, Y=y)}{P(Y=y)}$$

ex

The joint distribution of X and Y is drawn below:

| | | | | |
|---|---|---|---|---|
| | 3/8 | 1/2 | 1/8 | P(x) / P(y) |
| 1 | 1/4 | 1/3 | 1/12 | 2/3 |
| 0 | 1/8 | 1/6 | 1/24 | 1/3 |
| Y / X | 0 | 1 | 2 | |

a) X and Y are independent

b) If we divide both rows by their marginal probability we get the same answer.

c) $P(X = x|Y = 0) = P(X = x|Y = 1)$

d) All of the above