Stat 134 lec 1

Welcome to stat 134.

- section starts tomorrow
- do prelecture reading on bCourses/pages
- download pre-lecture notes
- daily quizzes to help you stay caught up
- weekly HW has an EC question from a previous midterm
- stat 198 adjunct run by Mike Leong

Today

① sec 1.1 equally likely outcomes
② sec 1.3 distributions

① Sec 1.1 Equally likely outcomes

We call the set of all outcomes of an experiment Ω, the outcome space, or the sample space.

let $A \subseteq \Omega$

$$P(A) = \frac{\#A}{\#\Omega}$$

Deck of cards: 4 suits H, C, D, S
13 ranks Ace, 2-10, J, Q, K
52 cards

#5e p10

ex Suppose a deck of cards is shuffled and the top 2 cards are dealt. What is the chance you get at least one ace among the 2 cards

A = get at least one ace among the 2 cards

$$P(A) = 1 - P(A^c) = 1 - \frac{48}{52}\cdot\frac{47}{51} = \boxed{.149}$$

complement rule

no ace in 2 cards

Alternatively

$$P(A) = \underset{\frac{4}{52}\cdot\frac{48}{51}}{\underset{\shortparallel}{P(\text{ace, non ace})}} + \underset{\frac{48}{52}\cdot\frac{4}{51}}{\underset{\shortparallel}{P(\text{non ace, ace})}} + \underset{\frac{4}{52}\cdot\frac{3}{51}}{\underset{\shortparallel}{P(\text{ace, ace})}}$$

$$= \frac{2(4\cdot 48) + 4\cdot 3}{52\cdot 51} = \boxed{.149}$$

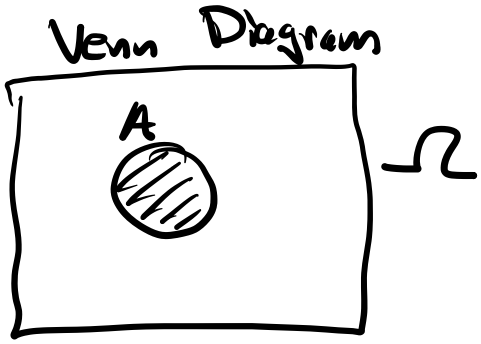
Sec 1.3 Distributions

To define probability we start with an outcome space, Ω, and assign to each element a nonnegative number and require that all numbers add up to 1.

Axioms

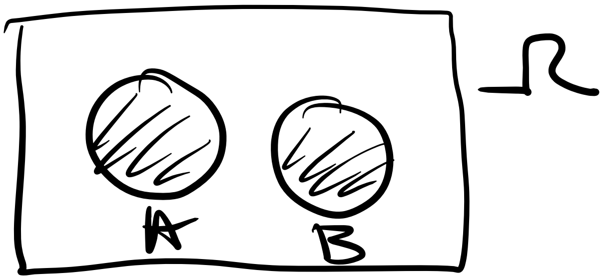
1) $P(A) \geq 0$ for all $A \subseteq \Omega$

2) $P(\Omega) = 1$

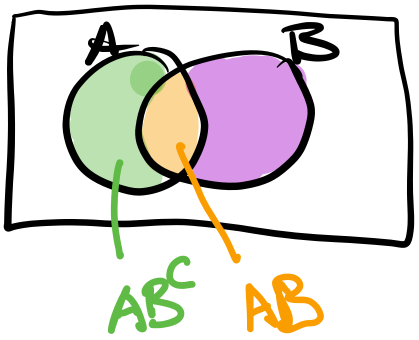


3) If A and B are mutually exclusive sets then $P(A \cup B) = P(A) + P(B)$

(addition rule)



Difference rule



Prove a formula for $P(AB^c)$ in terms of $P(A)$ and $P(AB)$

$$P(AB^c) = P(A) - P(AB)$$

$A = AB \cup AB^c$ disjoint union

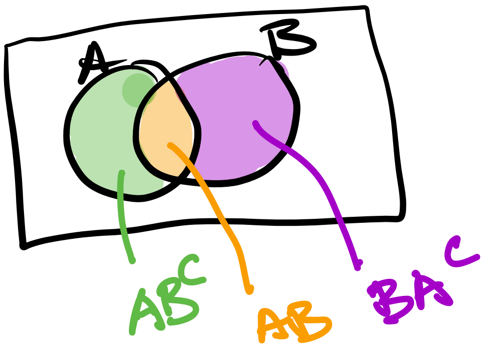
$P(A) = P(AB) + P(AB^c)$

$P(AB^c) = P(A) - P(AB)$

Inclusion exclusion rule

$P(A \cup B) = P(A) + P(B) - P(AB)$

Proof/



$A \cup B = AB^c \cup AB \cup BA^c$ disjoint union

$P(A \cup B) = \underset{P(A) - P(AB)}{\underset{\shortparallel}{P(AB^c)}} + P(AB) + \underset{P(B) - P(AB)}{\underset{\shortparallel}{P(BA^c)}}$ addition rule

$= P(A) + P(B) - P(AB)$ difference rule.

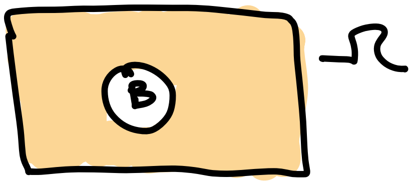
Prove the complement rule

$P(B^c) = 1 - P(B)$

Difference rule

$P(AB^c) = P(A) - P(AB)$

Picture



$\Omega = B \cup B^c$ disjoint union

$$\underset{\underset{1}{\parallel}}{P(\Omega)} = P(B) + P(B^c) \quad \text{addition rule}$$

$\Rightarrow P(B^c) = 1 - P(B).$

or let $A = \Omega$

$$\underset{\underset{P(B^c)}{\parallel}}{P(\Omega B^c)} = \underset{\underset{1}{\parallel}}{P(\Omega)} - \underset{\underset{P(B)}{\parallel}}{P(\Omega B)} \quad \text{difference rule}$$

$\Rightarrow P(B^c) = 1 - P(B)$

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Stat 134

1. A deck of cards is shuffled. What is the chance that the top card is the **king** of spades **or** the bottom card is the **king** of spades

   a $\frac{1}{52}+\frac{1}{52}-\boxed{\frac{1}{52}\times\frac{1}{52}}$ $P(AB)$ ← answer if with replacement,

   b $\frac{1}{52}+\frac{1}{51}$

   c $\frac{1}{52}+\frac{1}{52}-\frac{1}{52}\times\frac{1}{51}$

   [✓ **d**] none of the above $\frac{1}{52}+\frac{1}{52}=\frac{1}{2}$

Uniform distribution

Let $\{x_1, x_2, \dots, x_n\}$ be a finite set.

Suppose the probability of drawing each element is equally likely (i.e each has prob $\frac{1}{n}$)

We say $\{x_1, \dots, x_n\}$ has the uniform distribution.

We write $\text{Unif}(\{x_1, \dots, x_n\})$.

ex $\{1, 1, 2\}$ is a finite set.

$\text{Unif}(\{1,1,2\})$ means 1 has probability $\frac{2}{3}$ and 2 has probability $\frac{1}{3}$.

ex $\overset{7}{\text{Suppose}}$ $\overset{1}{\text{a}}$ $\overset{4}{\text{word}}$ $\overset{2}{\text{is}}$ $\overset{8}{\text{randomly}}$ $\overset{6}{\text{picked}}$ $\overset{4}{\text{from}}$ $\overset{4}{\text{this}}$ $\overset{8}{\text{sentance}}$.

Name the distribution of the length of the word picked?

$\text{Uniform}(\{7, 1, 4, 2, 8, 6, 4, 4, 8\})$

length of word picked

$P(X=7) = 1/9$

$P(X=1) = 1/9$

$P(X=4) = 3/9$