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DSP Exercise Set: Signals & Time-Domain Analysis

Theory Questions

1. Define the difference between energy signals and power signals. Give one real-world example of each.

Energy Signals	Power Signals
An energy signal is a signal if and only if its total energy E is finite and the average power P = 0. $0 < E < \infty$ These signals usually exist for a limited duration. A real-world example of an energy signal is the sound produced when a drum is hit during a concert. The sound exists only for a short duration and then disappears, meaning its total energy is finite.	A power signal is a signal if its average power P is finite and the total energy E = ∞. $0 < P < \infty$ These signals usually last forever (periodic signals). A real-world example of a power signal is the sinusoidal AC voltage supplied to homes. It exists continuously over time and has finite average power but infinite total energy.

2. State the Nyquist sampling criterion and explain what happens if it is not satisfied.

Nyquist Theorem also referred to as the Sampling Theorem is a principle of reproducing a sample rate, that must be at least twice the highest frequency present in the signal. This ensures that there are enough samples taken per unit of time to capture all the details of the original waveform without introducing aliasing, which can cause distortion or artifacts in the reconstructed signal.

$$f_s \geq 2f_m$$

Where:

f_s is frequency signal

f_m is max frequency

If the theorem is not satisfied, aliasing occurs. Aliasing refers to the distortion or unwanted noise that may destroy a signal's integral value.

3. Explain in your own words how time reversal and time shifting affect a signal's graph. Why are these operations important in signal analysis?

Time Reversal	Time Shifting
Flips a signal graph across the vertical axis (t = 0), exchanging positive and negative time. $x(t) \rightarrow x(-\ t)$ Graphically, the signal is mirrored around the vertical axis and shape of the signal does not change, but its orientation in time is reversed. This operation is important because it is a fundamental step in convolution and in the analysis of linear time-invariant (LTI) systems.	Translates the graph along the time axis: A positive shift moves it to the right (delay), and a negative shift moves it to the left (advance). $x(t) \rightarrow x(t\ -\ t_0)$ Graphically, the shape of the signal is not modified, only its position along the time axis changes. Time shifting is important because it models real-world delays in communication systems and signal transmission.

Exercises

1. For each of the following signals, determine whether it is continuous/discrete, periodic/aperiodic, and causal/non-causal:

a) x(t) = sin(2π10t)

- Continuous Time: Defined only at integer values of t.
- Periodic: With frequency f = 10Hz, T = 0.1 s
- Non-Casual: The signal has non-zero values for t<0

b) x[n] = u[n-3]

- Discrete Time: Defined only at integer values of n.
- Aperiodic
- Causal: Since x[n]=0 for n<0

c) x(t) = e^{-t}u(t)

- Continuous Time: Define for every real value of time t
- Aeriodic
- Casual: u(t) forces the signal to be zero for t<0

2. Is the signal x[n] = cos((5π/6)n) periodic? If yes, find its fundamental period.

$$w_0 = \frac{5\pi}{6}$$

$$\frac{2\pi}{w_o} = \frac{2\pi}{\frac{5\pi}{6}} = \frac{12}{5} \rightarrow \text{Rational number}$$

We have a rational number, our signal is periodic.

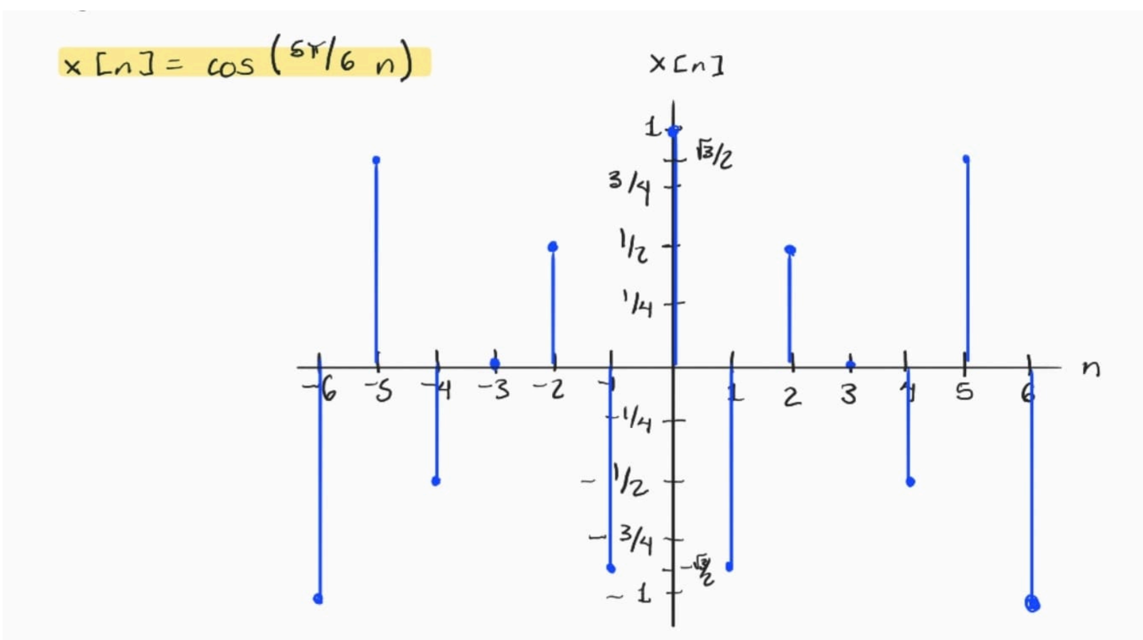
Obtaining our fundamental period:

$\frac{2\pi}{w_o} = \frac{N}{k} \Rightarrow N = \frac{2\pi}{w_o}k = \frac{12}{5}k$ where the minimum value of k for N to be an integer is 5

$$N = \frac{12}{5}(5) = 12$$

Result: N = 12

Using n:[-6,6]



3. A continuous-time signal x(t) = cos(200πt) is sampled at fs = 150 Hz.
- What is the Nyquist rate?
- Will aliasing occur? If so, what is the apparent frequency after sampling?

$2\pi f = 200\pi \Rightarrow f = \frac{200\pi}{2\pi} = 100$

$f_n = 2f_m = 2(100) = 200$

Nyquist rate: 200 Hz

$f_s < 2f_m \Rightarrow 150 < 2(100) \Rightarrow 150 < 200$ is true, so aliasing is going to occur

Obtaining apparent frequency after sampling

Using $0 \leq f_a \leq \frac{f_s}{2}] \Rightarrow [0, \frac{150}{2}] \Rightarrow [0, 75]$

$f_a = |f_o - mf_s|$ where m is an integer chosen to bring f_a within the range. In this case, k = 1

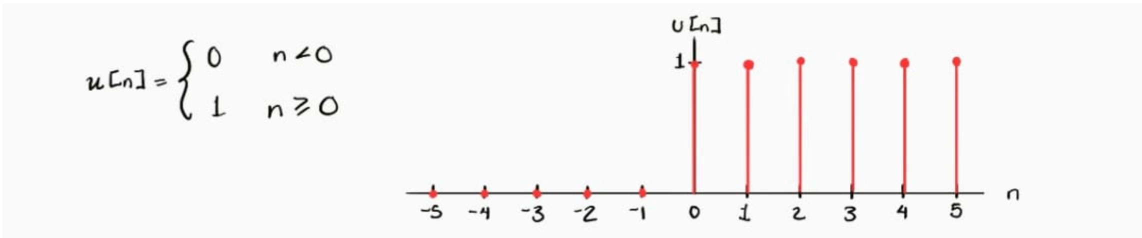
$f_a = |100 - 1(150)| = 50$

Apparent frequency after sampling: 50 Hz

4. Sketch the following signals for -5 ≤ n ≤ 5:
a) u[n]

Discrete Unit Step:

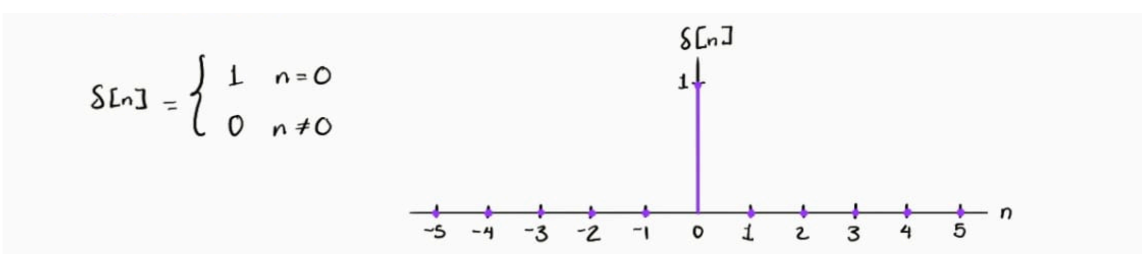
n	u[n]
-5	0
-4	0
-3	0
-2	0
-1	0
0	1
1	1
2	1
3	1
4	1
5	1



b) δ[n]

Discrete Delta Impulse:

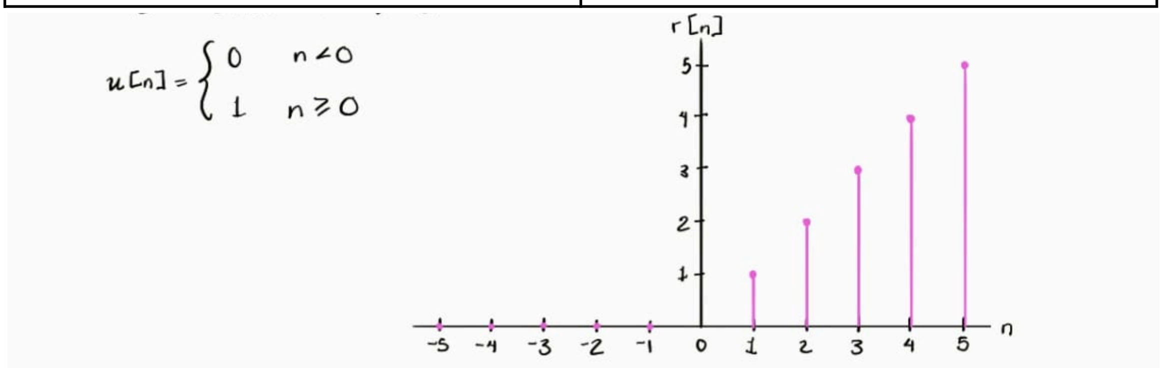
n	u[n]
-5	0
-4	0
-3	0
-2	0
-1	0
0	1
1	0
2	0
3	0
4	0
5	0



c) r[n] = n u[n]

Discrete ramp:

n	u[n]
-5	0
-4	0
-3	0
-2	0
-1	0
0	0
1	1
2	2
3	3
4	4
5	5

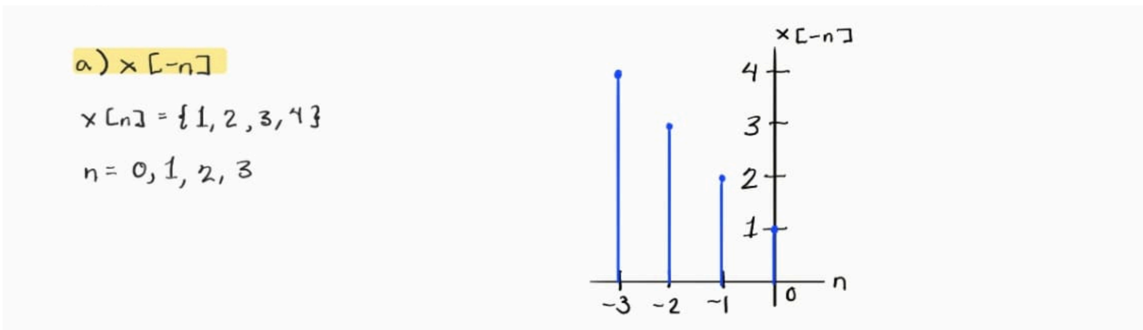


5. Given x[n] = {1,2,3,4} for n=0,1,2,3, compute and sketch x[-n].

n	x[n]
0	1
1	2
2	3
3	4

Knowing that $x[n] \rightarrow x[-n]$ represents time reversal in discrete time, we just need to reflect our signal around n = 0, exchanging positive and negative indices.

n	x[-n]
0	1
-1	2
-2	3
-3	4

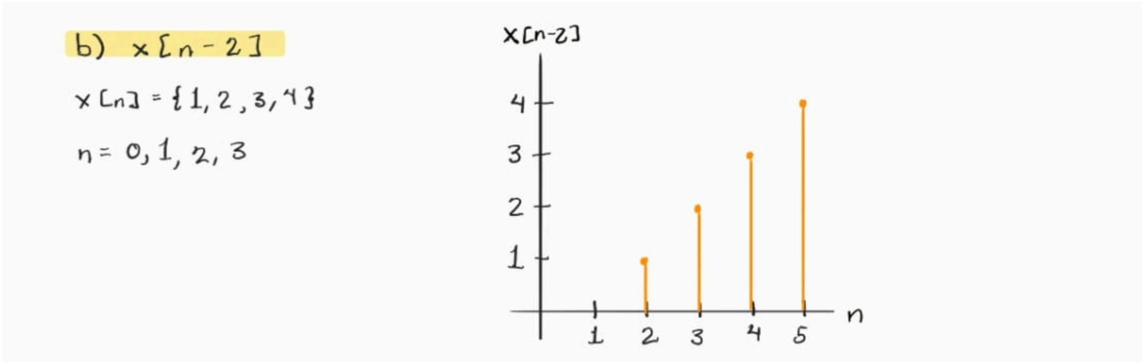


6. For the same signal x[n] = {1,2,3,4}, compute and sketch:
a) x[n-2]

n	x[n]
0	1
1	2
2	3
3	4

Knowing that x[n−k] represents time shifting in discrete time, we just need a delay of 2 samples (shift to the right).

n	x[n-2]
2	1
3	2
4	3
5	4

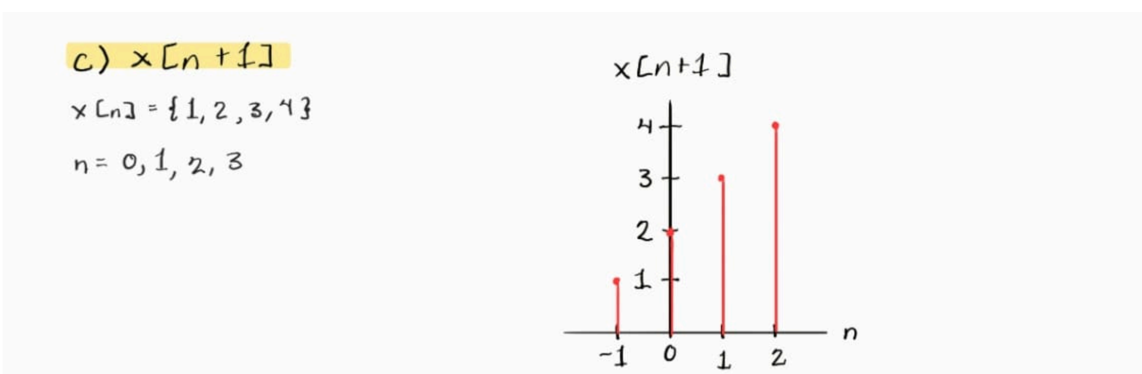


b) x[n+1]

n	x[n]
0	1
1	2
2	3
3	4

Knowing that x[n+k] represents time shifting in discrete time, we just need an advance of 1 sample (shift to the left).

n	x[n+1]
-1	1
0	2
1	3
2	4



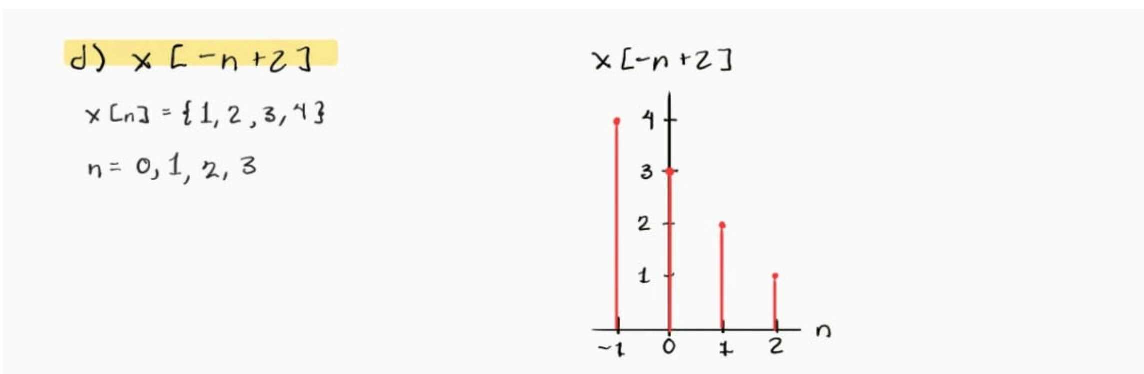
7. Take x[n] = {1,2,3,4}. Compute and sketch:

a) x[-n+2]

n	x[n]
0	1
1	2
2	3
3	4

Knowing that $x[n] \rightarrow x[-\ n]$ represents time reversal in discrete time and x[n−k] represents time shifting in discrete time, we first do an advance of 2 samples (shift to the left) and then reflect our signal around n = 0, exchanging positive and negative indices.

n	x[-n+2]
(time shifting): -2 ⇒ (time reversal): 2	1
(time shifting): -1 ⇒ (time reversal): 1	2
(time shifting): 0 ⇒ (time reversal): 0	3
(time shifting): 1 ⇒ (time reversal): -1	4



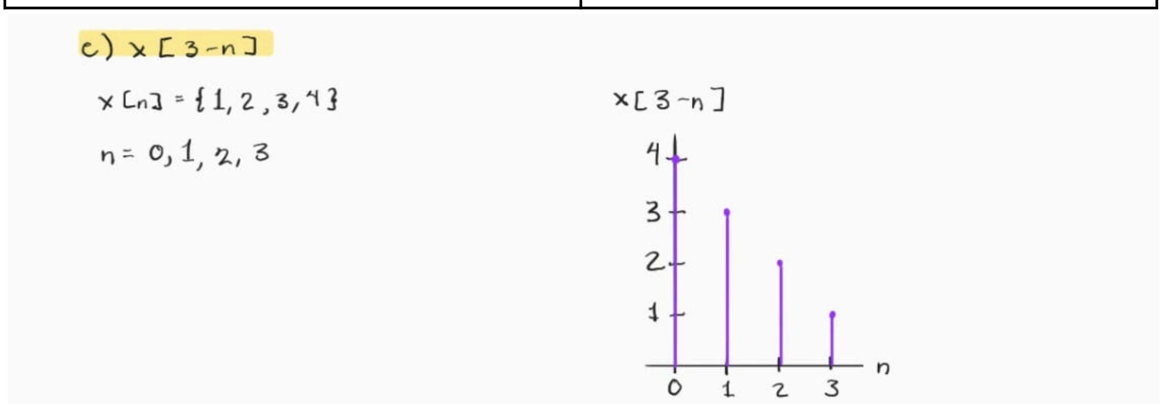
b) x[3-n]

n	x[n]
0	1
1	2

2	3
3	4

Knowing that $x[n] \rightarrow x[-\ n]$ represents time reversal in discrete time and x[n−k] represents time shifting in discrete time, we first do an advance of 3 samples (shift to the left) and then reflect our signal around n = 0, exchanging positive and negative indices.

n	x[3-n]
(time shifting): -3 ⇒ (time reversal): 3	1
(time shifting): -2 ⇒ (time reversal): 2	2
(time shifting): -1 ⇒ (time reversal): 1	3
(time shifting): 0 ⇒ (time reversal): 0	4



8. Determine whether the following signals are energy signals or power signals:
a) x[n] = cos((π/4)n)

$$w_0 = \frac{\pi}{4}$$

$$\frac{2\pi}{w_o} = \frac{2\pi}{\frac{\pi}{4}} = 8 \rightarrow \text{Rational number}$$

We have a rational number, our signal is periodic. A periodic signal never ends, so it has infinite energy.

E = ∞ is equal to a Power Signal

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} cos2(\frac{\pi}{4}n) \Rightarrow P = \frac{1}{2}$$

b) x[n] = (1/2)ⁿ u[n]

Since it decays and "turns off" towards 0, its energy is finite, so it's an energy signal.

E = 4/3

Compute energy/power as appropriate. (Pending)

9. Given x[n] = (0.9)ⁿ u[n]:
- **Is this an energy or power signal?**
- **Compute its total energy.**

Knowing we have a discrete unit step (n < 0: 0 and n >= 0:1), it only exists for n >= 0. Leading to $n \to \infty$ = finite energy. We have an Energy Signal.

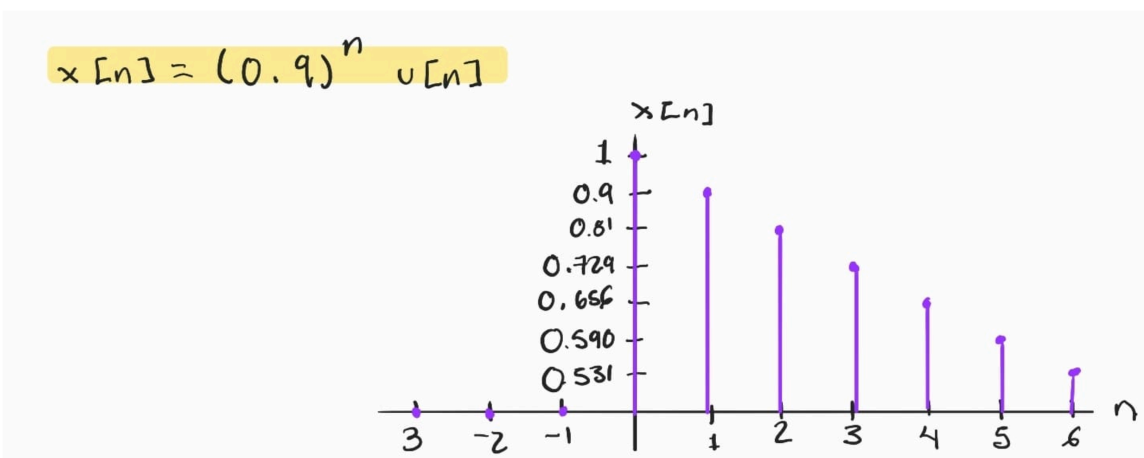
$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 \Rightarrow E = \sum_{n=0}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} \left|(0.9)^n\right|^2 = \sum_{n=0}^{\infty} (0.9)^{2n} = \sum_{n=0}^{\infty} (0.81)^n$$

We know that:

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \qquad where \qquad |r| < 1 \quad and\, r = 0.81$$

$$E = \frac{1}{1-0.81} = \frac{1}{0.19} = \frac{100}{19}$$

$$E = \frac{100}{19}$$



10. A signal x[n] = {1,-1,2,-2} for n=0,1,2,3.
- Plot the original signal.
- Apply time reversal → shift right by 2 samples → scale by factor 3.
- Write the resulting signal explicitly and sketch it.

n	x[n]
0	1
1	-1
2	2
3	-2

Applying time reversal:

n	x[n]
0	1
-1	-1
-2	2
-3	-2

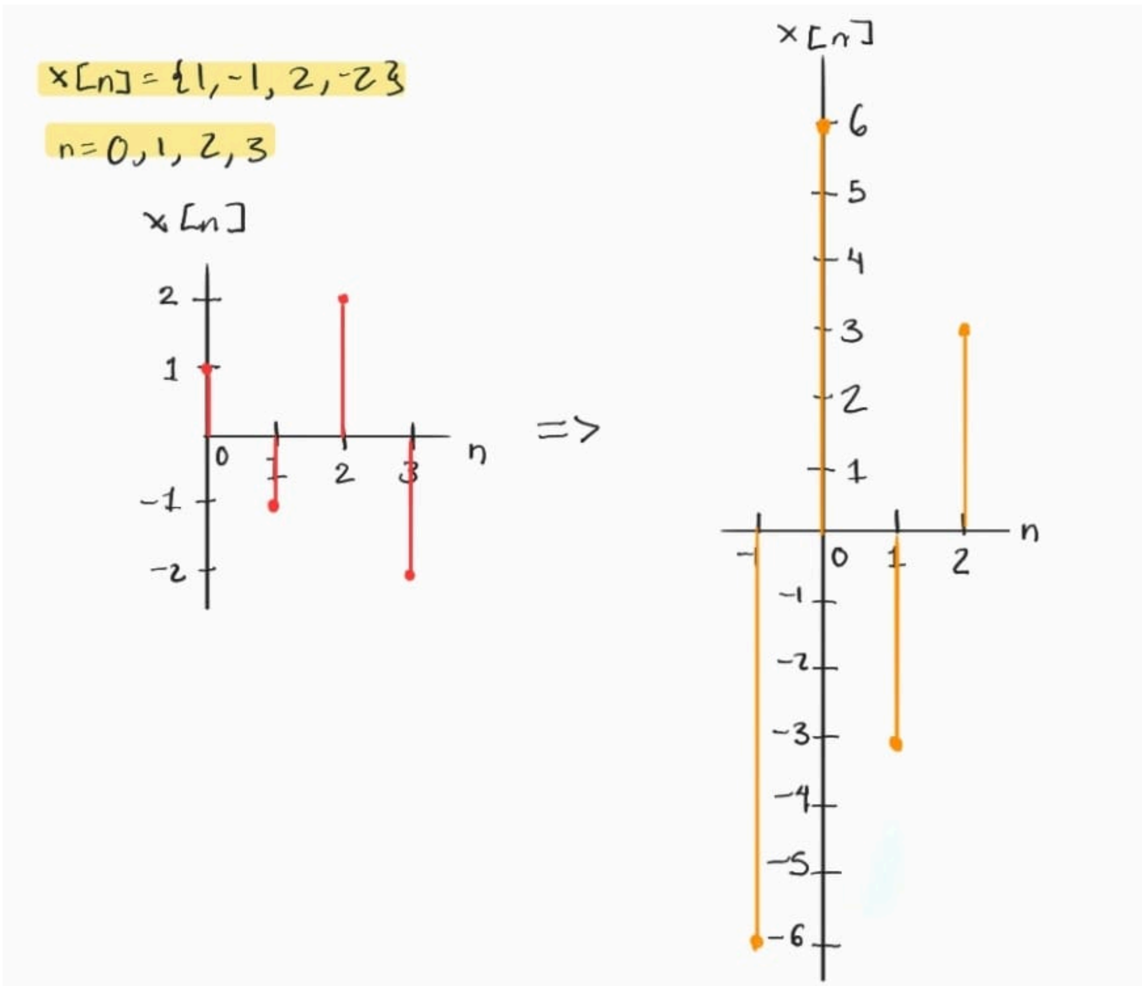
Shifting right by 2 samples:

n	x[n]
2	1
1	-1
0	2
-1	-2

Scale by factor 3:

n	x[n]
2	3

1	-3
0	6
-1	-6



11. Compute the mean and RMS of x[n] = {2, -2, 2, -2} for n=0...3.

Obtaining the mean in discrete time:

$$\mu = \frac{1}{N}\sum_{n=1}^{N} x[n] \qquad where\ N = 4$$

$$\mu = \frac{1}{4}(2 + (-2) + 2 + (-2)) = \frac{1}{4}(0) = 0 \qquad \mu = 0$$

Obtaining RMS in discrete time:

$$x_{rms} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}(x[n])^2} = \sqrt{\frac{1}{4}\sum_{n=1}^{N}(2^2 + (-2)^2 + 2^2 + (-2)^2)}$$

$$x_{rms} = \sqrt{\frac{1}{4}\sum_{n=1}^{N}(4 + 4 + 4 + 4)} = \sqrt{\frac{1}{4}(16)} = \sqrt{4} = 2$$

$$x_{rms} = 2$$

12. Compute the autocorrelation of x[n] = {1,2,1} and cross-correlation with y[n] = {1,0,-1}.

- Autocorrelation formula:

$$R_{xx}[k] = \sum_{n=-\infty}^{\infty} x[n]x[n-k]; \quad k = 0, \ \pm 1, \pm 2, \ \pm 3$$

We know that x[n] has 3 elements, so k = {-2, -1, 0, 1, 2}. We solve for every k

$$R_{xx}[-2] = \sum_{n=0}^{2} x[n]x[n-(-2)] = \sum_{n=0}^{2} x[n]x[n+2] = x[0]x[0+2] + x[1]x[1+2] + x[2]x[2+2]$$

$$R_{xx}[-2] = x[0]x[2] + x[1]x[3] + x[2]x[4] = 1(1) + 2(0) + 1(0) = 1$$

$$R_{xx}[-1] = \sum_{n=0}^{2} x[n]x[n-(-1)] = \sum_{n=0}^{2} x[n]x[n+1] = x[0]x[0+1] + x[1]x[1+1] + x[2]x[2+1]$$

$$R_{xx}[-1] = x[0]x[1] + x[1]x[2] + x[2]x[3] = 1(2) + 2(1) + 1(0) = 4$$

$$R_{xx}[0] = \sum_{n=0}^{2} x[n]x[n-(0)] = \sum_{n=0}^{2} x[n]x[n-0] = x[0]x[0-0] + x[1]x[1-0] + x[2]x[2-0]$$

$$R_{xx}[0] = x[0]x[0] + x[1]x[1] + x[2]x[2] = 1^2 + 2^2 + 1^2 = 6$$

$$R_{xx}[1] = \sum_{n=0}^{2} x[n]x[n-(1)] = \sum_{n=0}^{2} x[n]x[n-1] = x[0]x[0-1] + x[1]x[1-1] + x[2]x[2-1]$$

$$R_{xx}[1] = x[0]x[-1] + x[1]x[0] + x[2]x[1] = 1(0) + 2(1) + 1(2) = 4$$

$$R_{xx}[2] = \sum_{n=0}^{2} x[n]x[n-(2)] = \sum_{n=0}^{2} x[n]x[n-2] = x[0]x[0-2] + x[1]x[1-2] + x[2]x[2-2]$$

$R_{xx}[2] = x[0]x[-2] + x[1]x[-1] + x[2]x[0] = 1(0) + 2(0) + 1(1) = 1$

$R_{xx}[k] = \{1, 4, 6, 4, 1\}$ $\; where\ 6\ is\ k = 0$

- Crosscorrelation formula:

$$R_{xy}[k] = \sum_{n=-\infty}^{\infty} x[n]y[n-k]; \quad k = 0, \ \pm 1, \pm 2, \ \pm 3$$

We know that x[n] has 3 elements, so k = {-2, -1, 0, 1, 2}. We solve for every k:

$$R_{xy}[-2] = \sum_{n=0}^{2} x[n]y[n-(-2)] = \sum_{n=0}^{2} x[n]y[n+2] = x[0]y[0+2] + x[1]y[1+2] + x[2]y[2+2]$$

$$R_{xy}[-2] = x[0]y[2] + x[1]y[3] + x[2]y[4] = 1(-1) + 2(0) + 1(0) =- 1$$

$$R_{xy}[-1] = \sum_{n=0}^{2} x[n]y[n-(-1)] = \sum_{n=0}^{2} x[n]y[n+1] = x[0]y[0+1] + x[1]y[1+1] + x[2]y[2+1]$$

$$R_{xy}[-1] = x[0]y[1] + x[1]y[2] + x[2]y[3] = 1(0) + 2(-1) + 1(0) =- 2$$

$$R_{xy}[0] = \sum_{n=0}^{2} x[n]y[n-(0)] = \sum_{n=0}^{2} x[n]y[n-0] = x[0]y[0-0] + x[1]y[1-0] + x[2]y[2-0]$$

$$R_{xy}[0] = x[0]y[0] + x[1]y[1] + x[2]y[2] = 1(1) + 2(0) + 1(-1) = 0$$

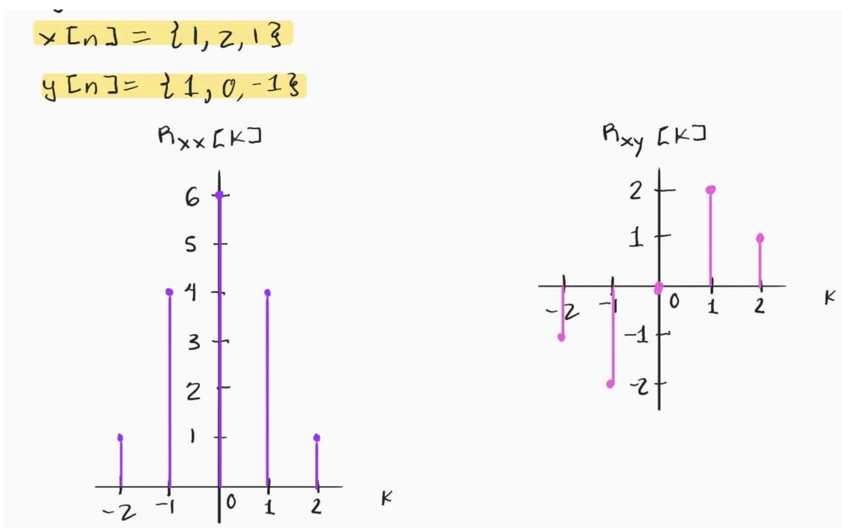
$$R_{xy}[1] = \sum_{n=0}^{2} x[n]y[n-(1)] = \sum_{n=0}^{2} x[n]y[n-1] = x[0]y[0-1] + x[1]y[1-1] + x[2]y[2-1]$$

$$R_{xy}[1] = x[0]y[-1] + x[1]y[0] + x[2]y[1] = 1(0) + 2(1) + 1(0) = 2$$

$$R_{xy}[2] = \sum_{n=0}^{2} x[n]y[n-(2)] = \sum_{n=0}^{2} x[n]y[n-2] = x[0]y[0-2] + x[1]y[1-2] + x[2]y[2-2]$$

$$R_{xy}[2] = x[0]y[-2] + x[1]y[-1] + x[2]y[0] = 1(0) + 2(0) + 1(1) = 1$$

$R_{xy}[k] = \{-1, -2, 0, 2, 1\}$



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