TC5 - Image Denoising in Wavelet Domain

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***Abstract*— This project aims at solving an inpainting problem using various and invariant wavelets transform with different thresholds. We evaluate the denoising performance using the measure of SNR(dB), which measures the estimation quality.**

We implemented the FISTA algorithm with the soft threshold at the first time, then with hard threshold and empirical Wiener. Consequently, soft thresholding gives the best performance in most cases. However, it requires relatively high threshold values for hard thresholding. On top of that, we explored the sparse representation with invariant wavelets, which improved the performance slightly. Finally, we tried different noise levels and painting levels, and it proved that as the noise level increases, it requires a larger threshold to fit the problem.

I. INTRODUCTION

A. Inpainting Problem

The inpainting problem consists in filling in missing or damaged image information. In the inpainting problem, the user has to select the area to be filled in so that the missing region does not stand out with respect to its surroundings. Sometimes, the areas missing or damaged in an image cannot be easily classified in an objective way.

For our project, we used the classical image processing image Barbara:

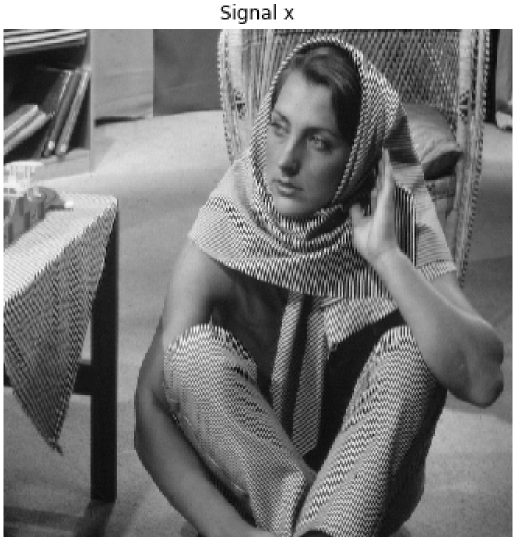

Fig. 1. Original image Barbara

We simulated the inpainting problem based on it by introducing a binary matrix A which has value $a_{i,j} = 0$ for missing pixel and $a_{i,j} = 1$ if the pixel is measured. A is of the same size of the image, with a parameter p controlling the Bernoulli law.

Besides, we added some white Gaussian noise $b \sim \mathcal{N}(0, std)$, thus we have the final simulation of observation y as:

$$y = A \times x + b \tag{1}$$

where all the computations above are element-wise.

During the experiments, we used the observation of simulation with Bernoulli parameter $p = 0.7$ for matrix A, and $std = 0.03$ for the added Gaussian noise. The signal-noise ratio equals 5.18

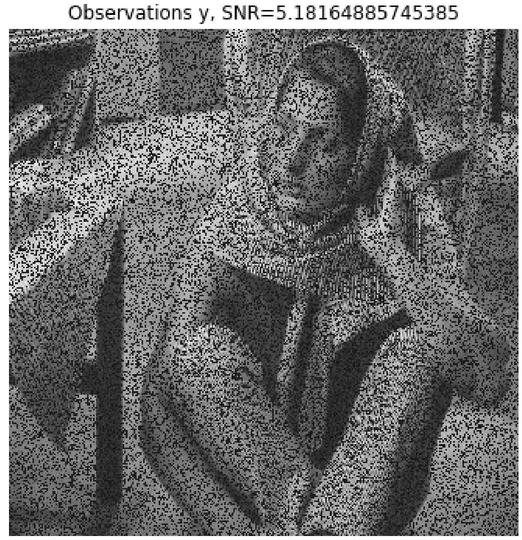

Fig. 2. Simulation of inpainting problem (p = 0.7, std = 0.03)

B. Fast Iterative Shrinkage/Thresholding algorithm (FISTA)

The original problem y can be computed as $y = A\Phi\alpha + b$. Here Φ is a Dictionary built on a bunch of orthogonal basis such as Wavelets or time-frequency transforms, and α are the synthesis coefficients. By using the Dictionary, we represent the original signal in a sparse way with few coefficients:

$$x = \Phi\alpha \tag{2}$$

During our experiments, we firstly applied the orthogonal wavelet sparsity, and then a translation of invariant wavelet sparsity. The difference is that the invariant wavelet transform considers Φ as a redundant tight frame where all the basis wavelet are normalized to have norm 1 (i.e., $||\Phi_m||$=1).

The goal is to have a sparse representation of our reconstructed signal, thus the problem can be well defined as a regularization regression problem:

$$\alpha = \frac{1}{2}||y - A\Phi\alpha||_2^2 + \lambda||\alpha||_1 \tag{3}$$

We chose to use the $l1$ norm as the regularization term because we want to have fewer number of coefficients.The solution is given by the so-called Soft-Thresholding operator :

$$\alpha = \mathcal{S}_\lambda(y) = y(1 - \frac{\lambda}{|y|})^+ \tag{4}$$

where $(x)^+ = \max(0,x)$, and all the operation are applied element-wise.

The algorithm is applied as the following structure:

Algorithm 1: FISTA

initialization: $\alpha^{(0)} = z^{(0)} = 0, L \le ||A\Phi||^2, t = 0$
while *not convergence* **do**
  $\alpha^{(t+1)} = \mathcal{S}_{\lambda/L}(z^{(t)} + \frac{1}{L}\Phi^* A^*(y - A\Phi z^t))$
  $z^{t+1} = \alpha^{(t+1)} + \frac{t}{t+5}(\alpha^{(t+1)} - \alpha^{(t)})$
  t = t+1
end

The output that we would retrieve are the synthesis coefficients, with the help of wavelet Dictionary we can finally inpaint the damaged picture as in equation (2)

Moreover, the soft threshold computation in the algorithm can be replaced by any thresholding rules, which help to fit in different cases. For example, we have hard threshold and empirical Wiener:

- Hard Thresholding : $\mathscr{H}_\lambda(\alpha) = \alpha \mathbf{1}_{|\alpha|>\lambda}$
- Empirical Wiener: $S_\lambda^{EW}(\alpha) = \alpha\left(1 - \frac{\lambda^2}{|\alpha|^2}\right)^+$

II. EXPERIMENTS

A. Inpainting using Orthogonal Wavelet Sparsity

In this part, we will present the results of variant wavelets sparsity representation. And we will also demonstrate the results with the invariant orthogonal wavelet basis in the section B.

1) Soft thresholding:

We simply implemented the soft threshold method, and we set $L = 1$. By default, we set the threshold value $= 0.1$, and we have the inpainting image as in Figure 3.

Besides, we also tried finding out the best value of λ by going through many choices. The results are shown in Figure 4. and we can see that when we have set $\lambda = 0.03$ we have the best performance on signal noise level.

Fig. 3. Reconstructed image with soft threshold ($\lambda = 0.1$)

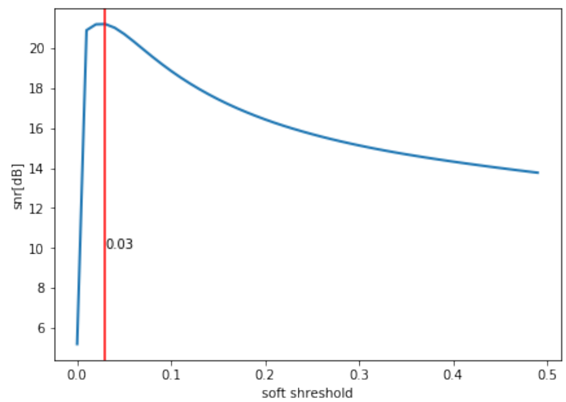

Fig. 4. SNR with different values of soft threshold λ

2) Hard thresholding:

As its name tells, the hard threshold works in a way cruder and less smooth than the soft threshold. So in our case, which requires doing a smooth regression, it did not work that well. We set $\lambda = 0.1$ by default as always and we get the reconstructed image in Figure 5.

On top of that, we also went through many possible of choices of λ to pick the one which fits best our problem. The results are shown in Figure 6.

3) Empirical Wiener:

The Empirical Wiener is also called non-negative garrote shrinkage. It preserves the energy in the significant coefficients while having the same support as the lasso. The corresponding results are shown in Figure 7. and Figure 8.

In conclusion, the soft thresholding gives the best value of SNR, which goes beyond 21 dB, then follows the Empirical Wiener and finally the hard thresholding. For the best deals of λ of each thresholding method, soft thresholding has the smallest value as $\lambda = 0.03$,

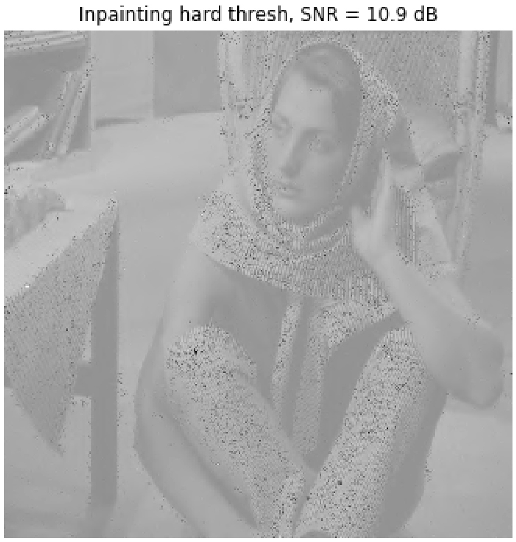

Fig. 5. Reconstructed image with hard threshold ($\lambda = 0.1$)

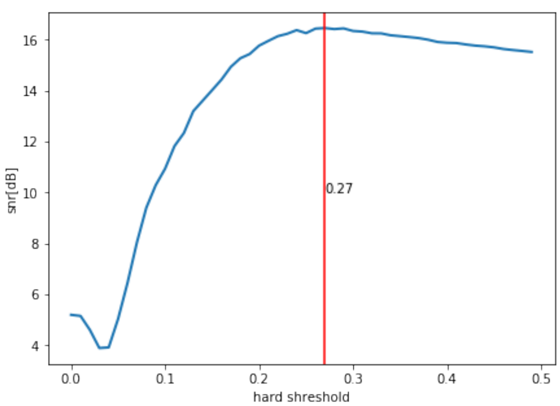

Fig. 6. SNR with different values of hard threshold λ

Fig. 7. Reconstructed image with EM threshold ($\lambda = 0.1$)

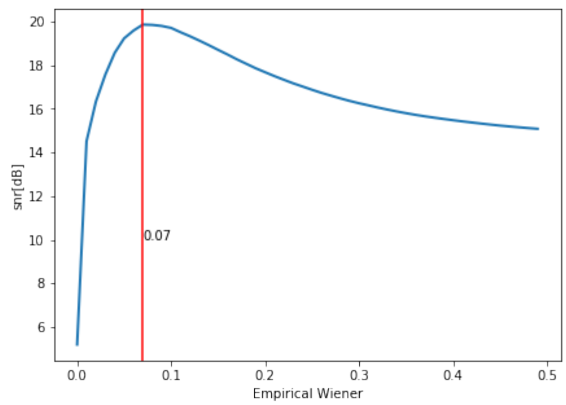

Fig. 8. SNR with different values of EM threshold λ

then comes that of the EM $\lambda = 0.07$. However, the hard thresholding has $\lambda = 0.27$, which is much higher than the previous ones.

B. Inpainting using invariant Orthogonal Wavelet Sparsity

Orthogonal sparsity performs a poor regularization because of the lack of translation invariance. The invariant orthogonal wavelet sparsity ensures each basis vector is normalized, allowing Ψ to be a redundant tight frame of translation invariant wavelets.

We have done almost the same explorations for soft and hard thresholding methods. This time turns out that the hard threshold works better than the soft threshold (Figure 9. - 12.), while the discrepancy of the best fit λ is smaller.

Fig. 9. Reconstructed image with soft threshold ($\lambda = 0.1$)

C. Influence of p and noise level

In this last section, we explored how different values of p and std of Gaussian noise level will affect the solution of inpainting problem. To remark, we only implemented

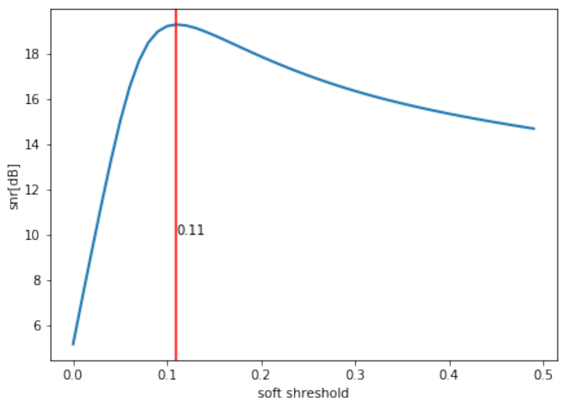

Fig. 10. SNR with different values of soft threshold λ

Fig. 11. Reconstructed image with hard threshold ($\lambda = 0.1$)

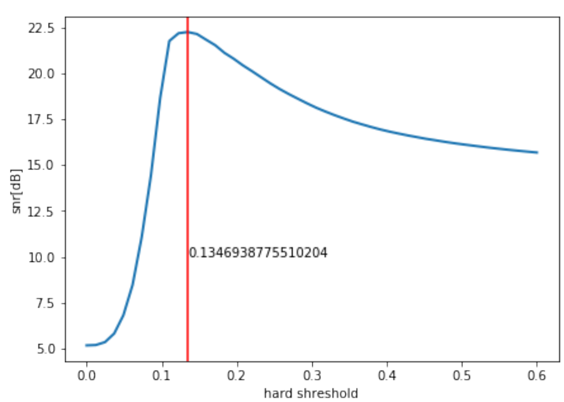

Fig. 12. SNR with different values of hard threshold λ

the soft thresholding method in this part. First, we tried with different p on fixing std = 0.01:

As we can see from Figure 14., the variation of p does not bother for the best threshold value $lambda$. For all the five p values, all of them gave the best choice of λ at about 0.1

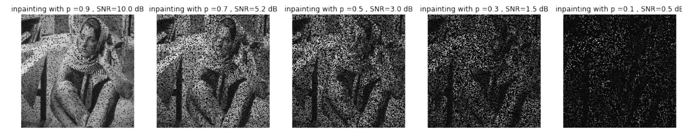

Fig. 13. Inpainting problems with different p

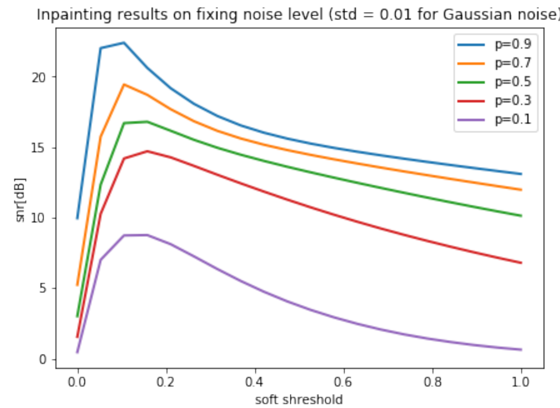

Fig. 14. SNRs with different soft threshold λ of different p value

Then we tried modifying the standard derivation of Gaussian noise std, from 0.01 to 0.5 on fixing p = 0.9. And as we can see from the results below, the best values of λ tend to move to the right side to have larger values as the noise level increases, which means it demands a higher threshold to inpaint noisier images. It means we will only keep the coefficients of the wavelet we are very sure of them to appear in the image,

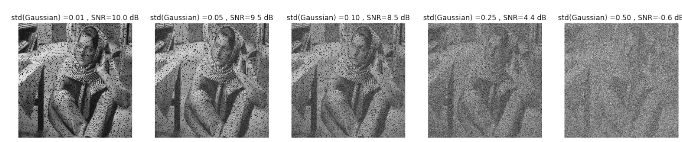

Fig. 15. Inpainting problems with different noise level

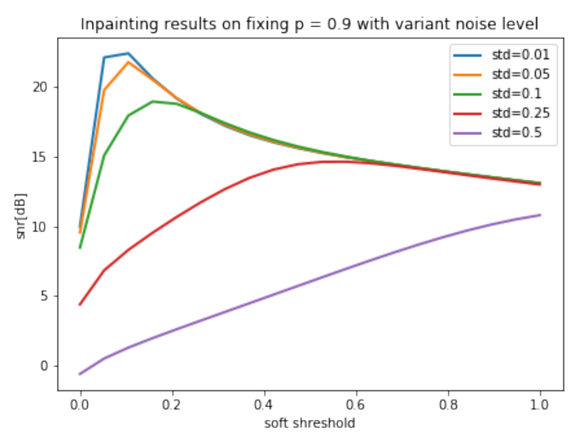

Fig. 16. SNRs with different soft threshold λ of different p value