The Chinese University of Hong Kong, Shenzhen
School of Data Science

MAT3007 · Homework 6

Due: 11:59 pm, Nov 24, 2022

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. If some problems need coding, you should paste your code in the file. Please upload only **one** file(pdf). The file name should be in the format **last name-first name-hw6**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.

Problem 1 (25pts). Consider the following function:

$$f(x, y, z) = 3x^2 - 2x - 2xy + 3y^2 - 2y - 2zy + 3z^2 - 2z - 2xz$$

(a). Considering the 1st-order necessary condition, try to find the candidate minimizers of $f(x, y, z)$.

(b). Considering the 2nd-order sufficient condition, whether these candidates are indeed local minimizers?

(c). Is (0,0,0) a local minimizer? Why?

Problem 2 (25pts). Given a symmetric matrix $A \in R^{n\times n}$, consider the following problem:

$$\begin{array}{rl} \min_{x\in R^n} & x^T Ax \\ \text{subject to} & 2 - x^T x = 0 \end{array}$$

(a). Give the KKT conditions of this problem.

(b). If A is positive definite without repeated eigenvalues, how many different KKT points are there at most?

(c). If A is positive definite without repeated eigenvalues, what is the minimum value of this problem, and how many local minimizers? (Hint:According to Rayleigh quotient, $min\{x^T Ax/(x^T x)\} = \lambda_{min}$, where λ_{min} is the minimum eigenvalue of A)

Problem 3 (25pts). Construct the KKT conditions for the following linear program:

$$\begin{aligned} \text{maximize} \quad & 3x_1 + x_2 + 4x_3 \\ \text{subject to} \quad & x_1 + 3x_2 + x_3 \leq 5 \\ & x_1 + 2x_2 + 2x_3 \leq 8 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Problem 4 (25pts). Construct the KKT conditions for the following nonlinear program:

$$\begin{aligned} \text{minimize} \quad & x_1 ln(x_1) + (x_2 - 2)^2 + x_3 \\ \text{subject to} \quad & x_1 + x_2 \leq 3 \\ & x_3 - x_2^2 \geq 3 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$