MAT3007 · Homework 2

Due: noon (12pm), October 9

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. If some problems need coding, you should paste your code in the file. Please upload only one file(pdf). The file name should be in the format **last name-first name-hw2. Any nonstandard assignment will not be graded.**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.

Problem 1 (20pts). Reformulate NLP as LP

Reformulate the following problems as linear programming

$$\begin{aligned} \text{minimize} \quad & 2x_2 + |x_1 - x_3| \\ \text{s.t.} \quad & |x_1 + 2| + |x_2| \le 5 \\ & x_3^2 \le 1 \end{aligned}$$

Please also write down its standard form.

Problem 2 (35pts). Basic solutions and basic feasible solutions

Consider the following linear optimization problem:

$$\begin{aligned} \text{maximize} \quad & x_1 + 2x_2 + 4x_3 \\ \text{s.t.} \quad & x_1 + x_3 \le 8 \\ & x_2 + 2x_3 \le 15 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

(a) Transform it into standard form;

(b) Argue without solving this LP that there must exist an optimal solution with no more than 2 positive variables;

(c) List all the basic solutions and basic feasible solutions (of the standard form);

(d) Find the optimal solution by using the results in step (c).

Problem 3 (45pts). A Robust LP Formulation

In this exercise, we consider the following optimization problem:

$$\min_{x \in \mathbf{R}^n} c^\top x \quad \text{subject to} \quad \|Ax - b\|_\infty \leq \delta, \quad x \geq 0 \tag{1}$$

where $A \in \mathbf{R}^{m\times n}$, $b \in \mathbf{R}m$, $c \in \mathbf{R}n$, and $\delta \geq 0$ are given and $\|y\|_\infty = \max_{1\leq i\leq p} |y_i|$ denotes the maximum norm of a vector $y \in \mathbf{R}^p$. In the case $\delta = 0$, problem (1) coincides with the standard form for linear programs. The choice $\delta > 0$ can be useful to model situations where A and/or b are not fully or exactly known, e.g., when A and/or b contains certain uncertainty (can be caused by noise). In this case, problem (1) belongs to the so-called robust optimization.

(a) Rewrite the optimization problem (1) as a linear problem.

(b) We now consider a specific application of problem (1).

The fruit store in Pandora is producing two different fruit salads A and B. The smaller fruit salad A consists of "1/4 mango, 1/8 pineapple, 3 strawberries"; the larger fruit salad B consists of "1/2 mango, 1/4 pineapple, 1 strawberry". The profits per fruit salad and the total number of fruits in stock are summarized in the following table:

	Mango	Pineapple	Strawberry	Net profit
Fruit salad A	1/4	1/8	3	10 RMB
Fruit salad B	1/2	1/4	1	20 RMB
Stock / Resources	25	10	120	

Suppose all fruits need to be processed and *completely used* to make the fruit salads A and B.

Given these constraints, formulate a linear program to maximize the total profits of the fruit store. Show that this program can be expressed in standard form

$$\min_{x \in \mathbf{R}^n} c^\top x \quad \text{subject to} \quad Ax = b, \quad x \geq 0,$$

with $n = 2$ and $m = 3$. In addition, is this linear programming solvable?

Note: Since we want to produce "complete fruit salads", the variables x_1 and x_2 should actually be modeled as integer variables: $x_1, x_2 \in \mathbb{Z}$. However, since we do not know how to deal with these integer constraints in general at this moment, you may just ignore them for now.

(c) One of the employee found some additional fruits in a storage crate and the manager of the fruit shop decides to determine the production plan by using the robust formulation (1). Consider the robust variant of the problem in part (b) with $\delta = 5$.

- Sketch the feasible set of this problem.

- Solve the problem graphically, i.e., Calculate the optimal value and the optimal solution set.
- Which constraints are active in the solution?
- Find one integer solution of this problem.