The Chinese University of Hong Kong, Shenzhen
School of Data Science

MAT3007 · Homework 9

Due: 11:59 pm, Dec 16, 2022

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. If some problems need coding, you should paste your code in the file. Please upload only **one** file(pdf). The file name should be in the format **last name-first name-hw6**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.

Problem 1 (30pts).
Use the branch-and-bound method to solve the following integer program.

$$\begin{array}{lll} \text{maximize} & 2x + y & \\ \text{subject to} & -3x + 2y & \leq 5 \\ & -x - 2y & \leq -2 \\ & 5x + 2y & \leq 17 \\ & x, y \in \mathbb{Z}. & \end{array}$$

You are allowed to use an LP solver to solve each of the relaxed linear program. Please specify the branch-and-bound tree and what you did at each node .

Problem 2 (30pts).
Consider a seller who sells m different products. For product j, there are B_j units in inventory. There are n customers, each customer i is interested in buying a bundle of the product S_i, where $S_i \subseteq \{1, ..., m\}$ and is willing to pay a price v_i for it. For each customer, the seller can only decide to accept his entire request S_i or reject him. The objective of the seller is to maximize the revenue.

- Formulate this problem as an integer program.
- Consider the following example $B_1 = 1$, $B_2 = 2$, $B_3 = 3$, $S_1 = \{1, 2\}$, $v_1 = 2$, $S_2 = \{3\}$, $v_2 = 1$, $S_3 = \{1, 3\}$, $v_3 = 3$, $S_4 = \{2, 3\}$, $v_4 = 2$, $S_5 = \{2\}$, $v_5 = 2$. What is one of the optimal solution to the LP (Linear programming) and IP respectively? What is the integrality gap?

Problem 3 (40pts).
Suppose we have a set of n many items and a set of m different knapsacks. For each item i and knapsack j, the following information is given:

- The item i has value (preference) v_i.
- The weight of item i is a_i.
- The capacity of knapsack j is at most C_j.

a) Formulate an integer program to maximize the total value of items that can be packed in the different knapsack while adhering to the capacity constraint (i.e., the total weight of items in each bag j is not allowed to be larger than C_j).

Hint: You can introduce variables x_{ij} to denote whether item i is placed in knapsack j.

b) Consider the following list of items and bags:

Item	Laptop	T-Shirt	Swim. Trunks	Sunglasses	Apples	Opt. Book	Water
Value	2	1	3	2	1	4	2
Weight	2	0.5	0.5	0.1	0.5	1	1.5

Knapsack 1	Knapsack 2
$C_1 = 3$	$C_2 = 2$

Formulate the corresponding IP in that case. What are the optimal solutions to the IP and its LP relaxation (you can use `MATLAB` or `CVX` to solve the problems)? Is there an integrality gap in this case?