



MAT3007 · Homework 5
Due: 11:59:59 pm, November 15

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. Please upload a file or a zip file. The file name should be in the format **last name-first name-hw5**. **Any nonstandard assignment will not be graded..**
- The homework must be written in English.
- **Late submission will not be graded.**
- Each student **must not copy** homework solutions from another student or from any other source.

Problem 1 (50pts). Consider the following linear program:

$$\begin{array}{llllllll} \text{maximize} & 3x_1 + 4x_2 + 3x_3 + 6x_4 & & & & & & \\ \text{subject to} & 2x_1 + x_2 - x_3 + x_4 & & & \geq & 12 & & \\ & x_1 + x_2 + x_3 + x_4 & & & = & 8 & & (1) \\ & -x_2 + 2x_3 + x_4 & & & \leq & 10 & & \\ & x_1, x_2, x_3, x_4 & & & \geq & 0. & & \end{array}$$

After transforming the problem into standard form and apply Simplex method, we obtain the final tableau as follow:

B	0	2	9	0	3	0	36
1	1	0	-2	0	-1	0	4
4	0	1	3	1	1	0	4
6	0	-2	-1	0	-1	1	6

- a) Derive the dual problem of the linear program (1) and calculate a dual solution based on complementarity conditions. Given that the optimal solution to the primal solution is unique, investigate whether the dual solution is unique.
- b) Do the optimal solution and the objective function value change if we
- decrease the objective function coefficient for x_3 to 0?
 - increase the objective function coefficient for x_3 to 9?
 - decrease the objective function coefficient for x_4 to 5?
 - increase the objective function coefficient for x_1 to 7?

- e) Find the possible range for adjusting the coefficient 8 of the second constraint such that the current basis is kept optimal.

Problem 2 (50pts). An insurance company is introducing three products: special risk insurance, mortgage insurance, and long-term care insurance. The expected profit is \$500 per unit on special risk insurance, \$250 per unit on mortgage insurance and \$600 per unit on long term care insurance. The work requirements are as follows:

Department	Working hours per unit			Working hours available
	Special risk	Mortgage	Long-term care	
Underwriting	2	1	1	240
Administration	3	1	2	150
Claims	1	2	4	180

The management team wants to establish sales quotas for each product to maximize the total expected profit.

1. Formulate this problem as a linear optimization problem. Specify the decision variables, objective function, and constraints.
2. After solving the problem, the final simplex tableau (for the standard form) is given as below (the variables are in the natural order as in the description of the problem):

B	0	50	0	0	140	80	35400
4	0	0.5	0	1	-0.7	0.1	153
1	1	0	0	0	0.4	-0.2	24
3	0	0.5	1	0	-0.1	0.3	39

Show the dual variables corresponding to the services of the three departments. Using complementarity conditions to explain why mortgage insurance is not sold.

3. Find the range of working hours available for underwriting to keep the current basis optimal.
4. Find the range of the expected profit on long-term care insurance such that the current basis remains optimal.