Write the equation for a binary lens.

```
zeta = z + m1 / (Conjugate[z1] - Conjugate[z]) + m2 / (Conjugate[z2] - Conjugate[z])
```

$$z + \frac{m1}{-\text{Conjugate}[z] + \text{Conjugate}[z1]} + \frac{m2}{-\text{Conjugate}[z] + \text{Conjugate}[z2]}$$

Write the conditions for m1 and m2 to work in parameter space involving m and dm.

```
m1 = m - dm
m2 = m + dm
```

$$-dm + m$$

$$dm + m$$

Take the conjugate of the binary lens equation. Solve for the conjugate of z, and substitute this back into binary lens equation. Assume the lensing bodies lie on the real axis, and that the masses are real. Then ζ is equal to the following expression:

```
Refine[zeta /. Solve[Conjugate[zeta] == Conjugate[ζ], Conjugate[z]], {m, dm, z1, z2} ∈ Reals]
```

$$\left\{z + \frac{-dm + m}{z1 + \frac{-dm+m}{-z+z1} + \frac{dm+m}{-z+z2} - \text{Conjugate}[\zeta]} + \frac{dm + m}{\frac{-dm+m}{-z+z1} + z2 + \frac{dm+m}{-z+z2} - \text{Conjugate}[\zeta]}\right\}$$

Move ζ to other side and combine the expression into one fraction. Collect only the numerator (since this polynomial is equal to zero) into powers of z, and mulitply by -1 for convention. This is the form of the polynomial in a general coordinate system.

$$\mathbf{pgeneral = Collect\Big[-Numerator\Big[Together\Big[-\zeta + z + \frac{-dm+m}{z1 + \frac{-dm+m}{-z+z1} + \frac{dm+m}{-z+z2} - Conjugate[\zeta]} + \frac{dm+m}{\frac{-dm+m}{-z+z1} + z2 + \frac{dm+m}{-z+z2} - Conjugate[\zeta]}\Big]\Big], z, Simplify\Big]}$$

$$\begin{aligned}
&-2\,m^2\,z1^2\,z2 - 2\,m^2\,z1\,z2^2 - m\,z1^3\,z2^2 - m\,z1^2\,z2^3 + m^2\,z1^2\,\zeta + dm^2\,(z1-z2)^2\,\zeta + 2\,m^2\,z1\,z2\,\zeta + m\,z1^3\,z2\,\zeta + m^2\,z2^2\,\zeta + 2\,m\,z1^2\,z2^2\,\zeta + m\,z1\,z2^3\,\zeta + z1^3\,z2^3\,\zeta -\\
&\quad dm\,(z1-z2)\,(2\,m + z1\,z2)\,(z1\,z2 - z1\,\zeta - z2\,\zeta) - z1\,z2\,((2\,dm\,(z1-z2) + z1\,z2\,(z1+z2))\,\zeta + m\,(-2\,z1\,z2 + 2\,z1\,\zeta + 2\,z2\,\zeta))\,\text{Conjugate}[\zeta] +\\
&\quad z1^2\,z2^2\,\zeta\,\text{Conjugate}[\zeta]^2 + z^5\,(z1 - \text{Conjugate}[\zeta])\,(-z2 + \text{Conjugate}[\zeta]) +\\
&\quad z^4\left(m\,z1 + m\,z2 + 2\,z1^2\,z2 + 2\,z1\,z2^2 + dm\,(-z1+z2) + z1\,z2\,\zeta - (2\,m + (z1+z2)\,(2\,z1 + 2\,z2 + \zeta))\,\text{Conjugate}[\zeta] + (2\,z1 + 2\,z2 + \zeta)\,\text{Conjugate}[\zeta]^2\right) +\\
&\quad z\left(m^2\,z1^2 - dm^2\,(z1-z2)^2 + 6\,m^2\,z1\,z2 + m\,z1^3\,z2 + m^2\,z2^2 + 2\,m\,z1^2\,z2^2 + m\,z1\,z2^3 - z1^3\,z2^3 - 4\,m^2\,z1\,\zeta - m\,z1^3\,\zeta - 4\,m^2\,z2\,\zeta -\right.\\
&\qquad 5\,m\,z1^2\,z2\,\zeta - 5\,m\,z1\,z2^2\,\zeta - 2\,z1^3\,z2^2\,\zeta - m\,z2^3\,\zeta - 2\,z1^2\,z2^3\,\zeta + dm\,(z1-z2)\left(z1\,z2\,(z2 - 2\,\zeta) + z1^2\,(z2-\zeta) - \left(4\,m + z2^2\right)\zeta\right) +\\
&\qquad \left(m\left(-2\,z1\,z2\,(z2 - 4\,\zeta) - 2\,z1^2\,(z2-\zeta) + 2\,z2^2\,\zeta\right) + 2\,dm\,(z1-z2)\,(z2\,\zeta + z1\,(z2+\zeta)) + z1\,z2\,(z1+z2)\,(2\,z2\,\zeta + z1\,(z2 + 2\,\zeta))\right)\text{Conjugate}[\zeta] -\\
&\qquad \left. z1\,z2\,(2\,z2\,\zeta + z1\,(z2 + 2\,\zeta))\,\text{Conjugate}[\zeta]^2\right) + z^3\left(dm\,z1^2 - m\,z1^2 - 2\,m\,z1\,z2 - z1^3\,z2 - dm\,z2^2 - m\,z2^2 - 4\,z1^2\,z2^2 - z1\,z2^3 - 2\,m\,z1\,\zeta -\right.\\
&\qquad 2\,m\,z2\,\zeta - 2\,z1^2\,z2\,\zeta - 2\,z1\,z2^2\,\zeta + \left(2\,dm\,(z1-z2) + 2\,m\,(z1 + z2 + 2\,\zeta) + (z1+z2)\left(z1^2 + 4\,z1\,z2 + z2^2 + 2\,z1\,\zeta + 2\,z2\,\zeta\right)\right)\text{Conjugate}[\zeta] -\\
&\qquad \left.\left(z1^2 + 2\,z1\,(2\,z2+\zeta) + z2\,(z2 + 2\,\zeta)\right)\text{Conjugate}[\zeta]^2\right) +\\
&\quad z^2\left(-2\,m^2\,z1 - 2\,m^2\,z2 + 2\,z1^3\,z2^2 + 2\,z1^2\,z2^3 + 4\,m^2\,\zeta + 3\,m\,z1^2\,\zeta + 6\,m\,z1\,z2\,\zeta + z1^3\,z2\,\zeta + 3\,m\,z2^2\,\zeta + 4\,z1^2\,z2^2\,\zeta + z1\,z2^3\,\zeta +\right.\\
&\qquad dm\,(z1-z2)\,(2\,m - 2\,z1\,z2 + z1\,\zeta + z2\,\zeta) - \left(2\,dm\,(z1-z2)\,(z1 + z2 + \zeta) + (z1+z2)\left(\left(6\,m + z2^2\right)\zeta + z1^2\,(2\,z2+\zeta) + 2\,z1\,z2\,(z2 + 2\,\zeta)\right)\right)\\
&\qquad\ \left.\text{Conjugate}[\zeta] + \left(z2^2\,\zeta + z1^2\,(2\,z2+\zeta) + 2\,z1\,z2\,(z2 + 2\,\zeta)\right)\text{Conjugate}[\zeta]^2\right)
\end{aligned}$$

Solve this in our first coordinate frame: geometric center; i.e. z1=-z2

$$\mathbf{pcenter = Collect[Expand[pgeneral\ /.\ z2 \to -z1, z], z, Simplify]}$$

$$\begin{aligned}
&z^5\left(z1^2 - \text{Conjugate}[\zeta]^2\right) + z^3\left(-2\,z1^4 + 4\,(dm\,z1 + m\,\zeta)\,\text{Conjugate}[\zeta] + 2\,z1^2\,\text{Conjugate}[\zeta]^2\right) +\\
&\quad z\,z1\left(-4\,dm^2\,z1 - 4\,m^2\,z1 + z1^5 - 8\,dm\,m\,\zeta - 4\,z1\,(dm\,z1 + m\,\zeta)\,\text{Conjugate}[\zeta] - z1^3\,\text{Conjugate}[\zeta]^2\right) +\\
&\quad z^4\left(-z1\,(2\,dm + z1\,\zeta) - 2\,m\,\text{Conjugate}[\zeta] + \zeta\,\text{Conjugate}[\zeta]^2\right) +\\
&\quad z^2\left(4\,dm\,z1\left(m + z1^2\right) + 2\left(2\,m^2 + z1^4\right)\zeta - 4\,dm\,z1\,\zeta\,\text{Conjugate}[\zeta] - 2\,z1^2\,\zeta\,\text{Conjugate}[\zeta]^2\right) +\\
&\quad z1^2\left(4\,dm\,m\,z1 - 2\,dm\,z1^3 + 4\,dm^2\,\zeta - z1^4\,\zeta + 2\,z1\,(m\,z1 + 2\,dm\,\zeta)\,\text{Conjugate}[\zeta] + z1^2\,\zeta\,\text{Conjugate}[\zeta]^2\right)
\end{aligned}$$

Solve this in our second coordinate frame : planet rest frame; i.e.z1 = 0

$$\mathbf{pbody1 = Collect[Expand[pgeneral\ /.\ z1 \to 0, z], z, Simplify]}$$

$$\begin{aligned}
&(dm - m)^2\,z2^2\,\zeta - z^5\,\text{Conjugate}[\zeta]\,(-z2 + \text{Conjugate}[\zeta]) - (dm - m)\,z\,z2\left(dm\,z2 + m\,z2 - 4\,m\,\zeta - z2^2\,\zeta + 2\,z2\,\zeta\,\text{Conjugate}[\zeta]\right) +\\
&\quad z^2\left(-dm\,z2\,(2\,m + z2\,\zeta) + m\left(-2\,m\,z2 + 4\,m\,\zeta + 3\,z2^2\,\zeta\right) + z2\left(-\left(6\,m + z2^2\right)\zeta + 2\,dm\,(z2+\zeta)\right)\text{Conjugate}[\zeta] + z2^2\,\zeta\,\text{Conjugate}[\zeta]^2\right) +\\
&\quad z^4\left((dm + m)\,z2 - (2\,m + z2\,(2\,z2 + \zeta))\,\text{Conjugate}[\zeta] + (2\,z2 + \zeta)\,\text{Conjugate}[\zeta]^2\right) +\\
&\quad z^3\left(-z2\,(dm\,z2 + m\,(z2 + 2\,\zeta)) + \left(-2\,dm\,z2 + \left(2\,m + z2^2\right)(z2 + 2\,\zeta)\right)\text{Conjugate}[\zeta] - z2\,(z2 + 2\,\zeta)\,\text{Conjugate}[\zeta]^2\right)
\end{aligned}$$

Solve this in our third coordinate frame : star rest frame; i.e.z2 = 0

pbody2 = Collect[Expand[pgeneral /. z2 → 0, z], z, Simplify]

$$(dm+m)^2\, z1^2\, \zeta + z^5\,(z1-\mathrm{Conjugate}[\zeta])\,\mathrm{Conjugate}[\zeta] - (dm+m)\, z\, z1\,\left(dm\, z1 - m\, z1 + 4\, m\, \zeta + z1^2\, \zeta - 2\, z1\, \zeta\, \mathrm{Conjugate}[\zeta]\right) + z^2\,\left(dm\, z1\,(2\, m + z1\, \zeta) + m\,\left(-2\, m\, z1 + 4\, m\, \zeta + 3\, z1^2\, \zeta\right) - z1\,\left(\left(6\, m + z1^2\right)\zeta + 2\, dm\,(z1+\zeta)\right)\mathrm{Conjugate}[\zeta] + z1^2\,\zeta\,\mathrm{Conjugate}[\zeta]^2\right) + z^4\,\left((-dm+m)\, z1 - (2\, m + z1\,(2\, z1+\zeta))\,\mathrm{Conjugate}[\zeta] + (2\, z1+\zeta)\,\mathrm{Conjugate}[\zeta]^2\right) + z^3\,\left(z1\,(dm\, z1 - m\,(z1+2\,\zeta)) + \left(2\, dm\, z1 + \left(2\, m + z1^2\right)(z1+2\,\zeta)\right)\mathrm{Conjugate}[\zeta] - z1\,(z1+2\,\zeta)\,\mathrm{Conjugate}[\zeta]^2\right)$$

Solve this in our fourth and final coordinate frame : center of mass; i.e. (z1m1 + z2m2)/(m1+m2) = 0

pcm = Collect[Expand[pgeneral /. z2 → −z1 (m − dm) / (m + dm), z], z, Simplify]

$$z^5\,(z1-\mathrm{Conjugate}[\zeta])\left(\frac{(-dm+m)\, z1}{dm+m} + \mathrm{Conjugate}[\zeta]\right) + \frac{1}{(dm+m)^3} z1^2\,\left(-m^3\, z1^4\,\zeta + dm\, m^2\, z1\,\left(8\, m^2 - 4\, m\, z1^2 + 3\, z1^3\,\zeta\right) + dm^2\, m\,\left(8\, m\, z1^2\,(z1-\zeta) + 16\, m^2\,\zeta - 3\, z1^4\,\zeta\right) + dm^3\,\left(-8\, m^2\,(z1-2\,\zeta) - 4\, m\, z1^2\,(z1-2\,\zeta) + z1^4\,\zeta\right) + 2\,(dm-m)\, z1\,\left(-m^3\, z1 + dm\, m\,\left(-4\, m + z1^2\right)\zeta + dm^2\,\left(m\, z1 - 4\, m\,\zeta - z1^2\,\zeta\right)\right)\mathrm{Conjugate}[\zeta] + (dm-m)^2\,(dm+m)\, z1^2\,\zeta\,\mathrm{Conjugate}[\zeta]^2\right) +$$
$$\frac{1}{(dm+m)^2} z^4\,\left((dm-m)\, z1^2\,(m\,\zeta + dm\,(4\, z1+\zeta)) - 2\,\left(m^3 + dm\, m\,(2\, m + z1\,\zeta) + dm^2\,(m + z1\,(4\, z1+\zeta))\right)\mathrm{Conjugate}[\zeta] + (dm+m)\,(m\,\zeta + dm\,(4\, z1+\zeta))\,\mathrm{Conjugate}[\zeta]^2\right) +$$
$$\frac{1}{(dm+m)^3} 2\, z^3\,\left(-z1\,\left(m^3\, z1^3 + dm^2\, m\,\left(-3\, z1^3 + 4\, m\,\zeta\right) + dm\, m^2\,\left(-z1^3 + 2\, m\,\zeta - 2\, z1^2\,\zeta\right) + dm^3\,\left(3\, z1^3 + 2\, m\,\zeta + 2\, z1^2\,\zeta\right)\right) + 2\,\left(m^4\,\zeta + dm\, m^2\,\left(2\, m\, z1 - z1^3 + 3\, m\,\zeta\right) + dm^3\,\left(2\, m\, z1 + 3\, z1^3 + m\,\zeta + 2\, z1^2\,\zeta\right) + dm^2\, m\,\left(4\, m\, z1 + 3\, m\,\zeta + 2\, z1^2\,\zeta\right)\right)\mathrm{Conjugate}[\zeta] - (dm+m)\, z1\,\left(-m^2\, z1 + 2\, dm\, m\,\zeta + dm^2\,(3\, z1 + 2\,\zeta)\right)\mathrm{Conjugate}[\zeta]^2\right) +$$
$$\frac{1}{(dm+m)^3} 2\, z^2\,\left(m^3\,\left(2\, m^2 + z1^4\right)\zeta + dm\, m^2\,\left(2\, m\, z1^3 + z1^4\,(2\, z1-\zeta) + 6\, m^2\,\zeta\right) + dm^2\, m\,\left(-4\, z1^5 + 6\, m^2\,\zeta + 8\, m\, z1^2\,\zeta - 3\, z1^4\,\zeta\right) + dm^3\,\left(-2\, m\, z1^2\,(z1-4\,\zeta) + 2\, m^2\,\zeta + z1^4\,(2\, z1 + 3\,\zeta)\right) - 2\, dm\, z1\,\left(m^2\,\left(4\, m - z1^2\right)\zeta + 2\, dm\, m\,\left(m\, z1 - z1^3 + 4\, m\,\zeta\right) + dm^2\,\left(2\, m\, z1 + 2\, z1^3 + 4\, m\,\zeta + 3\, z1^2\,\zeta\right)\right)\mathrm{Conjugate}[\zeta] + (dm+m)\, z1^2\,\left(-2\, dm\, m\, z1 - m^2\,\zeta + dm^2\,(2\, z1 + 3\,\zeta)\right)\mathrm{Conjugate}[\zeta]^2\right) -$$
$$\frac{1}{(dm+m)^3} z\, z1\,\left(4\, m^5\, z1 - m^3\, z1^5 + dm\, m^2\,\left(-4\, m\, z1^2\,\zeta + 4\, m^2\,(z1 + 4\,\zeta) + z1^4\,(3\, z1 + 4\,\zeta)\right) + dm^3\,\left(-4\, m^2\,(z1 - 4\,\zeta) + z1^4\,(z1 + 4\,\zeta) + 4\, m\, z1^2\,(-2\, z1 + 5\,\zeta)\right) + dm^2\, m\,\left(8\, m\, z1^3 - 4\, m^2\,(z1 - 8\,\zeta) - z1^4\,(3\, z1 + 8\,\zeta)\right) - 2\, z1\,\left(-2\, m^4\,\zeta + dm\, m^2\,\left(z1^3 - 2\, m\,\zeta\right) + 2\, dm^2\, m\,\left(-z1^3 + 5\, m\,\zeta - 2\, z1^2\,\zeta\right) + dm^3\,\left(z1^3 + 10\, m\,\zeta + 4\, z1^2\,\zeta\right)\right)\mathrm{Conjugate}[\zeta] + \left(dm^2 - m^2\right) z1^2\,(-m\, z1 + dm\,(z1 + 4\,\zeta))\,\mathrm{Conjugate}[\zeta]^2\right)$$