The first test determines which coordinate system(s) and which version of the derivation (general versus specific) ought to be used to produce the most reliable results as the mass ratio decreases. This test is done for the binary lens. In all cases, the two bodies are assumed to lie on the real axis. The coordinate frames that are being considered are: the geometric center frame, which assumes the vertical axis lies halfway between the star and the planet; the planetary caustic frame, which assumes the central point of the planetary caustic lies on the origin; the planet frame; the star frame; and the center-of-mass frame. All calculations were done using the complex polynomial root solver in Skowron and Gould 2012, for a constant value of s.

To test the success of each calculation, I simulated a grid of coordinates in the source plane, centered on the planetary caustic. At each point, I calculated the number of images that should be produced if the source were at that position. There should be three images when the source is outside the caustic, and five images when the source is inside the caustic. I also calculate the magnification at each of the points in the grid. These simulations are done for each of the aforementioned coordinate frames, for both the specifically-derived and general forms of the polynomial.

After making these plots, I checked whether they produced a result that agrees with theoretical expectations, using a pass-or-fail determination. This can be qualified visually, as it is clear when a method fails because the plot will reproduce a very noisey result with seemingly no regard to the caustic. **Figure 1** and **Figure 2** show the plots of the number of accepted images; **Figure 3** and **Figure 4** show the plots of the magnification for the same grid of points.

When using the general form of the polynomial equation, the planet frame performs slightly better than the other frames as the mass ratio approaches 10^{-8}. At mass ratios greater than 10^{-7}, none of the coordinate frames produce any errors. However, all of the frames result in high numbers of errors when the mass ratio drops below something of the order 10^{-8}.

On the other hand, when using the specifically-derived forms of the polynomial, the planet frame produces a correct plot of the number of images all the way down to a mass ratio of 10^{-15}; whereas the other frame fail. The speculated reason for this is that the form of the polynomial specifically derived for the planet frame elimates all instances of the term, q^2; whereas the derivations for all other frames have at least one q^2 term. Because 64-bit floating point numbers can only store around 16 digits, many errors tend to accumulate when adding terms of the order 10^0 with other terms of the order 10^{-16}. By eliminating every instance of q^2, it is possible to keep every significant digit throughout the calculation with mass ratios all the way down to 10^{-15}.

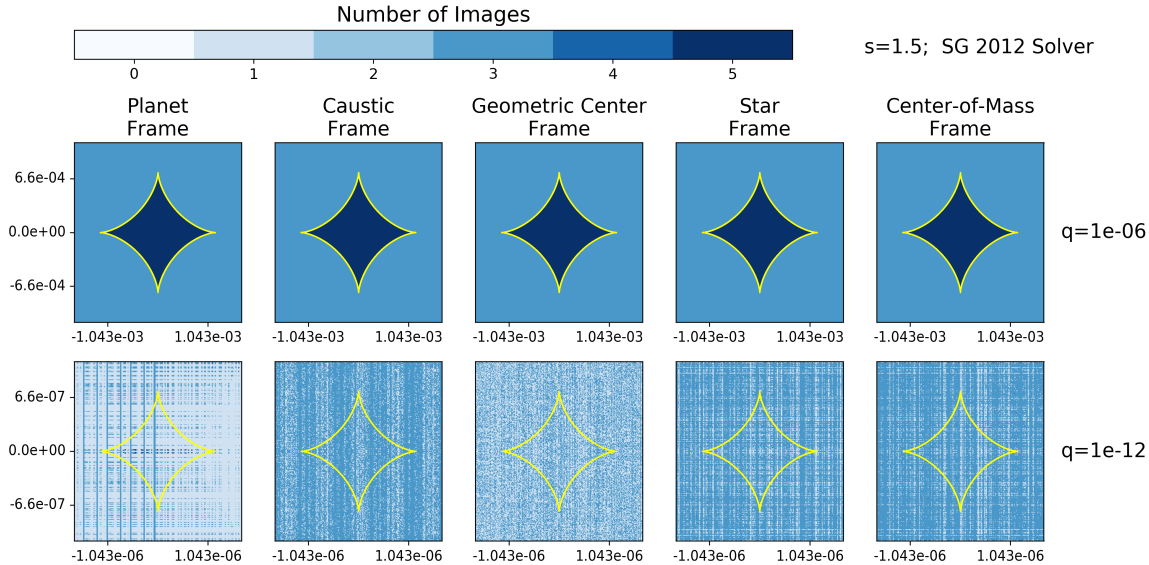

Figure 1: The number of images versus position for each coordinate frame using the general form of the polynomial. The top row shows the plots for a mass ratio, $q = 10^{-6}$. The bottom row shows the plots for a mass ratio, $q = 10^{-12}$. Notice that the scale on the x- and y-axes scale by orders of magnitude, an effect of the two given mass ratios. As expected, for $q = 10^{-6}$, the plots show the correct number of images for each coordinate system. However, for $q = 10^{-12}$, the plots fail for all coordinate frames.

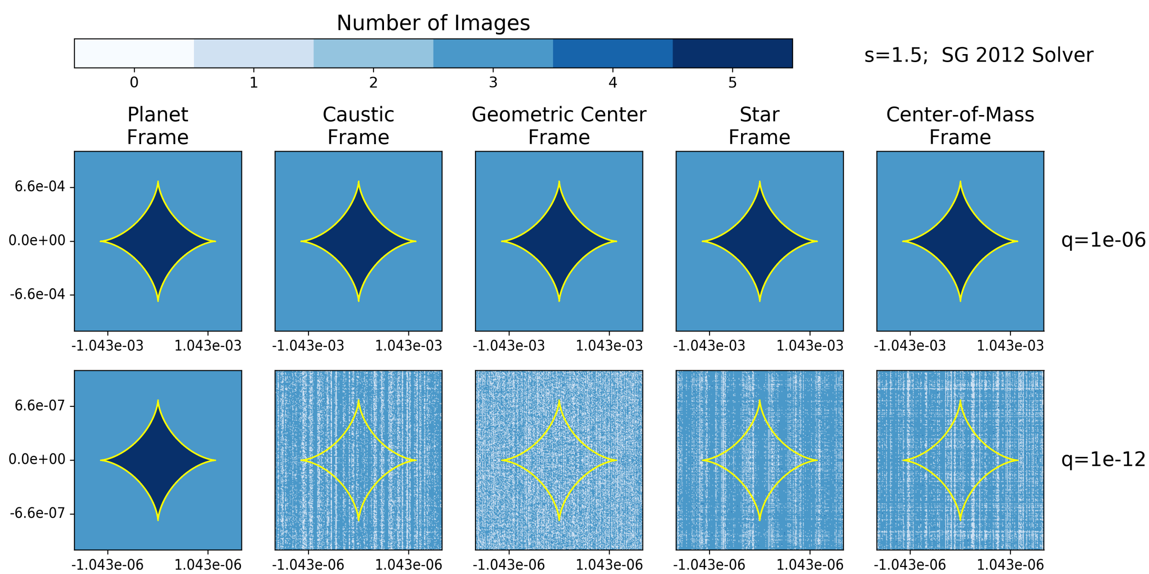

Figure 2: Same as **Figure 1** except using the specifically- derived forms of the polynomial. As expected, for $q = 10^{-6}$, the plots show the correct number of images for each coordinate system. For $q = 10^{-12}$, the plots fail for all coordinate frames except for the planet frame.

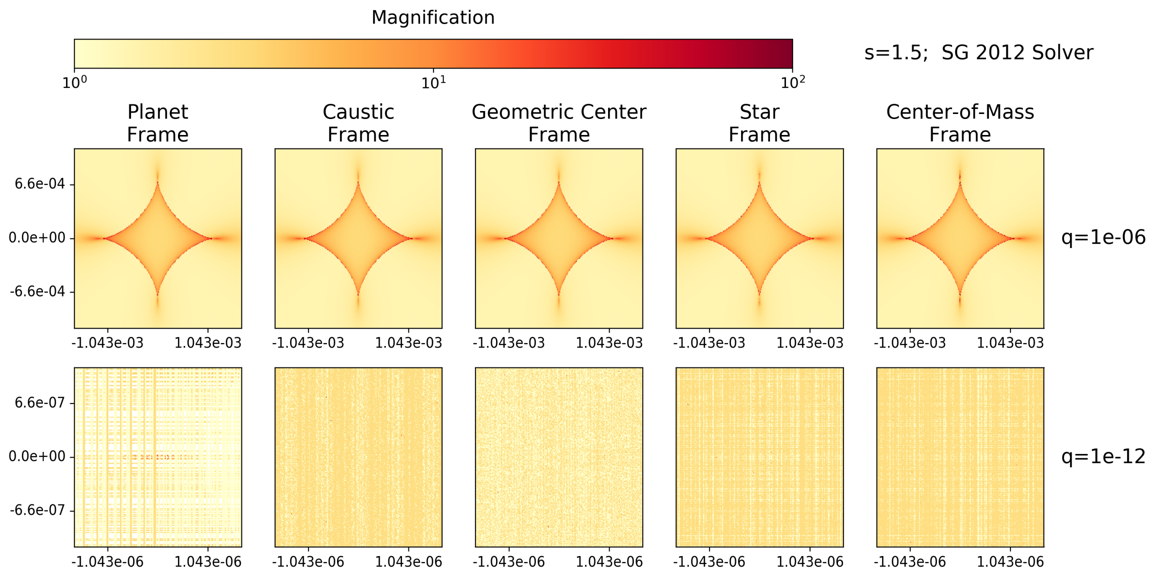

Figure 3: The magnification versus the position for each coordinate frame using the general form of the polynomial. As seen in plots of the number of images, the simulations pass for all coordinate frames when $q = 10^{-6}$, and fail for all coordinate frames when $q = 10^{-12}$.

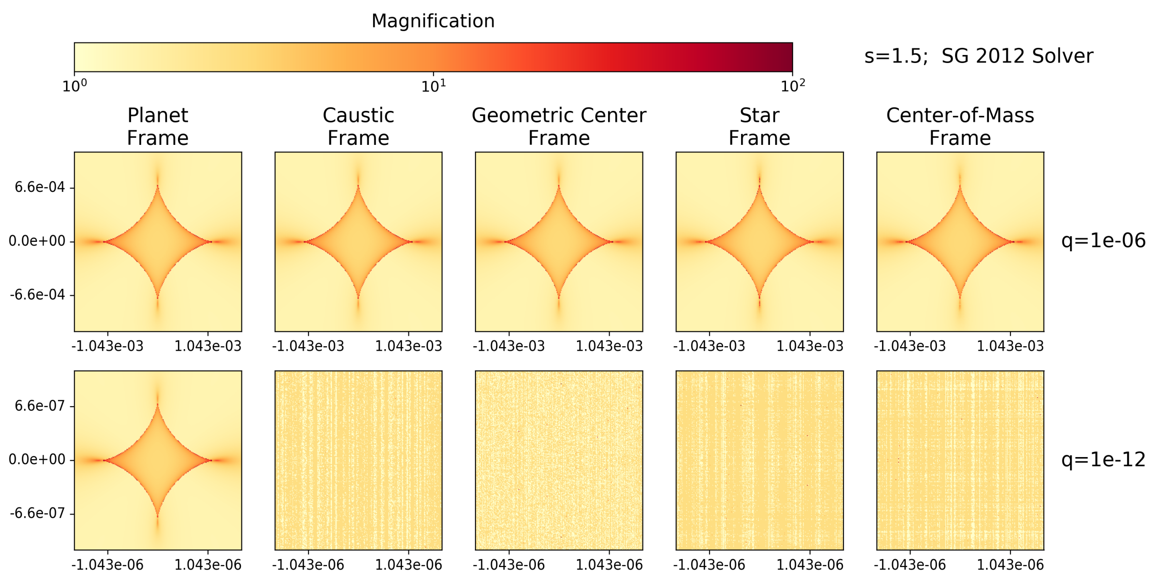

Figure 4: Same as **Figure 3** except using the specifically- derived forms of the polynomial. As seen in the plots of the number of images, when $q = 10^{-12}$, the simulation fails for all coordinate frames except for the planet frame.