Microlensing is becoming an increasingly popular method to search for planets because of its ability to discover more diverse distributions of planets than other common methods. Whereas the radial velocity method and the transit method are most sensitive to hot Jupiters, microlensing is most sensitive to distance Neptunes. Moreover, it has the potential to detect Earth-sized and Mercury-sized planets, something the other methods are not yet able to do. As WFIRST is expected to be launched in the mid-2020s, one of the goals of the mission is to discover more properties of planet formation, including for Earth-like planets.

A microlensing event occurs when a distant body, known as the source, passes behind another body or system of bodies, known as the lens. By the gravitational lens effect, The light of the source is bent around the lens and magnified. This produces an observable light curve that can indicate details about the lensing bodies, including the mass ratios and separations between each of the lensing bodies. Hereafter, this paper will focus on point-source point-lens microlensing events for binary and triple lenses.

In a binary lens, the mass ratio of the planet to the star is denoted by q, and the separation between the planet and the star is denoted by s. The separation is in units of the Einstein radius of the whole lens system, which means q and s are both unitless. In a triple lens, there will be two mass ratios, two separations, and an angle that is formed by the 3 bodies, which all have various definitions depending on the types of bodies in the system (e.g. stars, planets, or a moon). The definitions of these parameters are arbitrary on a case-by-case basis, so they will not be defined here.

The lensing equation is given by

$$\zeta = z + \sum_i \frac{m_i}{\bar{z}_i - \bar{z}} \tag{1}$$

where m_i is the mass of the lensing body divided by the total mass of the system, z_i is the position of the lensing body in units of the total system's Einstein radius, z is the position of each of the resulting images, $\bar{z}$ is the complex conjugate of z, and ζ is the position of the source. Each of the positions are given in complex space so that the subsequent calculations can be expressed in one polynomial, rather than 2 separate polynomials in the x- and y-directions. By solving for z, taking the complex conjugate, substituting this expression back into the lensing equation, and some algebra, we can obtain a very messy polynomial equation of variable z, which is equal to 0. This results in a 5th-degree polynomial for the binary lens, and a 10th-degree polynomial for the triple lens. Writing the equation in terms of a polynomial is favored because it more easily allows us to use polynomial root solvers to determine the positions and magnifications of the images. The lensing polynomial can alse be derived in different coordinate frames, which will be discussed later.

The problem with calculating microlensing events with a typical 64-bit computer is that floating point values are used to represent these variables. Because floating point values are written in binary code, there will inevitably be roundoff error as the terms of the polynomial are of extremely varying orders of magnitude. For reference, a 64-bit double floating point number carries approximately 16 significant digits. This means, if a computer were to make the computation $1+10^{-17}-1$, it would truncate the decimal and yield the value 0, where we would expect it to return 10^{-17}. On the other hand, using more precise calculation methods is not favorable because it will take much longer than necessary. One purpose of this project is to determine which coordinate frames minimize the effect of the roundoff error, without relying on more time-consuming methods.