

Write the equation for a triple lens.

$$\begin{aligned} \text{zeta} = & z + m1 / (\text{Conjugate}[z1] - \text{Conjugate}[z]) + m2 / (\text{Conjugate}[z2] - \text{Conjugate}[z]) + m3 / (\text{Conjugate}[z3] - \text{Conjugate}[z]) \\ & z + \frac{-dm + m}{-\text{Conjugate}[z] + \text{Conjugate}[z1]} + \frac{dm + m}{-\text{Conjugate}[z] + \text{Conjugate}[z2]} + \frac{m3}{-\text{Conjugate}[z] + \text{Conjugate}[z3]} \end{aligned}$$

Take the conjugate of the triple lens equation. Solve for the conjugate of z, and substitute this back into binary lens equation. Assume the lensing bodies lie on the real axis, and that the masses are real. Then ζ is equal to the following expression:

$$\text{zetaexpression} = \text{Refine}[\text{zeta} /. \text{Solve}[\text{Conjugate}[\text{zeta}] = \text{Conjugate}[\zeta], \text{Conjugate}[z]], \{m1, m2, m3, z1, z2, z3\} \in \text{Reals}]$$

$$\left\{ z + \frac{-dm + m}{z1 + \frac{dm+m}{-z+z2} + \frac{m3}{-z+z3} + \frac{-\text{Conjugate}[dm]+\text{Conjugate}[m]}{-z+z1}} - \text{Conjugate}[\zeta] \right. \\ \left. + \frac{dm + m}{z2 + \frac{dm+m}{-z+z2} + \frac{m3}{-z+z3} + \frac{-\text{Conjugate}[dm]+\text{Conjugate}[m]}{-z+z1}} - \text{Conjugate}[\zeta] + \frac{m3}{\frac{dm+m}{-z+z2} + z3 + \frac{m3}{-z+z3} + \frac{-\text{Conjugate}[dm]+\text{Conjugate}[m]}{-z+z1}} - \text{Conjugate}[\zeta] \right\}$$

Move ζ to other side and combine the expression into one fraction. Collect only the numerator (since this polynomial is equal to zero) into powers of z, and multiply by -1 for convention. This is the form of the polynomial in a general coordinate system, and it is equal to 0.

$$\text{pgeneral} = \text{Collect}[-\text{Numerator}[\text{Together}[\text{zetaexpression} - \zeta]], z, \text{Simplify}]$$

Substitute mass ratios into the expression.

$$\text{pgeneralEpsilon} = \text{Collect}[\text{Expand}[\text{pgeneral} /. \{m1 \rightarrow m, m2 \rightarrow \epsilon1 * m, m3 \rightarrow \epsilon2 * m\}], z, \text{Simplify}]$$

Output in a form more suited for Python.

$$\text{pgeneralEpsPython} = \text{FortranForm}[\text{Simplify}[\text{pgeneralEpsilon} /. \{\epsilon1 \rightarrow \text{"self.eps1"}, \epsilon2 \rightarrow \text{"self.eps2"}, m \rightarrow \text{"self.m"}, \zeta \rightarrow \text{"self.zeta"}, \text{Conjugate}[\zeta] \rightarrow \text{"self.zeta_conj"}, z1 \rightarrow \text{"self.z1"}, z2 \rightarrow \text{"self.z2"}, z3 \rightarrow \text{"self.z3"}\}]]$$