The first test determines, for the binary lens, which coordinate systems and which version of the derivation (general versus specific) ought to be used to produce reliable results as the mass ratio decreases. In all cases, the two bodies are assumed to lie on the real axis. The coordinate frames that are being considered are: the geometric center frame, which assumes the vertical axis lies halfway between the star and the planet; the planet frame, which assumes the planet lies on the origin; and the planetary caustic frame, which assumes the central point of the planetary caustic lies on the origin.

To test the success of each calculation, I simulated a grid of points centered on the planetary caustic. Each point represents a position for the source body. At each point, I calculated the number of images that should be seen by an observer. There should be three images when the source is outside the caustic, and five images when the source is inside the caustic. We can make these plots and check whether they produce a result that agrees with what we expect. However, checking whether the frames yield the correct number of solutions is insufficient to qualify whether they pass. I also calculate the magnification at each of the points in the grid. These simulations are done for each of the aforementioned coordinate frames, for both the specifically-derived and general forms of the polynomial. Success is determined by whether the plots produce results that agree with theoretical expectations. This can be qualified visually, as it is clear when a method fails because the plot will reproduce a very noisey result with seemingly no regard to the caustic. **Figure 1** and **Figure 2** show the results of this simulation.

When using the general form of the polynomial equation, the planet frame performs better than the geometric center frame and the caustic frame only when the mass ratio approaches 10^{-8}; at mass ratios greater than 10^{-7}, none of the coordinate frames produce any errors. However, all of the frames result in high numbers of errors when the mass ratio drops below something of the order 10^{-8}.

On the other hand, when using the specifically-derived forms of the polynomial, we see much different results. The planet frame now produces a correct plot of the number of images all the way down to a mass ratio of 10^{-15}; whereas the other frames' performance is slightly worse. The hypothesized reason for this is that the form of the polynomial specifically derived for the planet frame elimates many instances of terms with the mass ratio squared. Because 64-bit floating point numbers can only store around 16 digits, many errors tend to accumulate when adding terms of the order 10^{0} with other terms of the order 10^{-16}. By eliminating nearly every instance of the mass ratio squared, it is possible to keep every significant digit throughout the calculation with mass ratios all the way down to 10^{-15}.

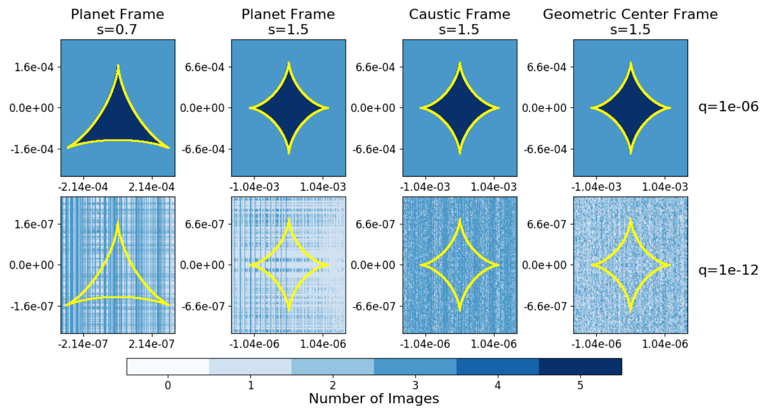

Figure 1: The number of images versus position for each coordinate frame using the general form of the polynomial. The top row shows the plots for a mass ratio, $q = 10^{-6}$. The bottom row shows the plots for a mass ratio, $q = 10^{-12}$. There are two values for separation for the planet frame; the reason will be justified in the next plot. As expected, for $q = 10^{-6}$, the plots show the correct number of images for each coordinate system. However, for $q = 10^{-12}$, the plots fail for all coordinate frames.

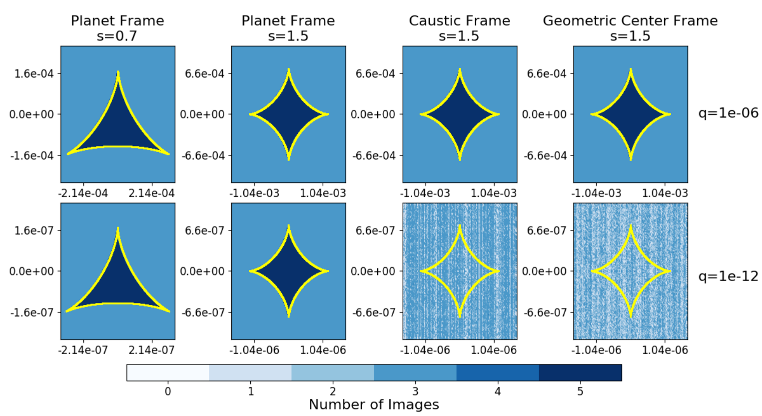

Figure 2: The number of images versus position for each coordinate frame using the specifically-derived forms of the polynomial. As expected, for $q = 10^{-6}$, the plots show the correct number of images for each coordinate system. For $q = 10^{-12}$, the plots fail for all coordinate frames except for the planet frame. Because it succeeds for planet frame with a separation $s > 1$, it it also worth checking if it succeeds for a separation, $s < 1$. It does succeed, and justifies that the planet frame is the only frame that succeeds for very small mass ratios.

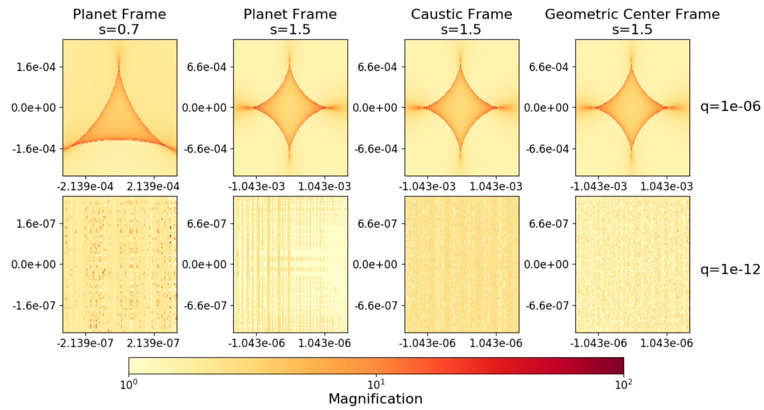

Figure 3: The magnification versus the position for each coordinate frame using the general form of the polynomial. As seen in plots of the number of images, the simulations pass for all coordinate frames when $q = 10^{-6}$, and fail for all coordinate frames when $q = 10^{-12}$.

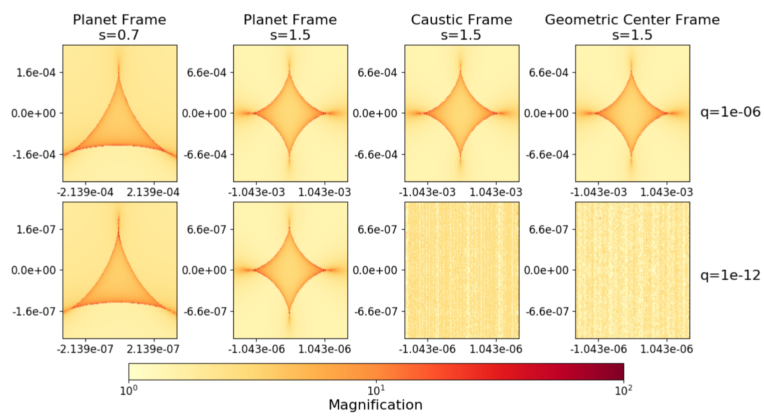

Figure 4: The magnification versus the position for each coordinate frame using the specifically-derived forms of the polynomial. As seen in the plots of the number of images, when $q = 10^{-12}$, the simulation fails for all coordinate frames except for the planet frame. Again, it passes for both $s > 1$ and $s < 1$ for the planet frame.