COMP3121 Homework 1

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Question 1

Statements 2 3 4 are correct.

1. Not correct, because there are two shortest paths between D - E, which are D - B - A - E, and D - C - F - E, and there are no common nodes in these two paths, then there is no node is pivotal for node pair of D - E. So, this statement is not correct.

2. Correct, because the shortest path between A - C is A - B - C, and the shortest path between A - D is A - B - D, so node B appears in the shortest path between A - C and A - D, so node B is pivotal of node pairs of A - C and A – D. So, this statement is not correct.

3. Correct, because there are two shortest paths between D - E, which are D - B - A - E, and D - C - F - E, and there are no common nodes in these two paths, then there is no node is pivotal for node pair of D - E. So, this statement is correct.

4. Correct, because if the path passes through node D, it should not be the shortest path, because it could pass through node B and node C to reach the goal of shortest path, so that node D is not pivotal for any nodes. So, this statement is not correct.

Question 2

Pentagon, hexagon, and octagon satisfied that every node in this polygon is a pivotal of at least one node pair.

Square: not satisfies, if we choose two nodes that are on the diagonal point, there are two different shortest paths, and these two-path pass through two left nodes, then there is no pivotal node. So, it is not satisfying that every node in this polygon is a pivotal of at least one node pair.

Pentagon: satisfies, if we choose two nodes that are separated by one node, the shortest path passes through the in-between one, so the in-between node could be the pivotal node. So, it satisfies that every node in this polygon is a pivotal of at least one node pair.

Hexagon: satisfies, if we choose two nodes that are separated by one node, the shortest path passes through the in-between one, so the in-between node could be the pivotal node. So, it satisfies that every node in this polygon is a pivotal of at least one node pair.

Octagon: satisfies, if we choose two nodes that are separated by one node, the shortest path passes through the in-between one, so the in-between node could be the pivotal node. So, it satisfies that every node in this polygon is a pivotal of at least one node pair.

Question 3

Octagon satisfied that every node in this polygon is a pivotal of at least two node pairs.

Square: not satisfied, if we choose two nodes that are on the diagonal point, there are two different shortest paths, and these two-path pass through two left nodes, then there is no pivotal node. So, it is not satisfying that every node in this polygon is a pivotal of at least two node pairs.

Pentagon: not satisfied, if we choose two nodes that are separated by one node, the shortest path passes through the in-between one, so the in-between node could be the pivotal node. So, it is one node pair. This node cannot be pivotal for other node pairs, so one node could only be a pivotal of one node pair. So, it is not satisfying that every node in this polygon is a pivotal of at least two node pairs.

Hexagon: not satisfied, if we choose two nodes that are separated by one node, the shortest path passes through the in-between one, so the in-between node could be the pivotal node. So, it is one node pair. And if we choose two nodes that are separated by two nodes, so there are two different shortest paths passing through different nodes, so there are no common nodes in these two shortest paths, so there is no pivotal node for nodes that are separated by two nodes. So, every node in hexagon is pivotal of only one node pair. So, it is not satisfying that every node in this polygon is a pivotal of at least two node pairs.

Octagon: satisfied, if we choose two nodes that are separated by one node, the shortest path passes through the in-between one, so the in-between node could be the pivotal node. So, it is one node pair. And if we choose two nodes that are separated by two nodes, there is only one shortest path passing through those in-between two nodes, so these two nodes could be pivotal for this node pair. So, every node in octagon is a pivotal of at least two node pair. So, it satisfied that every node in this polygon is a pivotal of at least two node pairs.

Question 4

Statement 3 and statement 4 are correct.

1. Not correct, there are two local gatekeepers, node A and D, if an edge between node B and E is added, node A is also a local gatekeeper, because its neighbor node C and F are not connected, node A is also a local gatekeeper after the edge is added. Node D is not influenced by added edge because D's neighbor node B and C are not connected. So, the local gatekeeper remains unchanged when an edge between node B and E is added. So, statement 1 is not correct.

2. Not correct, according to the definition of gatekeeper, every path from node Y to node Z passes through the gatekeeper node. For example, node D's neighbor, node B and node C, every path from node B to node C does not pass through node D, it also passes through node A, so node D is not gatekeeper. So, statement 2 is not correct.

3. Correct, according to the definition of local gatekeeper, the node A, local gatekeeper, two of its neighbors are not connected by an edge, node B and node E are not connected, so node A is a local gatekeeper. So, statement 3 is correct.

4. Correct, node A and node D are two local gatekeepers. For node A, its neighbor node C and node F are not connected, so node A is a local gatekeeper. For node B, its neighbor node A and node D are connected, so node B is not a local gatekeeper. For node C, its neighbor node D and node A are connected, so node C is not a local gatekeeper. For node D, its neighbor node B and node C are not connected, so node D is a local gatekeeper. For node E, its neighbor node A and node F are connected, so the node E is not a local gatekeeper. For node F, its neighbor node E and node A are connected, so the node F is not a local gatekeeper. So, in the above graph, there are 2 local gatekeepers, node D and node A. So, statement 4 is correct.

Question 5

Statement 1, 2, 3, and 4 are correct.

1. Correct, node C will be gatekeeper if the edge between node A and node B is removed. Every path from node A to node B will be pass through node C. So, statement 1 is correct.

2. Correct, node A, B, C, F, E are local gatekeepers. For node A, its neighbor node B and node E are not connected, so node A is a local gatekeeper. For node B, its neighbor node C and node A are not connected, so node B is a local gatekeeper. For node C, its neighbor node B and node F are not connected, so node C is a local gatekeeper. For node D, its neighbor node C and node B are connected, so node D is not a local gatekeeper. For node E, its neighbor node A and nod F are not connected, so node E is a local gatekeeper. For node F, its neighbor node C and node E are not connected, so node F is a local gatekeeper, so 5 nodes out of 6 nodes are local gatekeepers, so over half of the number of nodes are local gatekeepers. So, statement 2 is correct.

3. Correct, if the edge between node B and node C are removed, all the nodes will be local gatekeepers. For node A, its neighbor node B and node E are not connected, so node A is a local gatekeeper. For node B, its neighbor node C and node A are not connected, so node B is a local gatekeeper. For node C, its neighbor node B and node F are not connected, so node C is a local gatekeeper. For node D, its neighbor node C and node B are not connected, so node D is a local gatekeeper. For node E, its neighbor node A and nod F are not connected, so node E is a local gatekeeper. For node F, its neighbor node C and node E are not connected, so node F is a local gatekeeper. Then all the nodes will be local gatekeepers. So, statement 3 is correct.

4. Correct, node B will be gatekeeper if the edge between node C and node D is removed. Every path from node D to node C will pass through node B. So, statement 4 is correct.