AY 2022-23 Fall

COMP2012 (Fall 2022) Discrete Mathematics

Quiz 1. 10:00pm-11:00pm, 7th October 2022

Name: _____ Student ID: _____ Marks: _____ / 100

- This is an **individual** quiz.
- Please submit the **soft copy** of your answer to Blackboard (as a doc/docx/pdf file).

Q1) Let A be a set and B be a set. **[30 marks]**

Prove that $(A \cap B) \cup (\bar{A} \cap B) = B$.

LHS $\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \notin A \text{ and } x \in B)$

$\Rightarrow x \in U \text{ and } x \in B \Rightarrow x \in B \Rightarrow$ RHS

Q2) Show that $f(n) = 25 + 8n + log_2 n$ is $O(n)$. **[30 marks]**

$O(n)$ upper bound case

$c_1 n \geq 25 + 8n + \log_2 n$

$c_1 \geq 8 + \frac{25}{n} + \frac{\log_2 n}{n}$

$\frac{25}{n} > 0$ $\log_2 n > 0$ and $n > 0$, then $\frac{\log_2 n}{n} > 0$

then $c_1 > 0$

Hence statement is true

Q3) Prove by induction that for all $n \in \mathbb{N}$, $5^n - 1$ is divisible by 4. **[20 marks]**

$P(n) = 5^n - 1$

When $n=1$, $P(1) = 5^1 - 1 = 4$, which is divisible by 4.

$P(1)$ is true.

Assume that $P(k)$ is true, $P(k)$ is divisible by 4.

there exists an integer a such that $5^k - 1 = 4a$.

For case of $(k+1)$, $P(k+1) = 5^{k+1} - 1 = 5^k \cdot 5 - 1$

$= (4a+1) \cdot 5 - 1 = 20a + 5 - 1 = 20a - 4$

$= 4(5a - 1)$ $\therefore P(k+1)$ is divisible by 4

Thus $P(k+1)$ is true for $\forall x \in \mathbb{N}$

Q4) Prove by contradiction that in any group of 7 people there is a person who knows an even number of people. You may assume that if a person *A* "knows" a person *B*, then person *B* also "knows" person *A*. *(Hint: If for each person you count the number of acquaintances, what can you say about their total?)* **[20 marks]**

For the sake of contradiction, assume that in any group of 7 people, they all have odd number of acquaintances.

Sum of seven odd number should be an odd number.

But considering the hint, because if a person A "knows" a person B, then person B also "knows" person A, so every time "knows" action happens, the total number of each person's acquaintance should plus 2, then it should be an odd number.

It contradicts the initial assumption of sum (numbers of acquaintances)

End of Quiz 1