COMP2012 (Fall 2022) Discrete Mathematics

Individual Assignment 1   Due Date: 23:59, 28th October, 2022

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Student number	

Notes:

- This is an **individual** assignment.
- Please submit the **soft copy** of your answer to Blackboard
  (as a doc/docx/pdf file).
- You just need to write your answer. There is no need to copy questions.

Question 1.

1(a)

m is the average of integers $a_1, a_2, \ldots, a_n$.
If there are only numbers that is larger than m in $a_1, a_2, \ldots, a_n$, then the average of all these number will larger than m, so it contradicts that the average should be m. Hence, there exists some number in $a_1, a_2, \ldots, a_n$ such that it is smaller than or equal to m.

1(b)

Truth table for $\neg (p \vee q \vee r)$

p	q	r	$p \vee q$	$p \vee q \vee r$	$\neg (p \vee q \vee r)$
T	T	T	T	T	F
T	T	F	T	T	F
T	F	T	T	T	F
T	F	F	T	T	F
F	T	T	T	T	F
F	T	F	T	T	F
F	F	T	F	T	F
F	F	F	F	F	T

Truth table for $\neg p \wedge \neg q \wedge \neg r$

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg p \wedge \neg q$	$\neg p \wedge \neg q \wedge \neg r$
T	T	T	F	F	F	F	F
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	F
T	F	F	F	T	T	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	T	F	F
F	F	T	T	T	F	F	F
F	F	F	T	T	T	T	T

The last column of each truth table are identical to each other, so $\neg (p \vee q \vee r)$ and $\neg p \wedge \neg q \wedge \neg r$ are logically equivalent.

1(c)

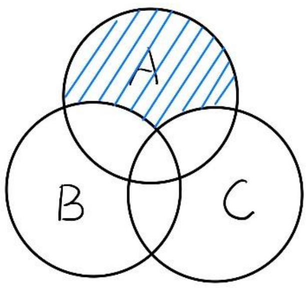
$P = x/4\ (P \in \mathbb{Z})$
$Q = x/3 (Q \in \mathbb{Z})$

For statement 1, if P is an integer, x should be the multiple of 4, some of x in domain, such as 4, 8, 12, 16 meets the requirement, so that this statement is true.

For statement 2, if Q is an integer, y should be the multiple of 3, some of x in domain, such as 6, 12, 18 meets the requirement, so that this statement is true.

For statement 3, if P Q are two integers, z should be multiple of 3 and be the multiple of 4, then only 12 meets the requirement of $\exists\ z \in Dom\ P(z) \wedge Q(z)$, then there's a $\neg$ sign, then all numbers other than 12 meets the requirement of this statement, so that statement 3 is true.

Question 2.
2(a)



2(b)
(I) $C(5,2) \times 2 = 20$
(II)
Reflexive means if $a \sim a$ should be in relation
Since we need to satisfy $d \sim e$ and d ~ a is the relation, then the $a \sim a$, $b \sim b$, $c \sim c$, $d \sim d$, $e \sim e$ is in relation

Symmetric means if $a \sim b$ is in relation, then $b \sim a$ should be in relation
Since $d \sim e$ and $d \sim a$ is the condition, then $e \sim d$ and $a \sim d$ should be in relation

Transitive means if $a \sim b$ is in relation and $b \sim c$ is in relation, then $a \sim c$ is in relation
Since $d \sim e$ and $d \sim a$ is the condition, then $e \sim a$ should be in relation
Then there will be one equivalence relation.

Question 3.
3(a)

3(a) $f(n) = O(g(n))$

$$f(n) \le cg(n)$$

$$\log_2 n^{\log_2 23} \le c \log_2 23^{\log_2 n}$$

$$\frac{\log_2 n^{\log_2 23}}{\log_2 23^{\log_2 n}} \le c$$

$$\frac{\log_2 23 \cdot \log_2 n}{\log_2 n \cdot \log_2 23} \le c$$

$$c \ge 1$$

$$f(n) = \Omega(g(n))$$

$$f(n) \ge cg(n)$$

$$\log_2 n^{\log_2 23} \ge c \log_2 23^{\log_2 n}$$

$$\frac{\log_2 23 \cdot \log_2 n}{\log_2 n \cdot \log_2 23} \ge c$$

$$c \le 1$$

$$\therefore f(n) = \Theta(g(n))$$

3(b)

$f(n) = O(g(n))$ then $f(n) \le c_1 g(n)$ $(c_1, n > 0)$

$g(n) = O(h(n))$ then $g(n) \le c_2 h(n)$ $(c_2, n > 0)$

$$f(n) \le c_1 g(n) \le c_1 \cdot c_2 h(n)$$

$$f(n) \le c_1 \cdot c_2 h(n)$$

then $f(n) = O(h(n))$

3(c)

FindMedian(Array A[1... n])

For integer $i \leftarrow 1$ to n-1
    $k \leftarrow i$
    for integer $j \leftarrow i$+1 to n
        if $A[k] > A[j]$ then
            $k \leftarrow j$
        swap $A[i]$ and $A[k]$
if n%2 == 0 then
    $median \leftarrow A[\lceil \frac{n-1}{2} \rceil]$ and $A[\lfloor \frac{n-1}{2} \rfloor]$
else
    $median \leftarrow A[\frac{n-1}{2}]$
return $median$

Question 4.

4(a)

Assume n = 1, P(1) :

LHS = $S_1 \cup T$ = RHS

So P(1) is true;

Assume n = k, and P(k) is true, then

LHS = $(S_1 \cup T) \cap (S_2 \cup T) \cap \ldots \cap (S_k \cup T)$

RHS = $(S_1 \cap S_2 \ldots \cap S_k) \cup T$

In case n = k + 1, P(k+1):

LHS = $(S_1 \cup T) \cap (S_2 \cup T) \cap \ldots \cap (S_k \cup T) \cap (S_{k+1} \cup T)$

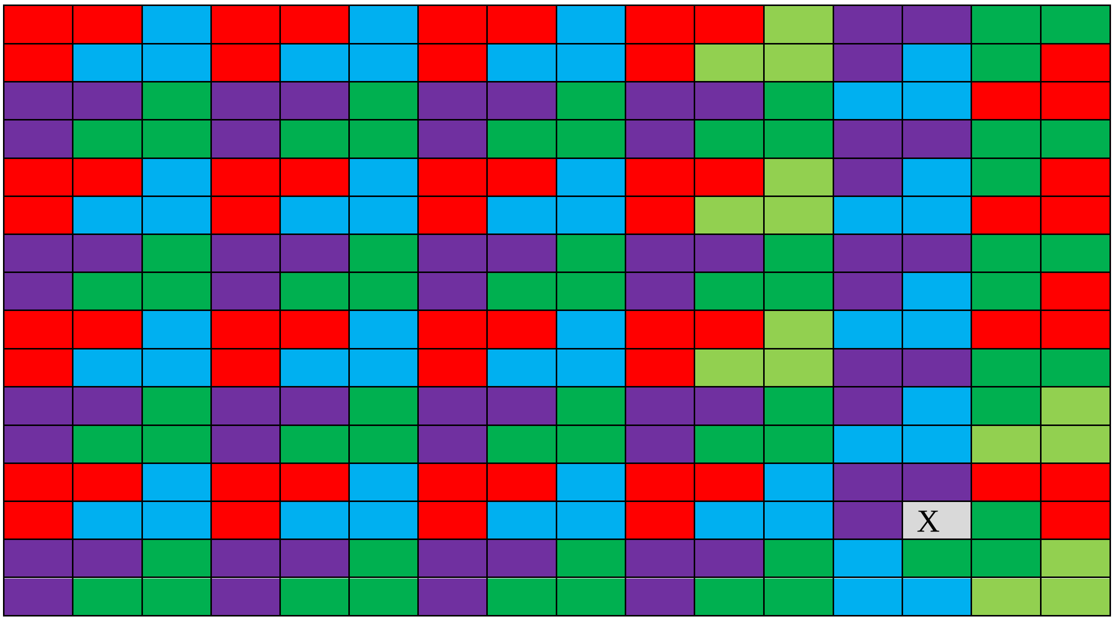
$= ((S_1 \cap S_2 \ldots \cap S_k) \cup T) \cap (S_{k+1} \cup T)$

$= (S_1 \cap S_2 \ldots \cap S_k \cap S_{k+1}) \cup T = RHS$

So P(k+1) is true for all integer k

Hence P(n) is true for $\forall$ x$\in \mathbb{N}$.

4(b)



End of Assignment 1