Lecture 1 Overview, Introduction, Graph

Network and Graph

- Graph = Object + Connection/Relationship
- Node - vertex
- Link - edge

Isomorphism

intrinsic structure of two graphs are the same

Lecture 2 - Graph, big data verification

Path, Connected Graph, Connected Component

- Path: sequence of nodes with property (each consecutive pair) in sequence connected by a edge
- *shortest path

Concepts:

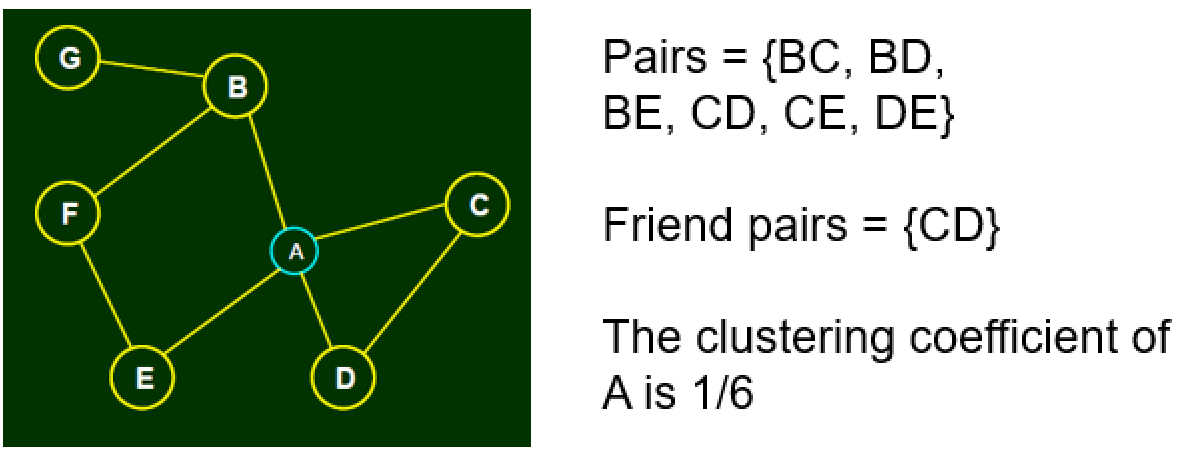
- connected graph/connected component: subset, maximal
- bipartite graph: does not contain cycle with an odd number of nodes
- breadth-first-search(BFS): Search from one node to another

General study methodologies

- properties of nodes, links, graphs
- real world problems transformed into graph
- how to interference from properties(solve problems)

Clustering Coefficient

- importance of certain node
- clustering coefficient pf A = friend pairs of node A/total pair of connected nodes of node A



Triadic Closure

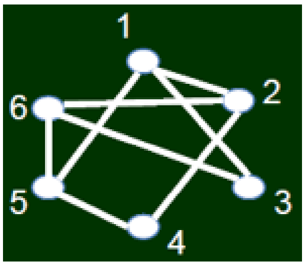
(evolution along times increase)

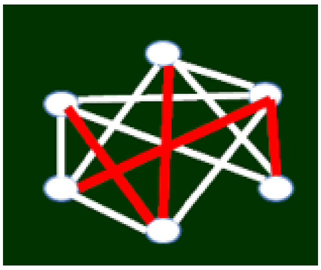
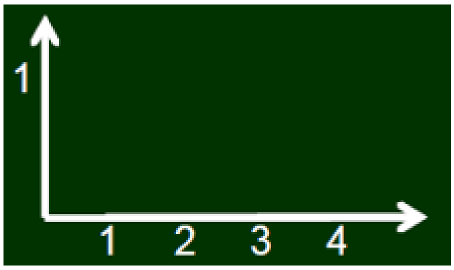
- if who originally don't know each other, they have an increasing chance to get to know each other

Triadic Closure Verification using Big Data

how many number of common friend

1. identify the edge don't pair together.
2. find common friends
3. probability to be friends(with number of common friends)



at different time

(1,4)	2	*
(1,6)	3	
(2,3)	2	*
(2,5)	3	*
(3,4)	0	
(3,5)	2	
(4,6)	2	*

Common friends	0	1	2	3	4
Probability to become friends	0	-	0.75	0.5	-

social interpretation -> qualitative -> qualitative description

- relationship of nodes changes along time.

Structural Holes

- After removing of which, makes the network become multiple connected components
- access to non-interacting parts
- less redundancy, more social capital
- e.g. 1 node is also a component.

Embeddedness

- number of common neighbors of a link
- e.g. embeddedness of (A, B) = 2

Strong tie, weak tie

- property of link

strong triadic closure

- (A, B) and (A, C) has strong tie, but there's no edge at all. so violate the strong triadic closure property.
- no common friend -> weak ties

Short cuts

connecting two nodes without which will lead to a long distance

bridge

after removal, number of connected components increases

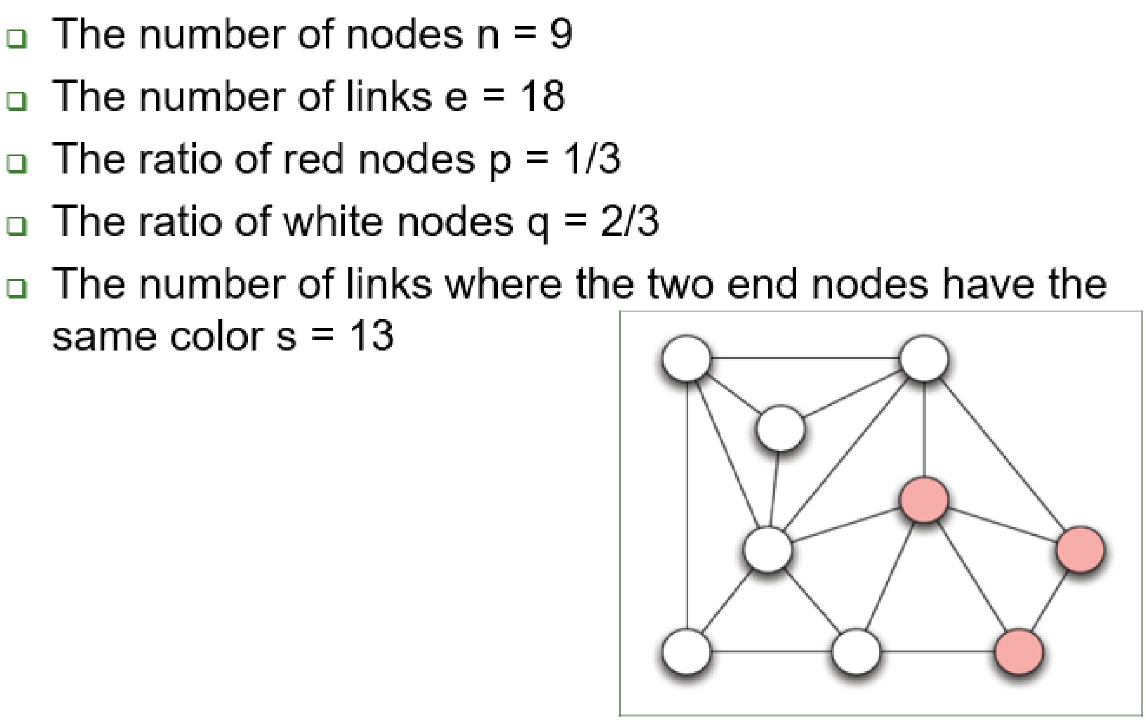
Lecture 3 - Homophily, big data analysis

Homophily

- common
- selection and social influence

Measuring homophily

definition of similarity can be different in different problems

- degree of similarity
    - 

    - more link with same color node, higher homophily
    - number of links(node color not same)/all links(s/e)
    - s/e vs. sum of ratio of different color nodes

Selection vs. Social Influence(mechanism)

selection:

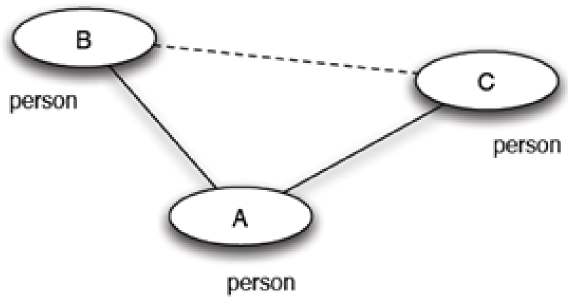
- become friends because of common interests //affiliation network

social influence:

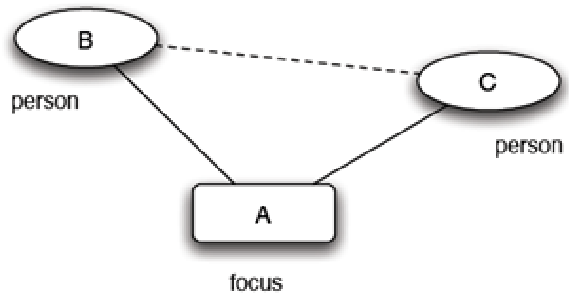
- join another club/organization together
- closure because of a common friend, but because of influence by a friend

3 types of closure

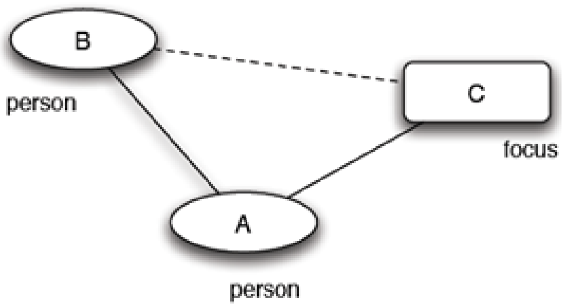
triadic closure Selection -> focal closure social influence -> membership closure



(a) *Triadic closure*



(b) *Focal closure*



(c) *Membership closure*

focal closure

- shared foci, connection
- more shared foci, higher chance of connection

Membership closure

- a friend has certain focus -> join the focus
- more friend in a focus, higher probability

big data analysis on origin of homophily

measure similarity: e.g. editor&wiki article: (X和Y都编辑的文章数量)/(X或Y中至少有一人编辑的文章数量)

Schelling Model

- agent live in a cell. 2 types of agents, x and o
- constraint: certain number of same-type neighbors

Lecture 4 - Small World

Small world problem

- median = 6
- six degree of seperation

Decentralized search

- every node can only see its neighboring nodes

decentralized search vs. breadth first search

- BFS: send to every friends, reach target
- DS: may fails to reach target person

phenomenon

- not straightforward
- network is quite sparse

limitation

- no short path

Watts-Strogatz-Kleinberg model

The probability that two nodes have a random link is inverse proportional to their grid distance with exponent q.(q=2)

Verification with big data

optimal q

Lecture 5 - Core-Periphery Structure, Directed Graph, Web Structure

Core-periphery Structure

- importance of people
- not reflected in decentralized search
- social status can be reflected by network structure

Directed Graph

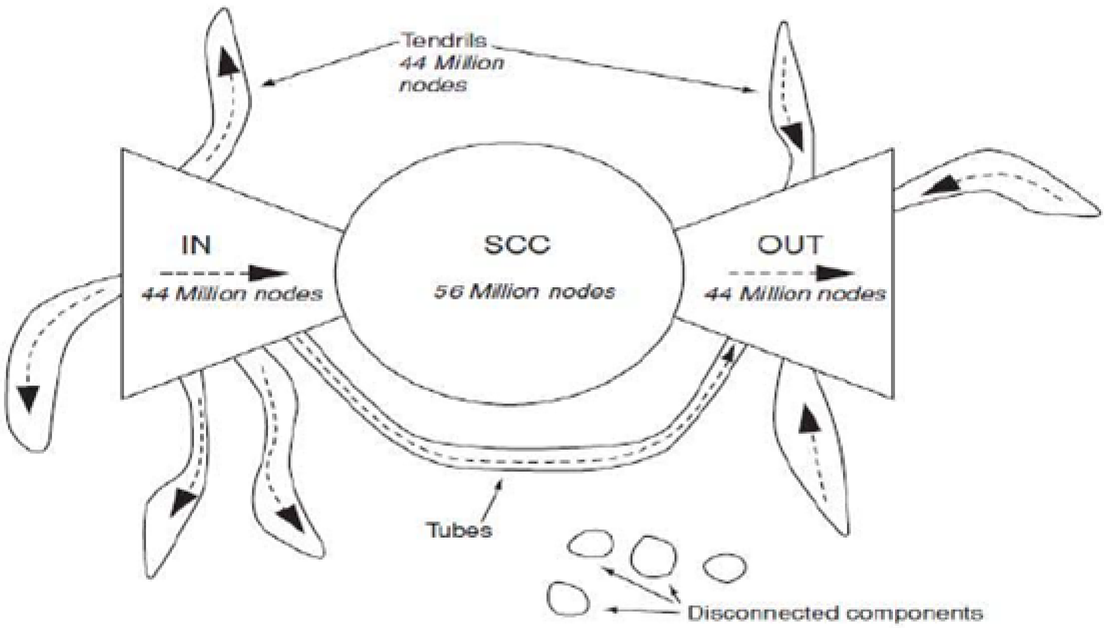
relationship with direction

- strong connectivity

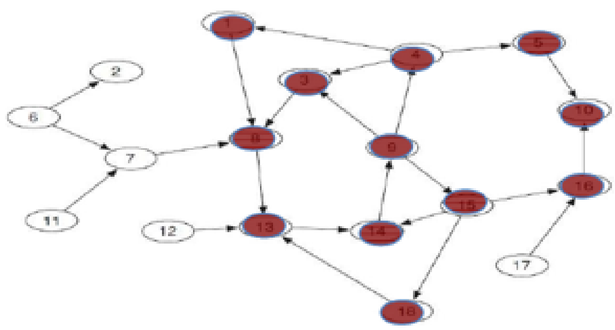
strong connected component

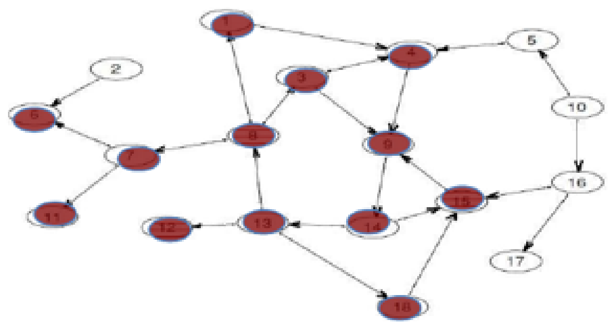
1. every node in subset has path to every other
2. subset is not part of some larger set with property that every node can reach every other

Bowtie Structure



How to obtain SCC, IN, OUT





FS = {1, 8, 3, 4, 9, 14, 13, 15, 18, 5, 10, 16}

BS = {1, 8, 3, 4, 9, 13, 14, 15, 18, 6, 7, 11, 12}

SCC = FS ∩ BS = {1, 3, 4, 8, 9, 13, 14, 15, 18}

IN = BS – SCC = {6, 7, 11, 12}

OUT = FS – SCC = {5, 10, 16}

proof SCC: all nodes in SCC are connected, no bigger SCC

Lecture 6 PageRank

Hub and Authorities

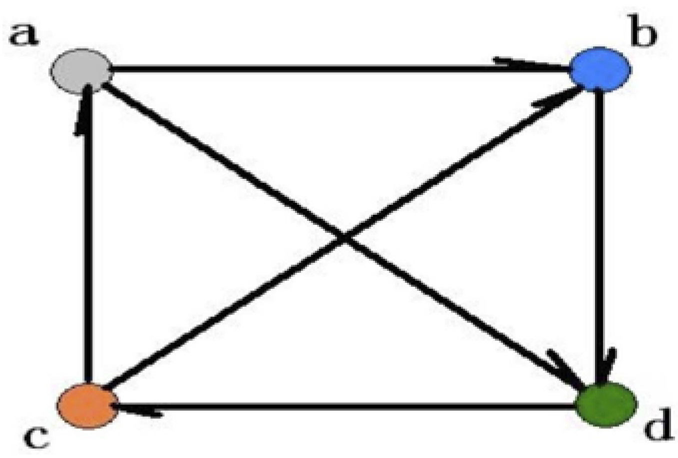
auth(p) hub(p)----HITS algorithem

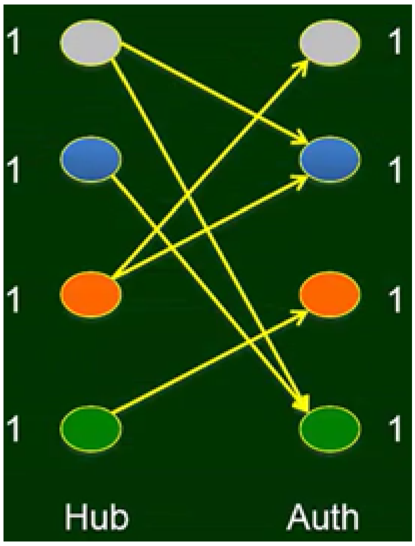
- input: directed graph
- initialization: for every p, auth(p)=1, hub(p)= 1
- authority update rule: auth(p) = sum of hub score of all page that point to it
- hub update rule: hub(p) = sum of authority score of all pages that it points to
- repeat

Information

- **Auth(a) = sum of Hub(a-in)**
- **Hub(a) = sum of Auth(a-out)**

- Compute the authority and hub scores of the following graph, run for 3 rounds



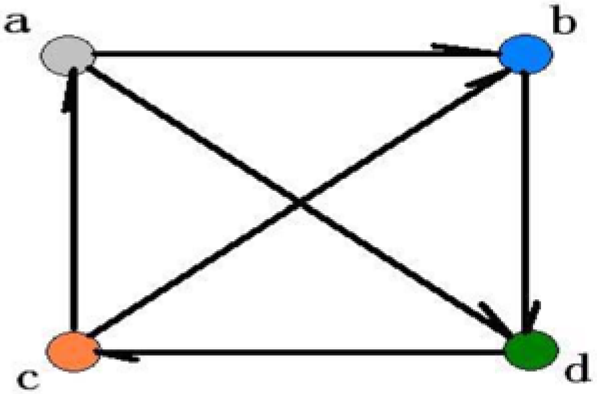


Auth				Hub			
a	b	c	d	a	b	c	d
a=H[c]	b=H[a]+H[c]	c=H[d]	d=H[a]+H[b]	a=A[b]+A[d]	b=A[d]	c=A[a]+A[b]	d=A[c]
1	1	1	1	1	1	1	1
1	2	1	2				
				4	2	3	1
3	7	1	6				
				13	6	10	1
10	23	1	19				
				42	19	33	1
33	75	1	61				
				136	61	108	1

PageRank

- share

- outgoing links with equal share



$$a = c/2$$
$$b = a/2 + c/2$$
$$c = d$$
$$d = a/2 + b$$

Lecture 6 & 7 & 8 - Game Theory

Game Theory(Pure Strategy)

		Your Partner: *Presentation*	Your Partner: *Exam*
You	*Presentation*	$90, 90$	$86, 92$
	Exam	$92, 86$	$88, 88$

basic ingredients of game

1. set of participants
2. set of option(strategies)
3. received payoff situation
4. received payoff matrix

(Strictly) Best Response & (Strictly) Dominant strategy

- Best response: A’s strategy can maximize A's payoff when response to one of B’s strategy
- strict best response: uniqueness
- Dominant strategy: A’s dominant strategy is the best response to every strategies of B
- Strictly dominant strategy: uniqueness
- Result
  - If both have strictly dominant strategy, both will adopt them
  - Only A has strictly dominant strategy, A will adopt it and B will adopt the best response to this strictly dominant strategy

Nash Equibilrium

- best response to each other

- no onw can be better by unilaterally change his own strategy; though both can become better if both changes

Multiple Equilibria

- need more information
- only narrow down the choice, but may not guarantee to predict

Mixed Strategies(no Nash Equilibrium)

- probability: randomness *distribution

- Expectation: E(X) = ap + b(1-p)

  - Expectation(A chose S1) = a1p+b1(1-p)
  - Expectation(A chose S2) = a2p+b2(1-p)
  - Ex(A-S1) = EX(A-S2)
  - Expectation(B chose S3) = c1q+d1(1-q)
  - Expectation(B chose S4) = c2q+d2(1-q)
  - Ex(B-S3) = EX(B-S3)

- set of probability distribution making the other player indifferent in choosing any his pure strategy

pure strategy vs. mixed strategy

1. Pure Strategy
2. Mixed Strategy: p & q

Pareto Optimality

- if there's no other choice of strategy in which all players receive payoff at least as high, at least one player receives a stritcly higher payoff

Social Optimality

- maximize sum of the players' payoffs

Lecture 9 & 10 - Network Traffic, Auction, Matching market

Game on Network Structure

Ingredients

1. Players: # of drives

2. Strategy set

3. Driver's Payoff: travel time(the less the better, depend on other's choice)

4. Equilibrium: no one has incentive to change

5. if anyone deviates, his pyoff will be: ...

Braess's Paradox

- Invest more resources may not get a good result

Auction

Ingredients in Game

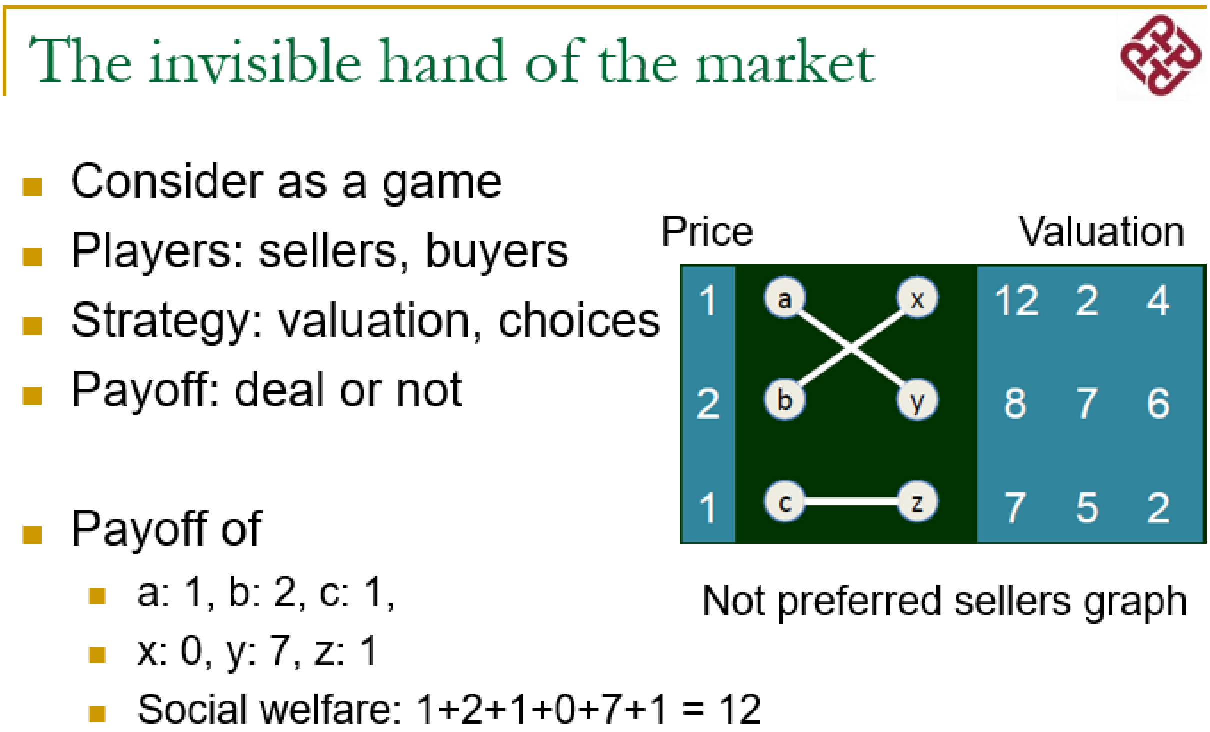
- Participant: sellers and buyers
- Strategy: bid
- Payoff: for buyers: value of the object; for sellers: the paid price for object
- Equilibrium:the best response for each other

Matching

- perfect matching: every one has object satisfied

Market invisible hand

- players: sellers, buyers
- strategy: valuation, choice
- payoff: deal or not
- Payoff of seller a, b, c & buyer x, y, z
- social welfare: sum of everyone's payoff



Market-clearing prices

- Prices combination set that maximize the social welfare

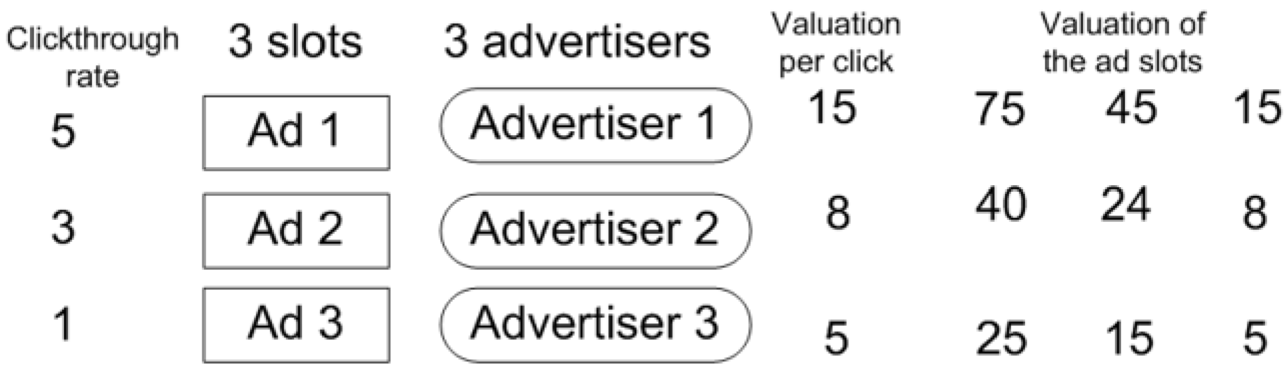
Lecture 10 & 11 - Sponsered search market

Sponsered search market

- combination of matching and auction

Concepts

- advertising slots
- clickthrough rate: expected click per hour on an ad slot
- advertiser's revenue per click: the expected revenue of every click
- advertiser's valuation: = clickthrough rate * revenue per click
- advertiser's payoff: (expected revenue - price) * clickthrough rate



Vickrey-Clareke-Groves(VCG) Mechanism

P(ij) = V(S,B-j) - B(S-i,B-j)

- buyer j bids true valuation and get slot i, payoff = v(ij)-p(ij)
- change to slot h, payoff = v(hj)-p(hj)
- need to show: v(ij)-p(ij) >= v(hj)-p(hj)
- translate to: v(ij) - [V(S,B-j) - V(S-i,B-j)] >= v(hj) - [V(S,B-j) - V(S-h,B-j)]
- which is: v(ij) + V(S-i, B-j) >= v(hj) + V(S-h, B-j)
- because we know: v(ij) + V(S-i, B-j) = V(S,B)
- and v(hj) + V(S-h, B-j) <= V(S,B)

Generalized Second-Price(GSP) Auction

payoff = v(i)r(i) - b(i+1)r(i)