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Subject: **OR**

Class Exercise

Case Study#1

Product Mix Problem:

The ABC Company produces two products: I and II. The raw material requirements, space needed for storage, production rate and selling prices for these products are given below:

\_\_\_\_\_\_\_\_\_\_Product\_\_\_\_\_\_\_\_\_\_\_\_

I II

|  |  |  |
| --- | --- | --- |
| Storage space(ft²/unit) | 4 | 5 |
| Raw material(lb/unit) | 5 | 3 |
| Production rate(units/hr) | 60 | 30 |
| Selling price($/unit) | 13 | 11 |

The total amount of raw material available per day for both products is 1575lb. The total storage space for all products is 1500 ft², and a maximum of 7 hours per day can be used for production. The company wants to determine how many units of each product to produce per day to maximize its total income.

**Solution:**

**Step-1 : Defining decision-variables:**

* let x1 be the no of units of product#1 produced per day.
* let x2 be the no of units of product#2 produced per day.

**Step-2 : Formulating objective function:**

maximize Z = 13.x1 + 11.x2

**Step-3: Formulating Constraints:**

1. Raw-material Constraint:

5.x1 + 3.x2 <=1575

1. Storage-space constraint:

4.x1 + 5x2 <=1500

1. Production-rate constraint:

x1/60 + x2/30 <=7

1. Non-negativity constraint:

x1, x2 >=0

**Complete LP-Model:**

maximize Z = 13x1 + 11x2 --------------🡪(i)

**subject to:**

5.x1 + 3.x2 <=1575 --------------🡪(ii)

4.x1 + 5x2 <=1500 --------------🡪(iii)

x1/60 + x2/30 <=7 --------------🡪(iv)

x1, x2 >=0 --------------🡪(v)

**Task#2:**

Maximize Z= $20A + $30C

subject to:

A+2C <=120 .....(i)

A<=60 .....(ii)

C<=50 .....(iii)

A,C >=0 .....(iv)

**find co-ordinate points, and locate on graph.**

**Step-1: Converting constraints in equations:**

**Constraint#1:**

A+2C <=120

After converting into equation:

A+2C = 120

We need two points **P1** & **P2** todraw the line of this equation:

1. **For P1 putting A=0 we get:**

0+2C = 120

2C= 120

C=60

Hence point **P1(A,C)= (0,60)**

1. **For P2 putting C=0 we get:**

A+2(0) = 120

A+0= 120

A=120

Hence point **P2(A,C)= (120,0)**

Now Taking Origin(0,0) as a trial point in Constraint#1 to identify its feasible region on graph:

A+2C <=120

0+0 <=120

0<=120 => since point below line(i.e origin) is

satisfying the constraint's inequality hence

feasible region for cons#1 is downward.

**Constraint#2:**

A<=60

After converting into equation:

A=60

**So the Point P1(A,C)= (60,0);**

Now Taking Origin(0,0) as a trial point in Constraint#2 to identify its feasible region on graph:

A<=60

0<=60 => since point below line(i.e origin) is

satisfying the constraint's inequality hence

feasible region for cons#2 is downward.

**Constraint#3:**

C<=50

After converting into equation:

C=50

**So the Point P1(A,C)= (0,50);**

Now Taking Origin(0,0) as a trial point in Constraint#3 to identify its feasible region on graph:

C<=50

0<=50 => since point below line(i.e origin) is

satisfying the constraint's inequality hence

feasible region for cons#3 is downward.

**Constraint#4:**

A,C >=0

**Since we are working only in first quadrant hence this constraint is already satisfied.**

**Task#3: E-9s and F-9s problem:**

Maximize Z= 5000.E + 4000.F

subject to:

10.E + 15.F <= 150 ......(i)

20.E + 10.F <= 160 ......(ii)

30.E + 10.F >=135 ......(iii)

E-3F <=0 ......(iv)

E+F>=5 ......(v)

E,F>=0 ......(vi)

**Converting constraints in equations:**

**Constraint#1:**

10.E + 15.F <= 150

After converting into equation:

10.E + 15.F = 150

We need two points **P1** & **P2** todraw the line of this equation:

1. **For P1 putting E=0 we get:**

10(0)+15F = 150

15F= 150

F=10

Hence point **P1(E,F)= (0,10)**

1. **For P2 putting F=0 we get:**

10E+15(0) = 150

10E= 150

E=15

Hence point **P2(E,F)= (15,0)**

Now Taking Origin(0,0) as a trial point in Constraint#1 to identify its feasible region on graph:

10.E + 15.F <= 150

0+0 <=150

0<=150 => since point below line(i.e origin) is

satisfying the constraint's inequality hence

feasible region for cons#1 is downward.

**Constraint#2:**

20.E + 10.F <= 160

After converting into equation:

20.E + 10.F = 160

**i) For P1 putting E=0 we get:**

20(0)+10F = 160

10F= 160

F=16

Hence point **P1(E,F)= (0,16)**

1. **For P2 putting F=0 we get:**

20E+10(0) = 160

20E= 160

E=8

Hence point **P2(E,F)= (8,0)**

Now Taking Origin(0,0) as a trial point in Constraint#2 to identify its feasible region on graph:

20.E + 10.F <= 160

0+0<=160 => since point below line(i.e origin) is

satisfying the constraint's inequality hence

feasible region for cons#2 is downward.

**Constraint#3:**

30.E + 10.F =135

After converting into equation:

30.E + 10.F =135

**i) For P1 putting E=0 we get:**

30(0)+10F = 135

10F= 135

F=13.5

Hence point **P1(E,F)= (0,13.5)**

1. **For P2 putting F=0 we get:**

30E+10(0) = 135

30E= 135

E=13.5

Hence point **P2(E,F)= (4.5,0)**

Now Taking Origin(0,0) as a trial point in Constraint#3 to identify its feasible region on graph:

30.E + 10.F >= 135

0+0>=135 => since point below line(i.e origin) is not

satisfying the constraint's inequality hence

feasible region for cons#3 is upwards.

**Constraint#4:**

E-3F <=0

After converting into equation:

E-3F =0

**i) For P1 putting E=0 we get:**

(0)-3F = 0

-3F= 0

F=0

Hence point **P1(E,F)= (0,0)**

1. **For P2 putting F=2(any value other than 0 so that we can plot on graph) we get:**

E-3(2) = 0

E-6= 0

E=6

Hence point **P2(E,F)= (6,2)**

Now Taking Point(6,4) as a trial point in Constraint#4 to identify its feasible region on graph:

E - 3.F <= 0

6-12<=0 => since point above line is satisfying

the constraint's inequality hence

feasible region for cons#4 is upwards.

**Constraint#5:**

E+F>=5

After converting into equation:

E+F =5

**i) For P1 putting E=0 we get:**

(0)+ F = 5

F= 5

Hence point **P1(E,F)= (0,5)**

1. **For P2 putting F=0 we get:**

E+(0) = 5

E= 5

Hence point **P2(E,F)= (5,0)**

Now Taking Origin(0,0) as a trial point in Constraint#4 to identify its feasible region on graph:

E + F >=5

0+0>=0 => since point below line(i.e origin) is not

satisfying the constraint's inequality hence

feasible region for cons#5 is upwards.

**Constraint#6:**

E,F >=0

**Since we are working only in first quadrant hence this constraint is already satisfied.**

**STEP-2: Plotting objective function:**

**Maximize Z= 5000.E + 4000.F**

Assuming Z=20000 we get

20000 = 5000.E + 4000.F

1. For point P1 put E=0:

20000 = 0 + 4000F

4000F= 20000

F= 5

(E,F)= (0,5)

1. For point P2 put F=0:

20000 = 5000E + 0

5000E= 20000

E= 4

(E,F)= (4,0)

Plotting points on Graph and joining both to form its line.

Move this Line in outward direction keeping slope constant since point C is last point touched by this objective function

line that's why C is the optimal point for given objective function.

Now to find point C :

As the C point is intersection of line of cons#1 & cons#2 lines:

cons#1:

10.E + 15.F = 150

cons#2:

20.E + 10.F = 160

solving simultaneously we get= C(4.5,7) this is the optimal point

Now for finding optimal solution we will put C(4.5,7) in objective function's line

Maximize Z= 5000.E + 4000.F

Z= 5000(4.5) + 4000(7)

Z= 22500+28000

Z= 50500

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Solution:

P(4.5,7) [Optimal Solution point]

Z= $50500 [Optimal Value]

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**Task#4: Multiple Optimal Solution Points problem:**

**Ans:**

**Max z= 3x1 - x2**

subject to:

15x1 - 5x2 <=30

10x1 + 30x2<=120

x1,x2>=0

**STEP-2: Plotting objective function:**

**Max z= 3x1 - x2**

Assuming z=3 we get

3 = 3.x1 - x2

i) For point P1 put x1=2:

3 = 3(2) - x2

-3=-x2

x2= 3

(x1,x2)= (2,3)

ii) For point P2 put x2=0:

3 = 3x1 - 0

3=3x1

x1= 1

(x1,x2)= (1,0)

Join these Points to form O.F's Line

Since we see that moving this Line in outward direction, the line overlaps the equation line of Cons#1

Hence we can say all points lying on cons#1 line will be the optimal solution points let's prove it:

As Point B(3,3) & C(2,0) also lie on this line so we will put both for finding optimal value:

Max z= 3x1 - x2

putting point B(3,3) we get:

z= 3(3)-3

z= 9-3

z=6

Max z= 3x1 - x2

putting point C(2,0) we get:

z= 3(2)-0

z= 6-0

z=6

Let's take Another point X(2.6,1.8) lying on this line for checking too:

Max z= 3x1 - x2

putting point X(2.6,1.8) we get:

z= 3(2.6)-1.8

z= 7.8-1.8

z=6

**Since the Value of O.F we get For each at these points is 6 hence**

**6 is the optimal value But O.F has many Optimal Points.**

**Task#4:**

**maximize z= 100x + 100y**

subject to:

10x + 5y <=80 (or 2x+y<=16) ……(i)

6x+6y<=66 (or x+y<=11) ……(ii)

4x+8y>=24 (or x+2y>=6) …..(iii)

5x+6y <=90 …..(iv)

x1,x2>=0 …..(v)

**Converting constraints in equations:**

**Constraint#1:**

2x+y <=16

**In Equation form:**

2x+y =16

i) For point P1 put x=0:

2(0)+y =16

y=16

(x,y)= (0,16)

ii) For point P2 put y=0:

2x+0 =16

2x=16

x=8

(x,y)= (8,0)

Now Taking Origin(0,0) as a trial point in Constraint#1 to identify its feasible region on graph:

2x+y <=16

0+0<=16 => since point below line(i.e origin) is

satisfying the constraint's inequality hence

feasible region for cons#1 is downward.

**Constraint#2**

x+y<=11

**In Equation form:**

x+y =11

i) For point P1 put x=0:

0+y=11

y=11

(x,y)= (0,11)

ii) For point P2 put y=0:

x+0=11

x=11

(x,y)= (11,0)

Now Taking Origin(0,0) as a trial point in Constraint#2 to identify its feasible region on graph:

x+y<=11

0+0<=11 => since point below line(i.e origin) is

satisfying the constraint's inequality hence

feasible region for cons#2 is downward.

**Constraint#3**

x+2y>=6

**In Equation form:**

x+2y=6

i) For point P1 put x=0:

0+2y=6

2y=6

y=3

(x,y)= (0,3)

ii) For point P2 put y=0:

x+2(0)=6

x=6

(x,y)= (6,0)

Now Taking Origin(0,0) as a trial point in Constraint#3 to identify its feasible region on graph:

x+2y>=6

0+0 >=6 => since point below line(i.e origin) is

Not satisfying the constraint's inequality hence

feasible region for cons#3 is upwards.

**Constraint#4**

5x+6y <=90

**In Equation form:**

5x+6y =90

i) For point P1 put x=0:

5(0)+6y =90

6y=90

y=15

(x,y)= (0,15)

ii) For point P2 put y=0:

5x+6(0) =90

5x=90

x=18

(x,y)= (18,0)

Now Taking Origin(0,0) as a trial point in Constraint#4 to identify its feasible region on graph:

5x+6y <=90

0+0<=90 => since point below line(i.e origin) is

satisfying the constraint's inequality hence

feasible region for cons#4 is downward.

**Constraint#5:**

x,y >=0

**Since we are working only in first quadrant hence this constraint is already satisfied.**

**STEP-2: Plotting objective function:**

maximize z= 100x + 100y

Assuming Z=200 we get

200= 100x + 100y

1. For point P1 put x=0:

200= 100(0) + 100y

200= 100y

100y=200

y=2

(x,y)= (0,2)

1. For point P2 put y=0:

200= 100x + 100(0)

200= 100x

100x=200

x=2

(x,y)= (2,0)

Plotting points on Graph and joining both to form its line.

Move this Line in outward direction keeping slope constant since point C is last point touched by this objective function

line that's why C is the optimal point for given objective function.

Now to find point C :

As the C point is intersection of line of cons#1 & cons#2 lines:

cons#1:

2x+y =16

cons#2:

x + y=11

solving simultaneously we get= C(5,6) this is the optimal solution point

Now for finding optimal value we will put C(5,6) in objective function's line

maximize z= 100x + 100y

Z= 100(5) + 100(6)

Z= 500+600

Z= 1100

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Solution:

P(5,6) [Optimal Solution point]

Z= $1100 [Optimal Value]

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**Task#5: Minimization Problem**

**minimize z= 8x1 + 6x2**

subject to:

4x1 + 2x2 >=20 or(2x1+x2>=10) ………(i)

-6x1 + 4x2 <=12 or(-3x1+2x2<=6) ……….(ii)

x1 + x2 >=6 ………..(iii)

x1,x2>=0 ……….(iv)

**Converting constraints in equations:**

**Constraint#1:**

2x1+x2 >=10

**In Equation form:**

2x1+x2 =10

i) For point P1 put x1=0:

2(0)+x2 =10

x2=10

(x1,x2)= (0,10)

ii) For point P2 put x2=0:

2x1+(0) =10

2x1=10

x1=5

(x1,x2)= (5,0)

Now Taking Origin(0,0) as a trial point in Constraint#1 to identify its feasible region on graph:

2x1+x2 >=10

0+0<=10 => since point below line(i.e origin) is

Not satisfying the constraint's inequality hence

feasible region for cons#1 is upwards.

**Constraint#2**

-3x1+2x2<=6

**In Equation form:**

-3x1+2x2 =6

i) For point P1 put x1=0:

-3(0)+2x2 =6

2x2=6

x2=3

(x1,x2)= (0,3)

ii) For point P2 put x2=6:

-3x1+2(6) =6

-3x1=6-12

-3x1=-6

x1=2

(x1,x2)= (2,6)

Now Taking Origin(0,0) as a trial point in Constraint#2 to identify its feasible region on graph:

-3x1+2x2<=6

0+0<=6 => since point below line(i.e origin) is

satisfying the constraint's inequality hence

feasible region for cons#2 is downward.

**Constraint#3**

x1+x2>=6

**In Equation form:**

x1+x2=6

i) For point P1 put x1=0:

0+x2=6

x2=6

(x1,x2)= (0,6)

ii) For point P2 put x2=0:

x1+0=6

x1=6

(x1,x2)= (6,0)

Now Taking Origin(0,0) as a trial point in Constraint#3 to identify its feasible region on graph:

x1+x2>=6

0+0 >=6 => since point below line(i.e origin) is

Not satisfying the constraint's inequality hence

feasible region for cons#3 is upwards.

**Constraint#4:**

x,y >=0

**Since we are working only in first quadrant hence this constraint is already satisfied.**

**STEP-2: Plotting objective function:**

minimize z= 8x1 + 6x2

Assuming Z=24 we get

24= 8x1 + 6x2

1. For point P1 put x1=0:

24= 8(0) + 6x2

24= 6x2

6x2=24

x2=4

(x1,x2)= (0,4)

1. For point P2 put x2=0:

24= 8x1 + 6(0)

24= 8x1

8x1=24

x1=3

(x1,x2)= (3,0)

Plotting points on Graph and joining both to form its line.

Move this Line in outward direction keeping slope constant since point C is firt point touched by this objective function line that's why C is the optimal point for given minimization objective function.

Now to find point C:

As the C point is intersection of line of cons#1 & cons#3 lines:

cons#1:

2x1+x2 >=10

cons#3:

x1 + x2=6

solving simultaneously we get= C(4,2) this is the optimal solution point

Now for finding optimal value we will put C(4,2) in objective function's line

minimize z= 8x1 + 6x2

Z= 8(4) + 6(2)

Z= 32+12

Z= 44

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Solution:

P(4,2) [Optimal Solution point]

Z= $44 [Optimal Value]

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**Task#6:** **Find conclusion of following objective function by changing right side of cons#1 from 40 to 80.**

max Z= 40x1 + 50x2

subject to:

x1 + 2x2 <=40 hr. of labor

4x1 + 3x2 <=120 120 lb. of clay

x1,x2>=0

**Ans:**

1. Changing Cons#1 coefficient:

x1 + 2x2<=80

in equation:

x1 + 2x2 =80

so p1 & p2 are P1(80,0) & P2(0,40)

Plot these points on graph.

For feasible region put Origin(0,0) as trial point:

x1 + 2x2 <=80

0 + 0 <=80

hence Feasible region=downwards;

Since point A'' is last point touched by O.F line while moving outwards.

As A'' point lies on x2 axis Hence no need to solve simultaneously the equations:

So the A'' point is A''(0,40)

To find optimal value put A''(0,40) in O.F:

Z= $40x1 + 50x2

z= 40(0) + 50(40)

z= 2000

So Now calculating difference b/w Old & New Z;

delta OV(delta Z)= Z(new) -Z(old)

delta objective value z= 2000-1360

delta OV z= 640

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So The objective Function's Optimal value changes by 640 by changing right side of cons#1 from 40 t0 80;

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Conclusion:

1. There is a parallel shift in the constraint#1's line:

2. Feasible region has been enlarged by changing right side of constraint#1 from 40 to 80.

3. Optimal value has been changed by 640 units from original value.

4. Optimal Solution Point has been changed from B(24,8) to A''(0,40).

**Task#7: Assume any Primal LP model and transform it into Dual LP Model using transformation rules.**

**ANS:**

Let’s assume E9 & F9 problem as Primal LP so the model is:

**Maximize Z= 5000.E + 4000.F**

**subject to:**

**2.E + 3.F <= 30 ......(i)**

**2.E + F <= 16 ......(ii)**

**6.E + 2.F >=27 ......(iii)**

**E-3F <=0 ......(iv)**

**E+F>=5 ......(v)**

**E,F>=0**

**Rule#1:** The no of variables in the dual problem(DP) is equal to number of constraints in the original(primal) solution. The number of constraints in the dual problem is equal to no of variables in primal problem(PP).

**Result of Rule#1:** 5 variables(y1,y2,y3,y4,y5) //as there were 5 constraints in PP

2 constraints //as there were 2 variables in PP

**Rule#2:** Co-efficient of the objective function in DP come from right hand side of constraints of primal problem:

**Result of Rule#2:** Objective function of dual problem:

**Z = 30y1 + 16y2 + 27y3 + 0.y4 +5y5**

* 30,16,27,0,5 came from right side of each constraint respectively.
* Still we are unaware of Z is either max or min objective function.

**Rule#3:** If primal problem is maximization problem then dual problem will be a minimization problem and vice versa.

**Result of Rule#3:**

* Dual will be minimization problem //as pp is a maximization problem.

**MIN Z = 30y1 + 16y2 + 27y3 + 0.y4 +5y5**

**Rule#4:** The coefficients of the first constraint function for the dual problem are the coefficients of first variable in the constraints for the pp & similarly for other constraints.

**Result of Rule#4:** left hand side of constraints

**cons#1:** 2.y1 + 2.y2 + 6.y3 + 1.y4 + 1.y5

**cons#2:** 3.y1 + 1.y2 + 2.y3 – 3.y4 + 1.y5

**Rule#5:** The right hand side of dual constraints come from the objective function coefficients in the original problem.

**Result of Rule#5:** Right hand side of constraints:

cons#1: 5000

cons#2: 4000

* Still we don't know about sign of left hand side & right hand side.

**L.H.S** **R.H.S**

cons#1: 2.y1 + 2.y2 + 6.y3 + 1.y4 + 1.y5 5000

cons#2: 3.y1 + 1.y2 + 2.y3 – 3.y4 + 1.y5 4000

**Task#8: Convert following primal model into dual, take a set(y1,y2,y3)=(0,1,6) in dual problem and then check is it a dual feasible set or not?, find Z(dual) and finally compare Z(primal) with Z(dual).**

**ANS:**

**Max Z= 3x1 + 4x2**

**subject to:**

**-2x1+3x2 <=6**

**5x1 - x2 <=40**

**x1+ x2 <=7**

**x1>=0, x2>=0**

* As we saw that P(x1,x2) =(3,2) was a Primal feasible set i.e satisfied all constraints and the Optimal value of O.F on this set of values is

**Z(primal) =17 ----------(1)**

**Step-1: Converting this Primal Model into a DUAL model:**

**Result of Rule#1:** 3 variables(y1,y2,y3)

2 constraints

**Result of rule#2:** Objective function of dual problem:

Z = 6y1 + 40y2 + 7y3

**Result of rule#3:** Dual will be minimization problem

MIN Z = 6y1 + 40y2 - 7y3

**Result of rule#4:** left hand side of constraints

cons#1: -2y1 + 5y2 + y3

cons#2: 3y1 - y2 + y3

**Result of Rule#5:** Right hand side of constraints:

cons#1: 3

cons#2: 4

L.H.S R.H.S

cons#1: -2y1 + 5y2 + y3 3

cons#2: 3y1 - y2 + y3 4

**Result of Rule#6:**

//as we see no variable is unconstrained in our original

primal problem so no any result of Rule#6

**Result of Rule#7:** Assigning signs to remaining constraints(For Max).

cons#1: -2y1 + 5y2 + y3 >= 3 :>as x1 was x1>=0

cons#2: 3y1 - y2 + y3 >=4 :>as x2 was x2>=0

**Result of Rule#8:**

//as we see no any constraint is having '=' in our original

primal problem so no any result of Rule#8

**Result of Rule#9:** Assigning signs to remaining variables(For Max).

Y1>=0;

Y2>=0;

Y3>=0;

Complete/Final DUAL LP Model:

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MIN Z = 6y1 + 40y2 + 7y3

subject to:

cons#1: -2y1 + 5y2 + y3 >= 3 -------(i)

cons#2: 3y1 - y2 + y3 >=4 -------(ii)

y1>=0 , y2>=0 , y3>=0

**Step-2: Checking if P(0,1,6) is a feasible set(i.e. it should satisfy every constraint):**

Cons#1: -2y1 + 5y2 + y3 >= 3 =>-2(0)+5(1)+6 >=6 => 11>=6

//cons#1 satisfied

Cons#2: 3y1 - y2 + y3 >=4 => 3(0)-1+6 >=4 => 5>=4

//cons#2 satisfied

Since **P(0,1,6)** is satisfying all the constraints of DUAL LP hence it’s a **Feasible set.**

**Step-3: To find optimal value on this set of values(i.e. P(0,1,6)):**

* In order to find optimal value put P(0,1,6) in DUAL Objective Function.

MIN Z = 6y1 + 40y2 + 7y3

Z(dual)= 6(0)+40(1) +7(6)

Z(dual)= 40+42

Z(dual)= 82

**Step-4: To compare Z(primal) with Z(dual):**

Z(primal)=17

Z(dual) =82

Hence **Z(dual) > Z(primal).**

**Task#9: Solve the assignment problem using Hungarian-method. The matrix entries are processing time of man in hours.**

**Men**

**1 2 3 4 5**

**I** 20 15 18 20 25

**II** 18 20 12 14 15

**(JOBS) III** 21 23 25 27 25

**IV** 17 18 21 23 20

**V** 18 18 16 19 20

1. **Step-1: As matrix is a square matrix so no need to add imaginary row/column.**
2. **Step-2: Row Reduction:**

1st row: smallest element=15

20-15= 5

15-15= 0

18-15= 3

20-15= 5

25-15= 10

2nd row: smallest element=12

18-12= 6

20-12= 8

12-12= 0

14-12= 2

15-12= 3

3rd row: smallest element=21

21-21= 0

23-21= 2

25-21= 4

27-21= 6

25-21= 4

4th row: smallest element=17

17-17= 0

18-17= 1

21-17= 4

23-17= 6

20-17= 3

5th row: smallest element=16

18-16= 2

18-16= 2

16-16= 0

19-16= 3

20-16= 4

**New Cost matrix:-**

5 0 3 5 10

6 8 0 2 3

0 2 4 6 4

1. 1 4 6 3

2 2 0 3 4

**3. Step-3: Column Reduction:**

1st column:- smallest element=0

=>As It is zero so no need to subtract.

5

6

0

0

2

2nd column:- smallest element=0

=>As It is zero so no need to subtract.

0

8

2

1

2

3rd column:- smallest element=0

=>As It is zero so no need to subtract.

3

0

4

4

0

4th column:- smallest element=2

5-2= 3

2-2= 0

6-2= 4

6-2= 4

3-2= 1

5th column:- smallest element=3

10-3= 7

3-3= 0

4-3= 1

3-3= 0

4-3= 1

**New Cost matrix:-**

5 0 3 3 7

6 8 0 0 0

0 2 4 4 1

0 1 4 4 0

2 2 0 1 1

**4. Step-4: Drawing lines to cover zeros:-**

5 0 3 3 7

6 8 0 0 0

0 2 4 4 1

0 1 4 4 0

2 2 0 1 1

**5. Step-5: Checking if matrix is reduced:-**

As we see that

No of lines=order of matrix => i.e. 5

Hence matrix is reduced so we will now directly move to that **Step-8**.

**6. Step-8: Select row that has single zero and assign that task to corresponding operator:-**

**1 2 3 4 5**

**I** 5 0 3 3 7

**II** 6 8 0 0 0

**III** 0 2 4 4 1

**IV** 0 1 4 4 0

**V** 2 2 0 1 1

Task -> Possible operator(s)

I -> 2

II -> 3,4,5

III -> 1

IV -> 1,5

V -> 3

=>Assign Task-I to operator-2.

=>Assign Task-III to operator-1.

=>Assign Task-V to operator-3.

=>Assign Task-IV to operator-5.

=>Assign Task-II to operator-4.

**Now from Original Cost Matrix:**

**1 2 3 4 5**

**I** 20 15 18 20 25

**II** 18 20 12 14 15

**(JOBS) III** 21 23 25 27 25

**IV** 17 18 21 23 20

**V** 18 18 16 19 20

**Total Cost is:-**

Assigned Task-Operator Cost

I-2 15

III-1 21

V-3 16

IV-5 20

II-4 14

+

Total Cost = Rs. $86.00