Conditional Probability

Lecture

Dated:

Conditional Probability

The Probability of event B given that A has already occurred is given by

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad where\ P(A) \neq 0$$

The Probability of event A given that B has already occurred is given by

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad where\ P(B) \neq 0$$

EXAMPLE 4–33   Parking Tickets

The probability that Sam parks in a no-parking zone *and* gets a parking ticket is 0.06, and the probability that Sam cannot find a legal parking space and has to park in the no-parking zone is 0.20. On Tuesday, Sam arrives at school and has to park in a no-parking zone. Find the probability that he will get a parking ticket.

SOLUTION

Let

$$N = \text{parking in a no-parking zone} \qquad T = \text{getting a ticket}$$

Then

$$P(T|N) = \frac{P(N \text{ and } T)}{P(N)} = \frac{0.06}{0.20} = 0.30$$

Hence, Sam has a 0.30 probability or 30% chance of getting a parking ticket, given that he parked in a no-parking zone.

EXAMPLE 4–34   Survey on Women in the Military

A recent survey asked 100 people if they thought women in the armed forces should be permitted to participate in combat. The results of the survey are shown.

Gender	Yes	No	Total
Male	32	18	50
Female	8	42	50
Total	40	60	100

Find these probabilities.

  a. The respondent answered yes, given that the respondent was a female.
  b. The respondent was a male, given that the respondent answered no.

Let

M = respondent was a male Y = respondent answered yes

F = respondent was a female N = respondent answered no

a. The problem is to find $P(Y|F)$. The rule states

$$P(Y|F) = \frac{P(F \text{ and } Y)}{P(F)}$$

The probability $P(F \text{ and } Y)$ is the number of females who responded yes, divided by the total number of respondents:

$$P(F \text{ and } Y) = \frac{8}{100}$$

The probability $P(F)$ is the probability of selecting a female:

$$P(F) = \frac{50}{100}$$

Then

$$P(Y|F) = \frac{P(F \text{ and } Y)}{P(F)} = \frac{8/100}{50/100}$$

$$= \frac{8}{100} \div \frac{50}{100} = \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{100}}} \cdot \frac{\overset{1}{\cancel{100}}}{\underset{25}{\cancel{50}}} = \frac{4}{25} = 0.16$$

b. The problem is to find $P(M|N)$.

$$P(M|N) = \frac{P(N \text{ and } M)}{P(N)} = \frac{18/100}{60/100}$$

$$= \frac{18}{100} \div \frac{60}{100} = \frac{\overset{3}{\cancel{18}}}{\underset{1}{\cancel{100}}} \cdot \frac{\overset{1}{\cancel{100}}}{\underset{10}{\cancel{60}}} = \frac{3}{10} = 0.3$$

Doctor Specialties Below are listed the numbers of doctors in various specialties by gender.

	Pathology	Pediatrics	Psychiatry
Male	12,575	33,020	27,803
Female	5,604	33,351	12,292

Choose one doctor at random.

a. Find P (male|pediatrician).

b. Find P (pathologist|female).

c. Are the characteristics "female" and "pathologist" independent? Explain.

Source: *World Almanac*.

	Pathology	Padiatrics	Psychiatry	
Male	12575	33020	27803	73398
Female	5604	33351	12292	51247
	18179	66371	40095	124645