Multiplication Rules of Probability

Lecture 11

Lecture 12

Rule 1: For Independent Events

When two events are independent, the probability of both occurring is

$$P(A \text{ and } B) = P(A) \, . \, P(B)$$

$$P(A \cap B) = P(A) . P(B)$$

Independent Events

Two events A and B are independent events if the fact that A occurs does not affect the probability of B occurring.

Tossing a coin and drawing a card from a deck

Having a large shoe size and having a high IQ

Parking in a no-parking zone and getting a parking ticket

Dependent Events

When the occurrence of the first event affects the occurrence of the second event in such a way that the probability is changed, the events are said to be dependent events.

- Having high grades and getting a scholarship
- Getting a raise in salary and purchasing a new car
- Driving on ice and having an accident
- A father being left-handed and a daughter being left-handed

Rule 2: For dependent Events

When two events are dependent, the probability of both occurring is

$$P(A\ and\ B) = P(A)\,.\,P(B|A)$$

$$P(A \cap B) = P(A).P(B|A)$$

A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

Solution: The Sample Space of coin is tossed $\{H, T\}$

The Sample Space of die is rolled $\{1, 2, 3, 4, 5, 6\}$

$$P(Head\ and\ 4) = \text{P(Head).P(4)}$$

$$P(A \cap B) = P(A).P(B)$$

$$P(A \cap B) = \frac{1}{2}.\frac{1}{6}$$

$$P(A \cap B) = 0.083$$

An urn contains 3 red balls, 2 blue balls, and 5 white balls. A ball is selected and its color noted. Then it is replaced. A second ball is selected and its color noted. Find the probability of each of these.

a. Selecting 2 blue balls

b. Selecting 1 blue ball and then 1 white ball

c. Selecting 1 red ball and then 1 blue ball

Solution: a

$$P(Blue\ and\ Blue) = P(Blue).P(Blue)$$

$$P(B \cap B) = P(B).P(B)$$

$$P(B \cap B) = \frac{2}{10}.\frac{2}{10}$$

$$P(B \cap B) = 0.04$$

b. Selecting 1 blue ball and then 1 white ball

$$P(Blue\ and\ white) = P(Blue).P(White)$$

$$P(B \cap W) = P(B).P(W)$$

$$P(B \cap W) = \frac{2}{10}.\frac{5}{10}$$

$$P(B \cap W) = 0.1$$

Solution: b

c. Selecting 1 red ball and then 1 blue ball

$$P(Red\ and\ blue) = P(Red).P(blue)$$

$$P(R \cap B) = P(R).P(B)$$

$$P(B \cap W) = \frac{3}{10}.\frac{2}{10}$$

$$P(R \cap B) = 0.06$$

EXAMPLE 4–26 Survey on Stress

A Harris poll found that 46% of Americans say they suffer great stress at least once a week. If three people are selected at random, find the probability that all three will say that they suffer great stress at least once a week.

Source: *100% American.*

SOLUTION

Let S denote stress. Then

$$\begin{aligned} P(S \text{ and } S \text{ and } S) &= P(S) \cdot P(S) \cdot P(S) \\ &= (0.46)(0.46)(0.46) \approx 0.097 \end{aligned}$$

There is a 9.7% chance that all three people will say they suffer great stress at least once a week.

EXAMPLE 4–27 Male Color Blindness

Approximately 9% of men have a type of color blindness that prevents them from distinguishing between red and green. If 3 men are selected at random, find the probability that all of them will have this type of red-green color blindness.

SOLUTION

Let C denote red-green color blindness. Then

$$\begin{aligned} P(C \text{ and } C \text{ and } C) &= P(C) \cdot P(C) \cdot P(C) \\ &= (0.09)(0.09)(0.09) \\ &= 0.000729 \end{aligned}$$

Hence, the rounded probability is 0.0007.

There is a 0.07% chance that all three men selected will have this type of red-green color blindness.

EXAMPLE 4–28 Overqualified Workers

In a recent survey, 33% of the respondents said that they feel that they are overqualified (O) for their present job. Of these, 24% said that they were looking for a new job (J). If a person is selected at random, find the probability that the person feels that he or she is overqualified and is also looking for a new job.

SOLUTION

$$P(\text{O and J}) = P(\text{O}) \cdot P(\text{J}|\text{O}) = (0.33)(0.24) \approx 0.079$$

There is about a 7.9% probability that the person feels that he or she is overqualified and is also looking for a new job.

EXAMPLE 4–29 Homeowner's and Automobile Insurance

World Wide Insurance Company found that 53% of the residents of a city had homeowner's insurance (H) with the company. Of these clients, 27% also had automobile insurance (A) with the company. If a resident is selected at random, find the probability that the resident has both homeowner's and automobile insurance with World Wide Insurance Company.

SOLUTION

$$P(\text{H and A}) = P(\text{H}) \cdot P(\text{A}|\text{H}) = (0.53)(0.27) = 0.1431 \approx 0.143$$

There is about a 14.3% probability that a resident has both homeowner's and automobile insurance with World Wide Insurance Company.

EXAMPLE 4–30 Drawing Cards

Three cards are drawn from an ordinary deck and not replaced. Find the probability of these events.

a. Getting 3 jacks

b. Getting an ace, a king, and a queen in order

c. Getting a club, a spade, and a heart in order

d. Getting 3 clubs

$$a.\ P(\text{3 jacks}) = \frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50} = \frac{24}{132{,}600} = \frac{1}{5525} \approx 0.0002$$

$$b.\ P(\text{ace and king and queen}) = \frac{4}{52}\cdot\frac{4}{51}\cdot\frac{4}{50} = \frac{64}{132{,}600} = \frac{8}{16{,}575} \approx 0.0005$$

$$c.\ P(\text{club and spade and heart}) = \frac{13}{52}\cdot\frac{13}{51}\cdot\frac{13}{50} = \frac{2197}{132.600} = \frac{169}{10.200} \approx 0.017$$

$$d.\ P(\text{3 clubs}) = \frac{13}{52}\cdot\frac{12}{51}\cdot\frac{11}{50} = \frac{1716}{132{,}600} = \frac{11}{850} \approx 0.013$$

Probabilities for "AtLeast"

EXAMPLE 4–35 Drawing Cards

A person selects 3 cards from an ordinary deck and replaces each card after it is drawn. Find the probability that the person will get at least one heart.

Let

E = at least 1 heart is drawn and $\bar{E}$ = no hearts are drawn

$$P(\bar{E}) = \frac{39}{52} \cdot \frac{39}{52} \cdot \frac{39}{52} = \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$$

$$P(E) = 1 - P(\bar{E})$$

$$= 1 - \frac{27}{64} = \frac{37}{64} \approx 0.578 = 57.8\%$$

Hence, a person will select at least one heart about 57.8% of the time.

EXAMPLE 4–36 Tossing Coins

A coin is tossed 5 times. Find the probability of getting at least 1 tail.

SOLUTION

It is easier to find the probability of the complement of the event, which is "all heads," and then subtract the probability from 1 to get the probability of at least 1 tail.

$$P(E) = 1 - P(\bar{E})$$

$$P(\text{at least 1 tail}) = 1 - P(\text{all heads})$$

$$P(\text{all heads}) = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$$

Hence,

$$P(\text{at least 1 tail}) = 1 - \frac{1}{32} = \frac{31}{32} \approx 0.969$$

There is a 96.9% chance of getting at least one tail.

35. Leisure Time Exercise Only 27% of U.S. adults get enough leisure time exercise to achieve cardiovascular fitness. Choose 3 adults at random. Find the probability that

a. All 3 get enough daily exercise

b. At least 1 of the 3 gets enough exercise

Source: www.infoplease.com

Solution: E for adults take exercise daily

a. All 3 get enough daily exercise

$$P(E \cap E \cap E) = P(E).P(E).P(E)$$

$$P(E \cap E \cap E) = (0.27)\ (0.27)\ (0.27)$$

$$P(E \cap E \cap E) = 0.0196$$

b. At least 1 of 3 gets enough exercise

Using At least rule , we have

$$P(at\ least\ 1\ of\ 3\ ) = 1 - P(\bar{E}).P(\bar{E}).P(\bar{E})$$

$$P(at\ least\ 1\ of\ 3) = 1 - (0.73)\ (0.73)\ (0.73)$$

$$P(at\ least\ 1\ of\ 3) = 0.610$$

42. Doctoral Assistantships Of Ph.D. students, 60% have paid assistantships. If 3 students are selected at random, find the probabilities that

a. All have assistantships

b. None has an assistantship

c. At least 1 has an assistantship

Source: U.S. Department of Education, *Chronicle of Higher Education*.

Conditional

Doctor Specialties Below are listed the numbers of doctors in various specialties by gender.

	Pathology	Pediatrics	Psychiatry
Male	12,575	33,020	27,803
Female	5,604	33,351	12,292

Choose one doctor at random.

a. Find *P* (male|pediatrician).

b. Find *P* (pathologist|female).

c. Are the characteristics "female" and "pathologist" independent? Explain.

Source: *World Almanac.*

	Pathology	Padiatrics	Psychiatry	
Male	12575	33020	27803	73398
Female	5604	33351	12292	51247
	18179	66371	40095	124645