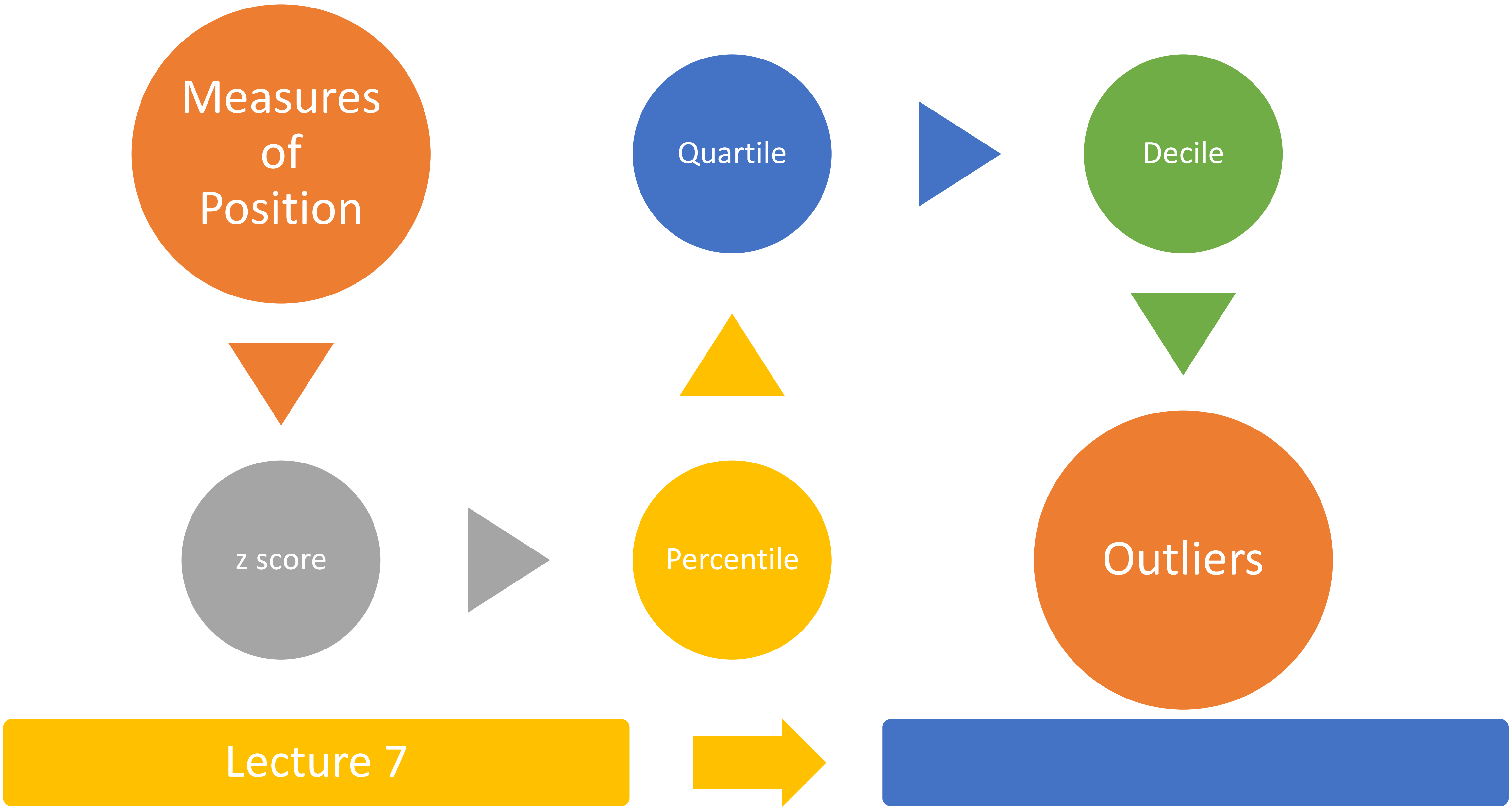


Measures of Position
Quartile
Decile
z score
Percentile
Outliers
Lecture 7

Standard Score Or Z Score

$$z = \frac{value - mean}{Standard\ Deviation}$$

For Sample $Z = \frac{x - \bar{x}}{s}$

For Population $Z = \frac{x - \mu}{\sigma}$

The z score represents the number of standard deviations that a data value falls above or below the mean.

Saying, “You can’t compare apples and oranges.”

A comparison of a relative standard similar to both can be made using Z Score with help of mean and Standard deviation

A z score, then, is actually the number of standard deviations each value is from the mean for a specific distribution.

EXAMPLE 3–27 Test Scores

A student scored 65 on a calculus test that had a mean of 50 and a standard deviation of 10; she scored 30 on a history test with a mean of 25 and a standard deviation of 5. Compare her relative positions on the two tests.

SOLUTION

First, find the z scores. For calculus the z score is

$$z = \frac{X - \overline{X}}{s} = \frac{65 - 50}{10} = 1.5$$

For history the z score is

$$z = \frac{30 - 25}{5} = 1.0$$

Since the z score for calculus is larger, her relative position in the calculus class is higher than her relative position in the history class.

EXAMPLE 3–28 Test Scores

Find the z score for each test, and state which is higher.

Test A	$X = 38$	$\bar{X} = 40$	$s = 5$
Test B	$X = 94$	$\bar{X} = 100$	$s = 10$

SOLUTION

For test A,

$$z = \frac{X - \bar{X}}{s} = \frac{38 - 40}{5} = -0.4$$

For test B,

$$z = \frac{94 - 100}{10} = -0.6$$

The score for test A is relatively higher than the score for test B.

Percentiles

Percentiles divide the data set into 100 equal groups.

Percentiles are symbolized by

$$P_1, P_2, P_3, \ldots, P_{99}$$

and divide the distribution into 100 groups.

Percentile Formula

The percentile corresponding to a given value X is computed by using the following formula:

$$\text{Percentile} = \frac{(\text{number of values below } X) + 0.5}{\text{total number of values}} \cdot 100$$

EXAMPLE 3–30 Test Scores

A teacher gives a 20-point test to 10 students. The scores are shown here. Find the percentile rank of a score of 12.

18, 15, 12, 6, 8, 2, 3, 5, 20, 10

SOLUTION

Arrange the data in order from lowest to highest.

2, 3, 5, 6, 8, 10, 12, 15, 18, 20

Then substitute into the formula.

$$\text{Percentile} = \frac{(\text{number of values below } X) + 0.5}{\text{total number of values}} \cdot 100$$

Since there are six values below a score of 12, the solution is

$$\text{Percentile} = \frac{6 + 0.5}{10} \cdot 100 = 65\text{th percentile}$$

Thus, a student whose score was 12 did better than 65% of the class.

(ii) find the percentile rank for a score of 6 .

Procedure Table

Finding a Data Value Corresponding to a Given Percentile

Step 1 Arrange the data in order from lowest to highest.

Step 2 Substitute into the formula

$$c = \frac{n \cdot p}{100}$$

where n = total number of values
p = percentile

Step 3A If c is not a whole number, round up to the next whole number. Starting at the lowest value, count over to the number that corresponds to the rounded-up value.

Step 3B If c is a whole number, use the value halfway between the cth and $(c + 1)$st values when counting up from the lowest value.

Example: Find the value corresponding to the 25th percentile

of a score of 18, 15, 12, 6, 8, 2, 3, 5, 20, 10

SOLUTION

Step 1 Arrange the data in order from lowest to highest.

$$2, 3, 5, 6, 8, 10, 12, 15, 18, 20$$

Step 2 Compute

$$c = \frac{n \cdot p}{100}$$

where n = total number of values
p = percentile

Thus,

$$c = \frac{10 \cdot 25}{100} = 2.5$$

Step 3 Since c is not a whole number, round it up to the next whole number; in this case, $c = 3$. Start at the lowest value and count over to the third value, which is 5. Hence, the value 5 corresponds to the 25th percentile.

Example: Find the value corresponding to the 60th percentile

of a score of 18, 15, 12, 6, 8, 2, 3, 5, 20, 10

SOLUTION

Step 1 Arrange the data in order from lowest to highest.

$$2, 3, 5, 6, 8, 10, 12, 15, 18, 20$$

Step 2 Substitute in the formula.

$$c = \frac{n \cdot p}{100} = \frac{10 \cdot 60}{100} = 6$$

Step 3 Since c is a whole number, use the value halfway between the c and $c + 1$ values when counting up from the lowest value—in this case, the 6th and 7th values.

$$2, 3, 5, 6, 8, \underset{\substack{\nearrow \\ \text{6th value}}}{10}, \underset{\substack{\nwarrow \\ \text{7th value}}}{12}, 15, 18, 20$$

The value halfway between 10 and 12 is 11. Find it by adding the two values and dividing by 2.

$$\frac{10 + 12}{2} = 11$$

Hence, 11 corresponds to the 60th percentile. Anyone scoring 11 would have done better than 60% of the class.

Percentile for Grouped Data

$$P_n = l + \frac{h}{f_n}\left(\frac{n.N}{100} - C\right)$$

Where n is nth percentile and N is total frequency of data , C is cummulative frequency before percentile class

The airborne speeds in miles per hour of 21 planes are shown.

Class	f
366-386	4
387-407	2
408-428	3
429-449	2
450-470	1
471-491	2
492-5012	3
513-533	4

Find the Percentiles (i) 20^{th} (ii) 45^{th} (iii) 60^{th} (iv) 75^{th} (v) 9^{th}

class	f	C.f
366-386	4	4
387-407	2	6
408-428	3	9
429-449	2	11
450-470	1	12
471-491	2	14
492-512	3	17
513-533	4	21

L	h	f	C
386.5	21	2	4

$$P_n = l + \frac{h}{f_n}\left(\frac{n.N}{100} - C\right)$$

Ans
388.6

$$\frac{n.N}{100}$$

For 20th Percentile

$$\frac{n.N}{100}$$

$$= \frac{20*21}{100} = 4.2$$

So 20th Percentile Class is 387-407 because CF>4.2

$$\frac{n.N}{100}$$

class	f	C.f
366-386	4	4
387-407	2	6
408-428	3	9
429-449	2	11
450-470	1	12
471-491	2	14
492-512	3	17
513-533	4	21

For 45th Percentile

$$\frac{n.N}{100}$$

$$=\frac{45*21}{100}=9.45$$

So 45th Percentile Class is 429-449 because CF>9.45

L	h	f	C
428.5	21	2	9

$$P_n = l + \frac{h}{f_n}\left(\frac{n.N}{100} - C\right)$$

Ans
433.225

class	f	C.f
366-386	4	4
387-407	2	6
408-428	3	9
429-449	2	11
450-470	1	12
471-491	2	14
492-512	3	17
513-533	4	21

L	h	f	C
491.5	21	3	14

$$P_n = l + \frac{h}{f_n}\left(\frac{n.N}{100} - C\right)$$

Ans
503.75

$$\frac{n.N}{100}$$

For 75th Percentile

$$\frac{n.N}{100}$$

$$= \frac{75*21}{100} = 15.75$$

So 75th Percentile Class is 492-512 because CF>15.75

$$\frac{n.N}{100}$$

class	f	C.f
366-386	4	4
387-407	2	6
408-428	3	9
429-449	2	11
450-470	1	12
471-491	2	14
492-512	3	17
513-533	4	21

L	h	f	C
366	21	4	0

$$P_n = l + \frac{h}{f_n}(\frac{n.N}{100} - C)$$

Ans
375.92

For 9th Percentile

$$\frac{n.N}{100}$$

$$= \frac{9*21}{100} = 1.89$$

So 75th Percentile Class is 366-386 because CF>1.89

class	f	C.f
366-386	4	4
387-407	2	6
408-428	3	9
429-449	2	11
450-470	1	12
471-491	2	14
492-512	3	17
513-533	4	21

L	h	f	C
470.5	21	2	12

$$P_n = l + \frac{h}{f_n}\left(\frac{n.N}{100} - C\right)$$

Ans
476.8

$$\frac{n.N}{100}$$

For 60th Percentile

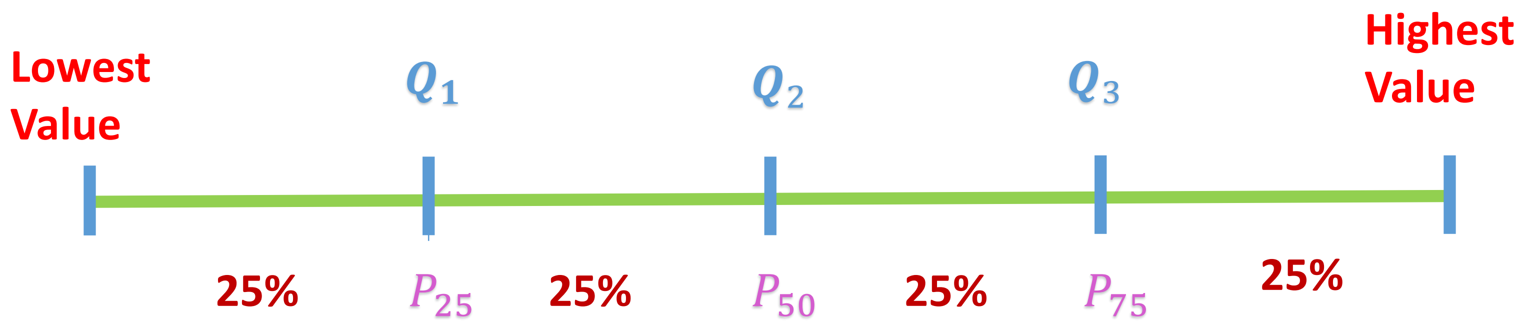
$$\frac{n.N}{100}$$

$$= \frac{60*21}{100} = 12.6$$

So 60th Percentile Class is 471-491 because CF>12.6

Quartiles

Quartile Divides data into four equal parts → Q_1 , Q_2 and Q_3



Find Q_1, Q_2, and Q_3 for the data set 15, 13, 6, 5, 12, 50, 22, 18.

SOLUTION

Step 1 Arrange the data in order from lowest to highest.

$$5, 6, 12, 13, 15, 18, 22, 50$$

Step 2 Find the median (Q_2).

$$5, 6, 12, 13, \underset{\substack{\uparrow \\ \text{MD}}}{\,} 15, 18, 22, 50$$

$$\text{MD} = \frac{13 + 15}{2} = 14$$

Step 3 Find the median of the data values less than 14.

$$5, 6, \underset{\substack{\uparrow \\ Q_1}}{\,} 12, 13$$

$$Q_1 = \frac{6 + 12}{2} = 9$$

So Q_1 is 9.

Step 4 Find the median of the data values greater than 14.

$$15, 18, \underset{\substack{\uparrow \\ Q_3}}{\,} 22, 50$$

$$Q_3 = \frac{18 + 22}{2} = 20$$

Here Q_3 is 20. Hence, $Q_1 = 9$, $Q_2 = 14$, and $Q_3 = 20$.

Quartile For Grouped Data

$$Q_n = l + \frac{h}{f_n}\left(\frac{n.N}{4} - C\right)$$

The airborne speeds in miles per hour of 21 planes are shown.

Class	f
366-386	4
387-407	2
408-428	3
429-449	2
450-470	1
471-491	2
492-5012	3
513-533	4

Find the Quartile (i) Q_1 (ii) Q_2(iii) Q_3

class	f	C.f
366-386	4	4
387-407	2	6
408-428	3	9
429-449	2	11
450-470	1	12
471-491	2	14
492-512	3	17
513-533	4	21

L	h	f	C
386.5	21	2	4

$$Q_n = l + \frac{h}{f_n}\left(\frac{n.N}{4} - C\right)$$

Ans
399.625

$$\frac{n.N}{4}$$

For Q_1

$$\frac{n.N}{4}$$

$$= \frac{1*21}{4} = 5.25$$

Quartile 1 Class is 387-407 because CF>5.25

$$\frac{n.N}{4}$$

class	f	C.f
366-386	4	4
387-407	2	6
408-428	3	9
429-449	2	11
450-470	1	12
471-491	2	14
492-512	3	17
513-533	4	21

L	h	f	C
428.5	21	2	9

$$Q_n = l + \frac{h}{f_n}\left(\frac{n.N}{4} - C\right)$$

Ans
444.25

For Q_2

$$\frac{n.N}{4}$$

$$= \frac{2*21}{4} = 10.5$$

Quartile 2 Class is 429-449 because CF>10.5

$$\frac{n.N}{4}$$

class	f	C.f
366-386	4	4
387-407	2	6
408-428	3	9
429-449	2	11
450-470	1	12
471-491	2	14
492-512	3	17
513-533	4	21

For Q_3

$$\frac{n.N}{4}$$

$$= \frac{3*21}{4} = 15.75$$

Quartile 3 Class is 492-512 because CF>15.75

L	h	f	C
491.5	21	3	14

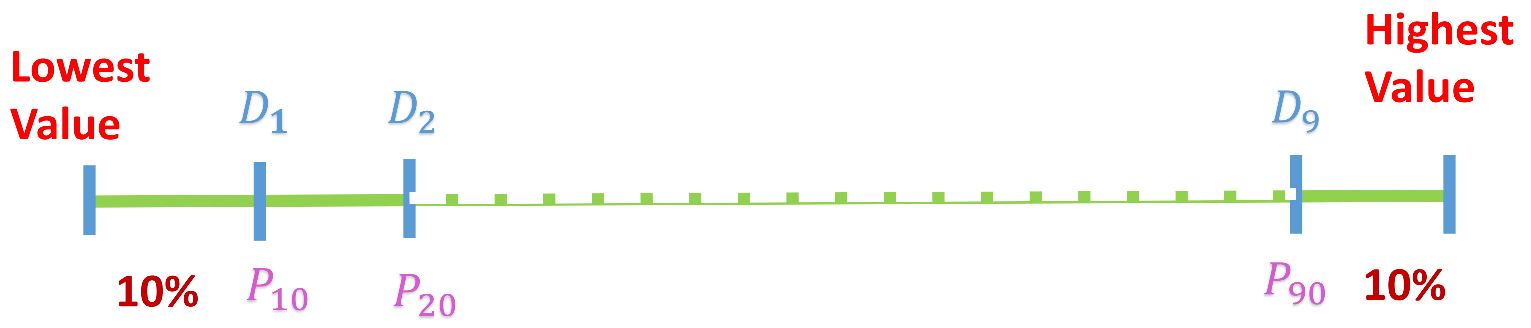
$$Q_n = l + \frac{h}{f_n}(\frac{n.N}{4} - C)$$

Ans
503.75

Deciles

Decile Divides data into ten equal parts

D_1 , D_2 … D_9



Decile For Grouped Data

$$D_n = l + \frac{h}{f_n}\left(\frac{n.N}{10} - C\right)$$

The airborne speeds in miles per hour of 21 planes are shown.

Class	f
366-386	4
387-407	2
408-428	3
429-449	2
450-470	1
471-491	2
492-5012	3
513-533	4

Find the Deciles (i) D_1 (ii) D_2(iii) D_3