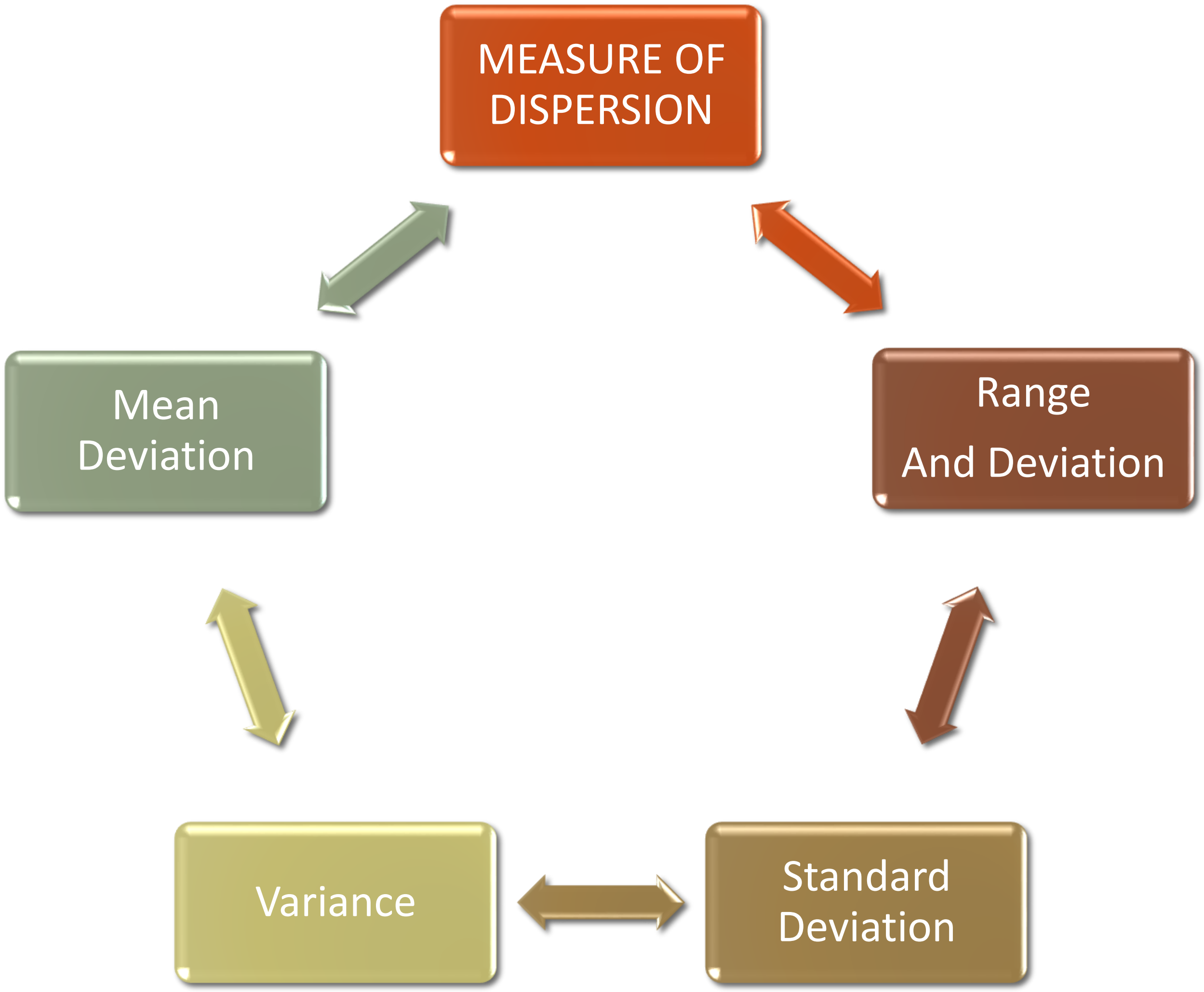



Lecture 5 + 6

Date 06-06-2020

## EXAMPLE 3–15 Comparison of Outdoor Paint

A testing lab wishes to test two experimental brands of outdoor paint to see how long each will last before fading. The testing lab makes 6 gallons of each paint to test. Since different chemical agents are added to each group and only six cans are involved, these two groups constitute two small populations. The results (in months) are shown. Find the mean of each group.

| Brand A | Brand B |
|---|---|
| 10 | 35 |
| 60 | 45 |
| 50 | 30 |
| 30 | 35 |
| 40 | 40 |
| 20 | 25 |

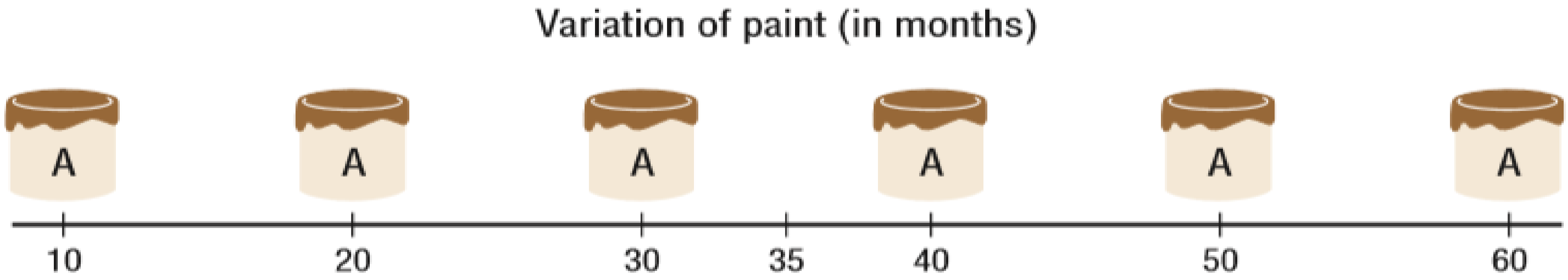
**SOLUTION**

The mean for brand A is

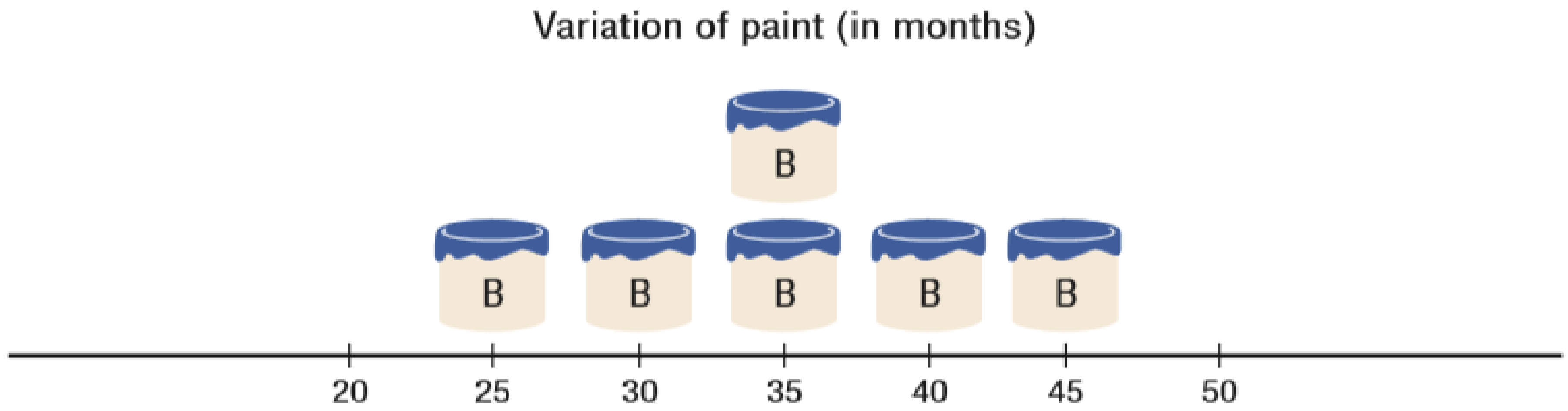
$$\mu = \frac{\Sigma X}{N} = \frac{210}{6} = 35 \text{ months}$$

The mean for brand B is

$$\mu = \frac{\Sigma X}{N} = \frac{210}{6} = 35 \text{ months}$$




(a) Brand A




(b) Brand B

# Range

The range is the highest value minus the lowest value. The symbol R is used for the range.

$$R = Largest\ value - Smallest\ Value$$

Example : Find Range in Out door paints given in previous example Solution

For brand A, the range is

$$R = 60 - 10 = 50 \text{ months}$$

For brand B, the range is

$$R = 45 - 25 = 20 \text{ months}$$

# Deviaton

Deviation is based on the difference or distance each data value is from the mean.

$$Deviation = Obsevarion - Mean$$

$$Deviation = x_i - \mu$$

Sum of Deviations is always zero i.e $\sum( x_i - \mu)$

Example : Find Deviations of 2 , 3 , 8 and 7

Solution:

| **Observations $x_i$** | **Deviation $x_i - \mu$** |
|---|---|
| 2 | 2-5=-3 |
| 3 | 3-5=-2 |
| 7 | 7-5=2 |
| 8 | 8-5=3 |

Example : Find Deviations of 4 , 5 , 5 and 6

Solution:

| **Observations $x_i$** | **Deviation $x_i - \mu$** |
|---|---|
| 2 | 2-5=-3 |
| 3 | 3-5=-2 |
| 7 | 7-5=2 |
| 8 | 8-5=3 |

Example : Find Deviations of 2 , 3 , 8 and 7

Solution:

| Observations $x_i$ | Deviation $x_i - \mu$ |
|---|---|
| 2 | 2-5=-3 |
| 3 | 3-5=-2 |
| 7 | 7-5=2 |
| 8 | 8-5=3 |

Example : Find Deviations of 4 , 5 , 5 and 6

Solution:

| Observations $x_i$ | Deviation $x_i - \mu$ |
|---|---|
| 4 | 4-5=-1 |
| 5 | 5-5=0 |
| 5 | 5-5=0 |
| 6 | 6-5= 1 |

Variance for Population

$$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N}$$

Standard Deviation for Population

$$\sigma = \sqrt{\frac{\Sigma(x_i - \mu)^2}{N}}$$

Example : Find Variance and SD of 2 , 3 , 8 and 7

Solution:

| Observations $x_i$ | Deviation | $(x_i - \mu)^2$ |
|---|---|---|
| 2 | 2-5=-3 | 9 |
| 3 | 3-5=-2 | 4 |
| 7 | 7-5=2 | 4 |
| 8 | 8-5=3 | 9 |
| | $\Sigma(x_i - \mu)^2 = 26$ | |

Example : Find Variance and SD of 4 , 5 , 5 and 6

Solution:

| Observations $x_i$ | Deviation | $(x_i - \mu)^2$ |
|---|---|---|
| 4 | 4-5=-1 | 1 |
| 5 | 5-5=0 | 0 |
| 5 | 5-5=0 | 0 |
| 6 | 6-5= 1 | 1 |
| | | $\Sigma(x_i - \mu)^2 = 2$ |

As Variance

$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N}$

So $\sigma^2 = \frac{26}{4} = 6.5$

S.D=$\sigma$=2.55

As Variance

$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N}$

So $\sigma^2 = \frac{2}{4} = 0.5$

S.D=$\sigma$=0.707

**Note : For Calculating Variance and SD of Sample replace $N$ by $n-1$ and $\mu$ by $\bar{x}$**

Mean → For Sample $\bar{x}$ → For Population $\mu$

Variance → For Sample $s^2$ → For Population $\sigma^2$

**Variance of Grouped Data**

For Population

$$\sigma^2 = \frac{\Sigma f_i (\boldsymbol{x}_m - \mu)^2}{N}$$

For Sample

$$s^2 = \frac{\Sigma f_i (\boldsymbol{x}_m - \bar{x})^2}{n-1}$$

| Class | Frequency $f_i$ | Midpoint $x_m$ | $f_i x_m$ | $(x_m - \mu)^2$ | $f_i\ (x_m - \mu)^2$ |
|---|---|---|---|---|---|
| **100-104** | 2 | 102 | 204 | 148.84 | **297.68** |
| **105-109** | 8 | 107 | 856 | 51.84 | **414.72** |
| **110-114** | 18 | 112 | 2016 | 4.84 | **87.12** |
| **115-119** | 13 | 117 | 1521 | 7.84 | **101.92** |
| **120-124** | 7 | 122 | 854 | 60.84 | **425.88** |
| **125-129** | 1 | 127 | 127 | 163.84 | **163.84** |
| **130-134** | 1 | 132 | 132 | 316.84 | **316.84** |
| Sum $\Sigma$ | =50 | | =5710 | | **=1808** |

$$\mu = \frac{\Sigma f_i x_m}{N} = \frac{5710}{50} = 114.2$$

*As Variance is*

$$\sigma^2 = \frac{\boldsymbol{\Sigma} f_i (\boldsymbol{x}_m - \mu)^{\boldsymbol{2}}}{N}$$

$$\sigma^2 = \frac{1808}{50}$$

$$\sigma^2 = 36.16$$

$$S.D = \sigma = 6.01$$

Coefficient of Variation

$$CVar = \frac{\sigma}{\mu} . 100$$

$$CVar = \frac{6.01}{114.2} . 100$$

CVar=5.26%

The **coefficient of variation,** denoted by CVar, is the standard deviation divided by the mean. The result is expressed as a percentage.

**For samples,**

$$\text{CVar} = \frac{s}{\overline{X}} \cdot 100$$

**For populations,**

$$\text{CVar} = \frac{\sigma}{\mu} \cdot 100$$

## Historical Note

Karl Pearson devised the coefficient of variation to compare the deviations of two different groups such as the heights of men and women.

### EXAMPLE 3–23 Sales of Automobiles

The mean of the number of sales of cars over a 3-month period is 87, and the standard deviation is 5. The mean of the commissions is $5225, and the standard deviation is $773. Compare the variations of the two.

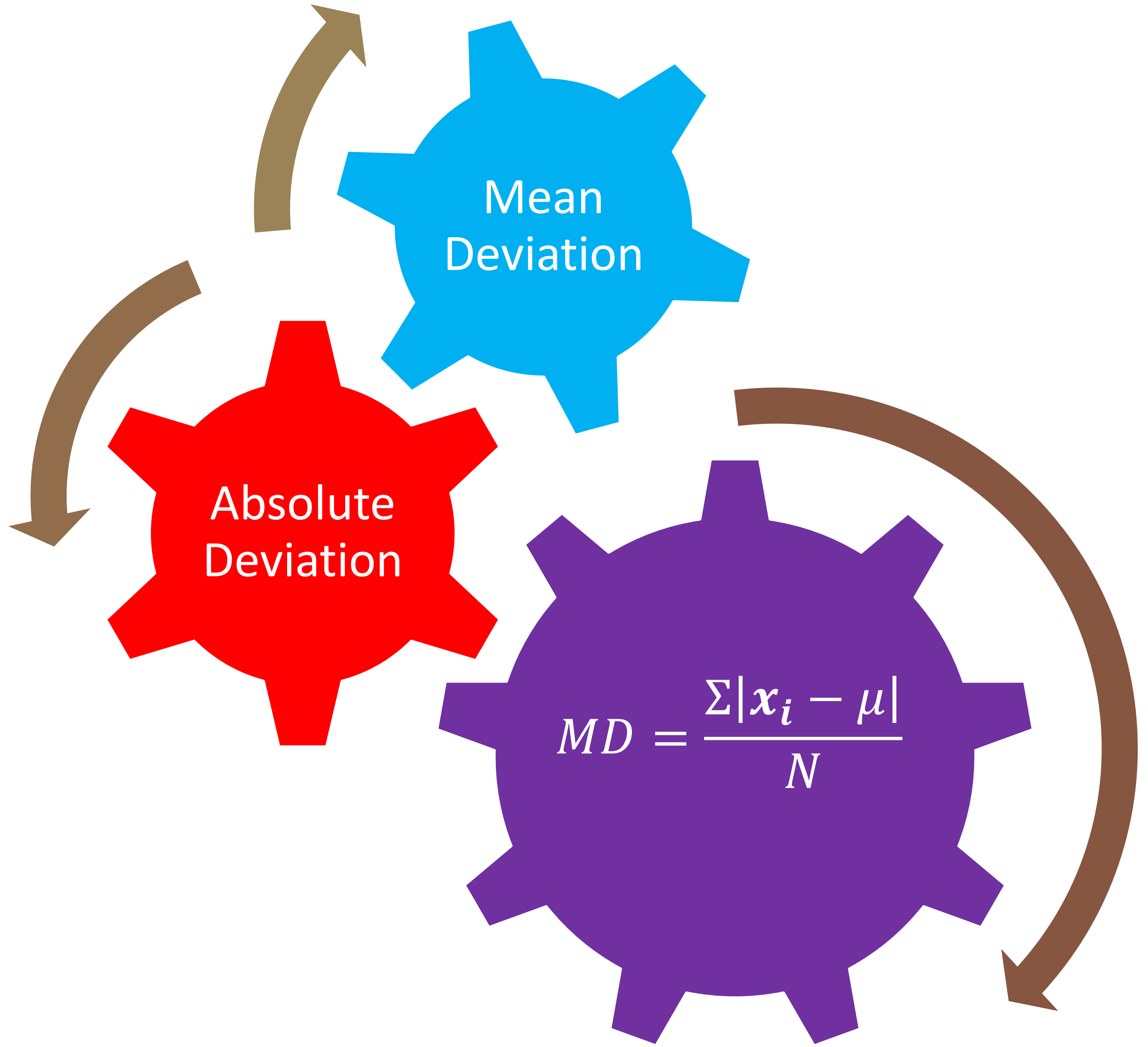
**SOLUTION**

The coefficients of variation are

$$\text{CVar} = \frac{s}{\overline{X}} = \frac{5}{87} \cdot 100 = 5.7\% \quad \text{sales}$$

$$\text{CVar} = \frac{773}{5225} \cdot 100 = 14.8\% \quad \text{commissions}$$

Since the coefficient of variation is larger for commissions, the commissions are more variable than the sales.


Mean Deviation
Absolute Deviation
$MD = \frac{\Sigma|\boldsymbol{x_i} - \mu|}{N}$

Example : Find Mean Deviations of 2 , 3 , 4 and 7

Solution:

| Observations $x_i$ | Deviation $x_i - \mu$ | $\lvert x_i - \mu \rvert$ |
|---|---|---|
| 2 | 2-5=-3 | 3 |
| 3 | 3-5=-2 | 2 |
| 7 | 7-5=2 | 2 |
| 8 | 8-5=3 | 3 |
| | | $\boldsymbol{\Sigma \lvert x_i - \mu \rvert = 10}$ |

Example : Find Mean Deviations of 4 , 5 , 5 and 6

Solution:

| Observations $x_i$ | Deviation $x_i - \mu$ | $\lvert x_i - \mu \rvert$ |
|---|---|---|
| 4 | 4-5=-1 | 1 |
| 5 | 5-5=0 | 0 |
| 5 | 5-5=0 | 0 |
| 6 | 6-5= 1 | 1 |
| | | $\boldsymbol{\Sigma \lvert x_i - \mu \rvert = 2}$ |

As $MD = \frac{\Sigma|x_i - \mu|}{N}$

So $\mathrm{MD} = \frac{10}{4}$

$= 2.5$

Coefficient of $MD = \frac{MD}{\mu}$

$= \frac{2.5}{5} = 0.5$

As $MD = \frac{\Sigma|x_i - \mu|}{N}$

So $\mathrm{MD} = \frac{2}{4}$

$= 0.5$

Coefficient of $MD = \frac{MD}{\mu}$

$= \frac{0.5}{5} = 0.1$

| Class | Frequency $f_i$ | Midpoint $x_m$ | $f_i x_m$ | $\lvert x_m - \mu \rvert$ | $f_i \lvert x_m - \mu \rvert$ |
|---|---|---|---|---|---|
| 100-104 | 2 | 102 | 204 | 12.2 | 24.4 |
| 105-109 | 8 | 107 | 856 | 7.2 | 57.6 |
| 110-114 | 18 | 112 | 2016 | 2.2 | 39.6 |
| 115-119 | 13 | 117 | 1521 | 2.8 | 36.4 |
| 120-124 | 7 | 122 | 854 | 7.8 | 54.6 |
| 125-129 | 1 | 127 | 127 | 12.8 | 12.8 |
| 130-134 | 1 | 132 | 132 | 17.8 | 17.8 |
| Sum Σ | =50 | | =5710 | | =243.2 |
| | | $\mu = \frac{\Sigma f_i x_m}{n} = \frac{5710}{50} = 114.2$ | | | |

Mean Deviation is

$$MD = \frac{\Sigma f_i |x_m - \mu|}{N}$$

$$MD = \frac{243.2}{50}$$

$$= 4.86$$

Coefficient of MD $= \frac{MD}{\mu}$

$$= \frac{4.86}{114.2}$$

$$= 0.043$$