Problem solving with *Peg solitaire*

Jenny Forsythe explores ways in which the game, *Solitaire,* can be used to develop mathematical thinking.

In my primary school years, I received the game *Solitaire*, which is now commonly called *Peg solitaire*, following American usage. At first glance it appears a straight forward challenge (figure 1). The instructions provided, read:

> The object of the game is to remove all the pegs from the board except the last one which must finish in the centre hole. Commence the game by removing the centre peg. You then take pieces by hopping over them into an empty space, removing from play the piece you have hopped over. Each move must be made in a straight line either vertically or horizontally.

Sounds simple enough, but I spent many hours and days without success. If by chance I had success, I certainly did not remember how to repeat it.

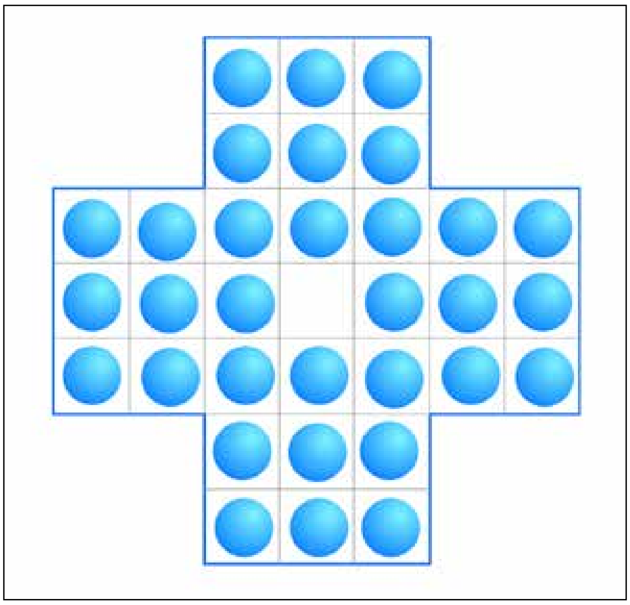


Figure 1: *Peg solitaire.*

It is encouraging to read three leading mathematicians, Berlekamp, Conway & Guy (2001, p. 805), writing "novices may come back in a week or so". A week or so to solve. Not the ideal turn around for a successful game. Elsewhere, Bergholt (1921, p. 5) commented "that the game has been allowed to fall into undeserved neglect is, I conceive, wholly due to the fact that no one up to the present, has ever provided the person of average intelligence with any adequate guide." Indeed, how long do you let a student work on a problem before stepping in? How much prior information should you provide? Over the years, I have come to believe that a key to unlock this problem would have been useful from the start; whether in the form of smaller challenges or the one I found whilst writing my undergraduate thesis on the analysis of solitaire, in the form of block removals expounded by Conway in the 1960's (Beasley 2019). Did I really need to wait until I was studying a mathematics degree to find this relatively straight forward key?

Incidentally the classic challenge of starting and finishing at the centre is just one of a number of challenges but no others were posed. Even in the court of Louis XIV, a monthly gazette, *Mercure Galant* (August & September 1697) provided sixteen challenges with solutions. No mention of the centre challenge here, it is not possible on the French board (thirty seven holes compared to the English board of 33 holes).

Block removals

The first published solution to the classic *Peg solitaire* problem that I found was in an article by J D Beasley (1962). What a eureka moment. I had set out to do a thesis on a problem I was not able to solve and here was a solution I could understand. This particular solution requires a three removal; a six removal; an L removal and a final hop.

The three removal

The three removal removes three consecutive pegs in a row or column leaving everything else the same. All the removals given require a catalyst. For this removal the catalyst is a peg (marked C) and a space either side of peg one (see figure 2 overleaf) both remain the same at the end of the removal. The process then proceeds as shown by figures two to five. The number on the pegs indicates the order of removal not the order of hopping.

Similarly, the six and L removal removes pegs as shown in the diagrams whilst leaving everything else the same.

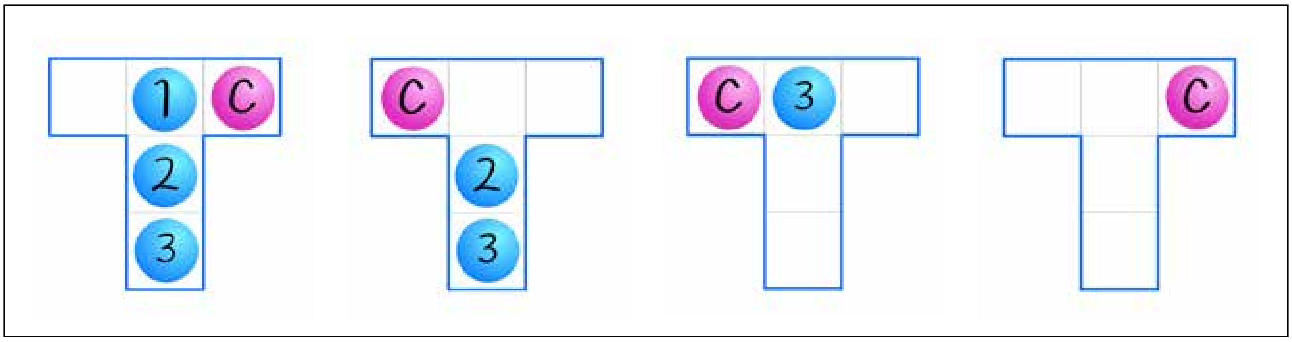


Figure 2: Remove peg one,by hopping the catalyst peg over it.

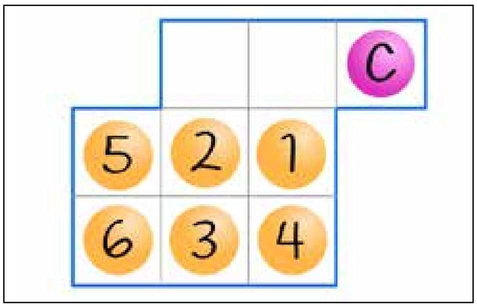
Figure 3: Remove peg two,by hopping three over it.

Figure 4 : Remove peg three, by hopping the catalyst peg over it.

Figure 5: RESULT Three pegs are removed. The catalyst peg ends up where it started.

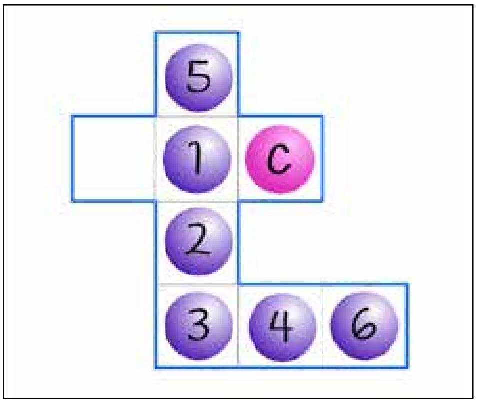
The QR code provides a video clip of the three removal.

The six removal



Steps to completion are left for the reader or see the QR code for a video clip.

The L removal



Steps to completion are left for the reader or see the QR code for a video clip.

Now the problem could be posed to split the board up using the block removals. This can be carried out as shown in figure 8. To complete the problem, remove the pegs in their blocks from one to seven.

Not only is this relatively straight forward to understand, it is easy to remember as a pattern and can be used to solve other *Solitaire* problems e.g., ending with a peg elsewhere; ending with the letter ‘H’ (Bergholt 1921) and many more.

Smaller steps

Starting with thirty-two pegs, the centre vacant and aiming to finish with just the centre filled is like starting with the fiendish *Sudoku.* Instead, why not start with just five pegs and finish with a peg in the centre as shown in figure 9.

In the classroom, I have used this example of problem solving to encourage my students to look for a key or to look for a simpler problem. Clearly, patterns are useful here but prior knowledge of block removals would be helpful. The other solutions that I know of are the popular *YouTube* solution ending with a sweep of six pegs which uses four L removals and the minimum solution which uses 18 seemingly random moves (A hop over more than one peg at a time being counted as one move). Given the number of solutions, it still surprises me how difficult they are to find by trial and improvement.

So, how long do I let a student struggle? As a mentor I want my student to be successful so, as a general rule, not long, usually just a few minutes. Then I will give them clues or a first step. Since reading Colin Foster’s article in MT265, I will endeavour to hold back a little longer and then try his suggested questioning. If I set problems outside lessons, they can message me through my website for clues. If appropriate, after two or three days, I provide model answers if they

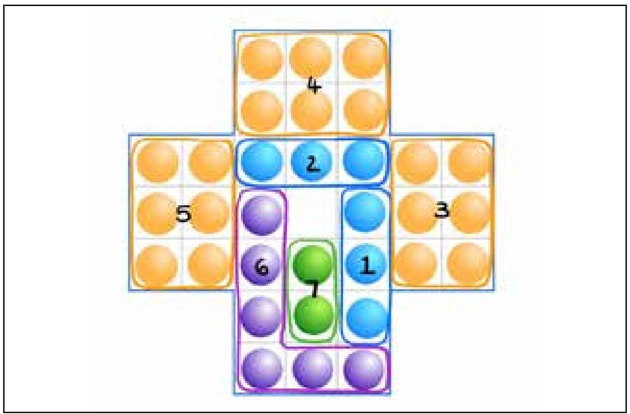


Figure 8

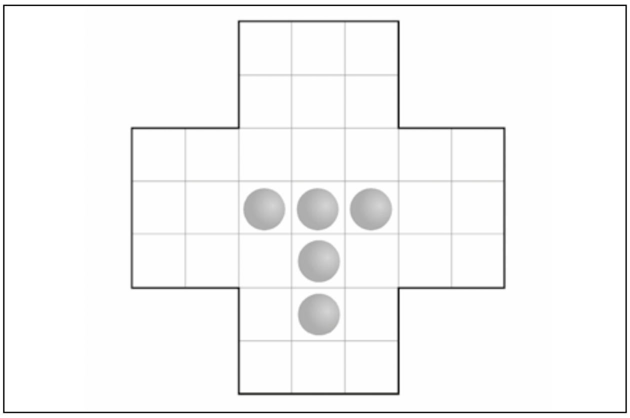


Figure 9

wish to use them. That said, it did me no harm to struggle for ten years before finding a clue.

Jenny Forsythe is a mathematics mentor and can be contacted at www.passion4maths.com.

References

Beasley, J. D. (2019). Personal conversation on Wednesday 13th February.

Beasley, J. D. (1962). Some notes on solitaire. *Eureka 25*. Cambridge: The Cambridge Archimedeans.

Bergholt, E. (1921). *Complete handbook to the game of solitaire*. London: Routledge.

Berlekamp, E. R. Conway, J. H. Guy, R. K. (2001). *Winnings ways for your mathematical plays* Volume 4. London: Routledge.

Acknowledgement

Whilst revisiting my references from my university thesis I found to my amazement that Beasley lives in the same town as myself. We have since met and I thank him for his encouragement and answering so many of my questions. Beasley proved the minimum number of moves to be 18 for the classic centre game on the 33-hole board on the 8th June 1964. I had the privilege to see his letter to Conway dated 11th June in which he provides the written proof. He is also the author of *The ins and outs of Peg solitaire* (1985/1992).