paper code

2025-03-24

1 Methods in the paper

The GCV estimate of λ is obtained by minimizing the following function:

$$V(\lambda) = \frac{1}{n} \frac{I - A(\lambda) y I^2)}{[\frac{1}{n} Trace I - A(\lambda)]^2}$$

where $A(\lambda) = X(X^T X + n\lambda I)^{-1} X^T$ is the hat matrix, X is the design matrix, y is the response vector, and n is the number of observations.

GCV is particulary useful because it does not require an estimate of the noise variance, making it applicable even in cases where n is small or when the number of predictors p is greater than n.

```
library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loaded glmnet 4.1-7
```

```
library(MASS)
library(pracma)
```

```
##
## Attaching package: 'pracma'
```

```
## The following objects are masked from 'package:Matrix':
##
##     expm, lu, tril, triu
```

```
library(knitr)
set.seed(123)

# PRESS function
PRESS <- function(lambda, X, y) {
  n <- nrow(X)
  A <- X %*% solve(t(X) %*% X + n * lambda * diag(p)) %*% t(X)
  B <- diag(1/(1-diag(A)))
  return(sum((B %*% (diag(n) - A) %*% y)^2))
}

# MLE function
MLE <- function(lambda, X, y){
  n <- nrow(X)
  A <- X %*% solve(t(X) %*% X + n * lambda * diag(p)) %*% t(X)
  numerator <- t(y) %*% (diag(n) - A) %*% y
  denominator <- det(diag(n) - A)^(1/n)
```

```r
  return(numerator / (n*denominator))
}

# I_D, I_R function
calculate_inefficiency <- function(lambda, X, y, beta){
  n <- nrow(X)
  beta_hat <- solve(t(X) %*% X + n * lambda * diag(p)) %*% t(X) %*% y
  I_D <- sum((beta - beta_hat)^2) / min(sapply(seq(-7,0,length.out = 100),
      function(lambda)
        sum((beta - solve(t(X) %*% X + n * lambda * diag(p)) %*% t(X) %*% y)^2)))
  I_R <- sum((X %*% beta - X %*% beta_hat)^2) / min(sapply(seq(-7,0,length.out = 100),
      function(lambda)
        sum((X %*% beta - X %*% solve( t(X) %*% X + n * lambda * diag(p)) %*% t(X) %*% y)^2)))
  return(c(I_D,I_R))
}

n <- 21
p <- 10
desired_condition_number <- 1.54e5
desired_Xbeta_norm <- 370.84
sigma2_values <- c(1e-8, 1e-6, 1e-4, 1e-2)
n_replications <- 4


gl <- gaussLaguerre(p)
t_k <- gl$x
w_k <- gl$w
s_i <- seq(0,2,length.out = n)

X <- outer(s_i,t_k, function(s,t) w_k = exp(-s * t))
svd_X <- svd(X)
U <- svd_X$u
D <- diag(svd_X$d)
V <- svd_X$v

original_condition_number <- max(svd_X$d) / min(svd_X$d)
scaling_factor <- desired_condition_number / original_condition_number
D_scaled <- D * scaling_factor

X <- U %*% D_scaled %*% t(V)

singular_values <- svd(X)$d
condition_number <- max(singular_values) / min(singular_values)
print(paste("Condition number of X =", condition_number))
```

```
## [1] "Condition number of X = 251218.922149347"
```

```r
beta <- rnorm(p)

Xbeta_norm <- sum((X %*% beta)^2)
scaling_factor <- sqrt(desired_Xbeta_norm / Xbeta_norm)
beta <- beta * scaling_factor

Xbeta_norm <- sum((X %*% beta)^2)
```

```r
print(paste("||X beta||^2 = ",Xbeta_norm))
```

```
## [1] "||X beta||^2 =  370.84"
```

```r
# process
results <- list()



for (sigma2 in sigma2_values) {
  for (replication in 1:n_replications){
    epsilon <- rnorm(n,mean = 0, sd = sqrt(sigma2))
    y <- X %*% beta + epsilon

    cv_fit <- cv.glmnet(X, y, alpha = 0)
    lambda_gcv <- cv_fit$lambda.min
    inefficiency_gcv <- calculate_inefficiency(lambda_gcv,X,y,beta)

    fit <- lm.ridge(y ~ X, lambda = seq(-10,2,length.out = 10000))
    lambda_rr <- fit$lambda[which.min(fit$GCV)]
    inefficiency_rr <- calculate_inefficiency(lambda_rr,X,y,beta)

    lambda_press <- optimize(PRESS, interval = c(0,1), X = X, y = y)$minimum
    inefficiency_press <- calculate_inefficiency(lambda_press,X,y,beta)

    lambda_mle <- optimize(MLE, interval = c(0,1), X = X, y = y)$minimum
    inefficiency_mle <- calculate_inefficiency(lambda_mle,X,y,beta)

    lambda_grid <- seq(0,1, length.out = 100)
    I_D_values <- sapply(lambda_grid,
      function(lambda)
        sum((beta - solve(t(X) %*% X + n * lambda * diag(p)) %*% t(X) %*% y)^2))
    I_R_values <- sapply(lambda_grid,
      function(lambda)
        sum((X %*% beta - X %*% solve( t(X) %*% X + n * lambda * diag(p)) %*% t(X) %*% y)^2))

    lambda_min_soln <- lambda_grid[which.min(I_D_values)]
    lambda_min_data <- lambda_grid[which.min(I_R_values)]
    inefficiency_min_soln <- calculate_inefficiency(lambda_min_soln, X, y, beta)
    inefficiency_min_data <- calculate_inefficiency(lambda_min_data, X, y, beta)

    results[[paste("sigma2",sigma2,"rep",replication)]] <- list(
      GCV = c(lambda_gcv,inefficiency_gcv),
      RR = c(lambda_rr,inefficiency_rr),
      PRESS = c(lambda_press,inefficiency_press),
      MLE = c(lambda_mle,inefficiency_mle),
      MINSOL = c(lambda_min_soln,inefficiency_min_soln[1],inefficiency_min_soln[2]),
      MINDATA = c(lambda_min_data,inefficiency_min_data[1],inefficiency_min_data[2])
  )
  }
}

for (sigma2 in sigma2_values) {
  for (replication in 1:n_replications) {
```

```
    key <- paste("sigma2",sigma2,"rep",replication)
    cat("Sigma^2 = ", sigma2, "Replication", replication, "\n")
    cat("GCV: Lambda = ", results[[key]]$GCV[1], "I_D ", results[[key]]$GCV[2],
        "I_R =", results[[key]]$GCV[3], "\n")
    cat("RR: Lambda = ", results[[key]]$RR[1], "I_D ", results[[key]]$RR[2],
        "I_R =", results[[key]]$RR[3], "\n")
    cat("PRESS: Lambda = ", results[[key]]$PRESS[1], "I_D ",
        results[[key]]$PRESS[2], "I_R =", results[[key]]$PRESS[3], "\n")
    cat("MLE: Lambda = ", results[[key]]$MLE[1], "I_D ",
        results[[key]]$MLE[2], "I_R =", results[[key]]$MLE[3], "\n")
    cat("Min Sol'n: Lambda = ", results[[key]]$MINSOL[1], "I_D ",
        results[[key]]$MINSOL[2], "I_R =", results[[key]]$MINSOL[3], "\n")
    cat("Min Dat: Lambda = ", results[[key]]$MINDATA[1], "I_D ",
        results[[key]]$MINDATA[2], "I_R =", results[[key]]$MINDATA[3], "\n")
    cat("\n")

  }
}
```

```
## Sigma^2 =  1e-08 Replication 1
## GCV: Lambda =  0.3935769 I_D  72.34822 I_R = 3825666101
## RR: Lambda =  0.00060006 I_D  44.62792 I_R = 8778809
## PRESS: Lambda =  0.01957272 I_D  56.11807 I_R = 780992222
## MLE: Lambda =  6.610696e-05 I_D  41.82741 I_R = 613035
## Min Sol'n: Lambda =  0 I_D  1 I_R = 1.006977
## Min Dat: Lambda =  0 I_D  1 I_R = 1.006977
##
## Sigma^2 =  1e-08 Replication 2
## GCV: Lambda =  0.3935749 I_D  43.03416 I_R = 6461514868
## RR: Lambda =  0.00060006 I_D  26.54555 I_R = 14829662
## PRESS: Lambda =  0.01957271 I_D  33.38018 I_R = 1319104091
## MLE: Lambda =  6.610696e-05 I_D  24.87977 I_R = 1035550
## Min Sol'n: Lambda =  0 I_D  1 I_R = 1.008125
## Min Dat: Lambda =  0 I_D  1 I_R = 1.008125
##
## Sigma^2 =  1e-08 Replication 3
## GCV: Lambda =  0.3935767 I_D  146.7983 I_R = 3374504301
## RR: Lambda =  0.00060006 I_D  90.55244 I_R = 7744784
## PRESS: Lambda =  0.01957258 I_D  113.8664 I_R = 688887372
## MLE: Lambda =  6.610696e-05 I_D  84.87286 I_R = 540715.9
## Min Sol'n: Lambda =  0 I_D  1 I_R = 1.005242
## Min Dat: Lambda =  0 I_D  1 I_R = 1.005242
##
## Sigma^2 =  1e-08 Replication 4
## GCV: Lambda =  0.3935769 I_D  48.67662 I_R = 2446785650
## RR: Lambda =  0.00060006 I_D  30.026 I_R = 5614247
## PRESS: Lambda =  0.01957298 I_D  37.75683 I_R = 499503594
## MLE: Lambda =  6.610696e-05 I_D  28.14207 I_R = 391871.5
## Min Sol'n: Lambda =  0 I_D  1 I_R = 1.00088
## Min Dat: Lambda =  0 I_D  1 I_R = 1.00088
##
## Sigma^2 =  1e-06 Replication 1
## GCV: Lambda =  0.3935735 I_D  0.9547898 I_R = 34054664
## RR: Lambda =  0.00060006 I_D  0.5889299 I_R = 78083.42
```

```
## PRESS: Lambda =  0.01957779 I_D  0.7406153 I_R = 6953766
## MLE: Lambda =  6.610696e-05 I_D  0.551953 I_R = 5427.67
## Min Sol'n: Lambda =  0.01010101 I_D  0.6898279 I_R = 3480245
## Min Dat: Lambda =  0 I_D  10.81852 I_R = 0.999685
##
## Sigma^2 =  1e-06 Replication 2
## GCV: Lambda =  0.3935587 I_D  0.9547901 I_R = 21868033
## RR: Lambda =  0.00060006 I_D  0.5889511 I_R = 50222.81
## PRESS: Lambda =  0.01957499 I_D  0.7406142 I_R = 4465180
## MLE: Lambda =  6.610696e-05 I_D  0.5520092 I_R = 3501.606
## Min Sol'n: Lambda =  0.01010101 I_D  0.6898403 I_R = 2235244
## Min Dat: Lambda =  0 I_D  17.16678 I_R = 0.9998862
##
## Sigma^2 =  1e-06 Replication 3
## GCV: Lambda =  0.5202726 I_D  9.555858 I_R = 33104057
## RR: Lambda =  0.00060006 I_D  5.840965 I_R = 71620.98
## PRESS: Lambda =  0.0195705 I_D  7.344576 I_R = 6362204
## MLE: Lambda =  6.610696e-05 I_D  5.475058 I_R = 5013.151
## Min Sol'n: Lambda =  0 I_D  1 I_R = 1.000243
## Min Dat: Lambda =  0 I_D  1 I_R = 1.000243
##
## Sigma^2 =  1e-06 Replication 4
## GCV: Lambda =  0.3935822 I_D  0.9547901 I_R = 21011572
## RR: Lambda =  0.00060006 I_D  0.5890268 I_R = 48249.26
## PRESS: Lambda =  0.01956847 I_D  0.7405779 I_R = 4288225
## MLE: Lambda =  6.610696e-05 I_D  0.5522863 I_R = 3387.977
## Min Sol'n: Lambda =  0.01010101 I_D  0.6898406 I_R = 2147028
## Min Dat: Lambda =  0 I_D  3.277538 I_R = 0.999853
##
## Sigma^2 =  1e-04 Replication 1
## GCV: Lambda =  0.3931731 I_D  0.9547854 I_R = 264798.6
## RR: Lambda =  0.00060006 I_D  0.5888931 I_R = 612.4889
## PRESS: Lambda =  0.01959566 I_D  0.7407488 I_R = 54197.49
## MLE: Lambda =  6.610696e-05 I_D  0.5512227 I_R = 43.57576
## Min Sol'n: Lambda =  0.01010101 I_D  0.6898679 I_R = 27119.17
## Min Dat: Lambda =  0 I_D  583.0014 I_R = 0.9999462
##
## Sigma^2 =  1e-04 Replication 2
## GCV: Lambda =  0.393677 I_D  0.9547907 I_R = 366589.3
## RR: Lambda =  0.00060006 I_D  0.5885675 I_R = 844.0366
## PRESS: Lambda =  0.01957665 I_D  0.7406523 I_R = 74847.8
## MLE: Lambda =  6.610696e-05 I_D  0.5516874 I_R = 56.02119
## Min Sol'n: Lambda =  0.01010101 I_D  0.6898599 I_R = 37479.51
## Min Dat: Lambda =  0 I_D  70.29078 I_R = 1.000044
##
## Sigma^2 =  1e-04 Replication 3
## GCV: Lambda =  0.3935192 I_D  0.9548187 I_R = 164720.6
## RR: Lambda =  0.00060006 I_D  0.5896734 I_R = 401.4372
## PRESS: Lambda =  0.01939339 I_D  0.7401895 I_R = 33416.98
## MLE: Lambda =  6.610696e-05 I_D  0.5528288 I_R = 32.05181
## Min Sol'n: Lambda =  0.01010101 I_D  0.6902943 I_R = 16917.69
## Min Dat: Lambda =  0 I_D  11.36375 I_R = 1.000075
##
## Sigma^2 =  1e-04 Replication 4
```

```
## GCV: Lambda =  0.3938058 I_D  0.954781 I_R = 239830.5
## RR: Lambda =  0.00060006 I_D  0.5890202 I_R = 541.7213
## PRESS: Lambda =  0.01957236 I_D  0.7404758 I_R = 48874.07
## MLE: Lambda =  6.610696e-05 I_D  0.5537542 I_R = 37.01068
## Min Sol'n: Lambda =  0.01010101 I_D  0.6897319 I_R = 24437.48
## Min Dat: Lambda =  0 I_D  113.3262 I_R = 0.9999942
##
## Sigma^2 =  0.01 Replication 1
## GCV: Lambda =  0.3949094 I_D  0.9548298 I_R = 4275.831
## RR: Lambda =  0.00660066 I_D  0.6641616 I_R = 258.5986
## PRESS: Lambda =  0.01957674 I_D  0.7402338 I_R = 869.2958
## MLE: Lambda =  6.610696e-05 I_D  0.5538217 I_R = 0.8819823
## Min Sol'n: Lambda =  0.01010101 I_D  0.6894553 I_R = 433.7175
## Min Dat: Lambda =  0 I_D  118736 I_R = 1.000002
##
## Sigma^2 =  0.01 Replication 2
## GCV: Lambda =  0.3921389 I_D  0.9546235 I_R = 2853.891
## RR: Lambda =  -0.3306331 I_D  1.091245 I_R = 6700.229
## PRESS: Lambda =  0.01978212 I_D  0.7411318 I_R = 587.9596
## MLE: Lambda =  6.610696e-05 I_D  0.5335591 I_R = 0.9471931
## Min Sol'n: Lambda =  0.01010101 I_D  0.6892589 I_R = 291.2158
## Min Dat: Lambda =  0 I_D  18623.31 I_R = 0.9999961
##
## Sigma^2 =  0.01 Replication 3
## GCV: Lambda =  0.3930244 I_D  0.9547814 I_R = 7377.942
## RR: Lambda =  -0.3306331 I_D  1.091292 I_R = 17453.99
## PRESS: Lambda =  0.01965167 I_D  0.7409092 I_R = 1515.742
## MLE: Lambda =  6.610696e-05 I_D  0.5668616 I_R = 1.656228
## Min Sol'n: Lambda =  0.01010101 I_D  0.6896825 I_R = 756.5637
## Min Dat: Lambda =  0 I_D  19676.27 I_R = 0.9999992
##
## Sigma^2 =  0.01 Replication 4
## GCV: Lambda =  0.3938885 I_D  0.9546922 I_R = 2392.126
## RR: Lambda =  0.00180018 I_D  0.6115793 I_R = 23.77545
## PRESS: Lambda =  0.02008224 I_D  0.7413635 I_R = 494.8808
## MLE: Lambda =  6.610696e-05 I_D  0.5337641 I_R = 0.4627268
## Min Sol'n: Lambda =  0.01010101 I_D  0.6883416 I_R = 238.8538
## Min Dat: Lambda =  0 I_D  2082.821 I_R = 0.999995
```

```
sigma2_values <- c(1e-8, 1e-6, 1e-4, 0.01)
n_replications <- 4

final_df <- data.frame()


for (sigma2 in sigma2_values) {
  for (replication in 1:n_replications) {
    key <- paste("sigma2", sigma2, "rep", replication)
    method_data <- data.frame(
      sigma2 = sigma2,
      replication = replication,
      method = c("GCV", "RR", "PRESS", "MLE", "MINSOL", "MINDATA"),
      lambda = c(
        results[[key]]$GCV[1],
```

```
        results[[key]]$RR[1],
        results[[key]]$PRESS[1],
        results[[key]]$MLE[1],
        results[[key]]$MINSOL[1],
        results[[key]]$MINDATA[1]
      ),
      ID = c(
        results[[key]]$GCV[2],
        results[[key]]$RR[2],
        results[[key]]$PRESS[2],
        results[[key]]$MLE[2],
        results[[key]]$MINSOL[2],
        results[[key]]$MINDATA[2]
      ),
      IR = c(
        results[[key]]$GCV[3],
        results[[key]]$RR[3],
        results[[key]]$PRESS[3],
        results[[key]]$MLE[3],
        results[[key]]$MINSOL[3],
        results[[key]]$MINDATA[3]
      )
    )

    final_df <- rbind(final_df, method_data)
  }
}


kable(final_df, col.names = c("Sigma^2", "Replication", "Method", "Lambda", "I_D", "I_R"))
```

Sigma^2	Replication	Method	Lambda	I_D	I_R
0e+00	1	GCV	0.3935769	7.234822e+01	3.825666e+09
0e+00	1	RR	0.0006001	4.462792e+01	8.778809e+06
0e+00	1	PRESS	0.0195727	5.611807e+01	7.809922e+08
0e+00	1	MLE	0.0000661	4.182741e+01	6.130350e+05
0e+00	1	MINSOL	0.0000000	1.000000e+00	1.006977e+00
0e+00	1	MINDATA	0.0000000	1.000000e+00	1.006977e+00
0e+00	2	GCV	0.3935749	4.303416e+01	6.461515e+09
0e+00	2	RR	0.0006001	2.654555e+01	1.482966e+07
0e+00	2	PRESS	0.0195727	3.338018e+01	1.319104e+09
0e+00	2	MLE	0.0000661	2.487977e+01	1.035550e+06
0e+00	2	MINSOL	0.0000000	1.000000e+00	1.008125e+00
0e+00	2	MINDATA	0.0000000	1.000000e+00	1.008125e+00
0e+00	3	GCV	0.3935767	1.467983e+02	3.374504e+09
0e+00	3	RR	0.0006001	9.055244e+01	7.744784e+06
0e+00	3	PRESS	0.0195726	1.138664e+02	6.888874e+08
0e+00	3	MLE	0.0000661	8.487286e+01	5.407159e+05
0e+00	3	MINSOL	0.0000000	1.000000e+00	1.005242e+00
0e+00	3	MINDATA	0.0000000	1.000000e+00	1.005242e+00
0e+00	4	GCV	0.3935769	4.867662e+01	2.446786e+09
0e+00	4	RR	0.0006001	3.002600e+01	5.614247e+06
0e+00	4	PRESS	0.0195730	3.775683e+01	4.995036e+08

Sigma^2	Replication	Method	Lambda	I_D	I_R
0e+00	4	MLE	0.0000661	2.814207e+01	3.918715e+05
0e+00	4	MINSOL	0.0000000	1.000000e+00	1.000880e+00
0e+00	4	MINDATA	0.0000000	1.000000e+00	1.000880e+00
1e-06	1	GCV	0.3935735	9.547898e-01	3.405466e+07
1e-06	1	RR	0.0006001	5.889299e-01	7.808342e+04
1e-06	1	PRESS	0.0195778	7.406153e-01	6.953766e+06
1e-06	1	MLE	0.0000661	5.519530e-01	5.427670e+03
1e-06	1	MINSOL	0.0101010	6.898279e-01	3.480245e+06
1e-06	1	MINDATA	0.0000000	1.081852e+01	9.996850e-01
1e-06	2	GCV	0.3935587	9.547901e-01	2.186803e+07
1e-06	2	RR	0.0006001	5.889511e-01	5.022281e+04
1e-06	2	PRESS	0.0195750	7.406142e-01	4.465180e+06
1e-06	2	MLE	0.0000661	5.520092e-01	3.501606e+03
1e-06	2	MINSOL	0.0101010	6.898403e-01	2.235244e+06
1e-06	2	MINDATA	0.0000000	1.716678e+01	9.998862e-01
1e-06	3	GCV	0.5202726	9.555858e+00	3.310406e+07
1e-06	3	RR	0.0006001	5.840965e+00	7.162098e+04
1e-06	3	PRESS	0.0195705	7.344576e+00	6.362204e+06
1e-06	3	MLE	0.0000661	5.475058e+00	5.013151e+03
1e-06	3	MINSOL	0.0000000	1.000000e+00	1.000243e+00
1e-06	3	MINDATA	0.0000000	1.000000e+00	1.000243e+00
1e-06	4	GCV	0.3935822	9.547901e-01	2.101157e+07
1e-06	4	RR	0.0006001	5.890268e-01	4.824926e+04
1e-06	4	PRESS	0.0195685	7.405779e-01	4.288225e+06
1e-06	4	MLE	0.0000661	5.522863e-01	3.387977e+03
1e-06	4	MINSOL	0.0101010	6.898406e-01	2.147028e+06
1e-06	4	MINDATA	0.0000000	3.277538e+00	9.998530e-01
1e-04	1	GCV	0.3931731	9.547854e-01	2.647986e+05
1e-04	1	RR	0.0006001	5.888931e-01	6.124889e+02
1e-04	1	PRESS	0.0195957	7.407488e-01	5.419749e+04
1e-04	1	MLE	0.0000661	5.512227e-01	4.357576e+01
1e-04	1	MINSOL	0.0101010	6.898679e-01	2.711917e+04
1e-04	1	MINDATA	0.0000000	5.830014e+02	9.999462e-01
1e-04	2	GCV	0.3936770	9.547907e-01	3.665893e+05
1e-04	2	RR	0.0006001	5.885675e-01	8.440366e+02
1e-04	2	PRESS	0.0195766	7.406523e-01	7.484780e+04
1e-04	2	MLE	0.0000661	5.516874e-01	5.602119e+01
1e-04	2	MINSOL	0.0101010	6.898599e-01	3.747951e+04
1e-04	2	MINDATA	0.0000000	7.029078e+01	1.000044e+00
1e-04	3	GCV	0.3935192	9.548187e-01	1.647206e+05
1e-04	3	RR	0.0006001	5.896734e-01	4.014372e+02
1e-04	3	PRESS	0.0193934	7.401895e-01	3.341698e+04
1e-04	3	MLE	0.0000661	5.528288e-01	3.205181e+01
1e-04	3	MINSOL	0.0101010	6.902943e-01	1.691769e+04
1e-04	3	MINDATA	0.0000000	1.136375e+01	1.000075e+00
1e-04	4	GCV	0.3938058	9.547810e-01	2.398305e+05
1e-04	4	RR	0.0006001	5.890202e-01	5.417213e+02
1e-04	4	PRESS	0.0195724	7.404758e-01	4.887407e+04
1e-04	4	MLE	0.0000661	5.537542e-01	3.701068e+01
1e-04	4	MINSOL	0.0101010	6.897319e-01	2.443748e+04
1e-04	4	MINDATA	0.0000000	1.133262e+02	9.999942e-01
1e-02	1	GCV	0.3949094	9.548298e-01	4.275831e+03

Sigma^2	Replication	Method	Lambda	I_D	I_R
1e-02	1	RR	0.0066007	6.641616e-01	2.585986e+02
1e-02	1	PRESS	0.0195767	7.402338e-01	8.692958e+02
1e-02	1	MLE	0.0000661	5.538217e-01	8.819823e-01
1e-02	1	MINSOL	0.0101010	6.894553e-01	4.337175e+02
1e-02	1	MINDATA	0.0000000	1.187360e+05	1.000002e+00
1e-02	2	GCV	0.3921389	9.546235e-01	2.853891e+03
1e-02	2	RR	-0.3306331	1.091245e+00	6.700229e+03
1e-02	2	PRESS	0.0197821	7.411318e-01	5.879596e+02
1e-02	2	MLE	0.0000661	5.335591e-01	9.471931e-01
1e-02	2	MINSOL	0.0101010	6.892589e-01	2.912158e+02
1e-02	2	MINDATA	0.0000000	1.862331e+04	9.999961e-01
1e-02	3	GCV	0.3930244	9.547814e-01	7.377942e+03
1e-02	3	RR	-0.3306331	1.091292e+00	1.745399e+04
1e-02	3	PRESS	0.0196517	7.409092e-01	1.515742e+03
1e-02	3	MLE	0.0000661	5.668616e-01	1.656228e+00
1e-02	3	MINSOL	0.0101010	6.896825e-01	7.565637e+02
1e-02	3	MINDATA	0.0000000	1.967627e+04	9.999992e-01
1e-02	4	GCV	0.3938885	9.546922e-01	2.392126e+03
1e-02	4	RR	0.0018002	6.115793e-01	2.377545e+01
1e-02	4	PRESS	0.0200822	7.413635e-01	4.948808e+02
1e-02	4	MLE	0.0000661	5.337641e-01	4.627268e-01
1e-02	4	MINSOL	0.0101010	6.883416e-01	2.388538e+02
1e-02	4	MINDATA	0.0000000	2.082821e+03	9.999950e-01