EconomiX

Optimal self-protection and health risk perception: bridging the gap between risk theory and the Health Belief Model

Emmanuelle Augeraud-Véron
Marc Leandri

2023-12 Document de Travail/ Working Paper

EconomiX - UMR 7235 Bâtiment Maurice Allais
Université Paris Nanterre 200, Avenue de la République
92001 Nanterre Cedex

Site Web : economix.fr
Contact : secreteriat@economix.fr
Twitter : @EconomixU

cnrs
Université Paris Nanterre

Optimal self-protection and health risk perception: bridging the gap between risk theory and the Health Belief Model

Emmanuelle Augeraud-Véron[*] and Marc Leandri[†‡§]

Abstract

In this contribution to the longstanding risk theory debate on optimal self-protection, we aim to bridge the gap between the microeconomic modeling of self-protection, in the wake of Ehrlich and Becker (1972), and the Health Belief Model, a conceptual framework extremely influential in Public Health studies (Janz and Becker, 1984). In doing so, we highlight the crucial role of risk perception in the individual decision to adopt a preventive behavior towards a generic health risk. We discuss the optimal prevention effort engaged by an agent displaying either imperfect knowledge of the susceptibility (probability of occurrence) or the severity (magnitude of the loss) of a health hazard, or facing uncertainty on these risk components. We assess the impact of risk aversion and prudence on the optimal level of self-protection, an issue at the core of the risk and insurance economic literature. Our results also pave the way for the design of efficient information instruments to improve health prevention when risk perceptions are biased.

Keywords: Prevention, Self-protection, Health Belief Model, Risk perception, Risk aversion, Prudence

JEL Codes: D81, I12, D9.

[*]Bordeaux School of Economics-UMR 6060, Université de Bordeaux.

[†]UMI SOURCE, Université Paris-Saclay, UVSQ, IRD.

[‡]CNRS, EconomiX–UMR 7235, Université Paris Nanterre.

[§]The authors thank the MSH-Paris Saclay for the Emergence funding of their project. Marc Leandri is grateful to the CNRS and the EconomiX research center for hosting him as a Visiting Researcher in 2022-23.

1 Introduction

Since the seminal paper of Ehlrich and Becker (1972), a vast literature in Risk Theory has addressed the topic of self-protection vs. self-insurance to better understand the decision mechanisms underlying the choice of a rational agent to dedicate resources to protect herself from a financial or a health risk. As shown in the extensive literature review of Courbage et al. (2013), this issue raises fascinating questions on the role risk aversion (Dionne and Eeckhoudt, 1985; Jullien et al., 1999), ambiguity aversion (Courbage and Peter, 2021) and prudence (Chiu, 2005; Eeckhoudt and Gollier, 2005) can play in optimal individual prevention. In addition, extending the time scale of the modeling framework to two periods can also yield contradictory results (Menegatti, 2009).

Beyond a perfect information setting, some contributions to this strand of research have explored the impact of imperfect information on optimal self-protection. On the one hand, a biased estimation by the agent of the probability component of the risk has been tackled by introducing probability weighting (Bleichrodt and Eeckhoudt, 2006; Baillon et al., 2019) and pessimism (Etner and Jeleva, 2012). On the other hand, Brianti et al. (2018) have introduced uncertainty on the actual effect of the disease and on the effectiveness of the cure.

In parallel to this vibrant theoretical literature in Economics, the determinants of individual prevention under subjective beliefs has been widely studied in the Public Health Literature through the lens of the Health Belief Model (HBM). Since its early developments in the 1950s by the U.S health public service, it has gained massive momentum within the Public Health discipline and has become "one of the most widely used social cognition models in health psychology" (Conner and Norman, 2005), at the conceptual root of numerous empirical studies of prevention behavior in diverse health issues[1]. Its predictive power for many health afflictions has been assessed regularly and remains satisfying (Janz and Becker, 1984; Carpenter, 2010). The HBM distinguishes various channels explaining the adoption of health-related prevention behavior and underlines in particular the prominent role of subjective risk perception in each individual's decision to engage in preventive behavior. This risk perception is broken down into two essential constructs: *severity perception* and *susceptibility perception*, reflecting respectively the utility loss sustained if the health risk materializes and the probability of this health impairment. Other complementary constructs such as *expected barriers*, *expected benefits*, and more recently *self-efficacy* (Rosentstock et al., 1988) are also included as potential determinants of health prevention behavior.

Although it has been designed initially for operative Public Health studies, this framework

[1]According to Google Scholar and Scopus approximately 5000 publications in the last decade resort to the HBM to ground their empirical design in a robust theoretical framework.

displays organic ties with expectancy theory since it addresses self-protection through the same expected-value prism as the optimal self-protection literature cited previously[2]. Given the large HBM-based empirical evidence gathered on the impact of risk perception on health prevention behavior, it seems only natural to bridge the gap between this Public Health concept and the modeling of optimal self-protection in Economics. In the latter strand of literature, Crainich and Eeckhoudt (2017) have pondered over the case of heterogeneous baseline probability of disease but to our knowledge no economic model has addressed subjective risk perception in line with the HBM by disentangling risk into two distinct channels: severity perception on the one side, susceptibility perception on the other. Hence our attempt to build a health-focused microeconomic self-protection model that isolates the role of these beliefs in the adherence to preventive behavior. The impact of these beliefs will be explored in two directions. First, we will assume that the agent has her own biased perceptions and second, we will confront the agent with uncertainty on severity and/or susceptibility.

The paper will present in Section 2 our original HBM-inspired optimal self-protection model suited for risk-averse and risk-neutral agents displaying subjective severity and susceptibility beliefs. Section 3 solves the model and investigates the comparative statics of optimal self-protection for the risk perception parameters. The impact of risk aversion on self-protection is explored in Section 4. In Section 5 we discuss how the agent's self-protection decision will shift when she is facing uncertainty on either or both severity or susceptibility. Section 6 concludes by highlighting the downstream research avenues opened up by our identification of the two risk perception channels on self-protection.

2 A model of risk perception-based self-protection

2.1 A two period health risk self-protection model based on the HBM constructs

In order to address the role of risk perceptions in optimal individual prevention, we assume that the agent has subjective beliefs regarding both the susceptibility of infection and the severity of the health risk. In line with the wide range of HBM-based empirical studies, our model can encompass a large spectrum of health risks: infectious[3] or chronic diseases against which a preventive behavior reducing the health risk probability can be adopted. This self-

[2]It must be reminded that alternative criteria to expected utility such as Rank Dependent Utility are also investigated in various economic contributions (Konrad and Skaperdas, 1993) but we focus here on the standard expected utility criteria.

[3]This approach is indeed well-fitted to address self-protection behaviors against COVID 19 (Dedonno et al., 2022) or vector-borne diseases such as Lyme disease (Leandri et al., 2023).

protection is costly in the broadest sense: it consists either in actual spending in goods or services reducing this health susceptibility or in intangible costs reflecting the tediousness of the prevention efforts ε (exercising, eating healthier food...), with $0 \leq \varepsilon \leq \varepsilon_{max}$, where ε_{max} is exogenous and strictly positive. The increasing convex cost of self-protection $C(\varepsilon)$ echoes the concept of *perceived barriers* in the HBM constructs (see Figure 1) as the cost of self-protection is not necessarily monetary but diminishes utility nonetheless. As our model focuses on health behaviors that can imply intangible adoption costs, neither consumption nor wealth are included explicitly in the utility function and saving from one period to another is not an option. However, the trade-off between the benefits of self-protection and its cost in terms of utility remains. Since our model aims at capturing the largest span possible of health issues, we need to operate a clear timing distinction between the prevention efforts and the occurrence of the health risk. Indeed, chronic diseases such as diabetes, or cardiovascular conditions demand prevention efforts to avoid health impairments in the long haul. Hence a two period model where the health loss potentially occurs in the second period, similar to Menegatti (2009).

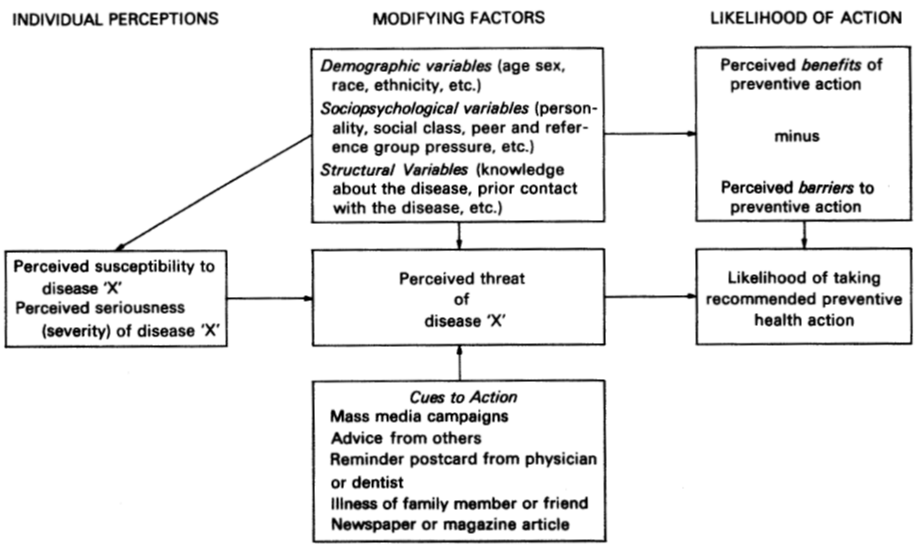

Figure 1: The Health Belief Model, Davidhizar (1983)

The first period utility U_1^d will thus be modeled as the difference between the baseline constant utility U_F of *business as usual* and the tangible and intangible costs of engaging in self-protection efforts $C(\varepsilon)$. The second period utility U_2^d, weighted in by a discount factor $0 < \beta < 1$, will be determined by an increasing concave health utility function u applied either to a healthy status $u(H)$ or to a degraded health status $u(H - M)$ where M is the severity of the disease[4] and $H \geq M > 0$. In the HBM, this severity encompasses "both medical/clinical consequences and possible social consequences" (Janz and Becker, 1984).

The probability to suffer the health condition, i.e. to catch an infectious disease or be affected by a chronic ailment, will be captured by the probability function $\alpha(\varepsilon;\pi)$ where π is the susceptibility, namely the baseline probability "of contracting a [health] condition" (Janz and Becker, 1984) with $\pi \in [0,1]$. Self-protection efforts decrease this probability with diminishing returns such that $\frac{\partial \alpha(\varepsilon;\pi)}{\partial \varepsilon} < 0$ and $\frac{\partial^2 \alpha(\varepsilon;\pi)}{\partial^2 \varepsilon} > 0$. α increases naturally with π. Considering that when the baseline susceptibility is higher, an additional protection effort reduces more strongly the probability to contract the condition, we have $\frac{\partial^2 \alpha(\varepsilon;\pi)}{\partial \varepsilon \partial \widehat{\pi}} < 0$. For instance in the case of an infectious disease, a higher prevalence rate mechanically raises the marginal efficiency of self-protection measures since it protects against a larger presence of infectious agents.

2.2 Risk perception and decision utility

We assume that the individual agent is characterized by her subjective susceptibility perception $\widehat{\pi}$ and her severity perception $\widehat{M}$, that differ from the true risk parameters. Consequently, her self-protection effort decision will be based on the maximization of a perception-based utility function, while the actual utility she will experience would result from the injection of this prevention effort into a utility function set on the actual parameters. The larger the information bias between risk perceptions and true risk parameters, the larger the gap between these two utilities. Hence the notation U^d we adopt to keep in mind that this utility function operates as the decision function of the agent but does not yield her actual welfare (Jimenez-Gomez, 2018).

Moreover, to disentangle health preferences and risk aversion, in our model we resort to an expected utility function *à la* Kihlstrom-Mirman (1974). Such a utility function enables to consider risk aversion in a two-period expected utility framework (Bommier et al., 2012; Bommier and Le Grand, 2014) as it keeps ordinal preferences unchanged and also satisfies the monotonicity property[5]. The intertemporal utility function for an agent with biased risk perceptions is thus written as follows.

$$U^d\left(\varepsilon;\widehat{M},\widehat{\pi}\right) = \mathbb{E}\left[\phi\left(U_1^d(\varepsilon) + \beta U_2^d(\varepsilon)\right)\right],$$

[4] We assume that the individual receives adequate treatment, the efficiency of healthcare and cure will not be addressed here.

[5] According to this latter property, an agent will not choose a decision if another available decision leads to better utility in all circumstances. Non-monotonicity yields counter-intuitive results about the role of risk aversion (Bommier and Legrand, 2019).

with ϕ an increasing and concave function. The risk aversion of this function is characterized by the concavity of ϕ.

Injecting the specific utility components presented in subsection 2.1 gives us

$$U^d\left(\varepsilon;\widehat{M},\widehat{\pi}\right)=\alpha\left(\varepsilon;\widehat{\pi}\right)\left(\phi(U_F-C\left(\varepsilon\right)+\beta U(H-\widehat{M})\right)+\left(1-\alpha\left(\varepsilon;\widehat{\pi}\right)\right)\phi(U_F-C\left(\varepsilon\right)+\beta U(H)). \quad (1)$$

The specific case of a risk neutral individual can be addressed by taking ϕ as the identity function such that

$$\begin{aligned} U^d\left(\varepsilon;\widehat{M},\widehat{\pi}\right) &= \mathbb{E}\left[U_1^d\left(\varepsilon\right)+\beta U_2^d\left(\varepsilon\right)\right], \\ &= U_1^d\left(\varepsilon\right)+\beta\mathbb{E}\left[U_2^d\left(\varepsilon\right)\right]. \end{aligned}$$

Corollary 1 *With $\Delta\left(\widehat{M}\right)=u\left(H\right)-u\left(H-\widehat{M}\right)$, the overall decision utility function of a risk neutral individual with subjective risk perceptions writes*

$$U^d\left(\varepsilon;\widehat{M},\widehat{\pi}\right)=U_F-C\left(\varepsilon\right)+\beta\left[u\left(H\right)-\alpha\left(\varepsilon;\widehat{\pi}\right)\Delta\left(\widehat{M}\right)\right]. \quad (2)$$

3 Optimal self-protection with risk perception

After solving our model in 3.1 we discuss the salient comparative static results in 3.2, thus contributing with our specific HBM model to the extensive debate animating the optimal self-protection literature.

3.1 Optimal self-protection with subjective risk perception

For the sake of clarity we shall use subsequently for a given function f the notation $f^{(i)}\left(\varepsilon\right)$ stands for $\frac{\partial^{(i)}f(\varepsilon)}{\partial\varepsilon^i}$, for $i=2,3$. For $i=1$, the notation $f'\left(\varepsilon\right)$ will be used.

We shall work under Assumption 1 defined as follows:

Assumption 1 $\alpha^{(2)}\left(\varepsilon;\widehat{\pi}\right)\alpha\left(\varepsilon;\widehat{\pi}\right)>2\left(\alpha'\left(\varepsilon;\widehat{\pi}\right)\right)^2$

Assumption 1 is required in a to ensure that self-protection can be a desirable choice and to guarantee the concavity in ε of the utility function U^d (Jullien et al., 1999, Condition C1, p. 23). Huber (2022) also stresses the need for this assumption when a Kihlstrom-Mirman expected utility function is used.

To facilitate the reading, we denote $w_1=U_F+\beta u\left(H-\widehat{M}\right)$ and $w_2=U_F+\beta u\left(H\right)$ the utility levels associated respectively with the infected state and the healthy state. We have $w_1<w_2$ by definition.

We can thus rewrite (1) as

$$U^d\left(\varepsilon;\widehat{M},\widehat{\pi}\right) = \alpha\left(\varepsilon;\widehat{\pi}\right)\phi\left(w_1\right) + \left(1-\alpha\left(\varepsilon;\widehat{\pi}\right)\right)\phi\left(w_2\right).$$

Then, the following Lemma holds.

Lemma 2 *For an agent with subjective risk perception* $(\widehat{\pi},\ \widehat{M})$ *and a risk aversion function* ϕ*, the optimal level of self protection* ε^*_ϕ *exists and is unique. If* ε^*_ϕ *is an interior solution,* $\varepsilon^*_\phi(\widehat{M},\widehat{\pi})$ *is the solution of the following equation:*

$$C'(\varepsilon)\left[\alpha(\varepsilon;\widehat{\pi})\phi'(w_1 - C(\varepsilon)) + (1-\alpha(\varepsilon;\widehat{\pi}))\phi'(w_2 - C(\varepsilon))\right] = -\alpha'(\varepsilon;\widehat{\pi})(\phi(w_2 - C(\varepsilon)) - \phi(w_1 - C(\varepsilon) \tag{3}$$

Proof. The proof is given in Appendix 6.1. ■

For a risk neutral agent, optimal effort is given by Corollary 3.

Corollary 3 *For a risk neutral agent with subjective risk perception* $(\widehat{\pi},\ \widehat{M})$ *the optimal level of self protection* ε^* *exists and is unique. If* ε^* *is an interior solution,* $\varepsilon^*(\widehat{M},\widehat{\pi})$ *is the solution of the following equation,*

$$C'(\varepsilon) = -\beta\Delta\left(\widehat{M}\right)\alpha'(\varepsilon;\widehat{\pi}). \tag{4}$$

Proof. A risk neutral individual determines her optimal self-protection effort $\varepsilon^*\left(\widehat{M},\widehat{\pi}\right) \in [0,\varepsilon_{\max}]$ that maximizes (2) and thus

$$\begin{aligned}
\frac{\partial U^d\left(\varepsilon;\widehat{M},\widehat{\pi}\right)}{\partial\varepsilon} &= -C'(\varepsilon) - \beta\Delta\left(\widehat{M}\right)\alpha'(\varepsilon;\widehat{\pi}),\\
\frac{\partial^2 U^d\left(\varepsilon;\widehat{M},\widehat{\pi}\right)}{\partial^2\varepsilon} &= -C^{(2)}(\varepsilon) - \beta\Delta\left(\widehat{M}\right)\frac{\partial^2\alpha(\varepsilon;\widehat{\pi})}{\partial\varepsilon^2} < 0. \blacksquare
\end{aligned}$$

Equation (3) captures the standard trade-off in optimal individual prevention between the marginal cost of the protection effort on the left-hand side and the marginal benefit of this protection on the right-hand side. The former reflects the reduction of expected utility in both states of the world induced by an increase in effort. The latter consists in the avoidance of the health loss risk through a decreased infection probability $\alpha'(\varepsilon;\widehat{\pi})$. This interpretation is particularly straightforward in the risk neutral case expressed in equation (4).

3.2 Comparative statics of the risk perception channels

By construction, the HBM implicitly assumes an unambiguous positive correlation between severity or susceptibility perception and individual prevention. While it has been confirmed empirically by a vast body of studies (Carpenter, 2010), this correlation lacks theoretical foundation. Our model allows us to explore the validity of this postulate through comparative statics on $\widehat{M}$ and $\widehat{\pi}$ for the risk averse and the risk neutral case.

As

$$\frac{\partial \varepsilon_{\phi}^{*}\left(\widehat{M},\widehat{\pi}\right)}{\partial \widehat{M}} = \frac{\beta u'\left(H-\widehat{M}\right)\left[C'\left(\varepsilon_{\phi}^{*}\right)\alpha\left(\varepsilon_{\phi}^{*};\widehat{\pi}\right)\phi^{(2)}\left(w_1-C\left(\varepsilon_{\phi}^{*}\right)\right)-\alpha'\left(\varepsilon_{\phi}^{*};\widehat{\pi}\right)\phi'\left(w_1-C\left(\varepsilon_{\phi}^{*}\right)\right)\right]}{-U^{d(2)}\left(\varepsilon_{\phi}^{*}\right)}, \tag{5}$$

it appears that the sign of $\frac{\partial \varepsilon^{*}\left(\widehat{M},\widehat{\pi}\right)}{\partial \widehat{M}}$ is ambiguous. Expression 5 yields straightforwardly the following Lemma:

Lemma 4 *For risk averse agents, there exist* $(\widehat{M},\widehat{\pi})$ *such that* $\frac{\partial \varepsilon_{\phi}^{*}\left(\widehat{M},\widehat{\pi}\right)}{\partial \widehat{M}} < 0$.
In particular, if $C'\left(\varepsilon_{\phi}^{*}\right) > \frac{\alpha'\left(\varepsilon_{\phi}^{*},\widehat{\pi}\right)}{\alpha\left(\varepsilon_{\phi}^{*},\widehat{\pi}\right)}\frac{\phi'\left(w_1-C\left(\varepsilon_{\phi}^{*}\right)\right)}{\phi^{(2)}\left(w_1-C\left(\varepsilon_{\phi}^{*}\right)\right)}$, *then* $\frac{\partial \varepsilon_{\phi}^{*}\left(\widehat{M},\widehat{\pi}\right)}{\partial \widehat{M}} < 0$.
If $C'\left(\varepsilon_{\phi}^{*}\right) < \frac{\alpha'\left(\varepsilon_{\phi}^{*},\widehat{\pi}\right)}{\alpha\left(\varepsilon_{\phi}^{*},\widehat{\pi}\right)}\frac{\phi'\left(w_1-C\left(\varepsilon_{\phi}^{*}\right)\right)}{\phi^{(2)}\left(w_1-C\left(\varepsilon_{\phi}^{*}\right)\right)}$, *then* $\frac{\partial \varepsilon_{\phi}^{*}\left(\widehat{M},\widehat{\pi}\right)}{\partial \widehat{M}} > 0$.

Regarding perceived susceptibility we have

$$\begin{aligned}\frac{\partial \varepsilon^{*}\left(\widehat{M},\widehat{\pi}\right)}{\partial \widehat{\pi}} &= \frac{-C'\left(\varepsilon\right)\left[\frac{\partial \alpha(\varepsilon;\widehat{\pi})}{\partial \widehat{\pi}}\left(\phi'\left(w_1-C\left(\varepsilon\right)\right)-\phi'\left(w_2-C\left(\varepsilon\right)\right)\right)\right]}{-U^{d(2)}\left(\varepsilon\right)} \\ &\quad -\frac{\frac{\partial^{(2)}\alpha(\varepsilon;\widehat{\pi})}{\partial\widehat{\pi}\partial\varepsilon}\left(\phi\left(w_2-C\left(\varepsilon\right)\right)-\phi\left(w_1-C\left(\varepsilon\right)\right)\right)}{-U^{d(2)}\left(\varepsilon\right)},\end{aligned} \tag{6}$$

which leads directly to the following Lemma:

Lemma 5 *For risk averse agents, there exist* $(\widehat{M},\widehat{\pi})$ *such that* $\frac{\partial \varepsilon_{\phi}^{*}\left(\widehat{M},\widehat{\pi}\right)}{\partial \widehat{\pi}} < 0$.
In particular, if $C'\left(\varepsilon\right) > \frac{-\frac{\partial^{(2)}\alpha(\varepsilon;\widehat{\pi})}{\partial\widehat{\pi}\partial\varepsilon}\left(\phi(w_2-C(\varepsilon))-\phi(w_1-C(\varepsilon))\right)}{\left[\frac{\partial\alpha(\varepsilon;\widehat{\pi})}{\partial\widehat{\pi}}\left(\phi'(w_1-C(\varepsilon))-\phi'(w_2-C(\varepsilon))\right)\right]}$, *then* $\frac{\partial \varepsilon_{\phi}^{*}\left(\widehat{M},\widehat{\pi}\right)}{\partial \widehat{\pi}} < 0$.
If $C'\left(\varepsilon\right) < \frac{-\frac{\partial^{(2)}\alpha(\varepsilon;\widehat{\pi})}{\partial\widehat{\pi}\partial\varepsilon}\left(\phi(w_2-C(\varepsilon))-\phi(w_1-C(\varepsilon))\right)}{\left[\frac{\partial\alpha(\varepsilon;\widehat{\pi})}{\partial\widehat{\pi}}\left(\phi'(w_1-C(\varepsilon))-\phi'(w_2-C(\varepsilon))\right)\right]}$, *then* $\frac{\partial \varepsilon_{\phi}^{*}\left(\widehat{M},\widehat{\pi}\right)}{\partial \widehat{\pi}} > 0$.

The first component of Lemmas 4 and 5 shows that for some sets of beliefs on severity and susceptibility for risk averse agents, an increase in perceived severity or susceptibility may have the counter-intuitive impact of decreasing optimal self-protection. This should prompt caution on the assumptions of the HBM on the causal role of risk perception on prevention. According to the second part of these Lemmas, if the marginal cost of self-protection is low enough, an increase in perceived severity will lead to more self-protection, as expected. Given the convexity of the protection cost function, risk averse agents that have already adopted a high level of self-protection measures will be less likely to respond positively to a stimulus increasing their perceived severity.

For a risk neutral individual, the following corollaries hold. Their proof comes from substituting Φ by the identity function respectively in (5) and (6).

Corollary 6 *For a risk neutral agent, the higher the perceived severity* $\widehat{M}$*, the higher the optimal protection effort* ε^**.*

Corollary 7 *For a risk neutral agent, the higher the perceived susceptibility* $\widehat{\pi}$*, the higher the optimal protection effort* ε^**.*

Corollaries 6 and 7 prove the positive impact of each channel of risk perception on optimal self-protection for risk neutral individuals, regardless of the marginal cost of self-protection. This independence from the initial level of self protection reflects the general definition of risk neutrality as preferences unaffected by the initial level wealth. Contrasting these results with the risk averse case in Lemmas 4 and 5 pinpoints the remarkable differences in sensitivity between risk averse and risk neutral agents. Risk averse agents can have a different reaction to increased perceived severity or susceptibility compared to risk neutral ones as they will be reluctant to risk suffering a higher net loss if they get infected despite the protection efforts already engaged.

Our analysis completes, from the perspective of risk perception, the established results in the literature (see below) on the self-protection effort of an agent more risk averse than the benchmark risk neutral one. When faced with an increase in the overall risk, either through higher severity or through higher susceptibility, a risk averse agent can modify her behavior in a counter-intuitive way. Our results also highlight the potential role of the self-protection cost function, which can deter risk averse agents already significantly protected to increase their protection. For public health policies, this result is key to encourage HBM studies to try and distinguish risk averse from risk neutral individuals in their empirical applications.

4 Preferences towards risk and self-protection under health risk perception

As mentioned in the Introduction, the theoretical economics literature has long pondered over the puzzling impact of risk-aversion on optimal self-protection. After Dionne and Eeckhoudt, (1985) showed that self-protection, contrary to self-insurance, can decrease when risk aversion increases, a loss probability threshold over which this effect takes place has been identified in a one-period setting by Jullien et al. (1999) and others (see the exhaustive review in Courbage et al., 2013). A similar result was confirmed by Menegatti (2009) and Huber (2022) for a two-period model. This probability threshold that determines if self-protection increases or decreases with risk aversion is considered endogenous in the sense that it depends jointly on the preferences of both the benchmark agent and the "more risk-averse" agent involved in the comparison. Through an elegant method based on risk neutral probabilities, Peter (2021) managed to remove this endogeneity in the definition of the threshold, now exclusively linked to the benchmark agent's preferences. We apply this approach to our Kihlstrom-Mirman function that respects ordinal preferences and define the following risk neutral probability $\overline{\alpha}$ for a risk aversion function ϕ, with ε_ϕ^* the corresponding optimal effort:

$$\overline{\alpha}\left(\varepsilon_\phi^*;\widehat{\pi}\right) = \frac{\alpha\left(\varepsilon_\phi^*,\widehat{\pi}\right)\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right)}{\alpha\left(\varepsilon_\phi^*,\widehat{\pi}\right)\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) + \left(1-\alpha\left(\varepsilon_\phi^*,\widehat{\pi}\right)\right)\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}.$$

$\overline{\alpha}$ corresponds to the loss probability that needs to be applied to a risk neutral agent for the latter to derive the same utility from the lottery as our benchmark agent does with the *real* probability $\alpha\left(\varepsilon_\phi^*;\widehat{\pi}\right)$ and the same payoffs (Heaton, 2018). In the case of a risk averse benchmark agent, $\overline{\alpha}$ is thus greater than $\alpha\left(\varepsilon_\phi^*;\widehat{\pi}\right)$, and they are equal for a risk neutral benchmark agent. Let us consider another agent whose risk preferences are characterized by φ, defined by $\varphi = k(\phi)$, where $k' > 0$ and $k^{(2)} < 0$ so that she is more risk averse than our benchmark agent. Her optimal effort is denoted $\varepsilon_{k\phi}^*$. In the wake of Peter (2021), we obtain the following Lemma:

Lemma 8 *1. If $\overline{\alpha}\left(\varepsilon_\phi^*,\widehat{\pi}\right) \geq \frac{1}{2}$ and if $k^{(3)} > 0$, then $\varepsilon_{k\phi}^* < \varepsilon_\phi^*$.*

2. If $\overline{\alpha}\left(\varepsilon_\phi^*,\widehat{\pi}\right) \leq \frac{1}{2}$ and if $k^{(3)} < 0$, then $\varepsilon_{k\phi}^* > \varepsilon_\phi^*$.

Proof. The proof is given in Appendix 6.2. ■

Lemma 8 confirms, under an additional condition on $k^{(3)}$, the ambiguous effect of risk aversion on self-protection in a two-period Kihlstrom-Mirman setting respecting ordinal preferences and with an exogenous threshold[6]. As spending in self-protection will reduce utility in both the loss and the no-loss state compared to abstaining from it, self-protection is not necessarily higher among agents that are more risk averse (Briys and Schlesinger, 1990). The third order preference on k, which can be interpreted as prudence in the economic sense, is needed to wave off the ambiguity. In particular, since decreasing the loss probability through self-protection increases variance when $\overline{\alpha} \in]\frac{1}{2}, 1]$, a greater risk aversion will lead prudent agents ($k^{(3)} > 0$) to lower self-protection. This behavior can be explained by their will to avoid the worst outcome: being sick and having borne additional self-protection costs. The opposite effect is at play for imprudent agents whose self-protection increases with risk aversion when $\overline{\alpha} \in [0, \frac{1}{2}[$. Our generic result holds for any type of benchmark agent and can be used to reformulate the seminal result of Eeckhoudt and Gollier (2005) for a risk neutral benchmark by substituting ϕ by the identity function and $\overline{\alpha}$ by α. Contrary to Menegatti (2009), we observe in our two period model a similar effect of prudence as in one period approaches, which questions Menegatti's conclusion that *"the impact of prudence on optimal prevention strictly depends on whether the effect of prevention is contemporaneous or lagged"* (p. 396).

It must be noted that there exist two cases for which the result is indeterminate due to conflicting effects. These two cases are the following: if $\overline{\alpha}\left(\varepsilon_{\phi}^{*}, \widehat{\pi}\right) > \frac{1}{2}$ and $k^{(3)} < 0$, and $\overline{\alpha}\left(\varepsilon_{\phi}^{*}, \widehat{\pi}\right) < \frac{1}{2}$ and $k^{(3)} > 0$. For the interested reader, Peter (2021) addresses these specific instances of indeterminacy through additional conditions on the relative curvature of the marginal transformation function (Proposition 3, p. 10).

5 Uncertainty on severity and susceptibility

Let us now address the issue of uncertainty which is central in the kind of health risk prevention we are looking at, given our focus on risk perception. As shown by the 2020 COVID 19 pandemic, uncertainty on severity and/or susceptibility can affect significantly preventive behaviors. In their one-period self-protection model, Brianti et al. (2018) have discussed the impact of applying uncertainty on either cure effectiveness or disease effect, while keeping the other parameter known. We expand their approach to study how the optimal self-protection effort changes when we introduce uncertainty on severity or on susceptibility, whether the other parameter is also uncertain or it is known. The comparison of a single source of uncertainty with a totally deterministic benchmark, as in Brianti et al. (2018), can be easily captured as a specific case of our general result.

Our risk components now take the form of random variables $\widetilde{M}$ (with support in $[0, H]$) and $\widetilde{\pi}$ (with support in $[0, 1]$). For this analysis that implies simultaneous uncertainty, we

[6]Extensive calculations left the impact of either $\widehat{M}$ or $\widehat{\pi}$ on the risk neutral probability $\overline{\alpha}$ undetermined.

need to make the following assumptions.

First we make an hypothesis in the spirit of Assumption 1.

Assumption 2 $\mathbb{E}\left[\alpha\left(\varepsilon;\widetilde{\pi}\right)\right]\mathbb{E}\left[\alpha^{(2)}\left(\varepsilon;\widetilde{\pi}\right)\right] > 2\mathbb{E}\left[\alpha'\left(\varepsilon;\widetilde{\pi}\right)\right]^{2}.$

Second we assume that

Assumption 3 *$\widetilde{M}$ and $\widetilde{\pi}$ are independent.*

This hypothesis is rather natural considering that each parameter constitutes a distinct component of the overall risk. It is backed up by empirical evidence on various forms of health impairments, in particular infectious diseases. Cummings et al. (1978) show that these two perceptions are substantially independent for a wide range of diseases. More recently, Dedonno et al. (2022) establish their independence in the case of COVID 19 and Leandri et al. (2022) observe independence between severity and susceptibility for Lyme disease prevention in a large sample of forest users in France.

We look first at the impact of introducing uncertainty on severity. To do so, we compare a setting with uncertainty weighing only on susceptibility with simultaneous uncertainty on both risk parameters, which amounts to comparing

$$\varepsilon^{*} = \arg\max_{\varepsilon}\mathbb{E}\left[U^{d}\left(\varepsilon;\mu_{M},\widetilde{\pi}\right)\right] \text{ and } \varepsilon^{**} = \arg\max_{\varepsilon}\mathbb{E}\left[U^{d}\left(\varepsilon;\widetilde{M},\widetilde{\pi}\right)\right],$$

where μ_M is defined as $\mathbb{E}\left[\widetilde{M}\right] = \mu_M$.

The following Lemma is then set.

Lemma 9 *Under Assumption 6.3 and if the agent is prudent (i.e. $\phi^{(3)} > 0$), introducing uncertainty on severity increases optimal self-protection ($\varepsilon^{**} \geq \varepsilon^{*}$).*

Proof. The proof is given in Appendix 6.3. This result holds whether the other parameter, susceptibility, is also a random variable or if it is known. In the latter specific case, Assumption 2 is not needed, Assumption 1 is sufficient. ■

To study the impact of uncertainty on susceptibility, we compare

$$\varepsilon^{*} = \arg\max_{\varepsilon}\mathbb{E}\left[U^{d}\left(\varepsilon;\widetilde{M},\mu_{\pi}\right)\right] \text{ and } \varepsilon^{**} = \arg\max_{\varepsilon}\mathbb{E}\left[U^{d}\left(\varepsilon;\widetilde{M},\widetilde{\pi}\right)\right],$$

where μ_π is defined as $\mathbb{E}\left[\widetilde{\pi}\right] = \mu_\pi$.

The following Lemma is then set.

Lemma 10 *Under Assumption 6.3, if $\alpha^{(3)}(\varepsilon;\mu_\pi) < 0$ and if the agent is prudent (i.e. $\phi^{(3)} > 0$), introducing uncertainty on susceptibility increases optimal self-protection ($\varepsilon^{**} \geq \varepsilon^{*}$).*

Proof. The proof is given in Appendix 6.4. As for Lemma '9, this result holds whether the other parameter, severity, is also a random variable or if it is known. In the latter specific case, Assumption 2 is not needed, Assumption 1 is sufficient. ■

Lemma 9 offers a complementary perspective on the behavior of prudent agents under Assumptions 6.3 and 3. When facing both an endogenous infection risk they can act upon (through ε), and an exogenous risk on the severity of the disease, they will increase the self-protection effort, whether the susceptibility is uncertain or not. If this result is unambiguous for severity, Lemma 10 shows that for susceptibility it requires an additional specific third order property of the probability function α, which can be interpreted as the prevention technology (Peter, 2021). The driving force of prudence in favor of self-protection facing uncertainty remains valid. A similar effect had been identified by Felder and Mayrhofer (2017) from the physician perspective in a model with comorbidity[7].

For risk-neutral individuals in the Kihlstrom-Mirman framework, the following Corollaries hold true.

Corollary 11 *For risk-neutral individuals, introducing uncertainty on severity strictly increases optimal self-protection.*

Proof. Proof is given in Appendix 6.5. This result holds whether the other parameter, susceptibility, is also a random variable or if it is known. In the latter specific case, Assumption 2 is not needed, Assumption 1 is sufficient. ■

This result is in line with Brianti et al. (2018, Proposition 2) for uncertainty on the disease effect.

Corollary 12 *For risk-neutral individuals, if $\alpha^{(3)} < 0$, introducing uncertainty on susceptibility strictly increases optimal self-protection. On the contrary, if $\alpha^{(3)} > 0$, uncertainty on susceptibility strictly decreases optimal self-protection.*

Proof. Proof is given in Appendix 6.6. This result holds whether the other parameter, severity, is also a random variable as well or if it is known. In the latter specific case, Assumption 2 is not needed, Assumption 1 is sufficient. ■

Corollaries 11 and 12 illustrate the interesting asymmetry between the two channels of risk perception. Whereas introducing uncertainty on severity drives unambiguously the risk neutral agent to increase her self-protection, uncertainty on susceptibility will have opposite effects depending on the properties of the prevention technology. When $\alpha^{(3)} > 0$ our agent will be reluctant to engage in self-protection costs that will yield a lower expected marginal

[7]Formally, our modeling of perceived severity amounts to a form of comorbidity in their setting.

efficiency. Under this assumption, in view of these contrasting reactions to uncertainty, it could be argued that health prevention policies should prioritize information campaigns on susceptibility before severity since dissipating uncertainty on the former is more likely to increase self-protection.

6 Conclusion

Through our original take on the HBM we have introduced two-way risk perception into the strand of economic literature on optimal self-protection. In doing so we broaden the theoretical perspective on self-protection and bring out complementary results to this issue. We were able to disentangle the severity perception and the susceptibility perception channel within the decision mechanism and we showed how this asymmetry generates differentiated reactions to risk aversion and to prudence, making the lasting debate on the role of these properties even more complex.

Combining the traditional study of risk aversion and prudence with the introduction of uncertainty has allowed us to characterize more thoroughly, under certain conditions, the behavior of prudent agents. Our findings emulate the results of Brianti el al. (2018) and extend their scope to the case of simultaneous uncertainty. We confirm that a more prudent agent can exert less self-protection effort when faced with a deterministic low infection risk. But we also find that if there is uncertainty on either component of the risk perception, a prudent agent will increase her self-protection.

Our comparative statics on the risk perception channels raises legitimate questioning of the theoretical foundations of the HBM, that assumes systematic unidirectional correlation between individual prevention and risk perception. Given the conceptual proximity of the HBM with expected value theory, our results call for a deeper dive in the assumption of this public health framework, in particular regarding risk aversion. The latter could be introduced more rigorously in empirical HBM-bases studies to better observe its effects on self-protection.

References

[1] Baillon, A., Bleichrodt, H., Emirmahmutoglu, A., Jaspersen, J., & Peter, R. (2022). When risk perception gets in the way: Probability weighting and underprevention. Operations Research, 70(3), 1371-1392.

[2] Bleichrodt, H., & Eeckhoudt, L. (2006). Willingness to pay for reductions in health risks when probabilities are distorted. Health Economics, 15(2), 211-214.

[3] Bommier, A., Chassagnon, A., & Le Grand, F. (2012). Comparative risk aversion: A formal approach with applications to saving behavior. Journal of Economic Theory, 147(4), 1614-1641.

[4] Bommier, A., & LeGrand, F. (2014). Too risk averse to purchase insurance? Journal of Risk and Uncertainty, 48(2), 135–166.

[5] Bommier, A., & Le Grand, F. L. (2019). Risk aversion and precautionary savings in dynamic settings. Management Science, 65(3), 1386-1397.

[6] Brianti, M., Magnani, M., & Menegatti, M. (2018). Optimal choice of prevention and cure under uncertainty on disease effect and cure effectiveness. Research in Economics, 72(2), 327-342.

[7] Briys, E., & Schlesinger, H. (1990). Risk aversion and the propensities for self-insurance and self-protection. Southern Economic Journal, 57(2), 458–467.

[8] Carpenter, C. J. (2010). A meta-analysis of the effectiveness of health belief model variables in predicting behavior. Health communication, 25(8), 661-669.

[9] Chiu, W. H. (2005). Degree of downside risk aversion and self-protection. Insurance: Mathematics and Economics, 36(1), 93-101.

[10] Conner M, Norman P. (2005). Predicting Health Behaviour: A Social Cognition Approach. In Conner M. and Norman P., Eds. Predicting Health Behaviour. 2nd ed. Berkshire: McGraw-Hill Education, 1-21.

[11] Courbage, C., & Peter, R. (2021). On the effect of uncertainty on personal vaccination decisions. Health Economics, 30(11), 2937-2942.

[12] Courbage, C., Rey, B., & Treich, N. (2013). Prevention and precaution. Handbook of insurance, 185-204.

[13] Crainich, D., & Eeckhoudt, L. (2017). Average willingness to pay for disease prevention with personalized health information. Journal of Risk and Uncertainty, 55(1), 29-39.

[14] Cummings, K. M., Jette, A. M., & Rosenstock, I. M. (1978). Construct validation of the health belief model. Health education monographs, 6(4), 394-405.

[15] Davidhizar, R. (1983). Critique of the health-belief model. Journal of advanced nursing, 8(6), 467-472.

[16] DeDonno, M. A., Longo, J., Levy, X., & Morris, J. D. (2022). Perceived Susceptibility and Severity of COVID-19 on Prevention Practices, Early in the Pandemic in the State of Florida. Journal of Community Health, 1-8.

[17] Dionne, G., & Eeckhoudt, L. (1985). Self-insurance, self-protection and increased risk aversion. Economics Letters, 17(1-2), 39–42.

[18] Eeckhoudt, L., & Gollier, C. (2005). The impact of prudence on optimal prevention. Economic Theory, 26(4), 989-994.

[19] Ehrlich, I., & Becker, G. S. (1972). Market insurance, self-insurance, and self-protection. Journal of political Economy, 80(4), 623-648.

[20] Etner, J., & Jeleva, M. (2013). Risk perception, prevention and diagnostic tests. Health economics, 22(2), 144-156.

[21] Felder, S., & Mayrhofer, T. (2017). Medical decision making. Springer Berlin Heidelberg.

[22] Heaton, J. B. (2018). Risk aversion as risk-neutral pessimism: A simple proof. International Review of Law and Economics, 56, 70-72.

[23] Huber, T. (2022). Comparative risk aversion in two periods: An application to self-insurance and self-protection. Journal of Risk and Insurance, 89(1), 97-130.

[24] Janz, N. K., & Becker, M. H. (1984). The health belief model: A decade later. Health education quarterly, 11(1), 1-47.

[25] Jimenez-Gomez, D. (2018). Nudging and phishing: A theory of behavioral welfare economics. Available at SSRN 3248503.

[26] Jullien, B., Salanié, B., & Salanié, F. (1999). Should more risk-averse agents exert more effort? The Geneva Papers on Risk and Insurance Theory, 24(1), 19–28.

[27] Kihlstrom, R. E., & Mirman, L. J. (1974). Risk aversion with many commodities. Journal of Economic Theory, 8(1), 361–388.

[28] Konrad, K. A., & Skaperdas, S. (1993). Self-Insurance and Self-Protection: A Nonexpected Utility Analysis. The Geneva Papers on Risk and Insurance Theory, 18(2), 131–146.

[29] Leandri M., Augeraud-Véron, E., Zugravu, N., Dalmas, L. & Kean, J. (2022). The determinants of Lyme disease prevention among French forest users, mimeo.

[30] Menegatti, M. (2009). Optimal prevention and prudence in a two-period model. Mathematical Social Sciences, 58(3), 393–397.

[31] Menegatti, M. (2021). Subsidizing risk prevention. Journal of Economics, 134(2), 175-193.

[32] Peter, R. (2021). Who should exert more effort? Risk aversion, downside risk aversion and optimal prevention. Economic Theory, 71(4), 1259-1281.

[33] Rosenstock, I. M., Strecher, V. J., & Becker, M. H. (1988). Social learning theory and the health belief model. Health education quarterly, 15(2), 175-183.

Appendix

6.1 Proof of Lemma 2

The aim of this proof is to show the concavity of $U^d(\varepsilon)$.

$$\begin{aligned} U^{d(2)}(\varepsilon) &= -C^{(2)}(\varepsilon)\left[\alpha(\varepsilon,\widehat{\pi})\,\phi'(w_1-C(\varepsilon))+(1-\alpha(\varepsilon,\widehat{\pi}))\,\phi'(w_2-C(\varepsilon))\right] \\ &\quad -\alpha^{(2)}(\varepsilon,\widehat{\pi})\left(\phi(w_2-C(\varepsilon))-\phi(w_1-C(\varepsilon))\right) \\ &\quad +2\alpha'(\varepsilon,\widehat{\pi})\,C'(\varepsilon)\left(\phi'(w_2-C(\varepsilon))-\phi'(w_1-C(\varepsilon))\right) \\ &\quad +\left(C'(\varepsilon)\right)^2\left[\alpha(\varepsilon,\widehat{\pi})\,\phi^{(2)}(w_1-C(\varepsilon))+(1-\alpha(\varepsilon,\widehat{\pi}))\,\phi^{(2)}(w_2-C(\varepsilon))\right]. \end{aligned}$$

The third term being positive, the sign of $U^{d(2)}(\varepsilon)$ is a priori undefined. However, using the interior optimality condition, it can be noticed that at the optimum the following equation holds.

$$\left(\phi(w_2-C(\varepsilon))-\phi(w_1-C(\varepsilon))\right)=\frac{-C'(\varepsilon)}{\alpha'(\varepsilon,\widehat{\pi})}\left[\alpha(\varepsilon,\widehat{\pi})\,\phi'(w_1-C(\varepsilon))+(1-\alpha(\varepsilon,\widehat{\pi}))\,\phi'(w_2-C(\varepsilon))\right].$$

Thus

$$\begin{aligned} \frac{\alpha'(\varepsilon,\widehat{\pi})\,U^{d(2)}(\varepsilon)}{C'(\varepsilon)} &> \left(\alpha^{(2)}(\varepsilon,\widehat{\pi})\,\alpha(\varepsilon,\widehat{\pi})-2\left(\alpha'(\varepsilon,\widehat{\pi})\right)^2\right)\left[\phi'(w_1-C(\varepsilon))-\phi'(w_2-C(\varepsilon))\right] \\ &\quad +\alpha^{(2)}(\varepsilon,\widehat{\pi})\,\phi'(w_2-C(\varepsilon)) \\ &\quad +\alpha'(\varepsilon,\widehat{\pi})\,C'(\varepsilon)\left[\alpha(\varepsilon,\widehat{\pi})\,\phi^{(2)}(w_1-C(\varepsilon))+(1-\alpha(\varepsilon,\widehat{\pi}))\,\phi^{(2)}(w_2-C(\varepsilon))\right]. \end{aligned}$$

Using Assumption 1, the previous expression enables us to conclude the proof that $U^d(\varepsilon)$ is concave at the optimum, and thus the optimum is a maximum.

6.2 Proof of Lemma 8

The proof is an adaptation of the proof of Proposition 2 in Peter (2021) for our framework of a two-period model with expected utility function *à la* Kihlstrom-Mirman. The optimal self-protection effort of a risk-averse agent with risk function ϕ is given as a solution of the following equation.

$$\begin{aligned} 0 &= -C'(\varepsilon)\left[\alpha(\varepsilon,\widehat{\pi})\phi'(w_1-C(\varepsilon))+(1-\alpha(\varepsilon,\widehat{\pi}))\phi'(w_2-C(\varepsilon))\right] \\ &\quad -\alpha'(\varepsilon,\widehat{\pi})\left(\phi(w_2-C(\varepsilon))-\phi(w_1-C(\varepsilon))\right). \end{aligned}$$

We denote $V^d(\varepsilon)$ the expected utility of a more risk-averse agent with utility function $k(\phi)$.

$$\begin{aligned} V^d(\varepsilon) &= \mathbb{E}\left[k\phi\left(U_1^d(\varepsilon)+\beta U_2^d(\varepsilon)\right)\right], \\ &= \alpha(\varepsilon,\widehat{\pi})\,k\phi\left(U_F-C(\varepsilon)+\beta u\left(H-\widehat{M}\right)\right) \\ &\quad +(1-\alpha(\varepsilon,\widehat{\pi}))\,k\phi\left(U_F-C(\varepsilon)+\beta u(H)\right). \end{aligned}$$

The results of Lemma 2 hold and V^d is a concave function at its extremum, which is thus a maximum. It implies that for ε smaller than the optimum V^d is increasing, and for ε greater that the optimum V^d is decreasing. Notice that V^d is not necessarily globally concave, as inflection points may exist.

$$\begin{aligned} V^{d'}(\varepsilon) &= \alpha'(\varepsilon,\widehat{\pi})\left[k\phi\left(U_F-C(\varepsilon)+\beta u\left(H-\widehat{M}\right)\right)-k\phi\left(U_F-C(\varepsilon)+\beta u(H)\right)\right] \\ &\quad -C'(\varepsilon)\left[\begin{array}{c} \alpha(\varepsilon,\widehat{\pi})\,k'\phi\left(U_F-C(\varepsilon)+\beta u\left(H-\widehat{M}\right)\right)\phi'\left(U_F-C(\varepsilon)+\beta u\left(H-\widehat{M}\right)\right) \\ +(1-\alpha(\varepsilon,\widehat{\pi}))\,k'\phi\left(U_F-C(\varepsilon)+\beta u(H)\right)\phi'\left(U_F-C(\varepsilon)+\beta u(H)\right) \end{array}\right]. \end{aligned}$$

According to our previous remark on V^d, $\varepsilon^*_{k\phi}$ is greater that ε^*_ϕ if and only if $V^{d'}\left(\varepsilon^*_\phi\right)\geq 0$. To ease the notation, we denote $\alpha^*_\phi=\alpha\left(\varepsilon^*_\phi,\widehat{\pi}\right)$. As

$$\alpha'\left(\varepsilon^*_\phi,\widehat{\pi}\right)=-C'\left(\varepsilon^*_\phi\right)\frac{\left[\alpha^*_\phi\phi'\left(w_1-C\left(\varepsilon^*_\phi\right)\right)+\left(1-\alpha^*_\phi\right)\phi'\left(w_2-C\left(\varepsilon^*_\phi\right)\right)\right]}{\left(\phi\left(w_2-C\left(\varepsilon^*_\phi\right)\right)-\phi\left(w_1-C\left(\varepsilon^*_\phi\right)\right)\right)},$$

$V^{d'}\left(\varepsilon^*_\phi\right)$ has the opposite sign as

$$\begin{aligned} f\left(\varepsilon^*_\phi\right) &= \frac{k\phi\left(w_1-C\left(\varepsilon^*_\phi\right)\right)-k\phi\left(w_2-C\left(\varepsilon^*_\phi\right)\right)}{\phi\left(w_2-C\left(\varepsilon^*_\phi\right)\right)-\phi\left(w_1-C\left(\varepsilon^*_\phi\right)\right)} \\ &\quad +\frac{\alpha^*_\phi k'\phi\left(w_1-C\left(\varepsilon^*_\phi\right)\right)\phi'\left(w_1-C\left(\varepsilon^*_\phi\right)\right)+\left(1-\alpha^*_\phi\right)k'\phi\left(w_2-C\left(\varepsilon^*_\phi\right)\right)\phi'\left(w_2-C\left(\varepsilon^*_\phi\right)\right)}{\alpha^*_\phi\phi'\left(w_1-C\left(\varepsilon^*_\phi\right)\right)+\left(1-\alpha^*_\phi\right)\phi'\left(w_2-C\left(\varepsilon^*_\phi\right)\right)}. \end{aligned}$$

Letting $\phi\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) = \varphi_1$ and $\phi\left(w_2 - C\left(\varepsilon_\phi^*\right)\right) = \varphi_2$, the previous expression can be rewritten as follows.

$$\begin{aligned}
f\left(\varepsilon_\phi^*\right) &= \frac{k\varphi_1 - k\varphi_2}{\varphi_2 - \varphi_1} + \frac{\alpha_\phi^* k'\left(\varphi_1\right)\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) + \left(1-\alpha_\phi^*\right)k'\left(\varphi_2\right)\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}{\alpha_\phi^*\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) + \left(1-\alpha_\phi^*\right)\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}, \\
&= \frac{1}{\varphi_2 - \varphi_1}\int_{\varphi_2}^{\varphi_1} k'\left(z\right)dz + \frac{\alpha_\phi^* k'\left(\varphi_1\right)\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) + \left(1-\alpha_\phi^*\right)k'\left(\varphi_2\right)\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}{\alpha_\phi^*\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) + \left(1-\alpha_\phi^*\right)\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}.
\end{aligned}$$

As

$$\begin{aligned}
\overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right) &= \frac{\alpha_\phi^*\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right)}{\alpha_\phi^*\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) + \left(1-\alpha_\phi^*\right)\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}, \\
1 - \overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right) &= \frac{\left(1-\alpha_\phi^*\right)\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}{\alpha_\phi^*\phi'\left(w_1 - C\left(\varepsilon_\phi^*\right)\right) + \left(1-\alpha_\phi^*\right)\phi'\left(w_2 - C\left(\varepsilon_\phi^*\right)\right)}.
\end{aligned}$$

$$f\left(\varepsilon_\phi^*\right) = \frac{1}{\varphi_2 - \varphi_1}\int_{\varphi_2}^{\varphi_1} k'\left(z\right)dz + \overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right)k'\left(\varphi_1\right) + \left(1 - \overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right)\right)k'\left(\varphi_2\right).$$

Let us rewrite

$$\overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right)k'\left(\varphi_1\right) + \left(1 - \overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right)\right)k'\left(\varphi_2\right) = \frac{1}{2}k'\left(\varphi_1\right) + \frac{1}{2}k'\left(\varphi_2\right) - \left(\frac{1}{2} - \overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right)\right)\left(k'\left(\varphi_1\right) - k'\left(\varphi_2\right)\right).$$

Thus, the sign of $V^{d'}\left(\varepsilon_\phi^*\right)$ is the same as the sign of $-f\left(\varepsilon_\phi^*\right)$, which is given by

$$\frac{1}{\varphi_2 - \varphi_1}\int_{\varphi_1}^{\varphi_2} k'\left(z\right)dz - \frac{1}{2}k'\left(\varphi_1\right) - \frac{1}{2}k'\left(\varphi_2\right) + \left(\frac{1}{2} - \overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right)\right)\left(k'\left(\varphi_1\right) - k'\left(\varphi_2\right)\right).$$

As

$$\begin{aligned}
\frac{1}{b-a}\int_a^b \frac{b-s}{b-a}ds &= \frac{1}{2}, \\
\frac{1}{b-a}\int_a^b \frac{s-a}{b-a}ds &= \frac{1}{2},
\end{aligned}$$

$$\begin{aligned}
-f\left(\varepsilon_\phi^*\right) &= \frac{1}{\varphi_2 - \varphi_1}\int_{\varphi_1}^{\varphi_2}\left[k'\left(s\right) - \frac{\varphi_2 - s}{\varphi_2 - \varphi_1}k'\left(\varphi_1\right) - \left(1 - \frac{\varphi_2 - s}{\varphi_2 - \varphi_1}\right)k'\left(\varphi_2\right)\right]ds \\
&\quad + \left(\frac{1}{2} - \overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right)\right)\left(k'\left(\varphi_1\right) - k'\left(\varphi_2\right)\right).
\end{aligned}$$

If $\overline{\alpha}\left(\varepsilon_\phi^*, \widehat{\pi}\right) = \frac{1}{2}$, then the last term is null. Moreover, if k' is concave (i.e. $k^{(3)} < 0$), then the term in brackets is positive, implying that $V^{d'}\left(\varepsilon_{\phi}^{*}\right) > 0$. As a consequence, $\varepsilon_{k\phi}^{*} > \varepsilon_{\phi}^{*}$. Similarly, if $k^{(3)} > 0$, $\varepsilon_{k\phi}^{*} < \varepsilon_{\phi}^{*}$.

If $\overline{\alpha}\left(\varepsilon_{\phi}^{*}, \widehat{\pi}\right) > \frac{1}{2}$, as $k^{(2)} < 0$, then the second term is negative. If $k^{(3)} > 0$, then $V^{d'}\left(\varepsilon_{\phi}^{*}\right) < 0$, thus $\varepsilon_{k\phi}^{*} < \varepsilon_{\phi}^{*}$.

If $\overline{\alpha}\left(\varepsilon_{\phi}^{*}, \widehat{\pi}\right) < \frac{1}{2}$, as $k^{(2)} < 0$, then the second term is positive. If $k^{(3)} < 0$, then $V^{d'}\left(\varepsilon_{\phi}^{*}\right) > 0$, thus $\varepsilon_{k\phi}^{*} > \varepsilon_{\phi}^{*}$.

6.3 Proof of Lemma 9

$$\mathbb{E}\left[U^{d}\left(\varepsilon, \widetilde{M}, \widetilde{\pi}\right)\right] = \mathbb{E}\left[\alpha\left(\varepsilon, \widetilde{\pi}\right)\phi\left(\widetilde{w}_1 - C\left(\varepsilon\right)\right) + \left(1 - \alpha\left(\varepsilon, \widetilde{\pi}\right)\right)\phi\left(w_2 - C\left(\varepsilon\right)\right)\right].$$

Since $\widetilde{M}$ and $\widetilde{\pi}$ are independent, denoting $\widetilde{w}_1 = U_F + \beta u\left(H - \widetilde{M}\right)$, we have

$$\mathbb{E}\left[U^{d}\left(\varepsilon, \widetilde{M}, \widetilde{\pi}\right)\right] = \mathbb{E}\left[\alpha\left(\varepsilon, \widetilde{\pi}\right)\right]\mathbb{E}\left[\phi\left(\widetilde{w}_1 - C\left(\varepsilon\right)\right)\right] + \left(1 - \mathbb{E}\left[\alpha\left(\varepsilon, \widetilde{\pi}\right)\right]\right)\phi\left(w_2 - C\left(\varepsilon\right)\right).$$

Let us recall that for this proof

$$\varepsilon^{*} = \arg\max_{\varepsilon}\mathbb{E}\left[U^{d}\left(\varepsilon, \mu_M, \widetilde{\pi}\right)\right] \text{ and } \varepsilon^{**} = \arg\max_{\varepsilon}\mathbb{E}\left[U^{d}\left(\varepsilon, \widetilde{M}, \widetilde{\pi}\right)\right].$$

If ε^{**} is an interior solution, it solves

$$\begin{aligned} & -\mathbb{E}\left[\alpha'\left(\varepsilon^{**}, \widetilde{\pi}\right)\right]\left(\phi\left(w_2 - C\left(\varepsilon^{**}\right)\right) - \mathbb{E}\left[\phi\left(\widetilde{w}_1 - C\left(\varepsilon^{**}\right)\right)\right]\right) \\ = \; & C'\left(\varepsilon^{**}\right)\left(\mathbb{E}\left[\alpha\left(\varepsilon^{**}, \widetilde{\pi}\right)\right]\mathbb{E}\left[\phi'\left(\widetilde{w}_1 - C\left(\varepsilon^{**}\right)\right)\right] + \left(1 - \mathbb{E}\left[\alpha\left(\varepsilon^{**}, \widetilde{\pi}\right)\right]\right)\phi'\left(w_2 - C\left(\varepsilon^{**}\right)\right)\right). \quad (7) \end{aligned}$$

Furthermore, denoting $\overline{w}_1 == U_F + \beta u\left(H - \mu_M\right)$, if ε^{*} is an interior solution, it solves

$$\begin{aligned} & -\mathbb{E}\left[\alpha'\left(\varepsilon^{*}, \widetilde{\pi}\right)\right]\left(\phi\left(w_2 - C\left(\varepsilon^{*}\right)\right) - \phi\left(\overline{w}_1 - C\left(\varepsilon^{*}\right)\right)\right) \\ = \; & C'\left(\varepsilon^{*}\right)\left(\mathbb{E}\left[\alpha\left(\varepsilon^{*}, \widetilde{\pi}\right)\right]\phi'\left(\overline{w}_1 - C\left(\varepsilon^{*}\right)\right) + \left(1 - \mathbb{E}\left[\alpha\left(\varepsilon^{*}, \widetilde{\pi}\right)\right]\right)\phi'\left(w_2 - C\left(\varepsilon^{*}\right)\right)\right). \end{aligned}$$

As ϕ is concave, $\mathbb{E}\left[\phi\left(\widetilde{w}_1 - C\left(\varepsilon\right)\right)\right] \leq \phi\left(U_F - C\left(\varepsilon\right) + \beta\mathbb{E}\left[u\left(H - \widetilde{M}\right)\right]\right)$.
As u is concave and ϕ increasing, then $\phi\left(U_F - C\left(\varepsilon\right) + \beta\mathbb{E}\left[u\left(H - \widetilde{M}\right)\right]\right) \leq \phi\left(\overline{w}_1 - C\left(\varepsilon\right)\right)$.
Thus

$$\begin{aligned} & -\mathbb{E}\left[\alpha'\left(\varepsilon, \widetilde{\pi}\right)\right]\left(\phi\left(w_2 - C\left(\varepsilon\right)\right) - \mathbb{E}\left[\phi\left(\widetilde{w}_1 - C\left(\varepsilon\right)\right)\right]\right) \\ \geq \; & -\mathbb{E}\left[\alpha'\left(\varepsilon, \widetilde{\pi}\right)\right]\left(\phi\left(w_2 - C\left(\varepsilon\right)\right) - \phi\left(\overline{w}_1 - C\left(\varepsilon\right)\right)\right). \end{aligned}$$

If $\phi^{(3)} \geq 0$, $\mathbb{E}\left[\phi'\left(\widetilde{w}_1 - C\left(\varepsilon\right)\right)\right] \geq \phi'\left(U_F - C\left(\varepsilon\right) + \beta\mathbb{E}\left[u\left(H - \widetilde{M}\right)\right]\right)$.

As u is concave and ϕ' decreasing, $\phi'\left(U_F - C(\varepsilon) + \beta\mathbb{E}\left[u\left(H - \widetilde{M}\right)\right]\right) \geq \phi'(\overline{w}_1 - C(\varepsilon))$. As a consequence,

$$\begin{aligned} & \frac{\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})](\phi(w_2 - C(\varepsilon)) - \mathbb{E}[\phi(\widetilde{w}_1 - C(\varepsilon))])}{\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]\phi'(\widetilde{w}_1 - C(\varepsilon)) + (1-\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})])\phi'(w_2 - C(\varepsilon))} \\ \leq & \frac{\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})](\phi(w_2 - C(\varepsilon)) - \phi(\overline{w}_1 - C(\varepsilon)))}{\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]\phi'(\overline{w}_1 - C(\varepsilon)) + (1-\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})])\phi'(w_2 - C(\varepsilon))} \end{aligned}$$

Let us denote $\widetilde{g}(\varepsilon) = \frac{\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})](\phi(w_2 - C(\varepsilon t)) - \mathbb{E}[\phi(\widetilde{w}_1 - C(\varepsilon))])}{\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]\phi'(\widetilde{w}_1 - C(\varepsilon)) + (1-\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})])\phi'(w_2 - C(\varepsilon))}$ and $\overline{g}(\varepsilon) = \frac{\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})](\phi(w_2 - C(\varepsilon)) - \phi(\overline{w}_1 - C(\varepsilon)))}{\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]\phi'(\overline{w}_1 - C(\varepsilon)) + (1-\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})])\phi'(w_2 - C(\varepsilon))}$.
According to (7) and (8) we have $-C'(\varepsilon^{**}) = \widetilde{g}(\varepsilon^{**})$ and $-C'(\varepsilon^{*}) = \overline{g}(\varepsilon^{*})$

We can then set the following intermediate Lemma.

Lemma 13 *Under the assumption $\phi^{(3)} > 0$ and Assumption 6.3, $\widetilde{g}(\varepsilon)$ and $\overline{g}(\varepsilon)$ are increasing in ε respectively when $\varepsilon = \varepsilon^{**}$ and when $\varepsilon = \varepsilon^{*}$.*

Proof. We begin with $\widetilde{g}(\varepsilon)$. $\widetilde{g}'(\varepsilon)$ has the same sign as

$$\begin{aligned} h(\varepsilon) &= \left(\mathbb{E}\left[\alpha^{(2)}(\varepsilon,\widetilde{\pi})\right]\varphi - C'(\varepsilon)\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]\varphi_1\right)(\phi'(w_2 - C(\varepsilon)) - A\varphi_1) \\ &\quad + \left(\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]\varphi_1 + C'(\varepsilon)\left(\phi^{(2)}(w_2 - C(\varepsilon)) - \mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]\varphi_2\right)\right)(\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]\varphi), \end{aligned}$$

where

$$\begin{aligned} \varphi &= \phi(w_2 - C(\varepsilon)) - \mathbb{E}[\phi(\widetilde{w}_1 - C(\varepsilon))] > 0, \\ \varphi_1 &= \phi'(w_2 - C(\varepsilon)) - \mathbb{E}[\phi'(\widetilde{w}_1 - C(\varepsilon))] < 0, \\ \varphi_2 &= \phi^{(2)}(w_2 - C(\varepsilon)) - \mathbb{E}\left[\phi^{(2)}(\widetilde{w}_1 - C(\varepsilon))\right]. \end{aligned}$$

As $\phi^{(3)} \geq 0$, then $\varphi_2 > 0$. Some computations enable us to rewrite $h(\varepsilon)$ as

$$\begin{aligned} h(\varepsilon) &= \varphi\varphi_1\left(\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]^2 - \mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]\mathbb{E}\left[\alpha^{(2)}(\varepsilon,\widetilde{\pi})\right]\right) + \phi'(w_2 - C(\varepsilon))\varphi\mathbb{E}\left[\alpha^{(2)}(\varepsilon,\widetilde{\pi})\right] \\ &\quad + C'(\varepsilon)\left(\varphi\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]\phi^{(2)}(w_2 - C(\varepsilon)) - \mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]\varphi\varphi_2\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]\right) \\ &\quad + C'(-\phi'(w_2 - C(\varepsilon))\varphi_1\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]) \\ &\quad + \mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]\varphi_1\varphi_1\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]C'(\varepsilon). \end{aligned}$$

At the optimum ε^{**}, the last term can be replaced using $\varphi_1\mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]C'(\varepsilon) = C'(\varepsilon)\phi'(w_2 - C(\varepsilon)) + \mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]\varphi$. Thus,

$$\begin{aligned} h(\varepsilon) &= \varphi\varphi_1\left(2\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]^2 - \mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]\mathbb{E}\left[\alpha^{(2)}(\varepsilon,\widetilde{\pi})\right]\right) + \phi'(w_2 - C(\varepsilon))\varphi\mathbb{E}\left[\alpha^{(2)}(\varepsilon,\widetilde{\pi})\right] \\ &\quad + \varphi\mathbb{E}[\alpha'(\varepsilon,\widetilde{\pi})]C'(\varepsilon)\left(\phi^{(2)}(w_2 - C(\varepsilon)) - \mathbb{E}[\alpha(\varepsilon,\widetilde{\pi})]\varphi_2\right). \end{aligned}$$

Then as $2\mathbb{E}\left[\alpha'\left(\varepsilon,\widetilde{\pi}\right)\right]^2-\mathbb{E}\left[\alpha\left(\varepsilon,\widetilde{\pi}\right)\right]\mathbb{E}\left[\alpha^{(2)}\left(\varepsilon,\widetilde{\pi}\right)\right]<0$, $\widetilde{g}'\left(\varepsilon\right)$ is positive at the optimum $\varepsilon=\varepsilon^{**}$, i.e. when it crosses $-C'\left(\varepsilon\right)$.

The proof for $\overline{g}\left(\varepsilon\right)$ is similar. ■

As a result, there can only be one crossing for each function $\widetilde{g}\left(\varepsilon\right)$ and $\overline{g}\left(\varepsilon\right)$. Indeed as $-C'\left(\varepsilon\right)$ is a decreasing function, it would mean that the other crossing would have to be negative, which is impossible.

According to (8) we have $-C'\left(\varepsilon^{**}\right)=\widetilde{g}\left(\varepsilon^{**}\right)\leq\overline{g}\left(\varepsilon^{*}\right)=-C'\left(\varepsilon^{*}\right)$.

As C is convex, we have shown that $\varepsilon^{**}\geq\varepsilon^{*}$.

It is straightforward that this result holds if susceptibility is a known parameter $\widehat{\pi}$ instead of a random variable $\widetilde{\pi}$. In that case Assumption 2 is not needed, Assumption 1 is sufficient.

6.4 Proof of Lemma 10

Let us recall that for this proof

$$\varepsilon^{*}=\arg\max_{\varepsilon}\mathbb{E}\left[U^{d}\left(\varepsilon;\widetilde{M},\mu_{\pi}\right)\right]\text{ and }\varepsilon^{**}=\arg\max_{\varepsilon}\mathbb{E}\left[U^{d}\left(\varepsilon;\widetilde{M},\widetilde{\pi}\right)\right].$$

where μ_{π} is defined as $\mathbb{E}\left[\widetilde{\pi}\right]=\mu_{\pi}$.

We have

$$\mathbb{E}\left[U^{d}\left(\varepsilon,\widetilde{M},\widetilde{\pi}\right)\right]=\mathbb{E}\left[\alpha\left(\varepsilon,\widetilde{\pi}\right)\phi\left(\widetilde{w}_{1}-C\left(\varepsilon\right)\right)+\left(1-\alpha\left(\varepsilon,\widetilde{\pi}\right)\right)\phi\left(w_{2}-C\left(\varepsilon\right)\right)\right].$$

Since $\widetilde{M}$ and $\widetilde{\pi}$ are independent,

$$\mathbb{E}\left[U^{d}\left(\varepsilon,\widetilde{M},\widetilde{\pi}\right)\right]=\mathbb{E}\left[\alpha\left(\varepsilon,\widetilde{\pi}\right)\right]\mathbb{E}\left[\phi\left(\widetilde{w}_{1}-C\left(\varepsilon\right)\right)\right]+\left(1-\mathbb{E}\left[\alpha\left(\varepsilon,\widetilde{\pi}\right)\right]\right)\phi\left(w_{2}-C\left(\varepsilon\right)\right).$$

If ε^{**} is an interior solution, it solves

$$\begin{aligned}&-\mathbb{E}\left[\alpha'\left(\varepsilon^{**},\widetilde{\pi}\right)\right]\left(\phi\left(w_{2}-C\left(\varepsilon^{**}\right)\right)-\mathbb{E}\left[\phi\left(\widetilde{w}_{1}-C\left(\varepsilon^{**}\right)\right)\right]\right)\\=\;&C'\left(\varepsilon^{**}\right)\left(\mathbb{E}\left[\alpha\left(\varepsilon^{**},\widetilde{\pi}\right)\right]\mathbb{E}\left[\phi'\left(\widetilde{w}_{1}-C\left(\varepsilon^{**}\right)\right)\right]+\left(1-\mathbb{E}\left[\alpha\left(\varepsilon^{**},\widetilde{\pi}\right)\right]\right)\phi'\left(w_{2}-C\left(\varepsilon^{**}\right)\right)\right).\end{aligned}$$

Furthermore if ε_{*} is an interior solution, it solves

$$\begin{aligned}&-\alpha'\left(\varepsilon_{*},\mu_{\pi}\right)\left(\phi\left(w_{2}-C\left(\varepsilon^{**}\right)\right)-\mathbb{E}\left[\phi\left(\widetilde{w}_{1}-C\left(\varepsilon^{**}\right)\right)\right]\right)\\=\;&C'\left(\varepsilon_{*}\right)\left(\alpha\left(\varepsilon_{*},\mu_{\pi}\right)\mathbb{E}\left[\phi'\left(\widetilde{w}_{1}-C\left(\varepsilon^{**}\right)\right)\right]+\left(1-\alpha\left(\varepsilon_{*},\mu_{\pi}\right)\right)\phi'\left(w_{2}-C\left(\varepsilon^{**}\right)\right)\right).\end{aligned}$$

As $\alpha^{(2)}>0$, then $\alpha\left(\varepsilon,\mu_{\pi}\right)\leq\mathbb{E}\left[\alpha\left(\varepsilon,\widetilde{\pi}\right)\right]$.

Thus

$$\alpha\left(\varepsilon,\mu_\pi\right)\left(\mathbb{E}\left[\phi'\left(\widetilde{w}_1-C\left(\varepsilon\right)\right)\right]-\phi'\left(w_2-C\left(\varepsilon\right)\right)\right)+\phi'\left(w_2-C\left(\varepsilon\right)\right)\geq\mathbb{E}\left[\alpha'\left(\varepsilon,\widetilde{\pi}\right)\right]\left(E\left[\phi'\left(\widetilde{w}_1-C\left(\varepsilon\right)\right)\right]\right.$$
$$\left.-\phi'\left(w_2-C\left(\varepsilon\right)\right)+\phi'\left(w_2-C\left(\varepsilon\right)\right)\right).$$

Moreover, $\phi\left(w_2-C\left(\varepsilon\right)\right)-\mathbb{E}\left[\phi\left(\widetilde{w}_1-C\left(\varepsilon\right)\right)\right]>0$. Thus if $-\alpha^{(3)}\left(\varepsilon,\mu_\pi\right)>0$, then

$$-\alpha'\left(\varepsilon,\mu_\pi\right)\left(\phi\left(w_2-C\left(\varepsilon\right)\right)-\mathbb{E}\left[\phi\left(\widetilde{w}_1-C\left(\varepsilon\right)\right)\right]\right)\leq-\mathbb{E}\left[\alpha'\left(\varepsilon,\widetilde{\pi}\right)\right]\left(\phi\left(w_2-C\left(\varepsilon\right)\right)-\mathbb{E}\left[\phi\left(\widetilde{w}_1-C\left(\varepsilon\right)\right)\right]\right).$$

As a consequence,

$$\begin{aligned}&\frac{\mathbb{E}\left[\alpha'\left(\varepsilon,\widetilde{\pi}\right)\right]\left(\phi\left(w_2-C\left(\varepsilon\right)\right)-\mathbb{E}\left[\phi\left(\widetilde{w}_1-C\left(\varepsilon\right)\right)\right]\right)}{\mathbb{E}\left[\alpha'\left(\varepsilon,\widetilde{\pi}\right)\right]\left(\mathbb{E}\left[\phi'\left(\widetilde{w}_1-C\left(\varepsilon\right)\right)\right]-\phi'\left(w_2-C\left(\varepsilon\right)\right)\right)+\phi'\left(w_2-C\left(\varepsilon\right)\right)}\\ \leq\ &\frac{\alpha'\left(\varepsilon,\mu_\pi\right)\left(\phi\left(w_2-C\left(\varepsilon\right)\right)-\mathbb{E}\left[\phi\left(\widetilde{w}_1-C\left(\varepsilon\right)\right)\right]\right)}{\alpha\left(\varepsilon,\mu_\pi\right)\left(\mathbb{E}\left[\phi'\left(\widetilde{w}_1-C\left(\varepsilon\right)\right)\right]-\phi'\left(w_2-C\left(\varepsilon\right)\right)\right)+\phi'\left(w_2-C\left(\varepsilon\right)\right)}.\end{aligned}$$

The end of the proof is then similar to the end of the proof of Lemma 9.

6.5 Proof of Corollary 11

Owing to the assumption of risk-neutrality, expected utilities can be written as follows.

$$\mathbb{E}\left[U^d\left(\varepsilon,\widetilde{M},\widetilde{\pi}\right)\right]=\mathbb{E}\left[U_F-C\left(\varepsilon\right)+\beta\left(\alpha\left(\varepsilon,\widetilde{\pi}\right)\left(u\left(H-\widetilde{M}\right)-u\left(H\right)\right)+u\left(H\right)\right)\right].$$

Since $\widetilde{M}$ and $\widetilde{\pi}$ are independent we have

$$\mathbb{E}\left[U^d\left(\varepsilon,\widetilde{M},\widetilde{\pi}\right)\right]=U_F-C\left(\varepsilon\right)+\beta\left(\mathbb{E}\left[\alpha\left(\varepsilon,\widetilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H-\widetilde{M}\right)\right]-u\left(H\right)\right)+u\left(H\right)\right).$$

Thus if ε^{**} is an interior solution, it is a solution of

$$C'\left(\varepsilon^{**}\right)=\beta\mathbb{E}\left[\alpha'\left(\varepsilon^{**},\widetilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H-\widetilde{M}\right)\right]-u\left(H\right)\right).$$

Furthermore, if ε^* is an interior solution, it is a solution of the following equation.

$$C'\left(\varepsilon^*\right)=\beta\mathbb{E}\left[\alpha'\left(\varepsilon^*,\widetilde{\pi}\right)\right]\left(u\left(H-\mu_M\right)-u\left(H\right)\right).$$

As u is concave, we have $\mathbb{E}\left[u\left(H-\widetilde{M}\right)\right]\leq u\left(H-\mu_M\right)$. And since $\mathbb{E}\left[\alpha'\left(\varepsilon^*,\widetilde{\pi}\right)\right]<0$, then

$$\mathbb{E}\left[\alpha'\left(\varepsilon^*,\widetilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H-\widetilde{M}\right)\right]-u\left(H\right)\right)\geq\mathbb{E}\left[\alpha'\left(\varepsilon^*,\widetilde{\pi}\right)\right]\left(u\left(H-\mu_M\right)-u\left(H\right)\right).$$

Since $\mathbb{E}\left[\alpha^{(2)}\left(\varepsilon^*, \widetilde{\pi}\right)\right] > 0$, the functions $\varphi\left(\varepsilon, \widetilde{M}, \widetilde{\pi}\right) = \mathbb{E}\left[\alpha'\left(\varepsilon^*, \widetilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H - \widetilde{M}\right)\right] - u\left(H\right)\right)$ and $\varphi\left(\varepsilon, \mu_M, \widetilde{\pi}\right) = \mathbb{E}\left[\alpha'\left(\varepsilon^*, \widetilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H - \mu_M\right)\right] - u\left(H\right)\right)$ are decreasing in ε.

Moreover, function C being convex ($C^{(2)} > 0$), we finally get $\varepsilon^* < \varepsilon^{**}$.

6.6 Proof of Corollary 12

As in the proof in subsection 6.5, if ε^{**} is an interior solution, it is a solution of the following equation:

$$C'\left(\varepsilon^{**}\right) = \beta\mathbb{E}\left[\alpha'\left(\varepsilon^{**}, \widetilde{\pi}\right)\right]\left(\mathbb{E}\left[u\left(H - \widetilde{M}\right)\right] - u\left(H\right)\right).$$

Furthermore, if ε^* is an interior solution, it is a solution of

$$C'\left(\varepsilon^*\right) = \beta\alpha'\left(\varepsilon^*, \mu_\pi\right)\left(\mathbb{E}\left[u\left(H - \widetilde{M}\right)\right] - u\left(H\right)\right).$$

Assuming $\alpha^{(3)} < 0$

$$\mathbb{E}\left[\alpha'\left(\varepsilon, \widetilde{\pi}\right)\right] \leq \alpha'\left(\varepsilon, \mu_\pi\right),$$

which yields

$$\mathbb{E}\left[\alpha'\left(\varepsilon, \widetilde{\pi}\right)\right]\left(E\left[u\left(H - \widetilde{M}\right)\right] - u\left(H\right)\right) \geq \alpha'\left(\varepsilon, \mu_\pi\right)\left(\mathbb{E}\left[u\left(H - \widetilde{M}\right)\right] - u\left(H\right)\right).$$

As C' is increasing in ε we finally get $\varepsilon^{**} > \varepsilon^*$.

Conversely, if $\alpha^{(3)} > 0$ then $\varepsilon^{**} < \varepsilon^*$.

It is straightforward that this result holds if severity is a known parameter $\widehat{M}$ instead of a random variable $\widetilde{M}$. In that case Assumption 2 is not needed, Assumption 1 is sufficient.