

Section 2.4

3. (a) $(\exists x \in \mathbb{Q})(x > \sqrt{2})$

Solution:

- In English, this statement says, "There exists a rational number x such that $x > \sqrt{2}$."
- The negation is $(\forall x \in \mathbb{Q})(x \leq \sqrt{2})$.
- In English, the negation says, "For all rational numbers x , $x \leq \sqrt{2}$."

(c) $(\forall x \in \mathbb{Z})(x \text{ is even or } x \text{ is odd})$

Solution:

- In English, this statement says, "For all integers x , x is even or odd."
- The negation is $(\exists x \in \mathbb{Z})(x \text{ is not even and } x \text{ is not odd})$
- In English, the negation says, "There exists an integer x , such that x is not even and not odd."

(e) $(\forall x \in \mathbb{Z})(\text{If } x^2 \text{ is odd, then } x \text{ is odd})$

Solution:

- In English, this statement says, "For all integers x , if x^2 is odd, then x is odd."
- The negation is $(\exists x \in \mathbb{Z})(x^2 \text{ is odd and } x \text{ is even})$.
- In English, the negation says, "There exists an integer x , such that x^2 is odd and x is even."

4. (e) $(\exists n \in \mathbb{Z})(\forall m \in \mathbb{Z})(m^2 > n)$

Solution:

- $(\forall n \in \mathbb{Z})(\exists m \in \mathbb{Z})(m^2 \leq n)$
- For all integers n , there exists an integer m , such that $m^2 \leq n$.

11. (a) A function f is continuous at the real number a provided that ...

Solution:

- $(\forall \epsilon > 0)(\exists \delta > 0)(\text{If } |x - a| < \delta, \text{ then } |f(x) - f(a)| < \epsilon)$

(b) A function f is not continuous at the real number a provided that ...

Solution:

- $(\exists \epsilon > 0)(\forall \delta > 0)(|x - a| < \delta \text{ and } |f(x) - f(a)| \geq \epsilon)$

(c) In English, A function f is not continuous at the real number a provided that . . .

Solution:

- There exists an $\epsilon > 0$, for all $\delta > 0$, such that $|x - a| < \delta$ and $|f(x) - f(a)| \geq \epsilon$.