

1 Portfolio Problem 1 - Divisibility by 4

1.1 Examples and Definitions

Problem to prove: Construct both a know-show table and a first attempt at a formal proof of the following biconditional statement.

Proposition. For each integer n , n is even if and only if 4 divides n^2 .

Exploratory work/Examples:

- Find four even integers a . At least one integer must be negative. Square each integer a and determine if 4 divides a^2 .
 - If we set $a = -2$, then $(-2)^2 = 4$, and $4 \mid 4$.
 - If we set $a = 0$, then $0^2 = 0$, and $4 \mid 0$.
 - If we set $a = 2$, then $2^2 = 4$, and $4 \mid 4$.
 - If we set $a = 4$, then $4^2 = 16$, and $4 \mid 16$.
- Find four perfect squares, b , such that $4 \mid b$. Then find the integer c such that $b = c^2$. Determine if c is even or odd.
 - If we set $b = 0$, then $4 \mid 0$. Using $b = c^2$, we get $0 = 0^2$, so $c = 0$, which is even.
 - If we set $b = 4$, then $4 \mid 4$. Using $b = c^2$, we get $4 = 2^2$, so $c = 2$, which is even.
 - If we set $b = 16$, then $4 \mid 16$. Using $b = c^2$, we get $16 = 4^2$, so $c = 4$, which is even.
 - If we set $b = 144$, then $4 \mid 144$. Using $b = c^2$, we get $144 = 12^2$, so $c = 12$, which is even.

1.2 Know-Show Table: If n is even, then $4 \mid n^2$ (Forward)

Step	Know	Reason
P1	n is even	Hypothesis
P2	$\exists k \in \mathbb{Z}$ s.t. $n = 2k$	Definition of even
P3	$n^2 = (2k)^2 = 4k^2$	Substitution
P4	$4k^2 = 2(2k^2)$	Factoring out 2
P5	$2k^2 \in \mathbb{Z}$	\mathbb{Z} closed under multiplication
P6	$n^2 = 2q$ where $q \in \mathbb{Z}$	Set $q = k^2$
Q1	4 divides n^2	Defn divides
Step	Show	Reason

1.3 Know-Show Table: If $4 \mid n^2$, then n is even (Reverse)

Step	Know	Reason
P1	$4 \mid n^2$	Hypothesis
P2	$n^2 = 4m$ for some $m \in \mathbb{Z}$	Definition of divisibility
P3	$n^2 \equiv 0 \pmod{4}$	Definition of congruence
P4	Assume n is odd	For contradiction
P5	$n = 2k + 1$ for some $k \in \mathbb{Z}$	Definition of odd
P6	$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1$	Expand and simplify n^2
P7	$n^2 = 4(k^2 + k) + 1$	Factor out 4
P8	$k^2 + k \in \mathbb{Z}$	\mathbb{Z} closed under multiplication
P9	$n^2 \equiv 1 \pmod{4}$	$4(k^2 + k) \equiv 0 \pmod{4}$
P10	Contradicts $n^2 \equiv 0 \pmod{4}$	From P2, since $n^2 = 4m$
Q1	n is even	Definition of even
Step	Show	Reason

1.4 First Draft

Theorem. For each integer n , n is even if and only if 4 divides n^2 .

Proof. Proofy McProoferson goes here. □

References

[1] Sundstrom, T., *Mathematical Reasoning: Writing and Proof*, Version 3, Creative Commons, 2020.

[2] Include any other references: images from the internet, definition or theorem from a book other than our class text, etc. Change the name in curly brackets to an appropriate reference name. If you have no other references, delete this item.