Math 3100 Lab 5

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1 Problem 1

1.1 Introductory Material

Definition (Even)**.** An integer a is an **even integer** provided that there exists an integer n such that $a = 2n$. [1, pg 16]

Definition (Odd)**.** An integer a is an **odd integer** provided there exists an integer n such that $a = 2n + 1$. [1, pg 16]

1.2 Final Draft

Theorem. *Let x and y be integers. If x is even and y is odd, then xy is even.*

Proof. First, we assume x is an even integer and y is an odd integer, and we will prove that the product xy is an even integer. By the definition of even, there exists an integer k such that $x = 2k$. And, by the definition of odd, there exists an integer j such that

$y = 2j + 1$. Then, substituting for x and y we get,

$$\begin{aligned} xy &= (2k)(2j+1) \\ &= 2k(2j+1) \\ &= 2(k(2j+1)) \\ &= 2q. \end{aligned}$$

Since integers are closed under multiplication and addition, $k(2j+1)$ is an integer. Since $xy = 2q$ for some integer q, then xy is an even integer. Therefore, we have proven that assuming x is even and y is odd, then the product xy is even. $\square$

References

[1] Sundstrom, T., *Mathematical Reasoning: Writing and Proof*, Version 3, Creative Commons, 2020.

[2] The work on this problem is the result of a class collaboration in which I was an active participant.