

Theorem. $(P \vee Q) \rightarrow R \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$

Solution:

Proof. Let P, Q , and R be statements. We begin with the left side and work towards the right side:

$$\begin{aligned}(P \vee Q) \rightarrow R &\equiv \neg(P \vee Q) \vee R && \text{[Conditional as Disjunction]} \\ &\equiv (\neg P \wedge \neg Q) \vee R && \text{[De Morgan's Law]} \\ &\equiv (\neg P \vee R) \wedge (\neg Q \vee R) && \text{[Distribution]} \\ &\equiv (P \rightarrow R) \wedge (Q \rightarrow R) && \text{[Conditional as Disjunction]}\end{aligned}$$

Therefore, $(P \vee Q) \rightarrow R \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$. □

Reflection:

- I worked from the left side and found the right.
- Remembering the problem we did in-class, the left side jumped out instantly at me.
- I did not get stuck, it just takes some time.
- I did not use Dr. Johnson's solution or help from any other source other than The Summary of Useful Logical Equivalences and my notes; I did however use ChatGPT to help clean up my Latex code.