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LaTeX Lab Assignment 1 09-09-2024

1. Which of the following sentences are statements? (Complete each justification.)

   (a) $(a+b)^2 = a^2 + b^2$

   Solution:
   This sentence is a not a statement because a and b are ambiguous; there is not enough information.

   (b) There exists a real number x such that $x^2 + 1 = 0$.

   Solution:
   This sentence is a statement because x is not ambiguous, the statement can be true or false. In this case, the statement is false because any number squared is positive, and adding 1 to any positive number will never add up to 0.

2. Identify the hypothesis and the conclusion of the conditional statement:

   If the numbers m and n are odd integers, then the product mn is an odd integer.

   Solution:
   The hypothesis is: "m and n are odd integers"

   The conclusion is: "the product mn is an odd integer"

3. **Problem 1.1.1(k)** Complete the exploration for the following conjecture:

   Conjecture. *Every even natural number greater than or equal to 4 is the sum of two prime numbers.*

   (a) The sentence above is a statement because...

   Solution:
   It's a declarative statement that can be true or false.

   (b) Let's explore to make a conjecture about the truth value of the statement. We will see if we can write the first few even integers as a sum of two primes. Continue the list for the even integers up to 20. (Note: the first few lines demonstrate three different ways to display equations. You should practice using each method)

   Solution:

$$4 = 2 + 2$$

$$6 = 3 + 3$$

$$8 = 3 + 5$$

$$10 = 3 + 7$$

$$12 = 5 + 7$$

$$14 = 3 + 11$$

$$16 = 11 + 5$$

$$18 = 7 + 11$$

$$20 = 7 + 13$$

(c) Choose your own even integer greater than 60 and try to write it as the sum of two primes.

Solution:

$$67 + 3 = 70$$

$$7 + 8,999,993 = 9,000,000$$

```python
# I couldn't help myself and wrote a basic Python program to calculate 9,000,000 systematically. This green is super hideous though.
from sympy import isprime

target_sum = 9000000

for p1 in range(2, target_sum):
if isprime(p1):
    p2 = target_sum - p1
    if isprime(p2):
        print(f"The two primes that add up to {target_sum} are {p1} and {p2}.")
        break
```

(d) Based on your explorations, do you think the conjecture above is true or false?

Solution:

Based on my explorations, I believe the conjecture is true.

(e) This conjecture has a name. Use the internet to find it's name and learn a little about the conjecture. Write at least two sentences about what you learned and create a footnote to cite your source.

Solution:

The conjecture is called Goldbach's Conjecture, and it is one of the oldest and best known problems in all of mathematics. While it has been shown to hold true for really big numbers (up to 4×10^{18}), it still remains unproven to this day.[1].

[1] https://en.wikipedia.org/wiki/Goldbach's_conjecture