



LUND
UNIVERSITY

**Written Examination
Discrete Mathematics
Wednesday, January 11, 2012**

Centre for Mathematical Sciences
Mathematics, Faculty of Science

No books, notes, computational devices, etc. are allowed. Use only paper supplied by the department. Use clear handwriting and give clear careful motivations. Fill in the form completely and write your name on each sheet of paper.

1. Solve the recurrence relation

$$a_{n+2} = -4a_n + 8n2^n$$

where $n \geq 0$ and $a_0 = 1, a_1 = 2$.

2. Determine the number of integer solutions to the equation

$$x_1 + x_2 + x_3 + x_4 = 15,$$

where $2 \leq x_i \leq 10$ for $1 \leq i \leq 4$.

3. Find a simultaneous solution for the system of three congruences:

$$\begin{aligned}x &\equiv 2 \pmod{5} \\x &\equiv 5 \pmod{6} \\x &\equiv 1 \pmod{7}.\end{aligned}$$

4. a) How many arrangements are there of all the letters in PEPPARKAKOR?
b) In how many of the arrangements in part (a) all vowels appear consecutively?
c) In how many of the arrangements in part (a) the words PARK or ORK do not appear?

5. Prove that the system of congruences

$$\begin{aligned}x &\equiv 1 \pmod{4} \\3x &\equiv 7 \pmod{9} \\2x &\equiv 10 \pmod{25}\end{aligned}$$

has no solution. Does this example contradict Chinese Remainder Theorem?

6. Consider the binary linear $(6, 3)$ code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

What can be said about the error-correction capability of the encoding function defined by G ? Can the received words 111111 or 101110 be properly decoded?