LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2013-04-02 kl 08-13

Students may use the formula sheet and a pocket calculator. All solutions should be properly justified.

1. Consider the function $f(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3)^2$.

a) Write the function as $f(x) = x^T Hx$ with a symmetric matrix H.
Is the matrix H positive/negative definite/semidefinite? (0.2)

b) Is the function $g(x) = \ln(1 + \exp(x^T Hx))$ convex on $\mathbf{R}^3$? (0.4)

c) Is the set $S = \{x \in \mathbf{R}^3 \colon x^T Hx \geq 1\}$ convex? (0.4)

2. a) Consider the matrix

$$H = \begin{pmatrix} 4 & 2 & 2 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}.$$

Show that the vectors $d_1 = (1, 1, 1)^T$ and $d_2 = (1, -1, -1)^T$ are conjugate with respect to H. Find a vector d_3 that is H-conjugate to d_1 and d_2. (0.5)

b) Draw the set $S = \{(x_1, x_2) \in \mathbf{R}^2 \colon x_1 + x_2 \geq 2,\ x_2 \geq 1\}$ and the set of all possible gradients $\nabla f(1, 1)$ that satisfies the necessary condition for the point $(1, 1)$ to be a local minimum in the problem $\min f(x)$ subject to $x \in S$. (0.5)

3. Consider the LP problem

$$\min\ 2x_1 + 5x_2 \qquad \text{subject to} \qquad \begin{cases} 2x_1 + x_2 & \geq & 4, \\ x_1 + 2x_2 & \geq & 5, \\ x_1 + 3x_2 & \geq & 6, \\ x_1 \geq 0, & & \\ x_2 \text{ free.} & & \end{cases}$$

a) Draw the constraint set in the plane and solve the problem graphically. (0.3)

b) State the dual problem and solve it by CSP. (0.4)

c) State the canonical form of the primal and write the simplex table. Pick a basic feasible solution of your choice from the picture in a) and carry out the initialization of the simplex table with this choice of basis. (0.3)

Please, turn over

4. Let $f(x) = \ln(1 + \|x\|^2)$.

a) Give the definition of a convex function. (0.2)

b) Is the function f convex on $\mathbf{R}^n$? (0.4)

c) Is the set $\{x \in \mathbf{R}^n \colon f(x) \le \alpha\}$ convex for $\alpha \in \mathbf{R}$? (0.4)

5. Solve the following problem by KKT method

$$\min\,(x+1)^2 + y^2 + z^2 \qquad \text{subject to} \qquad x^2 + y^2 \le 1,\ y + z = 6,\ x \ge 0.$$

6. a) Solve the problem in 5 by the duality method using the set $X = \{(x, y, z) \in \mathbf{R}^3 \colon x \ge 0\}$. (0.5)

b) State and prove the theorem that says that for a convex optimization problem any KKT point is the global minimizer. (0.5)

GOOD LUCK!