**LUNDS TEKNISKA HÖGSKOLA**
**MATEMATIK**

**TENTAMENSSKRIVNING**
**OPTIMERING**
**2012-08-31 kl 08-13**

*Students may use the formula sheet and a pocket calculator. All solutions should be properly justified.*

**1.** Which of the following statements are correct? Provide a short explanation.

a) The Dichotomous Search method always converges. (0.2)

b) The Steepest Descent method is faster than the Newton method. (0.2)

c) The modified Newton method does not use second derivatives of the function. (0.2)

d) For convex functions a global minimum always exists. (0.2)

e) There is no duality gap in LP problems. (0.2)

**2.** Consider the matrix

$$H = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

a) Is the matrix positive (negative) definite or semidefinite? (0.4)

b) Is the function $f(x) = \exp(x^T H x)$ convex on $\mathbf{R}^3$? (0.4)

c) State the Sylvester criterion. What does it tell us about $H$? (0.2)

**3.** a) Use the Phase 1 method for the LP set

$$\begin{cases} 2x_1 - 5x_2 - 3x_3 - 6x_4 & = & 2, \\ \qquad 2x_2 + \ x_3 + \ x_4 & = & 1, \\ 2x_1 + 2x_2 - 2x_3 - 4x_4 & = & 1, \\ \qquad\qquad \text{all } x_k & \geq & 0 \end{cases}$$

and find a feasible point in case the set is not empty. (0.5)

b) Does the vector $b = \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$ belong to the convex hull of the columns of the matrix $A = \begin{pmatrix} 2 & -5 & -3 & -6 \\ 0 & 2 & 1 & 1 \\ 2 & 2 & -2 & -4 \end{pmatrix}$? (0.5)

**Please, turn over**

**4.** a) Prove that if $f$ is a convex function on $S$ then for any constant $\alpha \in \mathbf{R}$ the set $M = \{x \in S\colon f(x) \leq \alpha\}$ is convex. (0.3)

b) Provide an example of a non-convex function $f(x, y)$ such that the set $M = \{(x, y) \in \mathbf{R}^2\colon f(x, y) \leq 1\}$ is nevertheless convex. (0.2)

c) Is the set $\{(x, y) \in \mathbf{R}^2\colon x^4 + 2x^2y^2 + y^4 \leq 1\}$ convex? (0.5)

**5.** Consider the problem

$$\min\,(x^2 + 2(y + 3)^2) \qquad \text{subject to} \quad 2xy + 2 \leq 0,\ y \geq 1.$$

Calculate the dual function if we treat the second constraint implicitly (that is, for $X = \{(x, y)\colon y \geq 1\}$), solve the dual problem and then solve the primal problem by showing that there is no duality gap.

**6.** Prove the existence and uniqueness of a global minimum for a quadratic polynomial with a positive definite Hessian.

# GOOD LUCK!