LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2014-08-28 kl 14-19

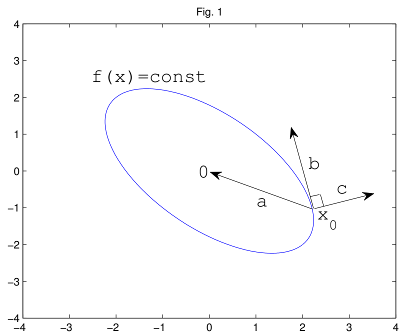
Students may use the formula sheet and a pocket calculator. All solutions should be properly justified.

1. a) Find out whether the following system of inequalities has a solution (0.5)

$$\begin{cases} x_1 - 3x_2 + 2x_3 & \leq & 0, \\ x_1 - 2x_2 + 4x_3 & \leq & 0, \\ x_1 + 3x_2 - 3x_3 & \leq & 0, \\ 6x_1 + x_2 - x_3 & > & 0. \end{cases}$$

b) Let f be a differentiable function on $\mathbf{R}^n$ and $x, d \in \mathbf{R}^n$ be given vectors. Prove that if $\lambda_0 \in \mathbf{R}$ is a stationary point of $F(\lambda) = f(x + \lambda d)$ then the gradient $\nabla f(x + \lambda_0 d)$ is orthogonal to the vector d. (0.5)

2. a) Let H be a positive definite 2×2 real matrix and $f(x) = x^T H x$. Which of the following three vectors **a**, **b** and **c** at the point x_0 on the level curve (see Fig. 1) could be search directions for



- the Steepest Descent method?
- Newton's method?
- the Conjugate Direction method?

(0.6)

b) Which two of the vectors **a**, **b** and **c** are H-conjugate? (0.4)

3. a) Make a proper guess of a basic feasible solution to the LP problem

$$\min(3x_1 + 2x_2 - x_3) \quad \text{subject to} \quad \begin{cases} 2x_1 + 3x_2 & & \leq & 5, \\ x_1 + 2x_2 - & x_3 & \geq & 1, \\ 2x_1 + 5x_2 - & 3x_3 & = & 2, \\ & \text{all } x_k & \geq & 0. \end{cases}$$

and solve it by the Simplex method (without Phase I). (0.5)

b) State the dual problem and solve it by the CSP[1]. (0.5)

Please, turn over

[1]Complementary Slackness Principle

4. a) Is the set $\{(x, y, z) \colon |x^2 + y^2 - z| \leq 1\}$ convex? (0.3)

b) Is the set $\{(x, y, z) \colon \max\{x^2 + y^2 - z, x^2 + y^2 + z\} \leq 1\}$ convex? (0.3)

c) For what values of $\alpha \in \mathbf{R}$ is the function $f(x, y) = x^\alpha y$ convex on $\{(x, y) \colon x > 0, y > 0\}$? (0.4)

5. Solve the following problem by KKT method

$$\min 2xy + 2yz + 2zx \qquad \text{subject to} \quad x^2 + y^2 \leq 2,\ 2x + 2y + z = 0.$$

6. a) Calculate the dual function for the Problem 5 using the set $X = \mathbf{R}^3$, solve the dual problem and explain how to solve the Probem 5 by duality. (0.6)

b) Let A be a $m \times n$ matrix with rank n and $b \in \mathbf{R}^m$. Prove that the minimum for $f(x) = \|Ax - b\|$ is given by the normal equation. (0.4)

GOOD LUCK!