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OPTIMERING
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Answers and Comments[1]

1. a) True. The Dichotomous method always converges (the uncertainty interval shrinks on each step almost by half).

b) False. SD may exhibit zigzagging (in fact, Newton is faster when converges).

c) False. The Hessian is used.

d) False (for example, $f(x) = x$).

e) True (see the book, Th. 7, p. 184).

2. a) Positive semidefinite (by definition), since $x^T Hx = (x_1 + x_2)^2 + x_3^2$.

b) Convex as $x^T Hx$ is convex (Hessian H is positive semidefinite) and f is a superposition of convex and growing exp and convex $x^T Hx$ functions.

c) Sylvester criterion fails (cannot say anything) since $\det \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 0$.

3. a) The set is empty. Simplex method for the auxiliary problem in Phase 1 ends with $\min = 2 \neq 0$.

b) No. In $A\lambda = b$ (with $\lambda \geq 0$) the second equation implies that $\lambda_2 = \lambda_3 = \lambda_4 = 0$, so $\lambda_1 = 1$. This λ fails to satisfy the other equations.

4. a) See the book, Th. 7, p. 210.

b) For example, $f(x, y) = -x^2$.

c) Convex. $x^4 + 2x^2y^2 + y^4 = (x^2 + y^2)^2 \leq 1 \Leftrightarrow x^2 + y^2 \leq 1$ (convex by 4a).

5. The dual function (minimum attains at $x = -uy$, $y = 1$)

$$\Theta(u) = \begin{cases} 32 + 2u - u^2 & \text{if } 0 \leq u \leq \sqrt{2}, \\ -\infty & \text{if } u > \sqrt{2}. \end{cases}$$

The dual problem: $\bar{u} = 1$, $\Theta(1) = 33$. The corresponding $\bar{x} = -1$, $\bar{y} = 1$ and $f(-1, 1) = \Theta(1)$, hence no duality gap.

6. See the book, Ex. 7 and Remark after it, p. 16.

[1] For re-exams only answers are provided.