LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2012-04-10 kl 08-13

Students may use the formulae sheet and a pocket calculator. All solutions should be properly justified.

1. Which of the following statements are correct? Provide a proper explanation.

a) The Newton method performs badly for any function with non-circular level curves. (0.2)

b) The Golden section method may diverge if the function is not unimodal. (0.2)

c) The Modified Newton method gives descent directions. (0.2)

d) The Conjugate Direction method converges in a finite number of steps (that equals to the number of variables). (0.2)

e) Quasi-Newton methods do not use the second derivative of the function. (0.2)

2. Consider the function $f(x, y) = (x^2 + y^2 - \frac{3}{2})e^{x+y}$.

a) Show that the global minimum in $\mathbf{R}^2$ exists and find the minimum point. (0.4)

b) Calculate several Newton steps starting from $(0, 0)$ to ascertain whether the method converges and, if it does, to what point. (0.3)

c) Make a modification of the Newton method in order to get the descent direction in the first step. (0.3)

3. a) Solve the LP problem below by the Simplex method

$$\max\,(x_2 + 2x_3) \quad \text{subject to} \quad \left\{ \begin{array}{rcl} x_2 + x_3 & \leq & 2, \\ -x_1 + 3x_2 - 2x_3 & \leq & 2, \\ -x_1 - x_2 + x_3 & = & 1, \\ x_1,\ x_2,\ x_3 & \geq & 0. \end{array} \right.$$

(0.5)

b) State the dual LP problem and solve it by the Complementary Slackness Principle. Is the optimal solution to the dual problem unique? (0.5)

Please turn over

4. a) Prove that if the function $f(x)$ is convex and increasing and if the function $g(x)$ is convex then the superposition $f(g(x))$ is convex. (0.2)

b) Prove or disprove convexity of the following functions

i. $f(x,y) = (x^2 + y^2)\ln(1 + x^2 + y^2)$, (0.2)

ii. $f(x,y) = x^2y^2\ln(1 + x^2y^2)$. (0.2)

c) Let A be an $n \times n$ matrix. Is the set

$$S = \{y \in \mathbf{R}^n \colon y = Ax,\ x \in \mathbf{R}^n,\ 0 \le x_k \le 1,\ k = 1 \dots n\}$$

convex? (0.4)

5. a) State the theorem on the necessary condition for a minimum in case of inequality constraints only

$$\min f(x) \quad \text{subject to} \quad x \in X,\ g_i(x) \le 0,\ i = 1, \dots, m.$$

(0.3)

b) Solve the problem

$$\min(x^2 + 2y^2 + z^2) \quad \text{subject to} \quad 1 + x^2 + y^2 \le z^2,\ z \ge 0.$$

(0.7)

6. a) Taking $X = \{z \ge 0\}$, calculate the dual function Θ and solve the dual problem to **5**b). Is there a duality gap here? (0.5)

b) Prove Farkas' theorem. (The theorem on separation hyperplanes may be used without proof.) (0.5)

Good luck!