Centre for Mathematical Sciences

Mathematics, Faculty of Science

No books, notes, computational devices, etc. are allowed. Use only paper supplied by the department. Use clear handwriting and give clear careful motivations. Fill in the form completely and write your name on each sheet of paper.

The oral exam will take place the days 1st, 2nd and 3rd of September. If you hadn't done before, please indicate on the answer sheet your preferred time. A list will be published on the department web page with the exact time for each student.

1. Determine the number of positive integers n smaller or equal than 3500 which are relatively prime with 3500. (Hint: you may decompose 3500 in its prime factors and then use the principle of inclusion/exclusion).

2. If we play with a dice shaker containing 5 dices, how many times do we need to throw them in order to ensure that we will obtain twice the same sum?

3. Solve the recursion equation

$$a_n = 2^{n-1} - a_{n-1} + 2a_{n-2}$$

4. Determine the number of integer solutions to the equation

$$x_1 + x_2 + x_3 + x_4 = 29$$

   under the constraints x_1 odd such that $x_1 \le 9$, $5 \le x_2 \le 10$, x_3 even such that $10 \le x_3$ and $10 \le x_4 \le 15$.

5. Solve the system of congruences

$$x \equiv -1 \mod 9$$
$$x \equiv 0 \mod 11$$
$$x \equiv 1 \mod 13$$

   by finding all its integer solutions x.

6. Consider the binary linear $(5, 3)$ code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

   What can be said about the error-correction capability of the encoding function defined by G? Whenever possible, decode the following received words: 11111 and 10101. Justify your answers.