LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2015-01-12 kl 08-13

Students may use the formulae sheet and a pocket calculator. All solutions should be properly justified.

1. Which of the following statements are correct? Provide a short explanation.

a) For the Newton method to converge in one step when applied to $f(x) = x^T Hx + c^T x + d$ the matrix H must be positive-definite. (0.2)

b) Approximation points from the penalty function method are not feasible. (0.2)

c) The Dichotomous search method always converges to the global minimum for a unimodal function. (0.2)

d) The global minimum always exists for a convex function. (0.2)

e) The Steepest Descent method converges fast for quadratic functions. (0.2)

2. Consider the function $f(x, y) = x^4 - 12xy + y^4$ on $S = \{(x, y) \colon x \geq 0,\ y \geq 0\}$.

a) Find the largest possible (convex) set $D_{max} \subset S$ such that $f(x, y)$ is convex on D_{max}. (0.5)

b) Show that the minimum of f over D_{max} exists and calculate it. (0.5)

3. Consider the LP problem

$$\max\,(4x_1 + 2x_2 + 3x_3) \qquad \text{subject to} \qquad \begin{cases} 5x_1 + x_2 + 4x_3 & \leq & 2, \\ x_1 + 2x_2 - x_3 & \leq & 6, \\ 2x_1 + \ \ x_2 + x_3 & \geq & 1, \\ x_1, x_2, x_3 & \geq & 0. \end{cases}$$

a) Guess a starting point[1] and solve the problem by the Simplex method. (0.5)

b) State the dual problem and solve it by the CSP[2] using the solution in 3a). (0.5)

Please, turn over

[1] without Phase 1

[2] Complementary Slackness Principle

4. a) Prove that an affine function $h\colon \mathbf{R}^n \to \mathbf{R}$ is convex. (0.2)

b) Which of the following sets are convex? (Prove or disprove.) (0.4)

- $\{(x, y, z)\colon x \geq y^2 + z^2,\ z > 0\}$,
- $\{(x, y, z)\colon x^2 \geq y^2 + z^2,\ y > 0\}$,
- $\{(x, y, z)\colon x^2 \geq y^2 + z^2,\ x > 0\}$.

c) Prove that the following function is convex in $\mathbf{R}^3$ (0.4)

$$f(x, y, z) = \sqrt{1 + x^2 + y^2 + z^2}.$$

5. Solve the optimization problem below using KKT conditions

$$\min\,(3x + 4y) \qquad \text{subject to} \qquad 2y \leq x^2 + y^2 \leq 4,\ x \geq 0.$$

6. a) Consider the problem in 5 and set $X = \{(x, y)\colon x \geq 0\}$. Calculate the dual function and make sure that there is no duality gap here. (You may use some calculations from Problem 5). (0.6)

b) In the course, a sufficient condition for a KKT point to be a global minimizer of a constrained problem is given. State the condition as a theorem and prove it.(0.4)

GOOD LUCK!