LUNDS TEKNISKA HÖGSKOLA
MATEMATISKA INSTITUTIONEN

LÖSNINGAR
OPTIMERING
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Answers and Comments[1]

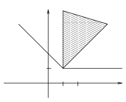
1.

a) $H = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, pos.-semidef. by definition.

b) Yes as a superposition of convex and increasing $\ln(1+\exp(t))$ and convex $f(x)$.

c) No, for example $(1,0,0)^T$ and $(-1,0,0)^T$ belong to the set, but not the middle of the interval (the origin).

2. a) For example, $d_3 = (0,1,-1)^T$ (there are many possibilities).

b) 

3. a) Min $= 11$ at $x = (3,\ 1)$.

b) The dual problem is

$$\max\,(4y_1 + 5y_2 + 6y_3) \quad \text{subject to} \quad \begin{cases} 2y_1 + y_2 + y_3 & \leq \ 2, \\ y_1 + 2y_2 + 3y_3 & = \ 5, \\ \text{all } y_k & \geq \ 0. \end{cases}$$

Max $= 11$ at $y = (0,\ 1,\ 1)$ (CSP: $y_1 = 0$, equality in the first).

c) Introduce $x_1 = z_1$, $x_2 = z_2 - z_3$, all $z_k \geq 0$, and pick the extreme point $(0,4)$

	*			*	*	
-2	-5	5	0	0	0	0
2	1	-1	-1	0	0	4
1	2	-2	0	-1	0	5
1	3	-3	0	0	-1	6

$\Rightarrow$

8	0	0	-5	0	0	20
2	1	-1	-1	0	0	4
3	0	0	-2	1	0	3
5	0	0	-3	0	1	6

4. a) See the book.

b) Not convex (draw $y = \ln(1+r^2)$ in (r,y)-plane to see, alt. calculate the Hessian).

c) Convex as equivalent to $\{x; \|x\| \leq \sqrt{\exp(\alpha) - 1}\}$ and the norm is convex.

5. A KKT point is $(0,1,5)$. The problem is convex, thus KKT is the global minimum. Min $= 27$.

6. a) The dual finction is $\Theta(u,v) = 1 - u - 6v - \frac{v^2(u+2)}{4(u+1)}$. $\max_{u\geq 0, v} \Theta(u,v) = 27$ at $u = 4$, $v = -10$. Since $\Theta(4,-10) = f(0,1,5)$ there is no duality gap.

b) See the book.

[1]For re-exams only answers are provided.