LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2013-08-30 kl 08-13

Students may use the formulae sheet and a pocket calculator. All solutions should be properly justified.

1. Decide whether the statement is correct or not and provide a short explanation.

a) The Golden Section method always converges to a local minimum on the initial interval. (0.2)

b) The Newton method may diverge even for a quadratic function. (0.2)

c) Two successive directions in the Steepest Descent method are orthogonal. (0.2)

d) The exact line search gives a point at which the search direction is tangent to the level set of the minimized function. (0.2)

e) If a is a global minimum for a constrained convex problem then the gradient at a of the minimized function is zero. (0.2)

2. a) Is the matrix

$$H = \begin{pmatrix} 1 & 1 & -2 \\ 1 & 3 & -4 \\ -2 & -4 & 9 \end{pmatrix}$$

positive-definite? (0.2)

b) Is the function $f(x,y) = x^4 + 2x^2y^2 + y^4$ convex on $\mathbf{R}^2$? (0.4)

c) Prove that the function $f(x,y) = e^{x^2+y^2} + \ln(e^{x+y}+1)$ is convex on $\mathbf{R}^2$. (0.4)

3. Let the set $S \subset \mathbf{R}^2$ be described by inequalities

$$S: \quad \begin{cases} x_1 - x_2 \geq 0, \\ -x_1 + 2x_2 \geq 2, \\ x_2 \leq 4, \\ x_1, x_2 \geq 0. \end{cases}$$

a) Convert the set of inequalities into the canonical form and use Simplex Phase 1 to find a feasible point. (0.5)

b) Using the LP duality and the CSP[1], show that $(4,4)$ is the optimal point for the problem: $\min(x_1 - 3x_2)$ subject to $(x_1, x_2) \in S$. (0.5)

Please, turn over

[1] Complementary Slackness Principle

4. a) Give the definition of the convex hull of a set $S \subset \mathbf{R}^n$. (0.2)

b) Solve graphically the optimization problem

$$\min ax - y \qquad \text{subject to } y \leq x^2,\ 0 \leq x \leq 1,\ y \geq 0$$

for *all* possible values of the real parameter a. (0.3)

c) Let $S \subset \mathbf{R}^n$, $c \in \mathbf{R}^n$ and let $f(x) = c^T x$ be a linear function. Denote by $H(S)$ the convex hull of the set S. Prove that

$$\min_{x \in S} f(x) = \min_{x \in H(S)} f(x).$$

(Hint: Prove $\geq$ and $\leq$ separately.) (0.5)

5. Consider the optimisation problem

$$\min\,(y^2 - 3x) \qquad \text{subject to} \qquad y \geq x^3,\ y \geq 0.$$

a) Solve the problem by the KKT-CQ necessary condition. (0.6)

b) Replace the condition $y \geq x^3$ with the equivalent $\sqrt[3]{y} \geq x$ and explain how the problem can be solved *without* knowing that the minimum exists. (0.4)

6. a) Consider the same problem as in 5b), that is

$$\min\,(y^2 - 3x) \qquad \text{subject to} \qquad \sqrt[3]{y} \geq x,\ y \geq 0.$$

Define $X = \{(x, y) \mid y \geq 0\}$. Calculate the dual function, solve the dual problem and check the duality gap. (0.5)

b) Let the Conjugate Direction method in combination with an exact line search be applied to a quadratic polynomial with a positive definite Hessian. Prove that the multidimensional search ends after a finite number of steps. (0.5)

GOOD LUCK!