LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2014-04-22 kl 08-13

Students may use the formula sheet and a pocket calculator. All solutions should be properly justified.

1. Consider the set $S = \{(x, y) \in \mathbf{R}^2 \colon y^2 \le x \le y\}$.

a) Draw the set S and the set of all feasible directions at $(1, 1)$. (0.2)

b) Assume that $(1, 1)$ is the local minimum point for some function $f(x, y)$ in S. Draw the set of all possible gradients $\nabla f(1, 1)$. Which of the eight vectors $(\pm 3, \pm 2)$ and $(\pm 2, \pm 3)$ belong to this set? (0.5)

c) Does the optimization problem

$$\text{minimize} \quad e^{(x-y)^2} - \cos(\ln(x^4 + y^{2014})) \quad \text{subject to } (x, y) \in S$$

have a solution? (0.3)

2. Consider the matrix A and the (column) vector b given by

$$A = \begin{pmatrix} 1 & 0 & 1 & -2 \\ -2 & 2 & 1 & -1 \\ 1 & 1 & -1 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 6 \\ 1 \end{pmatrix}.$$

a) Find out whether b belongs to the convex hull of the columns of A. (0.4)

b) Suggest three different methods (analytical or numerical) that you think may be applied to solve the optimization problem

$$\min \|Ax - b\|^2 \quad \text{subject to } x \ge 0.$$

Provide details and explanations why your choice is going to work. (0.6)

3. Consider the matrix A and the vector b from the Problem 2.

a) Use the Simplex method (Phase 1) to find out whether the set

$$S = \{x \in \mathbf{R}^4 \colon Ax = b, \ x \ge 0\}$$

is nonempty. In that case, calculate a basic feasible solution. (0.5)

b) Consider the problem of minimizing $f(x_1, x_2, x_3, x_4) = x_1 - x_4$ subject to $Ax = b$ and $x \ge 0$. Find out by the Complementary Slackness Principle whether the vector $(1, 0, 16, 8)$ is the optimal solution. (0.5)

Please, turn over

4. a) Prove that the sum of two convex functions is a convex function. (0.4)

b) Denote $f(x, y, z) = e^{x+y+z} - 1$. Are the following functions convex in $\mathbf{R}^3$

- $g(x, y, z) = \big|\max\{f(x, y, z), 0\}\big|$? (0.3)
- $h(x, y, z) = \max\{|f(x, y, z)|, 0\}$? (0.3)

Motivate your answer!

5. Solve the following problem by KKT method

$$\min\, xyz \qquad \text{subject to} \qquad x^2 + y^2 + z \leq 4,\ z \geq 0.$$

6. a) Solve the following optimization problem

$$\min\, x^2 + y^2 + xy^2 + y \qquad \text{subject to} \qquad 4 \leq y^2 - x^2,\ x \geq 0.$$

using the duality method with the set $X = \{(x, y) \in \mathbf{R}^2 \colon x \geq 0\}$. (0.5)

b) Prove the existence and uniqueness of the global minimum for a quadratic polynomial with a positive-definite Hessian. (0.5)

GOOD LUCK!