LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2015-05-07 kl 08-13

Students may use the formulae sheet and a pocket calculator. All solutions should be properly justified.

1. Define

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

a) Give the definition of a positive-definite matrix. (0.2)

b) Find out if the matrix H is positive definite/semidefinite or indefinite. (0.4)

c) Find all $c \in \mathbb{R}^3$ such that $Hx \leq 0$, $c^T x > 0$ has no solution. (0.4)

2. Consider the function $f(x,y) = x^2 + 2xy + 2y^2 - 2y$ on $S = \{(x,y) \colon x \geq 0,\ y \geq 0\}$.

a) Show that the minimum exists. (0.3)

b) Carry out one Newton's step from the starting point $(x_0, y_0) = (1,1)$ to get (x_1, y_1). How good/bad is it as a solution to $\min_{(x,y)\in S} f(x,y)$? (0.3)

c) Add a penalty to the function $f(x,y)$ and make one Newton's step from the point (x_1, y_1) that you have obtained in 2b). What kind of point do you get when the penalty goes to infinity? (0.4)

3. Consider the LP problem

$$\min\,(2x_1 + x_2) \qquad \text{subject to} \qquad \begin{cases} x_1 & \leq & 4, \\ x_2 & \geq & -2, \\ x_2 & \geq & -x_1 - 1, \\ x_1 & \geq & 0. \end{cases}$$

a) Solve the problem graphically. (0.2)

b) State the dual problem and solve it by the CSP[1] using the solution in 3a) (or 3c)). (0.4)

c) State the canonical form of the primal problem and solve it by the Simplex method. (0.4)

Please, turn over

[1]Complementary Slackness Principle

4. a) Is the function $f(x,y) = \frac{x}{y} + \frac{y}{x}$ convex on $\{(x,y)\colon x > 0,\ y > 0\}$? (0.3)

b) Is the set $\{(x,y)\colon \frac{x}{y} + \frac{y}{x} \leq 4,\ x > 0,\ y > 0\}$ convex? (0.3)

c) Let the function $F\colon \mathbb{R}^n \to [0, +\infty[$ be convex. Prove that the function

$$G(x) = (1 + F(x))^{F(x)}$$

is convex too. (0.4)

5. Consider the optimization problem

$$\min\,(3x^2 - 4xy + 2y^2) \qquad \text{subject to} \quad x^2 - y^2 \geq 1.$$

a) Solve the problem by KKT/CQ conditions. (0.6)

b) State the dual problem and solve the (primal) problem by duality. (0.4)

6. Prove *one* of the following theorems (of your choice).

- Farkas' theorem.
- If the matrix H is positive definite then the conjugate directions method for $f(x) = x^T Hx$ converges to the minimum in a finite number of steps.

GOOD LUCK!