Centre for Mathematical Sciences

Mathematics, Faculty of Science

Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Consider the set R of all matrices of form

$$\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$$

with $a, b, c \in \mathbb{Z}_5$.

Show that this is a ring (under ordinary matrix operations) and find a proper non-zero ideal I in this ring. Is R/I commutative?

2. Find all positive integers less then 2014 that have exactly 11 positive divisors.

3. Show that the two cycles $\sigma, \tau \in S_n$ have the same length if and only if there exists a permutation π for which $\sigma\pi = \pi\tau$.

4. Find all real polynomials for which $f(2) = 3$, $f(3) = 5$ and $f(5) = 2$.

5. Show that any group of order 10 contains a proper normal subgroup.

6. The complex numbers x, y, z are different roots of an irreducible (over $\mathbb{Q}$) cubic equation with rational coefficients. Show that if $x^3 + y^2 + z^2$ is a root of a quadratic equation with rational coefficients then $x + y + z = 1$.