LUNDS TEKNISKA HÖGSKOLA
MATEMATISKA INSTITUTIONEN

LÖSNINGAR
OPTIMERING
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Answers and Comments[1]

1. a) The cone of feasible directions is $\{(x,y)\colon x \le y < \frac{1}{2}x\}$ (def. on p. 235).

b) The cone of possible gradients is $\{(x,y)\colon y \le -x,\ y \le -2x\}$.
$(-2,-3)$ and $(-3,\pm 2)$ are possible gradients (Lemma 1, p. 235).

c) Set $y = x$ to minimize the first term. Then within the set $0 \le x^4 + x^{2014} \le 2$. Clearly there exists $0 \le x_0 \le 1$ such that $x_0^4 + x_0^{2014} = 1$ which minimizes the second term. So the minimum point $(x_0, x_0) \in S$ exists (and min=0).

2. a) No. The second equation in the convex combination $-2\lambda_1 + 2\lambda_2 + \lambda_3 - \lambda_4 = 6$ is impossible since $-2\lambda_1 + 2\lambda_2 + \lambda_3 - \lambda_4 \le 2\lambda_2 + \lambda_3 < 2 + 1 = 3 < 6$.

b) It is *constrained* nonlinear optimization due to $x \ge 0$. Analytical methods: KKT, duality (no gap by convexity), and numerical methods: penalty, barrier, are possible. One more trick is to use 3a) that gives $\min \|Ax - b\| = 0$.

3. a) The set is nonempty. A basic feasible solution is $(0,2,3,1)^T$.

b) Set $c = (1,0,0,-1)^T$. The dual problem is $\max b^T y$ subject to $A^T y \le c$. The CSP gives equality in the 1st, 3d and 4th rows. Solve it: $y = (0,-1,-1)^T$. The inequality in the 2d row is OK, hence, a feasible point for the dual. Finally $b^T y = c^T x = -7$ shows no duality gap, hence, the optimal points.

4. a) It follows immediately from the definition of convex function that for $h = f + g$

$$h(\lambda x + (1-\lambda)y) \le \lambda f(x) + (1-\lambda) f(y) + \lambda g(x) + (1-\lambda) g(y) = \lambda h(x) + (1-\lambda) h(y),$$

which proves convexity of h.

b) By Lemma 2(2), p. 211, $t = \max\{f, 0\} \ge 0$ convex $\Rightarrow g = |t| = t$ convex too.
$h = \max\{|f|, 0\} = |f|$. With $y = z = 0$ the function $|f(x,0,0)| = |e^x - 1|$ is not convex (*e.g.* can be seen by the graph) $\Rightarrow h$ is not convex.

5. The set is compact $\Rightarrow$ min exists. No CQ points. KKT points: many stationary and other KKT points that give $f = 0$ and two KKT points $(1,-1,2)$, $(-1,1,2)$ that give $f = -2$. The last two are the minimum points.

6. a) The dual finction is $\Theta(u) = 4u - \frac{1}{4(1-u)}$ for $0 \le u < 1$ (otherwise $-\infty$). The optimal $\bar{u} = 3/4$ and $\Theta(3/4) = 2$. Since $f(0,-2) = 2 = \Theta(3/4)$, there is no duality gap $\Rightarrow$ the minimum point is $(0,-2)$.

b) See the book, Ex 7, p. 16 (plus Th. 13, p. 216 and Corollary 1, p. 215).

[1] For re-exams only answers are provided.