LUNDS TEKNISKA HÖGSKOLA
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Answers and Comments[1]

1. a) No, since the dual system ((**) on page 135) has a solution $y = (2, 1, 3)^T$.

b) See the proof to Lemma 1, p. 43.

2. a) Vector $\mathbf{a}$ is the Newton direction (it minimizes the quadratic form in one step.) Vector $\mathbf{b}$ is none (it is a tangent vector). Vector $\mathbf{c}$ is none (it could be the Steepest descent direction, but it has the wrong sign).

b) Vectors $\mathbf{a}$ and $\mathbf{b}$, since $\mathbf{a} = -x_0$, $\mathbf{c} \parallel \nabla f(x_0) = 2Hx_0$ and $\mathbf{b}^T\mathbf{c} = 0 \Rightarrow \mathbf{b}^T H\mathbf{a} = 0$.

3. a) Pick (s_1, x_1, x_2) as a BFS, min $= 1$ at $x_{opt} = (0, 1, 1)$.

b) The dual problem has the solution $y_{opt} = (0, 1, 0)$, max $= 1$.

4. a) Not convex (e.g. draw the set profile for $y = 0$).

b) Convex, since $f = x^2+y^2-z$ and $g = x^2+y^2+z$ are both convex $\Rightarrow h = \max\{f, g\}$ convex $\Rightarrow \{h \leq 1\}$ is a convex set.

c) Only for $\alpha = 0$ (e.g. study the Hessian by (modified) Sylvester criterion for $\alpha \neq 0, 1$ and study cases $\alpha = 0$ and $\alpha = 1$ separately).

5. The set is compact $\Rightarrow$ min exists. No CQ points. KKT points: $(0, 0, 0)$ with $f = 0$, $\pm(1, -1, 0)$ with $f = -2$ and $\pm(1, 1, -4)$ with $f = -14$. The last two are the minimum points.

6. a) The Lagrange function L is a quadratic function with the indefinite Hessian $\Rightarrow$ $\Theta(u, v) = -\infty$ for all $u \geq 0$ and v. Maximization gives again $-\infty$. The obvious duality gap makes it impossible to use the dual problem in order to solve the primal one.

b) To minimize f is the same as to minimize $g = f^2$. The stationary point equation $\nabla g = 0$ is equivalent to the normal equation $\Rightarrow$ the solution to the normal equation is the stationary point + g is convex $\Rightarrow$ it is the global minimum.

[1] For re-exams only answers are provided.