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OPTIMERING
2013–08–30 kl 08–13

Answers and Comments[1]

1. a), b) and d) are true.

2. a) Yes (e.g. by Sylvester criterion).

b) $f(x,y) = (x^2+y^2)^2$ is convex (=superposition of convex and growing h^2 in $h \geq 0$ and convex $h = x^2+y^2$).

c) By Lemma 2, item 4, p. 211, $e^{x^2+y^2}$ is convex ($h = x^2+y^2$) and $\ln(e^{x+y}+1)$ is convex ($h = e^{x+y}$).

3. a) Possible answers in Phase 1 are $(2,2)$, $(4,4)$ and $(6,4)$. To test: draw the set S and check the corner points.

b) $(4,4)$ is feasible for the primal problem. Construct the dual problem, write down the CSP for $(4,4)$ and show that the dual solution candidate is feasible for the dual problem. Then use Th. 6, p. 182.

4. a) See the book, p. 122.

b) For $a \in]-\infty, 1[$ the minimum point is $(1,1)$. For $a \in]1, +\infty[$ the minimum point is $(0,0)$. For $a = 1$ both points are minimum points.

c) Inequality $\geq$ is trivial since $S \subset H(S)$ and the minimum over a large set is smaller. Now let $x \in H(S)$ then $x = \sum_{i=1}^{N} a_i x_i$ for some $x_i \in S$ and $a_i \geq 0$, $\sum_{i=1}^{N} a_i = 1$. Then $f(x) = \sum_{i=1}^{N} a_i f(x_i) \geq \sum_{i=1}^{N} a_i \cdot \min_S f(x) = \min_S f(x)$. It proves $\leq$.

5. a) Check that the minimum exists (add $y^2 - 3x \leq 0$ to the set + Th 2. p. 118). The set has no CQ points. The only KKT point $(\bar{x}, \bar{y}) = (\frac{1}{\sqrt[5]{2}}, \frac{1}{\sqrt[5]{2^3}})$ is then the minimum point. The minimum is $-\frac{5}{2\sqrt[5]{2}}$.

b) The problem becomes convex, so the KKT point is the global minimum by Corollary 1, p. 265.

6. a) The dual fiction is

$$\Theta(u) = \begin{cases} -\frac{5}{2\sqrt[5]{2}} \text{ if } u = 3, \\ -\infty \text{ if } u \neq 3. \end{cases}$$

The dual problem is trivial — the maximum is, of course, at $\bar{u} = 3$. Since $\Theta(\bar{u}) = f(\bar{x}, \bar{y})$ from 5a, there is no duality gap.

b) See the book, Th 4, p. 73.

[1] For re-exams only answers are provided.