LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2013-12-16 kl 08-13

Students may use the formulae sheet and a pocket calculator. All solutions should be properly justified.

1. Consider the matrix

$$H = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and the function $f(x) = \frac{1}{2}x^T Hx$.

a) Is the matrix H positive (or negative) definite, semi-definite or indefinite? (0.3)

b) Does the minimum of f in $\boldsymbol{R}^3$ exist? (0.2)

c) Take $x_0 = (1, 1, 1)^T$ and apply the Newton method to f. Does the method converge? If it does, find the limit. Is the limit a global minimum? (0.2)

d) Repeat 1c) for the modified Newton method with $\epsilon = 1$. (0.3)

2. a) Prove that the intersection of two convex sets is a convex set. (0.3)

b) Is the set $S = \{x_1 \leq x_2 \leq \ldots \leq x_n\}$ convex in $\boldsymbol{R}^n$? (0.3)

c) Find all $a \in \boldsymbol{R}$ such that the vector $(-a, a)$ belongs to the convex hull of $(-3, 1)$, $(2, 0)$ and $(0, 4)$. (0.4)

3. a) Make a proper guess of a basic feasible solution to the LP problem

$$\min(x_2 + 4x_3 + 2x_4) \quad \text{subject to} \quad \begin{cases} -2x_1 - 3x_2 + x_3 + 5x_4 & \leq & 4, \\ x_1 + x_2 - 2x_4 & \leq & 1, \\ -2x_1 - x_2 + x_3 + x_4 & = & 2, \\ \text{all } x_k & \geq & 0. \end{cases}$$

and solve it by the Simplex method (without Phase I). (0.5)

b) State the dual problem and solve it by the CSP[1]. (0.5)

Please, turn over

[1] Complementary Slackness Principle

4. a) State rigorously and prove that the superposition of a convex increasing function and a convex one is again convex. (0.5)

b) For what values of $a \in \boldsymbol{R}$ is the function

$$f(x) = x_1^2 + 4x_1x_2 + ax_2^2 - 2x_1x_3 - 2x_2x_3 + 2x_3^2$$

convex in $\boldsymbol{R}^3$? (0.5)

5. Consider the optimisation problem

$$\min\,(x^2(y-1) + y^4) \qquad \text{subject to } x^2 + y^3 \leq 4,\ \ y \geq 0.$$

a) Solve the problem using the KKT necessary condition. (0.6)

b) Explain what whould be the crucial difference in your solution if one took away the constraint $y \geq 0$. (0.4)

6. a) Consider again the problem in 5, that is

$$\min\,(x^2(y-1) + y^4) \qquad \text{subject to } x^2 + y^3 \leq 4,\ \ y \geq 0.$$

Define $X = \{(x,y) \mid y \geq 0\}$. Calculate the dual function, solve the dual problem and check the duality gap. (0.5)

b) State rigorously and prove the claim that for a convex problem a KKT point is the global minimizer. (If you refer to another lemma or theorem from the course, please, state it carefully.) (0.5)

Merry Christmas, Folks!