LUNDS UNIVERSITET

Matematikcentrum
Matematik NF

Tentamensskrivning
Diskret matematik
Lördag den 16 januari 2010
Skrivtid: 8.00–13.00

Inga hjälpmedel. Använd institutionens papper, skriv på bara den ena sidan och högst en uppgift på varje papper. Skriv tydligt, ge klara och kortfattade motiveringar, rita gärna figur i förekommande fall och ge tydliga svar. Fyll i omslaget fullständigt och skriv initialer på varje papper.

No books, notes, computational devices etc. are allowed. Use paper supplied by the Department, write only on one side of each sheet, and treat at most one exercise on each sheet of paper. Use clear handwriting, give clear and careful motivations. Fill in the form completely and write your name on each sheet of paper.

1. Solve the recursion problem

$$\begin{cases} a_{n+2} + a_{n+1} - 6a_n = 5 \cdot 2^{n+1}, \quad n = 0, 1, 2, \ldots, \\ a_0 = 0, \quad a_1 = -3. \end{cases}$$

2. Consider the binary $[5, 3]$ code C with generator matrix

$$G = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

Calculate the separation $d(C)$ for the code C, that is, calculate the minimum distance between code words in C with respect to the Hamming metric. Decide also which of the received words

$$10101 \quad \text{and} \quad 00111$$

are code words in C and which are not.

3. Calculate the number of positive integers n in the range $1 \le n \le 143$ that are not divisible by 2, 3 or 7.

4. Calculate the number of equivalence relations on the set $\{1, 2, 3, 4\}$ such that every equivalence class has at most 3 elements.

5. Find all polynomials $f(x)$ in $\mathbb{Z}_3[x]$ that satisfy the congruence relations

$$\begin{cases} f(x) \equiv 1 \mod x^2 + 1, \\ f(x) \equiv x \mod x^3 + 2x + 2, \end{cases}$$

in $\mathbb{Z}_3[x]$. Recall that $f_1(x) \equiv f_2(x) \mod g(x)$ means that the polynomial $f_1(x) - f_2(x)$ is divisible by $g(x)$.

Var god vänd!

6. Let n be a non-negative integer and denote by c_n the number of integer solutions $(x_1, x_2, x_3) \in \mathbb{Z}^3$ of the problem

$$\begin{cases} x_1 + x_2 + 2x_3 = n, \\ x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0. \end{cases}$$

Prove that

$$c_n = \frac{1}{4}n^2 + n + \frac{7}{8} + \frac{1}{8}(-1)^n$$

for integers $n \geq 0$.