LUND
UNIVERSITY

Centre for Mathematical Sciences

Mathematics, Faculty of Science

Use only the distributed paper sheets; write only on one side, and no more than one problem per sheet. Fill in the cover form fully and initialize each sheet. Write legibly. Give clear and brief arguments. Draw a picture if this helps.

For those who pass the written exam, an oral exam will take place January 10 and 11. The exact time and place for each student will be posted on a list at the department 08:00 January 7. Please indicate on the answer sheet your preferred time.

1. Solve the recurrence relation

$$a_{n+2} + a_{n+1} - 6a_n = 10 \cdot 2^n + 18 \cdot 3^n,$$

where $n \geq 0$ and $a_0 = 4$ and $a_1 = -2$.

2. Let a linear binary code be given by

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

What is the separation $d(C)$ with respect to the Hamming metric of the code? Determine whether the following are code-words, and correct as best you can those which are not:

$$1101010, \quad 1010100.$$

3. Find the smallest positive solution to the system of congruences

$$x \equiv 2 \ (\text{mod } 3)$$
$$x \equiv 3 \ (\text{mod } 4)$$
$$x \equiv 3 \ (\text{mod } 7).$$

4. In how many ways can you arrange the numbers $1, 2, 3, 4, 5, 6$ along a line so that none of the patterns 12, 23 or 34 appear?

5. (a) Show that $\mathbb{Z}_3[x]/(x^4 + 2x^3 + x^2 + 1)$ is a field.
  (b) Explain why every element in this field is represented by a polynomial of degree 3. Compute the equivalence class of $x^5 + 1$, and find its inverse.

6. You are playing a game of Scrabble, and at some point in the game you have 7 tiles with the letters

NXUNBET.

Please, turn over!

(a) How many 6 letter combinations can you construct?

(b) You manage to replace the tile 'X' with one that is blank. This means that the new tile may represent any letter in the (english) alphabet. How many 6 letter combinations are you able to construct from the new set of 7 tiles?