LUND UNIVERSITY

Written Examination
Discrete Mathematics
Friday, May 27, 2011
08.00–13.00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

No books, notes, computational devices, etc. are allowed. Use only paper supplied by the department. Use clear handwriting and give clear careful motivations. Fill in the form completely and write your name on each sheet of paper.

The oral exam will take place between the days 7^{th} until 11^{th} of June. If you hadn't done before, please indicate on the answer sheet your preferred time. A list will be published on the department web page with the exact time for each student.

1. Find the solution of the following recursion problem

$$a_{n+2} = -2a_{n+1} + 3a_n + (3n+2)2^n$$

for $n \geq 0$, with $a_0 = 0, a_1 = 1$.

2. Solve the system of congruences

$$\begin{aligned} x &\equiv 1 \mod 3 \\ x &\equiv 2 \mod 5 \\ x &\equiv 3 \mod 7 \end{aligned}$$

by finding all its integer solutions x.

3. Determine the number of positive integers n smaller or equal than 1000 such that n is not divisible by $3, 4, 5$ or 6.

4. Determine the number of integer solutions to the equation

$$x_1 + x_2 + x_3 = 14$$

under the constraints $1 \leq x_1 \leq 9$, x_2 odd such that $x_2 \geq 5$, and $x_3 \geq 0$.

5. Prove that the system of congruences

$$\begin{aligned} x &\equiv 5 \mod 6 \\ x &\equiv 7 \mod 15 \end{aligned}$$

has no solution. Does this example contradict Chinese Remainder Theorem?

6. Consider the binary linear $(6, 3)$ code with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

What can be said about the error-correction capability of the encoding function defined by G? Decode the following received word 110010. Can the received word 110001 be properly decoded?