LUNDS UNIVERSITET

Matematikcentrum
Matematik NF

Tentamensskrivning
Diskret matematik
Onsdag den 26 augusti 2009
Skrivtid: 8.00–13.00

Inga hjälpmedel. Använd institutionens papper, skriv på bara den ena sidan och högst en uppgift på varje papper. Skriv tydligt, ge klara och kortfattade motiveringar, rita gärna figur i förekommande fall och ge tydliga svar. Fyll i omslaget fullständigt och skriv initialer på varje papper.

No books, notes, computational devices etc. are allowed. Use paper supplied by the department, write only on one side of each paper, and treat at most one excercise on each paper. Use clear handwriting and give clear careful motivations. Fill in the form completely and write your name on each sheet of paper.

1. Solve the recursion problem

$$\begin{cases} a_{n+2} + 2a_{n+1} - 15a_n = 3^{n+1}, \quad n = 0, 1, 2, \ldots, \\ a_0 = 1, \quad a_1 = 1. \end{cases}$$

2. Consider the binary $[6, 3]$ code C with generator matrix

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

Calculate the separation $d(C)$ for the code C, that is, calculate the minimum distance between code words in C with respect to the Hamming metric. Decide which of the received words

$$011101 \quad \text{and} \quad 110101$$

are code words in C and which are not. For each of the above received words which is not a code word, find a nearest code word in C to this received word with respect to the Hamming metric, that is, find a nearest neighbour in C for every such word.

3. Calculate the number of integer solutions (x_1, x_2, x_3) for the problem

$$\begin{cases} x_1 + x_2 + x_3 = 20, \\ 0 \leq x_j \leq 8, \quad j = 1, 2, 3. \end{cases}$$

4. Let $p(x) = x^3 + 2x + 2$ in $\mathbb{Z}_3[x]$. Prove that the quotient ring

$$R = \mathbb{Z}_3[x]/(p(x))$$

is a field (in Swedish: kropp), and calculate the multiplicative inverse for the element $[x^2 + 1]$ in R.

Var god vänd!

5. Let k be a nonnegative integer and consider the formal power series (generating function) given by the formula

$$f_k(x) = \sum_{n=0}^{\infty} \binom{n}{k} x^n,$$

where $\binom{n}{k}$ are the usual binomial coefficients. Prove that every series $f_k(x)$ is a power series expansion around the origin of a rational function. Calculate the function $f_k(x)$ explicitly as a quotient of two relatively prime polynomials.

6. Prove the identity that

$$S(n,k) = \sum_{r=0}^{n-1} \binom{n-1}{r} S(r, k-1),$$

where the symbol $S(n,k)$ denotes Stirling numbers of the second kind.