LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2012-12-17 kl 08-13

Students may use the formula sheet and a pocket calculator. All solutions should be properly justified.

1. Consider the function $f(x, y) = x^2 - 2x - xy + e^{xy}$ in $\mathbf{R}^2$.

a) Show that the global minimum exists and find it. (0.4)

b) Do, if possible, one Newton step from the point $(0, 1)$. Explain. (0.3)

c) Do one Modified Newton step. Is the search direction descent? Do you get a smaller functional value? Explain. (0.3)

2. a) Prove using Farkas theorem that

$$\begin{cases} 2x - y & \leq & 0, \\ -x + 3y & \leq & 0 \end{cases} \qquad \Rightarrow \qquad 3x + 5y \leq 0. \qquad (0.5)$$

b) Replace $3x + 5y \leq 0$ in 2a) with $c_1 x + c_2 y \leq 0$ and plot all possible vectors $c = (c_1\ c_2)$ in the plane such that the implication still holds. (0.5)

3. a) Solve the LP problem using Simplex algorithm (without Phase I)

$$\max\,(2x_1 + 8x_2 + 6x_3) \quad \text{subject to} \begin{cases} x_1 + 3x_2 + x_3 & \leq & 6, \\ 3x_1 + 5x_2 + x_3 & \leq & 7, \\ 3x_1 + x_2 + x_3 & \geq & 2, \\ \text{all } x_k & \geq & 0 \end{cases} \qquad (0.5)$$

b) State the dual problem to 3a) and solve it by Complementary Slackness Principle. (0.5)

Please, turn over

4. a) Prove that if for $i = 1, 2, \ldots, m$ the functions g_i are convex then the set $S = \{x \in \mathbf{R}^n \colon g_i(x) \le \alpha_i, i = 1, 2, \ldots, m\}$ is convex for any $\alpha_i \in \mathbf{R}$. (0.3)

b) Prove that if the functions f and g_i, $i = 1, 2, \ldots, m$, are convex then for any constant $\epsilon > 0$ the barrier method function

$$F(x) = f(x) + \epsilon \sum_{i=1}^{m} \frac{-1}{g_i(x)}$$

is convex on S defined in 4a) when all $\alpha_i = 0$. (0.4)

c) Find all $a \in \mathbf{R}$ such that the function $f(x, y) = x^2 + 2xy^2 + ay^4$ is convex in $\{(x, y) \colon x \ge 0\}$. (0.3)

5. Solve the optimization problem

$$\min\,(x^2 - 2xy + 2y) \qquad \text{subject to} \qquad y^2 \le 2x,\ y \ge 0$$

using the KKT necessary condition.

6. a) State and prove the theorem that claims the equivalence of the existence of a saddle point and no duality gap. (0.4)

b) Solve the problem in 5 by duality method: take $X = \{y \ge 0\}$, calculate the dual function, solve the dual problem and prove no duality gap. (0.6)

GOOD LUCK!