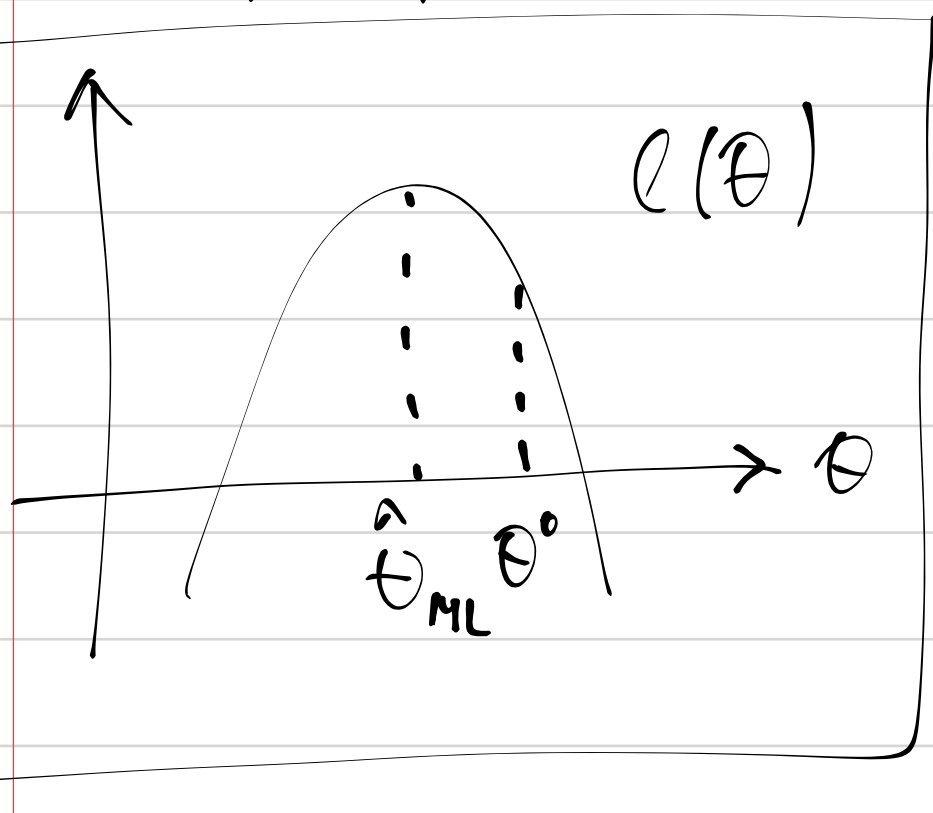
Лучев !!

Видно? Слышно? !!

LM/LR/W - тест !!



$$H_0: \theta_1^* = \theta_1^0$$
$$H_A: \theta_1^* \neq \theta_1^0$$

$$\max_{\theta} \ell(\theta) \qquad \hat{\theta} = \begin{pmatrix} \hat{\theta}_A \\ \hat{\theta}_B \end{pmatrix}$$

если θ_B меш: $I_F^{obs}(\hat{\theta})$

$$W = [\hat{\theta}_A - \theta_1^0]^T \cdot [\widehat{Var}(\hat{\theta}_A)]^{-1} \cdot [\hat{\theta}_A - \theta_1^0]$$

что это за зверь?

H_0 верна $\Rightarrow$ $W \xrightarrow{dist} \chi^2_{p_A}$

Аккуратно!

пробуй по всему θ

$$I_F(\hat{\theta})$$

проще: $$I_F^{obs}(\hat{\theta}) = -\frac{\partial^2 \ell}{\partial\theta\,\partial\theta^T}\Big|_{\hat{\theta}}$$

1. $I_F^{obs}(\hat{\theta})$
2. $(I_F^{obs})^{-1}$
3. вырежем блок.
4. $(\text{блок})^{-1}$

p_A, p_B, p

$$\left(I_F^{obs}(\hat{\theta})\right)^{-1} = \widehat{Var}(\hat{\theta})$$

в W-стат. этот блок оставляем!

$\widehat{Var}(\hat{\theta}_A)$ $\quad \widehat{Cov}(\hat{\theta}_A, \hat{\theta}_B)$

$\widehat{Cov}(\hat{\theta}_B, \hat{\theta}_A)$ $\quad \widehat{Var}(\hat{\theta}_B)$

p

LR, LM, W

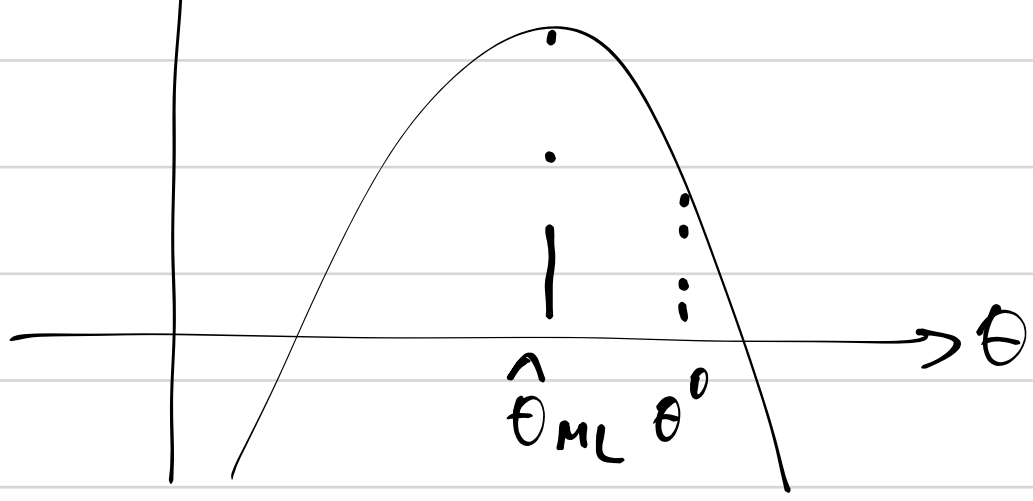
„пои - бо" Эмпирса.

Случай простой

θ - скаляр.

$H_0: \theta^* = \theta^0$

$\ell(\theta)$ - лог. ф. правд.

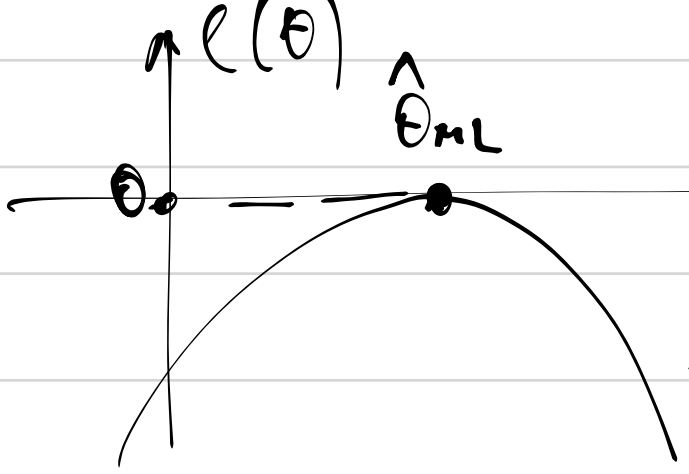


$\ell(\theta)$ = парабола!

$a\theta^2 + b\theta + c$

$(a\theta^2 + b\theta + c)$

$$W = (\hat{\theta}_m - \theta^0) \cdot I_F^{obs} \cdot (\hat{\theta}_m - \theta^0)$$



$\ell(\theta) = a(\theta - \hat{\theta}_{ML})^2$

$\frac{\partial^2 \ell}{\partial \theta^2} = 2a$

$W = (\hat{\theta}_{ML} - \theta^0)^2 \cdot (-2a)$

Упр: $W = -2\ell(\theta^0) \overset{\text{для пуассон-ой пар-лы:}}{=} 2(\ell(\hat{\theta}_{ML}) - \ell(\theta^0))$

↑ при особой „c" в формуле параболы.

Likelihood ratio test. (LR)

$\max \ell(\theta) \rightarrow \hat{\theta}_{ML} = \begin{pmatrix} \hat{\theta}_A \\ \hat{\theta}_B \end{pmatrix}$ ← макс. неогр.

$H_0: \theta_A^* = \theta_A^0$

$\max_{H_0} \ell(\theta) \rightarrow \hat{\theta}_R = \begin{pmatrix} \theta_A^0 \\ \hat{\theta}_B^R \end{pmatrix}$ ← макс с ограничением (R)

$LR = 2(\ell(\hat{\theta}_{ML}) - \ell(\hat{\theta}_R)) \xrightarrow[H_0, n\to\infty]{dist} \chi^2_{p_A}$ [теорема]

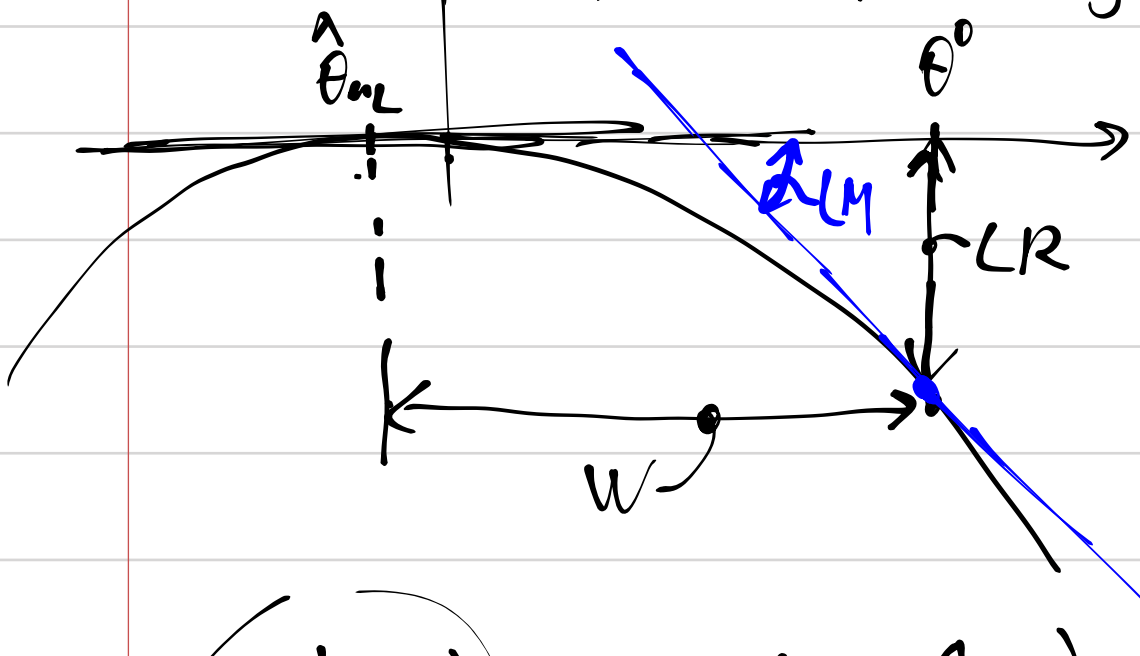
$\text{plim} \frac{LR}{W} = 1$ (H_0, регул)

почему LR и. быть проще вычисл-но?

⊕ нет $(\cdot)^{-1}$ [дорого, проблемно при $\frac{\text{число}}{\text{вып}} >> 0$]

⊖ в LR решает две оптим-ции, а в W – одна.

$\ell(\theta)$ – лог. фр прав.



$LR = 2\left(\ell(\hat{\theta}_M) - \ell(\hat{\theta}_R)\right)$

$\ell(\theta) = a(\theta - \hat{\theta}_{ML})^2$

макс. прав. бу. огр. | макс. прав. с огр.

$LR = -2a(\theta^0 - \hat{\theta}_{ML})^2$

$\ell'(\theta^0) = 2a(\theta - \hat{\theta}_{ML})$

$s = \frac{\partial \ell}{\partial \theta}$

$LM = [s(\hat{\theta}_R)]^T \cdot [\hat{V}_{ar}(s(\hat{\theta}_R))]^{-1} \cdot s(\hat{\theta}_R)$

$W = (\hat{\theta}_A - \theta_A^0)^T \cdot [\hat{V}_{ar}(\hat{\theta}_A)]^{-1} \cdot (\hat{\theta}_A - \theta_A^0)$

$H_0: \begin{cases} \theta_1 = 3 \\ \theta_2 = 9 \end{cases}$

$\hat{\vartheta}_R = \underset{H_0}{\arg\max}\, \ell(\theta)$

$LM = [s(\hat{\theta}_R)]^T \cdot \underbrace{[I_F^{obs}(\hat{\theta}_R)]^{-1}}_{T_F(\hat{\theta}_R)} \cdot s(\hat{\theta}_R) \xrightarrow[H_0 \text{ (прост)}]{\text{п-ство откл}} \chi^2_{p_A}$

⊕ LM не используется $\hat{\theta}_{ML}$

$\lim \frac{LM}{W} = 1$

Пример

Караси	Щуки	Вегеноры
p_K	$1-p_K-p_B$	p_B

данные по уловам $n=100$ 20К 50Щ 30Вег.

а) $p = \begin{pmatrix} p_K \\ p_B \end{pmatrix}$ $\hat{p}_{ML} =$

$$L = \frac{100!}{20!50!30!} \cdot p_K^{20} \cdot p_B^{50} (1-p_K-p_B)^{30}$$

$$l(p) = \ln L = \text{const} + 20 \ln p_K + 50 \ln p_B + 30 \ln (1-p_K-p_B)$$

$$s(p) = \begin{pmatrix} \frac{20}{p_K} - \frac{30}{1-p_K-p_B} \\ \frac{50}{p_B} - \frac{30}{1-p_K-p_B} \end{pmatrix} \rightarrow \hat{p} = \begin{pmatrix} \frac{20}{100} \\ \frac{50}{100} \end{pmatrix}$$

в) W ? Но $p_K = 1/3$, $p_B = 1/3$

Надо ли одно равенство нарушено

LR: $H_0: p_K = p_B$

LM: $H_1: p_K = \frac{1}{3}$

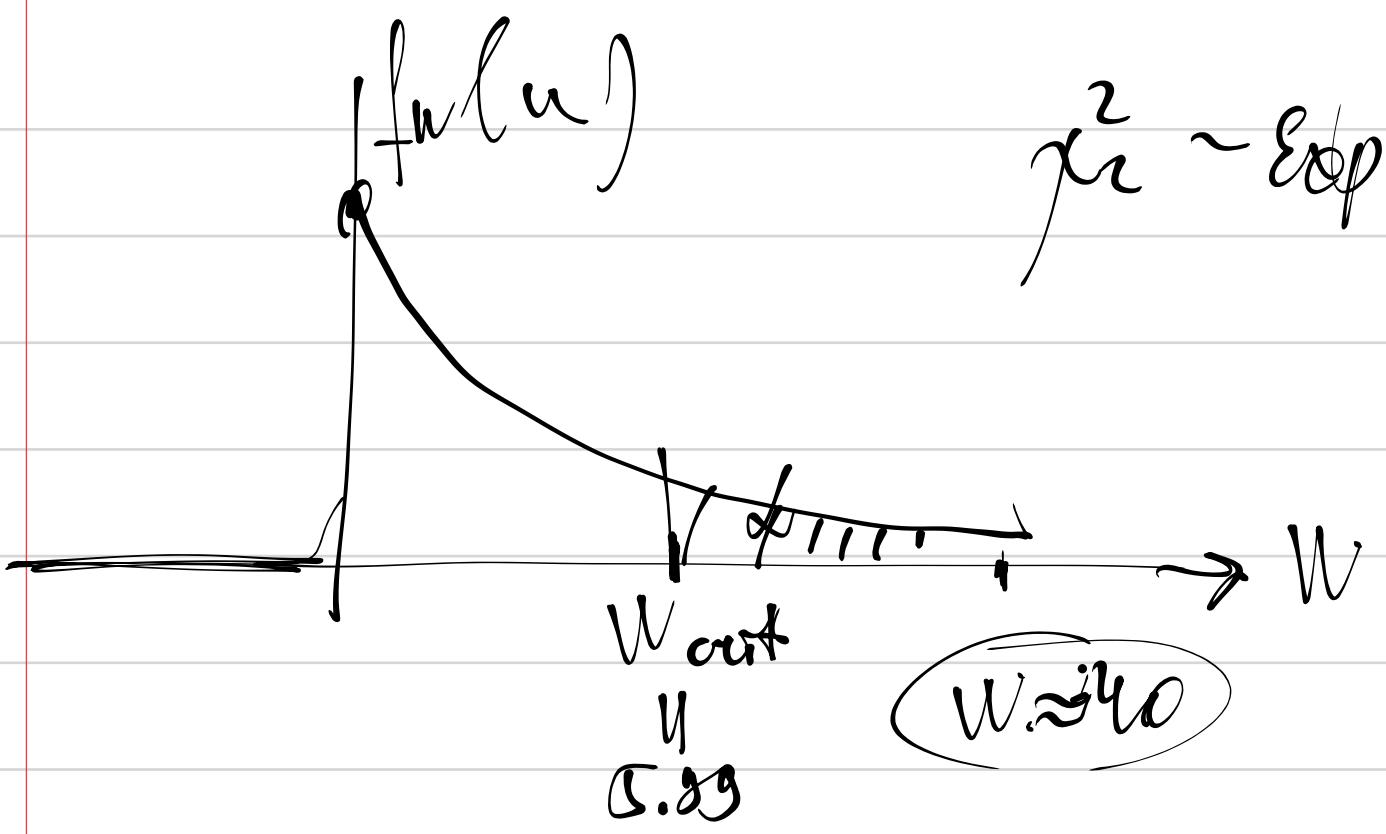
$$W = \begin{pmatrix} \frac{20}{100} - \frac{1}{3} & \frac{50}{100} - \frac{1}{3} \end{pmatrix} \cdot \hat{I} \cdot \begin{pmatrix} \frac{20}{100} - \frac{1}{3} \\ \frac{50}{100} - \frac{1}{3} \end{pmatrix}$$

$n=100$

χ^2_2

$$I_p^{obs}(p) = \begin{pmatrix} \frac{20}{p_K^2} + \frac{30}{(1-p_K-p_B)^2} & \frac{30}{(1-p_K-p_B)^2} \\ \frac{30}{(1-p_K-p_B)^2} & \frac{50}{p_B^2} + \frac{30}{(1-p_K-p_B)^2} \end{pmatrix}$$

$$\hat{I} = 100^2 \begin{pmatrix} \frac{1}{20} + \frac{1}{30} & \frac{1}{30} \\ \frac{1}{30} & \frac{1}{50} + \frac{1}{30} \end{pmatrix}$$



$\chi^2_2 \sim Exp$

$\alpha = 5\%$

$W \approx 40$

Вывод: Не отвергается!

$LR = 2\left(l(\hat{p}_{ML}) - l(\hat{p}_R)\right)$

$\hat{p}_{ML} = \begin{pmatrix} 0.2 \\ 0.5 \end{pmatrix}$

$\hat{p}_R$

const = $-\ln \frac{100!}{20!30!50!}$

max $p_K = p_B$

$20 \ln p_K + 50 \ln p_B + 30 \ln(1 - p_K - p_B)$

$70 \ln p_K + 30 \ln(1 - 2p_K)$

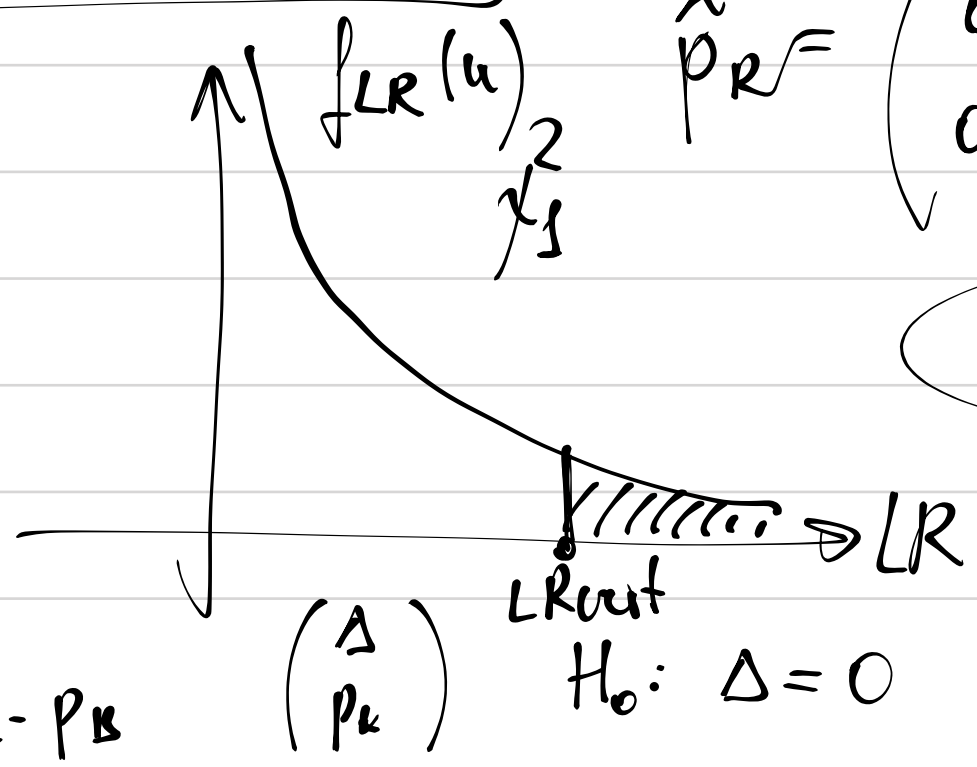
$\frac{70}{\hat{p}_K} = \frac{2 \cdot 30}{1 - 2\hat{p}_K}$

$70(1 - 2\hat{p}_K) = 60\hat{p}_K$

$\hat{p}_K = \frac{70}{200} = 0.35$

$\hat{p}_R = \begin{pmatrix} 0,35 \\ 0.35 \end{pmatrix}$

Если $LR > LR_{crit}$ то H_0 отверг



$\alpha = 5\%$

$H_0: p_K - p_B = 0$

1 уравнение

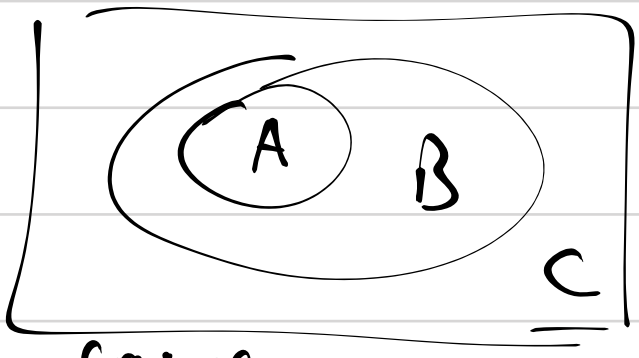
p_K

$\Delta = p_K - p_B$

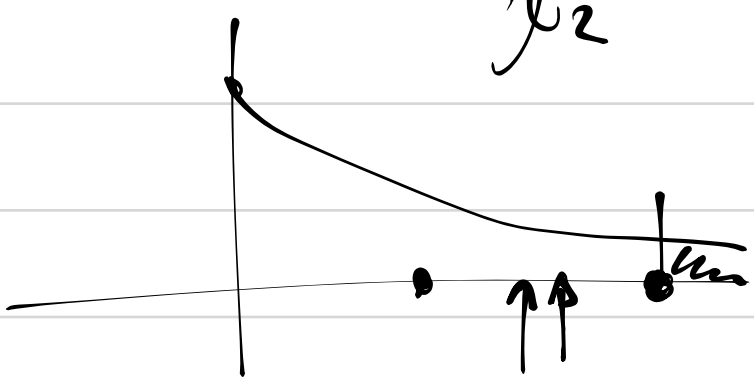
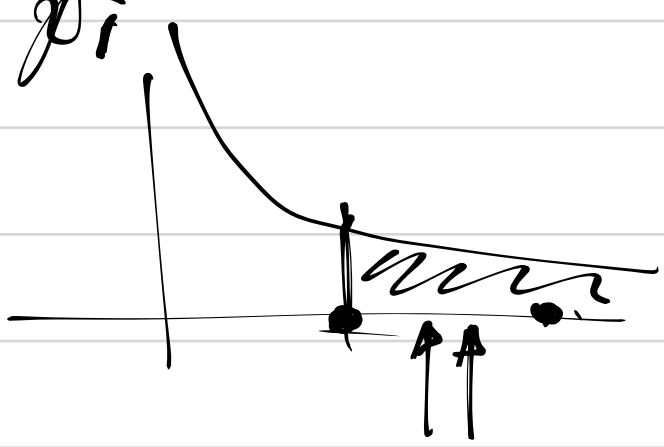
$\begin{pmatrix} \Delta \\ p_K \end{pmatrix}$

$H_0: \Delta = 0$

парадоксальное свойство проверки гипотез !!



Саша	Маша
H_0 : A	H_0 : B
H_A : не A, C-верна	H_A : не B, C-верна
$\alpha =$	α
A не отвергается	B отвергается

$\boxed{\text{Возм-но.}}$

H_0: $p_1 = \frac{1}{2}$ $p_2 = \frac{1}{2}$	H_0: $p_1 = \frac{1}{2}$
χ^2_2	χ^2_1