

Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 2500: Engineering Probability, Spring 2023

Homework #3: due Thursday, Feb. 16th, at 11:59PM.

Show all work for full credit!

Submit your work as a single PDF on Gradescope, labeling each problem number with a page.

1. The 4 members of Blackpink independently decide whether to show up to an event, each with probability 0.3. There are already 5 photographers at the event, but with every additional member that arrives, 10 more photographers show up. Let X be the number of photographers at the event.
 - (a) (5 points) Determine and sketch the PMF of X .
 - (b) (10 points) Compute $E(X)$, using the direct formula for expected value.
 - (c) (10 points) Compute $\text{Var}(X)$, using the direct formula for expected value.
 - (d) (5 points) Note that X is linearly related to Y , a binomial random variable. Determine the parameters of the binomial random variable Y and the relationship between X and Y .
 - (e) (10 points) Using this relationship, compute $E(X)$ and $\text{Var}(X)$ using the properties of expected value and the known mean and variance of a binomial random variable. If your answers don't agree with parts (b) and (c) you did something wrong!

2. Moon Knight is a man whose body can be shared between two different personalities: Marc or Stephen. Each personality has a different appetite for candy. If Marc possesses the body, the number of pieces of candy he eats in a day is modeled as (discrete) uniform distribution on $[10, 11, \dots, 30]$. If Stephen possesses the body, the number of pieces of candy he eats in a day is modeled as a binomial random variable with $n = 40$ and $p = \frac{3}{4}$. Suppose the probability that Marc possesses the body on a given day is 0.4, and let Y be the number of pieces of candy eaten in a day.

Use the moments of random variables to answer the below questions (e.g., using Table 3.1, pp. 115–116 of Leon-Garcia or a similar table). This problem should not require a lot of computation.

 - (a) (10 points) Compute $E(Y)$.
 - (b) (20 points) Compute $E(Y^2)$.

3. President Schmidt has been sighted walking around campus! Let X be the number of times he is sighted by students each day, modeled as a geometric random variable with parameter $p = 0.3$. (In this problem, we assume he is seen at least once every day.)
 - (a) (10 points) Compute the conditional PMF $p_X(x \mid X \leq 5)$.
 - (b) (10 points) Compute $E(X \mid X \leq 5)$.
 - (c) (10 points) Compute the conditional PMF $p_X(x \mid X > 5)$. (Hint: don't do any serious computation, use a property and say which one you use.)