Problem Set 5

Due: 11pm, Tuesday, October 18, 2022
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NOTES

1. Practice problems listed below and taken from the textbook are for your own practice, and are not to be turned in.
2. There are two parts of the Problem Set, an objective part consisting of multiple choice questions (with no partial credit available) and a subjective part (with partial credit possible). Please complete all questions.
3. Writing your solutions in LaTeX is preferred but not required.
4. Show all work for problems in the subjective part. Illegible or undecipherable solutions will not be graded.
5. Figures, if any, should be neatly drawn by hand, properly labelled and captioned.
6. Your completed work is to be submitted electronically to LMS as a single pdf file. Be sure that the pages are properly oriented and well lighted. (Please do not e-mail your work to Muhammad or me.)

Practice Problems from the textbook (Not to be turned in)

- Exercises from Chapter 3, pages 58–59: 1(d,f,h,p), 2(c,d,g), 3(c,j).
- Exercises from Chapter 3, page 63: 1(c,d,e), 3(b).

Objective part (Choose A, B, C or D; no work need be shown, no partial credit available)

1. (5 points) Consider the linear nonhomogeneous differential equation

$$y'' - 2y' + 5y = \cos t - \sin 2t$$

Select the correct form of a particular solution $y_p(t)$ for the DE

A $y_p(t) = A\cos t + B\sin 2t$
B X $y_p(t) = A\cos t + B\sin t + C\cos 2t + D\sin 2t$ X
C $y_p(t) = A\cos t + B\sin t + (C\cos 2t + D\sin 2t)t$
D None of these choices

2. (5 points) Consider the linear nonhomogeneous differential equation

$$y'' + y' = 5te^{-t} - t^2\cos t$$

Select the correct form of a particular solution $y_p(t)$ for the DE

A $y_p(t) = (A + Bt)e^{-t} + (C_0 + C_1 t + C_2 t^2)(D\cos t + E\sin t)$
B $y_p(t) = (At + Bt^2)e^{-t} + (C_0 + C_1 t + C_2 t^2)(D\cos t + E\sin t)$
C X $y_p(t) = (At + Bt^2)e^{-t} + (C_0 + C_1 t + C_2 t^2)\cos t + (D_0 + D_1 t + D_2 t^2)\sin t$ X
D None of these choices

3. (5 points) Consider the linear nonhomogeneous differential equation

$$y'' + 4y = te^{2t}\sin 2t + \cos 2t$$

Select the correct form of a particular solution $y_p(t)$ for the DE

A $y(t) = te^{2t}(A\cos 2t + B\sin 2t) + t(C\cos 2t + D\sin 2t)$

B $y(t) = te^{2t}[(At+B)\cos 2t + (Ct+D)\sin 2t] + (P\cos 2t + Q\sin 2t)$

C X $y(t) = e^{2t}[(At+B)\cos 2t + (Ct+D)\sin 2t] + t(P\cos 2t + Q\sin 2t)$ X

D None of these choices

Subjective part (Show work, partial credit available)

1. (15 points) Solve the initial-value problem

$$y'' + 3y' = e^{3t} + 4t, \qquad y(0) = 0, \quad y'(0) = 0$$

y_h
$y'' + 3y' = 0\,;\ r^2 + 3r = 0\,;\ (r)(r+3) = 0$
$r_1 = 0\,;\ r_2 = -3$
$y_h = C_1e^{0t} + C_2e^{-3t}$
$y_h = C_1 + C_2e^{-3t}$
y_p
$y_p = A_0 + A_1t + Be^{3t}$ resonates
$y_p = (A_0 + A_1t)t + Be^{3t}$
$y_p = A_0t + A_1t^2 + Be^{3t}$
$y_p' = A_0 + 2A_1t + 3Be^{3t}$
$y_p'' = 2A_1 + 9Be^{3t}$
$L[y_p] = (2A_1 + 9Be^{3t}) + 3(A_0 + 2A_1t + 3Be^{3t}) = e^{3t} + 4t$
$2A_1 + 9Be^{3t} + 3A_0 + 6A_1t + 9e^{3t} = e^{3t} + 4t$
$18Be^{3t} = e^{3t}\,;\ B = \frac{1}{18}$
$6A_1t = 4t\,;\ A_1 = \frac{2}{3}$
$2A_1 + 3A_0 = 0\,;\ \frac{4}{3} + 3A_0 = 0\,;\ 3A_0 = \frac{-4}{3}\,;\ A_0 = \frac{-4}{9}$
$y_p = \frac{-4}{9}t + \frac{2}{3}t^2 + \frac{1}{18}e^{3t}$
$y(t) = C_1 + C_2e^{-3t} + \frac{-4}{9}t + \frac{2}{3}t^2 + \frac{1}{18}e^{3t}$
$y'(t) = -3C_2e^{-3t} + \frac{-4}{9} + \frac{4}{3}t + \frac{1}{6}e^{3t}$
$y'(0) = 0 = -3C_2 + \frac{-4}{9} + \frac{1}{6}$
$3C_2 = \frac{-8}{9} + \frac{3}{18} = \frac{-5}{18}$
$C_2 = \frac{-5}{54}$
$y(0) = 0 = C_1 + \frac{-5}{54} + \frac{1}{18} = \frac{-2}{54} = \frac{-1}{27}$
$C_1 = \frac{1}{27}$
$y(t) = \frac{1}{27} + \frac{-5}{54}e^{-3t} + \frac{-4}{9}t + \frac{2}{3}t^2 + \frac{1}{18}e^{3t}$

2. (15 points) Solve the initial-value problem

$$y'' + 4y' + 5y = 2e^{-2t}\sin t \qquad y(0) = 0, \quad y'(0) = 0$$

y_h
$y'' + 4y' + 5y = 0\,;\ r^2 + 4r + 5 = 0\,;\ r = \frac{-4\pm\sqrt{16-4*1*5}}{2*1}\,;\ r = \frac{-4\pm\sqrt{-4}}{2}\,;\ r = -2 \pm i\,;\ \lambda = -2\,;\ \omega = 1$
$y_h = C_1e^{-2t}\cos(t) + C_2e^{-2t}\sin(t)$
y_p
$y_p = Ae^{-2t}\cos(t) + Be^{-2t}\sin(t)$ resonates
$y_p = Ate^{-2t}\cos(t) + Bte^{-2t}\sin(t)$
Let $s = \sin(t)$ and $c = \cos(t)$
$y_p = Ate^{-2t}c + Bte^{-2t}s$
$y_p = A(te^{-2t})c + B(te^{-2t})s$
dl*r+dr*l
(dll*lr+dlr*ll)*r+dr*l
$y_p' = A((1*e^{-2t} + -2e^{-2t}*t)*c + -s*te^{-2t}) + B((1*e^{-2t} + -2e^{-2t}*t)*s + c*te^{-2t})$
$y_p' = A(e^{-2t}c - 2te^{-2t}c - te^{-2t}s) + B(e^{-2t}s - 2te^{-2t}s + te^{-2t}c)$
$y_p'' = A((-2e^{-2t}c - e^{-2t}s) - 2(e^{-2t}c - 2te^{-2t}c - te^{-2t}s) - (e^{-2t}s - 2te^{-2t}s + te^{-2t}c))+$
$+B((-2e^{-2t}s + e^{-2}c) - 2(e^{-2t}s - 2te^{-2t}s + te^{-2t}c) + (e^{-2t}c - 2te^{-2t}c - te^{-2t}s))$
$y_p'' = A(-4e^{-2t}c - 2e^{-2t}s + 3te^{-2t}c + 4te^{-2t}s) + B(-4e^{-2t}s + 2e^{-2t}c + 3te^{-2t}s - 4te^{-2t}c)$
$L[y_p] = -A4e^{-2t}c - A2e^{-2t}s + A3te^{-2t}c + A4te^{-2t}s - B4e^{-2t}s + B2e^{-2t}c + B3te^{-2t}s - B4te^{-2t}c+$
$+A4e^{-2t}c - A8te^{-2t}c - A4te^{-2t}s + B4e^{-2t}s - B8te^{-2t}s + B4te^{-2t}c + A5te^{-2t}c + B5te^{-2t}s = 2e^{-2t}s$
$L[y_p] = e^{-2t}c(-4A+2B+4A) + e^{-2t}s(-2A-4B+4B) + te^{-2t}c(3A-4B-8A+4B+5A) + te^{-2t}s(4A+$
$3B - 4A - 8B + 5B) = 2e^{-2t}s$
$L[y_p] = e^{-2t}c(2B) + e^{-2t}s(-2A) + te^{-2t}c(0) + te^{-2t}s(0) = 2e^{-2t}s$
$L[y_p] = e^{-2t}c(2B) + e^{-2t}s(-2A) = 2e^{-2t}s$
$B = 0\,;\ A = -1$
$y_p = -te^{-2t}\cos(t)$
$y(t) = C_1e^{-2t}\cos(t) + C_2e^{-2t}\sin(t) - te^{-2t}\cos(t)$
$y(0) = 0 = C_1$
$C_1 = 0$
$y(t) = C_2e^{-2t}\sin(t) - te^{-2t}\cos(t)$
$y'(t) = C_2(-2e^{-2t}\sin(t) + \cos(t)e^{-2t}) - e^{-2t}\cos(t) + 2te^{-2t}\cos(t) + te^{-2t}\sin(t)$
$y'(0) = 0 = C_2 - 1$
$C_2 = 1$
$y(t) = e^{-2t}\sin(t) - te^{-2t}\cos(t)$

3. (15 points) Consider the linear, nonhomogeneous DE

$$ty'' + (t-1)y' - y = t^2e^{-2t}, \qquad t > 0$$

(a) Verify that $y_1(t) = e^{-t}$ and $y_2(t) = t - 1$ are homogeneous solutions of the differential equation and compute the Wronskian.
$y_1' = -e^{-t}\,;\ y_1'' = e^{-t}$
$L[y_1] = te^{-t} + (t-1)(-e^{-t}) - e^{-t}$
$L[y_1] = te^{-t} + e^{-t} - te^{-t} - e^{-t}$
$L[y_1] = 0$
$y_2' = 1\,;\ y_2'' = 0$
$L[y_2] = (t-1) - (t-1)$
$L[y_2] = 0$
$W = e^{-t}*1 - (t-1)(-e^{-t})$
$W = e^{-t} + te^{-t} - e^{-t}$
$W = te^{-t} \neq 0$

(b) Use Variation of Parameters to find the general solution of the DE.

$y'' + \frac{(t-1)}{t}y' - \frac{1}{t}y = te^{-2t}$

y_h

$y_h = C_1e^{-t} + C_2(t-1)$

y_p

$y_p = u_1y_1 + u_2y_2$

$u_1' = -\frac{(t-1)*te^{-2t}}{te^{-t}}$

$u_1' = -\frac{t^3e^{-2t}-te^{-2t}}{te^{-t}}$

$u_1' = -(te^{-t} - e^{-t})$

$u_1' = -te^{-t} + e^{-t}$

$u_1 = te^{-t} + C_3$

$u_2' = \frac{e^{-t}*te^{-2t}}{te^{-t}}$

$u_2' = \frac{e^{0}*te^{-t}}{t}$

$u_2' = e^{-t}$

$u_2 = -\frac{1}{2}e^{-2t} + C_4$

$y_p = (te^{-t} + C_3)e^{-t} + (-\frac{1}{2}e^{-2t} + C_4)(t-1)$

$y_p = te^{-2t} + C_3e^{-t} - \frac{1}{2}te^{-2t} + \frac{1}{2}e^{-2t} + C_4(t-1)$

choose $C_3 = 0\,;\ C_4 = 0$

$y_p = te^{-2t} - \frac{1}{2}te^{-2t} + \frac{1}{2}e^{-2t}$

$y_p = \frac{1}{2}te^{-2t} + \frac{1}{2}e^{-2t}$

$y_p = \frac{1}{2}(t+1)e^{-2t}$

$y(t) = C_1e^{-t} + C_2(t-1) + \frac{1}{2}(t+1)e^{-2t}$