

**Final Exam****Name:** \_\_\_\_\_**RIN:** \_\_\_\_\_**Skills you are retesting (filling this out is an optional reminder for you):**

1a	1b	1c	1d	1e	1f	1g	1h	1i	1j	1k	1l

2a	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l	2m	2n

3a	3b	3c	3d	3e	3f	3g	3h	3i	3j	3k	3l

4a	4b	4c	4d	4e	4f	4g	4h	4i	4j	4k	4l

**Instructions**

- 1.) Read all directions carefully.
- 2.) You have **4 hours** to complete this exam.
- 3.) Complete all material in Part A of this exam.
- 4.) Part B is the retest portion of the exam. You may complete as much or as little of this part of the exam as you wish. However, you should do your best to complete all calculations and analysis for any problem that you attempt. Whenever your answers demonstrate improved mastery of Core Skills, your previous scores for those skills will be replaced by the new scores from Part B of this exam.
- 5.) Show your work in enough detail to allow the graders to completely follow your thought process.
- 6.) Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use at least 2 significant digits.
- 7.) Make sure to write your answers legibly. You can write on the back of the exam pages or ask for scratch paper.

## Part 1 (Unit 4 Skills)

### A1. Capacitor Current

*(Skill 4a)*

A parallel plate capacitor has two square plates, each with an area of 9 square centimeters. The distance between the plates is 1mm and contained in that area is a dielectric with relative permittivity 10 and conductivity  $2 \times 10^{-3}$  S/m.

A voltage of  $10\cos(5000\pi t)$  is applied across the capacitor. Determine the conduction and displacement current through this capacitor.

(Skill 4b, 4c, 4d, 4e, 4f, 4g, 4i)

a.) Determine the power density of the incident wave.

b.) Determine the magnitude and direction of the H-field in the incident wave. Assume E points in +x and the wave is propagating in the -z direction.

c.) Determine whether the material the wave is striking is a good conductor, low-loss dielectric, or neither.

d.) Calculate alpha and beta inside the material, as well as the intrinsic impedance.

e.) Determine reflection coefficient at the material boundary.

f.) Determine the skin depth of this wave as it enters the material.

### A3. Wave Polarization

Determine the polarization of the following waves, i.e., linear, circular or elliptical, and propagation direction. If a wave is linearly polarized, determine the inclination angle. For circular polarization, determine the rotation direction. For elliptical polarization, determine the ellipticity and rotation angle.

a.)  $\vec{E}(z, t) = \hat{x}3\cos(\omega t - kz) + \hat{y}3\cos(\omega t - kz)$

b.)  $\vec{E}(z, t) = \hat{x}5\cos(\omega t - kz) - \hat{y}3\cos(\omega t - kz + \pi/8)$

**A4. Pool Illumination***(Skills 4j, 4k, 4l)*

You are swimming 8 meters underwater in a pool in which the water has a relative permittivity of 80. You are carrying a lantern which illuminates equally in every direction with randomly polarized light (i.e. a combination of parallel and perpendicular polarization).

a.) Is there a critical angle for the light being emitted from your lantern and hitting the air-water boundary from below? If so, describe that occurs when light hits the boundary at less than this angle and what happens when light hits the boundary at greater than this angle.

b.) Is there a Brewster angle for the light being emitted from your lantern and hitting the air-water boundary from below? If so, does it exist for both polarizations of emitted light or only for one polarization? (If this is the case, name which polarization has a Brewster angle.)

c.) Calculate the reflection coefficient of the light at exactly  $\frac{1}{2}$  of the critical angle. Is the reflection different for parallel vs. perpendicular polarization? Calculate the transmitted E-field amplitudes at parallel vs perpendicular polarization.

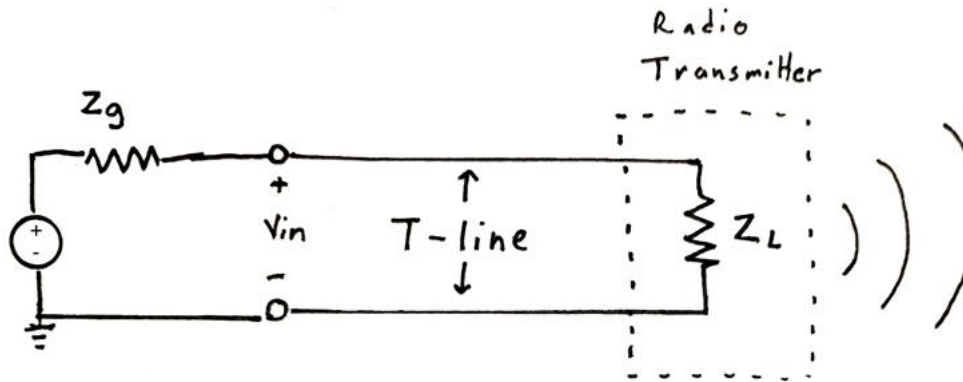
f.) Suppose that one particular ray of the emitted light is traveling partially in the x direction and partially in the z direction. The surface of the water is parallel to the x-y plane and the E-field points in the y direction. Is this parallel or perpendicular polarization?

## Part 2 (Retest Skills)

### B1. Radio Transmission

(Skills 1e, 1g, 1h, 1i, 1j, 1k, 1l)

The diagram below shows a voltage source attached to a transmission line. At the end of the line is a radio transmitter releasing an EM wave into the air. The source impedance  $Z_g$  of the voltage source is  $25\Omega$ . The characteristic impedance of the transmission line is  $50\Omega$ , and the radio transmitter behaves like a  $75\Omega$  resistive load.



a.) The voltage source is turned on and produces a DC voltage. (Choose an amplitude between 1V and 10V for this voltage.) What is the initial input voltage  $V_{in}$  that appears on the transmission line? What is the initial input current  $I_{in}$ ?

b.) Is the load matched to the transmission line?

c.) Calculate the amplitude of the first reflection at the load and the first reflection at the source.



d.) Determine the average power transmitted to the load when the signal hits it for the first time.

e.) Suppose that the transmission line has some nonzero level of loss. It is 5km long with a resistance per unit length of 0.05 ohms per meter and an inductance per unit length of 250 nanohenries per meter. If a 10V 1 MHz signal enters the line, will the losses be low? What amplitude will the signal have by the time it hits the load?

f.) Describe a change that you could make to the parameters of this transmission line ( $R'$ ,  $G'$ ,  $L'$  or  $C'$ ) in order to make it dispersionless. Support your statement with calculations. Assume that  $G' = 0$ .

g.) For a moment, ignore the losses on the transmission line. What is the input impedance of the line for the 1 MHz signal?

**B2. Wave Expressions***(Skills 1a, 1c, 1f)*

a.) Write a time-domain expression for a voltage traveling wave that moves in the  $+z$  direction. Then, rewrite it so that it is moving in the  $-z$  direction.

b.) Re-write the expression from part a in phasor notation.

c.) Suppose that the wave you described in part a hits a load for which the reflection coefficient is  $+0.5$ . Determine the standing wave ratio and draw the resulting standing wave pattern over one wavelength of transmission line.

**B3. Coaxial Cable**

(Skills 1b, 1d, 2c, 2d, 2e, 2f, 2i, 2j, 3a, 3b, 3e)

In this problem, you will design a coaxial cable. Its innermost layer should be a grounded cylindrical conductor of radius 1mm. Its outermost layer should be a conducting shell of thickness 1mm to which a voltage will be applied. In between these two layers is a dielectric. You may specify a thickness for the dielectric in the range 1-10mm and a relative permittivity for the dielectric that is non-infinite and greater than 1. Consider the permeability of the dielectric to be the same as free space. (*You may write your selected values below for reference.*)

a.) Suppose that a voltage is applied to the coaxial cable so that some charge density  $Q$  occurs on the inner and outer conductors. Write an expression for the D-field and E-field at all possible radii.

b.) Use your answer from part a to calculate the relationship between conductor charge and applied voltage for this cable, then calculate the capacitance per unit length of the cable.

- c.) Draw a picture of the E-field inside the cable.
- d.) On the picture you drew for part c, indicate where any surface or volume charge is distributed.
- e.) Suppose that a current  $I$  flows in the outer conductors and a current  $-I$  flows in the inner conductor. Write an expression for the H-field and B-field at all possible radii.

- f.) Use your answer from part d to calculate the inductance per unit length of the cable.
- g.) Draw a picture of the H-field inside the cable.
- h.) Use your answers from the earlier sections of this problem to calculate the cable's characteristic impedance.
- i.) Assume that this coaxial cable is lossless and calculate the velocity of a signal traveling on it. If the signal has a 10MHz frequency, calculate the signal's wavelength and phase constant.

**B4. Lightning in a Bottle***(Skills 2g, 2k, 2l, 2n, 3l)*

You manage to capture an entire bolt of lightning in a small bottle – in other words, you confine a 10C negative charge to a small volume.

a.) You are standing 100m away from the bottle and have a 1 microcoulomb positive charge on your body due to static electricity. What force will you experience due to the bottle of lightning, and what will be the direction of the force?

b.) What is the energy density of the electric field where you are standing?

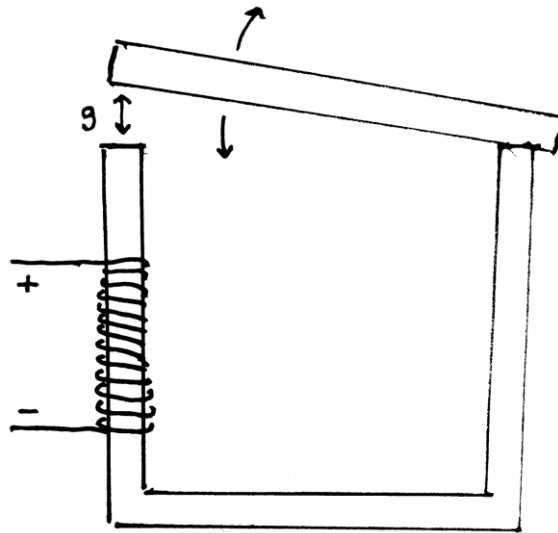
c.) Concentrating a lot of charge in a small volume like this will tend to lead to corona discharge – in other words, conduction of current through the air due to the electric field exceeding the dielectric breakdown strength of air, which is 3 MV/m. If the 10C coulomb negative charge is concentrated at a point, what is the radius around it within which corona discharge will occur?

d.) Suppose that right at this radius, the conductivity of air during corona discharge is  $10^{-5}$  S/m. Calculate the total amount of current that will flow outward.

e.) Suppose that the 10C lightning bottle moves horizontally (in the +x direction) at a velocity of 1 m/s. Earth's magnetic field has a strength of  $50 \times 10^{-6}$  T nearby and points in the +y direction. What will be the resulting magnetic force on the lightning bottle, and what direction will it point?

**B5. Magnetic Circuit***(Skills 3j, 3k, 3g)*

Below, a relay is shown. It consists of an iron core with permeability  $7000\mu_0$  with a top bar that can swivel open and closed, changing the air gap length  $g$ . When the bar is in the closed position, the iron core has a perimeter of 30cm (measuring through the middle.) The cross-section of the iron core is square and has area  $1\text{cm}^2$  and the cross-sectional area of the air gap is also  $1\text{cm}^2$ . The energizing inductor coil has 2000 turns.



a.) Initially, the top bar is open such that the air gap is 1cm in width. Calculate the total reluctance of the magnetic circuit. (Approximate the perimeter around the circuit as being uniform at all radii.)

b.) Calculate the total reluctance of the circuit assuming that the top bar is closed ( $g = 0\text{cm}$ ).



c.) If 1A of current is applied through the coil, calculate the flux density  $B$  inside the magnetic circuit assuming a 1cm air gap.

d.) Repeat part c, but assuming no air gap.

e.) Calculate the magnitude of the magnetic force that will tend to push the top bar closed assuming 1A of current in the inductor.

**B6. Electric + Magnetic Boundary***(Skills 2h, 3c, 3h, 3i)*

Suppose that a current of 100 mA/m (flowing in the +z direction) is flowing over the flat surface of a material with permeability  $1000\mu_0$ . Above the surface in free space, you measure a B-field of 0.0001 T in the x direction and 0.0002 T in the y direction. (The x direction runs parallel to the surface.) You also measure an H-field of 0.003 V/m in the x direction and 0.004 V/m in the y direction.

a.) Determine the x and y components of the B-field inside the material.

b.) Determine the x and y components of the E-field inside the material.

c.) Determine the magnetic vector potential expression for the B-field inside the material. *(Hint: Separately determine the magnetic vector potentials of the x and y components of the B-field, then add them together.)*

d.) Suppose that this particular magnetic material saturates very easily and the B-field you calculated inside of it for part a has caused it to saturate. Will the saturation cause the B-field to change inside the material? How about the H-field?

**B7. Inductor and Capacitor***(Skills 3d, 3f)*

A 1000-turn ideal solenoid is placed in an ambient magnetic field with amplitude 0.01T and frequency 10 kHz.

a.) Determine the flux and flux linkage through the coil.

b.) Determine the emf across the coil.

**B8. Finite Difference***(Skill 2m)*

Invent a simple problem that can be solved using the Finite Difference method, then solve it. The problem should involve a distribution of defined voltages in space and regions for which the voltage is initially undefined but for which you will find the voltage at a series of interspersed points in the space using Laplace's or Poisson's Equation.

**B9. Radio Transmission Part 2***(Skills 2a, 2b)*

Design an open-circuit stub that will match the radio transmitter in problem B1 to the load. Suppose that the signal in question is 3 MHz in frequency. The stub should be made of the same type of transmission line used for the main line in problem 1.

Write the length of the stub in wavelengths and its distance in wavelengths from the load here:

On the Smith Chart on the next page you should show the following:

- 1.) The normalized load impedance and admittance
- 2.) the input admittance of the transmission line at the place where the stub is located before the stub is added
- 3.) the distance in wavelengths between the load and the place where the stub is added
- 4.) the load admittance and input admittance of the stub, and the distance between the beginning and end of the stub