Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 2500: Engineering Probability, Fall 2022
Homework #5 Solutions

1. In this problem, X is a Gaussian random variable with mean 1.5 and standard deviation 0.25. We will also use Z, a Gaussian random variable with mean 0 and standard deviation 1. The idea is to convert every statement about X into a statement about Z so we can use the Q tables for the CDF of Z.

   (a)
$$\begin{aligned} P(X > 2) &= P\left(\frac{X-1.5}{0.25} > \frac{2-1.5}{0.25}\right) \\ &= P(Z > 2) \\ &= Q(2) \\ &= 0.0228 \end{aligned}$$

   (b)
$$\begin{aligned} P(|X-1.5| > 0.1) &= P\left(\frac{|X-210|}{0.25} > \frac{0.1}{0.25}\right) \\ &= P(|Z| > 0.4) \\ &= 2Q(0.4) \\ &= 2(0.3446) \\ &= 0.6892 \end{aligned}$$

   (c)
$$\begin{aligned} P(X \in [1.2, 1.75]) &= P\left(\frac{X-1.5}{0.25} \in \left[\frac{1.2-1.5}{0.25}, \frac{1.75-1.5}{0.25}\right]\right) \\ &= P(Z \in [-1.2, 1]) \\ &= 1 - Q(1.2) - Q(1) \\ &= 1 - 0.1151 - 0.1587 \\ &= 0.7269 \end{aligned}$$

2. Here, Y is a Gaussian random variable with mean 50 and standard deviation 6. We again must convert every statement about Y into a statement about a standard Gaussian Z, but this time we're using the inverse of the Q tables.

   (a)
   $$P(Y > a) = P\left(Z > \frac{a-50}{6}\right)$$

   This means $Q\left(\frac{a-50}{6}\right) = 0.2$. Eyeballing from the table the value z such that $Q(z) = 0.2$ is close to 0.8 (WolframAlpha tells us it's more like 0.84). Thus

   $$a = 6Q^{-1}(0.2) + 50 = 6(0.84) + 50 = 55.04$$

   (b)
   $$P(|Y - 50| < b) = P\left(|Z| < \frac{b}{6}\right)$$

   This means $Q\left(\frac{b}{6}\right) = 0.4$. That is, we want 0.4 of the total probability within the range $Z \pm \frac{b}{6}$, so there is 0.3 of the probability left in each tail. Eyeballing from the table the value z such that $Q(z) = 0.3$ is close to 0.5 (WolframAlpha tells us it's more like 0.53). Thus

   $$b = 6Q^{-1}(0.3) = 6(0.53) = 3.18$$

3. Here we have V, a Gaussian with mean $\mu_V = 424$ and variance 100 (hence $\sigma_V = 10$), and J, a Gaussian with mean $\mu_J = 400$ and variance 144 (hence $\sigma_J = 12$).

   (a) We can compute that

$$P(\text{damage } > 407 \mid \text{Vi}) = Q\left(\frac{407-404}{3}\right) \qquad = Q(1) = 0.1587 \tag{1}$$

$$P(\text{damage } > 407 \mid \text{Jinx}) = Q\left(\frac{407-400}{5}\right) \qquad = Q(1.4) = 0.0808 \tag{2}$$

   So Vi is more likely to do more than 407 damage.

   (b)

$$P(\text{damage } > 416 \mid \text{Vi}) = Q\left(\frac{416-404}{3}\right) \qquad = Q(4) = 0.0000317 \tag{3}$$

$$P(\text{damage } > 416 \mid \text{Jinx}) = Q\left(\frac{416-400}{5}\right) \qquad = Q(3.2) = 0.000687 \tag{4}$$

   So Jinx is more likely to do more than 416 damage. (Note that we don't actually have to compute the Q values in either problem, just compare the number of standard deviations away from the mean.)

   (c) Basically, we're trying to find D that is the same number of standard deviations away from the respective means, so that the areas in the two tails of the distributions are the same. This results in the simple equation

$$\frac{D-404}{3} = \frac{D-400}{5}$$

   which means that $D = 410$, i.e., that 410 is 2 standard deviations away from each of the means.

   (d) This is a conditional probability problem.

$$\begin{aligned} P(J > 412 \mid J > 406) &= \frac{P(J > 412 \cap J > 406)}{P(J > 406)} \\ &= \frac{P(J > 412)}{P(J > 406)} \\ &= \frac{P(Z > 2.4)}{P(Z > 1.2)} \\ &= \frac{Q(2.4)}{Q(1.3)} \\ &= \frac{8.198e-3}{9.680e-2} \\ &= 0.0847 \end{aligned}$$