ITE - Homework 8 - Solution

Problem 1 Current mirror

(a) $V_{BE1} = 0.7\,V$

Note that $V_{BE1} = V_{BE2} = V_{BE} = 0.7\,V$

$\Rightarrow I_{B1} = I_{B2} = I_B$ (T_1 and T_2 are identical)

Voltage drop across R_1

$$V_{R1} = \underline{\underline{V_{CC} - V_{BE}}} = \underline{\underline{V_{CC} - 0.7V}}$$

(b) Current through R_1

$$V_{R1} = I_{R1} R_1 = V_{CC} - V_{BE}$$

$$\Rightarrow \quad I_{R1} = \underline{\underline{\frac{V_{CC} - V_{BE}}{R_1}}}$$

$$I_{E1} = I_{C1} + I_{B1} = I_{C1} + I_B = \beta I_B + I_B$$

$$= (\beta + 1)\, I_B$$

$$I_{R1} = I_{C1} + I_{B1} + I_{B2} = I_{C1} + 2 I_B$$

$$= \beta I_B + 2 I_B = (\beta + 2)\, I_B$$

Eliminating I_B from the two eqns. above yields

$$I_{E1} = (\beta+1)\, I_B = (\beta+1)\,\frac{I_{R1}}{\beta+2}$$
$$= \underline{\underline{I_{R1}\,\frac{\beta+1}{\beta+2}}} \approx \underline{\underline{I_{R1}}}$$

(c) $$I_{B1} = \frac{I_{E1}}{\beta+1} = \frac{I_{R1}\,\frac{\beta+1}{\beta+2}}{\beta+1} = \underline{\underline{\frac{I_{R1}}{\beta+2}}} \approx \underline{\underline{\frac{I_{R1}}{\beta}}}$$

$$I_{C1} = I_{B1}\,\beta = \frac{I_{R1}}{\beta+2}\,\beta = \underline{\underline{I_{R1}\,\frac{\beta}{\beta+2}}} \approx \underline{\underline{I_{R1}}}$$

(d) V_{B1} and V_{B2} are the same

T_1 and T_2 are identical transistors

$\Rightarrow I_{B1} = I_{B2} = I_B$

$\Rightarrow I_{E1} = I_{E2}$

$\Rightarrow I_{C1} = I_{C2}$

(e) Current through load $= I_{C2}$

$$\underbrace{I_{C2}}_{= I_{Load}} = \beta I_B = I_{C1} = \underline{\underline{I_{R1}\,\frac{\beta}{\beta+2}}} \approx \underline{\underline{I_{R1}}}$$

$\Rightarrow I_{Load} \approx I_{R1}$

(f) Current I_{R1} is "<u>mirrored</u>" to load

$\Rightarrow \underline{\underline{I_{Load} \approx I_{R1}}}$

(g) Yes, Z_{Load} can be zero. The eqns. derived above are valid for $Z_{Load} = 0$.

(h) No, Z_{Load} cannot be ∞. No I_{C2} current would flow and the current I_{R1} would not be "mirrored".

(i) Maximum voltage across Z_{Load} must be less than $(V_{CC} - \underbrace{V_{CE,sat}}_{0.2V})$

$$\Rightarrow Z_{Load} I_{R1} < V_{CC} - V_{CE,sat}$$

$$\Rightarrow Z_{Load} < \frac{1}{I_{R1}} (V_{CC} - \underbrace{V_{CE,sat}}_{0.2V})$$

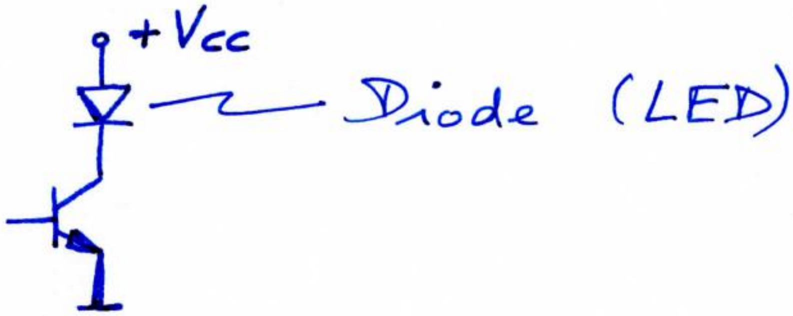
(j) $V_{CC} = 10V \qquad \beta = 100 \qquad R_1 = 500\Omega$

$$\Rightarrow I_{R1} = \frac{10V - 0.7V}{500\Omega} = \frac{9.3V}{500\Omega} = \underline{18.6\,mA}$$

$$I_{C1} = I_{R1} \frac{\beta}{\beta+2} = 18.6\,mA \frac{100}{100+2} = \underline{18.2\,mA}$$

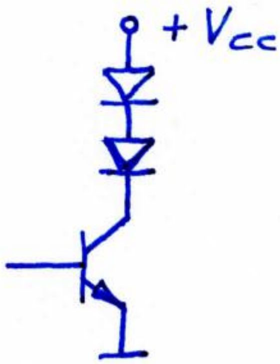
$$I_{C2} = I_{C1} = I_{Load} = \underline{18.2\,mA}$$

(k)



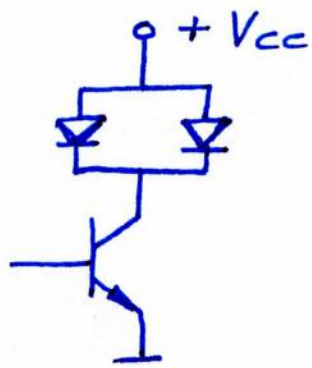
$$I_{Diode} = I_{C2} = I_{C1} = \underline{\underline{18.2\ mA}}$$

(l)



$$I_{Diode\,1} = I_{Diode\,2} = I_{C1}$$
$$= \underline{\underline{18.2\ mA}}$$

(m)



Due to symmetry, each diode carries $I_{C1}/2$ $= 18.2\ mA/2 = \underline{\underline{9.1\ mA}}$

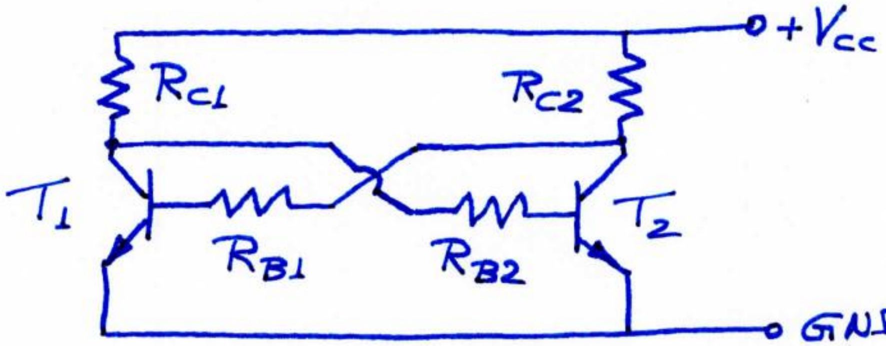
(n) We appreciate that the current through the load is 18.2 mA, independent of the type of load.

Problem 2 SRAM cell 4R 2T

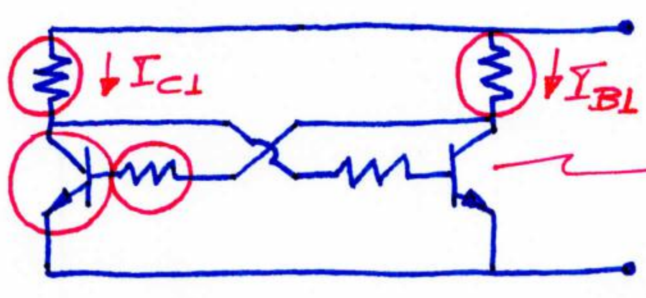
4R ↳ 4 resistors 2T ↳ 2 transistors

$V_{cc} = 5V \qquad \beta = 50 \qquad V_{CE,sat} = 0.2V$

(a) Circuit diagram



(b) Let us choose the state where T_1 is conductive $\Rightarrow T_1$ is ON $\Rightarrow V_{CE1,sat} = 0.2V$ $\Rightarrow T_2 = \text{OFF}$. Which elements consume power?



Circled elements consume power

(c) $R_{C1} \Rightarrow P = I_{C1}^2 R_{C1}$

$R_{B1} \Rightarrow P = \left(\frac{I_{C1}}{\beta}\right)^2 R_{B1} \Rightarrow$ small

$R_{C2} \Rightarrow P = \left(\frac{I_{C1}}{\beta}\right)^2 R_{C2} \Rightarrow$ small

$T_1 \Rightarrow P = V_{CE,sat} \times I_C \Rightarrow$ small

↳ 0.2V

Total power

$$P = P_{RC1} + P_{RB1} + P_{RC2} + P_{T1}$$

$\hookrightarrow$ Dominant summand

(d) R_{C1} is greatest consumer of power.

$$P \approx 1\mu W = I_{C1}^2 R_{C1}$$

$$P \approx 1\mu W = \frac{V_{RC1}^2}{R_{C1}} = \frac{(V_{CC} - V_{CE,sat})^2}{R_{C1}}$$

$$= \frac{(5V - 0.2V)^2}{R_{C1}}$$

Solve for R_{C1}: $R_{C1} = \frac{(5V - 0.2V)^2}{1\mu W} = \frac{(4.8V)^2}{1\mu W}$

$$= \underline{\underline{23.0\,M\Omega}}$$

$$\Rightarrow I_{C1} = \frac{1\mu W}{4.8V} = \underline{\underline{0.208\,\mu A}}$$

$$\Rightarrow I_{B1} = \frac{0.208\,\mu A}{\beta} = \underline{\underline{4\,nA}} \quad \text{(very small)}$$

How to choose R_{B1}? The choice is not critical as long as it keeps T_1 in the ON state. We therefore choose $R_{B1} = 23\,M\Omega$ (same vallue as R_{C1}). Other choices are acceptable.

$$\Rightarrow R_{C1} = R_{C2} = \underline{\underline{23\,M\Omega}}$$

$$R_{B1} = R_{B2} = \underline{\underline{23\,M\Omega}}$$

(e) $1\ \text{Gbit} = 10^9\ \text{bit}$

SRAM memory would consume

$$10^9 \times 1\,\mu W = \underline{\underline{1\,kW}}$$

Power is too high. Circuit consumes too much power for large-scale memory.