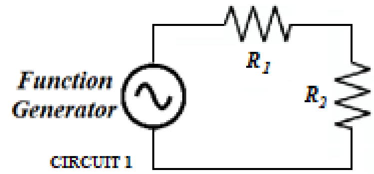
31A – AC Circuits

Background



In lecture, we deduced the following relationships for AC reactance and the relationship between current and voltage for inductors, capacitors, and resistors.

Consider a system in which the current is given by

$i(t) = I_0 \cos(\omega t)$ and $v(t) = V_0 \cos(\omega t + \varphi)$.

For all devices - $V_0 = I_0 X$ where X is the Reactance for inductors or capacitors and the Resistance for resistors. The reactance has the same units (ohms) as resistance.

For capacitors: $X = X_C = 1/\omega C$ and φ = -π/2.
For inductors: $X = X_L = \omega L$ and φ = +π/2.
For resistors: $X = R$.

When components are wired in series with one another, the current through all components must be the same: $I = \frac{V_C}{X_C} = \frac{V_L}{X_L} = \frac{V_R}{R}$, but the voltages will be out of phase with one another.

For a resistor R and capacitor C wired in series with current $i(t) = I_0 \cos(\omega t)$, the relationship between ac potential and ac current is: $V_R = I_0 R$ and $V_C = I_0 X_C$ and $V_{total} = \sqrt{(V_C^2 + V_R^2)} = I_0\sqrt{(R^2 + X_C^2)}$. For an inductor and resistor in series the equations are the same, substituting L for C in the subscripts. (Note that the reactances have different frequency dependence.)

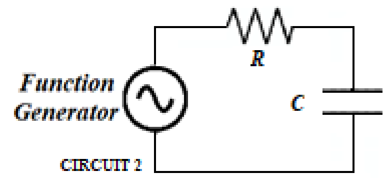
When all three components are wired together in series, the inductive and capacitive reactances are added taking the fact that one lags and one leads into account.

$V_{total} = \sqrt{(V_R^2 + (V_L - V_C)^2)} = I_0\sqrt{(R^2 + (X_L - X_C)^2)}$.

Model and Experiment: RC Circuit

Model

1. Circuit A: A sinusoidal voltage of constant amplitude V_0 = 3V and varying frequency is applied across a 100 ohm resistor and a 2.2 µF capacitor in series.



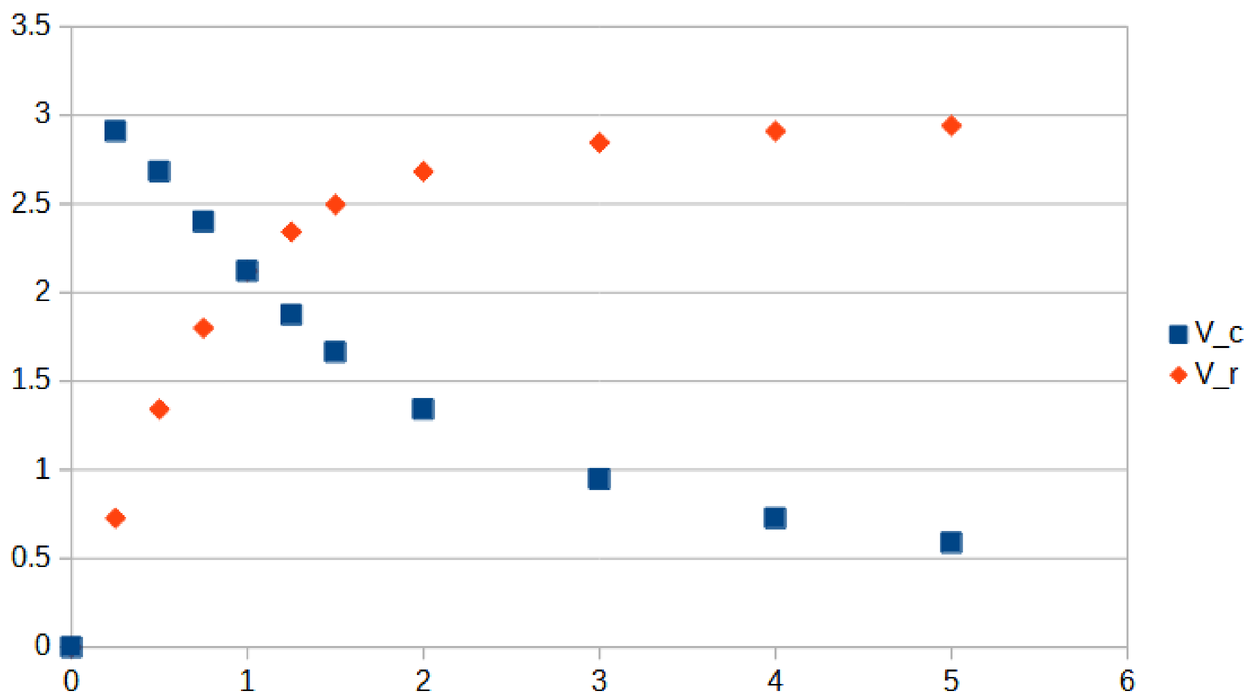
2. Calculate $\tau = RC$ and $\omega_1 = 1/RC$ for this circuit and the fill in the table.

τ = 220u s ω_1 = 4550 rad/s f_1 = 723 Hz
4545.45 723.43

$\frac{\omega}{\omega_1}$ $\left(=\frac{f}{f_1}\right)$	Frequency (cycles/s = Hz)	Frequency (radian/s)	X_C (Ω)	R (Ω)	$I = \frac{V}{\sqrt{X_C^2 + R^2}}$	$V_C = IX_C$ (V)	$V_R = IR$ (V)
0	0	0	###	100	0	0	0
0.25	180.8578899	1136.363636	400	100	0.0072760687511	2.9104275	0.727606875
0.50	361.7157798	2272.727273	200	100	0.013416407865	2.683281573	1.341640786
0.75	542.5736696	3409.090909	133	100	0.018	2.4	1.8
1.0	723.4315595	4545.454545	100	100	0.0212132034356	2.121320344	2.121320344
1.25	904.2894494	5681.818182	80	100	0.0234260642833	1.874085143	2.342606428
1.5	1085.147339	6818.181818	66.67	100	0.0249615088301	1.664100589	2.496150883
2.0	1446.863119	9090.909091	50	100	0.02683281573	1.341640786	2.683281573
3.0	2170.294679	13636.36364	33.33	100	0.0284604989415	0.948683298	2.846049894
4.0	2893.726238	18181.81818	25	100	0.0291042750044	0.727606875	2.9104275
5.0	3617.157798	22727.27273	20	100	0.0294174202707	0.588348405	2.941742027

Plot your calculated V_C and V_R against ω/ω_1 in the space below. (Excel is your friend here.)

PLOT HERE



RC Experiment

Equipment: M1K board; Electronic Breadboard; Capacitor (2.2 μF preferred), Resistor (100 Ω preferred).

3. Preliminary information.

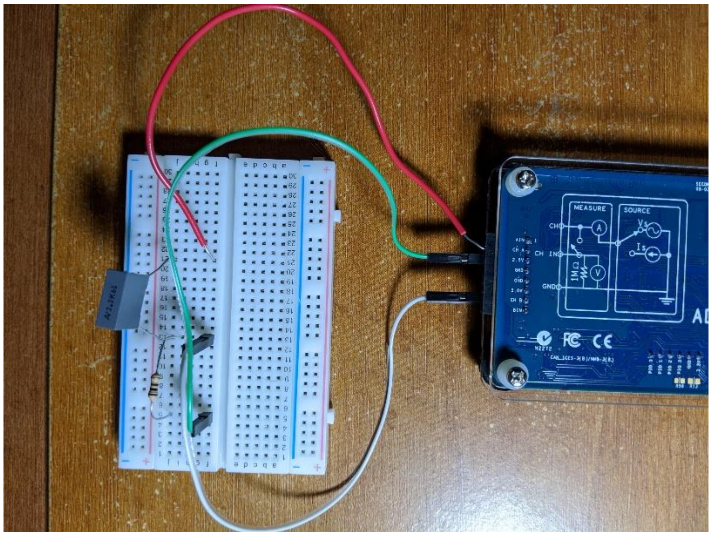
a) Record the values of your resistor (R) and capacitor (C):

$R =$ 100 ohm C 2.2uF

b) Calculate the angular frequency ω_1 and the frequency f_1 corresponding to $1/RC$ for your RC pair.

$\omega_1 =$ 4545.45 rad/s $f_1 =$ 723.43 Hz

- Set up the resistor R and capacitor C in series with the Ch A SVMI source as the function generator as shown in the diagram above. Use Ch B to measure the voltage at the center junction of the resistor and capacitor. An image of the set-up is given here.



- Set up the ALICE oscilloscope using Ch A as the potential source.
  - Use the AWG Configuration Panel to set up the measurement channels.
    - AWG CH A
      - Mode = SVMI with Term = To 2.5
      - Shape=Sine; Freq Ch A = f_1 from question 3 above.
      - Min Ch A = 1.0; Max Ch A = 4.0
      - Make sure the SYNC AWG Box at the bottom of the menu is unchecked.
    - AWG CH B to Hi-Z with Term=Open (This disconnects B from its source voltage. Other settings are unimportant.) Also, uncheck the Sync AWG box.
  - Here are suggested settings for getting started. You should vary them in order to optimize your view.
    - On the menus bar across the top of the oscilloscope frame
      - Curves menu, choose CA-V and CB-V
      - Trigger menu select CA V and Auto level.
      - Edge menu = Falling
      - Time mS/Div = 1.0 mS/Div

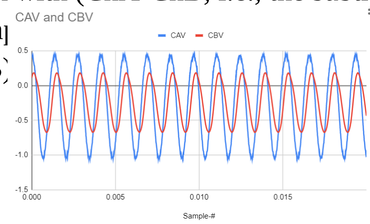
- On the settings bar across the bottom of the oscilloscope select CA V/Div, CA V Pos, CB V V/Div, and CB V Pos to get both Channel A and Channel B Voltage on the screen with each signal filling most of the vertical range. (You will find it best to set the CA and CB V Position to 2.5 V.) Set the Time/div to get a few cycles on the screen.

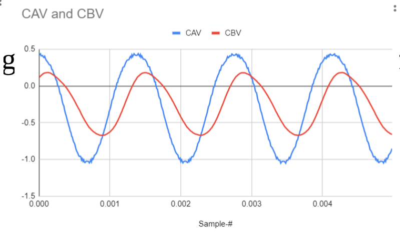
- Save the data to CSV.

- Import the data into a program such as Excel.
  - Subtract 2.5 V from both channel A and channel B to set the center of the wave at 0 V.
  - Create a new column with (ChA-ChB, i.e., the subtraction of ChB from ChA). This is the voltage across the ca

4a) Plot Ch A, Ch B, and (ChA-ChB) g nsert the plot here.

PLOT HERE





- On the right hand (CONN) menu panel in ALICE, select CA V P-P ($V_{pp}(A)$ measurement) and CB V P-P ($V_{pp}(B)$ measurement). ($V_{pp}(A)$ is the driving voltage. $V_{pp}(B)$ is the voltage across the resistor.)

  $\sqrt{V_{pp}(A)^2 - V_{pp}(B)^2}$ is the voltage across the capacitor.

5) Fill in the table below by changing the Ch A frequency and measuring the Ch B peak to peak voltage for each frequency.

$\frac{\omega}{\omega_1}\left(=\frac{f}{f_1}\right)$	Frequency (cycles/s =Hz)	Frequency (radian/s)	Ch A $V_{pp}(A)$ (V)	Ch B $V_{pp}(B)$ (V)	$\sqrt{V_{pp}(A)^2 - V_{pp}(B)^2}$ (V)
0.25					
0.50					
0.75					
1.0					
1.25					
1.5					
2.0					
3.0					
4.0					
5.0					

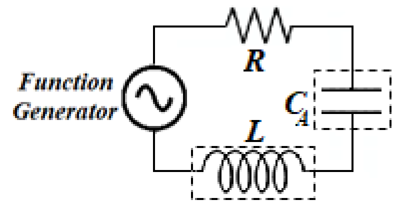
6) a) Plot $V_R = V_{pp}(B)$ and $V_C = \sqrt{V_{pp}(A)^2 - V_{pp}(B)^2}$ as functions of $\frac{\omega}{\omega_1}$ here:

PLOT HERE

7) Describe the behavior of $V_{pp}(B)$ and $\sqrt{V_{pp}(A)^2 - V_{pp}(B)^2}$ as functions of frequency. Is this consistent with your plot in the model at the start of this section?

Model and Experiment: LRC Circuit

Model



8. A sinusoidal voltage of constant amplitude V_0 = 3V and varying frequency is applied across a 100 ohm resistor, a 2.2 μF capacitor, and a 4.7 mH inductor in series.

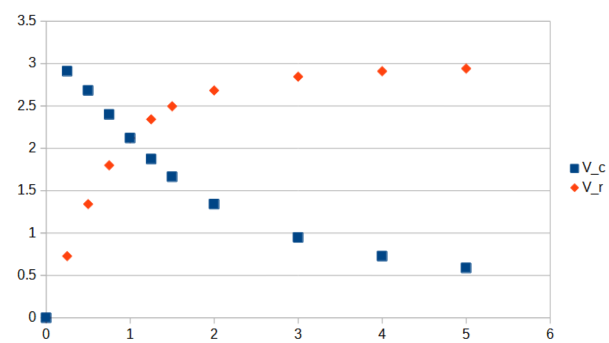
9. Calculate $\omega_N = 1/\sqrt{LC}$ for this circuit and the fill in the table.

ω_N = 9834.21 rad/s f_N = 1565.16 Hz

$\frac{\omega}{\omega_N}$ $\left(=\frac{f}{f_N}\right)$	Frequency (cycles/s = Hz)	Frequency (radian/s)	X_C (Ω)	R (Ω)	X_L (Ω)	$V_R = \frac{V_0 R}{\sqrt{R^2 + (X_C - X_L)^2}}$ (V)
0	0	0	0	100	0	0
0.25	180.85788988	1136.3636364	400	100	5.341	0.7368632383
0.50	361.71577975	2272.7272727	200	100	10.68	1.4011751994
0.75	542.57366963	3409.0909091	133	100	16	1.946174532
1.0	723.43155951	4545.4545455	100	100	21.36	2.3582109113
1.25	904.28944939	5681.8181818	80	100	26.7	2.647475109
1.5	1085.1473393	6818.1818182	66.67	100	32	2.8349071493
2.0	1446.863119	9090.9090909	50	100	42.73	2.9920974509
3.0	2170.2946785	13636.363636	33.33	100	64.09	2.867430947
4.0	2893.726238	18181.818182	25	100	85.45	2.5673161055
5.0	3617.1577975	22727.272727	20	100	107	2.2653676366

10. Plot your calculated V_R against frequency in the space below. (Excel is your friend here.)

PLOT HERE



11) Describe the behavior of V_R as a function of frequency.

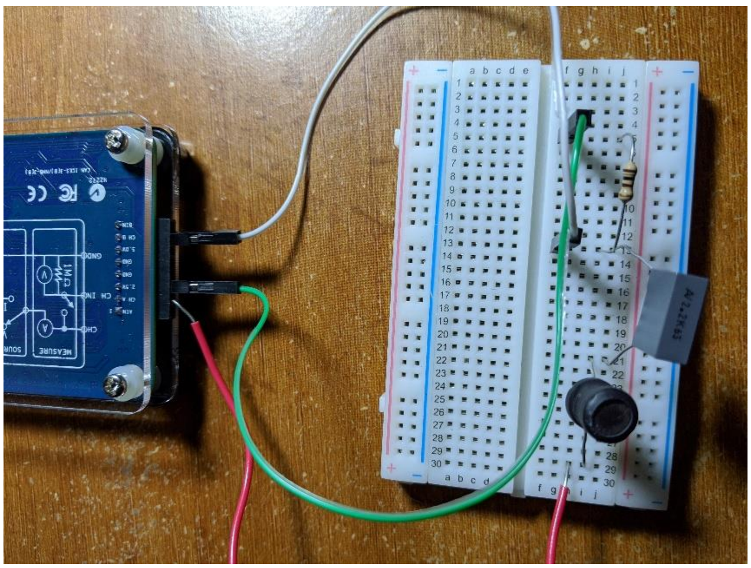
LRC Circuit Experiment

Equipment: M1K board; Electronic Breadboard; Capacitor (2.2 µF preferred); Inductor (4.7 mH preferred); Resistor (100 Ω preferred).

12) Estimate the natural frequency f_N in Hz for this circuit using *your values for L and C.*

f_N = __________________Hz

- Set up the RCL circuit shown above, consistent with the image below. Once again ChA serves as the source and Ch B serves to measure the voltage across the resistor.



  - Set the CH A frequency to f_N .
  - Set up ALICE as you did for the RC circuit above.

13) Fill in the table below by changing the Ch A frequency and measuring the Ch B peak to peak voltage for each frequency.

$\frac{\omega}{\omega_N}$ $\left(=\frac{f}{f_N}\right)$	Frequency (cycles/s = Hz)	Frequency (radian/s)	Ch A $V_{pp}(A)$ (V)	Ch B $V_{pp}(B)$ (V)
0.25				
0.50				
0.75				
1.0				
1.25				
1.5				
2.0				
3.0				
4.0				
5.0				

14) Channel B is the peak to peak voltage across the reistor. It is a measure of the peak to peak current. Plot the Ch B voltage from the table above as a function of frequency.

PLOT HERE

15) Describe the shape of the plotted data as a function of ω/ω_N. Is its behavior consistent with your model calculations in the previous section?