# Exam 1 Practice Solutions

## 1. Transient Analysis

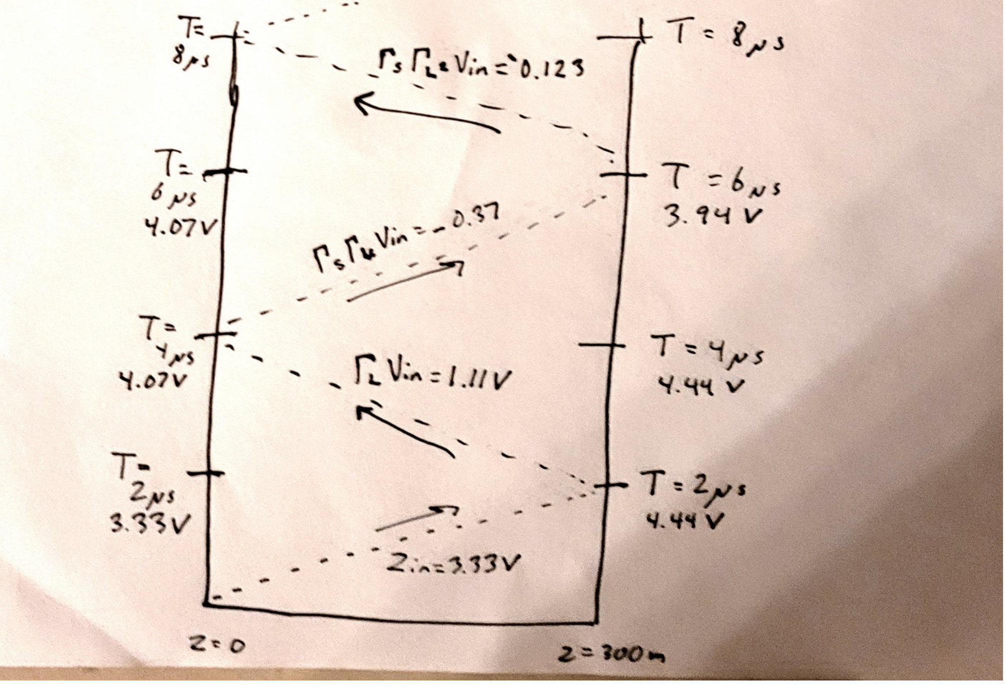
1a. Impedance of T-Line just looks like a $40\Omega$ resistor.

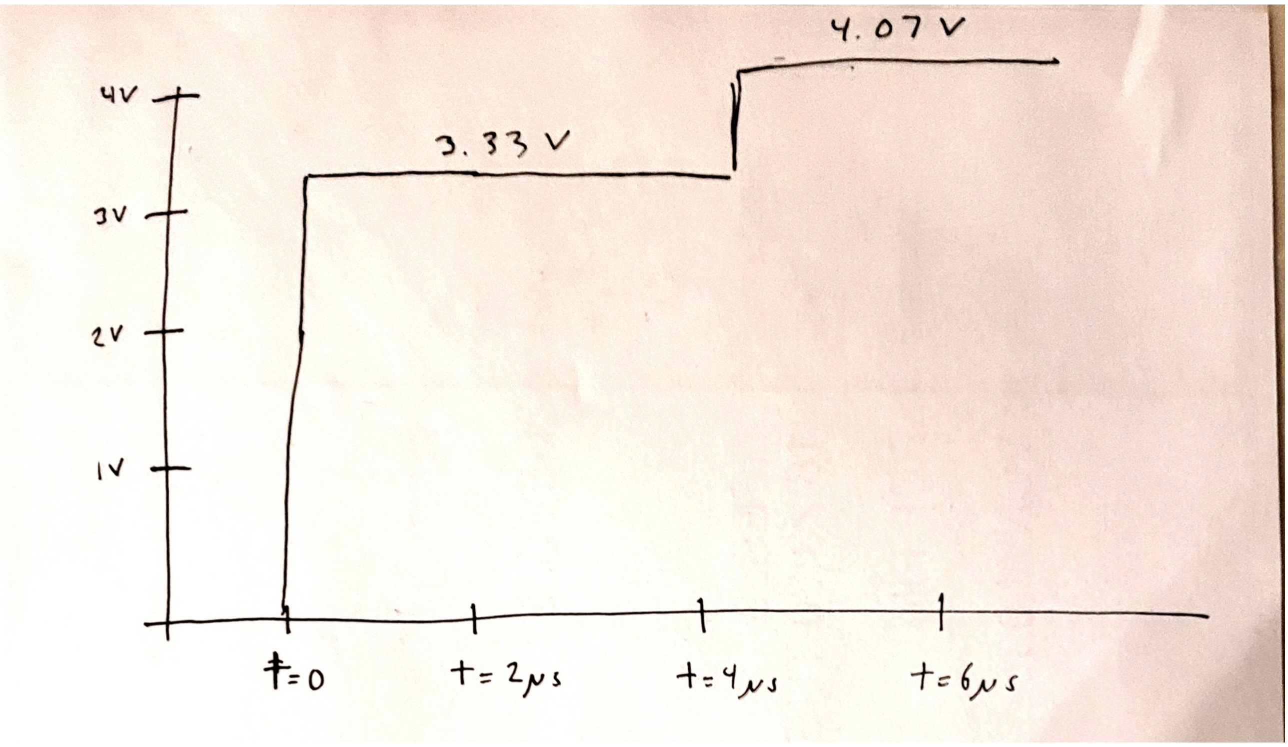
$$V^+ = 5V \cdot \frac{40}{20+40} = 3.33\ V$$

b. $$\text{time delay} = \frac{300m}{1.5 \times 10^8 m/s} = 2\mu s$$

c. $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = \frac{80-40}{80+40} = \frac{1}{3} \qquad \Gamma_S = \frac{20-40}{20+40} = -\frac{1}{3}$$




4.07 V
3.33 V
4V
3V
2V
1V
t=0
t= 2μs
t=4μs
t=6μs

**2.) Input Impedance, Reflection, and Power**

a.) $1V + \Gamma_L (1V) = 0.75\,V$

$\Gamma_L = \; = -0.25$

$$-0.25 = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L - 50}{Z_L + 50}$$

$Z_L = 30\Omega$

b.) velocity factor $= 0.6$, light speed $= 3.0 \times 10^8\,m/s$

$$\frac{0.6 \times 3 \times 10^8\,m/s}{10^6\,Hz} = 180m$$

c.) $\beta = \frac{2\pi}{\lambda} = \frac{\pi}{90}$ $L = 300m$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L}$$

$$Z_{in} = 50 \frac{(30) + j(50) \tan\left(\frac{300\pi}{90}\right)}{(50) + j(30) \tan\left(\frac{300\pi}{90}\right)} = 57.7 + j26.6\Omega$$

d.) $\frac{270m}{\lambda} = \frac{270}{180} = 3/2.$ The new section of line has length $3/2\,\lambda$. Since this is a multiple of $\frac{\lambda}{2}$, $Z_{in} = Z_L$. So the $Z_{in}$ will be the same as in part c.

e.)

$$P_{av} = \frac{|V_0^+|^2}{2Z_0}\left[1 - |\Gamma_L|^2\right]$$

$$P_{av} = \frac{(1)^2}{2(50)}\left[1 - 0.25^2\right] = 9.375\,mW$$

If the load were matched, $\Gamma_L$ would be 0.

$$P_{av} = \frac{(1)^2}{2(50)} = 10mW$$

**3.) Standing Waves**

a.) This is not a standing wave pattern. Standing wave patterns are defined by the equation 2.64 on page 70 of Ulaby. This pattern has magnitude 0 at some wavelengths, which suggests that it must correspond to a reflection coefficient of 1 or -1. But reflection coefficients of 1 and -1 generate standing wave patterns that look different from this. You can see this by playing with the standing wave simulator (https://www.rfmentor.com/node/138?no_cache=1612764476). You'll note that there is no value of reflection coefficient for which you can generate a standing wave pattern that has this shape.

b.)

3b. Yes

$$SWR = \frac{1}{0} = \infty$$

Since SWR is infinite and we have a minimum at the load, $\Gamma = -1$

$$\Gamma = -1 = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad Z_L = 0$$

**4.) Lossy Transmission Line**

4.

$$5e^{-\alpha(50)} = 2$$

$$e^{-\alpha(50)} = \frac{2}{5}$$

$$-\alpha(50) = \ln\frac{2}{5}$$

$$50\alpha = 0.916$$

$$\alpha = 0.0183$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{25} = 0.251$$

**5.) Low-Loss Transmission Line**

5a. Series impedance is $r' + j\omega l'$

$r = 0.2\ \Omega/m$

at 1K, $j\omega l' = j(2\pi \cdot 1000)(100 \times 10^{-6})$

$= j\,0.628$

Low-loss approximation holds for $\frac{r'}{j\omega l'} \ll 1$. But at 1K, it does not hold.

5b. At 1 MHz, $j\omega l' = j(2\pi \times 10^{6})(100)(10^{-6})$

$\approx j\,628$

$\frac{r'}{j\omega l'} = \frac{0.2}{j628} \ll 1,$

so low-loss approximation holds.

5c. We need to choose g such that the Heaviside condition is satisfied:

$$\frac{r'}{l'} = \frac{g'}{c'}$$

given the specified parameters for this line:

$$\frac{0.2}{100 \times 10^{-6}} = \frac{g'}{100 \times 10^{-12}}$$

$$g' = 2 \times 10^{-7} \text{ S/m}$$

$$(2 \times 10^{-7} \text{ S/m})(10 \text{ m}) =$$
$$2 \times 10^{-6} \text{ S}$$

$$\frac{1}{g'} = 500 \text{ k}\Omega$$