Hayden Fuller

fulleh

22A – Gauss' Law Concepts

Background

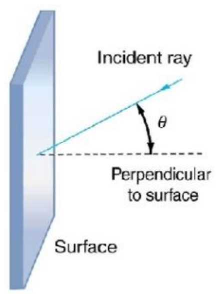
Gauss' Law relates the charge enclosed in a volume to the net electric field flux that passes through the surface that encloses the volume. The double integral with the circle means one is integrating over a closed surface.

$$\oint\!\!\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0} \qquad \text{Eq. 22a}$$

Gauss' Law can be applied to find charge enclosed from the field or to find field from charge enclosed. In order to determine the electric field at a point using Gauss' Law it is usually necessary for the problem to have a simple symmetry (spherical, cylindrical…). Note the vector dot product in the integral.

Calculating Flux

1. Calculate the flux of an electric field of magnitude 3 N/C passing through a flat surface of area 2m^2 when the surface normal is at an angle of :



0 degrees with respect to the field 6

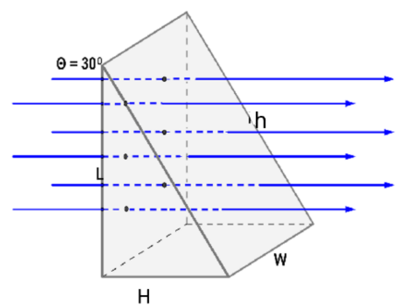
20 degrees with respect to the field 5.638

45 degrees with respect to the field 3sqrt2

60 degrees with respect to the field 3

90 degrees with respect to the field 0

2. Consider a right angle prism as sketched to the right. The apex angle is θ=30 degrees. The height is L, the width is W and the base is H. Another side of the right face has a length h. A constant electric field E_0 passes normally into the left face of the prism. Calculate the flux through each face of the prism and show that the sum of the flux through *ALL* faces of the prism is zero. (Give your answers in terms of L, W, and E_0).



$\Phi_{bottom} =$ 0

$\Phi_{left\ face} =$ -LWE

$\Phi_{right\ face} =$ hWcos(30)E=LWE

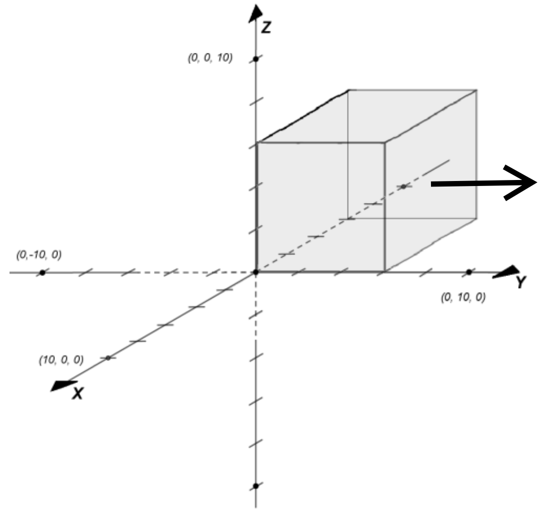
$\Phi_{back} =$ 0

$\Phi_{front} =$ 0

$\Phi_{total} =$ 0-LWE+LWE+0+0=0

Calculating Charge Enclosed from Flux

3. Given an electric field of $\vec{E} = (10\hat{\imath} + 20y\hat{\jmath} + 30\hat{k})\left(\frac{N}{C}\right)$ in the region of the cube to the right, you will solve for the charge enclosed. (Yes, the *y* is intended.) Distances are in meters.



The cube is shown here on *x*-*y*-*z* axes. The front surface of the cube is in the *y*-*z* plane.

a. Draw a normal unit vector in the center of the right hand surface.
b. What is the Cartesian unit vector that represents the normal to the right hand surface? ($\pm\hat{\imath}, \pm\hat{\jmath}, \pm\hat{k}$)
   Normal vector=__+j__

c. Write the normal vector($\pm\hat{\imath}, \pm\hat{\jmath}, \pm\hat{k}$) for the

   left__-j__, bottom__-k__, top__k__, front__i__, and back__-i__ surfaces of the cube..

Now solve for the flux through the surfaces of the cube.

d. Find the flux $\Phi = \int \vec{E} \cdot d\vec{A}$ for the left hand (*y* = *0*) surface. $\Phi_{y=0} =$ __-720__

f. Find the flux $\Phi = \int \vec{E} \cdot d\vec{A}$ for the right hand (*y* = *L*) surface. $\Phi_{y=L} =$ __720__

g. Find the flux $\Phi = \int \vec{E} \cdot d\vec{A}$ for the bottom (*z* = 0) surface. $\Phi_{z=0} =$ __-1080__

h. Find the flux $\Phi = \int \vec{E} \cdot d\vec{A}$ through the top (*z* = *L*) surface. $\Phi_{z=L} =$ __1080__

i. Find the flux $\Phi = \int \vec{E} \cdot d\vec{A}$ through the front (*x* = 0) surface. $\Phi_{x=0} =$ __360__

j. Find the flux $\Phi = \int \vec{E} \cdot d\vec{A}$ through the back (*x* = - L) surface. $\Phi_{x=-L} =$ __-360__

Now find the net flux through the cube surface and the resultant charge enclosed.

k. Find the total flux through all of the surfaces of the cube. $\Phi_{Total} =$ __0__

l. Using the total flux, find the charge enclosed by the cube surfaces. $q_{enclosed} =$ __0__

22B – Applications of Gauss' Law

1. Consider a spherical charge distribution of radius where the charge density increases with distance from the center, $\rho(r) = \begin{cases} ar^2 \text{ for } r < R \\ 0 \text{ for } r > R \end{cases}$ where a is a positive constant and R is the radius of the charged sphere. Find the electric field as a function of distance from the center ($\vec{E}(\vec{r})$).

a. Do you want to do the full three-dimensional vector integral 21d?

A) Not if I can help it. B) No. C) Sure, why not?

Answer a.__A______

Let's try using Gauss' Law instead.

$$\oiint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}} = \oiint \vec{\boldsymbol{E}} \cdot \hat{\boldsymbol{n}} dA = \frac{q_{enclosed}}{\varepsilon_0} \qquad \text{(Eq. 22a).}$$

The double integral reminds us that the integration take place over an area while the circle reminds us that the area is a closed surface!

You will simplify the left-hand side of Gauss' Law by addressing the "standard" set of questions below and simplifying in steps.

b. You want to choose a Gaussian surface of simple symmetry. Explain why a spherical surface of radius r centered on the center of charge is more appropriate than a cubical surface.

Because everything is of equal radius and all normals are parallel to the field

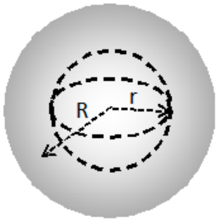
c. For your spherical surface, the electric field and the surface normal are parallel to one another at every point on the surface. Rewrite the left-hand side of equation 22a taking only this observation into account.

\int\int E dA

d. The electric field is independent of position on a spherical surface of radius r. This means that E can be taken outside the integral. Complete your simplification of the left hand side of equation 22a by completing the integral of dA over the surface i.e., $\Phi = E \oiint d\,A$.

E\int\int dA = E 4 pi R^2

e. In general for a spherically symmetric charge distribution where the charge density $\rho(r')$, the charge enclosed in a sphere of radius r is $q_{enclosed\ inside\ r} = \int_0^r \rho(r') 4\pi r'^2 dr'$.



ρ Charge Density

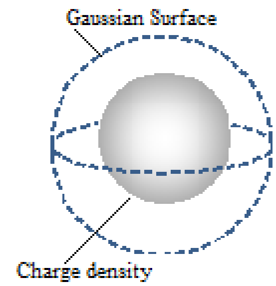
Complete this integral for the charge distribution above for $r < R$.

$q_{enclosed} =$ \int_0^R a r^2 4 pi r^2 dr = a4pi \int_0^R r^4 dr= a 4 pi 1/5 (R^5)-(0^5)
=a 4 pi 1/5 R^5

f. Solve for $E(r)$ for $r < R$ in terms of r, R, and a.

$E(r < R) =$ E 4 pi r^2 = a 4 pi 1/5 r^5 1/e0
E = a r^3 1/5 1/e0

The arguments from question 1) can be repeated to find $\oint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$ for $r > R$. In this case the Gaussian surface encloses the total charge density.



g. Using $q_{enclosed} = \int_0^R \rho(r')4\pi r'^2 dr'$, find the total charge Q of the sphere.

$Q_{total} =$ 4 pi a R^5 1/5

h. Solve for $E(r)$ for $r > R$ in terms of r, R, and a.

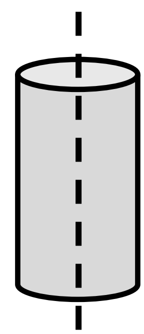
$E(r) =$ a R^2 /(5 e0 r^2)

2) Gauss' Law can be used to find the field for other simple symmetries by choosing a Gaussian surface appropriate to each charge distribution. IN this question you will solve for the field for a line of charge.

a) For an infinitely long, uniform, straight cylinder of charge of charge density λ, the appropriate Gaussian surface is:
A) A sphere centered on the center of the line.
B) A circular cylinder coaxial with the line.
C) A cube.

Answer: a). B

Explain your choice: everything is parallel and of equal radius



b) Assume a Gaussian surface that is a cylinder of radius r and length L centered on the line. It can be argued that the field from the line must point radially outward from the line, perpendicular to the line.

i. The electric flux through the top and bottom surfaces is zero. Explain why.

Because these surfaces are perpendicular to the electric field

ii. Simplify the expression $\iint \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{A}}$ for the curved cylindrical surface when it is assumed that $\vec{E}$ is normal to the surface. (Write your answer in terms of r, L, and $E(r)$.)

\int\int E dA E(r) L 2 pi r

iii. Write the charge, $q_{enclosed}$, enclosed in the cylinder in terms of λ and L.

lamba * L

iv. Write the electric field at distance r from the line in terms of $\lambda, r,$ and constants.

$E(r) =$ 2k lambda/r

4) Gauss' Law can also be used to calculate the field due to an infinitely large diameter sheet with uniform charge density. Write down the equation for the electric field for an infinite flat sheet of positive area charge density σ and identify the terms. (You can get this equation from the textbook or class notes! Identify your source.)

E = sigma /(2 e0)

Terms: sigma=surface charge density
e0=permittivity of free space

"Good Enough for Most Applications"

5) Gauss Law is only approximate for finite lines and sheets. In this section you will compare the results of Gauss' Law, calculated for ideal symmetries, with exact calculations using Coulomb's Law. (If the difference between Gauss' Law and exact calculation is less than 2%, it's "good enough for Physics 2 and MasteringPhysics.")

a. Circular Disk: The electric field on the axis of a circular disk of charge density σ is $E_z(z) = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{\left(\frac{R}{z}\right)^2+1}}\right]$ at a distance of z from the disk. Gauss' Law predicts a field of $E_z(z) = \frac{\sigma}{2\varepsilon_0}$. At what value of z/R does the prediction of Gauss' Law differ from Coulomb's law by 2%? Explain your logic in finding this number.

sigma/(2e0)~= sigma/(2e0)(1-1/sqrt((R/z)^2+1))
0.98=1-1/sqrt((R/z)^2+1))
sqrt(2499)~=50
to find where they differ by 2%, since the sigma/(2e0) terms cancel out, just find where the [1-1/sqrt((R/z)^2+1)] term =0.98, which is R/z=49.9899, or z/R~=0.020004

b. Straight Line: In activity 21D we dragged you through the calculation of the electric field a distance r from center of a line of charge of length L. The result was $E = \frac{\lambda}{2\pi r\varepsilon_0}\frac{1}{\sqrt{\left(4\frac{r^2}{L^2}+1\right)}}$. The prediction for an infinite line is $E = \frac{\lambda}{2\pi r\varepsilon_0}$. At what value of r/L does the prediction of Gauss' Law differ from Coulomb's law by 2%? Explain your logic in finding this number.

lambda/(2pi r e0)~=lambda/(2pi r e0) 1/sqrt(4(r^2/L^2)+1)
.98=1/sqrt(4 (r^2/L^2) +1)
r/L~=0.101
to find where they differ by 2%, since the lambda/(2 pi r e0) terms cancel out, just find where the 1/sqrt(4 (r^2/L^2) +1) term =.98, which is r/L~=0.101