1) Equivalent circuits

a. For $I = 8\angle 45°\text{mA}$ in phasor form with a 1.59kHz frequency, determine the voltage V1 in the time domain form.

$$\omega_1 := 2\cdot\pi 1.59\cdot 10^3 = 9.99\times 10^3$$

$$L_1 := 25\cdot 10^{-3}\cdot H$$

$$Z_{EQ} = Z_{R1} + Z_{L1} = R + j\omega_1\cdot L_1$$

$$\omega_1\cdot L_1 = 249.757\,H$$

$$R_1 := 6k\Omega \qquad Z_{L1} := \left(10j\cdot 10^3 L_1\right) = 250i\,H$$

$$Z_{EQ} = 6000 + 250j$$

in polar form the next part is easier.

$$\sqrt{6000^2 + (250)^2} = 6.005\times 10^3$$

$$\operatorname{atan}\left(\frac{250}{6000}\right) = 2.386\cdot\deg$$

$$Z.EQ = 6005\angle 2.386°$$

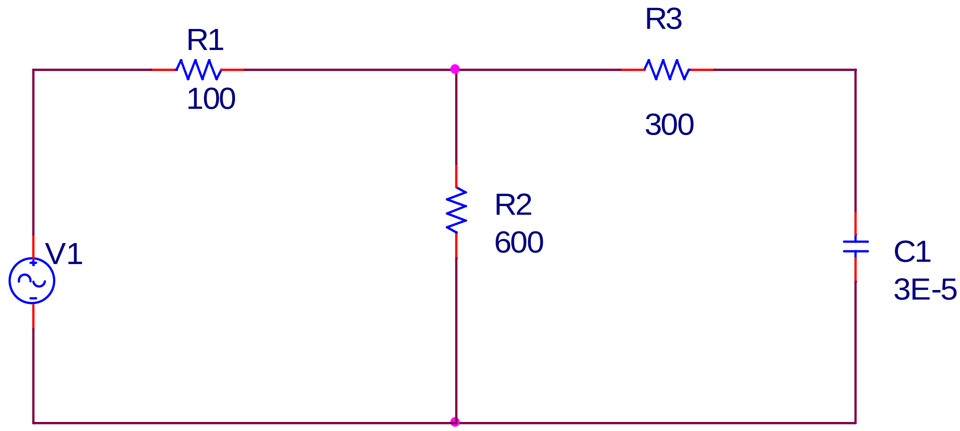
$$V_1 = I\cdot Z_{EQ} = (0.008\angle 45°)\cdot(6005\angle 2°) = 48.04\angle 47°$$

$$6005\cdot 0.008 = 48.04$$

$$\omega_1 = 9.99\times 10^3$$

$$V_1 = 48.04\cdot\cos\left(9.99\times 10^3 t + 47°\right)$$

2) First order circuits



The source is a 15V sinusoidal signal with a frequency of 31.83Hz and has zero phase.

$Z_1 := 100\Omega$ $Z_2 := 600\Omega$ $Z_3 := 300\Omega$ $C_{1a} := 3\cdot 10^{-5}$ $\omega_2 := 2\cdot\pi\cdot 31.83\text{Hz}$

$V_{1a} := 15\text{V}$ $\frac{-i}{C_{1a}\cdot\omega_2} = -166.672i\,\text{s}$ $\omega_2 = 199.994\frac{1}{\text{s}}$

$Z_{C1a} := -166.7i\Omega$

a. Determine the phasor expression for the voltage source.

$V_1 = 15\angle 0°$

b. Determine the equivalent impedance seen by the source.

$$\frac{(Z_{C1a}+Z_3)\cdot Z_2}{Z_{C1a}+Z_3+Z_2}+Z_1 = (313.268 - 71.631i)\ \Omega$$

Either rectangular or phasor/polar form is fine.

$\sqrt{(313)^2+(-71.6)^2} = 321.085$ $\text{atan}\left(\frac{-71.6}{313}\right) = -12.885\cdot\text{deg}$

$Z_{EQ2phasor} := 321 < -12.9\text{deg}$

c. Determine the phasor expression for the current through the source.

$$I_s = \frac{V_1}{Z_{EQ2phasor}} \qquad \frac{15\angle 0°}{321 < -13°} = 47\angle\cdot 13°\,\text{mA} \qquad \frac{15}{321} = 0.047$$

d. Determine the phasor expression for the voltage across C1.

Using a current divider, we can obtain the current through C1.

$$I_{C1a} = \frac{Z_2}{Z_2 + Z_3 + Z_{C1a}}\cdot(47\angle\cdot 13°\,\text{mA}) \qquad \frac{Z_2}{Z_2 + Z_3 + Z_{C1a}} = 0.645 + 0.119i$$

$$\sqrt{0.645^2 + 0.119^2} = 0.656$$

$$\text{atan}\left(\frac{0.119}{0.645}\right) = 10.453\cdot\text{deg}$$

$$(0.047 < 13°)\cdot(0.656 < 10.45°) \qquad 0.047\cdot 0.656 = 0.031$$

$$13 + 10.45 = 23.45$$

$$I_{C1a} = 31 < 23.45° \quad \text{mA}$$

$$V_{C1a} = Z_{C1a}\cdot I_{C1a} = 166\cdot(\angle - 90°)\Omega\cdot(0.031 < 23.45°)\text{A} = 5.146 < -66.55\text{deg} \qquad \text{V}$$

$$166\cdot 0.031 = 5.146$$
$$-90 + 23.45 = -66.55$$

e. Determine the time domain expression for the voltage across C1.

$$V_{C1a} = 5.146\cdot\cos(200t - 66.55°)$$

f. Determine the transfer function, H(s) = VC1(s) / Vs(s), for the above RC circuit.

Find the Thevinin equivalent

$$V_{TH} := \frac{Z_2}{Z_1 + Z_2} \cdot V_{1a} = 12.857\,V$$

$$R_{TH} := \frac{Z_1 \cdot Z_2}{Z_1 + Z_2} + Z_3$$

$$R_{TH} = 385.714\,\Omega$$

$$V_{C1a} = \frac{\frac{1}{sC_{1a}}}{385.7 + \frac{1}{sC_{1a}}} \cdot V_{TH}$$

because we need Vs subsitute in Vth equation

$$V_{C1a} = \frac{\frac{1}{sC_{1a}}}{385.7 + \frac{1}{sC_{1a}}} \cdot \left(\frac{Z_2}{Z_1 + Z_2} \cdot V_s \right)$$

$$\frac{Z_2}{Z_1 + Z_2} = 0.857$$

$$H(s) = \frac{V_{C1}}{V_s} = \frac{-166.7j}{385.7 - 166.7j} \cdot 12.857$$

g. Verify your soltuion to part d. using the transfer function (remember s = jω in AC steady state).

$$H(j200) = \frac{-166.7j}{385.7 - 166.7j} \cdot 12.857$$

for ω = 200 rad/s

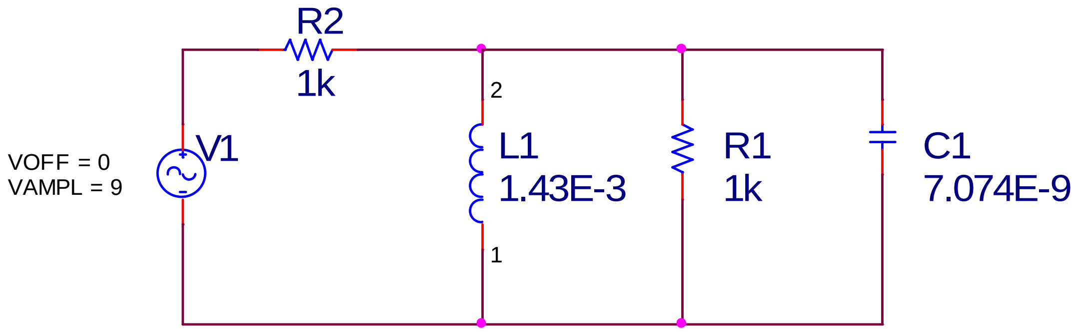
$$\frac{-166.7j}{385.7 - 166.7j} \cdot 12.857 = 2.024 - 4.682i$$

$$\sqrt{2.024^2 + (-4.682)^2} = 5.101$$

$$\operatorname{atan}\left(\frac{-4.682}{2.024} \right) = -66.621 \cdot deg$$

$$H_{j200} = 5.1 < -66deg$$

3) Phasors- RLC



a. Using phasor analysis, determine the voltage across the capacitor when the source is 50kHz.

Find the admittance, the invert to get inductance. Use this in your analysis.

$$Y_{EQ} = \frac{1}{R_1} + \frac{1}{s \cdot L_1} + s \cdot C_1$$

$$\omega_3 := 2 \cdot \pi \cdot 50 \times 10^3$$

$$\omega_3 = 3.142 \times 10^5$$

$$Y_{EQ} = \frac{1}{R_1} + \frac{1}{j\omega L_1} + j\omega C_1$$

$$Y_{EQ} := \left[\frac{1}{1000} + \frac{-j}{(\omega_3) \cdot (1.43 \cdot 10)^{-3}} + (\omega_3) \cdot 7.07j \cdot 10^{-9} \right]$$

$$\left(\omega_3 \cdot 1.433 \cdot 10^{-3}\right)^{-1} = 2.221 \times 10^{-3}$$

$$(\omega_3) \cdot 7.07j \cdot 10^{-9} = 2.221i \times 10^{-3}$$

$$Y_{EQ} = 0.001 - 0.0022j + 0.0022j$$

they cancelled

$$Y_{EQ} = 0.001$$

$$Z_{EQ} = 1000$$

$$V_C = \frac{Z_{EQ}}{Z_{EQ} + R_2} \cdot V_1 = \frac{1000}{1000 + 1000} \cdot (9\angle 0°)$$

$$V_C = 4.5\angle 0°$$

b. Using phasor analysis, determine the votlage across the capacitor when the source is 50 Hz. (reminder: -90degrees is -j) ***Partial answer check: ZRLC = 0.45j***

$$\omega_{3b} := 2\cdot\pi\cdot 50$$

$$Y_{EQ} = \frac{1}{1000} + \frac{-j}{(314)\cdot 1.43\cdot 10^{-3}} + j\cdot(314)\cdot 7.07\cdot 10^{-9}$$

$$\omega_{3b} = 314.159$$

$$Y_{EQ} = 0.001 - 2.22j + 0.0000022j$$

$$Y_{EQ} = 0.001 - 2.22j \qquad \sqrt{0.001^2 + (-2.22)^2} = 2.22$$

$$YEQ = 2.22\angle -90° \qquad \operatorname{atan}\left(\frac{-2.22}{0.001}\right) = -89.974\cdot deg$$

$$Y_{EQ} = -2.22j$$

$Z_{EQ} = 0.45j$ Answer check

$$V_C = \frac{Z_{EQ}}{Z_{EQ} + R_1}\cdot V_1 = \frac{0.45j}{0.45j + 1000}\cdot(9\angle 0°)$$

Very close to 0

At relatively low frequencies, the inductor looks like a short circuit.

c. Using phasor analysis, determine the voltage across the capacitor when the source is 50MHz (50E6Hz).(reminder: 90degrees is j)

$$\omega_{3c} := 2\cdot\pi\cdot 50\cdot 10^6$$

$$Y_{EQ} = \frac{1}{1000} + \frac{-j}{\left(3.14\cdot 10^8\right)\cdot 1.43\cdot 10^{-3}} + j\cdot\left(3.14\cdot 10^8\right)\cdot 7.07\cdot 10^{-9}$$

$$\omega_{3c} = 3.142\times 10^8$$

$$Y_{EQ} = 0.001 - 0.00000022j + 2.22j$$

$$Y_{EQ} = 0.001 + 2.22j \qquad \sqrt{0.001^2 + (-2.22)^2} = 2.22$$

$$YEQ = 2.22\angle 90° \qquad \operatorname{atan}\left(\frac{2.22}{0.001}\right) = 89.974\cdot deg$$

$$Y_{EQ} = 2.22j$$

$Z_{EQ} = -0.45j$ Answer check

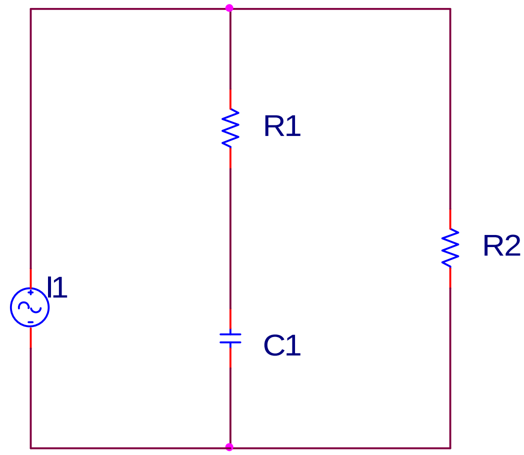
$$V_C = \frac{Z_{EQ}}{Z_{EQ} + R_1}\cdot V_1 = \frac{-0.45j}{-0.45j + 1000}\cdot(9\angle 0°)$$

Very close to 0

At relatively high frequencies, the capacitor looks like a short circuit.

4) Transfer functions

Determine the transfer functions in the following circuit. Determine the behavior of the transfer function as $\omega \to 0$ and $\omega \to \infty$



a. Voltage across C1 relative to the source voltage $H(s) = \dfrac{V_{C1}(s)}{I_1(s)}$
Using the current divider formula (then ohms law)

$$V_{C1} = \frac{1}{sC} \cdot \left(\frac{R_2}{R_2 + R_1 + \frac{1}{sC}} \cdot I_1 \right)$$

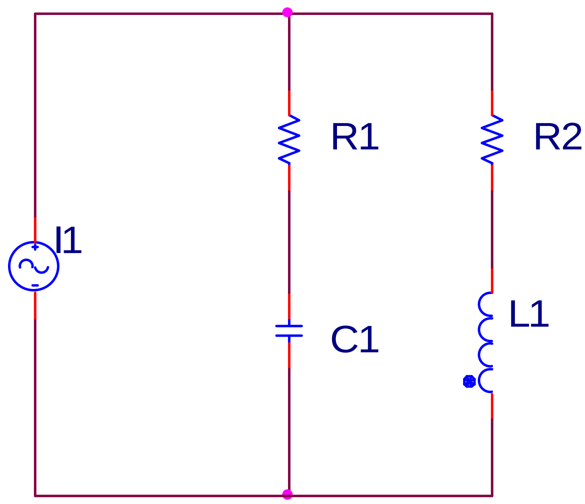
$$H(s) = \frac{V_{C1}}{I_1} = \frac{R_2}{s(R_2 + R_1) \cdot C + 1} = \frac{\frac{R_2}{\left[(R_1+R_2) \cdot C\right]}}{s + \frac{1}{(R_1 + R_2)C}}$$

b. Determine the magnitude of the transfer function as frequency approaches zero, $|H(s \to 0)|$

$$H(s \to 0) = \frac{\frac{R_2}{\left[(R_1+R_2) \cdot C\right]}}{s + \frac{1}{(R_1 + R_2)C}} = \frac{\frac{R_2}{\left[(R_1+R_2) \cdot C\right]}}{\frac{1}{(R_1+R_2)C}} = R_2$$

c. Determine the magnitude of the transfer function as frequency approaches infinity, $|H(s \to \infty)|$

$$H(s \to \infty) = \frac{\frac{R_2}{\left[(R_1+R_2) \cdot C\right]}}{s + \frac{1}{(R_1 + R_2)C}} = \frac{\frac{R_2}{\left[(R_1+R_2) \cdot C\right]}}{s} = 0$$



d. Voltage across L1 relative to the source current $H(s) = \dfrac{V_{L1}(s)}{I_1(s)}$

This is a current divider

$$H(s)=\frac{V_L(s)}{I_1(s)}=\frac{R1+\dfrac{1}{sC1}}{R1+\dfrac{1}{sC1}+R2+sL1}sL1=\frac{s^2R1C1L1+sL1}{s^2L1C1+sC1(R1+R2)+1}$$

e. Determine the magnitude of the transfer function as frequency approaches zero, $|H(s\to 0)|$

$$H(s\to 0)\approx\frac{s^2R1C1L1+sL1}{s^2L1C1+sC1(R1+R2)+1}=sL1\to 0$$

As frequency goes to zero, an inductor becomes a short circuit and the voltage across the inductor goes to zero.

f. Determine the magnitude of the transfer function as frequency approaches infinity, $|H(s\to\infty)|$

$$H(s\to\infty)\approx\frac{s^2R1C1L1}{s^2L1C1}=R1$$

As frequnecy goes to infinity, an inductor becomes an open circuit and the current goes through R1, with the associated voltage drop.