Exam 4 Crib Sheet

Full Version of Maxwell's Equations

Integral Form	Differential Form
$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
$\oint \vec{B} \cdot d\vec{S} = 0$	$\nabla \cdot \vec{B} = 0$
$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\oint \vec{D} \cdot dS = \oint \rho dv = Q_{encl}$	$\nabla \cdot \vec{D} = \rho$

Complex Permittivity and EM Waves In Media

$\epsilon_c = \epsilon' - j\epsilon''$	$\epsilon' = \epsilon \quad \epsilon'' = \frac{\sigma}{\omega}$
Skin Depth: $\delta_s = \frac{1}{\alpha}$	$\eta = \frac{\lvert\tilde{E}\rvert}{\lvert\tilde{H}\rvert}$

Low-Loss Dielectric

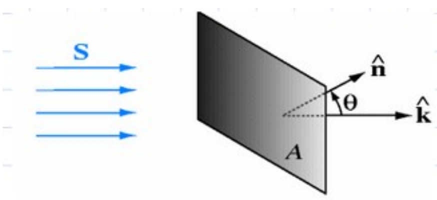
$\frac{\epsilon''}{\epsilon'} < 0.01$	$\eta = \sqrt{\frac{\mu}{\epsilon}}$
$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}$	$\beta = \omega\sqrt{\mu\epsilon}$

Good Conductor

$\frac{\epsilon''}{\epsilon'} > 100$	$\eta = (1+j)\frac{\alpha}{\sigma}$
$\alpha = \sqrt{\pi f \mu \sigma}$	$\beta = \sqrt{\pi f \mu \sigma}$

EM Waves and Energy

Poynting Vector: W/m^2 $\vec{S} = \vec{E} \times \vec{H}$	$P = \int_A (\vec{S} \cdot \hat{a}_n) dA$
$P = \lvert S \rvert A cos\theta$	$S_{av} = \hat{a}_z \frac{\lvert\tilde{E}\rvert^2}{2\eta}$ W/m^2



Wave Polarization

Polarization phasor expression:

$$\tilde{E}(z) = (a_x\hat{a}_x + a_y e^{j\delta}\hat{a}_y)e^{-jkz}$$

Linear polarization inclination angle:

$$\psi = tan^{-1}(\frac{a_y}{a_x})$$

For circular polarization,

when $\delta = +\pi/2$, polarization is left-handed (rotates clockwise)
when $\delta = -\pi/2$, polarization is right-handed (rotates counter-clockwise)

Elliptical polarization auxiliary angle:

$$\psi_0 = tan^{-1}(\frac{a_y}{a_x})$$

Elliptical polarization rotation angle (γ):

$$tan2\gamma = (tan2\psi_0)cos\delta$$

(for $-\pi/2 \leq \gamma \leq \pi/2$)

Elliptical polarization ellipticity angle (χ):

$$sin2\chi = (sin2\psi_0)sin\delta$$

(for $-\pi/4 \leq \chi \leq \pi/4$)

Magnetic field direction:

$$\tilde{H}(z) = \hat{a}_z \times \frac{\tilde{E}_z}{\eta}$$

Wave Reflection

Wave Phasor with E pointed in x and propagating in z: $$\tilde{E}_1(z) = \hat{a}_x E_0^i (e^{-\gamma_1 z} + \Gamma e^{\gamma_1 z})$$
$$\gamma_1 = k_1 = \alpha_1 + j\beta_1$$
Normal Incidence Reflection Coefficient: $$\Gamma = \frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\epsilon_{r1}} - \sqrt{\epsilon_{r2}}}{\sqrt{\epsilon_{r1}} + \sqrt{\epsilon_{r2}}}$$
Normal Incidence Transmission Coefficient: $$\tau = 1 + \Gamma = \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$
Snell's Law $$k_1 sin\theta_1 = k_2 sin\theta_2 \quad n_1 sin\theta_1 = n_2 sin\theta_2$$ $$\eta = \frac{\eta_0}{n}$$
Parallel Polarization Reflection: $$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} \qquad \tau_{\parallel} = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1}$$ $$1 + \Gamma_{\|} = \tau_{\|} \frac{cos\theta_2}{cos\theta_1}$$

Perpendicular Polarization Reflection:

$$\Gamma_{\perp} = \frac{E_{m1}^{-}}{E_{m1}^{+}} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

$$\tau_{\perp} = \frac{E_{m2}^{+}}{E_{m1}^{+}} = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$

$$1 + \Gamma_{\perp} = \tau_{\perp}$$

Critical Angle:

$$\theta_C = sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Perpendicular Brewster Angle:

$$\sin\theta_{B\perp} = \sqrt{\frac{1-(\mu_1\epsilon_2/\mu_2\epsilon_1)}{1-(\mu_1/\mu_2)^2}}\ .$$

Parallel Brewster Angle

$$\sin\theta_{B\parallel} = \sqrt{\frac{1-(\epsilon_1\mu_2/\epsilon_2\mu_1)}{1-(\epsilon_1/\epsilon_2)^2}}\ .$$