Homework # 5

Due: Wednesday, July 19th

Problem 1. (25 pts) Express the following signal in terms of sinusoids.

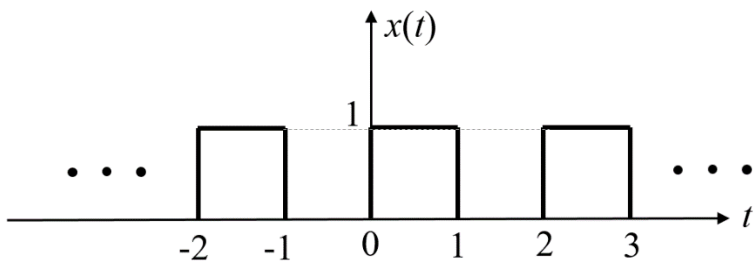
$$x(t) = je^{jt} - je^{-jt} + (1+j)e^{2jt} + (1-j)e^{-2jt}$$

That is, find real numbers A_1, A_2, ϕ_1, ϕ_2 in the sinusoidal representation below:

$$x(t) = A_1 \cos(t - \phi_1) + A_2 \cos(2t - \phi_2).$$

The amplitude A_1 and A_2 have to be positive.

Problem 2. (25 pts) Consider the periodic signal $x(t)$ shown below. It can be expressed using the exponential Fourier series as



$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where $\omega_0 = {}^{2\pi}/_{T}$ with T being the period of the signal. The Fourier Series coefficients a_k are calculated by

$$a_k = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t}\, dt.$$

Calculate the Fourier Series coefficients of $x(t)$. You must show your integral calculation (i.e., do not refer to table or textbook). Calculate the zeroth coefficient (*k=0*) separately if you encounter division by zero in your calculation.

Problem 3. (25 pts) Consider the following signal

$$x(t) = 2\cos\left(300\pi t + \frac{\pi}{3}\right) + 5\sin(600\pi t) - 10\cos\left(900\pi t + \frac{\pi}{4}\right)$$

a) (10 points) Find the fundamental period of $x(t)$.
b) (20 points) Use Euler's formula to find the Fourier Series $x(t)$.

Problem 4. (25 pts) Consider a complex signal $x(t)$ with Fourier coefficients $\{X_k\}$.

a) (15 points) Find the Fourier coefficients of Even$\{x(t)\}$ in terms of $\{X_k\}$.
b) (15 points) Find the Fourier coefficients of Odd$\{x(t)\}$ in terms of $\{X_k\}$.

1)
e^jx=cosx+jsinx
cosx=(e^jx+e^-jx)/2, 2cosx=e^jx+e^-jx
sinx=(e^jx-e^-jx)/2j, 2jsinx=e^jx-e^-jx
j e^jt - j e^-jt
e^jpi/2 e^jt + e^-jpi/2 e^-jt + e^2jt + e^-2jt + e^jpi/2 e^2jt + e^-jpi/2 e^-2jt
e^j(t+pi/2) + e^-j(t+pi/2) + e^j(2t) + e^-j(2t) + e^j(2t+pi/2) + e^-j(2t+pi/2)
2cos(t+pi/2) + 2cos(2t) + 2cos(2t+pi/2)
2cos(t+pi/2) + 2cos(2t) + 2cos(2t+pi/2)
2cos(t+pi/2) + 2sqrt(2)cos(2t+pi/4)
A1=2
A2=2sqrt2
phi1=0
phi2=pi/4

2)
T=2, w0=pi, a0=1/2
ak=1/2 int_0^2 x(t)e^(-j k pi t) dt
ak=1/2 int_0^1 e^(-j k pi t) dt
ak=1/2 1/(-j k pi) e^(-j k pi t)|_0^1
ak=1/2 1/(-j k pi) (1-e^(-j k pi))
ak=j/(2 k pi) (1-e^(-j k pi))
ak=j/(2 k pi) (1-(-1)^k)
ak=(1-(-1)^k) j / (2 k pi)

3)
a)
T1,T2,T3=1/150,1/300, 1/450
T=LCM=1/150, f0=150, w0=300pi
b)
x(t)=2(e^(j(300pit+pi/3))+e^(-j(300pit+pi/3)))/2
+5(e^(j(600pit))-e^(-j(600pit)))/2j
+-10(e^(j(900pit+pi/4))+e^(-j(900pit+pi/4)))/2
=sum(xk e^(j k 300pi t))
=(x1=e^(j(pi/3))) e^(j (1) 300pi t) + (x-1=e^(-j(pi/3))) e^(j (-1) 300pi t)
+(x2=5/2j) e^(j (2) 300pi t) + (x-2=-5/2j) e^(j (-2) 300pi t)
+(x3=-5e^(j(pi/4))) e^(j (3) 300pi t) + (x-3=-5e^(-j(pi/4))) e^(j (-3) 300pi t)
xk=[-5e^(-j pi/4) 5j/2 e^(-j pi/3) _0_ e^(j pi/3) -5j/2 -5e^(j pi/4)]

4)
a)
Even(x(t))=(x(t)+x(-t))/2
(Xk + X-k)/2
b)
Odd(x(t))=(x(t)-x(-t))/2
(Xk - X-k)/2