Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 2500: Engineering Probability, Fall 2022
Homework #1 Solutions

1. (a) The sample space contains 13 outcomes, given by the following set:

$$\mathcal{S} = \{RR, RF, RDD, FR, FFD, FDD, FDF, DRD, DFF, DFD, DDR, DDF, DDDD\}$$

Note that 5 of these outcomes don't take up the whole 8-hour period since there's not enough time to squeeze in another full activity.

> Grading criteria: 6 points in total
> -1 point: 1-2 outcomes are incorrect or missing
> -2 point: 3-4 outcomes are incorrect or missing
> -3 point: 5-6 outcomes are incorrect or missing
> -4 point: 7-9 outcomes are incorrect or missing
> -5 point: 10-12 outcomes are incorrect or missing
> -6 point: all outcomes are incorrect or missing

(b) The event A is the subset

$$A = \{RDD, FDD, FDF, DDR, DDF, DDDD\}$$

> Grading criteria: 6 points in total, one point for each element

(c) The event B is the subset

$$B = \{RR, RF, RDD, FR, DRD, DDR\}$$

> Grading criteria: 6 points in total, one point for each element

(d) The event $A \cap B^c$ is the set of outcomes of A that are not also in B:

$$A \cap B^c = \{FDD, FDF, DDF, DDDD\}$$

> Grading criteria: 6 points in total, 1.5 point for each element

(e) Clearly the minimum value of X is 0 since the friends may not have fought at all yet. The maximum value of X is 6 since it's only possibly to have 2 full 3-hour fights in the 8-hour window. Any other real value of X in this interval is possible. So the sample space for X is the continuous interval $[0, 6]$.

> Grading criteria: 6 points in total
> -3 point: some efforts, but incorrect
> -6 point: no answer or incorrect

2. (a) One way to think about this is how many ways there are to choose 4 of the 10 people for the 4-person team. Since order doesn't matter, the answer is $\binom{10}{4} = 210$.

Grading criteria: 10 points in total
-5 point: if you show the process but the final answer in incorrect
-10 point: no answer or incorrect

(b) There are 4 choices for the first arrival, 3 choices for the next arrival, and so on. That is, there are $4! = 24$ unique orders of arrival.

Grading criteria: 10 points in total, partial efforts will be given if the process is shown

(c) If Max and Lucas are both on the 6-person team, there are $\binom{8}{4} = 70$ ways to fill the remaining 4 slots; thus the probability is the number of desirable outcomes divided by the number of possible outcomes, or $\frac{70}{210} = \frac{1}{3}$.

Grading criteria: 10 points in total, partial efforts will be given if the process is shown. If your final answer is incorrect but it is due to error in your previous parts, the full credit will be given.

(d) Let's let ML denote the event that Max and Lucas are on the same team, and SD denote the event that Steve and Dustin are on the same team. Conditional probability tells us that

$$P(\text{ML} \mid \text{SD}) = \frac{P(\text{ML} \cap \text{SD})}{P(\text{SD})}$$

Let's compute $P(\text{SD})$ first. There are two mutually exclusive ways this can happen: either Steve and Dustin are on the 4-person team (let's call this event SD4), or they are on the 6-person team (let's call this event SD6). Remember from part (a) that there are 210 ways of choosing the teams to begin with.

How many outcomes are in SD6? Of the remaining 8 kids, we need to choose 4 to fill the remaining slots on the 6-person team, so there are $\binom{8}{4} = 70$ ways to do this. Similarly, there are $\binom{8}{2} = 28$ outcomes in SD4, so the overall probability of SD is $\frac{70+28}{210} = \frac{98}{210}$.

Now let's look at the numerator $P(\text{ML}\cap\text{SD})$. There are 4 mutually exclusive ways this can happen: all 4 kids are either on the 6-person or the 4-person team, or 2 of the kids are on one team, and 2 of the kids are on the other team.

There's only 1 outcome where all 4 kids are on the 4-person team. For the other three outcomes, we can think of it as choosing 2 of the 6 kids for the empty slots. That is, if the pairs of kids are on different teams, we need 2 kids to fill out the 4-person team, and if the 4 kids are all on the 6-person team, we need 2 kids to fill out the 6-person team. So the total number of outcomes in the numerator is $3 \cdot \binom{6}{2} + 1 = 3(15) + 1 = 46$, and the probability of the numerator event is $\frac{46}{210}$.

Putting it all together, we divide the numerator probability by the denominator probability to get the final answer, $\frac{46}{98} = \frac{23}{49}$.

Grading criteria: 20 points in total, partial efforts will be given based on the process you show. 5 points for $P(SD)$, and 5 points for each case. If you only accomplish part of the question, 10 points will be given.

3. (a) By the **Law of Total Probability** or **Total Probability Theorem**, we have

$$
\begin{aligned}
P(\text{superhero}) &= P(\text{superhero}|\text{Netflix})P(\text{Netflix}) + P(\text{superhero}|\text{HBO})P(\text{HBO}) \\
&\quad + P(\text{superhero}|\text{Disney})P(\text{Disney}) \\
&= (0.2)(0.4) + (0.3)(0.35) + (0.8)(0.25) \\
&= 0.08 + 0.105 + 0.2 \\
&= 0.385
\end{aligned}
$$

> Grading criteria: 10 points in total, 2 points for the theory name. 8 points for the probability. 2 points will be deducted if you equation is correct but there is computation error.

(b) Here we use **Bayes' Rule**:

$$
\begin{aligned}
P(\text{Disney}|\text{superhero}) &= \frac{P(\text{superhero}|\text{Disney})P(\text{Disney})}{P(\text{superhero})} \\
&= \frac{(0.8)(0.25)}{0.385} \\
&= 0.519
\end{aligned}
$$

> Grading criteria: 10 points in total, 2 points for the theory name. 8 points for the probability. 2 points will be deducted if you equation is correct but there is computation error.