Fields and Waves I
Studio Session 7
Spring 2024
Due 11:59pm, March 13th
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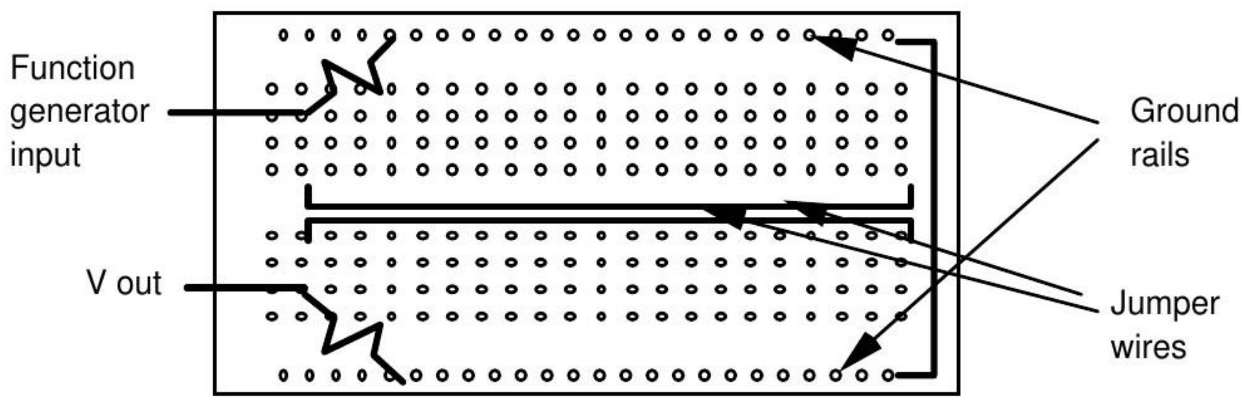
When you have completed the lab, submit the answers to the underlined questions on Gradescope. If you wish, you may work with a partner and submit one report for both of you. There is no need for a “formal” lab report.

Part 1: Capacitive Coupling

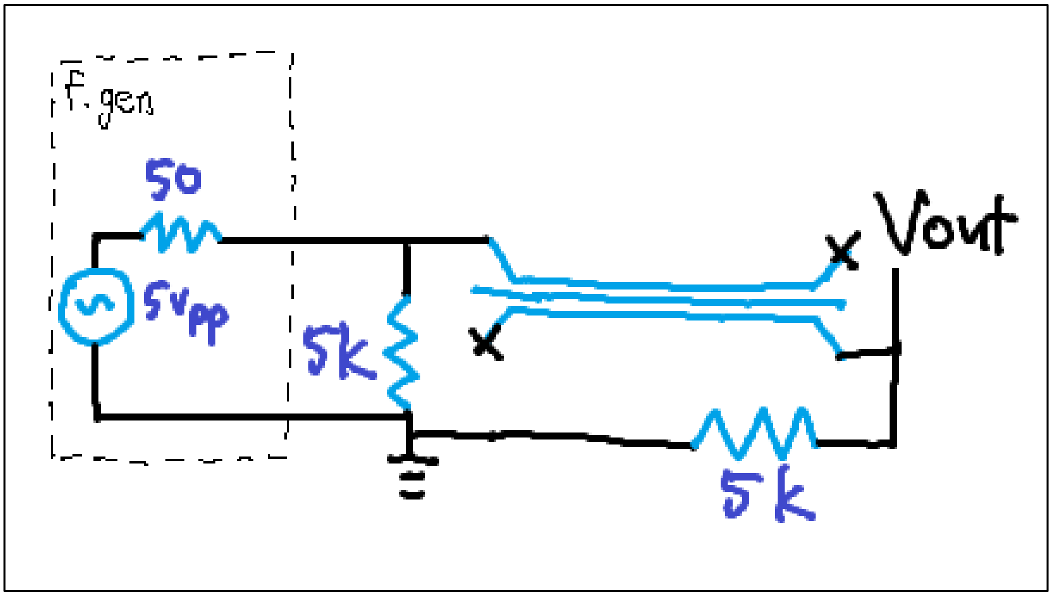
Connect the circuit together as shown in the diagram below. The jumper wires may need to be cut down to size. Cut them to be about the length of your breadboard. Note that the spatial layout of the jumper wires is important in this experiment. Use resistors that are approximately 5kΩ. Set the function generator to at least 10 kHz with a 5V P-P output. Monitor the function generator output on one scope channel and the output voltage on the other scope channel.



Question 1: How large a voltage do you measure at Vout? Is it what you expected?

100 mV yes from the capacitor we created we expect a small charge generated from the sharp increase of the square wave as if it is a very high frequency signal and then the capacitor discharges to ground.

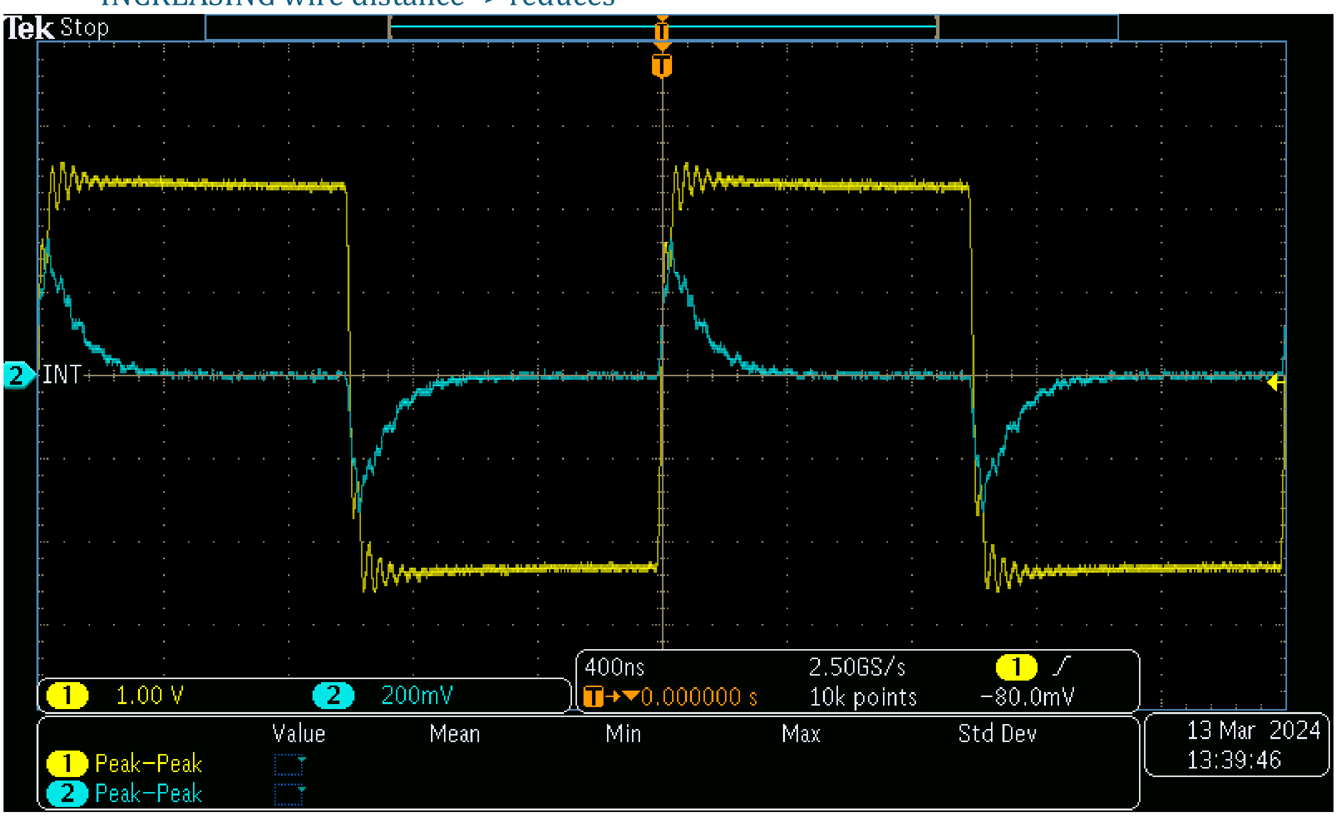
Question 2: The two wires form a capacitor. Draw a circuit diagram with the capacitance of the wires unspecified. Be sure you include the internal impedance of the function generator.



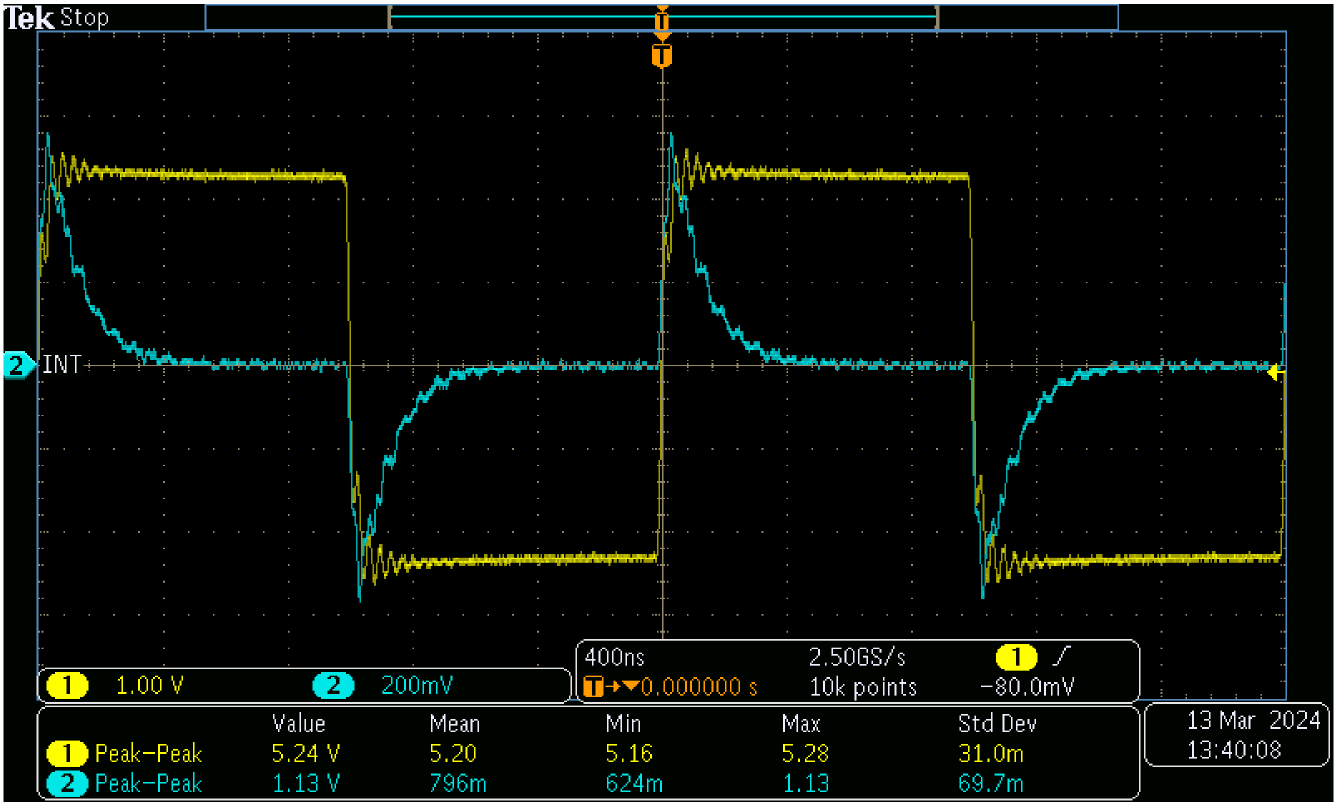
Question 3: Vary the spacing between the wires and the frequency. What changes occur? Is the peak-to-peak amplitude of the input voltage you observe on the scope equal to 5V?

For changing wire spacing:

INCREASING wire distance -> reduces

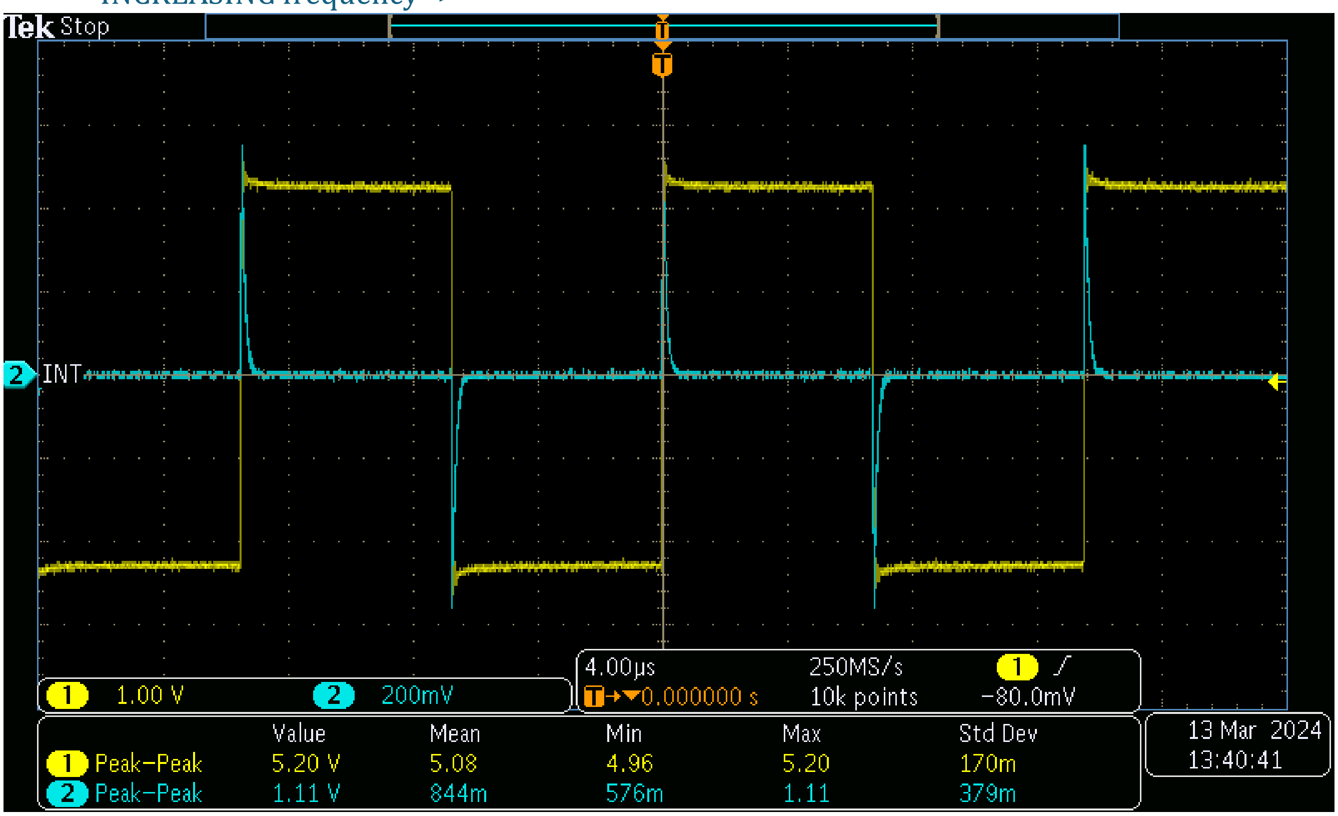


DECREASING wire distance ->

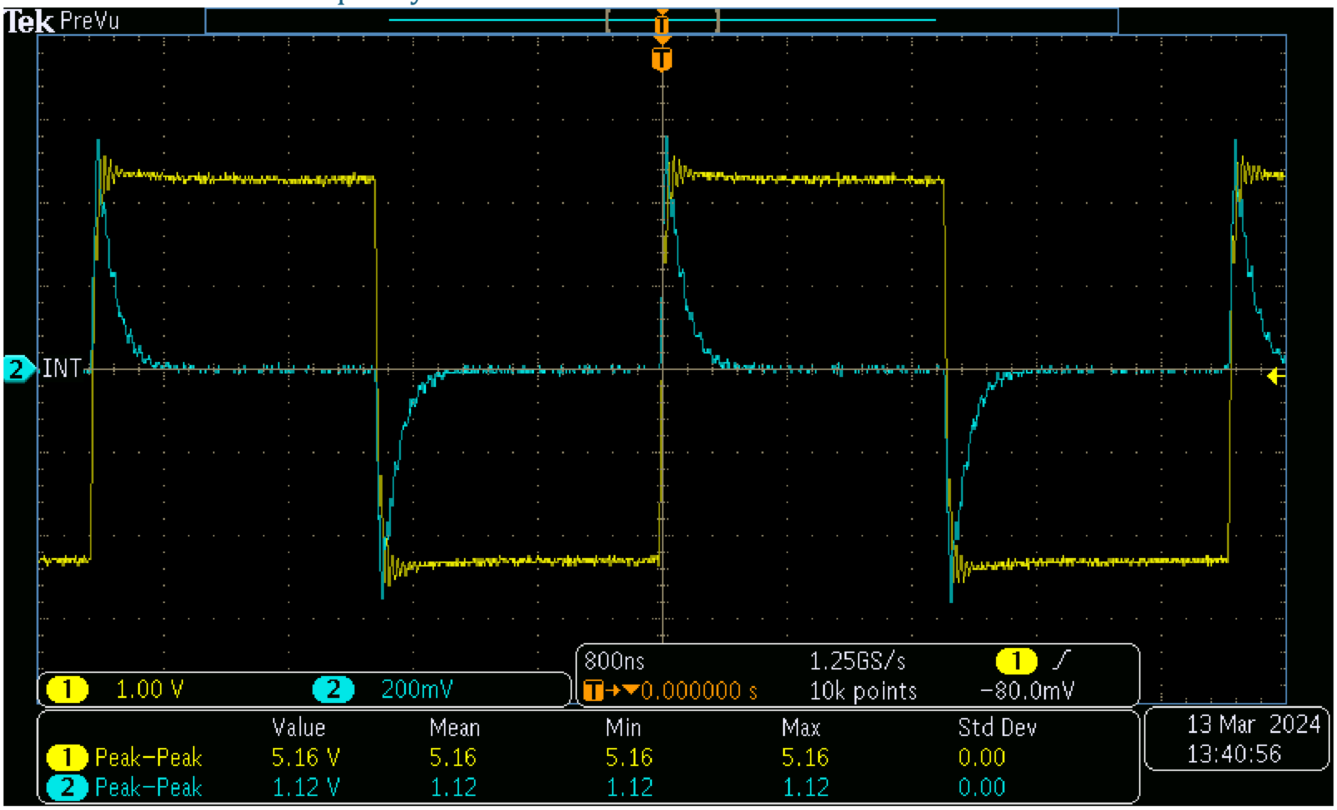


For changing function gen frequency:

INCREASING frequency ->



DECREASING frequency ->

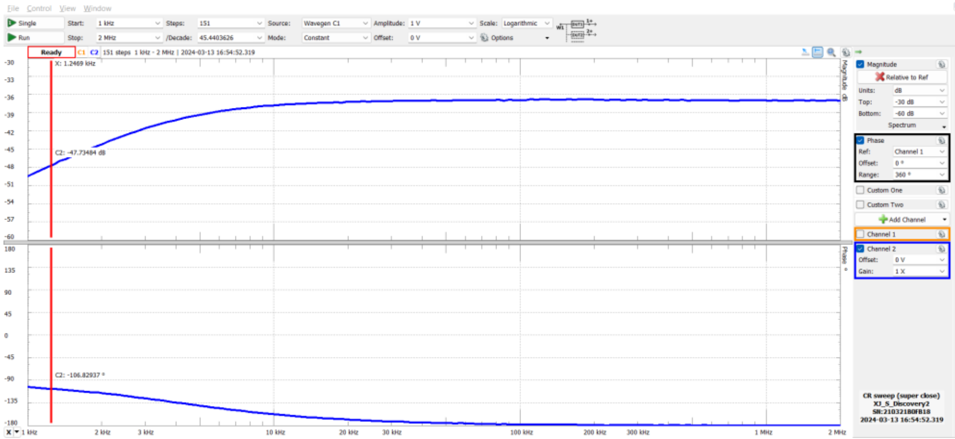


The peak-to-peak amplitude of the input voltage Vin is 5.2V

<u>Question 4:</u> Try to maximize the capacitance by choosing the optimal wire separation, then estimate the value of the two wires' capacitance as best you can. The best way to do this would be to look at the transfer function of the circuit (output amplitude over input amplitude) at a range of frequencies.

$$f_{cutoff} = \frac{1}{2\pi RC}$$
$$C = \frac{1}{2\pi \cdot R \cdot f_{cutoff}}$$

For the CR (high-pass) filter formed by the circuit, with R=5.1k, we found the transfer function/bode plot as:



When the wires are as close as possible, the 3dB cutoff was at ~4.2 kHz, so:

$$C = \frac{1}{2\pi \cdot (5100) \cdot (4200)} = 7.43\, nF$$

For further separated capacitor wires, cutoff at 4.4922 kHz, so:

$$C = \frac{1}{2\pi \cdot (5100) \cdot (4492.2)} = 6.95\, nF$$

Increasing distance between wires/plates lowered the capacitance.

$$\mathbf{C} = \frac{\boldsymbol{\varepsilon}\mathbf{A}}{\mathbf{d}}$$

These results match our expectations, since increasing the distance between plates should make the capacitance go DOWN (which is what we found).

<u>Question 5:</u> Now try and increase the capacitance further using some tin foil and/or paper. Can you estimate the new capacitance?

For same separation with foil added in between, cutoff at 4.1282 kHz, so:

$$C = \frac{1}{2\pi \cdot (5100) \cdot (4128.2)} = 7.56\, nF$$

Adding the foil in between effectively raised the capacitance of the circuit.

$$\mathbf{C} = \frac{\boldsymbol{\varepsilon}\mathbf{A}}{\mathbf{d}}$$

Adding the copper foil effectively increases the permittivity of the dielectric (epsilon), which RAISED the capacitance (again, agrees with what we found).

Part 2: Theremin

A theremin is a musical instrument that relies on the beat frequency of an LRC oscillating circuit versus a reference circuit to produce a tone. It is controlled by waving your hands near two antennas. One is a loop antenna (on the left in the image above) that controls volume. The other antenna (on the right) controls pitch. The principle of control for a theremin is that the two antennas form two variable capacitors, with each antenna operating as one plate of a capacitor and one of your hands operating as the other plate. As you move your hands, these capacitances change, changing the operation of the circuit.

In this lab, you are provided with a Burns B3 theremin to experiment with. In addition to controlling the theremin with the two antennas, there are volume and pitch adjustment knobs on the front of the instrument.

<u>Question 6:</u> Attempt to play the theremin. How do you generate a higher pitch vs. a lower pitch using the right antenna? How about a higher volume vs. a lower volume using the left antenna?

Getting closer to the right antenna generates a higher pitch. Getting closer to the left antenna reduces the volume.

<u>Question 7:</u> Explain how moving your hands changes the capacitance of the two hand-theremin systems.

The Dialectric material for the theremin is air and when moving your hand closer and farther increases and decreases the distance between the two “plates” one plate being your hand and the other being the receiver.

Question 8: Try playing the theremin with foil in your hands. Does this change the sounds that you produce from the theremin? Why or why not?

3/13/24 the TA said that we do not have foil and to skip this portion.

Part 3: Finite Difference Theremin

You will now investigate a simulated theremin using a spreadsheet and the finite difference Poisson's Equation method.

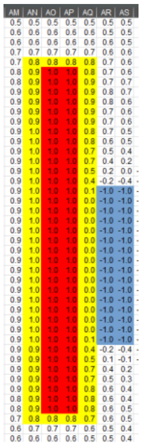
Download the spreadsheet called "Finite Difference Theremin" from the Studio Session folder on the shared drive. In this spreadsheet, the grid is set up such that each square represents one square centimeter. The red rectangle represents a theremin antenna, which we will represent as being at a voltage of 1V. The blue rectangle represents the hand of a theremin operator, represented at -1V. Note that we can only represent two dimensions in a spreadsheet, so we assume the depth (into the screen) represented by the spreadsheet is 2cm and we assume that all fields are uniform in the into-the-screen direction.

Question 9: Examine the spreadsheet's cells. In roughly a paragraph, explain as best you can how the spreadsheet works. You should include an explanation of the grid cells as well as the calculation cells below it, such as the D-field and charge.

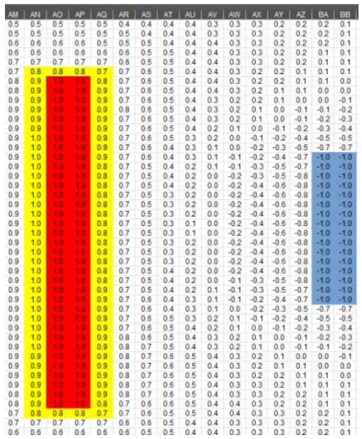
The antenna is hard coded at a value of 1V, The hand is hard coded at a value of –1V and the surrounding cells are hard coded as 0. Each other cell is an average of the 4 cardinal cells surrounding the cell. The length of cell, depth of cell and permittivity of free space are all constants. The average voltage near the antenna takes the average of the voltage on the cells on the perimeter of the antenna. The voltage difference near the antenna is then 1v - the average we just took. The E field is then the voltage difference divided by the length of the cells (so, Volts/meter = electric field strength). The D-field is the electric displacement field and it is the magnetic field times the permittivity of free space. The Antenna charge is the D-field multiplied by the area of the electrode. This makes sense, because the D-field is flux charge density, and multiplying density by some area gives the charge amount.

The Capacitance is determined using the formula $C = \frac{Q}{E}$, where Q is the amount of charge, and E is the applied voltage between the "plates" of the capacitor. Since we have the antenna at 1V and the hand at –1V, the applied voltage difference between these is 2V. So, the formula in the spreadsheet divides our Antenna Charge by 2V, then multiplies by a factor of 10^12 to convert Farads to picoFarads.

Question 10: Now, consider the case in which the theremin operator moves their hand. What is the minimum and maximum antenna capacitance that the operator can achieve? Show the capacitance for 5 different hand distances from the antenna, including the distances corresponding to the minimum and maximum capacitances you found. (You can calculate this by moving the blue cells around the grid. Don't change the shape or size of the collection of blue cells; just change where they are located. Don't move the blue cells past the green ground plane, and don't move them into the yellow cells surrounding the antenna.)

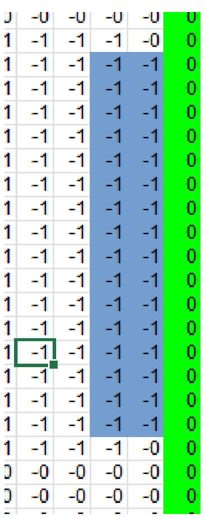


2.187472024 pF

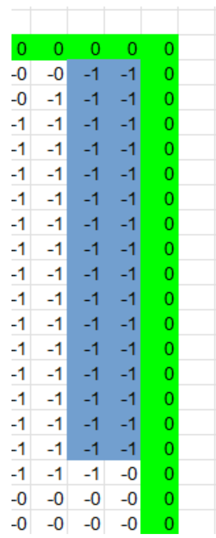


0.777567039 pF

0.53448075 pF



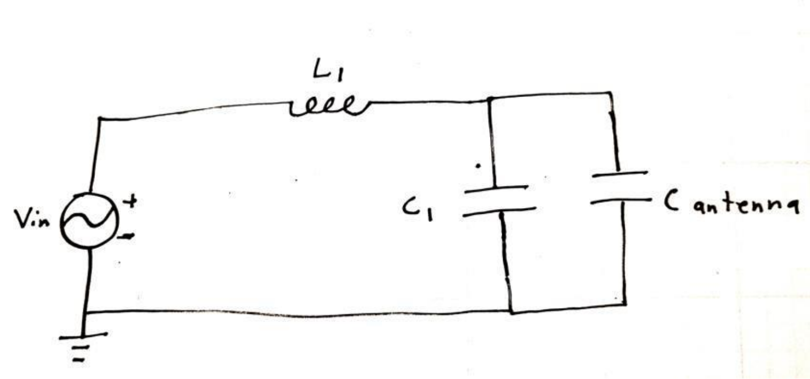
0.521537126 pF (hand all the way to the right side)



0.499719624 pF (in top right corner)

The highest capacitance found was ~2.2 pF, while the lowest was ~0.5 pF

Question 11: Assume that the theremin antenna is part of a variable capacitance oscillator circuit that looks like this.



Assume that L_1 is 1mH and C_1 is 150pF. $C_{antenna}$ is the variable capacitance of the antenna and the hand together. For each of the capacitance values you calculated in part b, use your knowledge of circuits to calculate the resonant frequency of this circuit.

$$f_{resonant} = \frac{1}{2\pi\sqrt{LC}}$$

Where $C = C_1 + C_{antenna} = (150 \cdot 10^{-12}) + C_{antenna}$

$$f_{resonant} = \frac{1}{2\pi\sqrt{(1 \cdot 10^{-3})(150 + 2.1874) \cdot 10^{-12}}} = 407.97\ kHz$$

$$f_{resonant} = \frac{1}{2\pi\sqrt{(1 \cdot 10^{-3})(150 + 0.7776) \cdot 10^{-12}}} = 409.88\ kHz$$

$$f_{resonant} = \frac{1}{2\pi\sqrt{(1 \cdot 10^{-3})(150 + 0.5345) \cdot 10^{-12}}} = 410.21\ kHz$$

$$f_{resonant} = \frac{1}{2\pi\sqrt{(1 \cdot 10^{-3})(150 + 0.5215) \cdot 10^{-12}}} = 410.22\ kHz$$

$$f_{resonant} = \frac{1}{2\pi\sqrt{(1 \cdot 10^{-3})(150 + 0.4997) \cdot 10^{-12}}} = 410.25\ kHz$$

So the resonant frequency of the circuit ranges from ~407.97 kHz to ~410.25 kHz

Question 12: Consider the theremin's reference resonator circuit, which will be identical to the circuit in part C except with $C_{antenna}$ removed so that only L_1 and C_1 are present. For each of the resonant frequencies you calculated in part c, calculate the beat frequency between the variable capacitance circuit and the reference circuit. (The beat frequency is the difference between the frequencies of the two circuits, and it is the frequency you will actually hear for a given hand position relative to the theremin's antenna.) By doing a bit of research, state whether each beat frequency is in the audible range, and if so, what musical note is closest to it. The reference below may be helpful.

https://www.liutaiomottola.com/formulae/freqtab.htm

$$f_{resonant,\,reference} = \frac{1}{2\pi\sqrt{(1\cdot10^{-3})(150\cdot10^{-12})}} = 410.94\ kHz$$

So the reference resonant frequency of the theremin (without hand) is ~410.94 kHz.
Beat frequency (Hz) = (410.94 – Resonant Frequency)*10^3

Resonant Frequency (kHz)	**Beat Frequency (Hz)**	**Audible? Musical Note?**
407.97	2970	Yes, F# / Gb, 7th Octave
409.88	1060	Yes, C, 6th Octave
410.21	730	Yes, F# / Gb, 5th Octave
410.22	720	Yes, F# / Gb, 5th Octave
410.25	690	Yes, F, 5th Octave