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CSCI 2200 — Foundations of Computer Science (FoCS) — Exam 1

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36 total /51

Q1-18 (2 POINTS EACH) — Mark one answer for each question....

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Q1	(a)	(b)	(c)	(d)	(e)
Q2	(a)	(b)	(c)	(d)	(e)
~~Q3~~	~~(a)~~	~~(b)~~	~~(c)~~	~~(d)~~	~~(e)~~
Q4	(a)	(b)	(c)	(d)	(e)
Q5	(a)	(b)	(c)	(d)	(e)
Q6	(a)	(b)	(c)	(d)	(e)
Q7	(a)	(b)	(c)	(d)	(e)
Q8	(a)	(b)	(c)	(d)	(e)
Q9	(a)	(b)	(c)	(d)	(e)

Q10	(a)	(b)	(c)	(d)	(e)
Q11	(a)	(b)	(c)	(d)	(e)
Q12	(a)	(b)	(c)	(d)	(e)
Q13	(a)	(b)	(c)	(d)	(e)
X Q14	(a)	(b)	(c)	(d)	(e)
Q15	(a)	(b)	(c)	(d)	(e)
Q16	(a)	(b)	(c)	(d)	(e)
Q17	(a)	(b)	(c)	(d)	(e)
Q18	(a)	(b)	(c)	(d)	(e)

I forgot my log rules...

Q19 (4 POINTS)

Proof, we prove the claim by contradiction

Assume $\log_2 9$ is a rational number, $\log_2 9 = \frac{a}{b}$ $a \in \mathbb{Z}$ $b \in \mathbb{N}$, a_*, b_* are minimums with no common factors,

$9 = 2^{\frac{a}{b}} = 2^a 2^{\frac{1}{b}}$

Q20 (5 POINTS)

Proof. We use leaping induction to prove $n^2 - 1$ is divisible by 8 for all odd $n \in N$

[base cases] For $P(1)$ we have $1^2 - 1 = 0$ is divisible by 8, which is true.
And for $P(3)$ we have $3^2 - 1 = 8$ is divisible by 8, which is true.

[induction step] We show $P(n) \to P(n+4)$ using a direct proof. Assume n^2+1 is divisible by 8. We must prove $P(n+4)$, i.e. $(n+4)^2 - 1$ is divisible by 8.

LHS $(n+4)^2 - 1 = n^2 + 8n + 16 - 1$

Since n is odd, let $n = 2k+1$ for $k \in N_0$

LHS $(n+4^2) - 1 = (2k+1)^2 + 8(2k+1) + 16 - 1 = (2k)^2 + 4k + 1 + 8(2k+1) - 16 - 1 = 4k^2 + 4k + 8(2k+1) + 16$

if k is even, let $k = 2j$. $4(2j)^2 + 4(2j) = 4 \cdot 4j^2 + 8j = 8(2j^2 + j)$

if k is odd, let $k = 2j+1$. $4(2j+1)^2 + 4(2j+1) = 4(4j^2 + 4j + 1) + 4 \cdot (2j+1) = 4(4j^2 + 6j + 2) = 8(2j^2 + 3j +$

therefore $4k^2 + 4k$ is divisible by 8, and can be written as $8(m)$

$(n+4)^2 = 8m + 8(2k+1) + 16 = 8(m + 2k + 1) + 2)$, so $(n+4)^2$ is divisible by 8, and $P(n) \to P(n+4)$

therefore, by induction $n^2 - 1$ is divisible by 8 for all odd $n \in \mathbb{N}$ ■

Q21 (5 POINTS)

Proof. We use strong induction to prove $P(n)$:

[base case] For $P(3)$ we have $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$

[induction step]