1.1 Microelectronics Technology S 2024 Crib Sheet Exam 1+2+F Hayden Fuller PN

Basic:

n-type, majority e minority h, donors, 5 electron, P, As, Sb, p-type, majority h minority e, acceptors, 3 electron, B, Al, Ga, In

p_n holes in n side, minority

$1eV = 1.6 \times 10^{-19} J$

$k = 1.38 \times 10^{-23} J/K = 8.6 \times 10^{-5} eV/K$, $kT = 0.025eV$

$E_{G,Si} = 1.12eV$

$n_i = 10^{10}$, $n_i^2 = np$, $n = n_i e^{(E_F - E_i)/kT}$, $p = n_i e^{(E_i - E_F)/kT}$

$p - n + N_D - N_A = 0$, $n^2 - n(N_D - N_A) - n_i^2 = 0$

$N_D > N_A \;=>\; n = N_D - N_A$; $p = n_i^2/n$ $\;N_D \approx N_A \;=>\; n = p = n_i$

Band diagrams: n-type: E_C, E_F, E_i, E_v, p-type: E_C, E_i, E_F, E_v

Point defect: one atom missing. Electron generation: one electron missing

Electron moving: breaks off and moves. Hole moving: electron line rotates into hole.

effective mass:

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$, Steps from 1 to 0 at E_F at 0K, smoothe at temp.

$E_F = 1 - e^{\frac{E-E_F}{kT}} = \frac{E_C + E_V}{2}$ in intrinsic

Distribution of carriers = distribution of states * probability of occupancy = $g(E)f(E)$

Conduction band electrons: $n_0 = \int_{E_C}^{E_top} g_C(E)f(E)dE$, holes in VB: $p_0 = \int_{E_bottom}^{E_v} g_V(E)(1-f(E))dE$

total free electron concentration $3kT$ away from edges (non-degenerate): $n = N_c e^{-\frac{E_C - E_F}{kT}}$, hole: $p = N_v e^{-\frac{E_F - E_V}{kT}}$

where effective density of states $N_C = 2.8 \times 10^{19} cm^{-3}$ and $N_C = 1 \times 10^{19} cm^{-3}$, $3kT$ around $N_{AorD} = 2 \times 10^{17}$

Drift: caused by electric field, drift velocity $v_d = \mu_p E \quad cm/sec = cm^2/Vs * V/cm$

$I = Q/T$, $J_{P|drift} = I/A = qp\mu_p E = \frac{E}{\rho}$

resistivity: $\rho = 1/(1p\mu_p + qn\mu_n)$

resistivity measurement: 4 point probe, eddy current apparatus

Diffusion: random thermal mothion, high to low concentration, must be a concentration gradient

Flux $F = -D\frac{d\eta}{dx}$, $\eta =$ particle concentration, $D =$ diffusion coefficient

holes/electrons go high to low,that's flux, but diffusion current is negative for electrons

$J_{p|diff} = -qD_p \frac{dp}{dx}$, $J_{n|diff} = qD_n \frac{dn}{dx}$

$J_p = J_{p|drift} + J_{p|diff} = q\mu_p pE + -qD_p \frac{dp}{dx}$, $J_n = J_{n|drift} + J_{n|diff} = q\mu_n nE + qD_n \frac{dn}{dx}$, $J = J_n + J_p$

Band bending: electric field bends the band diagram

$KE = E - E_C$, $PE = E_C - E_{ref} = -qV$ (for electrons), $V = -(E_C - E_{ref})/q$, $E = -\frac{dV}{dx} = \frac{dE_{C,V,i}}{dx}/q$

Hot point measurement: Hot end makes particles move away.

p-type: holes move away, current goes out hot probe. n-type: electrons move away, current goes into hot probe

in thermal equilibrium: E_F is constant, net current $J_{p|drift} + J_{p|diff} = 0$, recombination and generation cancel

Einstein: $J_{n|drift} + J_{n|diff} = q\mu_n nE + qD_n \frac{dn}{dx} = 0$, $E = \frac{dE_i}{dx}/q$, $n = n_i e^{(E_F - E_i)/kT}$

electrons: $\frac{D_n}{\mu_n} = \frac{kT}{q}$, holes: $\frac{D_p}{\mu_p} = \frac{kT}{q}$

recombination:

band to band recombination gives off light, band to band generation through thermal and light absorption, RG center is indirect-middle step

auger recombination, electron drops, and gives another electron KE. Impact ionization, on a slope, electron moves and falls

SI is mostly RG recombination due to impurities

direct semiconductors: k is matched so with less energy there's a photon. With a difference in k, more energy, phonon.

RG statistics:

if photon energy hv is greater than band gap E_G, iti's absorbed and an electron is moved up.

absorption: $I = I_0 e^{-\alpha x}$, each photon creates an e-h pair. $\frac{dn}{dt}|_{light} = \frac{dp}{dt}|_{light} = G_L(x,\lambda) = G_{L0} e^{-\alpha x}$

α drops off with wavelength. Higher wavelength, lower frequency, lower energy, doesn't get absorbed

indirect thermal recombination-generation, n_0, p_0 under thermal equilibrium, n, p as functions of t.

$\Delta n = n - n_0$, $\Delta p = p - p_0$, Δ's are deviations from equilibrium. N_t is number of RG centers/cm^3

low level injection condition assumed, change in majority carrier concentration negligable, $\Delta p << n_0$, $n \approx n_0$

$\frac{dp}{dt} = \frac{dp}{dt}|_R + \frac{dp}{dt}|_G + G_L(x,\lambda)$, hole build up = recomb loss + gen gain + external light

$\frac{dp}{dt}|_R = -C_p N_t p$

thermal equilibrium: $\frac{dp}{dt}|_G = -\frac{dp}{dt}|R = C_p N_t p_0$

generally when $G_L = 0$, $\frac{dp}{dt} = -\frac{\Delta p}{\tau_p}$, minority carrier lifetime $\tau_p = \frac{1}{C_p N_t}$

1.2 $\frac{\delta\Delta p}{\delta p} = -\frac{\Delta p}{\tau_p}$
perturbation removed at $t=0$: $\Delta p = \Delta p(0)e^{-t/\tau_p}$
$\frac{dp}{dt} = fracdpdt|_{drift} + \frac{dp}{dt}|_{diff} + \frac{dp}{dt}|_{thermalRG} + \frac{dp}{dt}|_{light/other}$
current input: holes: $\frac{dp}{dt} = \frac{1}{q}\frac{dJ_p}{dx} + \frac{dp}{dt}|_{thermalRG} + \frac{dp}{dt}|_{light/other}$, electrons: first term is positive
Minority carrier diffusion equiations: electrons for p type, simplifications
$J_n = q\mu_n nE + qD_n\frac{dn}{dx} \approx qD_n\frac{dn}{dx}$
$\frac{dn}{dx} = \frac{d}{dx}(n_0+\Delta n) = \frac{d\Delta n}{dx}$
$\frac{dn}{dt}|_{thermalRG} = -\frac{\Delta n}{\tau_n}$, $\frac{dn}{dt}|_{light} = G_L$
$\frac{dn}{dt} = \frac{d}{dt}(n_0+\Delta n) = \frac{d\Delta n}{dt}$
$\frac{d\Delta n_p}{dt} = D_n\frac{d^2\Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$
$\frac{d\Delta p_n}{dt} = D_p\frac{d^2\Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} + G_L$
Minority carrier diffusion length: $L_p = (D_p\tau_p)^{1/2}$, average distance minority carriers can diffuse
misc
low level injection assumption, majority carriers don't change significantly
p+ n- is forward biased
L_p is minority p, so n side Microelectronics Technology S 2024 Crib Sheet Exam 2+F Hayden Fuller BJT
Equilibrium energy band diagram for pn junction $kT/q = .0256V$
$n = n_ie^{(E_F-E_i)/kT}$, $p = n_ie^{(E_i-E_F)/kT}$, E_F low for p, high for n
$V = (E_{ref}-E_C)/q$, $E_{ref}-E_C = qV$, $E = 1/qdE_C/dx = 1/qdE_i/dx$, $\rho/\epsilon = dE/dx$, $\epsilon = K_s\epsilon_0$
conceptual pn junction formation
p gives some positive to n and n gives some electrons to p, creating negative region in p and positive region in n
Built in voltage V_{bi} , after formation net drift and diffusion currents sum to zero
E field from nN_D to pN_A , $V_{bi} = 1/q[(E_i-E_F)_p + (E_F-E_i)_n] = kT/q\ln(p_pn_n/n_i^2)$
$(E_i-E_F)_p = kT\ln(p/n_i)$, $(E_F-E_i)_n = kT\ln(n/n_i)$, $p_p/p_n = n_n/n_p = e^{V_{bi}q/kT}$
Depletion approximation
Poisson $dE/dx = \rho/(K_s\epsilon_0) = q/(K_s\epsilon_0)(N_D-N_A)$ for $-x_p < x < x_n$, 0 elsewhere
Quantitative analysis: E field
$dE/dx = \rho/\epsilon = -qN_A/\epsilon = qN_D/\epsilon$
$E(x) = \{-qN_A(x_p+x)/\epsilon\} - x_p < x < 0, \{-qN_D(x_n-x)/\epsilon\}0 < x < x_n, 0x < --x_p, x > x_n$
Relationship between x_n and x_p
$E_{max} = -qN_Ax_p/\epsilon = -qN_Dx_n/\epsilon$, $N_Ax_p = N_Dx_n$ (equal net charge)
$W = x_n + x_p$, $x_n = WN_A/(N_A+N_D)$, $x_p = WN_D/(N_A+N_D)$, if $N_A >> N_D$ then $W \approx x_n$, viceversa
$E = -dV/dx$, $V_{bi} = -\int_{-xp}^{xn} E(x)dx = N_Dx_nWq/(2\epsilon) = W^2qN_AN_D/(2\epsilon(N_A+N_D))$
$W = \sqrt{V_{bi}2\epsilon(N_A+N_D)/(qN_AN_D)} = \sqrt{2\epsilon(N_A+N_D)(V_{bi}-V_A)/(qN_AN_D)}$
$dV/dx = \{qN_A(x_p+x)/\epsilon\}$, $-x_p < x < 0$, $\{qN_D(x_n-x)/\epsilon\}$, $0 < x < x_n$
$V(x) = \{qN_A(x_p+x)^2/2\epsilon\}$, $-x_p < x < 0$, $\{V_{bi} - qN_D(x_n-x)^2/2\epsilon\}$, $0 < x < x_n$
Drift due to E field n to p, holes to p, constant. Diffusion due to added minority carriers, holes to n. E
$V_A = 0$, med E field, med diffusion currents. $V_A > 0$, small E, large diff. $V_A < 0$, large E, small diff
V_A breaks E_F , + to p, smaller gap, p side lowers, n side raises
V_A up linear, E_i gap down linear, carrier concentration exp dec, diffusion current incr exp with V_A
drift constant because limited by how often, not how fast
net $= I_{diff} - I_{drift}$. $V_A = 0$ $I_{diff} = I_{drift} = I_0$. $I = I_0e^{V_A/V_{ref}} - I_{drift} = I_0(e^{V_A/V_{ref}} - 1)$
carrier concentrations under equilibrium, $carrier_{side}$. p side minority electron n_p
$p_p/p_n = e^{(V_{bi}-V_A)q/kT}$, low level injection $p_n = p_{n0}e^{V_Aq/kT}$, $n_p = n_{p0}e^{V_Aq/kT}$
minority carrier concentration under bias graph
p side has n_{p0} , slopes up into $n_p = n_{p0} + \Delta n_p(x'')$ for total $\Delta n_p(0)$, $\Delta n_p(x'') = \Delta n_p(0)e^{-x''/L_n}$
$\Delta p_n(x_n) = p_n(x_n - p_{n0} = p_{n0}(e^{V_Aq/kT} - 1)$, $\Delta n_p(-x_p) = n_{p0}(e^{V_Aq/kT} - 1)$
carrier injection under forward bias
x" axis $\Delta n_p(0) = n_{p0}(e^{V_Aq/kT} - 1)$, $\Delta n_p(x'') = \Delta n_p(0)e^{-x''/L_n}$
x' axis $\Delta p_n(0) = p_{n0}(e^{V_Aq/kT} - 1)$, $\Delta p_n(x') = \Delta p_n(0)e^{-x'/L_p}$
Current and minority carrier diffusion
$J_p(x) = qp\mu_pE - qD_pdp/dx$, $J_n(x) = qn\mu_nE - qD_ndn/dx$, simplified $J_p(x) = -qD_pdp/dx$
$\delta\Delta p/\delta t = D_p\delta^2\Delta p/\delta x^2 - \Delta p/\tau_p + G_L$, $\delta\Delta n/\delta t = D_n\delta^2\Delta n/\delta x^2 - \Delta n/\tau_n + G_L$, simplified $0 = D_p\delta^2\Delta p/\delta x^2 - \Delta p/\tau_p$
diode: $J_p(x'=0) = \Delta p_n(0)qD_p/L_p = p_{n0}qD_p/L_p(e^{V_Aq/kT} - 1)$ and $J_n(x''=0) = -n_{p0}qD_n/L_n(e^{V_Aq/kT} - 1)$
for total current $J = J_0(e^{V_Aq/kT} - 1) = (p_{n0}qD_p/L_p + n_{p0}qD_n/L_n)(e^{V_Aq/kT} - 1)$

2.1 large forward $V_A >> kT/q$, $J = J_0 e^{V_A q/kT}$. Large reverse $V_A << -kT/q$, $J = -J_0$
Avalanching, Zener, RG current, if V_A approaches $V_b i$, high current. Series current, high level injection
IV Reverse- Breakdown to G-R part
IV Forward- G-R part(1/2kT) to Ideal(q/kT) to High Level Injection to Series Resistance Effect
reverse breakdown: $V_{BR} \propto 1/N_B$, V_{BR} is breakdown voltage, N_B is bulk doping on lightly doped side
Avalanching: lightly doped diodes, diff current flips direction, impact ionization, one e from p to n creates more
Electric field must hit critical E_{CR} . steep fall, multiplication factor $M = 1/[1 - (|V_A|/V_{BR})^m]$, m 3 to 6
$E(x=0) = -qN_D x_n/\epsilon_{Si} = -\sqrt{(V_{bi} - V_A)2qN_A N_D/[\epsilon_{Si}(N_A + N_D)]}$
Breakdown when $E(0) = E_{CR}$, $\sqrt{V_{BR} 2qN_A N_D/[\epsilon_{Si}(N_A + N_D)]}$
Zener: tunneling, wall becomes thin when tall,
I_{R-G} increases with depletion layer volume W increases with reverse voltage.
$I_{R-G} = -qAn_i W/2\tau_0$ where $\tau = (\tau_p + \tau_n)/2$
in forward bias: $I_{R-G} = I_0'(e^{V_A q/2kT} - 1)$, total forward current $= I_{diff} + I_{R-G}$, $I_{diff} = I_0(e^{V_A q/kt} - 1)$ where $I_0 = qA(D_n + n_i^2/L_n N_A + D_p n_i^2/L_p N_D)$
since $I_{diff} \propto n_i^2$ grows faster than $I_{R-G} \propto n_i$, RG is negligable in forward bias, more ideal in Ge and high temp
V_A approaches V_{bi} , $I \approx I_0 e^{(V_A - IR_s)q/kt}$
$log(I)$ vs V_A is slope q/kT but veers right by ΔV . ΔV vs I gives linear slope R_s
High level injection: when V_A within $0.2V$ ish of V_{bi} , $I = e^{V_A q/2kT}$, minority hits majority and they increase linearly together
$log(I)$ vs V_A shikanes with Avalanch/Zener breakdown, thermal gen in depletion, origin, thermal recombonation in depletion, ideal q/kT in middle, high level injection q/2kT above, serries resistance above
Small signal admittance $Y = i/v_a = G + j\omega C$, res RS to cap CD+ cap DJ + res GD
$C_j = \epsilon_{Si} A/W = A\sqrt{\epsilon_{Si} qN_B/2(V_{bi} - V_A)}$, up with $\sqrt{N_B}$, down with reverse bias
$W = \sqrt{2\epsilon_{Si}(N_A + N_D)(V_{bi} - V_A)/(qN_A N_D)} = \sqrt{2\epsilon_{Si}(V_{bi} - V_A)/(qN_B)}$
$1/C_J^2 = 2(V_{bi} - V_A)/(A^2 qN_B \epsilon_{Si})$, vs V_A , slope first part, $= 0$ at V_{bi}
C_D charge storage cap dominant in forward bias. $p+n$ has $I = Q_p/\tau_p$ where Q_p total excess charge n side
$Q_p = I\tau_p = qAD_p \tau_p p_{n0}/L_p * [e^{V_A q/kT} - 1] \approx qAL_p p_{n0} e^{V_A q/kT}$
$C_D = dQ_p/dV = I\tau_p q/kT$, $G_D = Iq/kT$
Transient response, charge Q_p goes zero when turned off from current flow and recomb, $dQ_p/dt = i(t) - Q_p/\tau_p$
$Q_p = qAL_p \Delta p_n(0)$, to maintain charge, current $I = qAL_p \Delta p_n(0)/\tau_p$ must be supplied at $x' = 0$
$Q_p(t) = I\tau_p e^{-t/\tau_p}$, $I_F = V_F - V_{on}/R_F \approx V_F/R_F$, $I_R = V_R + v_A(t)/R_R \approx V_R/R_R$
charge between reverse and forward curves needs to be moved, drop over time is pulled from axis
storage delay time: $dQ_p/dt = i - Q_p/\tau_p = -I_R - Q_p/\tau_p$ for $0 < t < t_s$, $t_s = \tau_p \ln(1 + I_F/I_R)$
applications: rectifiers, low R in forward, p+ n n+ prefered, reduce parasitic resistance, low I_0 in reverse, High voltage breakdown, p+nn+high band gap materials
switching, fast, dope with gold to reduce lifetimes, narrow base for small stored charge
Zener, heavy dope p+ and n+ for low breakdown, reference voltage
Varactor, variable resistance, V controlled C for tuning radio or TV, $C_J \propto V_A^- 1/2$ (abrupt, dope to linear)
Opto-elect, photodetect, solar cells, LED, laser diodes. PhotoD: $I_L = -qAG_L(L_N + W + L_P)$, $I = I_{dark} + I_L$
BJT: pnp: IE in IB+IC out. npn: IB+IC in, IE out
biasing modes: B is expected to be - for pnp, Mode, EB polarity, CB polarity. Saturation, F, F. ACTIVE, F, R. Inverted, R, F. Cutoff, R, R. A S
n C I. Vert+ VEB pnp VBE npn. Horiz+ VCB pnp VBC npn
electrostatic equilibrium p+ n p EBC,
$V = -1/q(E_C - E_{ref})$, up and flatens in B, drops to flat in C
$E = 1/qdE_C/dx = 1/qdE_i/dx$, sharp negative triangle left B, smaller positive left C
$dE/dx = \rho/\epsilon$
forward, p+ thinB n, small E to p+, big h/e and small e/h, same thinB small E, minority e lower than minority h, both going up
reverse, n wideB p, large E to p, e/h and h/e, minorities drop off to 0
combine for p+ n p, curve up to thin, curve down and drop to wide, up to e
make B very thin, curve up to thin, drop to zero for rev bias, back up a bit, D and CS $I = \alpha I_E$, B has $I = (1-\alpha)I_E$
emitter efficiency $\gamma = I_{EP}/(I_{EP} + I_{EN}) = I_{EP}/I_E$
base transport factor $\alpha_T = I_C/I_{Ep}$
$I_C = \alpha_T I_{EP} = \alpha_T \gamma I_E = \alpha_{dc} I_E$, $\alpha_{dc} = \alpha_T \gamma$
$I_C = \beta_{dc} I_B$, $\beta_{dc} = \alpha_{dc}/(1 - \alpha_{dc}) = \alpha_T \gamma/(1 - \alpha_T \gamma)$
detailed quantitative analysis, assume pnp, steady state, low level, only drift and diff, no gen, one dimension, etc.
solve minority carrier diffusion equations for each of the three regions
$\delta\Delta p/\delta t = D_p \delta^2 \Delta p/\delta x^2 - \Delta p/\tau_p + G_L$, $\delta\Delta n/\delta t = D_n \delta^2 \Delta n/\delta x^2 - \Delta n/\tau_n + G_L$

2.2 under steady state $G_L = 0$, $0 = D_p\delta^2\Delta p/\delta x^2 - \Delta p/\tau_p$, $0 = D_n\delta^2\Delta n/\delta x^2 - \Delta n/\tau_n$
for pnp base, only interested in holes (current in E and split)
$\Delta n = n - n_0$ excess carriers above equilibrium, area of excess carriers$= Q_n$.
X_B and X_E flow awayfrom BE junction, $I_E = I_P - I_N \approx (qAp_{B0}D_B/L_B * e^{V_{EB}q/kT}) + (qAn_{E0}D_E/L_E * e^{V_{EB}q/kT})$
$I_P = Q_p/\tau_B$, $I_P = Q_n/\tau_E$. n_E curve up, p_B linear down, n_C collecter curve up
I_E broken down into $I_n = qaD_ndn/dx$ and $I_p = -qAD_pdp/dx$
$I_C = qAD_Bp_B(0)/W_B = qAp_{B0}D_B/W_B * e^{V_{EB}q/kT}$
I_E made up of I_{EP} and I_{EN}
$I_{EP} = I_c + qAW_B\Delta p_B(0)/2\tau_B \approx qAp_{B0}D_B/W_Be^{V_{EB}q/kT} + qAp_{B0}W_B/2\tau_Be^{V_{EB}q/kT}$
$I_B = qAp_{B0}W_B/2\tau_Be^{V_{EB}q/kT} + qAn_{E0}D_E/L_Ee^{V_{EB}q/kT}$ (recombination + e injection to E)
$\alpha_T = 1/[1 + (W_B/L_B)^2/2]$, $\gamma = 1/[1 + D_En_{E0}W_B/D_Bp_{B0}L_E] = 1/[1 + D_EW_BN_B/D_BL_EN_E]$
BJT in cutoff, minority carriers drop off on E and C, zero in B.
BJT in saturation, E and C curve up, p_{B0} is linear down but still high, above E below C.
Base width modulation $I_C \approx qAD_B\Delta p_B(0)/W_Be^{V_{EB}q/kT}$, B drops to 0 at C
Early effect, CB reverse bias up, depletion width up, W down, I_C up
punch through, W approaches 0. for high reverse CB, EB barrier lowers, and large I_C at high V_{CE0} due to either punchthrough or avalanch

workfunction Φ, difference between Fermi and vacuum, energy to free e's from metal, $\Phi_s = X + (E_C - E_F)_{FB}$
$X = (E_0 - E_C)_{SURFACE}$, $X_{Si} = 4.03\,\text{eV}$
Contact is sticky, but Si side is dragged until Fermi matches. Curves up if $\Phi_M > \Phi_S$, down if $\Phi_M < \Phi_S$
barrier height Φ_B is the barrier for flow from M to S, $\Phi_B = \Phi_M - X$ in ideam MS n-type
barrier of $\Phi_M - \Phi_S$ when flowing S to M
applied voltage brings M down (lowers S to M barrier), nagative brings M up, drags S with it

	n-type	p-type
$\Phi_M > \Phi_S$	rectifying	ohmic
$\Phi_M < \Phi_S$	ohmic	rectifying

Schottky diode $V_{bi} = 1/q[\Phi_B - (E_C - E_F)_{FB}]$, $\rho \approx qN_D$ for $0 < x < W$, ≈ 0 for $x > W$
$dE/dx = \rho/\epsilon_{Si} = qN_D/\epsilon_{Si}$ for $0 < x < W$, $E(x=0) = qN_DW/\epsilon_{Si}$, $W = \sqrt{(V_{bi} - V_A)2\epsilon_{Si}/qN_D}$
p^+n vs MS: p^+n dom current from recomb in depletion under small forward bias and hole injection from p^+ under larger forward (holes from p to n); MS dom current from electron injection from S to M (e from S to M)
$I = I_S(e^{V_A q/kT} - 1)$ where $I_S = AA^*T^2e^{-\Phi_B/kT}$
more reverse leakage for Schottky than p^+n, but majority carrier allows to be faster
MOS
current D to S, electrons S to D, from N+ to N+ through p, past SiO_2
larger VG forms a larger channel for e flow, increasing saturation of ID from VD
ideal Cap: $\Phi_M = \Phi_S = X + (E_C - E_F)_{FB}$, EF's match, just a barrier in between
$E_0 - E_F = \Phi_M$, barrier to $E_0 = X_i$, $\Phi'_M = \Phi_M - X_i$, same forr X and X'
$E_{FM} - E_{FS} = -qV_G$
$dE_{oxide}/dx = \rho/\epsilon = 0$, E field is constant in the oxide
Accumulation: negative $V_G < 0$
neg app V to M brings M up, holes accum on Si side sloping up, O sloped up towards M to match, F moves flat
ρ vs x, tall thin sheet of electrons below M side of MO, slightly shorter thicker sheet of holes on p side of OS
E vs x, E=0 in M, jumps to negative constant in O, drops quickly and curvs off quickly to zero in S
Depletion: $V_G > 0$, brings M down, O sloped down to M, p sloped the same,
ρ vs x, tall sheet holes left, short finite depletion layer width wide block of electrons under right
E vs x, sharp slopee up in M to O, constant positive O, drop off and 45 linear to zero S
$E_{ox} = \epsilon_{Si}/\epsilon_{ox} * E_{Si}$
Inversion: large positive gate voltage, E_i goes below E_F (at boundary/curve, still C i F V at FB for p)
ρ vs x, taller sheet h left, short very wide block of immobile acept under right, short thin sheet of mobile e under
E vs x, sharp slopee up in M to O, constant positive O, large drop off and slight linear to zero S
E_i (sruface) $-E_i$ (bulk) $= 2[E_F - E_i$ (bulk) $]$, $\phi_S = 2\phi_F$, onset inversion, that V_G is threshhold voltage V_T
Quantitative analysis:
$\phi(x)$ is potential (Voltage) at any point in the semiconductor
$\phi(x) = 1/q[E_{i,bulk} - E_i(x)]$ potential at any point x; $\phi_S = 1/q[E_{i,bulk} - E_{i,surface}]$ Surface potential
$\phi_F = 1/q[E_{i,bulk} - E_F]$ for doping concentration; $\phi_F > 0$ means p type
ϕ bends up from S to O meets at positive ϕ_S; $E_i - E_F = q\phi_F$
$\phi_S = 2\phi_F$ at depletion-inversion point
Delta-depletion solution, consider p type, accumulation
mobile holes in S near O, assume it's a pulse, Q on M $= -Q_M$, Q on S = -Q on M = Q_M, $|Q_{accumulation}| = |Q_M|$
assume depletion, apply V_G such $\phi_S < 2\phi_F$, immobile ions in Si, $|qN_AAW| = |Q_M|$
$W = \sqrt{\phi_S 2\epsilon_{Si}/qN_A}$ and $E_{Si} = W|qN_A/\epsilon|$
at start of inversion, $\phi_S = 2\phi_F$, $W = W_T = \sqrt{4\phi_F\epsilon_{Si}/qN_A}$
for both p^+n and MS (n-Si), $W = \sqrt{2V_{bi}/qN_D}$, V_{bi} in V is the numerical same as the band bending in eV
pn and MS $E_{max} - qN_DW/\epsilon_{Si} = -\sqrt{2V_{bi}qN_D/\epsilon_{Si}}$
for MOS, same but replace $V_{bi} = \phi_S$, $E_{max} = -\sqrt{|\phi_S|2qN_D/\epsilon_{Si}}(n) = \sqrt{|\phi_S|2qN_A/\epsilon_{Si}}(p)$
as we get stronger inversion, W stays the same, extra charges are in delta function (thin pulse), max $W = W_T$
Gate Voltage relationship:
$V_G = \Delta\phi_{ox} + \Delta\phi_{Semi}$ (full potential differnce across the region)
$\Delta\phi_{Semi} = \phi(x=0) - \phi(bulk) = \phi_S$; $\Delta\phi_{ox} = x_{ox}E_{ox}$
since no interface charges up to inversion, $\epsilon_{ox}E_{ox} = \epsilon_{Si}E_{Si}$, $E_{ox} = E_{Si}\epsilon_{Si}/\epsilon_{ox}$
3.2 $E_{Si} = |qN_A/\epsilon_{Si}|W = |qN_A/\epsilon_{Si}|\sqrt{\phi_S 2\epsilon_{Si}/qN_A} = \sqrt{\phi_S 2qN_A/\epsilon_{Si}}$
$V_G = \phi_S + x_{ox}E_{ox} = \phi_S + x_{ox}E_{Si}\epsilon_{Si}/\epsilon_{ox} = \phi_S + x_{ox}\epsilon_{Si}/\epsilon_{ox}\sqrt{\phi_S 2qN_A/\epsilon_{Si}}$
alternative gate voltage: consider p-type: $\Delta\phi_{ox} = Q_M/C_{ox} = -Q_S/C_{ox}$, $Q_S = -qAN_AW$, $C_{ox} = \epsilon_{ox}A/x_{ox}$
MOS C-V charicteristics
Gate cap varies with gate V, useful for diagnosing deviations from ideal in O and S during fab
Measure: apply DC bias+small AC (high 1MHz or 1k-1Meg), vary bias to get quasi-continuous C-V characteristics

V_G vs C_G; p-type: flat, shicane starts high before zero, drops low after 0V, low frequency goes back up sharper. n-type: flip over vertical axix. V_T at near bottom split point
accumulation: assume p $V_G < 0$, h in S, e in M, $C_G = C_{ox} = \epsilon_{ox}A/x_{ox}$
depletion: assume p $V_G > 0$, h in M, W width depletion of holes, gives both C_O and C_S.
$C_{ox} = \epsilon_{ox}A/x_{ox}$; $C_S = \epsilon_{Si}A/W$; $C_G = C_{ox}C_S/(C_{ox}+C_S)$; $W = \sqrt{\phi_S 2\epsilon_{Si}/qN_A}$
inversion: $V_G >= V_T$, $\phi_S = 2\phi_F$, $W = W_T = \sqrt{\phi_F 4\epsilon_{Si}/qN_A}$, at high frequncy, electrons in delta function aren't able to respond, so W varries with AC, $C_{ox} = \epsilon_{ox}A/x_{ox}$; $C_S = \epsilon_{Si}A/W$; $C_G(\omega \to \infty) = C_{ox}C_S/(C_{ox}+C_S)$
at low frequency, electrons can respond, and ; $C_G(\omega \to 0) = C_{ox}$
Deep Depletion: fast ramp rate, inversion layer doesn't form, no equilibrium, W will go past W_T and C_G will decrase
V_T + for p, - for n. $V_T = 2\phi_F + (\pm x_{ox}\epsilon_{Si}/\epsilon_{ox} * \sqrt{|\phi_F| 4qN_A/\epsilon_{Si}})$
Higher doping higher $|V_T|$, $C_{max} = C_{ox}$, $C_{min} = C_{ox}C_S/(C_{ox}+C_S)$
C/C_O vs V_G, n-type: starts mid for heavy doping and low ramp rate, low for low doping and high ramp total deep depletion, both go high and right
under deep depletion: $V_G = \phi_S + x_{ox}\epsilon_{Si}/\epsilon_{ox}\sqrt{\phi_S 2qN_A/\epsilon_{Si}}$, $W = \sqrt{\phi 2\epsilon_{Si}/qN_A}$
MOSFET:nmos example:
$0 < V_G < V_T$, open, V_{DS} doen't matter, no channel, no current
$V_G > V_T$, $V_{DS} \approx 0$, I_D increases with V_{DS}, full channel of electrons
$V_G > V_T$, V_{DS} small, I_D increases slowly with V_{DS}, channel getting pinched
$V_G > V_T$, $V_{DS} \approx$ pinch off, I_D reches saturation of $I_{D,sat}$ at $V_{DS,sat}$, channel just barely pinched off
$V_G > V_T$, $V_{DS} > V_{DS,sat}$, $I_{D,sat}$ already saturated, channel totally pinched off with horizontal gap ΔL
I_D vs V_D, log curve start line, curve off (still "linear"), sat at $V_{D,sat}$ slope if $\Delta L \approx L$, flat if $\Delta L << L$
increasing V_G, $I_D = 0$ for $V_G < V_T$, lineas start at $V_G > V_T$
$V_G = V_T$ means $\phi_S = 2\phi_F$ (note: $\epsilon_{Si}/\epsilon_{ox} = 11.9/3.9 = 3.05$
n channel (p silicon) $V_T = 2\phi_F + x_{ox}\epsilon_{Si}/\epsilon_{ox} * \sqrt{\phi_F 4qN_A/\epsilon_{Si}}$
p channel (n silicon) $V_T = 2\phi_F - x_{ox}\epsilon_{Si}/\epsilon_{ox} * \sqrt{|\phi_F| 4qN_D/\epsilon_{Si}}$
ground S and D= V_{DS}, ϕ along channel is 0 - V_{DS}; For $V_G < V_T$ inversion layer change is zero
For $V_G > V_T$, $Q_n(y) = -Q_G = -C_{ox}(V_G - \phi - V_T)$. $J_n = q\mu_n nE = -q\mu_n n d\phi/dy$ when diff current is neglected
I_D is same everywhere, but J_n can vary. $I_D = -Z/L * \mu_n \int_0^{V_{DS}} Q_n(y)d\phi = Z\mu_n/L * C_{ox}[(V_G - V_T)V_{DS} - V_{DS}^2/2]$
$I_{D,sat} = (V_G - V_T)^2 Z\mu C_{ox}/2L$
ac response: $I_D = f(V_G, V_{DS})$; $i_d = g_m v_g + g_d v_d$, transcond $g_m = \frac{\delta I_D}{\delta V_G}|_{V_{DS}}$, drain/channel cond $g_d = \frac{\delta I_D}{\delta V_{DS}}|_{V_G}$
low freqency equiv: shared S, D to S current source of $g_m v_g$ and D to S resistor g_d
high frequency: add G to S cap C_{gs} and G to D cap C_{gd}
when $V_{DS} < V_{DS,sat}$, $g_d = Z\mu_n C_{ox}/L * (V_G - V_T - V_{DS})$ and $g_m = Z\mu_n C_{ox} V_{DS}/L$
when $V_{DS} > V_{DS,sat}$, $g_d = 0$ and $g_m = Z\mu_n C_{ox}/L * (V_G - V_T)$
cut off frequency when current gain is 1. input= $j\omega C_G v_G$, output= $g_m v_G$, $f_T = g_m/(2\pi C_{GS})$; $C_{GS} \approx ZLC_{ox}$
these are all enhancement mode so far: NMOS: V_T is positive, zero is off. PMOS: V_T is negative, zero is off
REAL MOS:
ideally Fermi levels lign up when made, irl, ϕ_M and ϕ_S rely on the metal and doping, need to apply $V_G = \phi_{MS}/q$ to get flat band. (Assume $E_{F,M} = E_V$)
we use heavily doped polysilicon for gate, p: $E_{FM} = E_V$, n: $E_{FM} = E_C$
interface charges: loose charges in the metal Q_i will induce $-Q_i$ in S. acts as a positive gate voltage, negative S charges bend bands. apply $-Q_i/C_{ox}$ to get flat band
these all mean a correction needs to be made to V_T. $V_{FB} = 1/q * \phi_{MS} - Q_i/C_{ox}$, $V_T = V_T' + V_{FB}$
shifts horizontally C_G vs V_G curve so zero is at low rather than high C
Enhancement VS Depletion:
enhancement: $V_G = 0$ is off. all I_D is from positive V_D
depletion: $V_G = 0$ is on. more I_D from positive V_D, but you get some as long as V_G isn't too negative
V_T adjustment with ion implantation: Boron (+), Phosphorus (-)
$\Delta V_T = Q_{ion}/C_{ox} = qB_{dose}/C_{ox}$ (positive for B), $= -qP_{dose}/C_{ox}$ (negative for P)
Frequently used
$(E_i - E_F)_p = kT\ln(p/n_i)$, $(E_F - E_i)_n = kT\ln(n/n_i)$, $p_p/p_n = n_n/n_p = e^{V_{bi}q/kT}$
$C_{ox} = \epsilon_{ox}A/x_{ox}$
$V_T = 2\phi_F + (\pm x_{ox}\epsilon_{Si}/\epsilon_{ox} * \sqrt{|\phi_F| 4qN_A/\epsilon_{Si}}) + V_{FB}$