Fields and Waves I

Lecture 20

Displacement Current
Lossless and Lossy EM Waves

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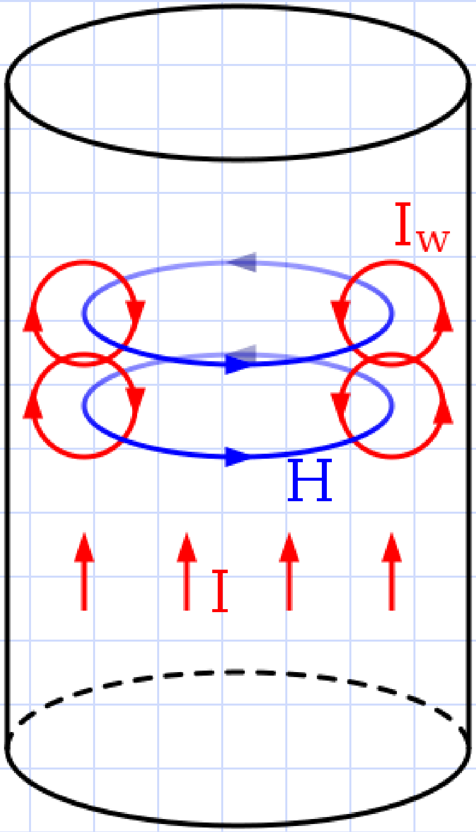
Maxwell’s Equations

Eddy Currents

Maxwell’s Equations

Eddy Currents + Skin Effect



Maxwell's Equations

Full Version

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{S} + \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S}$$

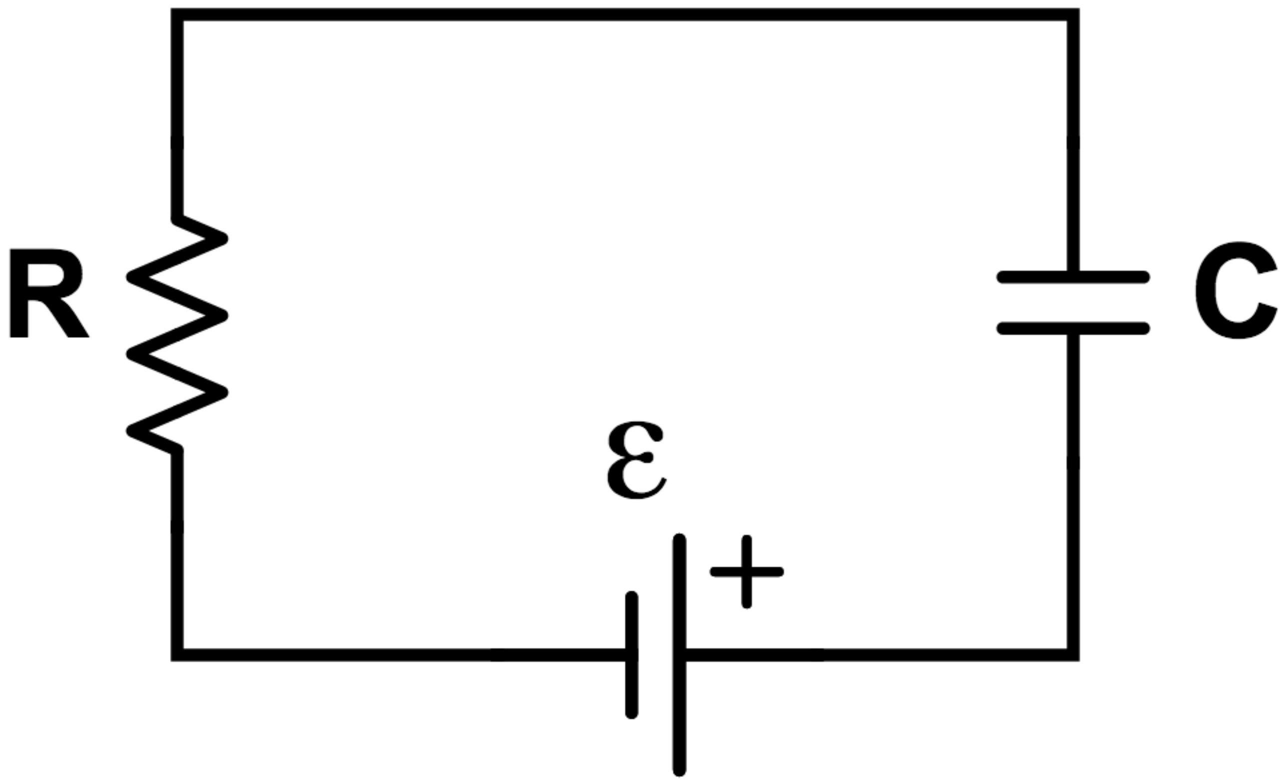
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{D} \cdot dS = \oint \rho dv = Q_{encl}$$

$$\nabla \cdot \vec{D} = \rho$$

Note that the time-varying terms couple electric and magnetic fields in both directions. Thus, in general, we cannot have one without the other.

Displacement Current



Conductors vs. Dielectrics

The analysis of the capacitor under time-varying conditions assumed that the insulator had no conductivity. If we generalize our results to include both σ and ε we will have both a conduction and a displacement current.

$$I = I_C + I_D = \sigma\pi a^2 \frac{V_o}{d} + j\omega \frac{\varepsilon\pi a^2}{d} V_o = (\sigma + j\omega\varepsilon)\pi a^2 \frac{V_o}{d}$$

Note that the conduction current has a phase angle of zero degrees while the displacement current has an angle of 90 degrees.

Conductors vs. Dielectrics

The material will behave mostly like a conductor when

$$\frac{|I_C|}{|I_D|} = \frac{\sigma}{\omega\varepsilon} >> 1$$

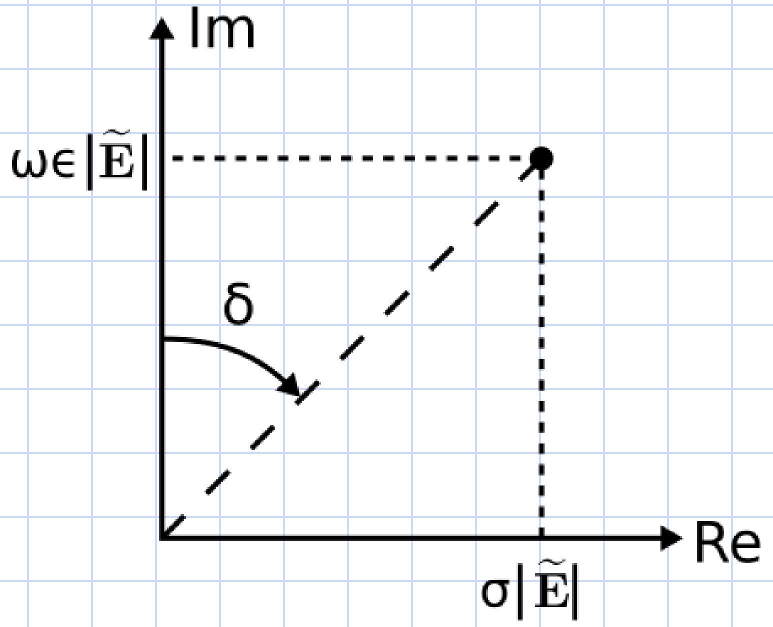
The material will behave mostly like a dielectric when

$$\frac{|I_C|}{|I_D|} = \frac{\sigma}{\omega\varepsilon} << 1$$

Conductors vs. Dielectrics

Loss tangent of the material: $\tan\delta = \dfrac{\sigma}{\omega\varepsilon}$

This tells us the phasor-domain angle of the current that results from the conduction and displacement currents combined.



Source: LibreTexts

Displacement Current

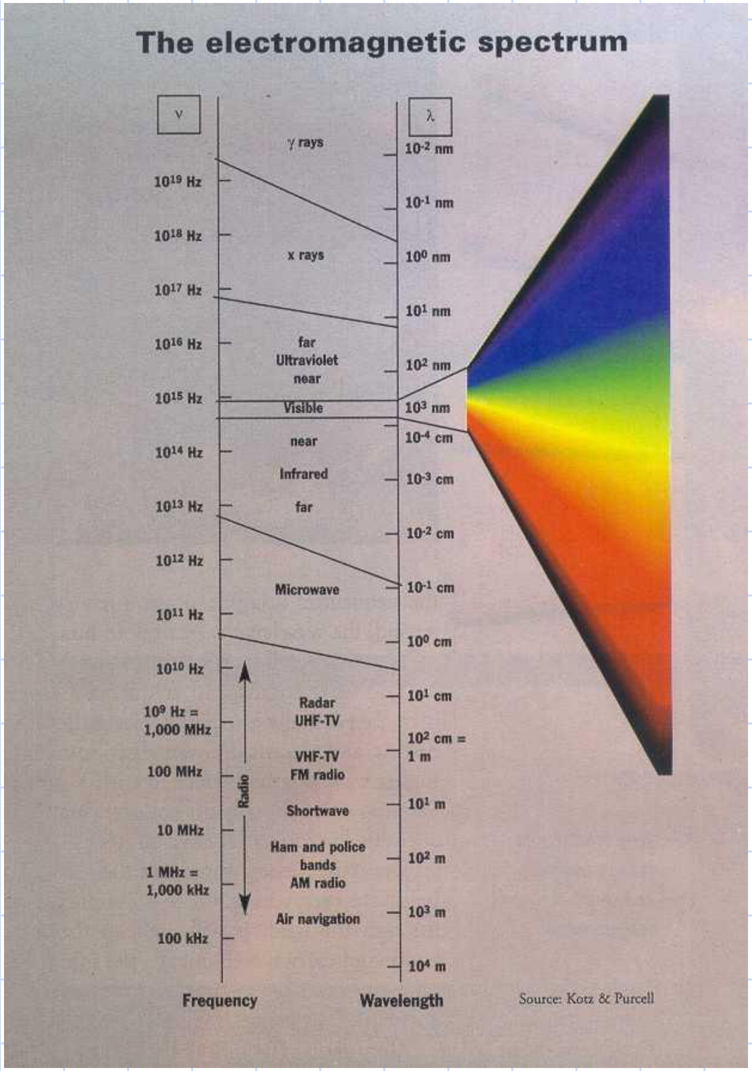
Do Lecture 20 Exercise 1 in groups of up to 4.

Overview

Tesla's wireless power transmission experiments

[The remnants of Tesla’s Wardencliff Laboratory in Long Island are still standing.
(but not open to the public)]

Electromagnetic Waves



Typical values of f, β, λ for X-rays, visible light, microwaves, and FM radio in free space

	f (Hz)	β (m⁻¹)	λ (m)
X-rays	10^{19}	2.1×10^{11}	3×10^{-11}
visible light	6×10^{14}	1.3×10^{7}	5×10^{-7}
microwaves	10^{10}	210	0.03
FM radio	10^{8}	2.1	3.0

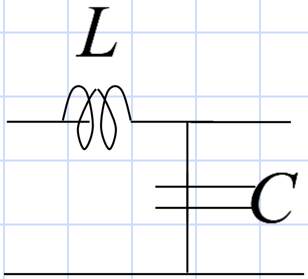
$\beta = \frac{\omega}{c}$ for free space $\quad \lambda = \frac{2\pi}{\beta} = \frac{1}{f\sqrt{\mu_0\varepsilon_0}}$

http://www.esat.kuleuven.ac.be/sista/education/techecon/

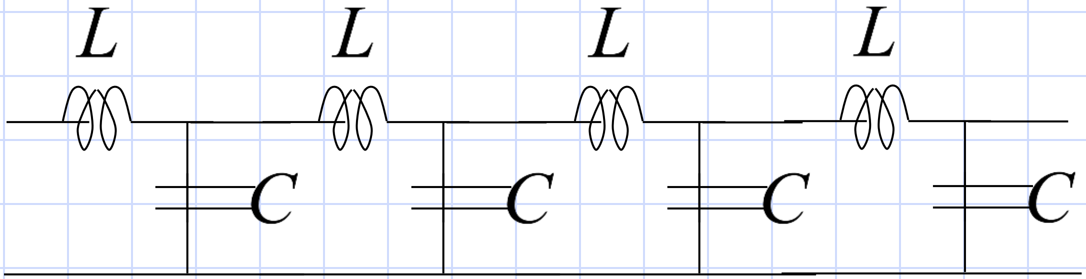
Lossless EM Waves

Transmission Line Review

Model of a short section:



Model the full length as:

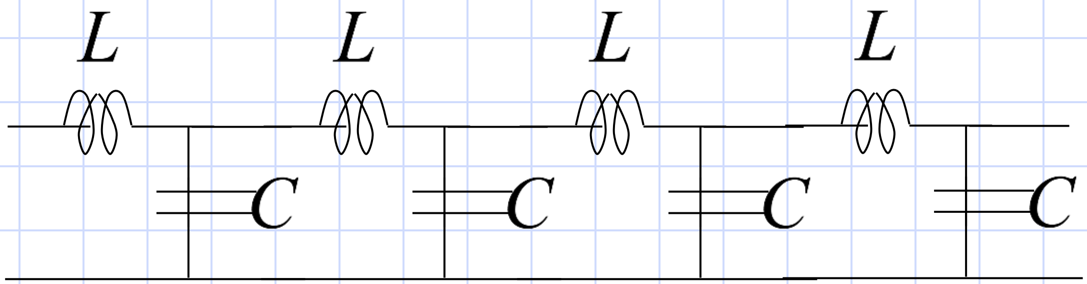


....etc.

Lossless EM Waves

Transmission Line Review

- One of the definitions of ε_0 is "the capacitance of the vacuum"; likewise, μ_0 is "the inductance of the vacuum".
- To visualize EM waves propagating through space, we can think of space as a T-line with $C' = \varepsilon_0$ and $L' = \mu_0$.



….etc.

Lossless EM Waves

Transmission Line Review

$$\frac{\partial^2 V}{\partial z^2} = lc\frac{\partial^2 V}{\partial t^2} \rightarrow \frac{\partial^2 V}{\partial t^2} = \frac{1}{lc}\frac{\partial^2 V}{\partial z^2} \rightarrow \frac{\partial^2 V}{\partial t^2} = u^2\frac{\partial^2 V}{\partial z^2}$$

For free space,

$$u = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3.0 \times 10^8 m/s = c$$

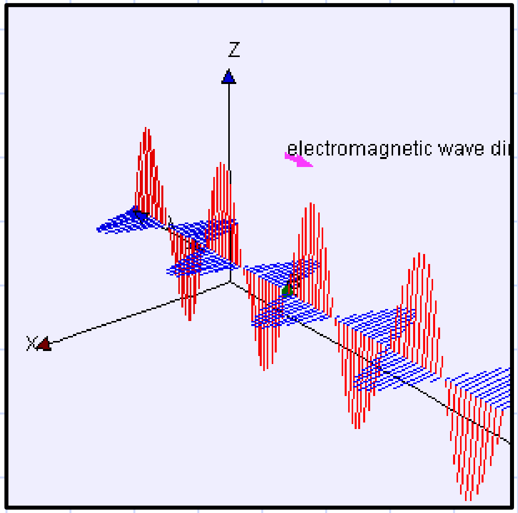
Electromagnetic Waves

The “Waves” Part of Fields & Waves

In Lecture 1 we briefly described electromagnetic waves propagating in space according to the following equations:

$$\frac{\partial^2 \vec{E}}{\partial t^2} = c^2 \nabla^2 \vec{E} \qquad \frac{\partial^2 \vec{B}}{\partial t^2} = c^2 \nabla^2 \vec{B}$$

Equipped with the full version of Maxwell’s Equations, we can now turn our attention to the propagation of EM waves.



Source: https://en.wikipedia.org/wiki/Electromagnetic_radiation

Electromagnetic Waves

EM wave propagation involves electric and magnetic fields having 3 components, each dependent on all three coordinates, in addition to time.

e.g. Electric field

$$\underbrace{E(x,y,z,t)}_{\text{instantaneous field}} = \mathrm{Re}\left\{ \underbrace{\tilde{E}(x,y,z)}_{\text{vector phasor}} e^{j\omega t} \right\}$$

Valid for the other fields D, H, B and their sources J, ρ_v

Electromagnetic Waves

Maxwell's Equations in Phasor Domain

time domain

$$\nabla \bullet \tilde{E} = \tilde{\rho}_v / \varepsilon \qquad \nabla \bullet E = \rho_v / \varepsilon$$

$$\nabla \times \tilde{E} = -j\omega\mu\tilde{H} \qquad \nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \bullet \tilde{H} = 0 \qquad \nabla \bullet H = 0$$

$$\nabla \times \tilde{H} = \tilde{J} + j\omega\varepsilon\tilde{E} \qquad \nabla \times \vec{H} = \vec{J} + \frac{\partial D}{\partial t}$$

remember $D = \varepsilon E$ $\quad B = \mu H$ $\quad J = \sigma E$

Lossless EM Waves

Homogenous wave equation (charge free)

Combining $\nabla \times \tilde{E} = -j\omega\mu\tilde{H}$ and $\nabla \times \tilde{H} = j\omega\varepsilon_c \tilde{E}$

$$\begin{cases} \nabla^2 \tilde{E} - \gamma^2 \tilde{E} = 0 \\ \nabla^2 \tilde{H} - \gamma^2 \tilde{H} = 0 \end{cases}$$

$$\gamma^2 = -\omega^2 \mu\varepsilon_c$$

γ propagation constant

- Thus we get the same results using either Maxwell's Equations or T-line equations!
- This also means that we could do transmission line analysis using Maxwell's Equations if we wished.

Lossy EM Waves

Transmission Line Review

For lossless systems: $$\beta = \omega\sqrt{lc}$$

For lossy systems:

$$\gamma = \alpha + j\beta = \sqrt{(r + j\omega l)(g + j\omega c)}$$

The phasors have the factor:

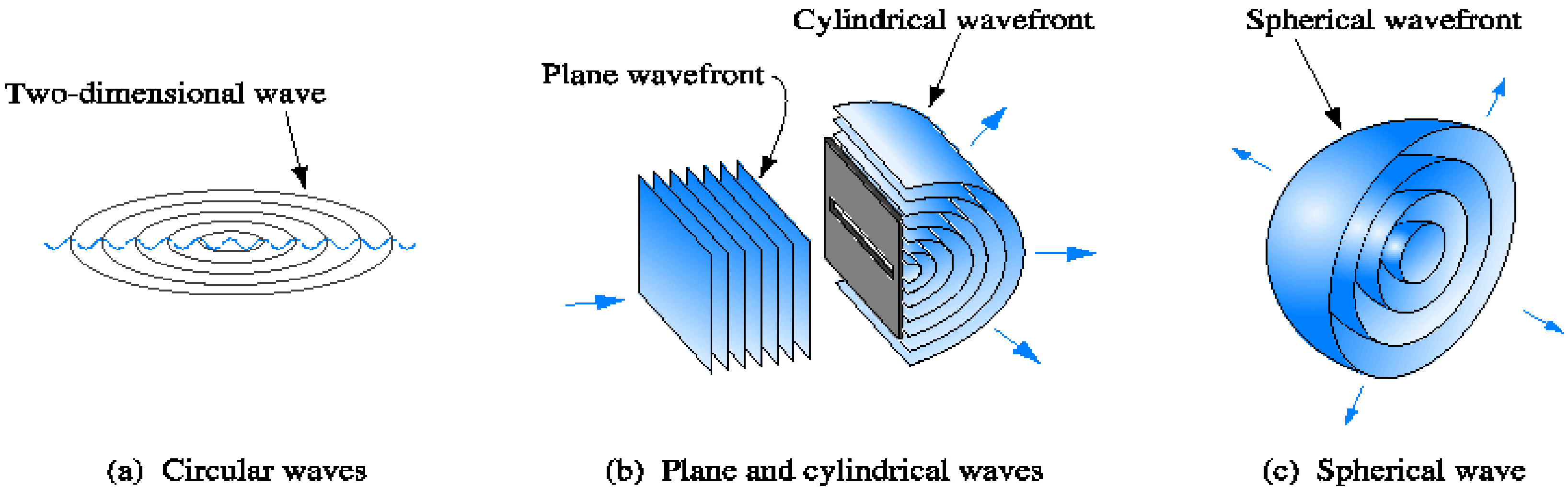
$$e^{-\gamma z} = e^{-\alpha z} \cdot e^{-j\beta z}$$

Attenuation/loss factor due to resistance

- How do we translate the treatment of losses from T-lines to EM waves in general?

Electromagnetic Waves

Some Typical Waves

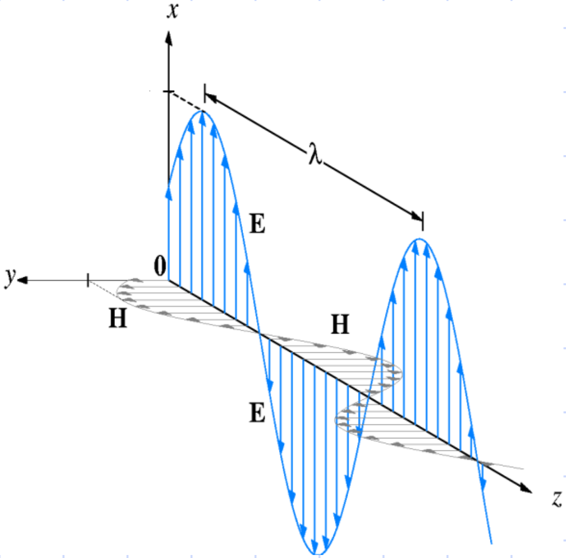


Ulaby

Figure 1-10

Transverse Electromagnetic Wave (TEM)

- A plane wave has no electric or magnetic field components along the direction of propagation
- Electric and magnetic fields are perpendicular to each other and to the direction of propagation
- They are uniform in planes perpendicular to the direction of propagation



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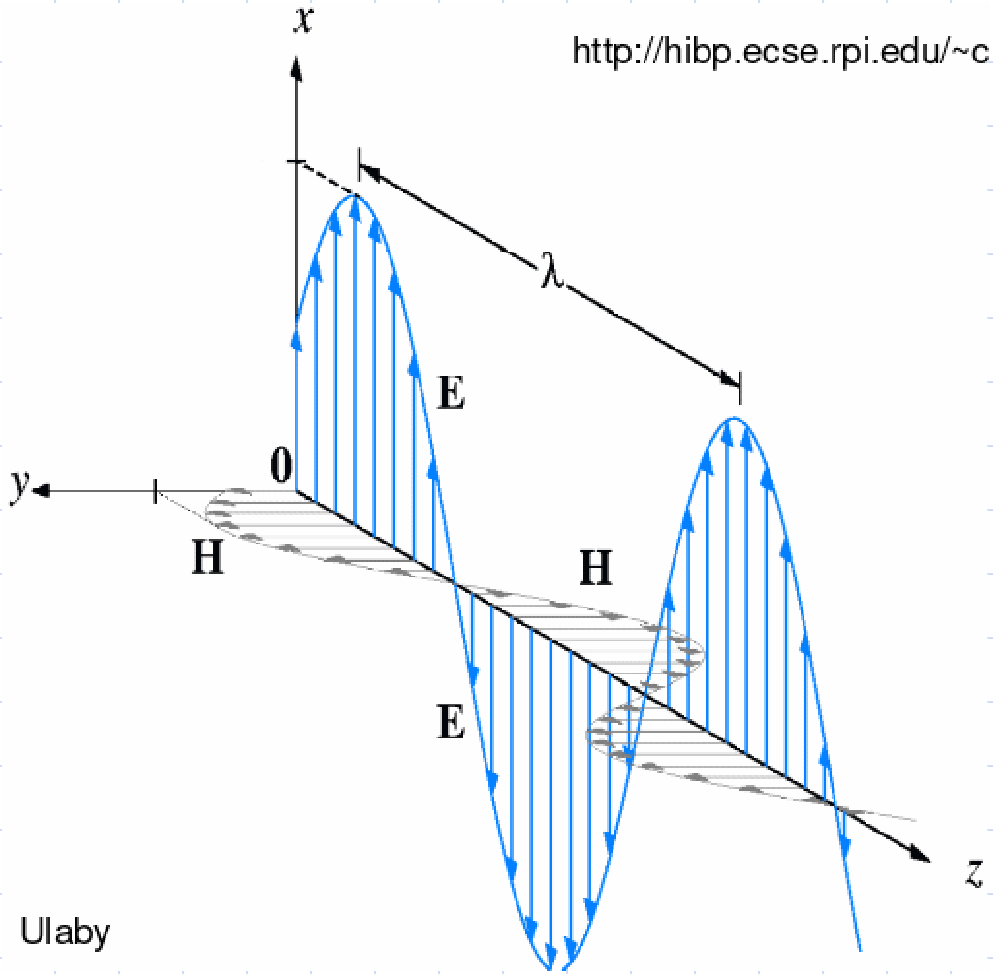
Electromagnetic Waves

Transverse Electromagnetic Wave



http://hibp.ecse.rpi.edu/~crowley/java/EMWave/emwave.html

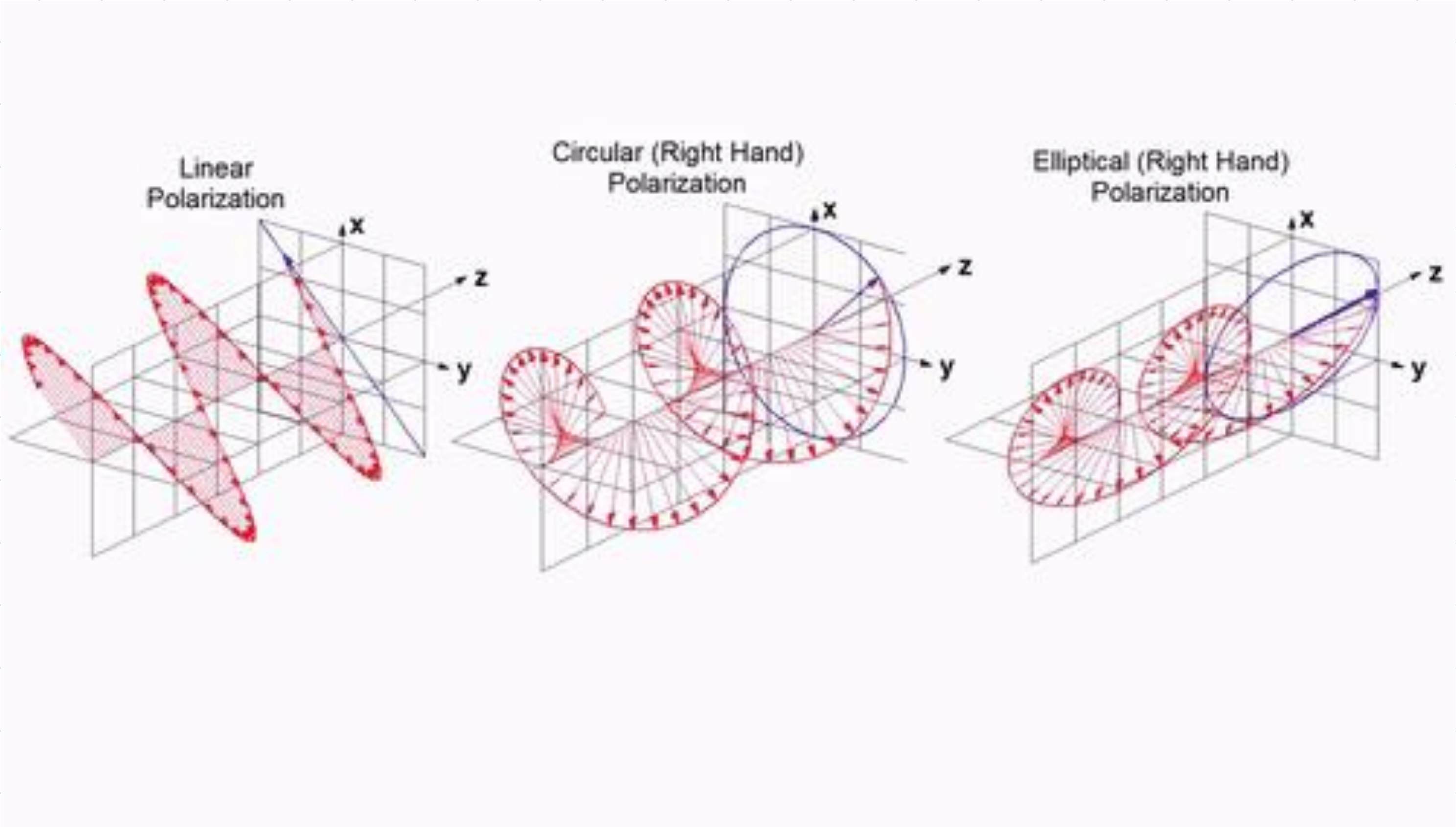
Spatial variation of E and H at t=0

$$\lambda = \frac{2\pi}{k}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

Electromagnetic Waves

Some Typical Waves



Source: [Gfycat]

Lossy EM Waves

Traveling Plane Waves

The Electric Field in phasor form (only x component)

$$\frac{d^2 \tilde{E}_x}{dz^2} + k^2 \tilde{E}_x = 0$$

General solution of the differential equation

$$\tilde{E}_x(z) = \tilde{E}_x^+(z) + \tilde{E}_x^-(z) = \underbrace{E_{x0}^+}_{\text{amplitudes (constant)}} e^{-jkz} + \underbrace{\cancel{E_{x0}^- e^{jkz}}}_{0}$$

amplitudes (constant)

0

For a traveling direction in the +z direction only

Lossy EM Waves

Solution of the Wave Equation

The Electric Field in phasor form (only x component)

$$\frac{d^2 \tilde{E}_x}{dz^2} - \gamma^2 \tilde{E}_x = 0$$

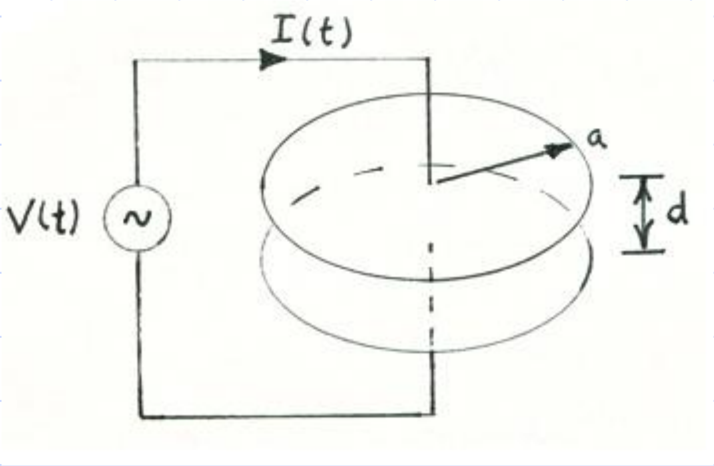
General solution of the differential equation for a lossy medium

$$\tilde{E}_x(z) = \tilde{E}_x^+(z) + \tilde{E}_x^-(z) = \underbrace{E_{x0}^+ e^{-(\alpha + j\beta)z}}_{\text{forward traveling in +z direction}} + \underbrace{E_{x0}^- e^{(\alpha + j\beta)z}}_{\text{backward traveling in -z direction}}$$

Lossy EM Waves

Capacitor Current Review

- Remember how we treated the two types of current flowing through this capacitor:



$$I = I_C + I_D = \sigma \pi a^2 \frac{V_o}{d} + j\omega \frac{\varepsilon \pi a^2}{d} V_o = (\sigma + j\omega\varepsilon)\pi a^2 \frac{V_o}{d}$$

Note that the conduction current has a phase angle of zero degrees while the displacement current has an angle of 90 degrees.

Lossy EM Waves

Complex Permittivity

$$\nabla \times \tilde{H} = (\sigma + j\omega\varepsilon)\tilde{E} = j\omega(\varepsilon - j\frac{\sigma}{\omega})\tilde{E}$$

ε_c complex permittivity

$$\varepsilon_c = \varepsilon' - j\varepsilon''$$

For lossless medium $\sigma=0$ $\varepsilon''=0$ $\varepsilon_c = \varepsilon' = \varepsilon$

Lossy EM Waves

Complex Permittivity

$$\vec{J} = \sigma E$$

$$\nabla \times \tilde{H} = \vec{J} + j\omega\varepsilon\tilde{E} = (\sigma + j\omega\varepsilon)\tilde{E} = j\omega\left(\varepsilon - j\frac{\sigma}{\omega}\right)\tilde{E}$$

$$\boxed{\varepsilon_c = \varepsilon' - j\varepsilon''}$$

complex permittivity ε_c

Homogenous wave equation (charge free)

Combining $\nabla \times \tilde{E} = -j\omega\mu\tilde{H}$ and $\nabla \times \tilde{H} = j\omega\varepsilon_c\tilde{E}$

$$\begin{cases} \nabla^2\tilde{E} - \gamma^2\tilde{E} = 0 \\ \nabla^2\tilde{H} - \gamma^2\tilde{H} = 0 \end{cases}$$

$$\gamma^2 = -\omega^2\mu\varepsilon_c$$

γ propagation constant

Lossy EM Waves

Wave Equations for a Conducting Medium

$$\nabla^2 \tilde{E} - \gamma^2 \tilde{E} = 0$$ Homogenous wave equation for $\tilde{E}$

$$\nabla^2 \tilde{H} - \gamma^2 \tilde{H} = 0$$ Homogenous wave equation for $\tilde{H}$

γ ; propagation constant is complex

$$\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu(\varepsilon' - j\varepsilon'')$$

$$\varepsilon' = \varepsilon \qquad \varepsilon'' = \frac{\sigma}{\omega}$$

Can also have this term in a lossy dielectric

Lossy EM Waves

Propagation Constant

$$\gamma = \alpha + j\beta$$

Attenuation constant

Phase constant

$$\alpha = \omega \left\{ \frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} - 1 \right] \right\}^{1/2}$$ *[Np/m]*

(for a lossy medium)

$$\beta = \omega \left\{ \frac{\mu\varepsilon'}{2} \left[\sqrt{1 + \left(\frac{\varepsilon''}{\varepsilon'} \right)^2} + 1 \right] \right\}^{1/2}$$ *[rad/m]*

Lossy EM Waves

- For T-line analysis we defined characteristic impedance Z_0, the ratio of voltage to current:

$$Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}} \approx \sqrt{\frac{r + j\omega l}{j\omega c}} \approx \sqrt{\frac{j\omega l}{j\omega c}}\sqrt{1 + \frac{r}{j\omega l}} \approx \sqrt{\frac{l}{c}}\left(1 - j\frac{r}{2\omega l}\right)$$

- For EM wave analysis we define intrinsic impedance η (also called wave impedance), which is the ratio of E to H.

$$\eta = \frac{E(x,y,z)}{H(x,y,z)} \qquad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$$

Lossy EM Waves

Intrinsic Impedance, η_c

The relationship between electric and magnetic field phasors is the same but the intrinsic impedance of lossy medium, η_c is different

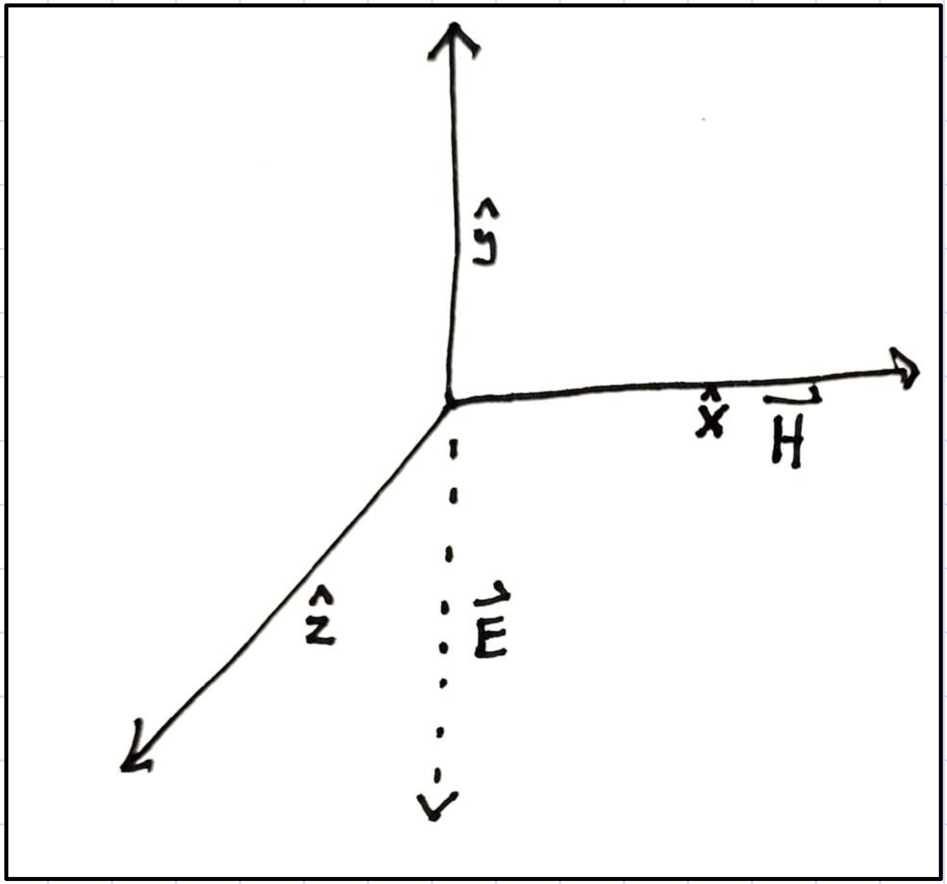
If +z is the direction of the propagation

$$\tilde{E} = -\eta_c \hat{a}_z \times \tilde{H} \qquad \tilde{H} = \frac{1}{\eta_c} \hat{a}_z \times \tilde{E}$$

intrinsic impedance $$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$$

Lossy EM Waves

Intrinsic Impedance, η_c



$$\tilde{E} = -\eta_c \hat{a}_z \times \tilde{H}$$

$$\tilde{H} = \frac{1}{\eta_c} \hat{a}_z \times \tilde{E}$$

Lossy EM Waves

Low-Loss Dielectric

defined when ε''/ε'<<1

practically if ε''/ε'<10^{-2}, the medium can be considered as a low-loss dielectric

$$\alpha = \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$ *[Np/m]*

$$\beta \cong \omega\sqrt{\mu\varepsilon'} = \omega\sqrt{\mu\varepsilon}$$ *[rad/m]*

$$\eta_c \cong \sqrt{\frac{\mu}{\varepsilon'}}$$ *[Ω]*

Note that these two terms have the same function but have different frequency dependence

Lossy EM Waves

Good Conductor

defined when $\varepsilon''/\varepsilon' >> 1$

practically if $\varepsilon''/\varepsilon' > 100$, the medium can be considered as a good conductor

$$\alpha \cong \omega\sqrt{\frac{\omega\varepsilon''}{2}} = \omega\sqrt{\frac{\sigma\mu}{2\omega}} = \sqrt{\pi f\mu\sigma} \qquad [Np/m]$$

$$\beta = \alpha \cong \sqrt{\pi f\mu\sigma} \qquad [rad/m]$$

$$\eta_c \cong \sqrt{j\frac{\mu}{\varepsilon''}} = (1+j)\sqrt{\frac{\pi f\mu}{\sigma}} = (1+j)\frac{\alpha}{\sigma} \qquad [\Omega]$$

- When $10^{-2} \leq \varepsilon''/\varepsilon' \leq 100$, the medium is considered as a "Quasi-Conductor".

Lossy EM Waves

Skin Depth, δ_s

shows how well an electromagnetic wave can penetrate into a conducting medium

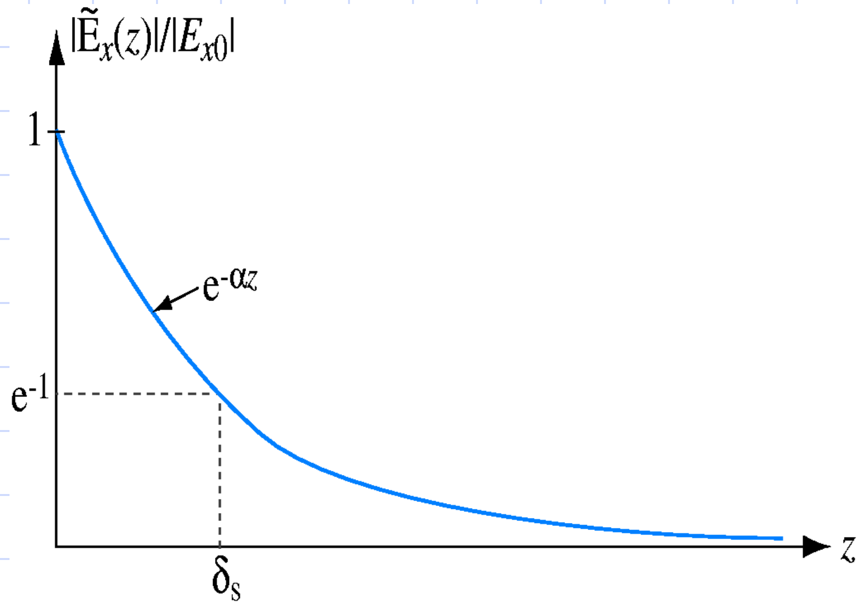
Skin Depth $$\delta_s = \frac{1}{\alpha} \quad \text{[m]}$$

Perfect dielectric:

$\sigma=0 \qquad \alpha=0 \qquad \delta_s=\infty$

Perfect Conductor:

$\sigma=\infty \qquad \alpha=\infty \qquad \delta_s=0$



Lossy EM Waves

Example 1

Find α, β, λ , and η of an electromagnetic wave traveling through seawater ($\varepsilon_r = 72$ $\sigma = 4$) at 10 MHz and 100 GHz.

Lossy EM Waves

Example 1

$\underline{f = 100\text{ GHz}}$; $\frac{\sigma}{\omega\varepsilon} = \frac{4}{(2\pi \times 10^{11})\, 72\varepsilon_0} \cong .0100 \Rightarrow$ low-loss dielectric

$\eta \cong \sqrt{\frac{\mu}{\varepsilon'}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right) = 44.4(1 + j.005) = 44.4 \angle .005 \text{ rad.}$

$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{72\varepsilon_0}} = \boxed{88.8\text{ m}^{-1} = \alpha}$

$\beta \cong \omega\sqrt{\mu_0\varepsilon'} = 2\pi \times 10^{4}\sqrt{\mu_0 72\varepsilon_0}$

$\boxed{\beta = 1.78 \times 10^{4}}$ $\boxed{\lambda = \frac{2\pi}{\beta} = 3.5 \times 10^{-4}}$

$\underline{f = 10\text{ MHz}}$

$\frac{\sigma}{\omega\varepsilon} \cong 100$; good conductor

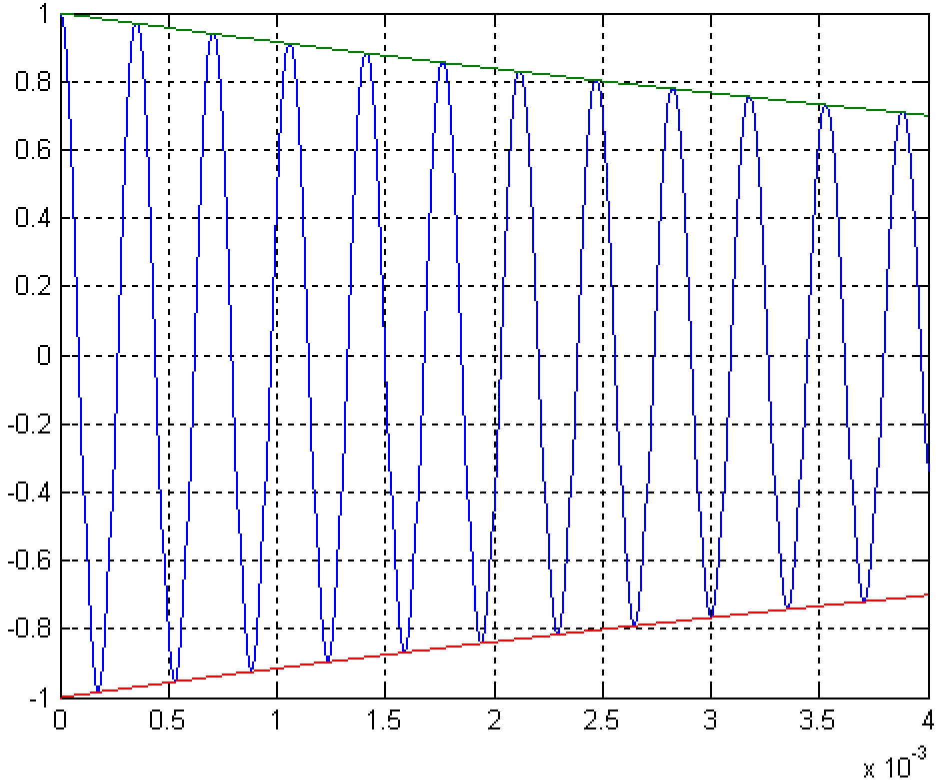
$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi f\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma} = \sqrt{\pi \times 10^{7}\mu_0\, 4} = \boxed{12.6\text{ m}^{-1}}$

$\lambda = \frac{2\pi}{\beta} = \boxed{0.5\text{ m}}$

$\eta = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = \sqrt{\frac{\pi f\mu}{\sigma}}(1+j) = \sqrt{\frac{\pi\, 10^{7}\mu_0}{4}}(1+j) = \boxed{\pi(1+j)}$ or $\sqrt{2}\pi e^{j\pi/4}$

Lossy EM Waves

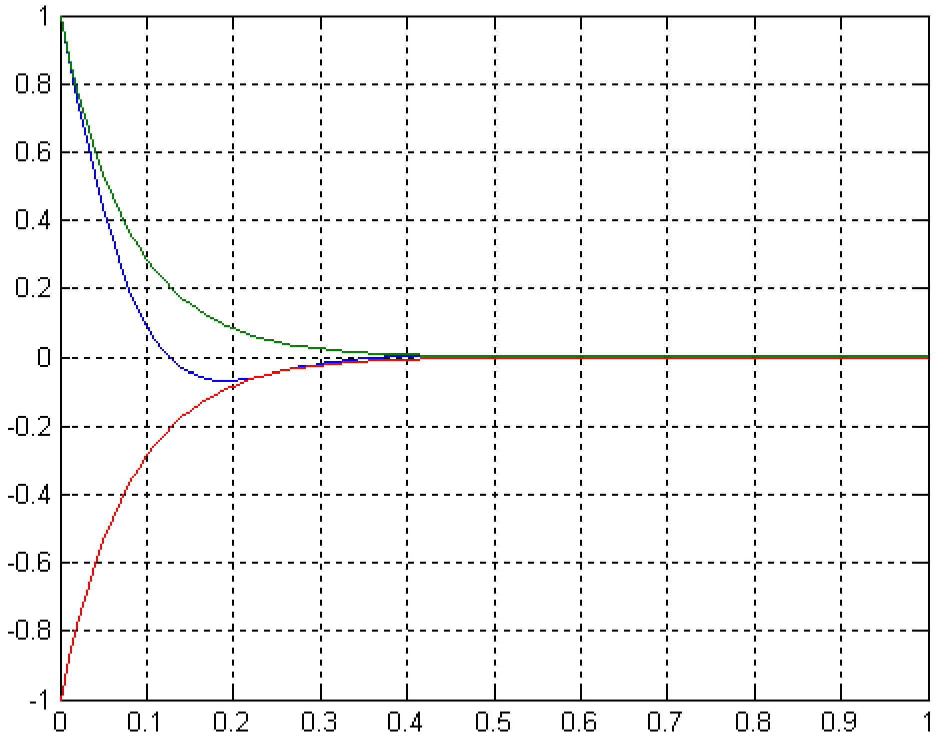
Example 1 – 100 GHz



What features do you observe in this wave?

Lossy EM Waves

Example 1 – 10 MHz



What features do you observe in this wave?

Lossy EM Waves

Example 3

The electric field of a plane wave is given by $\vec{E} = E_m cos(\omega t - \beta z)\hat{a}_x$

a. Write **E** in phasor form.

b. Is **E** the solution of this wave equation? $\frac{\partial^2 E_x}{\partial z^2} = -\mu\varepsilon\omega^2 E_x$

c. Find **H** using the phasor form of the ∇ x **E** equation. Assume the **E** and **H** phasors are only a function of z.

d. Evaluate the amplitude ratio, η = |**E**| / |**H**| in terms of material properties.

e. If **E** was in the **a**$_y$ direction, what direction would **H** be in?

f. How many independent parameters are there in the following set?

$$\omega, \beta, \mu, \varepsilon, \eta, \lambda, T$$

Lossy EM Waves

Example 3

a. $\boxed{\vec{E} = E_m e^{-j\beta z}\,\hat{a}_x}$

b. $\frac{\partial^2 E_x}{\partial z^2} = -\mu\varepsilon\omega^2 E_x \Rightarrow \frac{\partial E_x}{\partial z} = -j\beta E_m e^{-j\beta z} \quad \frac{\partial^2 E_x}{\partial z^2} = -\beta^2 E_m e^{-j\beta z}$

$\Downarrow$

$-\beta^2 E_x = -\mu\varepsilon\omega^2 E_x \Rightarrow \boxed{\beta^2 = \mu\varepsilon\omega^2}$

c. $\nabla \times \vec{E} = -j\omega\mu\vec{H} \Rightarrow \vec{H} = \frac{j}{\omega\mu}\nabla\times\vec{E},$

$\nabla \times \vec{E} = \frac{\partial E_x}{\partial z}\hat{a}_y + \underbrace{5 \text{ terms}}_{\to 0} = -j\beta E_m e^{-j\beta z}$

$\vec{H} = \frac{j(-j)}{\omega\mu}\beta E_m e^{-j\beta z}\hat{a}_y = \frac{\beta}{\omega\mu}E_m e^{-j\beta z}\hat{a}_y = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\mu}E_m e^{-j\beta z}\hat{a}_y$

$\boxed{\vec{H} = \sqrt{\frac{\varepsilon}{\mu}}\,E_m e^{-j\beta z}\,\hat{a}_y}$ ← several ways to write

Lossy EM Waves

Example 3

d. $\eta = \dfrac{|\vec{E}|}{|\vec{H}|} = \dfrac{E_m}{\sqrt{\frac{\varepsilon}{\mu}}\,E_m} = \boxed{\sqrt{\frac{\mu}{\varepsilon}} = \eta}$

e. If $\vec{E}$ is in $\hat{a}_y \Rightarrow \nabla \times \vec{E} = -\dfrac{\partial E_y}{\partial z}\hat{a}_x = -j\omega\mu\vec{H}$

extra minus from $\nabla \times \vec{E}$ $+\hat{a}_x$ $\Rightarrow$ $\boxed{\vec{H} \text{ is in } -\hat{a}_x \text{ direction}}$

There are 3 independent parameters

2 are material-related (i.e. ε & μ)

1 is frequency-related (f or λ or β)

Plane Waves

Example 4

WRPI broadcasts at 91.5 MHz. The amplitude of **E** on campus is roughly 0.08 V/m. Assume a coordinate system in which the wave is polarized in the $\mathbf{a}_y$ direction and propagating in the $\mathbf{a}_z$ direction.

Assume the phase is 0 at z = 0.

a. What are β, η and λ for this wave?

b. Write the electric and magnetic fields in phasor form.

c. Write the electric field in time domain form.

Lossy EM Waves

Example 3

d. $\eta = \frac{|\vec{E}|}{|\vec{H}|} = \frac{E_m}{\sqrt{\frac{\varepsilon}{\mu}}\, E_m} = \boxed{\sqrt{\frac{\mu}{\varepsilon}} = \eta}$

e. If $\vec{E}$ is in $\hat{a}_y \Rightarrow \nabla \times \vec{E} = -\frac{\partial E_y}{\partial z}\hat{a}_x = -j\omega\mu \vec{H}$

extra minus from $\nabla \times \vec{E}$ $+\hat{a}_x$ $\Rightarrow$ $\boxed{\vec{H} \text{ is in } -\hat{a}_x \text{ direction}}$

There are 3 independent parameters

2 are material-related (i.e. ε & μ)

1 is frequency-related (f or λ or β)

Plane Waves

Example 4

a. $\beta = \omega\sqrt{\mu\varepsilon} = 2\pi(91.5\times10^6)\sqrt{\mu_o\varepsilon_o} = \boxed{1.92\ m^{-1}}$

$\eta = \sqrt{\frac{\mu_o}{\varepsilon_o}} = \boxed{377}$ $\qquad \lambda = \frac{2\pi}{\beta} = \boxed{3.28\ m}$

b. generic $\hat{\vec{E}} = E_m e^{-j\beta z}\,\hat{a}_y$ $\qquad \vec{H} = -H_m e^{-j\beta z}\,\hat{a}_x$

$$\boxed{\hat{\vec{E}} = 0.08\, e^{-j1.92z}\,\hat{a}_y\ \frac{V}{m}}$$

$H_m = \frac{E_m}{\eta} = \frac{0.08}{377} = 2.12\times10^{-4}$ $\qquad \boxed{\hat{\vec{H}} = -2.12\times10^{-4} e^{-j1.92z}\,\hat{a}_x\ \frac{A}{m}}$

c. $\vec{E}(z,t) = \mathrm{Re}\left(\hat{\vec{E}}(z)\,e^{j\omega t}\right) = E_m\cos(\omega t - \beta z)\,\hat{a}_y$

$\omega = 2\pi f = 2\pi(91.5\times10^6) = 5.75\times10^8$

$$\boxed{\vec{E}(z,t) = 0.08\cos(5.75\times10^8 t - 1.92z)\,\hat{a}_y}$$

EM Wave Power Transmission

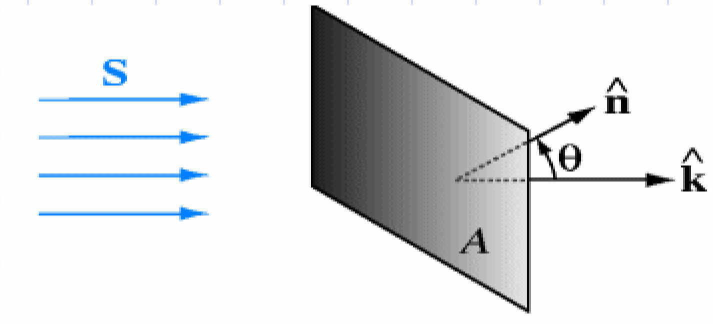
- Poynting Vector S , is defined

$$S = E \times H \qquad \text{[W/unit area]}$$

S is along the propagation direction of the wave

Ulaby

Total power



$$P = \int_A S \cdot \hat{a}_n \, dA \qquad \text{[W]}$$

OR $\quad P = |S| A \cos\theta \qquad$ [W]

Average power density of the wave [W/m²]

EM Wave Power Transmission

Average Power Density

$$\tilde{E}(z) = \tilde{E}_x^+(z)\hat{a}_x + \tilde{E}_y^+(z)\hat{a}_y$$

$$\tilde{E}(z) = (E_{x0}\hat{a}_x + E_{y0}\hat{a}_y)e^{-(\alpha+j\beta)z}$$

$$\tilde{H}(z) = \frac{1}{\eta_c}\hat{a}_Z \times \tilde{E} = \frac{1}{\eta_c}(-E_{y0}\hat{a}_x + E_{x0}\hat{a}_y)e^{-\alpha z}e^{-j\beta z}$$

Average power density

$$S_{av} = \frac{1}{2}\mathrm{Re}\left\{\tilde{E} \times \tilde{H}^*\right\} = \hat{a}_z \frac{1}{2}(|E_{x0}|^2 + |E_{y0}|^2)e^{-2\alpha z}\,\mathrm{Re}\left\{\frac{1}{\eta_c^*}\right\} \text{ [W/m}^2\text{]}$$

NOTE

EM Power Transmission

Plane wave in a Lossless Medium

$$\tilde{E}(z) = \tilde{E}_x^+(z)\hat{a}_x + \tilde{E}_y^+(z)\hat{a}_y$$

$$\tilde{E}(z) = (E_{x0}\hat{a}_x + E_{y0}\hat{a}_y)e^{-jkz}$$

$$\tilde{H}(z) = \frac{1}{\eta}\hat{a}_Z \times \tilde{E} = \frac{1}{\eta}(-E_{y0}\hat{a}_x + E_{x0}\hat{a}_y)e^{-jkz}$$

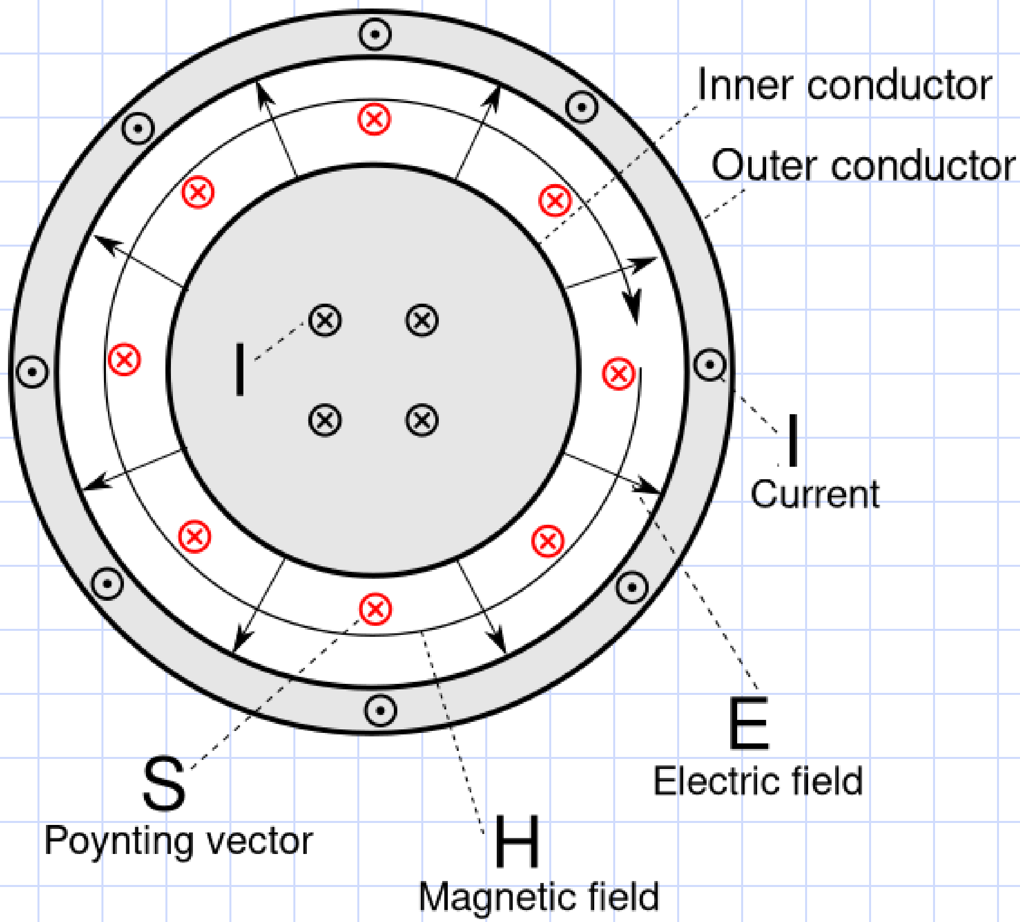
$$S_{av} = \hat{a}_z \frac{1}{2\eta}\left(\left|E_{x0}\right|^2 + \left|E_{y0}\right|^2\right)$$

$$\boxed{S_{av} = \hat{a}_z \frac{\left|\tilde{E}\right|^2}{2\eta}} \qquad [\mathrm{W/m^2}]$$

EM Wave Power Transmission

Coaxial Cable Poynting Vector

$$\vec{S} = \vec{E} \times \vec{H}$$

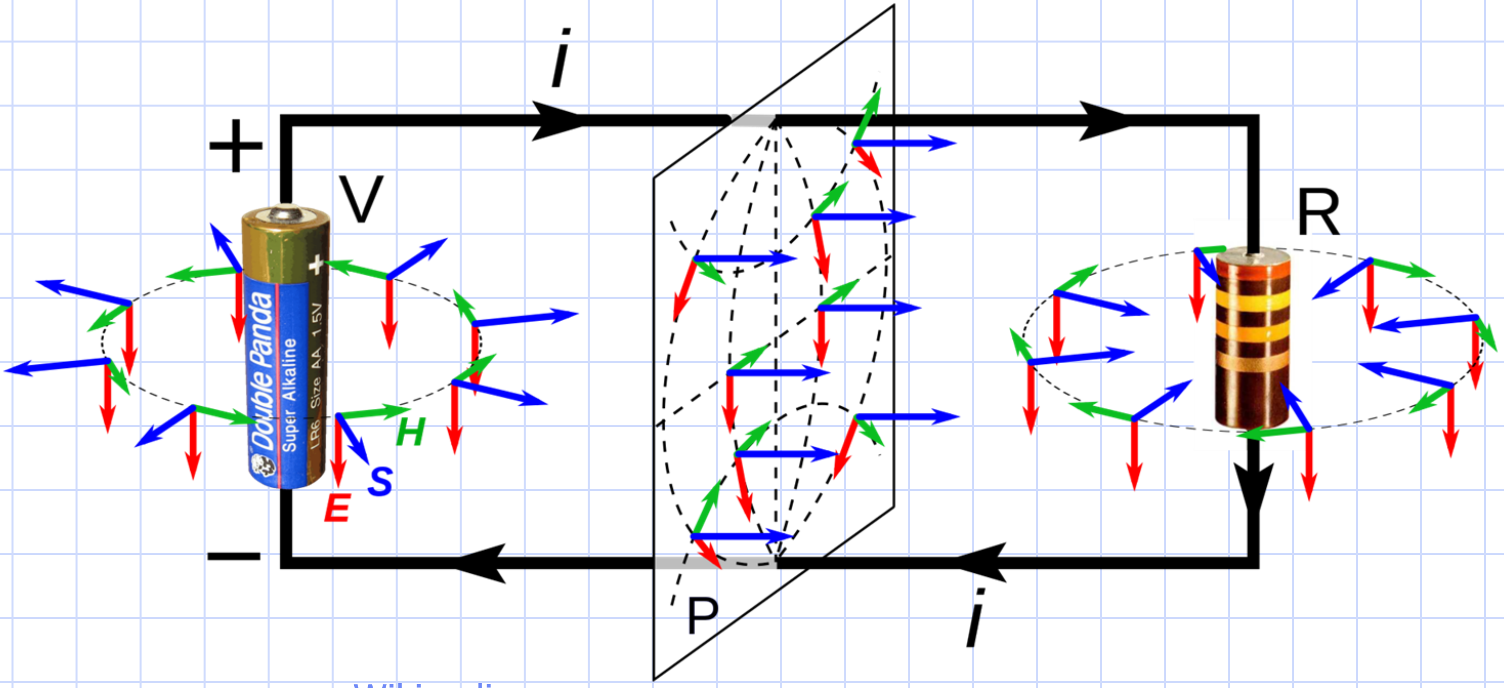


Wikipedia

EM Wave Power Transmission

Electric Circuit Poynting Vector

$$\vec{S} = \vec{E} \times \vec{H}$$



Wikipedia

EM Wave Power Transmission

Average Power Density

If η_c is written in polar form

$$\eta_c = |\eta_c| e^{j\theta_\eta}$$

Average power density

$$S_{av} = \hat{a}_z \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos\theta_\eta \qquad [\mathrm{W/m^2}]$$

where

$$|E_0| = \left[|E_{x0}|^2 + |E_{y0}|^2 \right]^{1/2}$$

EM Wave Power Transmission

The transmitter is about 10 km from campus. What transmitter power is required to radiate the same power density into a sphere of radius 10 km?

EM Wave Power Transmission

The transmitter is about 10 km from campus. What transmitter power is required to radiate the same power density into a sphere of radius 10 km?

$$P_{total} = \left|\vec{S}_{av}\right| 4\pi R^2 = (8.5\times10^{-6})(4\pi)(10^4\,m)^2 = \boxed{10.7\ kW}$$