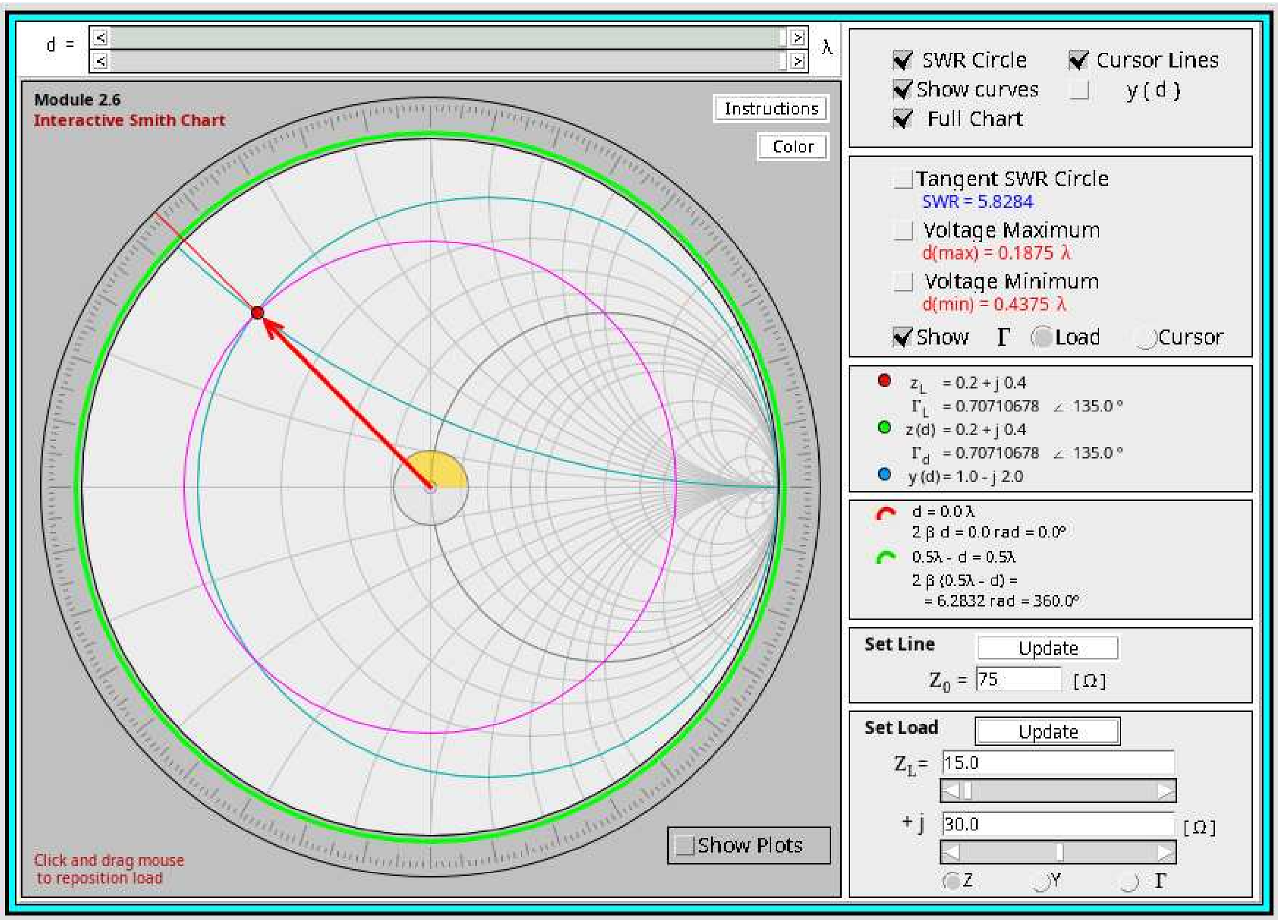
Exam 2 Practice Solutions

(no due date; just for practice)

1. Impedance Matching and Smith Charts

a.) This corresponds to an input impedance of 15+j30 Ω. Normalized for a 75 Ω transmission line, this becomes 0.2+j0.4.

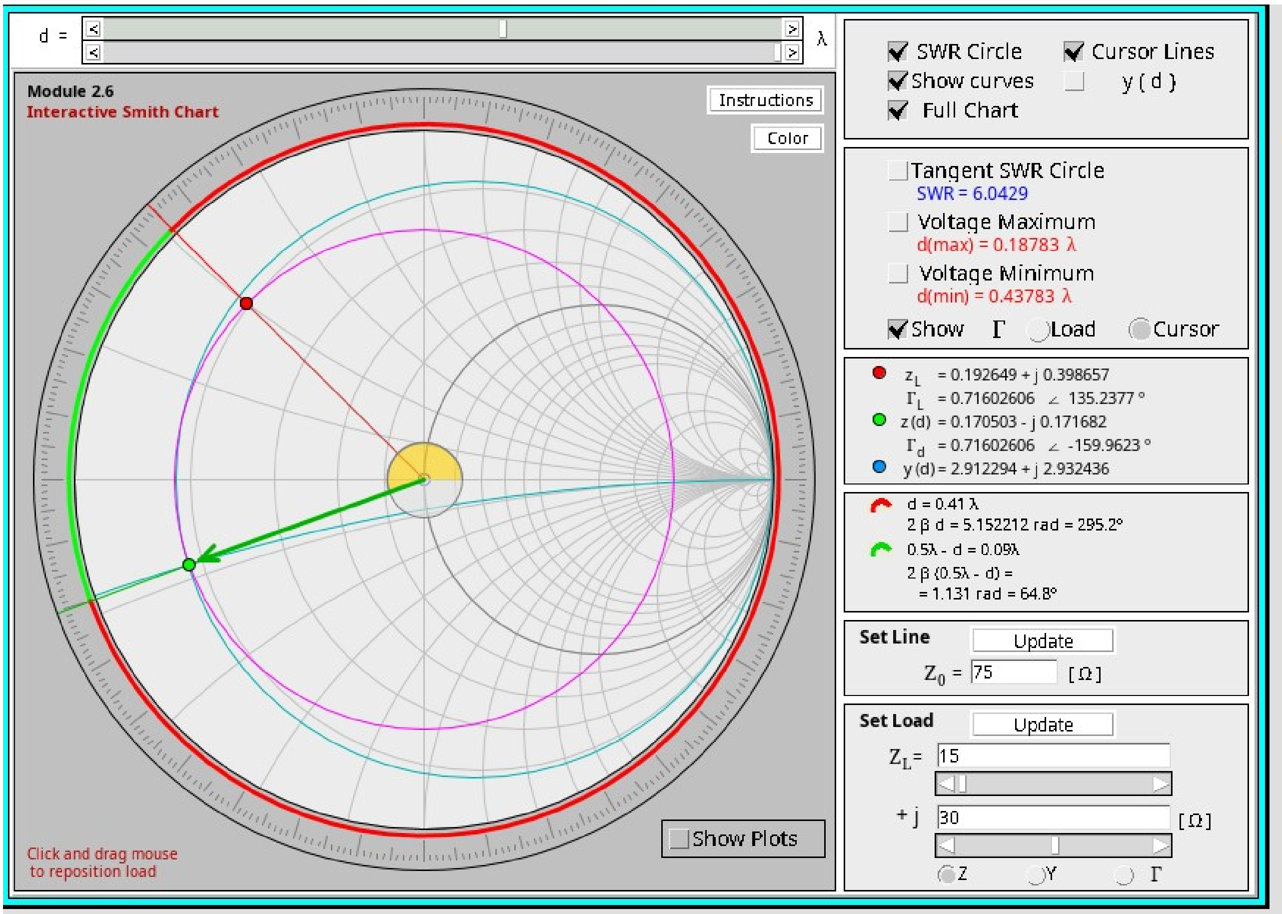


b.)

One wavelength on this line is 0.349 meters, so 10 meters represents 28.59 wavelengths.

Recall that one revolution around the Smith chart represents a half wavelength. To figure out how to represent the load impedance on the Smith Chart, we first figure out how many whole multiples of 0.5 are in 28.59. The answer is 57. (28.59 – 57*0.5 = 0.09). So as we move from the point 10m back to the load, we will rotate toward the load by 57 complete revolutions of the Smith chart, plus 0.09 wavelengths. Since one half wavelength has the property Zin = ZL, these 57 half-wavelengths will not change Zin; only the final 0.09 wavelengths will. So we can find the load impdance by taking the impedance we calculated in part a and rotating it 0.09 wavelengths toward the load. (Below, I show this on the Smith Chart widget by rotating 0.5-0.09 = 0.41 wavelengths toward the generator, which puts us in the same position on the Smith Chart as rotating 0.09 wavelengths toward the load.)

The normalized load impedance is shown to be 0.193 + j 0.399. Denormalizing for 75 Ω, this becomes 14.5+ j29.9 Ω.



c.) In part b, we determined that the wavelength of the 600MHz signal on this transmission line was 0.349 meters. Since the stub will be made of the same type of cable, the wavelength will be the same for the stub. We want the open-circuit stub to be a quarter wavelength long such that its input impedance at 600 MHz will be zero. This it should be 0.349m/4, or 8.725 centimeters, long.

This stub will work regardless of where we place it on the cable. Note that this stub is acting as a filter, and it is designed to have an input impedance of 0 regardless of where you put it on the cable. This is different from the matching stub described in the lecture, which is designed to have a specific admittance that matches the input admittance of the transmission line at a specific point on the line.

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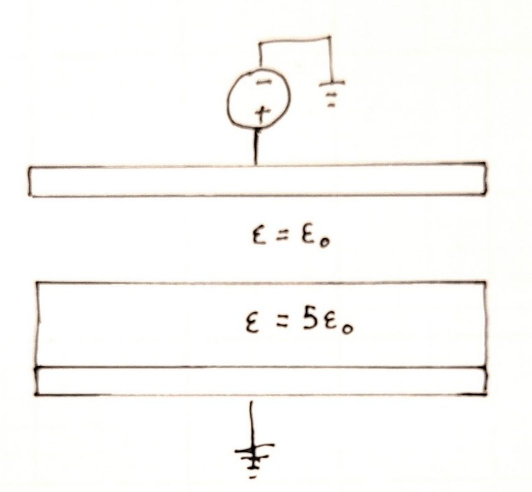
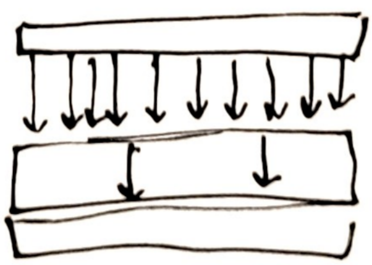
2. Capacitance

Consider the parallel plate capacitor shown below. The top and bottom plates are conductors, and half the space between them is filled with a dielectric. The plates are 10cm long and 10cm wide, and the width of the gap between them is 2cm. (1cm is free space and 1cm is dielectric). Assume that there is never any surface charge at the interface between the air gap and the dielectric.



a.)



5 times more field lines in the air gap than in the dielectric.

6.)

For an air-gap-only parallel plate capacitor, $\vec{E} = \sigma/\varepsilon_0$. Therefore $\vec{D} = \sigma$. For the air gap region of this capacitor, $\vec{E} = \sigma/\varepsilon_0$. For the dielectric region, $\vec{E} = \sigma/5\varepsilon_0$.

The area of the plates is $10cm \cdot 10cm = 0.01m^2$.

So $\sigma = \frac{1nC}{0.01m^2} = 100nC/m^2$

For air gap region,

$$\vec{E} = \frac{\sigma}{\varepsilon_0} = -(11.29\ kV/m)\,\hat{z}$$

For dielectric region,

$$\vec{E} = -(2.25\ kV/m)\,\hat{z}$$

(Note that we have defined the +z direction as pointing upwards. Since the electric field will point from the region of positive charge toward the region of negative charge, this field will point in the -z direction.)

c.) If we measure voltage from the top to the bottom plate:

$$V_{top} - V_{bottom} = -\int_{bottom}^{top} \vec{E} \cdot \vec{dl}$$

$$= -\left((-11.29\ kV/m)(0.01m) + (-2.25\ kV/m)(0.01m)\right)$$

$$= 135.4\ V$$

d.) $C = \frac{Q}{V} = \frac{1\,nC}{135.4\,V} = 7.386\ pF$

$$E = \frac{1}{2} C V^2 = \frac{1}{2}(7.386pF)(135.4V)^2$$

$$E = 67.7\ nJ$$

e.) As long as we continue to assume the same parallel plate capacitor condition, the fields will be the same.

The dielectric voltage will also be the same.

The air ~~gap~~ voltage is now

$$(11.29\ kV/m)(0.005\,m) + (2.25\,kV/m)(0.01\,m)$$

$$= 78.9\ V$$