

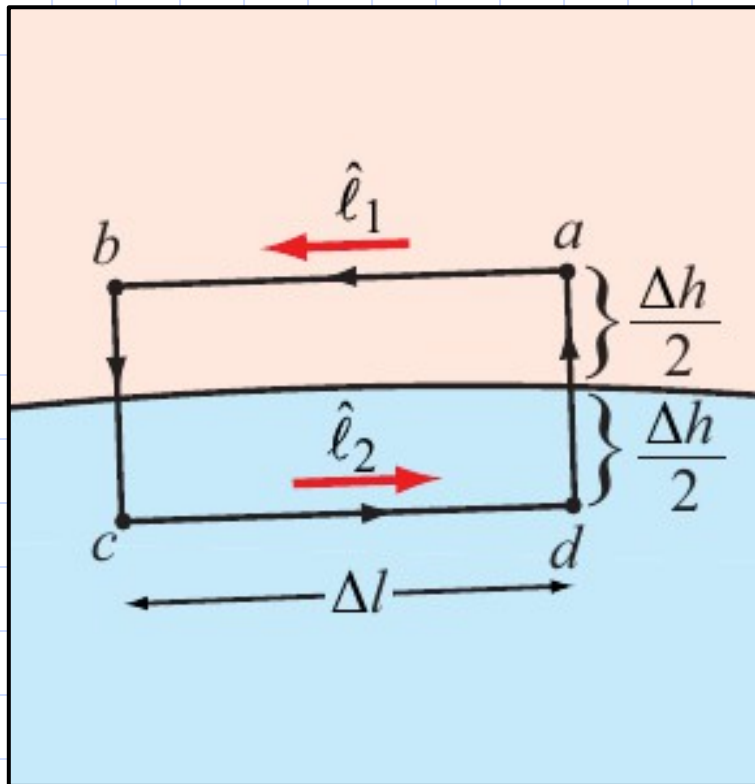
# Fields and Waves I

## Exam 2 Review

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# Boundary Conditions



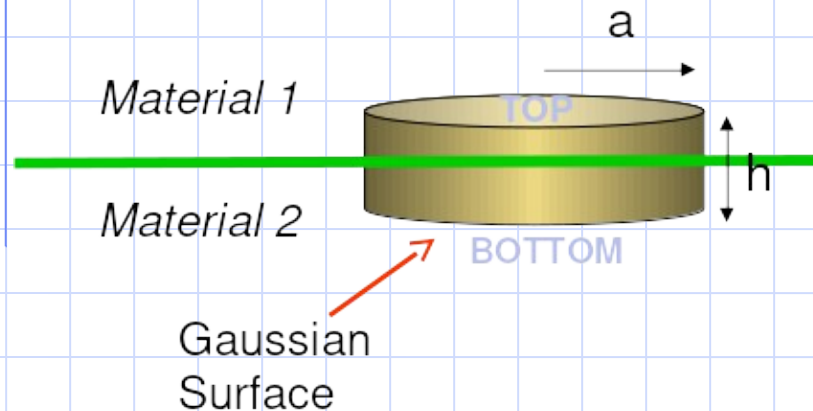
Ulaby

$$\vec{E}_{1t} = \vec{E}_{2t}$$

- So component of the E-field that is tangent to a media boundary is continuous across it.
- What about normal to the boundary?

# Boundary Conditions

## NORMAL COMPONENT



$$\oint \mathbf{D} \cdot d\mathbf{s} = Q_{\text{enclosed}}$$

Take  $h \ll a$  (a thin disc)

$$Q_{\text{enclosed}} = \rho_s \cdot A$$

$$\oint \mathbf{D} \cdot d\mathbf{s} = \int_{\text{TOP}} \mathbf{D} \cdot d\mathbf{s} + \int_{\text{BOTTOM}} \mathbf{D} \cdot d\mathbf{s}$$

$$= (D_{1n} - D_{2n}) \cdot A$$

$$\therefore D_{1n} - D_{2n} = \rho_s$$

# Dielectrics + Boundary Conditions

- 1.) An electric field of  $4 \text{ V/m}$  strikes a dielectric of relative permittivity  $10$  at an angle of  $45$  degrees. Determine the magnitude of the E-field inside the material as well as its angle relative to the boundary.
- 2.) Replace the dielectric with a conducting material. What is its surface charge density where the electric field touches it?

# Dielectrics + Boundary Conditions

# Dielectrics + Boundary Conditions

# Laplace and Poisson Equations

■ Laplace's Equation:

$$\nabla^2 V = 0$$

■ Poisson's Equation

$$\nabla^2 V = -\frac{\rho}{\epsilon}$$

$$\nabla^2 = \begin{bmatrix} \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

*(in cartesian coordinates)*

# Laplace and Poisson Equations

- A dielectric sphere with 10cm radius, relative permittivity 5, and a bound charge of  $1 \text{ C/m}^3$  is grounded at its center. Use Laplace and Poisson's Equations to find the voltage at all  $r$ .



# Laplace and Poisson Equations

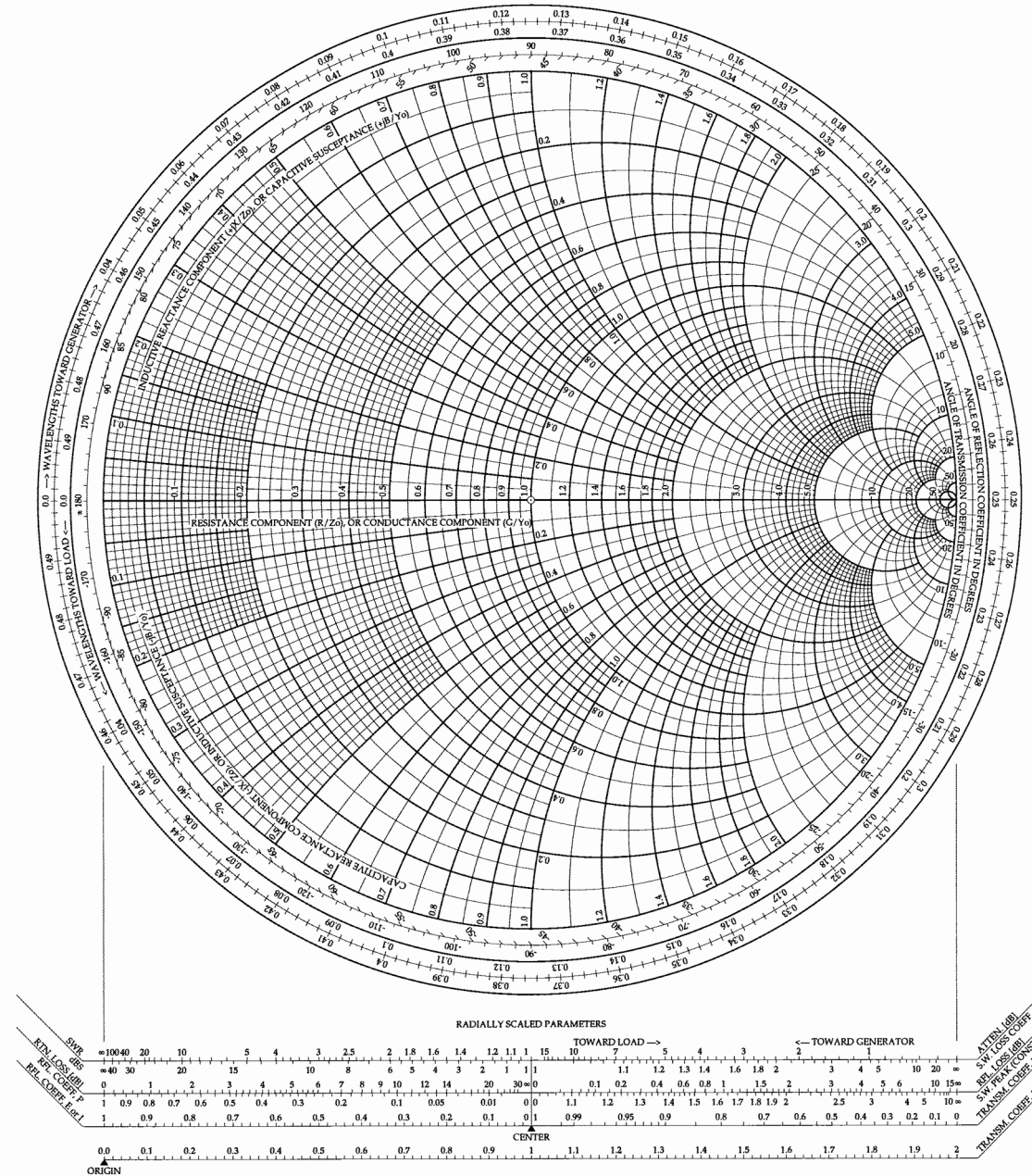
# Laplace and Poisson Equations

# Smith Charts

- 1.) Suppose the input impedance of a  $0.2$  wavelength transmission line is  $4$  ohms. Find the load impedance.
  
- 2.) You have a length of transmission line with a load at the end. Its input admittance is  $1 + 0.7j$ . You wish to match it to the line's characteristic impedance. What length of open-circuit stub should be added in parallel at this point to match the line?

# Smith Charts

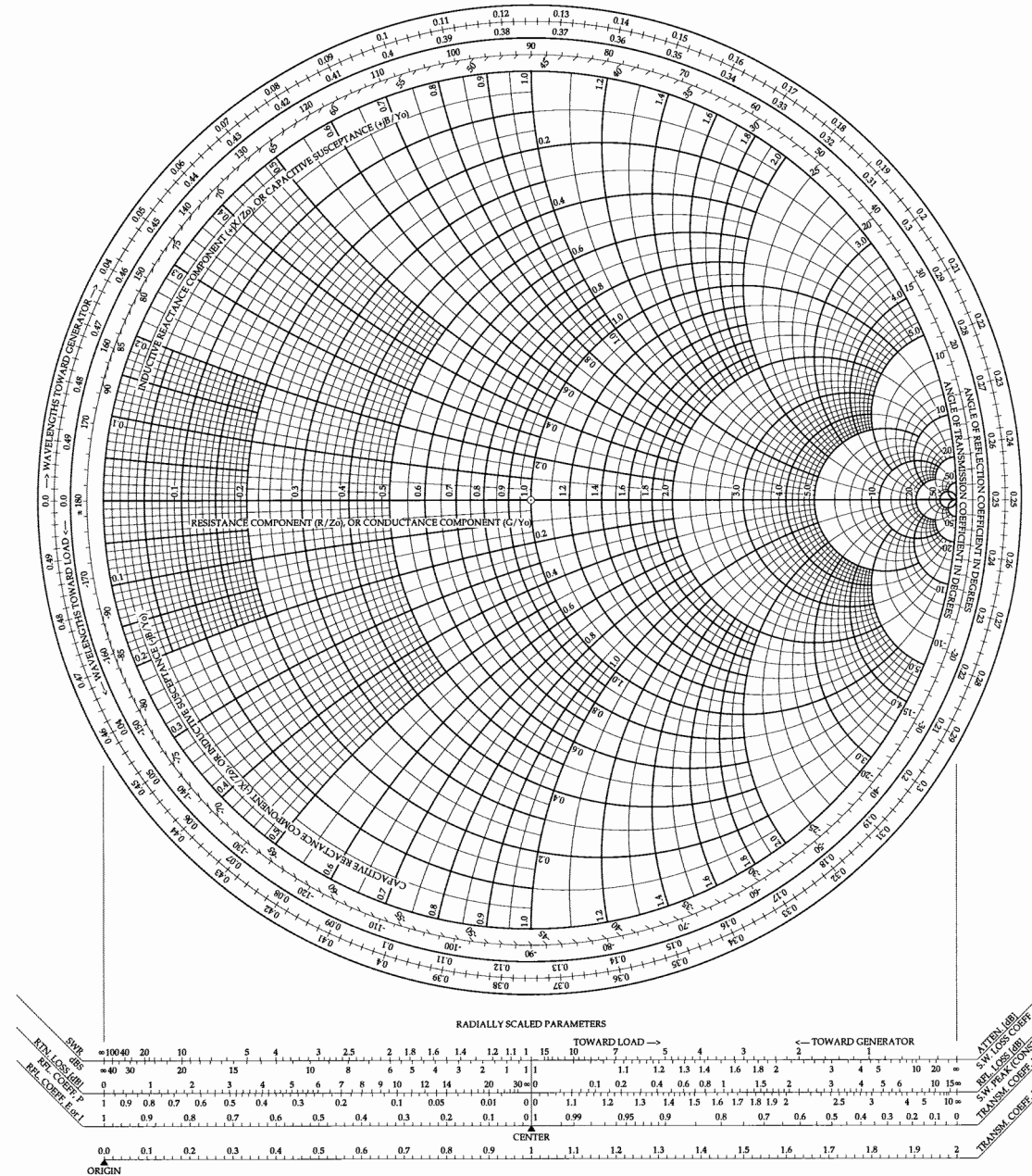
## The Complete Smith Chart Black Magic Design



Field

# Smith Charts

## The Complete Smith Chart Black Magic Design



Field

# Field Math

Differential Surfaces and  
Volumes

**How do you describe the shapes of all the  $ds$  surfaces in the following coordinate systems?**

■ Cartesian coordinates:

[https://mathinsight.org/cartesian\\_coordinates](https://mathinsight.org/cartesian_coordinates)

■ Cylindrical coordinates:

[https://mathinsight.org/cylindrical\\_coordinates](https://mathinsight.org/cylindrical_coordinates)

■ Spherical coordinates:

[https://mathinsight.org/spherical\\_coordinates](https://mathinsight.org/spherical_coordinates)