Exam 2

Graded

Student

Hayden Fuller

Total Points

35 / 60 pts

Question 1

Problem 1 — **9** / 10 pts

1.1 **Skill 2f** — **5** / 5 pts

- ✔ **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- ✔ **+ 1 pt** Correct units
- **+ 0 pts** Incorrect/incomplete units

1.2 **Skill 2i** — **4** / 5 pts

- **+ 4 pts** Complete mastery w/ calculations
- ✔ **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- ✔ **+ 1 pt** Correct units
- **+ 0 pts** Incorrect/incomplete units

Question 2

Problem 2 **16** / 30 pts

2.1 **Skill 2b** **3** / 5 pts

- **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- ✔ **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- ✔ **+ 1 pt** Correct units
- **+ 0 pts** Incorrect or no units

2.2 **Skill 2c** **4** / 5 pts

- ✔ **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- **+ 1 pt** Correct units
- ✔ **+ 0 pts** Incorrect or incomplete units

2.3 **Skill 2d** **2** / 5 pts

- **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- ✔ **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- **+ 1 pt** Correct units
- ✔ **+ 0 pts** Incorrect units / no units (units = field direction indicator)

2.4 **Skill 2g** **2** / 5 pts

- **+ 4 pts** Complete mastery w/calculations
- **+ 3 pts** Mastery (some math errors)
- ✔ **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery / Blank
- ✔ **+ 0 pts** No units / incomplete units / units error
- **+ 1 pt** Units included

2.5 **Skill 2h** **4** / 5 pts

- **+ 4 pts** Complete mastery w/ calculations
- ✔ **+ 3 pts** Mastery w/ math errors or incomplete math
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- ✔ **+ 1 pt** Units included
- **+ 0 pts** No units / insufficient units justification

2.6 **Skill 2j** **1** / 5 pts

- **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- ✔ **+ 0 pts** No mastery
- ✔ **+ 1 pt** Correct units
- **+ 0 pts** No units

Question 3

Problem 3 **4** / 5 pts

3.1 **Skill 2e** **4** / 5 pts

- ✔ **+ 1 pt** included Unit
- **+ 4 pts** Complete mastery w/ calculations
- ✔ **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery

Question 4

Problem 4 **3** / 10 pts

4.1 **Skill 2k** **0** / 5 pts

- **+ 1 pt** included Unit
- **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- ✔ **+ 0 pts** No mastery

4.2 **Skill 2l** **3** / 5 pts

- ✔ **+ 1 pt** included Unit
- **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- ✔ **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery

Question 5

Problem 5 **3** / 5 pts

5.1 **Skill 21** **3** / 5 pts

- **+ 1 pt** Unit included
- **+ 4 pts** Complete mastery
- ✔ **+ 3 pts** Mastery w / math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery

Exam 2

Instructions

1.) Unless otherwise specified, you have one class period to complete the questions below.

2.) Read all directions carefully.

3.) Show your work in enough detail to allow the graders to completely follow your thought process.

4.) Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use at least 2 significant digits.

5.) Make sure to write your answers legibly. You can write on the back of the exam pages or ask for scratch paper

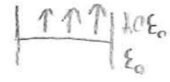
1. Boundary Conditions

Consider the boundary between two different materials. The top material has permittivity $\varepsilon_r = 20$ and the bottom material has the same permittivity as free space. Between the top and bottom materials there is a surface charge with density +200 nC / m^2.

$\varepsilon_0 = 8.85 \times 10^{-12}$

a.) (6 pts) Define some electric field in the top region. It may be any field you choose as along as you specify both magnitude and direction and the magnitude is not zero. Then calculate the electric field in the bottom region.

upwards facing magnitude 1 V/m



$D = \varepsilon E$ $\quad$ $C/m^2 = F/m \cdot V/m$

$D_{top} - D_{bottom} = \rho$

$D_{top} = 20\varepsilon_0 \cdot 1$

$20\varepsilon_0 - D_{bottom} = 200 \times 10^{-9}$

$D_{bottom} = 20\varepsilon_0 - 200 \times 10^{-9} = -1.998 \times 10^{-7} = \varepsilon_0 \cdot E_{bottom}$

$E_{bottom} = D_{bottom}/\varepsilon_0 = -22398$ V/m

b.) (6 pts) Now assume that each of the regions consists of a cube 1 meter in length on each side. Calculate the electric field energy density in each region and the total energy stored.

$w_e = \frac{1}{2}\varepsilon E^2 \qquad W_e = \frac{1}{2}\int \varepsilon E^2 \, dV$

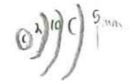
$w_{e\,top} = \frac{1}{2} 20\varepsilon_0 1^2 = 10\varepsilon_0 = 8.85 \times 10^{-11}$ J/m^3

$w_{e\,bottom} = \frac{1}{2}\varepsilon_0 \cdot 22398^2 = 250 \times 10^6 \varepsilon_0 = 0.00222$ J/m^3

$W_{e\,top} = 8.85 \times 10^{-11}$ J

$W_{e\,bottom} = 0.00222$ J

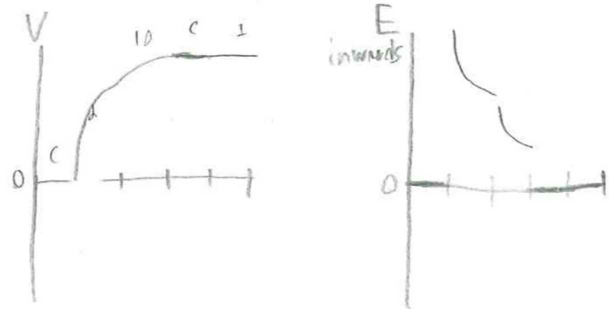
$C \quad 10\varepsilon_0$
$2\varepsilon_0 \quad C$



2. Capacitance

Consider a spherical capacitor with a layered structure. Its innermost layer is a grounded spherical conductor of radius 1mm. This is covered by a 1mm thick layer of dielectric with permittivity $2\varepsilon_0$, then a 1mm thick layer of permittivity $10\varepsilon_0$, and finally at the outermost layer, 1mm thick shell of conductor. At any given moment of operation, the capacitor has some charge +Q on the outer conductor and charge -Q on the inner conductor, which we will represent as the charge magnitude Q.

a.) Write an expression for the electric field inside all regions in the interval 0mm ≤ r≤5mm as a function of Q. Be sure to specify the direction of the field.



$$E = \frac{Q}{4\pi\varepsilon r^2}$$

$$E_{out} = \begin{cases} 0<r<1 & 0 \\ 1<r<2 & -Q/(8\pi\cdot\varepsilon_0 r^2) \\ 2<r<3 & -Q/(40\pi\varepsilon_0 r^2) \\ 3<r<4 & 0 \\ 4<r<5 & 0 \end{cases}$$

result is negative, so the field is facing inwards

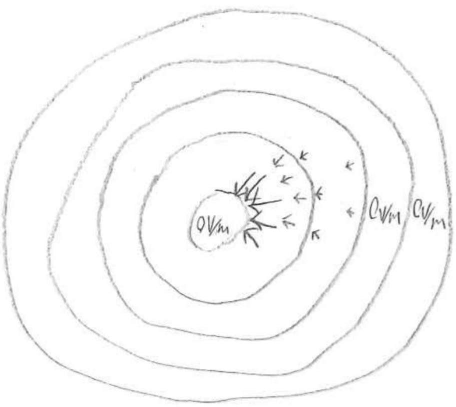
b.) For a given Q, where is there a nonzero surface charge density and what is its value?

$E_{2-} = 1.1\times10^9 \qquad D_{2-} = 2\varepsilon_0 E_{2-} = 0.01989$

$E_{2+} = 2.2\times10^8 \qquad D_{2+} = 10\varepsilon_0 E_{2+} = 0.01979 \qquad \rho = 0$

1mm, $\frac{3Q}{4\pi} \text{mm}^2 = \rho \qquad SA = \frac{4}{3}\pi\, 1\text{mm}^2$

c.) Draw a cross-section of this capacitor and sketch the field inside. Be sure to show the direction of the field and do your best to draw the field line density as being proportional to the field magnitude.



d.) Write an expression relating the voltage of this capacitor to Q, and calculate its capacitance.

$C = \frac{Q}{V}$ $Q = 2Q$ $V = 2Q/C$

$V = Q \int_{1mm}^{2mm} \frac{1}{8\pi\varepsilon_0 r^2} dr + \int_{2mm}^{3mm} \frac{1}{40\pi\varepsilon_0 r^2} dr = 2.397 \times 10^{12}$

$2.397 \times 10^{12} Q = 2Q/C$ $C = \frac{2}{2.397 \times 10^{11}} = 8.34 \times 10^{-13}$

$= 0.83 pF$

e.) How much voltage must be applied to this capacitor before dielectric breakdown occurs, and where will it occur? Assume that the dielectric strength of all dielectric materials is 20 MV/m. (That's megavolts, not millivolts.)

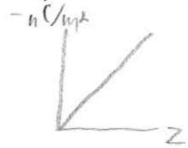
20×10^6 V/m
20×10^3 V/mm

40 kV, near the center conductor where the E field is stronger

3. Electric Charge

A cylinder is 1m tall and has a radius of 0.5m. The surface of the cylinder has a charge density of -z nC/m2. (the z axis points along the center of the cylinder and the bottom of the cylinder is at z=0, so the charge density starts as zero at the bottom of the cylinder and becomes increasingly negative toward the top.)

a.) What is the total charge Q on the surface of the cylinder?

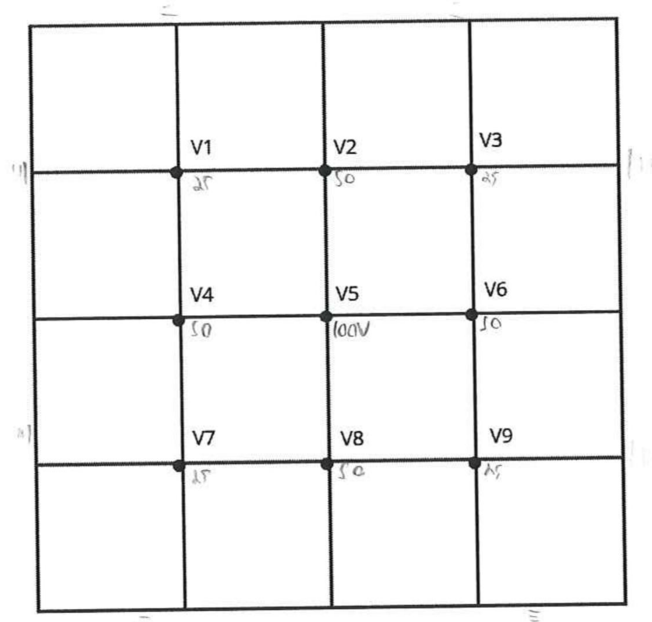


$Q = 2\pi(.5)\cdot\frac{1}{2} = \pi/2 \text{ nC}$

b.) If a +100nC charge is located at a height of 50m above the cylinder, what is the magnitude and direction of the force it will experience? (For this calculation you may approximate the cylinder as being a point charge.)

$F = \frac{100n \cdot \pi/2\, n}{4\pi\varepsilon_0 50^2} = 5.6497 \times 10^{-10}\text{ N}$

4. Finite Difference



Consider the box shown above, which is subdivided into 16 smaller regions. The box contains a material with the same permittivity as free space. At the center of the box, V5, there is a voltage source of voltage 100V. The outer edges of the box are grounded.

a.) Should you use Laplace's Equation or Poisson's Equation to solve for the voltage inside the box?

Poisson's

b.) Use 1 iteration of the Finite Difference Method to find the voltages at points V1 through V9.

V

c.) Suppose that the material inside the box has conductivity 5×10^{-5} S/m. Calculate the magnitude and direction of the density of current flowing between point V9 and the bottom grounded plate of the box.

downwards A/m

conductivity $S\,m/m^2$

$S \cdot L/A$

assume the whole box is $1m \times 1m$

$V=IR$

$I=VY$

$I = 100^V \cdot 5\times10^{-5}\ S/m = 5\ mA/m$ downwards

5. Impedance Matching

A telecom system has a transmission line with a 50Ω characteristic impedance. The line has a velocity of 0.6c. (c = $3x10^8$ m/s). The load consists of a 100Ω resistance and a 5μH inductance in series.

50Ω $(100 + 5 \times 10^{-6}$

You are sending a sinusoidal signal down this line toward the load. Choose a frequency for this signal in the range 1MHz – 5MHz.

$1\ MHz = 10^6\ Hz = 2\pi \times 10^6$ rad/sec

a.) What is the impedance of the load at this frequency? What is the normalized load impedance? Plot this impedance of the Smith chart on the next page.

$Z = 100 + j\omega L = 100 + j10\pi = 100 + j31.4$

$2 + j.628$ ohms

b.) What is the normalized load admittance? Plot it on the Smith Chart.

$Y = \frac{1}{z} = .455 - j.1429$

c.) Match the load to the 50Ω line using a parallel short-circuit stub made out of the same type of cable as the main line. You must report the distance (in meters) from the load at which the stub is located and the length of the stub. On the Smith Chart you should label the following:

1.) the input admittance of the transmission line at the place where the stub is located before the stub is added

2.) the distance in wavelengths between the load and the place where the stub is added

3.) the load admittance and input admittance of the stub, and the distance between the beginning and end of the stub

The Complete Smith Chart

Black Magic Design