21C – PhET Activity – Electric Field

The electric force on a charged object can be thought of as resulting from an interaction between that object and an electric field produced by all other charges.

In symbols, $\vec{F_0} = q_0\vec{E}_{all\ other\ charges}$, and in words, "The force on particle 0 due to all other charges is equal to the charge q_0 multiplied by the field due to all other charges." The force on charge q_0 due to point charge q_1 is $\vec{\boldsymbol{F}}_{1\ on\ 0} = k\frac{q_0q_1}{r_{10}^2}\frac{\vec{\boldsymbol{r}}_{10}}{|\vec{\boldsymbol{r}}_{10}|}$, so the field at the position of charge 0 due to point charge q_1 is

$$\vec{\boldsymbol{E}}_0 = \frac{1}{q_0}k\frac{q_0q_1}{r_{10}^2}\frac{\vec{\boldsymbol{r}}_{10}}{|\vec{\boldsymbol{r}}_{10}|} = k\frac{q_1}{r_{10}^2}\frac{\vec{\boldsymbol{r}}_{10}}{|\vec{\boldsymbol{r}}_{10}|}. \qquad \text{(Eq. 21.b)}$$

The field points away from a positive charge and decreases in strength as the inverse square of the distance from the source charge. The field at a point is the vector sum of the fields from all charges (except the test probe charge) in the problem.

You will now explore some of the properties of the electric field using simulation software.

Equipment: Personal computer that can run the PhET (HTML5) application Charges and Fields

- Download and run the PhET simulation, "Charges and Fields". https://phet.colorado.edu/en/simulation/charges-and-fields
- Click/Check "Electric Field", "Values", and "Grid" in upper right box.
- Drag a +1 nC charge to a grid point near the center of the grid. The arrows that appear indicate the direction of the field at the points at the center of the arrows.
- Drag the sensor tool (from the box at the bottom of the screen) around the grid and observe how the direction and magnitude change with position. You can drag many sensors to various locations to make it easier to visualize the vector field.

1) What do you think the intensity of the field vectors indicates?

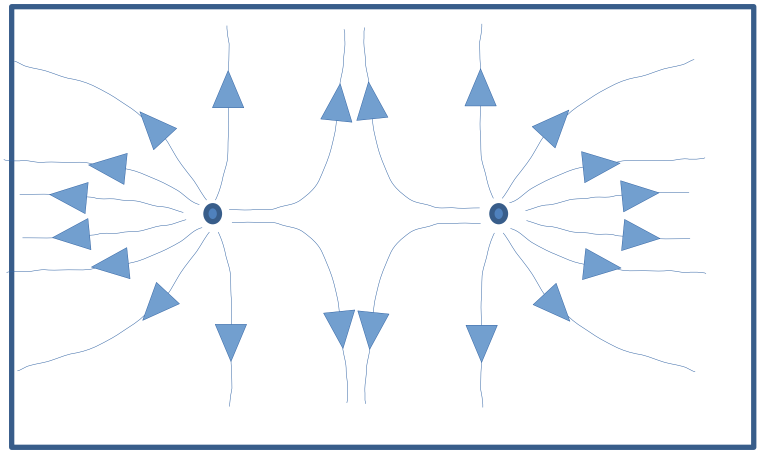
The intensity of the electric field at a given point.

2) Drag a field sensor (bottom tool box) to distances of 1, 2, and 3 meters from the 1 nC charge. What is field strength at each of these points (Note that 1V/m = 1N/C.)? Compare your simulation result with the field strength computed directly from equation 21b.

Distance	1 m	2 m	3 m
Observed Field (N/C)	8.99	2.24	1.00
Calculated Field (N/C)	9	2.25	1

- Clear the field and charges by clicking the button in the lower right screen. Click on Electric Field, Values, and Grid again. Drag a +1 nC charge to a grid point slightly to the left of center. Drag a second charge to a grid point 3 m (6 major grid lines) to the right of the first.
- Drag several *Sensors* to various locations on the grid so that you can get a sense of the strength of the field.

3) Sketch field lines to represent the field in the space to the right. Remember that field line density is greatest where the field is largest.



4) Drag a second + 1 nC charge on top of the first charge on the left (making + 2 nC at that point). There is a point at which the field is zero. Find its distance from the left-hand charge using the tape measure tool.

The zero point is ___1.80___m from the left-hand charge.

5) Use equation 21b to find the point on the x-axis at which the field is expected to be zero. (Show your work.)

2/r^2=1/(3-r)^2
2(3-r)^2=r^2
2(r^2-6r+9)=r^2
2r^2-12r+18=r^2
r^2-12r+18=0
r=1.757

6) Explain why the field cannot be zero for points off the x-axis.

Becacuse both particles are the same charge, they will have y components that don't cancel out.

21D – Calculating Fields using Vector Notation

Equipment: Paper, pencil, and brain.

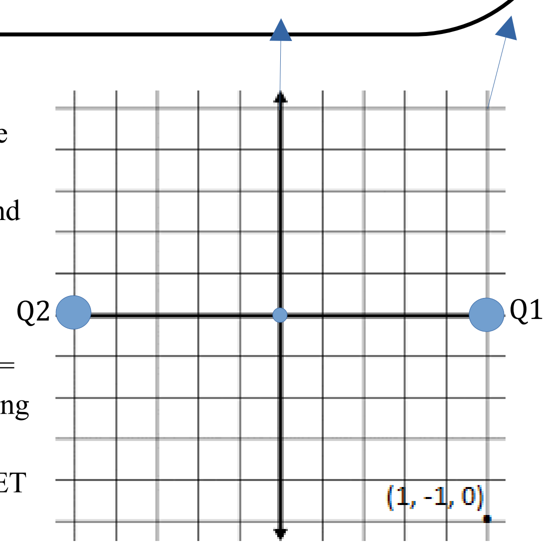
Key Theoretical Ideas and Text Source:
Reading: Lecture Notes 2. Young and Freedman Section 21.5.

- The electric field at the point (x, y, z) due to a collection of charges at positions (x_i, y_i, z_i) can be written as:
  $$\vec{E}(x,y,z) = \sum_i \frac{q_i}{4\pi\varepsilon_0}\frac{(x-x_i)\hat{\boldsymbol{i}} + (y-y_i)\hat{\boldsymbol{j}} + (z-z_i)\hat{\boldsymbol{k}}}{\left((x-x_i)^2+(y-y_i)^2+(z-z_i)^2\right)^{3/2}} \qquad \text{(Eq. 21c)}$$
- For continuous distributions of charge, the q_i above are represented by a differential dq', which is the charge in an infinitesimal volume dV', where d$q' = \rho$dV'. (The prime is added to indicate that the variable describes where the charge is.)
- For continuous distributions:
  $$\vec{\boldsymbol{E}}(x,y,z) = \iiint \frac{\rho(x',y',z')}{4\pi\varepsilon_0}\frac{(x-x')\hat{\boldsymbol{i}} + (y-y')\hat{\boldsymbol{j}} + (z-z')\hat{\boldsymbol{k}}}{\left((x-x')^2+(y-y')^2+(z-z')^2\right)^{3/2}}\,dx'dy'dz' \qquad \text{(Eq. 21d).}$$
  - The integral is over the primed coordinates, which denote where the charge is located. This can be a difficult calculation for the general case, but can be simplified for many special cases. Two examples:
    For a sheet of charge in the x-y (z = 0) plane with area charge density $\sigma(x', y')$:
    $$\vec{\boldsymbol{E}}(x,y,z) = \iint \frac{\sigma(x',y')}{4\pi\varepsilon_0}\frac{(x-x')\hat{\boldsymbol{i}} + (y-y')\hat{\boldsymbol{j}} + z\hat{\boldsymbol{k}}}{\left((x-x')^2+(y-y')^2+z^2\right)^{3/2}}\,dx'dy' \qquad \text{(Eq. 21e)}$$
  - For a line of charge along the x-axis with charge linear charge density $\lambda(x')$ on the x axis:
    $$\vec{\boldsymbol{E}}(x,y,z) = \int \frac{\lambda(x')}{4\pi\varepsilon_0}\frac{(x-x')\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}} + z\hat{\boldsymbol{k}}}{\left((x-x')^2+y^2+z^2\right)^{3/2}}\,dx' \qquad \text{(Eq. 21f)}$$

Questions 1a-g will help you to interpret the relations above and practice with Cartesian vector components. They all deal with the same physical situation, which you will sketch in question 1. In problems 1a-g, substitute numbers for variables when possible and simplify by eliminating terms that drop out or cancel (become zero).

1)

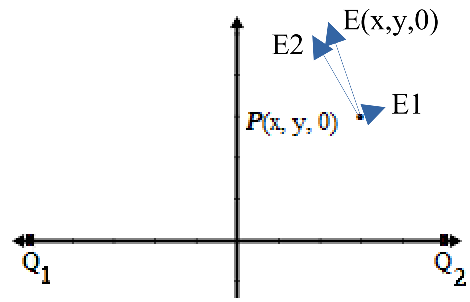
a. Sketch the charges Q_1 = +1nC at point $\boldsymbol{P_1}$'= (+1, 0, 0) and Q_2 = +1nC at point $\boldsymbol{P_2}$'= (-1, 0, 0) on the x-y plane to the right, along with sketches of the direction for the electric field vectors at points $(\pm1, \pm1, 0)$, $(0, \pm1, 0)$, $(0, 0, 0)$. (You can use the PhET simulation from the last module if you like.)



You now have an indication of what the resultant $\vec{E}$ field looks like at each of the specific points. The point of observation will now be located at an arbitrary point ***P*** = (*x, y,* 0).

b. *Field vector at any point* ***P*** = (*x, y,* 0):

i. On the diagram, sketch the vectors $\vec{\boldsymbol{E}}_1$ and $\vec{\boldsymbol{E}}_2$ and the resultant vector $\vec{\boldsymbol{E}}(x, y, 0)$ at the point ***P*** = (*x, y,* 0).

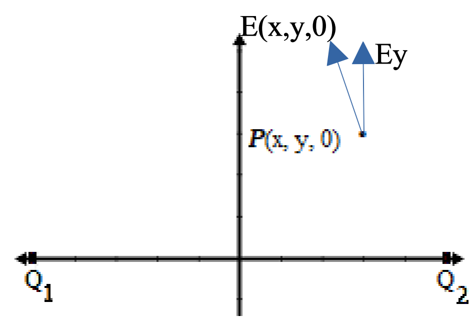


ii. For the charges in question 1, $\boldsymbol{P}_1{}'$ = (-1, 0, 0) and $\boldsymbol{P}_2{}'$ = (1, 0, 0), rewrite equation 21c to find the field at **any** point ***P*** = (*x, y,* 0) in the *x-y* (*z* = 0) plane.

$$\vec{E}(x, y, 0) = \sum_i \frac{q_i}{4\pi\varepsilon_0} \frac{(x - x_i)\hat{\boldsymbol{i}} + (y - y_i)\hat{\boldsymbol{j}}}{\left((x-x_i)^2+(y-y_i)^2\right)^{3/2}}$$

c. *Component of the field in the y – direction at any point* ***P*** = (*x, y,* 0):

i. On the diagram, sketch the resultant vector $\vec{\boldsymbol{E}}(x, y, 0)$ and the *y*-component, $\vec{\boldsymbol{E}}_y$, at the point ***P*** = (*x, y,* 0).



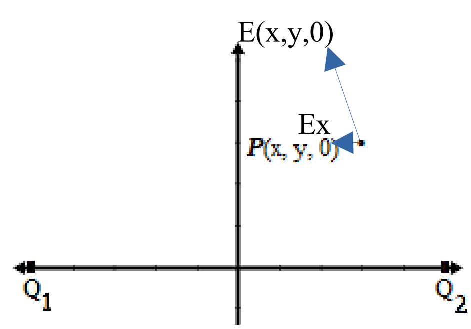
ii. Write the equation for the *y*-component ($\vec{\boldsymbol{E}}_y$) (Simplify your answer from question 1b, above), for the electric field.

$$E_y(x, y, 0) = \sum_i \frac{q_i}{4\pi\varepsilon_0} \frac{(y - y_i)\hat{\boldsymbol{j}}}{\left((x-x_i)^2+(y-y_i)^2\right)^{3/2}}$$

d. *Component of the field in the x – direction at any point* ***P*** = (*x, y,* 0):

i. On the diagram, sketch the resultant vector $\vec{\boldsymbol{E}}(x, y, 0)$ and the *x*-component, $\vec{\boldsymbol{E}}_x$. at the point ***P*** = (*x, y,* 0).

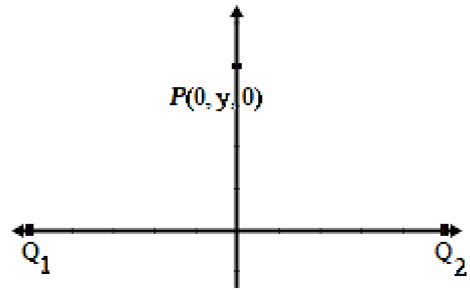
ii. Write the equation for the *x*-component ($\vec{\boldsymbol{E}}_x$) (Simplify your answer to question 1b, above), for the electric field



$$E_x(x, y, 0) = \sum_i \frac{q_i}{4\pi\varepsilon_0} \frac{(x - x_i)\hat{\boldsymbol{i}}}{\left((x-x_i)^2+(y-y_i)^2\right)^{3/2}}$$

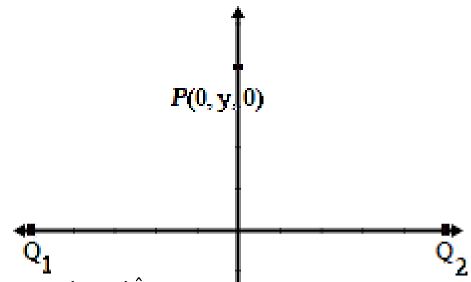
You now have an indication of what the resultant $\vec{E}$ field looks like at the arbitrary point $\boldsymbol{P} = (x, y, 0)$. ***The point of observation will now be shifted to P = (0, y, 0).***

e. Find the field in the y-direction at the point $\boldsymbol{P} = (0,y,0)$: Write the y-component (simplify your answer to question 1b, above), of the field in the y-direction ($\vec{E}_y$).



$$E_y(0, y, 0) = \sum_i \frac{q_i}{4\pi\varepsilon_0} \frac{(y-y_i)\hat{j}}{((y-y_i)^2)^{3/2}}$$

f. At the same point, $\boldsymbol{P} = (0, y, 0)$ find the *Field in the x-direction*: Write the x-component (question 1b, above), of the field in the x-direction ($\vec{E}_x$).



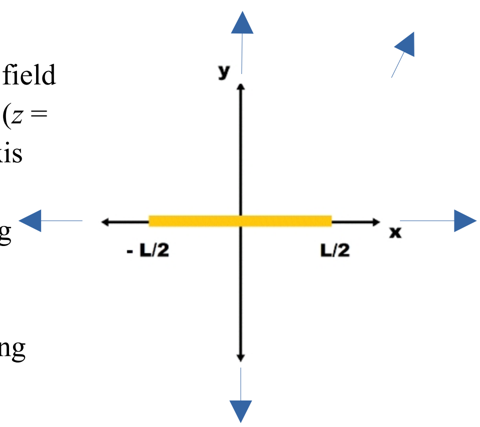
$$E_x(0, y, 0) = 0$$

g. Show that the x-component of the field is zero at the point (0, y, 0).

$$E_x(0, y, 0) = \sum_i \frac{q_i}{4\pi\varepsilon_0} \frac{(x-x_i)\hat{i}}{((x-x_i)^2+(y-y_i)^2)^{3/2}} = \frac{q_i}{4\pi\varepsilon_0} \frac{(x-x_i)\hat{i}}{((x-x_i)^2+(y-y_i)^2)^{3/2}} + \frac{q_i}{4\pi\varepsilon_0} \frac{-(x+x_i)\hat{i}}{((x-x_i)^2+(y-y_i)^2)^{3/2}} = 0$$

<u>Note that you could have avoided doing the actual vector calculation by noting that the x-components of the field from the two charges oppose and cancel one another along the y-axis</u>. This is a common trick in field calculations and takes advantage of the symmetry of the problem.

2) Now let's tackle a harder problem. We will use equation 21f to find the field for a continuous distribution of charge. We want to find the field in the x-y (z = 0) plane due to a line of total charge Q distributed uniformly along the x-axis $\boldsymbol{P'} = (x', 0, 0)$ between points $x' = +L/2$ and $x' = -L/2$. Just as in question 1) above, we will use the same physical situation for all parts (a-i), simplifying when possible as we proceed.



a. Sketch your guesses for the resultant electric field vector at the following points including (L,0,0), (-L,0,0), (0,L,0), (0,-L,0), (L,L,0).

b. Linear charge density λ is defined as the ratio of charge to length.

   a) Write the linear charge density, λ in terms of Q and L.

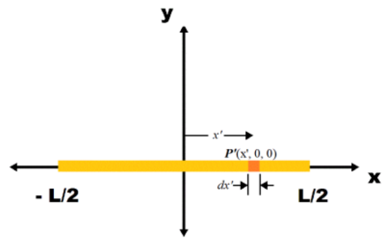
   $\boxed{\lambda = Q/L}$

   b) Write the linear charge density, λ in terms of increment of charge dq' and the corresponding infinitesimal length dx'.

   $\boxed{\lambda = dq'/dx'}$

c. Express dq' in terms of the corresponding infinitesimal length element dx', located at x', and the linear charge density: λ

$dq' = \lambda dx'$



Eq. 21c has to be slightly modified to give the increment of field $d\vec{E}$ at point $\boldsymbol{P} = (x, y, z)$, due to charge dq' at point $\boldsymbol{P'} = (x', y', z')$, yielding,

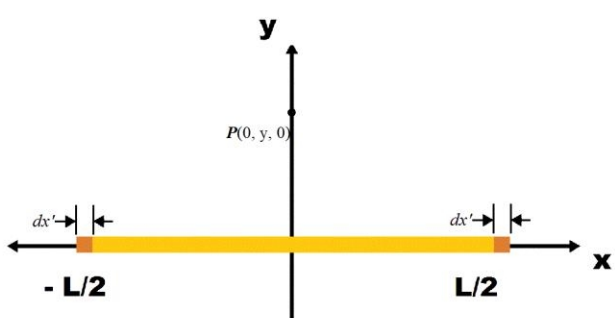
$$d\vec{\boldsymbol{E}}(x, y, z) = d\vec{\boldsymbol{E}}_x + d\vec{\boldsymbol{E}}_y + d\vec{\boldsymbol{E}}_z = \frac{dq'}{4\pi\varepsilon_0} \frac{(x - x')\hat{\boldsymbol{i}} + (y - y')\hat{\boldsymbol{j}} + (z - z')\hat{\boldsymbol{k}}}{((x - x')^2 + (y - y')^2 + (z - z')^2)^{3/2}}$$

d. Simplify the above equation for the increment of electric field $d\vec{\boldsymbol{E}}$ at any point in the plane $\boldsymbol{P} = (x, y, 0)$ due to an increment of charge dq' located at point $\boldsymbol{P'} = (x', 0, 0)$. Use dq' from answer 2c above.

$$d\vec{\boldsymbol{E}}(x, y, 0) = \frac{\lambda dx'}{4\pi\varepsilon_0} \frac{(x - x')\hat{\boldsymbol{i}} + (y - y')\hat{\boldsymbol{j}}}{((x - x')^2 + (y - y')^2)^{3/2}}$$

e. If you want to calculate the total field at any point $\boldsymbol{P} = (0, y, 0)$ along the y axis, do you need to calculate both the x and y components? (Is one of the components zero? If one is zero, identify which one is zero, and why it is zero. A sketch can provide a good explanation.)

Zero Component is x because the left and right sides cancel out



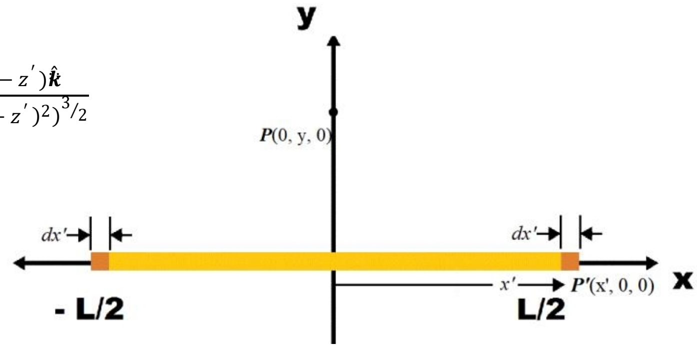
f. (Ok, so the x-component is zero.) Now write the electric field, in the form of the modified equation shown above at point $\boldsymbol{P} = (0, y, 0)$ (in the non-zero direction) due to a charge dq' at point $\boldsymbol{P'} = (x', 0, 0)$.

$$d\vec{\boldsymbol{E}}(0, y, 0) = \frac{\lambda dx'}{4\pi\varepsilon_0} \frac{(y - y')\hat{\boldsymbol{j}}}{((y - y')^2)^{3/2}}$$

g. To find the total field you have to sum over all dq's corresponding to each increment of length dx'. Summing of infinitesimals is called integration. Set up the integral over the length of the line of charge, including correct limits and integration variables. (Do not actually solve the integral.).

$$\int_{-L/2}^{L/2} \frac{\lambda dx'}{4\pi\varepsilon_0} \frac{(x-x')\hat{i} + (y-y')\hat{j} + (z-z')\hat{k}}{((x-x')^2 + (y-y')^2 + (z-z')^2)^{3/2}}$$

$$k \int_{-L/2}^{L/2} \frac{(y-y')\hat{j}}{(x'^2+y^2)^{3/2}} \lambda dx'$$



(Here is the result of your integral if you set it up correctly. $\vec{E} = \frac{1}{4\pi\varepsilon_0 y} \frac{Q}{\left(y^2+\left(\frac{L}{2}\right)^2\right)^{1/2}} \hat{j}$ (Eq. 21g).)

h. Suppose now that the length of the line segment L is much, much less than y, i.e., $y >> L$. If done correctly, Eq. 21g should reduce to a familiar form in the limit ($y >> L$).

Go ahead and simplify Eq. 21g for this limit. Hint: Look at the terms shown in the denominator to the right - $\frac{1}{\left(y^2+\left(\frac{L}{2}\right)^2\right)^{1/2}}$. Which one can be ignored? Finally substitute the result back into the equation

$$\vec{E} \cong \quad k \frac{Q}{y^2} \hat{j}, \text{ and solve.}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0 y} \frac{Q}{{y^2}^{1/2}} \hat{j}}$$

i. Is this limit of Eq. 21g physically sensible? Compare it to Coulomb's Law for a point charge Q at the point (0,0,0). Express why the two results coincide!

At small L or large y, the line charge approaches the same behavior as a point charge