MATH-2400 INTRODUCTION TO DIFFERENTIAL EQUATIONS FALL 2022

Problem Set 1

Assigned: Friday, September 2, 2022
Due: 5pm, Friday, September 9, 2022
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NOTES

1. Practice problems listed below and taken from the textbook are for your own practice, and are not to be turned in.
2. There are two parts of the Problem Set, an objective part consisting of multiple choice questions (with no partial credit available) and a subjective part (with partial credit possible). Please complete all questions.
3. Writing your solutions in LaTeX is preferred but not required.
4. Show all work for problems in the subjective part. Illegible or undecipherable solutions will not be graded.
5. Figures, if any, should be neatly drawn by hand, properly labelled and captioned.
6. Your completed work is to be submitted electronically to LMS as a single pdf file. Be sure that the pages are properly oriented and well lighted. (Please do not e-mail your work to Muhammad or me.)

Practice Problems from the textbook (Not to be turned in)

- Exercises from Chapter 1, pages 4–6: 1(b,c,g), 2(g,i), 3(f), 5(d), 6(a).
- Exercises from Chapter 2, pages 12–14: 1(c,f,l,m), 2(d,f,h), 3(f,g,h), 4(c,d).

Objective part (Choose A, B, C or D; no work need be shown, no partial credit available)

1. (5 points) Consider the DE $t^2y'' + y'\ln(t) = y\cos(t)$. Choose the best answer among the following classifications of the DE.

   A Linear and homogeneous

   B Nonlinear

   C Linear

   DX Linear and non-homogeneous

2. (5 points) For what value of a does the DE $ty'' + ay' + ty + 2\sin(2t) = 0$ admit $y(t) = t\sin(2t)$ as a solution?

   A $a = 2$

   B $a = -2$

   C $a = -1$

   DX No value of a

3. (5 points) Consider the DE $y' = 2y + 4$. A plot of the direction field reveals the qualitative behavior of solutions $y(t)$ of the DE. Suppose a solution is chosen such that $y(0) = C$. Select the value of C for which the remains bounded as $t \to \infty$.

   A $C = 2$

   BX $C = -2$

   C All values of C

   D No value of C

Subjective part (Show work, partial credit available)

1. (15 points)

   (a) Find all values of the constant a, if any, such that $u(t) = (3 + a\sin t)^{-1/2}$ is a solution of the first-order ODE $4u' + u^3 \cos t = 0$.
   $u' = -1/2(3 + a\sin t)^{-3/2} a\cos t$
   $-2a + 1 = 0$
   $a = 1/2$

   (b) Find all values of the constant b, if any, such that $v(t) = 6e^{bt}$ is a solution of the second-order ODE $v'' + 5v' + 6v = 0$.
   $v' = 6be^{bt}$
   $v'' = 6b^2e^{bt}$
   $6b^2e^{bt} + 30be^{bt} + 36e^{bt} = 0$
   $b^2 + 5b + 6 = 0$
   $b = -3 \qquad b = -2$

   (c) Find all values of the constant c, if any, such that $w(x,t) = \sin(x - ct)$ is a solution of the second-order PDE $w_{tt} = 9w_{xx}$.
   $w_t = -c\cos(x - ct) \qquad w_{xx} = -c^2\sin(x - ct)$
   $w_x = \cos(x - ct) \qquad w_{xx} = -\sin(x - ct)$
   $-c^2\sin(x - ct) = -9\sin(x - ct)$
   $c = 3 \qquad c = -3$

2. (15 points) Consider the DE

$$y' + 5y = 3$$

   (a) Find constants a and b such that $y(t) = Ce^{at} + b$ is a solution of the DE for any value of the constant C.
   $y' = Cae^{at}$
   $Cae^{at} + 5Ce^{at} + 5b = 3$
   $a = -5 \qquad b = -3/5$

   (b) Find C such that $y(t)$ satisfies the DE and the initial condition $y(0) = -2$.
   $y(0) = -2 = Ce^{-5*0} - 3/5$
   $-2 = C - 3/5$
   $C = -7/5$

3. (15 points) Consider the DE

$$yy' = \sin(2t)$$

Integrate both sides of the equation to determine $y(t)$ satisfying the DE and the initial condition $y(0) = -1$.

$\int(y\frac{dy}{dt})dt = \int(\sin(2t))dt$

$\int(y)dy = \int(\sin(2t))dt$

$\frac{y^2}{2} = -\frac{1}{2}\cos(2t) + C$

$\frac{(-1)^2}{2} = -\frac{1}{2}\cos(2*0) + C$

$1/2 = -1/2 + C$

$C = 1$

$\frac{y^2}{2} = -\frac{1}{2}\cos(2t) + 1$

$y^2 = -cos(2t) + 2$

$y = \sqrt{-cos(2t) + 2}$