MATH-2400 DIFFEQ F 2022 Crib Sheet Exam 1 Tuesday, October 4, 2022 Hayden Fuller Notes:

CLASIFICATION/FIRST ORDER

ODE-one var PDE-multi var

linear: $F(t, y(t), y'(t))$: $f(t, y) = p(t)y + q(t) = g(t)$

homo: $y(t) = 0$ is a solution, $g(t) = 0$

seperable: y on left, t on right, $y' = F(t)G(y)$

$y' = F(t)G(y)$; $\frac{dy}{G(y)dt} = F(t)$; $\int \frac{1}{G(y)} dy = \int F(t)dt + C$; plug in IC, solve for C; plug in C, solve for y(t)

$y' + p(t)y = q(t)$; $\mu(t) = e^{\int p(t)dt}$; steps: $\mu'(t) = p(t)\mu(t) = p(t)e^{\int p(t)dt}$; $\mu y' + \mu' y = \mu q$; $y(t) = \frac{1}{\mu(t)} \int \mu(t)q(t)dt + \frac{C}{\mu(t)}$

interval where t solution exists? $y' + p(t)y = q(t)$ where $p(t)$ and $q(t)$ are continuous.

Interval with t_0 has unique solution for $y(t)$ for any t on interval and any $y(t_0) = a$ and $y'(t_0) = b$

Modeling: rate constant(only): $Q' = kQ$; $Q(t) = Ce^{kt}$; (with addition): $Q' = kQ + R$; $Q = Ce^{kt} - \frac{R}{k}$

Population: direction field: ind:t,y dep:y'

phase plot: $y' = f(y)$, 0=solution, decreasing 0=stable, increasing 0=unstable.

SECOND ORDER: $y'' + p(t)y' + q(t)y = g(t)$; $y(t) = C_1 y_1(t) + C_2 y_2(t)$

wronskion: $w(t) = det[y_1, y_2, //, y_1', y_2'] = y_1 y_2' - y_1' y_2 \neq 0$ (linearly independent, not multiples of each other)

$L[y_1] = y_1'' + p(t)y_1' + q(t)y = g(t)$

CONSTANT COEFFICIENT: $ay'' + by' + cy = 0$; $y(t) = e^{rt}$; $ar^2 + br + c = 0$

CASE 1 $r_1, r_2 \in R$, $r_1 \neq r_2$; $b^2 - 4ac > 0$; $y(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

CASE 2 $r_1, r_2 \in R$, $r_1 = r_2$; $b^2 - 4ac = 0$; $y(t) = C_1 e^{rt} + C_2 t e^{rt}$

REDUCTION OF ORDER:

$y_2 = y_1 h$, $y_2' = y_1' h + y_1 h'$, $y_2'' = y_1'' h + 2y_2' h' + y_1 h''$

$y_1'' h + 2y_2' h' + y_1 h'' + p(t)(y_1' h + y_1 h') + q(t)(y_1 h) = (y'' + py' + qy)h + (2y' + py)h' + yh''$ (should) $= (2y' + py)h' + yh''$, $u = h'$; $yu' + (2y' + py)u = 0$

get u to one side, integrate to find u (has C); integrate to get h (has D) and plug in to $y_2 = y_1 h$; choose C and D to be easy.

CASE 3 $r_1, r_2 \notin R$; $b^2 - 4ac < 0$; $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = r = \lambda \pm i\omega$;

$y_1^c = e^{\lambda + i\omega} = e^{\lambda t}(\cos(\omega t) + i \sin(\omega t))$

$y_2^c = e^{\lambda - i\omega} = e^{\lambda t}(\cos(\omega t) - i \sin(\omega t))$

$y_1(t) = e^{\lambda t} \cos(\omega t)$ $y_2(t) = e^{\lambda t} \sin(\omega t)$

CAUCHY-EULER

$ax^2 y'' + bxy' + cy = 0$; $y = x^r$; $ar(r-1) + br + c = 0$ roots r_1 , r_2

CASE 1: $y = C_1 x^{r_1} + C_2 x^{r_2}$

CASE 2: $y = C_1 x^r + C_2 x^r \ln(x)$

CASE 3: $y = C_1 x^{\lambda} \cos(\omega \ln(x)) + C_2 x^{\lambda} \sin(\omega \ln(x))$

polar: $y = Re^{\lambda t} \cos(\omega t - \phi)$; $y(t) = (R\cos(\phi))e^{\lambda t} \cos(\omega t) + (\sin(\phi))e^{\lambda t} \sin(\omega t)$

$R \cos \phi = C_1$, $R \sin \phi = C_2$

Examples:

$3y'' + 4y' - 7y = 0$; $y(t) = 7e^{at}$; $y'(t) = yae^{at}$; $y''(t) = 7a^2 e^{at}$

$21a^2 e^{at} + 28ae^{at} - 49e^{at} = 0$; $21a^2 + 28a - 49 = 0$; $a = 1$

1: $t^3 y' + 4t^2 y = e^t$; $y(1) = 0$, $t > 0$, $y' + 4t^{-1} y = e^t t^{-3}$, use $\mu = \exp(\int 4t^{-1} dtt) = \exp(4 \ln(t)) = t^4$

$(t^4 y)' = te^t$