Fields and Waves I
Studio Session 3
Spring 2024
Due 11:59pm, February 7th
Joseph Hutchinson, Xavier Jiang , Gavin Divincint, Hayden Fuller

When you have completed the lab, submit the answers to the underlined questions on Gradescope. If you wish, you may work with a partner and submit one report for both of you. There is no need for a "formal" lab report.

Question 1: Obtain a long spool of coaxial table. Figure out its time delay. You can use the same methods that you used in Studio Session 2. While doing this, determine whether or not your oscilloscope needs a 50-ohm terminating resistor at the input. If so, add one and leave it in place for the remainder of the lab.

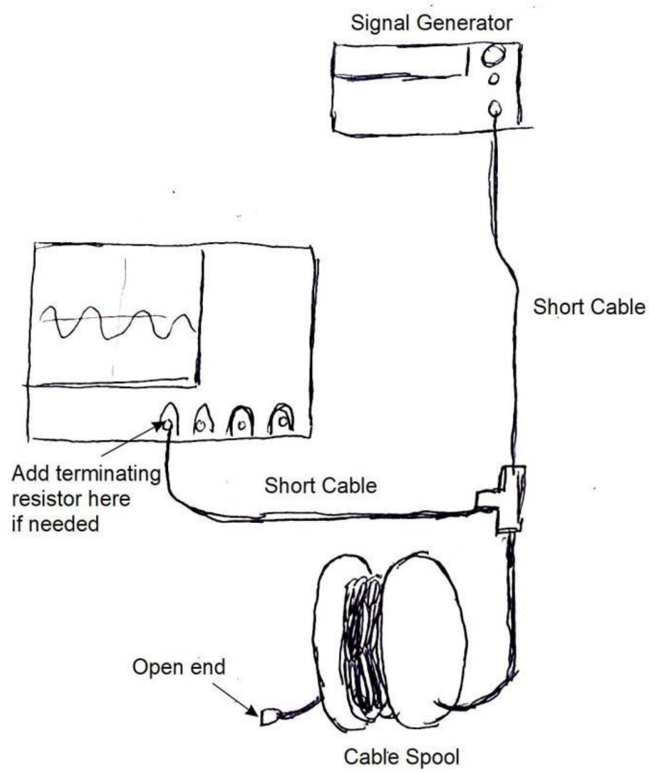
Using a 1MHz input signal, we measured a time delay of:

$$t_d = t_{coax} - t_{source} = 416\ ns$$

$length = \text{signal velocity} * \text{time delay} = (0.66 * 3 * 10^8 m/s) * (416 * 10^{-9} s) = 82.368\ \text{meter}s$

We have no need for the terminating 50 Ω resistor, since the nominal impedance of our RJ-58 cable is 50 Ω, and the oscilloscope is set to terminate at 50 Ω.

Question 2: Next, connect the spool to the signal generator and the scope in the manner shown in the diagram below. One end of the spool will be unconnected (and therefore an open circuit).



If you consider this setup using conventional circuit theory, you may be inclined to think that the long cable spool will not affect the circuit because the end is an open circuit. This will most likely be true for small frequencies. Set the signal generator to output a 1kHz sinusoidal signal. Do you see any effect from the spool?

There is no noticeable effect on the signal measured by the oscilloscope. The scope is measuring a perfect 1kHz, 1Vp-p sinusoidal signal. So at a low frequency, the open circuit of the coax line does not seem to affect things.

<u>Question 3</u>: Now we will consider higher frequencies. Remember in class that we stated that a quarter-wavelength transmission line will make an open circuit look like a short circuit. In other words, if we input a signal into the spool with wavelength four times as long as the spool, the signal will appear to be shorted (i.e. the amplitude will drop to zero).

Figure out the frequency for which your spool length is a quarter of a wavelength. What is this frequency? *(An easy way to do this is to multiply the time delay of your spool by four. This the is period of the signal.)*

$$f = \frac{v_f * c}{4 * \lambda}$$

$$f = \frac{0.66 * 3 * 10^8 m/s}{4 * 82.368 \text{ m}} = 600961\, Hz \;=\; 600.961\, kHz$$

Input a sinusoidal signal from the signal generator. Start at about 100 kHz and increase in 100 kHz increments until you see a decrease in the amplitude of the signal you measure. Then increase / decrease in smaller increments. Keep tweaking the frequency until you find the frequency that produces the minimum signal amplitude. Does this agree with your calculation of the quarter-wavelength frequency? (*Note: You may not see the amplitude go all the way to zero due to various effects we won't discuss here.)*

We found that ~575 kHz led to the minimum signal amplitude achievable.

This agrees fairly well with our calculated quarter-wavelength frequency of 600 kHz ! Our calculated value has a ~4.5% error from the actual measured frequency.

<u>Question 4</u>: Attach a 50-ohm terminating resistor on the end of the long spool of cable, then try sweeping the signal frequency past the quarter-wavelength frequency. What effect does the resistor have on the change in frequency with amplitude? Why is this?

Prior to adding the 50-ohm terminating resistor, changing the frequency could lead to reflection that made the measured Vpp up to ~1.6v (even when our original signal is 1Vpp).

But, with the 50-ohm resistor connected to the end of the long cable, most of the reflection is now avoided. The maximum measured Vpp is now ~1.06 V. Rather than having an open circuit with "infinite impedance" at the end of the cable, we now have an impedance-matched 50 Ω which minimizes the reflections! The reflection coefficient is minimized when $Z_L \approx Z_0$, so this matches our theoretical expectation.

<u>Question 5</u>: Remove the 50-ohm terminating resistor from the end of the spool. Finally, consider two frequencies: the frequency that is 50% of your quarter-wavelength frequency and the one that is 90% of your quarter-wavelength frequency.

a.) What are these two frequencies? If you are running out of time, skip to part h to record the data you'll need to complete the rest of question 5. Parts b through g can be completed outside the lab.

Our $\frac{\lambda}{4}$ frequency is 575 kHz, so the two frequencies are:

$50\% \times 575\, kHz = 287.5\, kHz$

$90\% \times 575\, kHz = 517.5\, kHz$

b.) For each frequency, what fraction of the frequency's wavelength does the transmission line represent? *(An easy way to calculate this is by dividing the line's time delay by the signal's period.)*

$t_d = 416ns$

For 287.5 kHz:

Fraction $= \frac{416 \times 10^{-9} s}{1/287.5kHz} = 0.1196$

of the frequency's wavelength.

So for 287.5 kHz, the transmission line represents 0.1196 of the wavelength.

For 517.5 kHz:

Fraction $= \frac{416 * 10^{-9} s}{1/517.5kHz} = 0.21528$

of the frequency's wavelength.

So for 517.5 kHz, the transmission line represents 0.21528 of the wavelength.

c.) Using a Smith Chart, figure out what the normalized impedance is at the two frequencies. Do this by putting your pen at the open circuit (right-hand side) of the chart and rotating toward the generator (clockwise) by the number of wavelengths you calculated in part b. Then make a mark and read off the impedance. **Attach the Smith chart when you hand in your report, marking both points.**

For 287.5 kHz: Impedance $= 1.05j$

For 517.5 kHz: Impedance $= 0.25j$

.22j

d.) What is the input impedance of the transmission line at these two frequencies? Calculate this by multiplying your answers in part c by the characteristic impedance of your cable, which you may estimate as 50 ohms.

For 287.5 kHz:

Input Impedance $= (1.05 * 50)j = 52.5j$

For 517.5 kHz:

Input Impedance $= (0.25 * 50)j = 12.5j$

11j

e.) What are the effective impedances that your signal sees at the oscilloscope at the two frequencies? Because the oscilloscope and the cable spool are in parallel, use the formula below. Use 50 ohms for the input impedance of the scope.

$$Z_{eff} = \frac{Z_{scope} \cdot Z_{cable}}{Z_{scope} + Z_{cable}}$$

For 287.5 kHz:

$$Z_{eff} = \frac{Z_{scope} * Z_{cable}}{Z_{scope} + Z_{cable}} = \frac{50 * 52.5j}{50 + 52.5j} = 26.219 + 24.97\text{j}$$

$$\boldsymbol{Z_{eff} = 26.219 + 24.97\text{j}}$$

For 517.5 kHz:

$$Z_{eff} = \frac{Z_{scope} \times Z_{cable}}{Z_{scope} + Z_{cable}} = \frac{50 \times 12.5j}{50 + 12.5j} = 2.94 + 11.76\text{j}$$

$$\boldsymbol{Z_{eff} = 2.94 + 11.76\text{j}}$$

2.3+10.49j

f.) When the signal hits the oscilloscope, it will behave as though it is hitting a load of Zeff as calculated in part e. What is the reflection coefficient for this load at each frequency?

For 287.5 kHz:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(26.219 + 24.97j) - 50}{(26.219 + 24.97j) + 50} = -0.18484 + 0.38817j$$

$$\boldsymbol{\Gamma_L = -0.18484 + 0.38817j}$$

For 517.5 kHz:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(2.94 + 11.76j) - 50}{(2.94 + 11.76j) + 50} = -0.8 + 0.4j$$

$$\boldsymbol{\Gamma_L = -0.8 + 0.4j}$$

g.) What signal amplitude do you expect to see on the oscilloscope for the two frequencies? Calculate this using the following equation and the reflection coefficients from part f.

$$|V_{oscilloscope}| = |V_{source}| \cdot (1 + \Gamma)$$

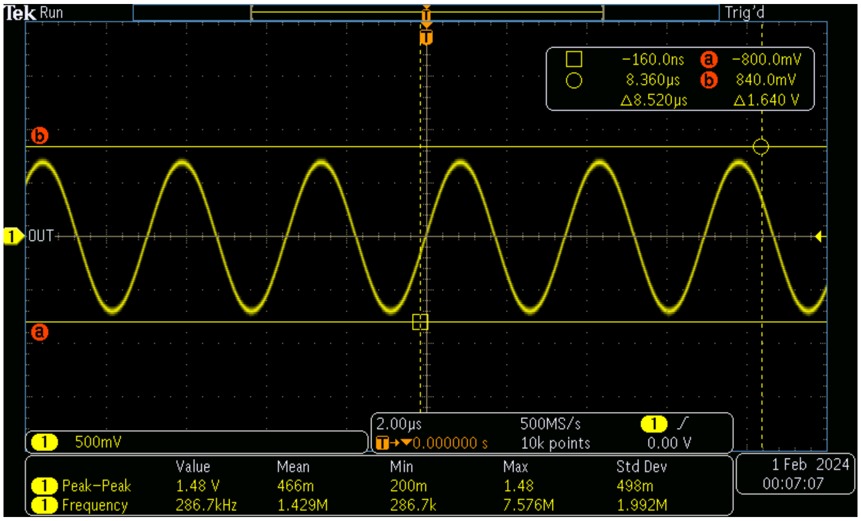
For 287.5 kHz:

$$Vscope \; = \; 1\text{V} \times (1 + (-0.18484 + 0.38817j))$$
$$= 1V \times (.815 + 0.388j)$$
$$= \Re(.815 + .388j) \approx .815\text{V}$$

For 517.5 kHz:

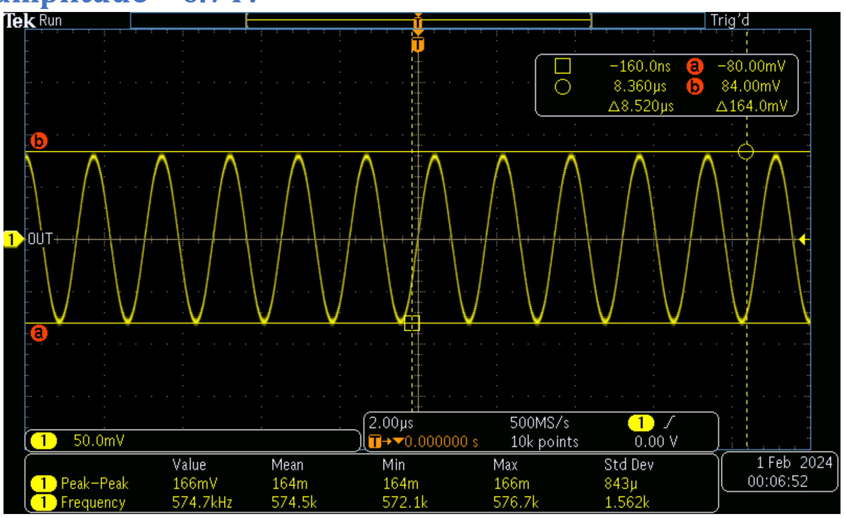
$$Vscope \; = \; 1\text{V} \times (1 - .8 + .4j)$$
$$= \Re(.2 + .4j) = .2V$$

h.) Finally, dial the two frequencies from part A into the signal generator and measure the amplitude at the oscilloscope. Compare these measured amplitudes to the ones you calculated in part g



For 287.5 kHz:

Vpp=1.48 V, so **V amplitude = 0.74V**



For 517.5 kHz:

Vpp=0.166V, so **V amplitude = 0.083V**

After reviewing our screenshots of the data for this part, we've noticed that the scope impedance was set to 1Mohms. In previous parts of this studio, it was correctly set to 50 ohms, but the setting must have been changed because _____. So, our experimental results do not align with