Fields and Waves I

Lecture 4

Input Impedance

Pulses on Transmission Lines

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Materials from other sources are referenced where they are used.
Those listed as Ulaby are figures from Ulaby's textbook.

Review

We have saw that:

Minimum occurs at LOAD for $Z_L \rightarrow 0$

Is it also true that:

Maximum occurs at LOAD for $Z_L \rightarrow \infty$

Or, in general, that:

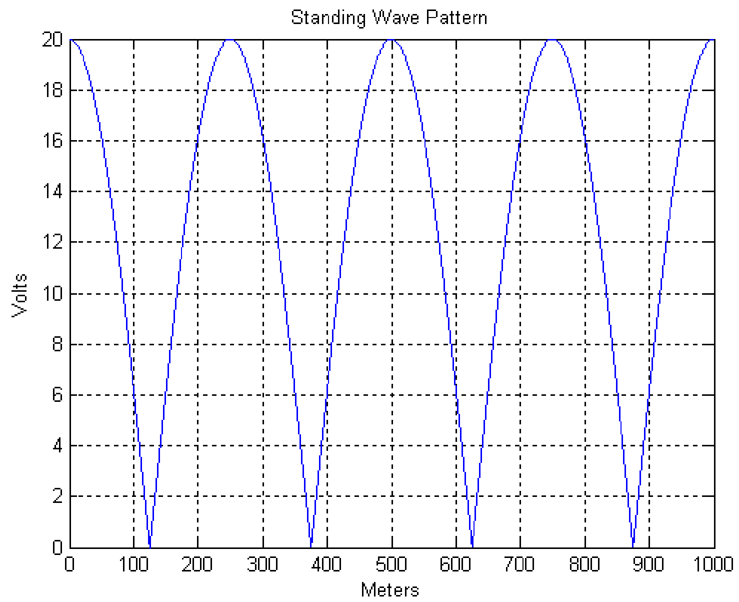
$\Gamma_L > 0 \quad \Rightarrow Z_L > Z_0$ *Max at LOAD*

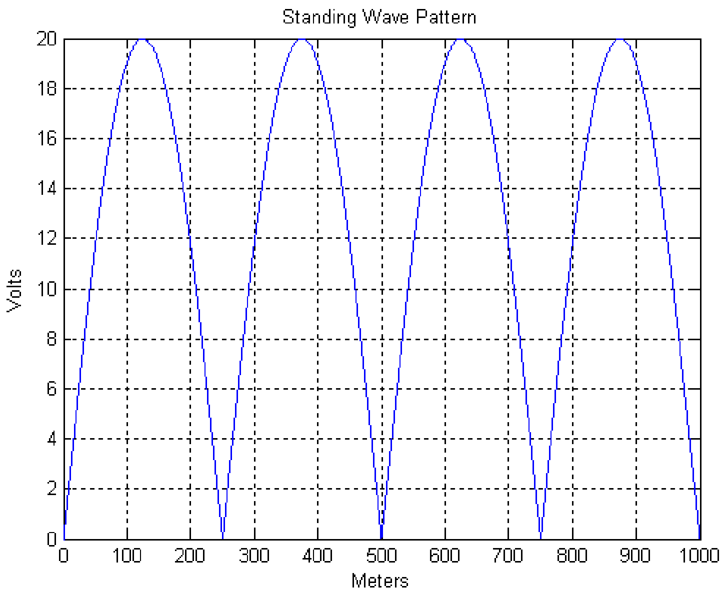
$\Gamma_L < 0 \quad \Rightarrow Z_L < Z_0$ *Min at LOAD*

IF Z_L is REAL

Generalized Impedance

$$\Gamma(z) = \Gamma_L \cdot e^{-j\cdot 2\cdot \beta\cdot (L-z)}$$





Generalized Impedance

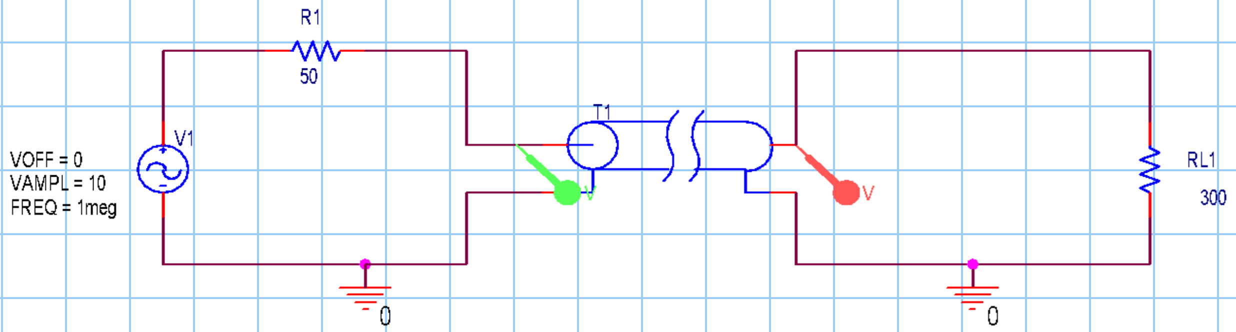
$$\Gamma(z) = \Gamma_L \cdot e^{-j \cdot 2 \cdot \beta \cdot (L-z)}$$

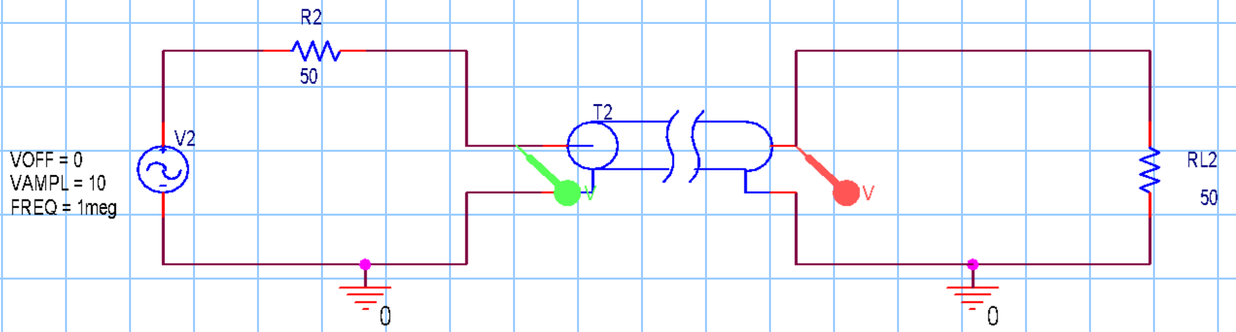
- We see that if you go backward a quarter wavelength from an open circuit load, you will see at that spot the <u>same voltage pattern</u> you would see if there were a short circuit load to the right of that point.

- In other words, as you move backward from the load, it begins to "look like" a different load. The expression for Γ(z), the <u>phase-shifted reflection coefficient</u>, reflects this by taking into account the load as well as the distance from it.

Generalized Impedance

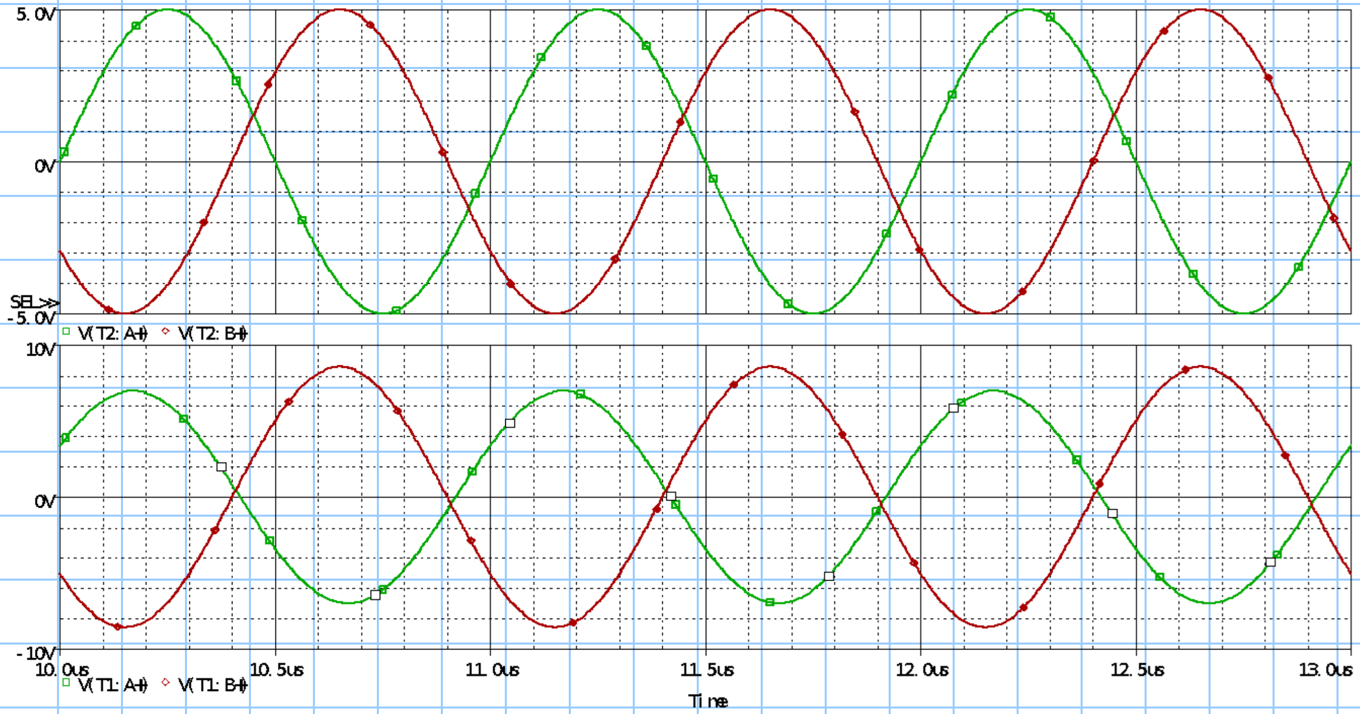
For the same source and line, but different load:





Input Impedance

A change in the load results in a change in the voltages we measure at the input. If we define an "input impedance", this will also change.



Input Impedance

What does $Z_{in} = \dfrac{V_{in}}{I_{in}}$ look like?

To figure this out, we can use our expression for phase-shifted reflection coefficient.

$$\Gamma(z) = \frac{V^{-} \cdot e^{+j \cdot \beta \cdot z}}{V^{+} \cdot e^{-j \cdot \beta \cdot z}} = \frac{V^{-}}{V^{+}} \cdot e^{j \cdot 2 \cdot \beta \cdot z}$$

$$= \Gamma_L \cdot e^{j \cdot 2 \cdot \beta \cdot z}$$ if z =0 at LOAD

Input Impedance

Previously, we have seen:

$$\hat{V}(z) = V^+(z) + V^-(z) = V^+ \cdot e^{-j\cdot\beta\cdot z} \cdot (1 + \Gamma(z))$$

What about I?

$$\hat{I}(z) = \frac{V^+(z)}{Z_o} - \frac{V^-(z)}{Z_o} = \frac{V^+}{Z_o} \cdot e^{-j\cdot\beta\cdot z} \cdot (1 - \Gamma(z))$$

Input Impedance

Form the Ratio (the generalized impedance):

$$\frac{\hat{V}(z)}{\hat{I}(z)} = Z_o \cdot \frac{1+\Gamma(z)}{1-\Gamma(z)} = Z(z)$$

We are primarily interested in the z=0 value (if we define z=0 as the source end.)

$$Z_{in}(z=0) = Z_o \cdot \frac{1+\Gamma(z=0)}{1-\Gamma(z=0)}$$

Input Impedance

Using algebra and trigonometry (Ulaby pg. 76), we can write this as:

$$Z_{in}(z=0) = Z_o \cdot \frac{Z_L + j \cdot Z_o \cdot \tan(\beta \cdot L)}{Z_o + j \cdot Z_L \cdot \tan(\beta \cdot L)}$$

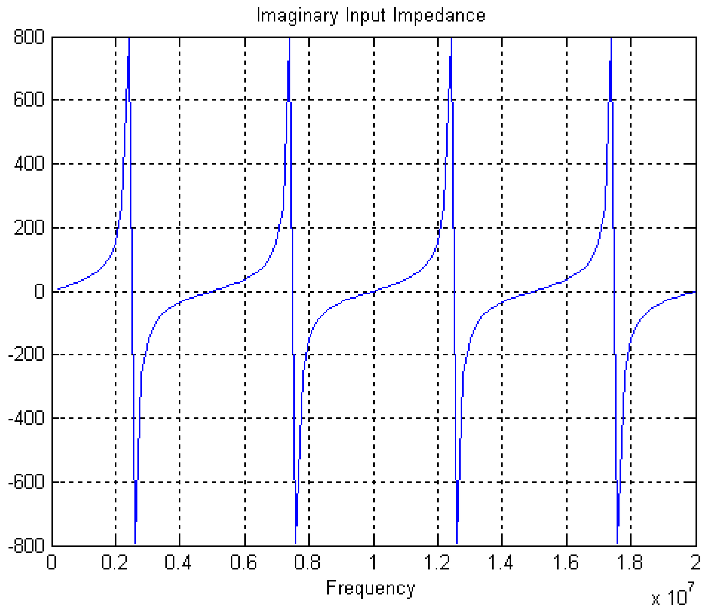
Special Case example: Z_L=0 (short circuit)

Can change Z_{in} by changing these two parameters

$$Z_{in}(z=0) = Z_o \cdot \frac{0 + j \cdot Z_o \cdot \tan(\beta \cdot L)}{Z_o + j \cdot 0 \cdot \tan(\beta \cdot L)} = j \cdot Z_o \cdot \tan(\beta \cdot L)$$

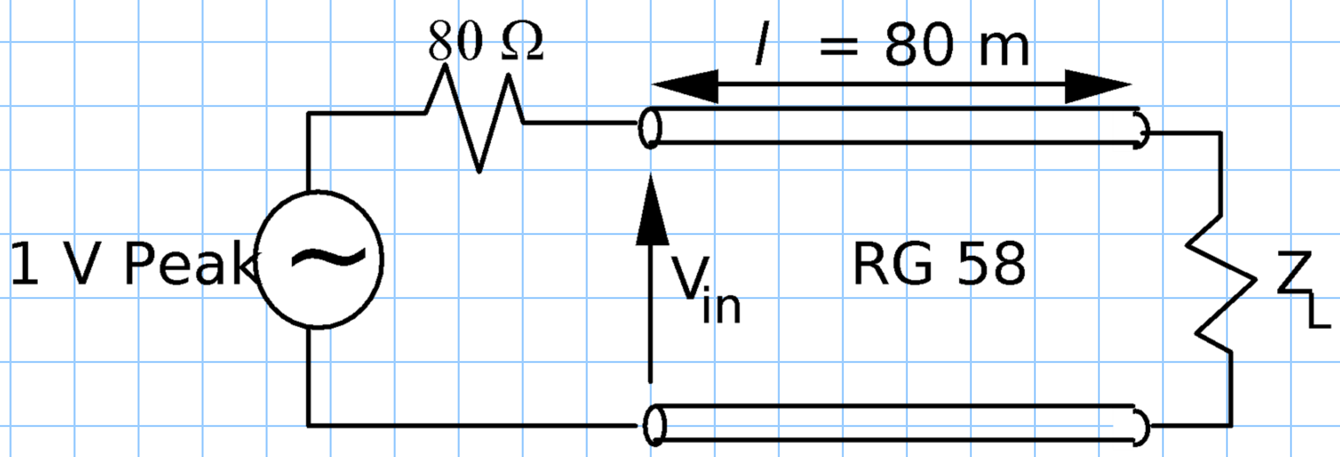
Input Impedance

For this short circuit load case, this impedance is imaginary (reactive) and you can vary frequency (and therefore β) to achieve any magnitude you want.



Input Impedance

Consider some other cases…



Input Impedance

Open Circuit Case:

$$Z_L = \infty$$

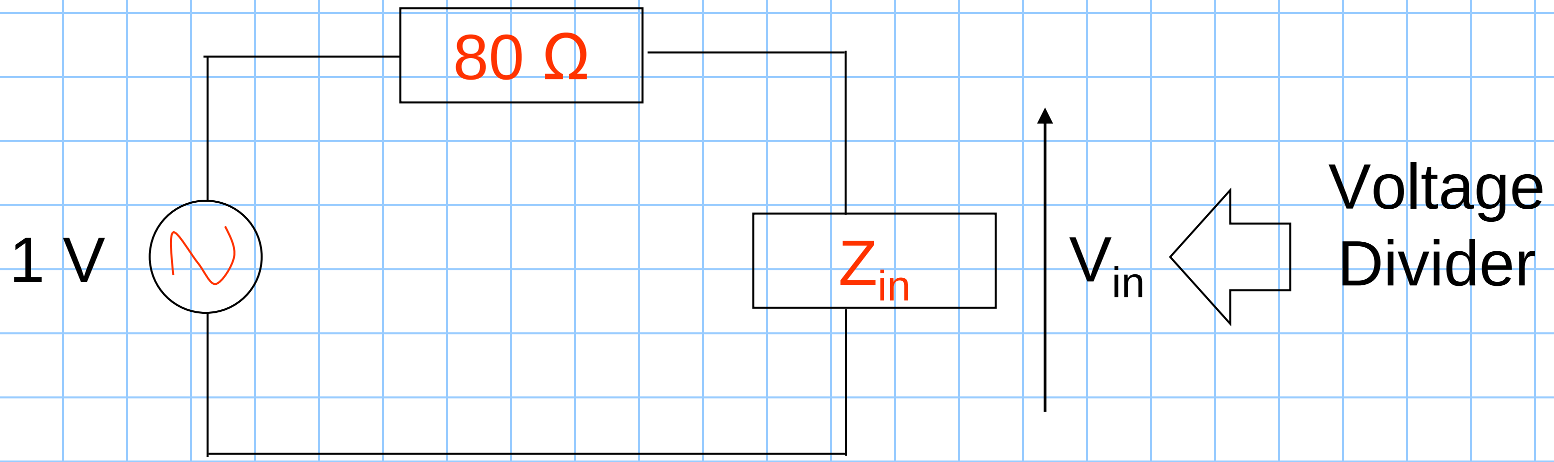
$$Z_{in} = Z_o \frac{Z_L + j \cdot Z_o \cdot \tan(\beta \cdot L)}{Z_o + j \cdot Z_L \cdot \tan(\beta \cdot L)}$$

Z_L = 93Ω:

Lots of complex algebra, but straightforward.

Input Impedance

Knowing Z_{in} (z=0), we can use it to represent an entire circuit network in calculations.



$$Power = \frac{1}{2} Re\left\{V_{in} \times I_{in}^{*}\right\} = \frac{1}{2} Re\left\{\frac{V_{in} \times V_{in}^{*}}{Z_{in}^{*}}\right\} = \frac{1}{2} Re\left\{\frac{\left|V_{in}\right|^{2}}{Z_{in}^{*}}\right\}$$

Input Impedance

Special Cases

- Recall that the standing wave pattern repeated every half wavelength. Thus, we expect that this will also happen for $Z_{in.}$ First, consider the trivial case of *L=0.*

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta L}{Z_o + jZ_L \tan \beta L} = Z_L$$

- Now let the line be a half wavelength long

$$\tan \beta L = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right) = \tan(\pi) = 0 \qquad Z_{in} = Z_o \frac{Z_L + 0}{Z_o + 0} = Z_L$$

Input Impedance

Special Cases

- Thus, for a line that is exactly an integer number of half wavelengths long

$$Z_{in} = Z_L$$

- Thus, if you have a transmission line with the wrong characteristic impedance, you can match the load to the source by selecting a length equal to a half wavelength.

Input Impedance

Special Cases

- If the line is an odd multiple of a quarter wavelength, we also get an interesting result.

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta L}{Z_o + jZ_L \tan \beta L} = Z_o \frac{jZ_o \tan \beta L}{jZ_L \tan \beta L} = \frac{Z_o^{\,2}}{Z_L}$$

$$\tan \beta L = \tan \frac{2\pi}{\lambda} \frac{\lambda}{4} = \tan \frac{\pi}{2} \rightarrow \infty$$

Input Impedance

Special Cases

For the quarter-wave transmission line:

$$\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}$$

Thus, such a transmission line works like an "impedance transformer".

In applications, a quarter-wave T-line can be useful when you want to make one impedance look like another one.

Review

- Suppose a transmission line has $Z_0 = 40\Omega$ and is terminated in an open circuit. The transmission line is 100m long.

- Suppose that a sinusoidal signal on the line has wavelength 200m. What is the input impedance of the line?

- What will the input impedance of the line be if we double the signal's frequency?

- What about if we halve the signal's frequency?

Review

- Recall that the standing wave pattern repeated every half wavelength. Thus, we expect that this will also happen for $Z_{in.}$ First, consider the trivial case of $L=0$.

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta L}{Z_o + jZ_L \tan \beta L} = Z_L$$

- Now let the line be a half wavelength long

$$\tan \beta L = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{2}\right) = \tan(\pi) = 0 \qquad Z_{in} = Z_o \frac{Z_L + 0}{Z_o + 0} = Z_L$$

Review

- Thus, for a line that is exactly an integer number of half wavelengths long

$$Z_{in} = Z_L$$

- Thus, if you have a transmission line with the wrong characteristic impedance, you can match the load to the source by selecting a length equal to a half wavelength.

Review

For the quarter-wave transmission line:

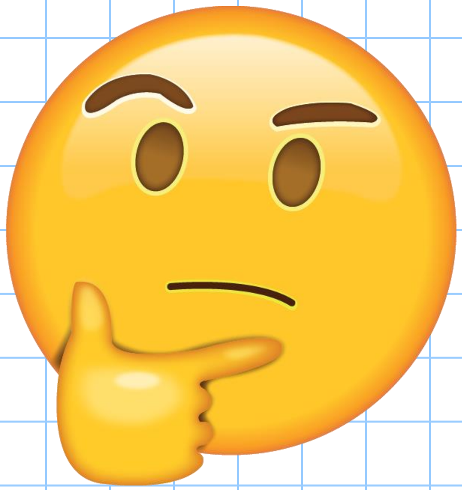
$$\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}$$

Thus, such a transmission line works like an "impedance transformer".

In applications, a quarter-wave T-line can be useful when you want to make one impedance look like another one - **for instance, an inductor could be make to look like a capacitor.**

Review

...but do you really trust the math?



Let's try a simulation. **Complete Lecture 4 Exercise 1 on Gradescope. You may work in groups of up to 4.**

Transients on T-Lines

The transmission line impedance transformer has some limitations:

- Input signal needs to be one frequency
- Only works at steady state

To better understand transmission lines, we need to broaden our understanding to include transient behavior.

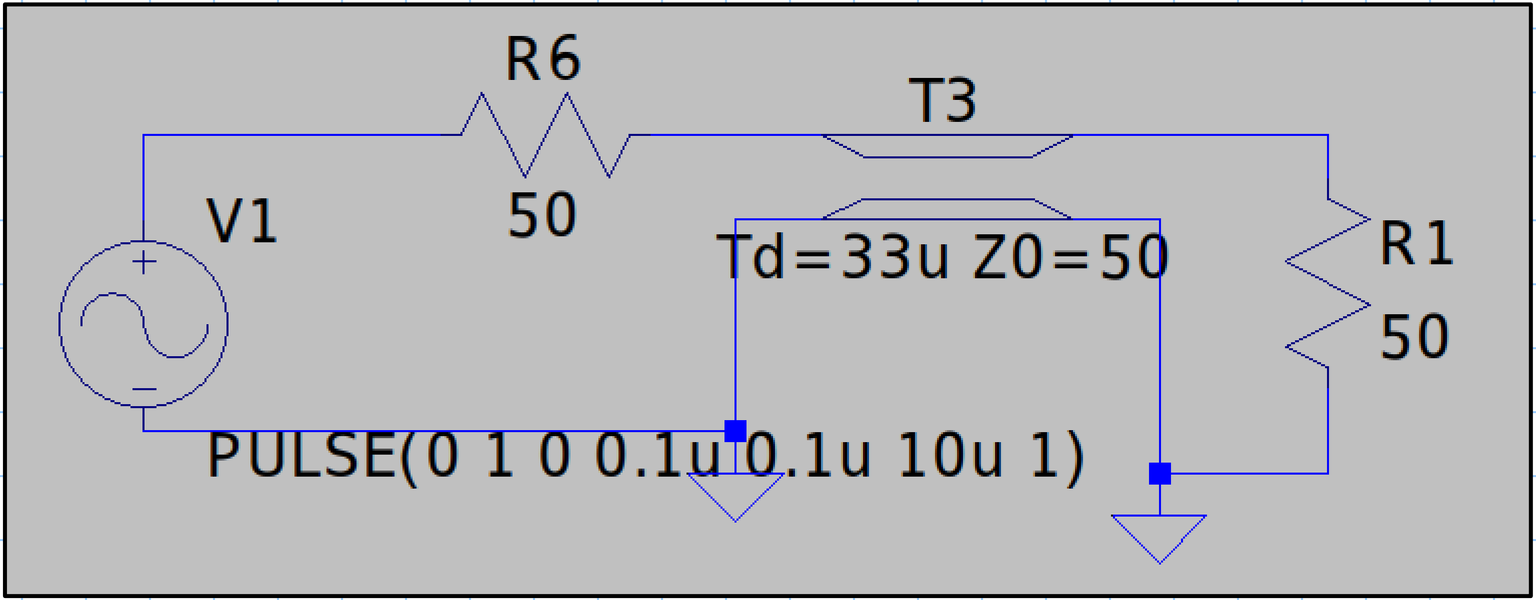
Transients On T-Lines

What do transient signals do on a lossless t-line?

- Pulses will look the same at the input and output except for a delay.
- This is independent of the shape of the pulse.

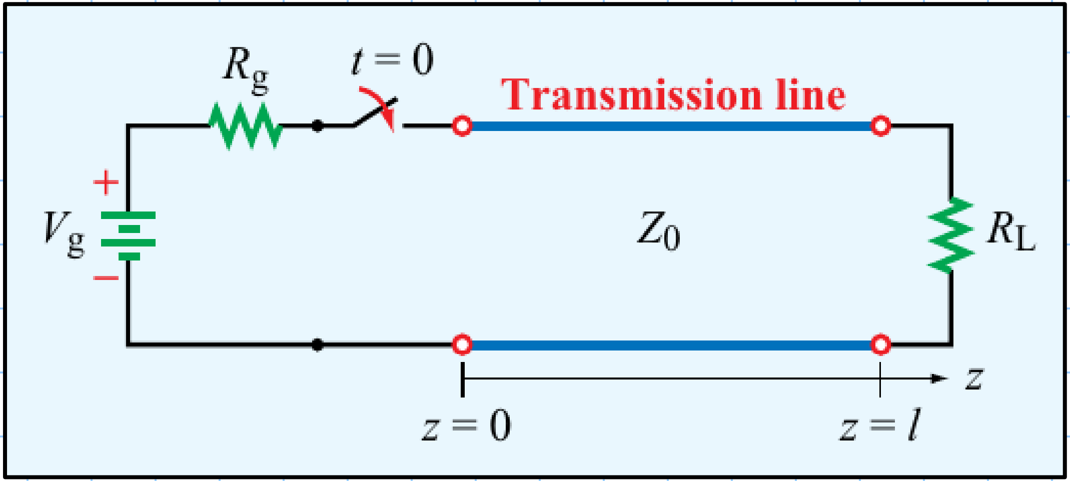
Transients On T-Lines

Basic simulated transmission line with pulsed source



How do we understand the voltage drop across R6?

Transients On T-Lines

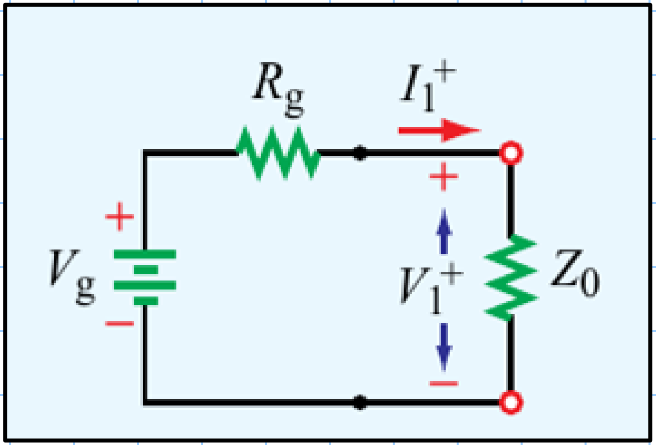


Ulaby pg. 115

Consider the circuit above. At t=0, the switch closes and a voltage pulse begins moving forward down the line.

- Is there a backwards traveling wave at this point?
- Are there any standing waves?
- What is the ratio of current and voltage on the line at this moment?

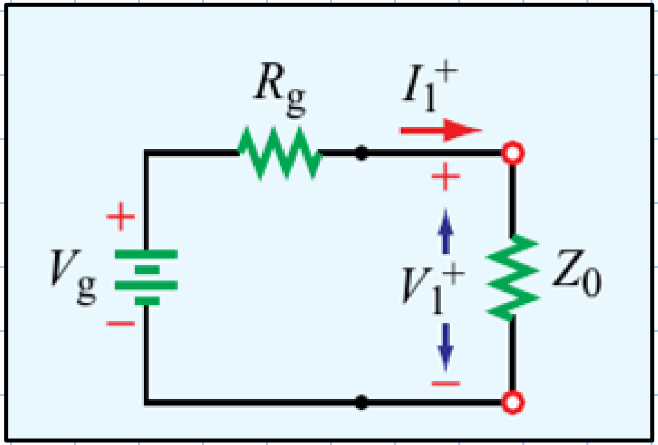
Transients On T-Lines



Ulaby pg. 115

At t=0, the t-line circuit therefore behaves equivalently to this one. So what are the current and voltage expressions for the load?

Transients On T-Lines

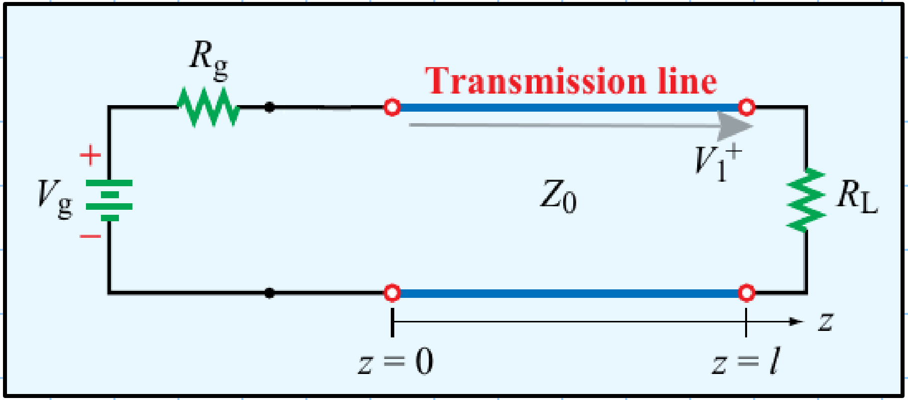


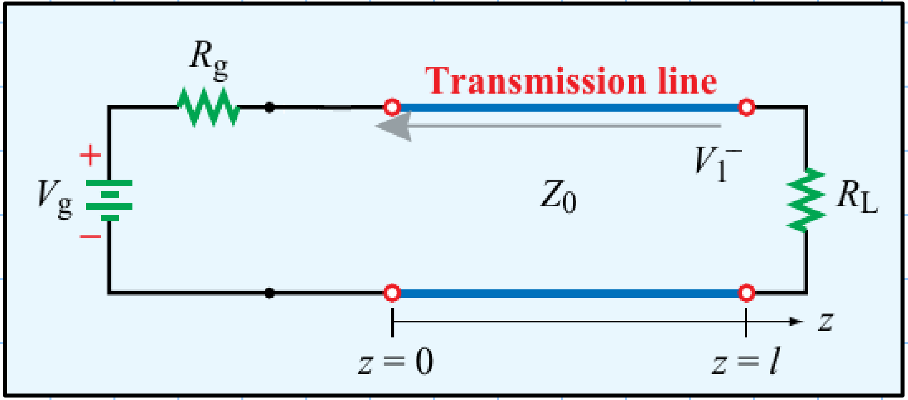
Ulaby pg. 115

At t=0, the t-line circuit therefore behaves equivalently to this one. So what are the current and voltage expressions for the load?

$$V_1^+ = \frac{V_g Z_0}{Z_g + Z_0} \qquad I_1^+ = \frac{V_g}{Z_g + Z_0}$$

Transients On T-Lines





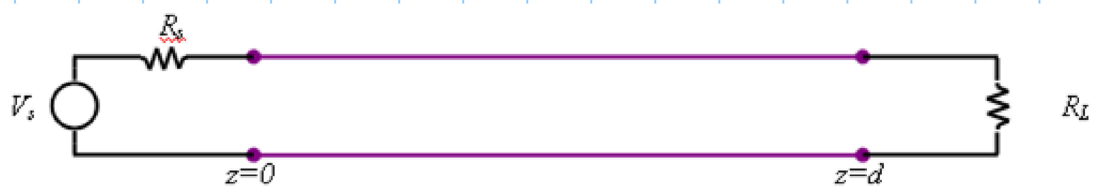
$$V_1^- = \Gamma V_1^+$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{V_0^-}{V_0^+} = -\frac{I_0^-}{I_0^+}$$

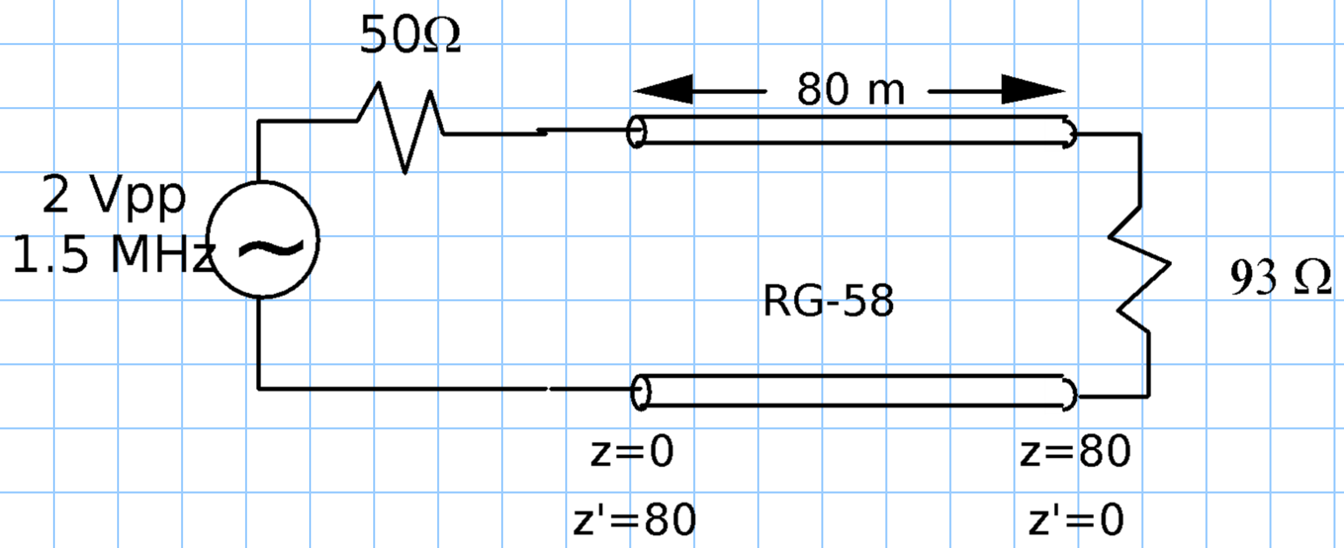
This voltage pulse still reflects at the load as governed by the reflection coefficient.

Transients On T-Lines



So the pulse travels down the line, hits the load, reflects if there is an impedance mismatch, and the reflection goes back toward the source.

Transients On T-Lines

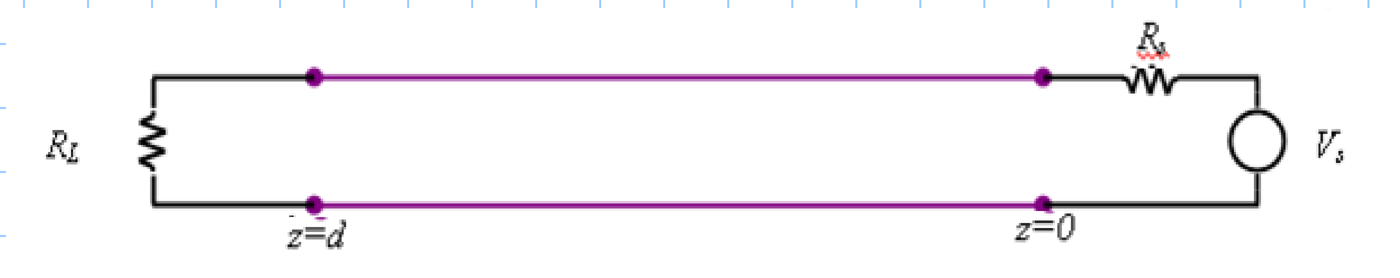


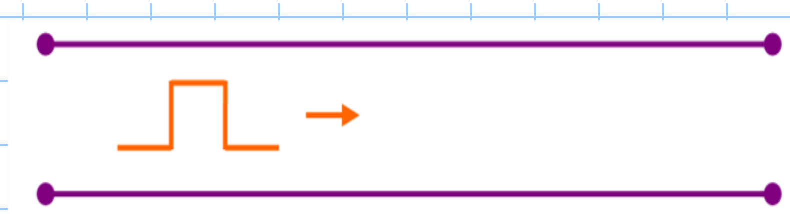
The model transmission line circuits we have considered thus far have had a source impedance, and that source impedance has tended to be equal to the characteristic impedance of the line. **Why is this?**

Transients On T-Lines

Mathematically, there is no difference between a signal hitting a load impedance and a signal hitting a source impedance. In both cases, the behavior will be governed by the reflection coefficient.

Transients On T-Lines





Mathematically, there is no difference between a signal hitting a load impedance and a signal hitting a source impedance. In both cases, the behavior will be governed by the reflection coefficient.

Transients On T-Lines

So what happens when both the source and load impedance are mismatched? Time for another simulation!

Code:

```
n 5 5
n 15 5
n 35 5
n 42 5
g 5 15
g 15 15
g 35 15
g 42 15
v 5:-1 p:5000:20:60 4 0
r 25 0 1
z 50 25 1 2 5 6
s 2 3
r 25 7 3
```

Transients On T-Lines

Our backwards-traveling hits the source and reflects again!

$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+ \qquad \Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$$

Transients On T-Lines

... and again!

$$V_2^- = \Gamma_L V_2^+ = \Gamma_L \Gamma_g V_1^- = \Gamma_g (\Gamma_L)^2 V_1^+ \qquad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Transients On T-Lines

In the case of the transmission line with an impedance mismatch at both the source and the load, how long should the circuit take (in principle) to reach steady state?

Transients On T-Lines

In the case of the transmission line with an impedance mismatch at both the source and the load, how long should the circuit take (in principle) to reach steady state?

Answer: Forever.

(In principle the pulse will continue bouncing between the source and the load without limit. In practice the pulse will fall within the measurement error of a steady-state solution after a finite number of reflections.)

Transients On T-Lines

$$\begin{aligned} V_\infty &= V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \cdots \\ &= V_1^+[1 + \Gamma_\mathrm{L} + \Gamma_\mathrm{L}\Gamma_\mathrm{g} + \Gamma_\mathrm{L}^2\Gamma_\mathrm{g} + \Gamma_\mathrm{L}^2\Gamma_\mathrm{g}^2 + \Gamma_\mathrm{L}^3\Gamma_\mathrm{g}^2 + \cdots] \\ &= V_1^+[(1+\Gamma_\mathrm{L})(1 + \Gamma_\mathrm{L}\Gamma_\mathrm{g} + \Gamma_\mathrm{L}^2\Gamma_\mathrm{g}^2 + \cdots)] \\ &= V_1^+(1+\Gamma_\mathrm{L})[1 + x + x^2 + \cdots], \end{aligned}$$

Ulaby bg. 117

$$x = \Gamma_L \Gamma_g$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$$ (for |x| < 1)

Transients On T-Lines

So even if our pulse theoretically bounces around forever, it *does* converge to a voltage, and that voltage is :

$$V_\infty = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_g}$$

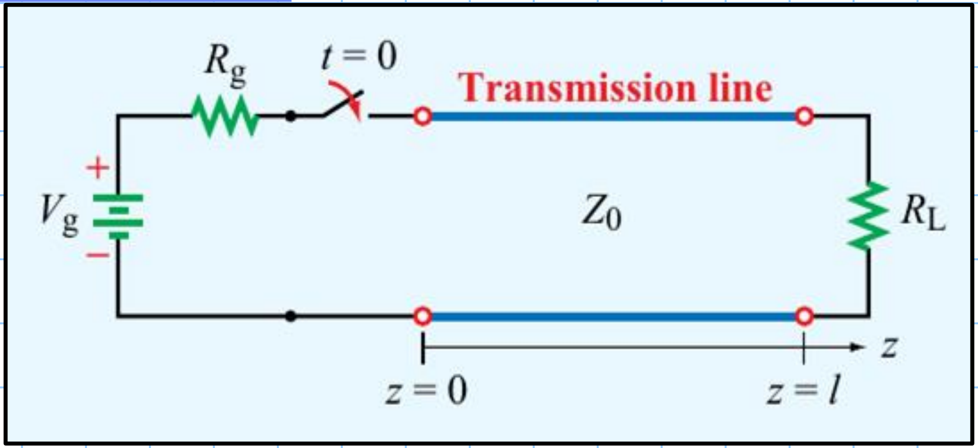
Using our earlier equations:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad V_1^+ = \frac{V_g Z_0}{Z_g + Z_0}$$

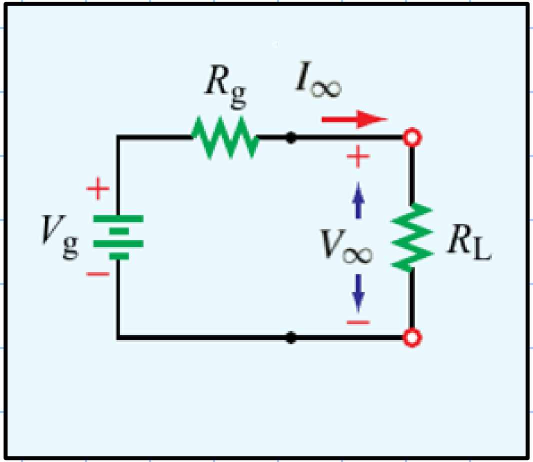
we can write:

$$V_\infty = \frac{V_g R_L}{R_g + R_L}$$

Transients On T-Lines



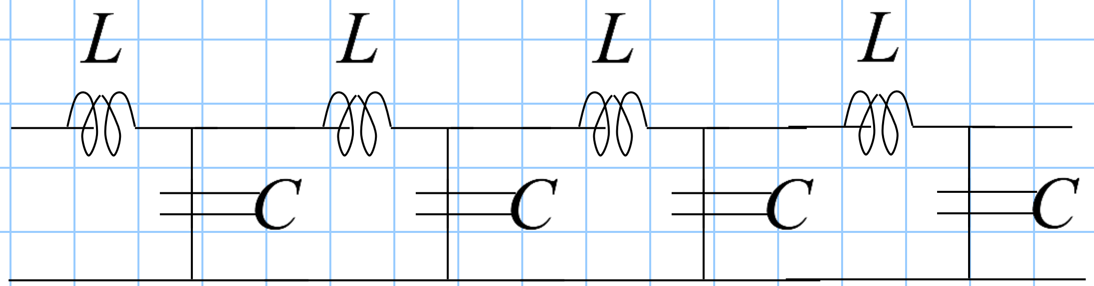
$$V_\infty = \frac{V_g R_L}{R_g + R_L}$$



So as time increases, this transmission line approaches a state in which the transmission line effectively doesn't exist anymore.

The pulse that we put into the line is DC (doesn't change after t=0). Why don't we see this same steady-state behavior when using a sinusoidal source voltage?

Transients on T-Lines



Keep the lumped model of the transition line in mind. Applying a DC voltage to this line and the inductor impedance will eventually go to 0 while the capacitor impedance becomes infinite. So, in classical circuit terms, the lumped line just becomes two wires.

For a sinusoidal input voltage, an excitation is continually applied to the inductors and capacitors such that they never reach steady state.

Transients On T-Lines

<u>Some things in common between transient and steady state analysis:</u>

- Reflection at the end of the line (*either end*) is governed by the reflection coefficient.
- Both forward and backward traveling can be present in general.
- Z_0 is needed to understand relationship between current and voltage on the line.

Transients On T-Lines

A major difference between transient and steady state analysis:

- In transient analysis, there is no backwards-traveling wave **until enough time has passed for a reflection to occur at the end of the line.**
- Without forwards and backwards waves on the line at the same time, a lot of our steady-state concepts (standing wave ratio, phase-shifted reflection coefficient) become meaningless.

Transients On T-Lines

Next example - Square pulse, mismatched source and load