Chapter 11-1 Detailed Quantitative Analysis

The goal is to relate transistor performance parameters (γ, α_T , β_{dc} etc.) to doping, lifetimes, base-widths etc.

Assumptions:

pnp transistor, steady state, low-level injection.

Only drift and diffusion, no external generations

One dimensional etc.

General approach is to solve minority carrier diffusion equations for each of the three regions:

$$\frac{\partial \Delta p}{\partial t} = D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_p} + G_L \quad \text{and} \quad \frac{\partial \Delta n}{\partial t} = D_n \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$$

General Quantitative Analysis

Under steady state and when $\boldsymbol{G}_{\mathrm{L}} = 0$,

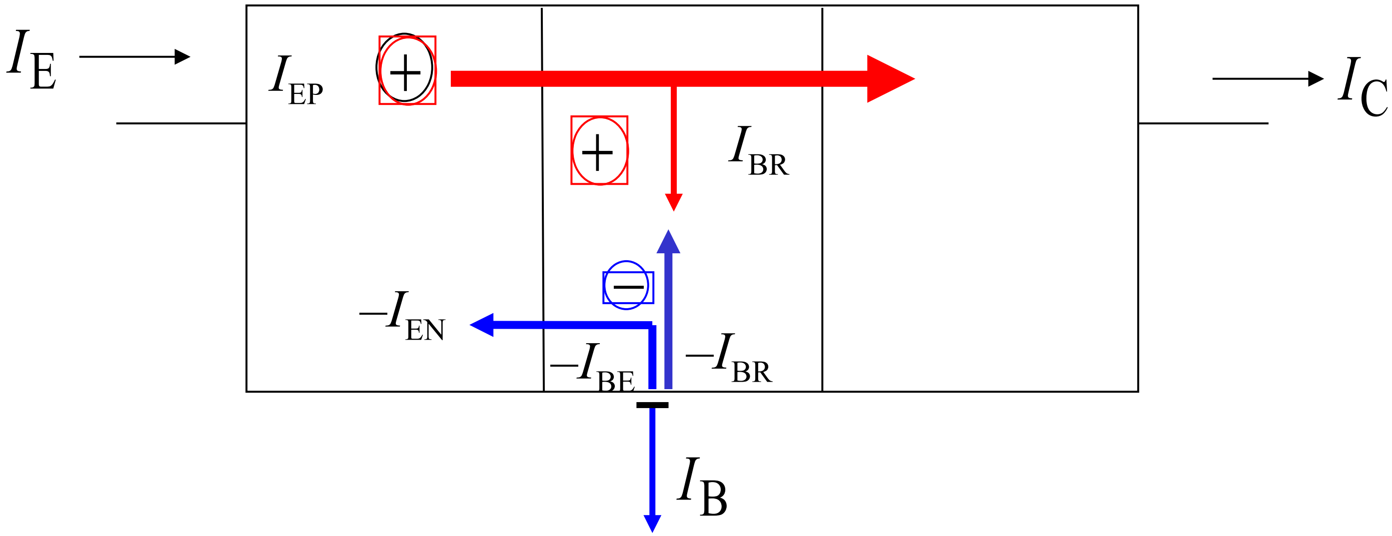
$$D_{\mathrm{p}} \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_{\mathrm{p}}} = 0 \qquad \text{and} \qquad D_{\mathrm{n}} \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_{\mathrm{n}}} = 0$$

For the **base** in pnp, we are interested only in holes.

$$D_{\mathrm{p}} \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau_{\mathrm{p}}} = 0$$

The rigorous analysis is carried out in chapter 11, but we are going to take a more simplified approach.

Review: Operational Parameters



Injection Efficiency : $\gamma = I_{EP}/(I_{EP} + I_{EN})$

Base transport factor : $\alpha_T = I_C / I_{EP}$

Collector to emitter current gain: $\alpha_{DC} = \alpha_T \gamma$

Collector to base current gain: $\beta_{DC} = \alpha_{DC} / (1 - \alpha_{DC})$

These parameters can be related to device parameters such as doping, lifetimes, diffusion lengths, etc.

Review: Indirect thermal recombination-generation

n_0, p_0 – under thermal equilibrium

n, p – under arbitrary conditions, functions of t

$$\left.\begin{aligned}\Delta n &= n - n_0\\ \Delta p &= p - p_0\end{aligned}\right\}$$

Δn and Δp are deviations in carrier concentrations from their equilibrium values. Δn and Δp can be both positive or negative. Δn and Δp are termed excess carriers – excess above the equilibrium concentration.

Low-level injection condition is assumed.

– Change in the majority carrier concentration is negligible.

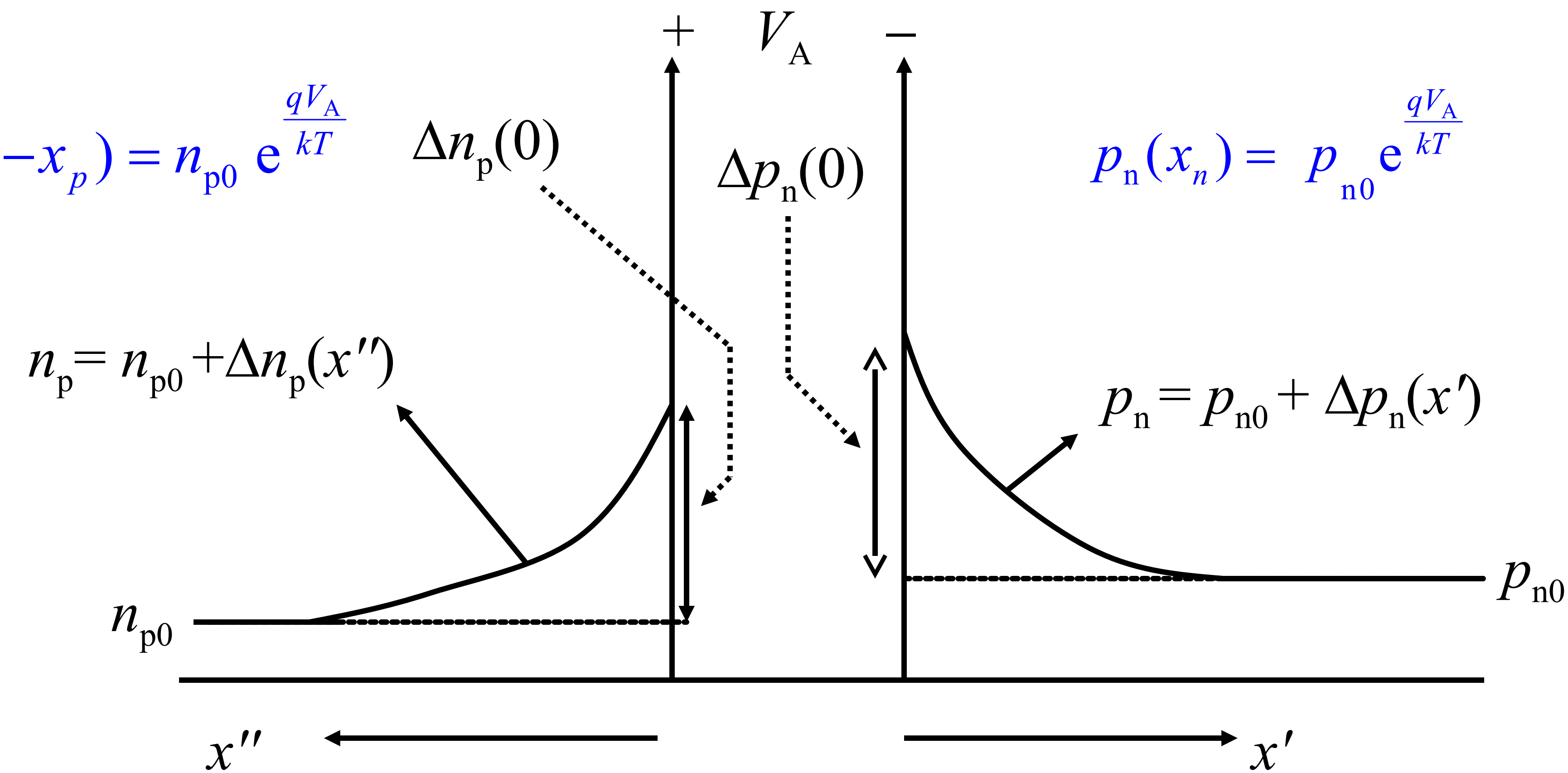
For example, in n-type material, $\Delta p \ll n_0$; $n \approx n_0$.

in p-type material, $\Delta n \ll p_0$; $p \approx p_0$.

Majority carriers are electrons in n-type material and holes in p-type material.

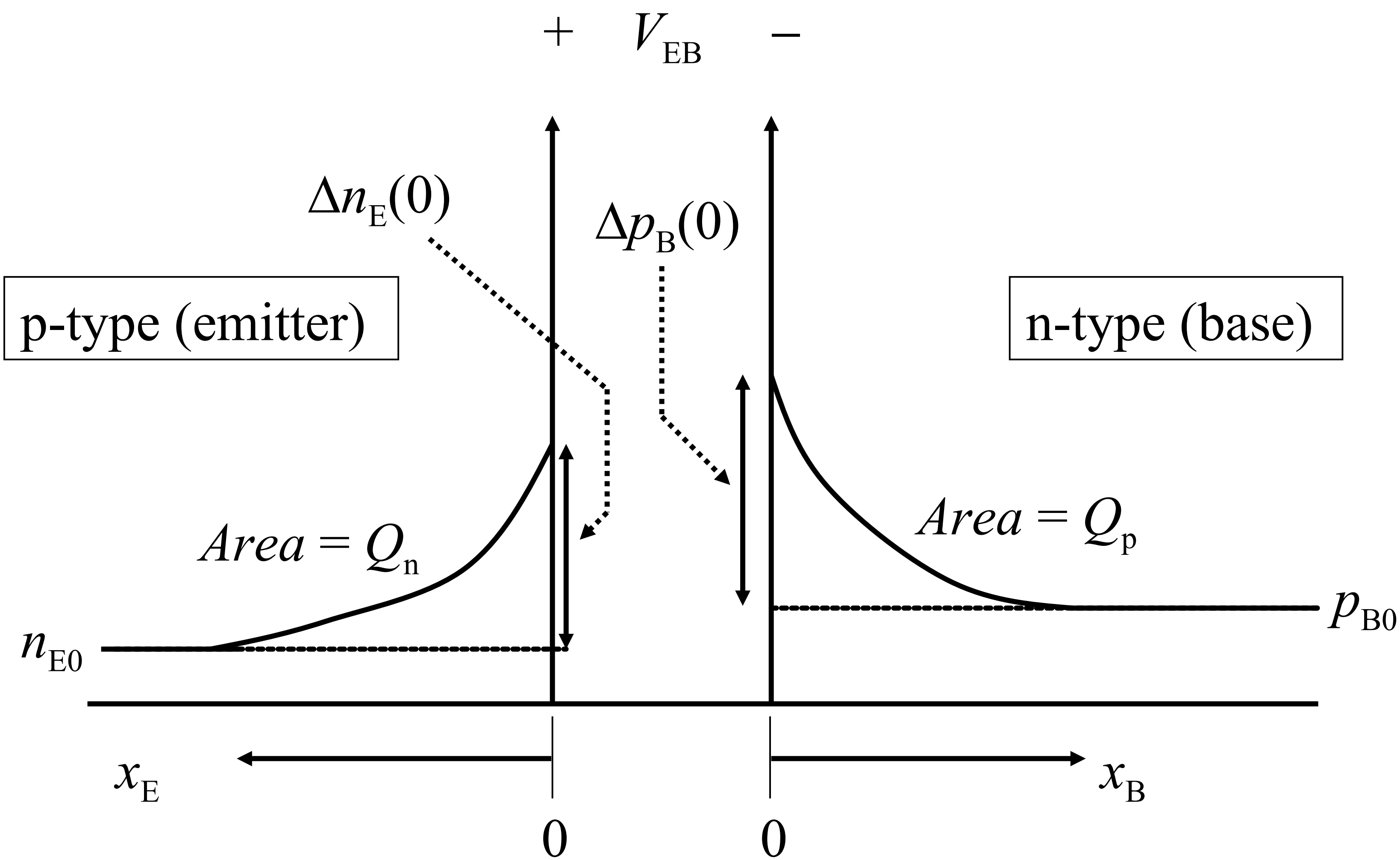
Review: Minority carrier concentration profile under bias

$$n_p(-x_p) = n_{p0}\, e^{\frac{qV_A}{kT}} \qquad\qquad p_n(x_n) = p_{n0}\, e^{\frac{qV_A}{kT}}$$



$$\Delta n_p(x'') = \Delta n_p(0)\; e^{-\frac{x''}{L_n}} \qquad\qquad \Delta p_n(x') = \Delta p_n(0)\, e^{-\frac{x'}{L_p}}$$

Review of P-N Junction Under Forward Bias



Review of P-N Junction Under Forward Bias (cont.)

$$I_n = q\,A\,D_E\ \ d\Delta n/dx_E \ \ = -\,(q\,A\,D_E/L_E)\ \Delta n_E(0)$$

$$I_p = -\,q\,A\,D_B\ \ d\Delta p/dx_B \ \ = \ (q\,A\,D_B/L_B)\ \Delta p_B(0)$$

Total current

$I \ = \ I_P + (-\,I_N)$ ("–" because x_E and x_B point in opposite directions)

$$= (q\,A\,D_B/L_B)\ \Delta p_B(0) + (q\,A\,D_E/L_E)\ \Delta n_E\,(0)$$

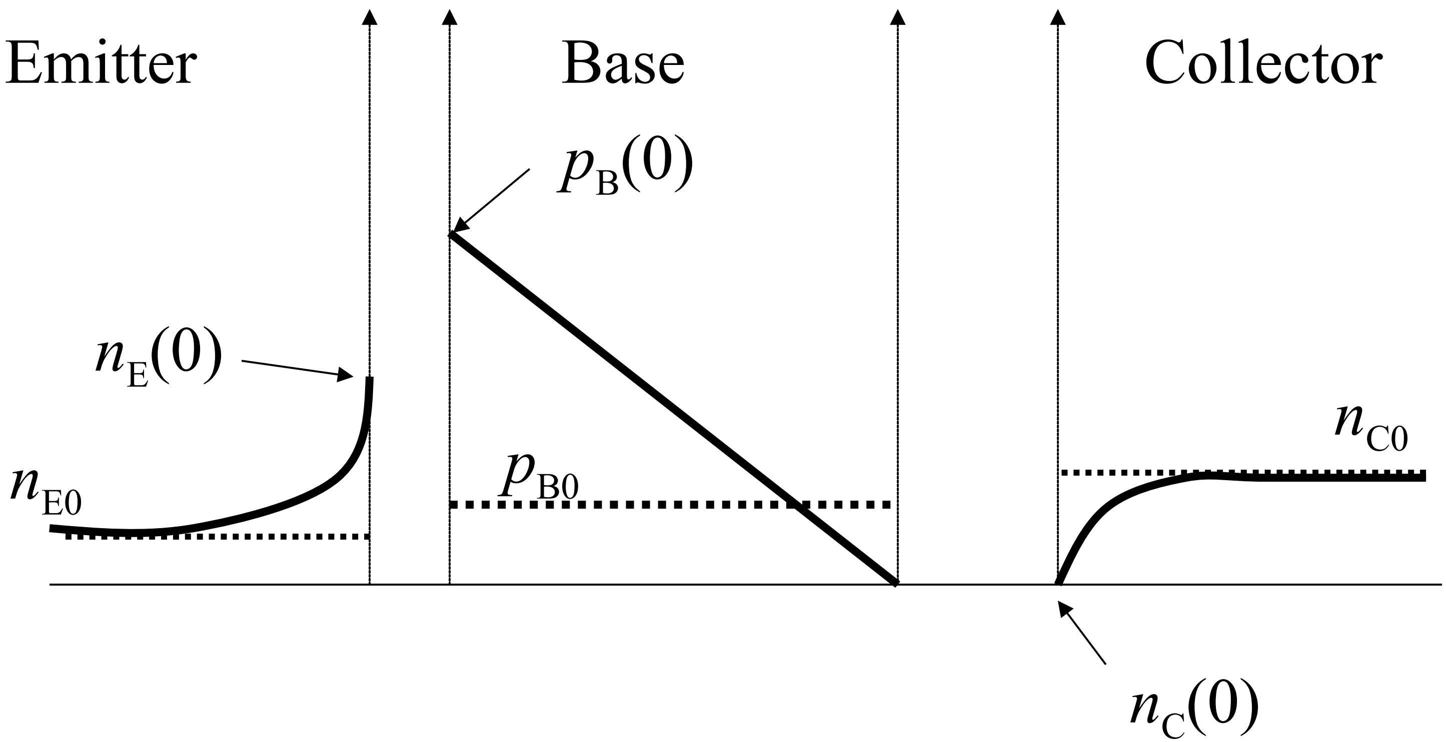
$$= (q\,A\,D_B/L_B)\ p_{B0}\,[\exp\,(q\,V_{EB}/kT) - 1\,] + \\ + (q\,A\,D_E/L_E)\ n_{E0}\,[\exp\,(q\,V_{EB}/kT) - 1\,]$$

$$\approx (q\,A\,D_B/L_B)\ p_{B0}\,\exp\,(q\,V_{EB}/kT) + (q\,A\,D_E/L_E)\ n_{E0}\,\exp\,(q\,V_{EB}/kT)$$

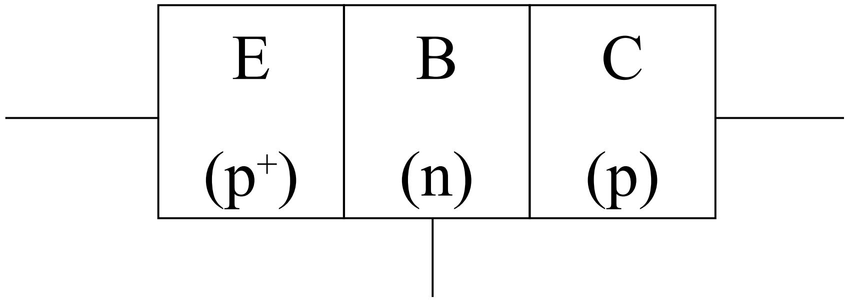
***Note*: I_n and I_p can also be calculated based on the fact that Q_p has to be**

Simplified Analysis

Consider the carrier distribution in a forward active pnp transistor



Simplified Analysis (cont.)



n_{E0}, p_{B0} and n_{C0} = equilibrium concentration of minority carriers in emitter, base and collector

$n_E(0)$, $p_B(0)$ and $n_C(0)$ = minority carrier concentration under forward active conditions at the edge of the respective depletion layers

$\Delta n_E(0)$, $\Delta p_B(0)$ and $\Delta n_C(0)$ = excess carrier concentration at the edge of the depletion layers

Simplified Analysis (cont.)

$$\Delta n_{\mathrm{E}}(0) = n_{\mathrm{E}}(0) - n_{\mathrm{E0}} = n_{\mathrm{E0}}\ [\exp(q\,V_{\mathrm{EB}}/kT) - 1]$$

$$\Delta p_{\mathrm{B}}(0) = p_{\mathrm{B}}(0) - p_{\mathrm{B0}} = p_{\mathrm{B0}}\ [\exp(\mathrm{q}\,V_{\mathrm{EB}}/kT) - 1]$$

By taking the slopes of these minority carrier distribution at the depletion layer edges and multiplying it by "$q\,A\,D_{\mathrm{n,p}}$", we can get hole and electron currents.

Note that $I_{\mathrm{n}} = q\,A\,D_{\mathrm{n}}\,(\mathrm{d}n/\mathrm{d}x)$ and $I_{\mathrm{p}} = -\,q\,A\,D_{\mathrm{p}}\,(\mathrm{d}p/\mathrm{d}x)$

Calculation of Currents

Collector current, $\boldsymbol{I_C}$

$I_C = q\ A\ D_B\ (dp/dx_B)$ (slope must be taken at end of base)

$= q\ A\ D_B\ [p_B(0) - 0]\ /\ W_B$

$= q\ A\ D_B\ p_B(0)\ /\ W_B$

$I_C = q\ A\ (D_B/W_B)\ p_{B0} \exp\,(qV_{EB}\ /\ kT)$ **---- (A)**

(only hole current if we neglect the small reverse saturation current of reverse biased C-B junction)

Calculation of Currents (cont.)

Emitter Current, I_E

I_E is made up of two components, namely I_{EP} and I_{EN}

$$
\begin{aligned}
I_{EP} &= I_c + \text{current lost in base due to recombination} \\
&= I_c + \text{excess charge stored in base}/\tau_B \\
&= I_c + q\,A\,W_B\,\Delta p_B(0)\,/\,(2\tau_B) \\
&\approx q\,A\,(D_B/W_B)\;p_{B0}\,[\exp(qV_{EB}/kT)\,] + \\
&\quad + q\,A\,[W_B/(2\tau_B)]\;p_{B0}\;[\exp(qV_{EB}/kT)] \quad \text{----}\ \ \textbf{(B)}
\end{aligned}
$$

[Assuming $\exp(q\,V_{EB}/kT) - 1 \approx \exp(q\,V_{EB}/kT)$
when V_{EB} is positive, i.e forward biased]

Calculation of Currents (cont.)

Emitter Current (cont.)

I_{EN} corresponds to electron current injection from base to emitter since E-B junction is forward biased.

$$I_{EN} = q\,A\,(D_E / L_E)\, n_{E0}\,[\exp\,(q\,V_{EB} / kT) - 1\,]$$
$$\approx q\,A\,(D_E / L_E)\, n_{E0}\,[\exp\,(q\,V_{EB} / kT)] \quad \text{----} \quad \textbf{(C)}$$

Calculation of Currents (cont.)

Base Current, I_B

-supplies electrons for recombination in base
-supplies electrons for injection to emitter

$$I_B = q A p_{B0} [W_B / (2\tau_B)] [\exp (qV_{EB} / kT)]$$
$$+$$
$$q A (D_E / L_E) n_{E0} \exp (qV_{EB} / kT)$$

(recombination) + (electron injection to emitter)

Now we can find transistor parameter easily.

Calculation of Currents (cont.)

Base transport factor, α_T

$$\alpha_T = \frac{I_C}{I_{EP}} = \frac{\dfrac{qAD_B}{W_B}\, p_{B0} \exp\left(\dfrac{qV_{EB}}{kT}\right)}{\dfrac{qAD_B}{W_B}\, p_{B0} \exp\left(\dfrac{qV_{EB}}{kT}\right) + \dfrac{qAW_B}{2\tau_B}\, p_{B0} \exp\dfrac{qV_{EB}}{kT}} = \frac{1}{1 + \dfrac{1}{2}\left(\dfrac{W_B}{L_B}\right)^2}$$

(same as eq. 11.42 in text)

Emitter injection efficiency, γ

$$\gamma = I_{EP} / [\, I_{EP} + I_{EN}\,]$$

$$= 1 / [\, 1 + I_{EN} / I_{EP}\,]$$

$$= 1 / [\, 1 + \mathbf{(C)} / \mathbf{(B)}\,]$$

$$= \frac{1}{1 + \dfrac{D_E\, n_{E0} / L_E}{D_B\, p_{B0} / W_B}}$$

Calculation of Currents (cont.)

$$\boxed{\gamma = \frac{1}{1 + \dfrac{D_E\, W_B\, n_{E0}}{D_B\, L_E\, p_{B0}}} = \frac{1}{1 + \dfrac{D_E\, W_B\, N_B}{D_B\, L_E\, N_E}}}$$

(Eq 11.41 in textbook)

➔ $n_{E0} = n_i^{\,2} / N_E$ … doping in emitter

➔ $p_{B0} = n_i^{\,2} / N_B$ … doping in base

$$\boxed{\alpha_{dc} = \gamma\ \alpha_T}$$

$$\boxed{\beta_{DC} = \alpha_{DC} / (1 - \alpha_{DC})}$$