Fields and Waves I

Lecture 6

Lossy Transmission Lines
Power
Smith Charts

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Exam 1

- Practice problems + solutions now on the shared drive
- Exam 1 crib sheet on shared drive
- Core Skill Report on shared drive
- Exam 1 Study Survey on Gradescope
- *Remember units!*

Homework 2 – Conceptual Questions

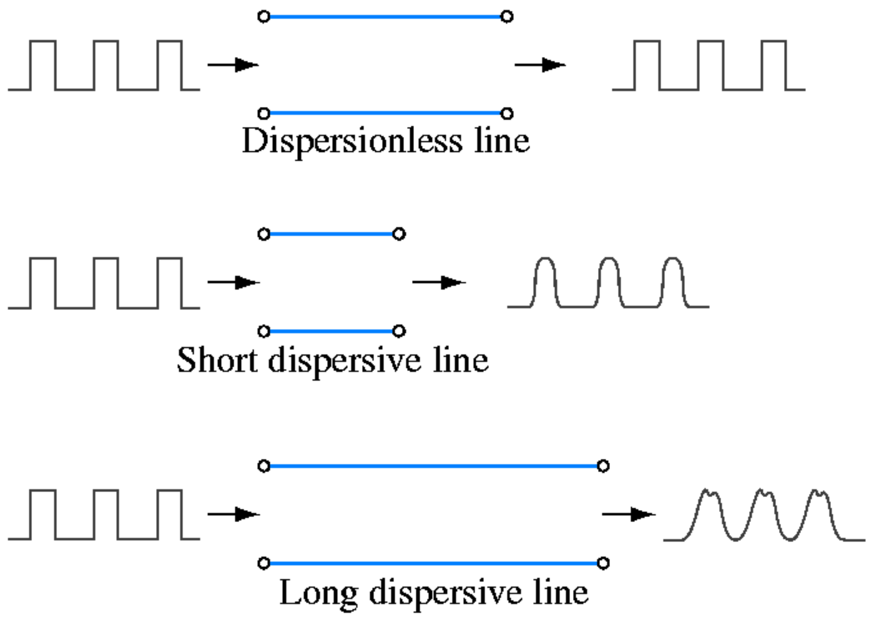
"The standing wave ratio can be found using the absolute value of reflection coefficient at the load. Why is the absolute value used? Why doesn't the sign of the reflection coefficient matter?"

Homework 2 – Conceptual Questions

"In principle, a standing wave pattern could exist for a non-sinusoidal input signal as well. Choose some non-sinusoidal input signal, length of line, Z0, and load impedance and draw the standing wave pattern that will result from it. Show calculations to justify the correctness of your drawing."

T-Line Parameters

Dispersion: A dispersive transmission line will have frequency-dependent impedance behavior, leading to distortion of signals. (Keep in mind that a square pulse is composed of a series of harmonic frequencies)



Distortionless Lines

For practical lines, the conductance per unit length *g* is negligible. Thus, we will add loss between the conductors so that

$$\frac{r}{l} = \frac{g}{c}$$

This is called the **Heaviside condition** and it can be achieved with periodic lumped shunt resistors.

Distortionless Lines

- Consider a "bad RG-58" cable.

What g is required to make it distortionless?

$$\frac{\frac{10\Omega}{50m}}{0.25\mu H/m} = \frac{g}{100pF/m}$$

Distortionless Lines

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What g is required to make it distortionless?

$$\frac{\frac{10\Omega}{50m}}{0.25\mu H/m} = \frac{g}{100pF/m}$$

$$g = 0.2mS/m$$

What resistance would this be per unit length?

Distortionless Lines

What resistance would this be per unit length?

$$g = 0.2mS/m$$

$$(0.2mS/m)(50m) = 10mS$$

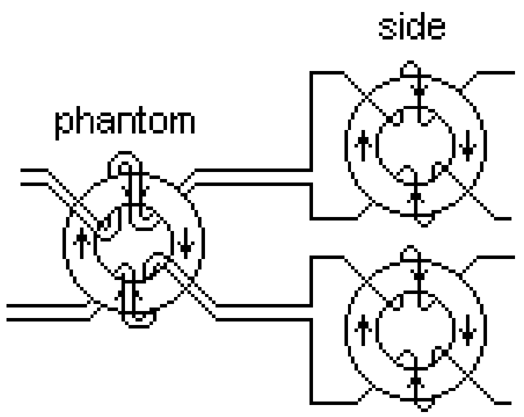
$$\frac{1}{10mS} = 100\Omega$$

(value of each parallel resistor in the t-line simulation, equivalent to 50m)

$$r = \frac{100\Omega}{50m} = 2\Omega/m$$

Distortionless Lines

- In the early days of telephony, Heaviside proposed making lines distortionless. This was done by adding inductance rather than conductance since the losses were not increased significantly.



Campbell-Shaw Loading

http://www.du.edu/~jcalvert/tech/cable.htm

Distortionless Lines

- Adding these components made it possible for phone calls to go from NY to Chicago.
- Then in the 1850s an even more impressive engineering feat was achieved: the first transatlantic undersea cable. This would not have been possible without knowledge of the Heaviside Condition.

Distortionless Lines

Modern submarine cable with repeaters

prog.world

Power Analysis

- How do we define power in EE?
- Basic definition:

$$P = IV \qquad\qquad P = \frac{V^2}{R}$$

- We can find instantaneous power on the transmission line in the same way:

$$P(d,t) = v(d,t)i(d,t)$$

Power Analysis

$$P(d,t) = v(d,t)i(d,t)$$

To find incident power (i.e. we do not consider reflection yet):

$$P_i(d,t) = v_0^+(d,t)i_0^+(d,t)$$

$$v_0^+(d,t) = |V_0^+|cos(\omega t + \beta d + \phi^+)$$

$$i_0^+(d,t) = \frac{|V_0^+|}{Z_0}cos(\omega t + \beta d + \phi^+)$$

$$P_i(d,t) = \frac{|V_0^+|^2}{Z_0}cos^2(\omega t + \beta d + \phi^+)$$

Note: this assumes a lossless t-line. In the lossy case we would need to deal with attenuation and the phase angle of Z_0.

Power Analysis

To find reflected power:

$$P_i(d,t) = v_0^+(d,t)\,i_0^+(d,t)$$

Minus sign appears due the sign flip of the reflected current wave (see lecture 2). Why does this sign flip make sense? Because the current must change direction!

$$v_0^-(d,t) = |\Gamma||V_0^+|cos(\omega t + \beta d + \phi^+ + \theta^r)$$

$$i_0^-(d,t) = -|\Gamma|\frac{|V_0^+|}{Z_0}cos(\omega t + \beta d + \phi^+ + \theta^r)$$

$$P_r(d,t) = -|\Gamma|^2\frac{|V_0^+|^2}{Z_0}cos^2(\omega t + \beta d + \phi^+ + \theta^r)$$

Power flow also changes direction @ reflection, hence the sign change

Power Analysis

To find time-averaged incident power:

$$P^i_{av} = \frac{1}{T}\int_0^T P^i(d,t)dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} P^i(d,t)dt$$

$$= \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{|V_0^+|^2}{Z_0} cos^2(\omega t + \beta d + \phi^+)dt$$

$$P^i_{av} = \frac{\omega}{2\pi}\frac{\pi}{\omega}\frac{|V_0^+|^2}{Z_0} = \frac{|V_0^+|^2}{2Z_0}$$

Power Analysis

To find time-averaged reflected power:

$$P_{av}^r = \frac{1}{T}\int_0^T P^r(d,t)dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} P^r(d,t)dt$$

$$= \frac{\omega}{2\pi}\int_0^{2\pi/\omega} -|\Gamma|^2\frac{|V_0^+|^2}{Z_0}cos^2(\omega t+\beta d+\phi^+ +\theta^r)dt$$

$$P_{av}^r = -|\Gamma|^2\frac{\omega}{2\pi}\frac{\pi}{\omega}\frac{|V_0^+|^2}{Z_0} = -|\Gamma|^2\frac{|V_0^+|^2}{2Z_0} = -|\Gamma|^2 P_{av}^i$$

Power Analysis

Therefore:

$$P_{av} = P_{av}^{i} + P_{av}^{r} = \frac{|V_0^+|^2}{2Z_0}[1 - |\Gamma|^2]$$

Do Lecture 6 Exercise 1 on Gradescope in groups of up to 4.

Impedance Matching

- For a transmission line to work well, we must not only engineer the line parameters properly but also match it to its load.
- Why is this? Think about the following cases:
  - A generator receiving power
  - A speaker
  - An antenna

Impedance Matching

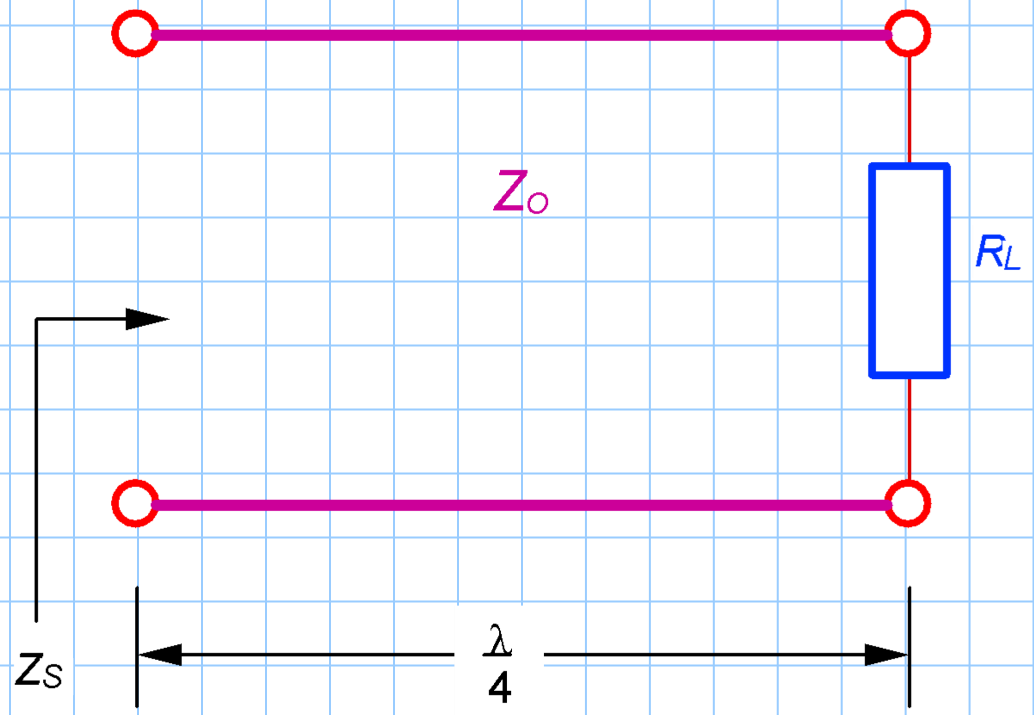
- Reflections lead to variations in the input impedance of the line. The input impedance changes with line length and frequency.
- Power is wasted. An impedance match provides maximum power transfer to the load.
- A VSWR > 1 means there will be voltage maxima on the line. These can lead to voltage breakdown at high power levels.

Impedance Matching

- If we match the load to Z_O, the input impedance remains constant at the value Z_O. Therefore, the input impedance is independent of line length and frequency (over the bandwidth of the matching network).
- VSWR = 1. Therefore there are no voltage peaks on the line.
- Maximum power transfer to the load is achieved.

Impedance Matching

Consider a lossless quarter-wave length of line terminated by a resistance R_L:



Impedance Matching

Assuming the line is lossless:

$$Z_S = Z_O \frac{Z_L + jZ_O \tan \beta l}{Z_O + jZ_L \tan \beta l}$$

and $\quad \tan \beta l = \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right) = \tan\left(\frac{\pi}{2}\right) = \infty$

so $\boxed{Z_S = Z_O \frac{jZ_O}{jZ_L} = \frac{Z_O^2}{R_L}}$

Note that Z_S is purely real, so the line allows us to transform one resistance value to another resistance.

Impedance Matching

Key Properties of λ/4 Lines

- Impedance inversion:

$$Z_S = \frac{Z_O^2}{Z_L}$$

- We can therefore convert an open circuit to a short circuit, and vice versa:
  - short circuit termination: $Z_{in,\ sc} = \infty\ \Omega$
  - open circuit termination: $Z_{in,\ oc} = 0\ \Omega$

Impedance Matching

Key Properties of λ/4 Lines

- "Normalized impedance" is a way to express impedances as a ratio of characteristic impedance. So in normalized terms, a matched impedance would be 1. (This will be useful soon.)

  If $Z_0 = 25\Omega$, what is the impedance $50+75j\ \Omega$ expressed in normalized terms?

Impedance Matching

Key Properties of λ/4 Lines

- “Normalized impedance” is a way to express impedances as a ratio of characteristic impedance. So in normalized terms, a matched impedance would be 1. (This will be useful soon.)

  If $Z_0 = 25\Omega$, what is the impedance $50+75j\ \Omega$ expressed in normalized terms?

  Answer: $2+3j$

Impedance Matching

Key Properties of λ/4 Lines

A mismatched load can be matched to a transmission line using a quarter-wave transformer of suitable characteristic impedance.

e.g.: match a 100 Ω resistor to a 50 Ω line.

$$R_L = 100 \ \Omega$$
$$R_S = 50 \ \Omega$$

∴ the transformer characteristic impedance Z_{OT} must be:

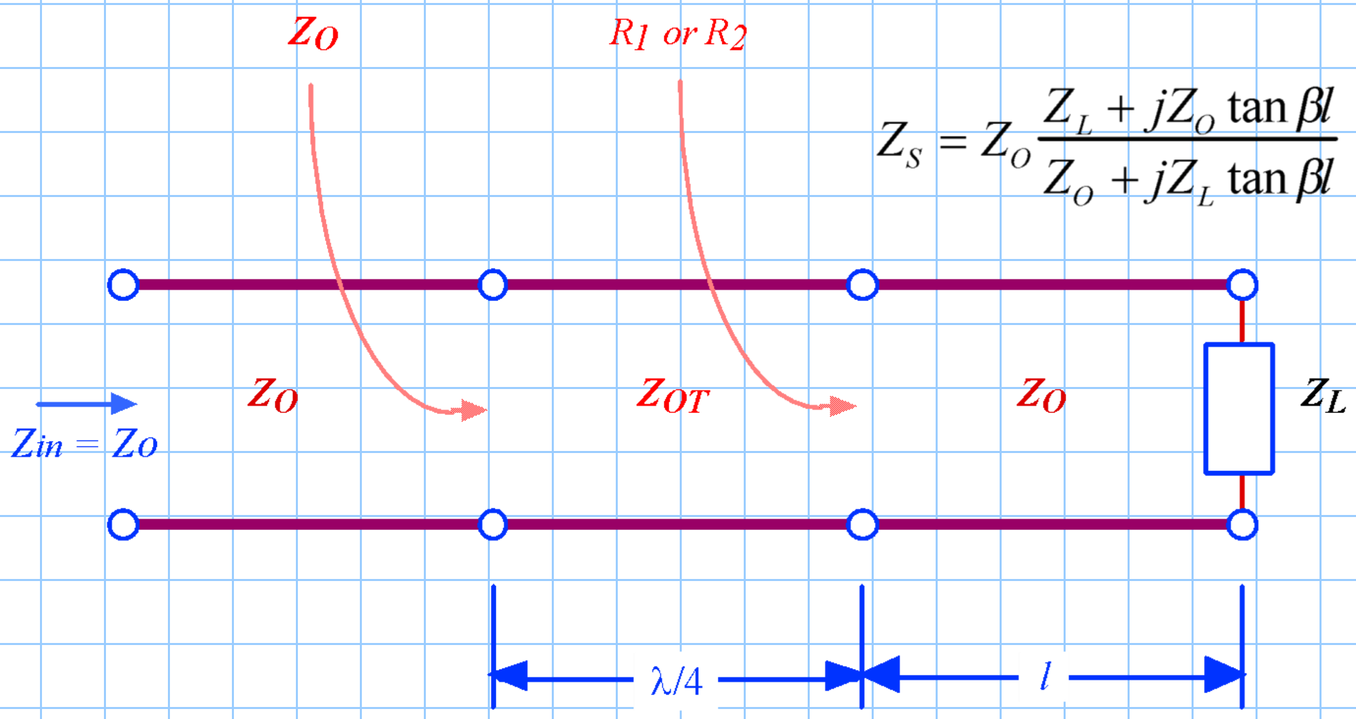
$$\boxed{\begin{aligned} Z_{OT} &= \sqrt{R_L R_S} \\ &= \sqrt{100 \times 50} \\ &= 70.7 \ \Omega \end{aligned}}$$

$$\boxed{Z_{in} = \frac{Z_o^2}{Z_L} = \frac{(70.7)^2}{100} = 50 \, \Omega}$$

Impedance Matching

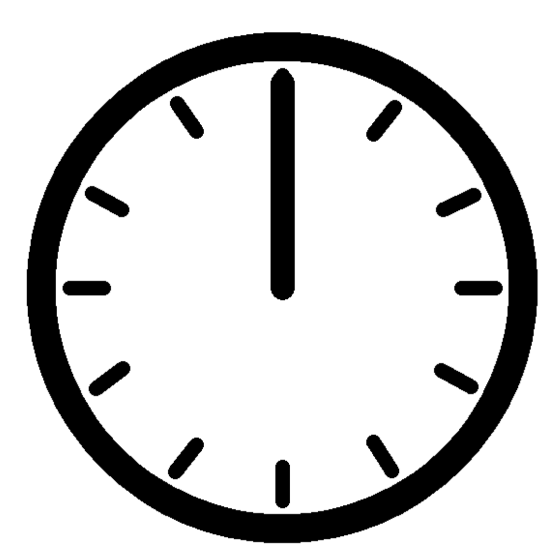
If Z_L is not real, a length of line (with characteristic impedance Z_O) may be used to transform Z_L to a real impedance, which can then be converted to Z_O by the quarter-wave transformer, of characteristic impedance Z_{OT}.

$$Z_S = Z_O \frac{Z_L + jZ_O \tan \beta l}{Z_O + jZ_L \tan \beta l}$$



Impedance Matching

- How do we find a length of line that makes the input impedance of that first segment real?
- We know that the input impedance is periodic in $\lambda/2$ with line length. But the math is hard.
- What if we had a system where we could map a load impedance onto a circle, rotate it by βL like we rotate a clock, and read out the input impedance?
- There's a chart that does this, and it's called a Smith Chart.

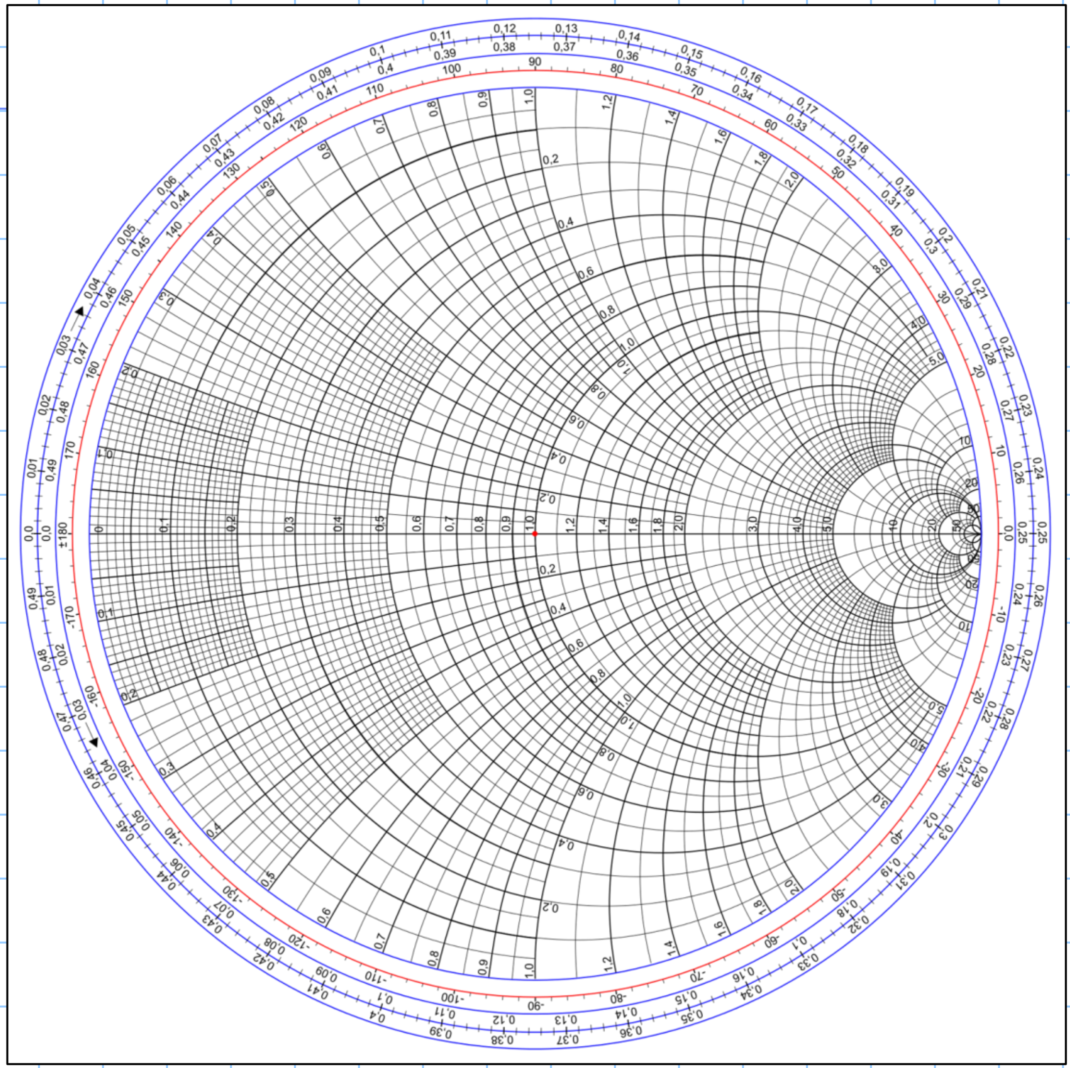


Smith Chart

- Invented by American engineer Philip Hagar Smith in 1939 for antenna input impedance matching
- Also invented in parallel by Japanese engineer Mizuhashi Tosaku in 1937 (the chart is also called the Mizuhashi Chart or Smith-Mizuhashi Chart)
- Allows us to graphically solve the relation between reflection coefficients and load impedance and/or find input impedance, saving us time.

Philip H. Smith

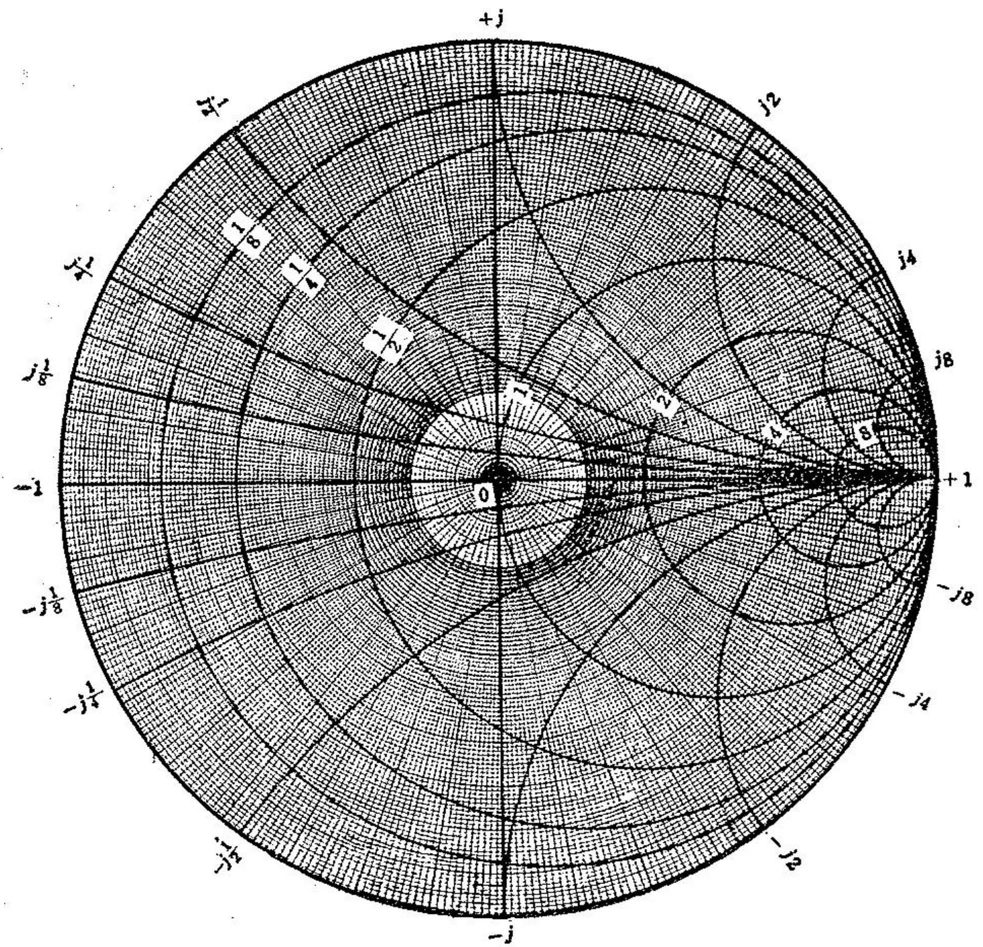
Smith Chart



Mizuhashi Chart

Source: Wayback Machine

Note that both US and Japanese versions use the same curved coordinate system



第一圖　反射係數 γ の Z_{01}（及 Z_{02}）に對する圓線圖

Smith Chart

Reflection coefficient was a complex quantity.

$$\Gamma = |\Gamma| e^{j\theta_r} = \Gamma_r + j\Gamma_i \qquad \text{...(6.1)}$$

We can thus plot reflection coefficients in the complex Γ plane. The components are:

$$\Gamma_r = |\Gamma| \cos\theta_r$$

$$\Gamma_i = |\Gamma| \sin\theta_r$$

Complex Γ-plane

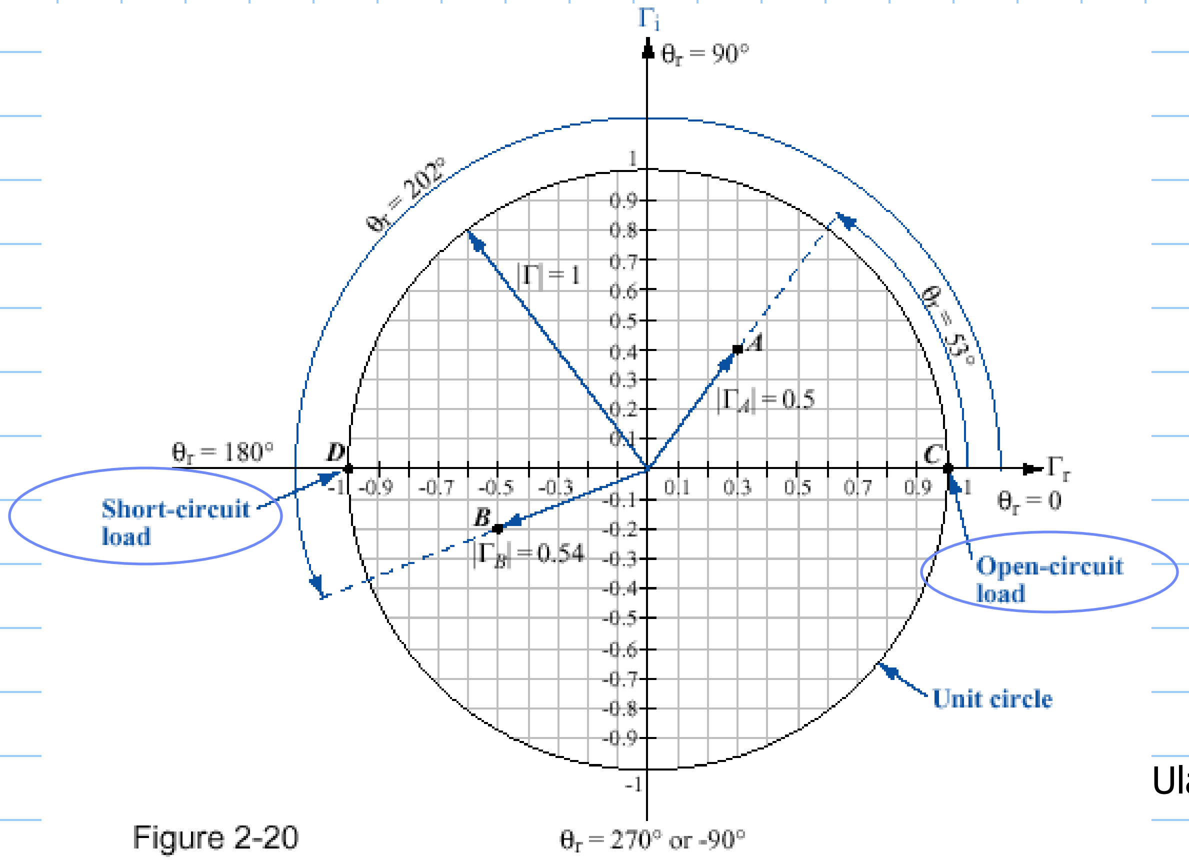


Figure 2-20

Ulaby

Smith Chart

We need to relate impedances to reflection coefficients:

First, we normalize all impedances with respect to the characteristic impedance of the line:

$$z = \frac{Z}{Z_0} \qquad \text{e.g.} \quad z_L = \frac{Z_L}{Z_0}$$

For an impedance of Z_R becomes:

$$\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_R/Z_0 - 1}{Z_R/Z_0 + 1} = \boxed{\frac{z_R - 1}{z_R + 1}} \Leftrightarrow z_R = \boxed{\frac{1+\Gamma}{1-\Gamma}} \quad ..(6.2)$$

Smith Chart

Now since the normalized impedance can be written as:

$$z_R = r_R + jx_R \qquad ..(6.3)$$

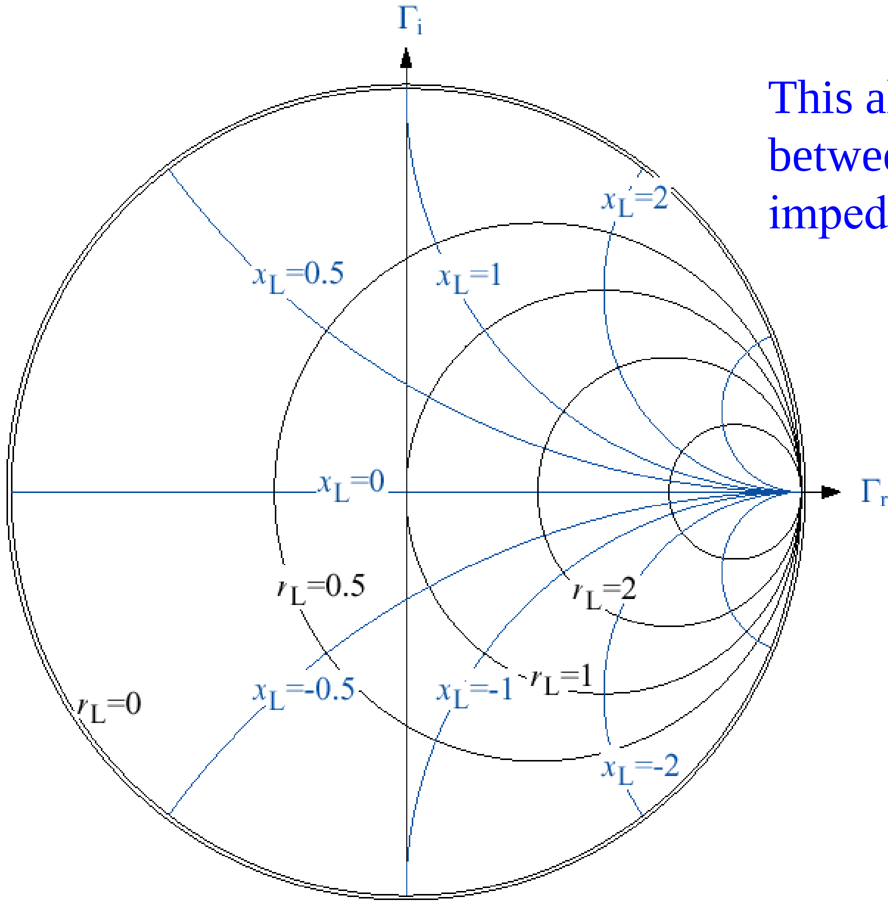
we set (6.3) equal to (6.2) using the real and imaginary parts of (6.1). This gives:

$$r_R + jx_R = \frac{1+\Gamma}{1-\Gamma}$$

We can then solve for the r_R and x_R in terms of Γ. Graphical families of all possible solutions to this equation constitute the Smith Chart.

Smith Chart

A Smith chart is therefore a polar plot of Γ, with contours of real and imaginary parts of z superimposed on top.



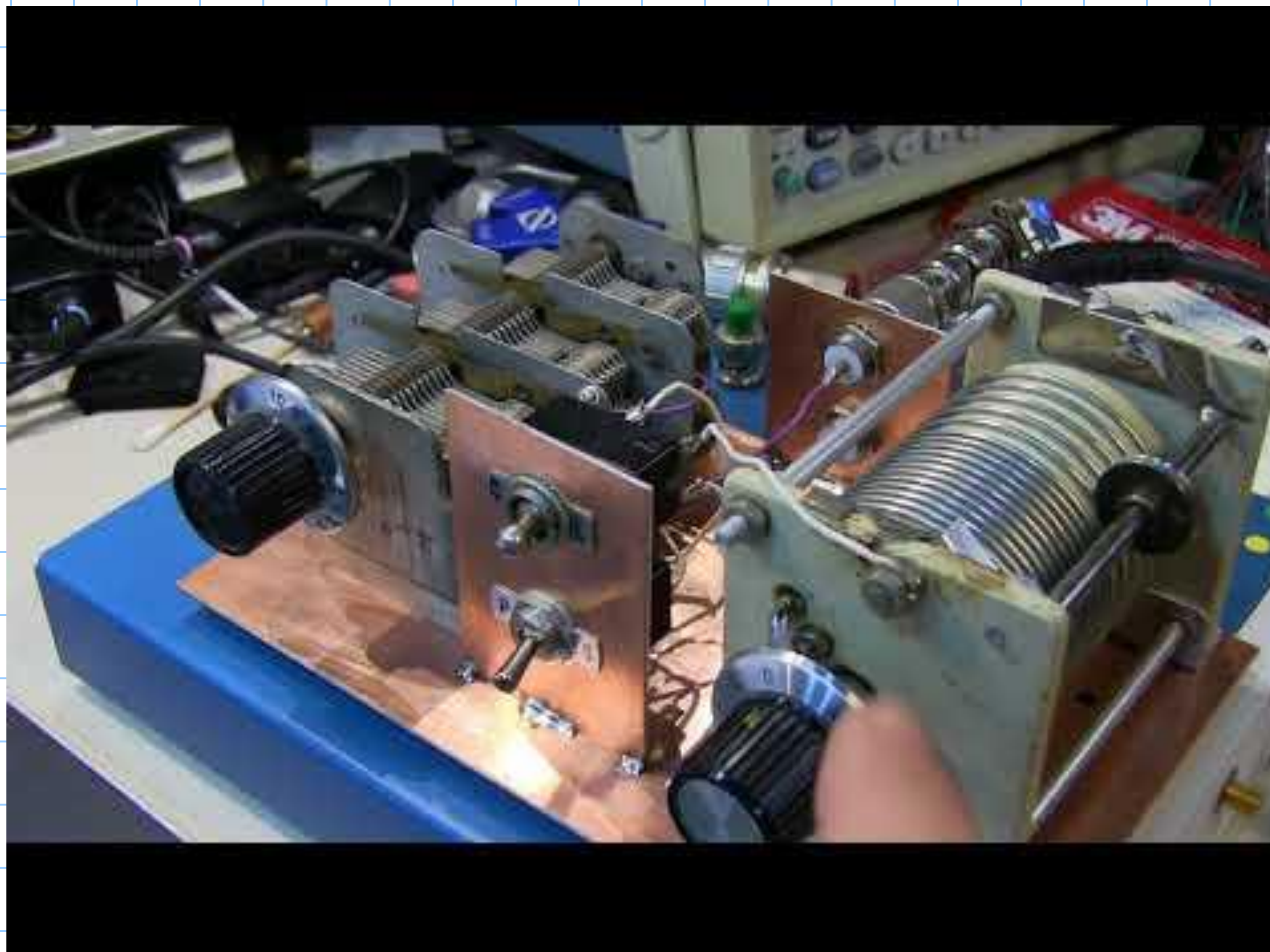
This allows easy conversion between normalized impedance z and Γ

Ulaby

Smith Chart

- A Smith chart printout is available on the shared drive
- This chart is designed to be printed out and done by hand with a ruler
- However, you can also use it on your computer using digital tools
- ImageJ is a useful tool for this

Smith Chart



Frequency Sweep @ 4:00

Smith Chart

- Example:
  - Let's say that a t-line has a characteristic impedance of 40Ω and a load impedance of 20+40jΩ.
  - What is the magnitude and phase of the reflection coefficient?

Smith Chart

- Example:
  - Let's say that a t-line has a characteristic impedance of 40Ω and a load impedance of 20+40jΩ.
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Normalized load impedance is 0.5+j

Smith Chart

- Example:
  - Let's say that a t-line has a characteristic impedance of 40Ω and a load impedance of 20+40jΩ.
  - What is the magnitude and phase of the reflection coefficient?

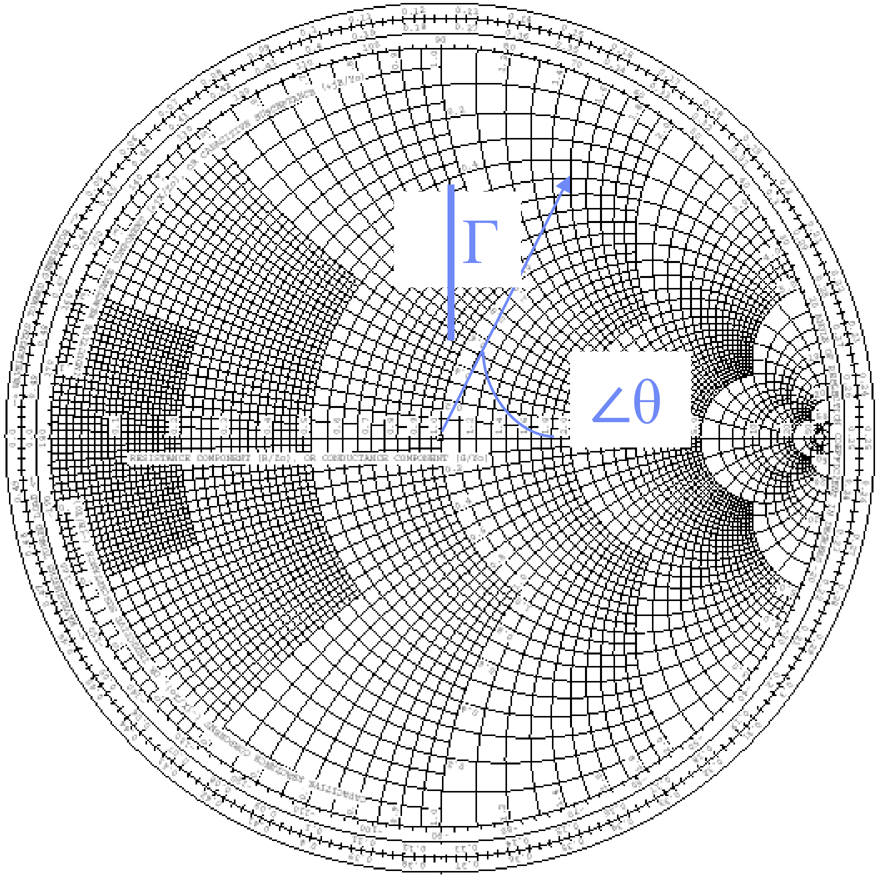
Normalized load impedance is 0.5+j

$\Gamma \approx 0.63 \angle 85°$

Smith Chart

The reflection coefficient is proportional to the length of the radial vector on the chart. The length of the vector to the periphery corresponds to $|\Gamma| = 1$.

The phase angle of the reflection coefficient is measured from the positive
direction of the horizontal axis



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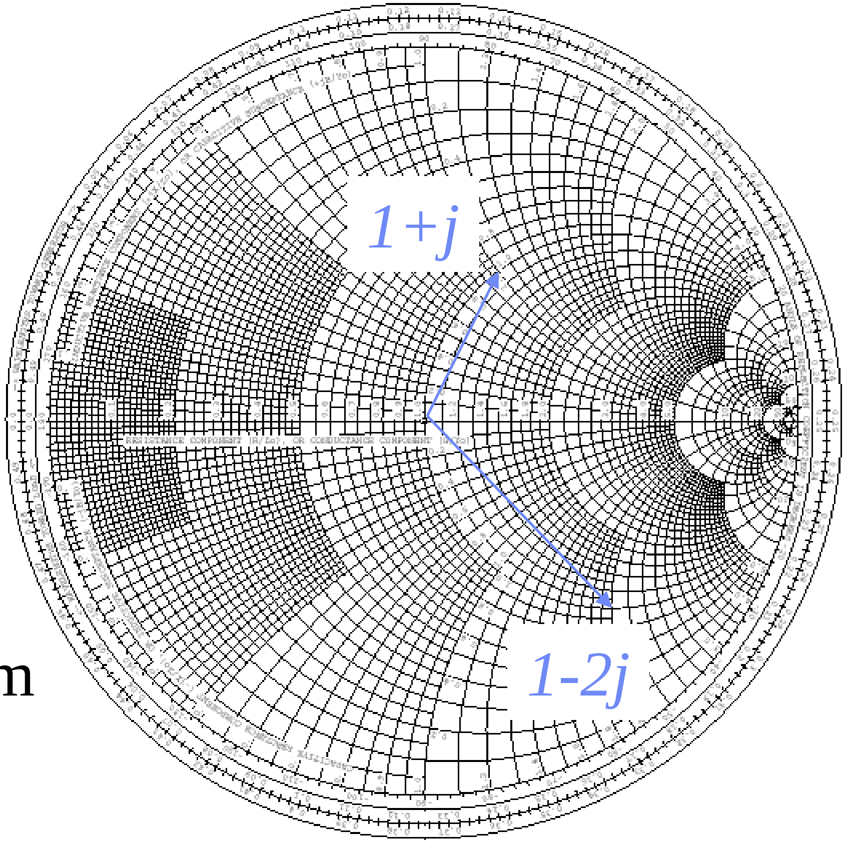
Smith Chart

All impedances in the top half are inductive e.g. 1+j



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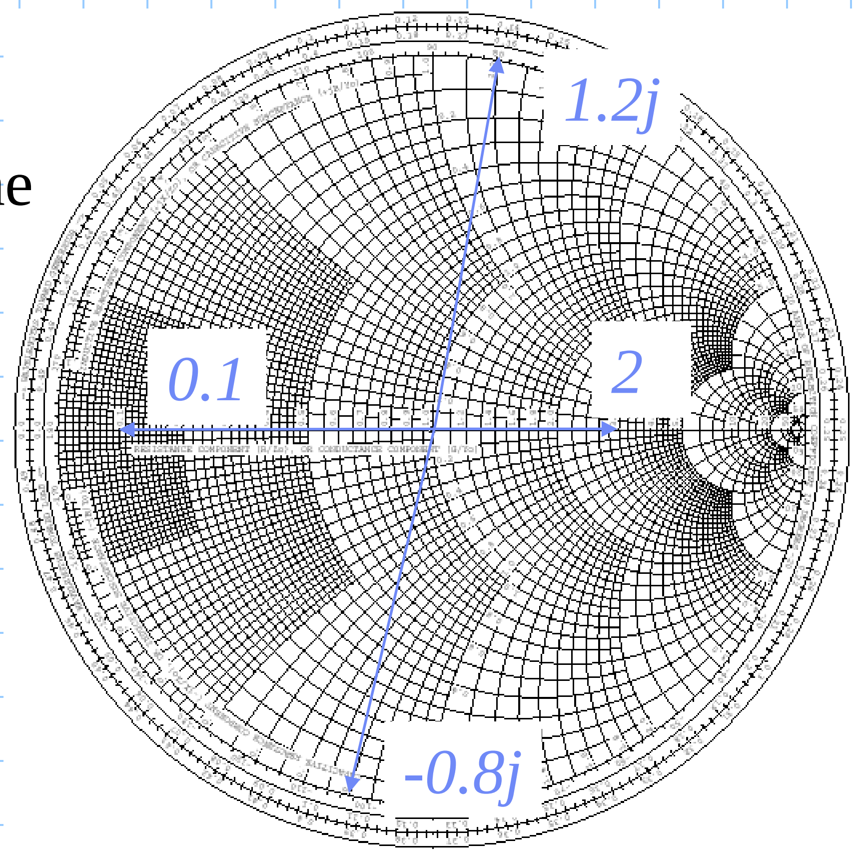
All impedances in the bottom half are capacitive e.g. 1-2j

Smith Chart

purely real impedances are along the horizontal center line

purely imaginary impedances are along the periphery



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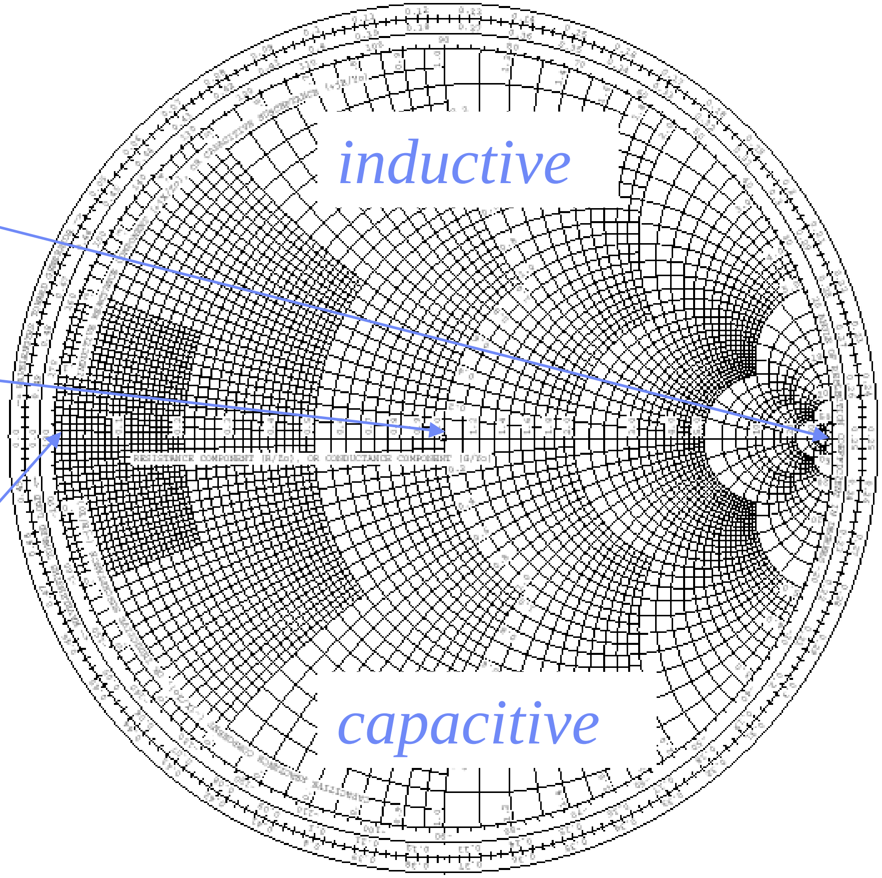
UQ

Smith Chart

open circuit point
(infinite impedance)

unity impedance z =1
(match point)

short circuit point
(zero impedance)



Ulaby

Smith Chart

What do you notice about the angle between the open and short circuit on the Smith Chart?

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They are 180 degrees away from one another.

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What length of transmission line is required to make an open circuit look like a short circuit or vice versa?

Smith Chart

What do you notice about the angle between the open and short circuit on the Smith Chart?

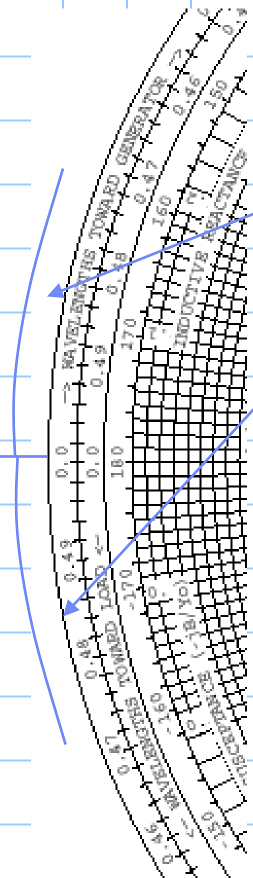
They are 180 degrees away from one another.

What length of transmission line is required to make an open circuit look like a short circuit of vice versa?

A quarter wavelength.

Thus, the angle on a Smith chart is also measured in wavelength (of the AC input signal).

Smith Chart



two scales on the periphery (in wavelengths)
1 towards generator (clockwise)
1 towards load (counterclockwise)

Note also that once around the whole chart is a total length of $\lambda/2$

Ulaby

Smith Chart

If the normalized impedance of a load is +j, what length of transmission line (in wavelengths) is required to make the load look like a real impedance?

Smith Chart

If the normalized impedance of a load is +j, what length of transmission line (in wavelengths) is required to make the load look like a real impedance?

An eighth of a wavelength. (which will cause it to look like an open circuit)

Smith Chart

If the normalized impedance of a load is 3+3j, what length of transmission line (in wavelengths) is required to make the load look like a real impedance?

Smith Chart

If the normalized impedance of a load is 3+3j, what length of transmission line (in wavelengths) is required to make the load look like a real impedance?

Reflection coefficient is **≈0.7 ∠20°**

20/360 = 5.5% of a half wavelength = 2.7% of a wavelength

Smith Chart

For the impedance we found in our first example (0.5 + j), what is the standing wave ratio?

(Read from the bottom of the chart)

Smith Chart

SUMMARY

- The Smith Chart allows the graphical solution of the transmission line equation for *Z*.
- The Chart gives direct conversion between Γ and *Z*.