Problem Set 7

Due: 11pm, Tuesday, November 1, 2022

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Section 17

NOTES

1. Practice problems listed below and taken from the textbook are for your own practice, and are not to be turned in.
2. There are two parts of the Problem Set, an objective part consisting of multiple choice questions (with no partial credit available) and a subjective part (with partial credit possible). Please complete all questions.
3. Writing your solutions in LaTeX is preferred but not required.
4. Show all work for problems in the subjective part. Illegible or undecipherable solutions will not be graded.
5. Figures, if any, should be neatly drawn by hand, properly labelled and captioned.
6. Your completed work is to be submitted electronically to LMS as a single pdf file. Be sure that the pages are properly oriented and well lighted. (Please do not e-mail your work to Muhammad or me.)

Practice Problems from the textbook (Not to be turned in)

- Exercises from Chapter 6, pages 149–150: 1(c,d,f,i), 2(a,d), 4(a,d).
- Exercises from Chapter 6, pages 155–157: 1(b,d), 2(a,d,g,k).
- Exercises from Chapter 6, page 163: 1(c,h), 2(b,c,e).

Objective part (Choose A, B, C or D; no work need be shown, no partial credit available)

1. (5 points) Let $f(t) = e^t$, $g(t) = e^{2t}\sin 3t$ and $H(t)$ be the Heaviside step function. Identify the correct statement, or select "All of these choices" if all of the statements are correct.

   A If $\hat{F} = \int_0^T f(t)e^{-st}\,dt$, then $\lim_{T\to\infty} \hat{F}$ exists for $s > 1$.

   B If $G(s) = \int_0^\infty g(t)e^{-st}\,dt$, then $G(s) = \dfrac{3}{(s-2)^2+9}$ assuming $s > 2$.

   C If $h(t) = \cos t + tH(t-1)$, then $h(t)$ is piecewise continuous for $t \in [0,3]$.

   [D] All of these choices

2. (5 points) Which of the following is correct, or select "None of these choices" if none are correct.

B is not correct, because $\mathcal{L}\big(u''(t)\big)$ would have an extra term $u'(0)$ as well.
C is correct by T.8 and T.2 .

A $\mathcal{L}\big(\sin^2 t\big) = \dfrac{1}{(s^2+1)^2}$

B $\mathcal{L}\big(u''(t)\big) = s\mathcal{L}\big(u'(t)\big)$

[C] $\boldsymbol{\mathcal{L}\big(t^2 e^{-t}\big) = \dfrac{2}{(s+1)^3}}$

D None of these choices

3. (5 points) Let $f(t) = [1 - H(t-\pi)]\sin t + H(t-\pi)\cos t$, where $H(t)$ is the Heaviside step function. Which of the following is correct, or select "None of these choices" if none are correct.

A $f(0) = f(3\pi)$

[B] $\boldsymbol{f(\pi/2) = f(2\pi)}$

C $f(2\pi) = f(3\pi)$

D None of these choices

Subjective part (Show work, partial credit available)

1. (15 points) Use the properties of the Laplace transform and the table of Laplace transforms discussed in class (or the one given in the text) to find the following:

   (a) Find $F(s)$ and $G(s)$ if

$$f(t) = 2te^{-2t}\sin 3t, \qquad g(t) = \begin{cases} 0 & \text{if } t < 3 \\ e^t\cos(t-3) & \text{if } t \geq 3 \end{cases}$$

For $\boldsymbol{F(s)}$:

Let $f_1(t) = e^{-2t}\sin(3t)$.

By T.7, with $a = -2$ and $b = 3$:
$F_1(s) = \frac{b}{(s-a)^2+b^2} = \frac{3}{(s+2)^2+9}$
$F_1(s) = \frac{3}{(s+2)^2+9}$

By T.9, where $n = 1$:
$F(s) = 2(-1)^n F_1{}^n(s) = -2F_1{}'(s)$

Need to find $F_1{}'(s)$:
$F_1(s) = \frac{3}{(s+2)^2+9} = 3[\ (s+2)^2 + 9\ ]^{-1}$

$F_1{}'(s) = -3[\ (s+2)^2 + 9\ ]^{-2} * (2s+4)$

$F_1{}'(s) = \frac{-3(2s+4)}{[(s+2)^2+9]^2}$

$F(s) = -2F_1{}'(s) = \frac{6(2s+4)}{[(s+2)^2+9]^2}$

$\boldsymbol{F(s) = \frac{6(2s+4)}{[(s+2)^2+9]^2}}$

For $\boldsymbol{G(s)}$:

$g(t) = e^t\cos(t-3) * H(t-3) = e^3 * e^{(t-3)}\cos(t-3) * H(t-3)$

Let $g_1(t-3) = e^{(t-3)}\cos(t-3)$
So: $g_1(t) = e^{(t)}\cos(t)$
Use T.6, where $a = 1$ and $b = 1$, to find that:
$G_1(s) = \frac{(s-a)}{(s-a)^2+b^2} = \frac{(s-1)}{(s-1)^2+1}$

$G(s) = e^3 * G_1(s) * e^{-3s} = G_1(s) * e^{3(1-s)}$ by T.12
$G(s) = \frac{(s-1)}{(s-1)^2+1} * e^{3(1-s)}$

$\boldsymbol{G(s) = \frac{(s-1)e^{3(1-s)}}{(s-1)^2+1}}$

(b) Find $u(t)$ and $v(t)$ if

$$U(s) = \frac{2s+7}{s^2+4s+5}\,, \qquad V(s) = \frac{e^{-s}}{s(s-2)}$$

For $\boldsymbol{u(t)}$:
Need to modify the form of this fraction, because PFD would not immediately work. Looks like we can "Complete The Square":
$U(s) = \frac{2s+7}{s^2+4s+5} = \frac{2s+7}{s^2+4s+4+1} = \frac{2s+7}{(s+2)^2+1}$
We notice that these terms are coming close to the form of T.6 and T.7 in the table. So, rearrange a bit more:
$U(s) = \frac{2s+4}{(s+2)^2+1} + \frac{3}{(s+2)^2+1} = 2\frac{(s+2)}{(s+2)^2+1} + 3\frac{1}{(s+2)^2+1} = 2U_1(s) + 3U_2(s)$

The first term appears as:
$U_1(s) = \frac{(s+2)}{(s+2)^2+1}$
By T.6 with $a = -2$, and $b = 1$:
$u_1(t) = e^{at}\cos(bt) = \boldsymbol{e^{-2t}\cos(t)}$

The second term appears as:
$U_2(s) = \frac{1}{(s+2)^2+1}$
By T.6 with $a = -2$ and $b = 1$:
$u_2(t) = e^{at}\sin(bt) = \boldsymbol{e^{-2t}\sin(t)}$

So overall, combining $u_1(t)$ and $u_2(t)$ with their respective coefficients from above:
$u(t) = 2u_1(t) + 3u_2(t)$
$\boldsymbol{u(t) = 2e^{-2t}\cos(t) + 3e^{-2t}\sin(t)}$

For $\boldsymbol{v(t)}$:
Need to perform Partial Fraction Decomposition (PFD) on $V(s)$ in order to split it into multiple, manageable pieces. So:
Let $V_1(s) = \frac{1}{s(s-2)} = \frac{A}{s} + \frac{B}{(s-2)}$
Multiply by common denominator $s(s-2)$:
$1 = A(s-2) + Bs$
$1 = (A+B)s - 2A$

So $A = -1/2$ and $B = 1/2$. Plug these back into the original decomposition:
$V_1(s) = -(\frac{1}{2})\frac{1}{s} + (\frac{1}{2})\frac{1}{(s-2)}$
$v_1(t) = -(\frac{1}{2}) + (\frac{1}{2})e^{2t}$

So $v(t) = v_1(t-c)H(t-c)$ by T.12, where $c = 1$.
$v(t) = (-\frac{1}{2} + \frac{1}{2}e^{2(t-1)})H(t-1)$

$\boldsymbol{v(t) = \frac{1}{2}(e^{2t-2} - 1)H(t-1)}$

2. (15 points) Consider the initial-value problem

$$y'' + 2y' = 4t, \qquad y(0) = -1, \quad y'(0) = 3$$

(a) Use $y(t) = e^{rt}$ to determine the homogeneous solution $y_h(t)$, and use the method of undetermined coefficients to determine the particular solution $y_p(t)$. Apply the initial conditions to determine the solution $y(t)$ of the IVP.

By substituting $y(t) = e^{rt}$ into the DE, simplify to get:
$r^2 + 2r = 0$
$r(r+2) = 0$
$r_1 = 0$ and $r_2 = -2$

Because these roots are distinct and real, homogeneous solution will be of the form:
$y_h(t) = C_1 e^{r_1 t} + C_2 e^{r_2 t}$
$y_h(t) = C_1 + C_2 e^{-2t}$

Make a guess for $y_p(t)$:
$y_p(t) = A_1 t + A_2 t^2$

Find first and second derivatives of our guess, in order to plug into the left-hand side DE:
$y_p(t) = A_1 t + A_2 t^2$
$y_p{}'(t) = A_1 + 2A_2 t$
$y_p{}''(t) = 2A_2$

Plug in our guessed $y_p(t)$ to DE:
$y'' + 2y' = 4t$
$2A_2 + 2(A_1 + 2A_2 t) = 4t$
$(2A_2 + 2A_1) + (4A_2)t = 4t$

$(2A_2 + 2A_1) = 0$ and $4A_2 = 4$
$\boldsymbol{A_2 = 1}$, so $\boldsymbol{A_1 = -1}$

Thus, our particular solution $y_p(t)$ is:
$y_p(t) = A_1 t + A_2 t^2$
$\boldsymbol{y_p(t) = t^2 - t}$

The solution $y(t)$ is a combination of $y_h(t)$ and $y_p(t)$:
$y(t) = C_1 + C_2 e^{-2t} + t^2 - t$

Apply the first IC, that $y(0) = -1$:
$-1 = C_1 + C_2 e^0 + (0)^2 - (0)$
$\boldsymbol{C_1 + C_2 = -1}$ **(eq. 1)**

Find $y'(t)$ in order to apply the second IC:
$y'(t) = -2C_2 e^{-2t} + 2t - 1$
Plug in $y'(0) = 3$:
$3 = -2C_2 e^0 + 2(0) - 1$
$-2C_2 - 1 = 3$
$\boldsymbol{C_2 = -2}$
So: $C_1 + C_2 = -1$
$\boldsymbol{C_1 = 1}$

With $\boldsymbol{C_1 = 1}$ and $\boldsymbol{C_2 = -2}$, the solution $\boldsymbol{y(t)}$ is:
$\boldsymbol{y(t) = 1 - 2e^{-2t} + t^2 - t}$

(b) Take a Laplace transform of the DE and use the ICs to determine $Y(s)$, the Laplace transform of $y(t)$. Use the properties of Laplace transforms and the table of Laplace transforms discussed in class (or the one given in the text) to find $y(t)$. Confirm that the solution found here agrees with the one found in part (a).

The DE is:
$y'' + 2y' = 4t$

Based on the linear nature of Laplace transformations, the overall transform could be considered a sum of individual ones.
$\mathcal{L}(y'') + 2\mathcal{L}(y') = 4\mathcal{L}(t)$

By Table Entry 11 (T.11), the first term becomes:
$s^2Y(s) - sy(0) - y'(0)$
Substitute in the ICs $y(0) = -1$ and $y'(0) = 3$:
$s^2Y(s) + s - 3$

By T.10, the second term becomes:
$2(sY(s) - y(0))$
$2sY(s) + 2$

By T.2 with $n = 1$, the third term becomes:
$4(\frac{n!}{s^{n+1}}) = 4(\frac{1!}{s^{1+1}})$
$4/s^2$

Return these terms to the full DE, and let "Y" indicate $Y(s)$:
$s^2Y + s - 3 + 2sY + 2 = \frac{4}{s^2}$
$sY(s+2) = \frac{4}{s^2} + 1 - s$

$sY = \frac{(4/s^2)}{(s+2)} + \frac{1}{(s+2)} - \frac{s}{(s+2)} = \frac{4}{s^2(s+2)} + \frac{s^2}{s^2(s+2)} - \frac{s^3}{s^2(s+2)}$
$Y = \frac{4}{s^3(s+2)} + \frac{s^2}{s^3(s+2)} - \frac{s^3}{s^3(s+2)}$
$Y(s) = \frac{4+s^2-s^3}{s^3(s+2)}$

Must use Partial Fraction Decomposition in order to break up the above into manageable terms:
$Y(s) = \frac{4+s^2-s^3}{s^3(s+2)} = \frac{B_0}{s} + \frac{B_1}{s^2} + \frac{B_2}{s^3} + \frac{B_3}{(s+2)}$

Multiply all terms by the left hand side's denominator, $s^3(s+2)$:
$4 + s^2 - s^3 = \frac{s^3(s+2)B_0}{s} + \frac{s^3(s+2)B_1}{s^2} + \frac{s^3(s+2)B_2}{s^3} + \frac{s^3(s+2)B_3}{(s+2)}$
$4 + s^2 - s^3 = s^2(s+2)B_0 + s(s+2)B_1 + (s+2)B_2 + s^3B_3$

Expand terms then group (should have 7 terms on RHS):
$4 + s^2 - s^3 = s^3B_0 + 2s^2B_0 + s^2B_1 + 2sB_1 + sB_2 + 2B_2 + s^3B_3$
$4 + s^2 - s^3 = s^3(B_0 + B_3) + s^2(2B_0 + B_1) + s(2B_1 + B_2) + (2B_2)$

From these equalities, we get the following equations:
$2B_2 = 4$ (eq. 0)
$2B_1 + B_2 = 0$ (eq. 1)
$2B_0 + B_1 = 1$ (eq. 2)
$B_0 + B_3 = -1$ (eq. 3)

Use (eq. 0) to solve for B_2:
$2B_2 = 4$
$\boldsymbol{B_2 = 2}$

Plug $B_2 = 2$ into (eq. 1):
$2B_1 + B_2 = 0$
$2B_1 + 2 = 0$
$\boldsymbol{B_1 = -1}$

Plug $B_1 = -1$ into (eq. 2):
$2B_0 + B_1 = 1$
$2B_0 - 1 = 1$
$\boldsymbol{B_0 = 1}$

Plug $B_0 = 1$ into (eq. 3):
$B_0 + B_3 = -1$
$1 + B_3 = -1$
$\boldsymbol{B_3 = -2}$

Plug the constants B_0, B_1, B_2, B_3 back into the original PFD:
$Y(s) = \frac{B_0}{s} + \frac{B_1}{s^2} + \frac{B_2}{s^3} + \frac{B_3}{(s+2)}$

$\boldsymbol{Y(s) = \frac{1}{s} + \frac{-1}{s^2} + \frac{2}{s^3} + \frac{-2}{(s+2)}}$
Now that we have $Y(s)$ in a more manageable, simplified form, we can begin to apply Laplace Transformation rules from the table to each term.
$Y(s) = \frac{1}{s} + \frac{-1}{s^2} + \frac{2}{s^3} + \frac{-2}{(s+2)}$

Y_1 looks like it can be easily simplified using T.1 :
$Y_1 = \frac{1}{s}$, so:
$\boldsymbol{y_1 = 1}$

Y_2 looks like it can be reverse transformed using T.2, with $n = 1$:
$Y_2 = -\frac{1}{s^2}$, so: $y_2 = -t^n$
$\boldsymbol{y_2 = -t}$

For Y_3 apply T.2, with $n = 2$:
$Y_3 = \frac{2}{s^3}$, so: $y_3 = t^n$
$\boldsymbol{y_3 = t^2}$

The last term, Y_4 , most resembles the form of T.3 with $a = -2$ and a constant of -2 in front.
$Y_4 = (-2)\frac{1}{(s-(-2))}$, so: $y_4 = (-2)e^{at}$
$\boldsymbol{y_4 = -2e^{-2t}}$

$y(t) = y_1 + y_2 + y_3 + y_4$
$y(t) = 1 - t + t^2 - 2e^{-2t}$

After combining all terms the solution $y(t)$ of the DE, found via Laplace Transformations and properties, is:
$\boldsymbol{y(t) = 1 - 2e^{-2t} + t^2 - t}$

This matches my solution found in part (a)!

3. (15 points) The displacement $u(t)$ of a forced mass-spring-damper system is governed by

$$u'' + 2u' + 10u = f(t), \qquad u(0) = 0, \quad u'(0) = 1$$

where the external forcing is given by

$$f(t) = \begin{cases} 0 & \text{if } t < 1 \\ 10e^{-2(t-1)} & \text{if } t \geq 1 \end{cases}$$

(a) Take a Laplace transform of the DE and use the ICs to determine $U(s)$, the Laplace transform of $u(t)$.

The RHS, $f(t)$, can be re-expressed as:
$f(t) = 10e^{-2(t-1)}H(t-1)$

The DE is:
$u'' + 2u' + 10u = 10e^{-2(t-1)}H(t-1)$

Based on the linear nature of Laplace transformations, the overall transform could be considered a sum of individual ones:
$\mathcal{L}(u'') + \mathcal{L}(2u') + \mathcal{L}(10u) = \mathcal{L}(10e^{-2(t-1)}H(t-1))$

By Table Entry 11 (T.11), the first term becomes:
$s^2U(s) - su(0) - u'(0)$
Substitute in the ICs $u(0) = 0$ and $u'(0) = 1$:
$s^2U(s) - 1$

By T.10, the second term becomes:
$2(\ sU(s) - u(0)\ )$
Substitute in the IC $u(0) = 0$:
$2sU(s)$

By T.2 with $n = 1$, the third term becomes:
$10(\frac{n!}{s^{n+1}})$
$\frac{10}{s^2}$

The fourth term will follow T.12 with $c = 1$. Let $a(t-1) = 10e^{-2(t-1)}$. So:
$a(t) = 10e^{-2t}$
By T.3 with $a = -2$, the transform of $a(t)$ is:
$A(s) = \frac{10}{s-a} = \frac{10}{s+2}$
fourth term $= \mathcal{L}(\ a(t-1)H(t-1)\ ) = A(s)e^{-s} = \frac{10}{s+2}e^{-s}$

Return these terms to the full DE, and let "U" indicate $U(s)$. Solve for U:
$s^2U - 1 + 2sU + \frac{10}{s^2} = \frac{10}{s+2}e^{-s}$
$s^2U + 2sU = \frac{10}{s+2}e^{-s} - \frac{10}{s^2} + 1$
$sU(s+2) = \frac{10}{s+2}e^{-s} - \frac{10}{s^2} + 1$
$sU = \frac{10}{(s+2)^2}e^{-s} - \frac{10}{s^2(s+2)} + \frac{1}{(s+2)}$

$U = \frac{10}{s(s+2)^2}e^{-s} - \frac{10}{s^3(s+2)} + \frac{1}{s(s+2)}$

Each of the above colored terms can be simplified via PFD, then reverse transformed to find $u(t)$.

First term (U_1):
$U_1 = \frac{10}{s(s+2)^2} * e^{-s}$
The green term can be simplified via PFD:
$\frac{10}{s(s+2)^2} = \frac{G_0}{s} + \frac{G_1}{(s+2)} + \frac{G_2}{(s+2)^2}$
$10 = (s+2)^2 G_0 + s(s+2)G_1 + sG_2$
$10 = (s^2+4s+4)G_0 + (s^2+2s)G_1 + sG_2$
$10 = s^2(G_0+G_1) + s(4G_0+2G_1+G_2) + (4G_0)$
End up with 3 equations to solve for the constants:
$4G_0 = 10$ (eq. 0)
$4G_0 + 2G_1 + G_2 = 0$ (eq. 1)
$G_0 + G_1 = 0$ (eq. 2)

Solve (eq . 1):
$G_0 = 10/4$
$\boldsymbol{G_0 = 5/2}$

Plug $\boldsymbol{G_0 = 5/2}$ into (eq. 2):
$(5/2) + G_1 = 0$
$\boldsymbol{G_1 = -5/2}$

Plug G_0 and G_1 into (eq. 1):
$4G_0 + 2G_1 + G_2 = 0$
$4(5/2) + 2(-5/2) + G_2 = 0$
$10/2 + G_2 = 0$
$\boldsymbol{G_2 = -5}$

So, plug constants back into U_1 :
$U_1 = (\frac{G_0}{s} + \frac{G_1}{(s+2)} + \frac{G_2}{(s+2)^2})e^{-s}$
$\boldsymbol{U_1 = [(\frac{5}{2})\frac{1}{s} + (-\frac{5}{2})\frac{1}{(s+2)} + (-5)\frac{1}{(s+2)^2}]e^{-s}}$

Second term (U_2):
$U_2 = \frac{10}{s^3(s+2)}$
Simplify via PFD:
$\frac{10}{s^3(s+2)} = \frac{H_0}{s} + \frac{H_1}{s^2} + \frac{H_2}{s^3} + \frac{H_3}{(s+2)}$
$10 = s^2(s+2)H_0 + s(s+2)H_1 + (s+2)H_2 + s^3 H_3$
$10 = (s^3+2s^2)H_0 + (s^2+2s)H_1 + (s+2)H_2 + s^3 H_3$
$10 = s^3(H_0+H_3) + s^2(2H_0+H_1) + s(2H_1+H_2) + (2H_2)$

Get 4 equations from this:
$2H_2 = 10$ (eq. 0)
$2H_1 + H_2 = 0$ (eq. 1)
$2H_0 + H_1 = 0$ (eq. 2)
$H_0 + H_3 = 0$ (eq. 3)

$\boldsymbol{H_2 = 5}$
So $\boldsymbol{H_1 = -5/2}$, and $\boldsymbol{H_0 = 5/4}$, and $\boldsymbol{H_3 = -5/4}$

So plug constants back into U_2 :
$U_2 = \frac{H_0}{s} + \frac{H_1}{s^2} + \frac{H_2}{s^3} + \frac{H_3}{(s+2)}$
$\boldsymbol{U_2 = (\frac{5}{4})\frac{1}{s} + (-\frac{5}{2})\frac{1}{s^2} + (5)\frac{1}{s^3} + (-\frac{5}{4})\frac{1}{(s+2)}}$

Third term (U_3):
$U_3 = \frac{1}{s(s+2)}$
Simplify via PFD:
$\frac{1}{s(s+2)} = \frac{T_0}{s} + \frac{T_1}{(s+2)}$
$1 = (s+2)T_0 + sT_1$
$1 = s(T_0 + T_1) + 2T_0$
Get 2 equations:
$T_0 + T_1 = 0$ (eq. 0)
$2T_0 = 1$ (eq. 0)

So $\boldsymbol{T_0 = 1/2}$ and $\boldsymbol{T_1 = -1/2}$

Plug constants back into U_3:
$U_3 = \frac{T_0}{s} + \frac{T_1}{(s+2)}$
$\boldsymbol{U_3 = (\frac{1}{2})\frac{1}{s} + (-\frac{1}{2})\frac{1}{(s+2)}}$

Overall, $U(s)$ comes to be:
$\boldsymbol{U(s) = U_1 - U_2 + U_3}$

$\boldsymbol{U(s) = [(\frac{5}{2})\frac{1}{s} + (-\frac{5}{2})\frac{1}{(s+2)} + (-5)\frac{1}{(s+2)^2}]e^{-s} - [(\frac{5}{4})\frac{1}{s} + (-\frac{5}{2})\frac{1}{s^2} + (5)\frac{1}{s^3} + (-\frac{5}{4})\frac{1}{(s+2)}] + [(\frac{1}{2})\frac{1}{s} + (-\frac{1}{2})\frac{1}{(s+2)}]}$

(b) Use the properties of Laplace transforms and the table of Laplace transforms discussed in class (or the one given in the text) to find $u(t)$.

$U(s) = [(\frac{5}{2})\frac{1}{s} + (-\frac{5}{2})\frac{1}{(s+2)} + (-5)\frac{1}{(s+2)^2}]e^{-s} - [(\frac{5}{4})\frac{1}{s} + (-\frac{5}{2})\frac{1}{s^2} + (5)\frac{1}{s^3} + (-\frac{5}{4})\frac{1}{(s+2)}] + [(\frac{1}{2})\frac{1}{s} + (-\frac{1}{2})\frac{1}{(s+2)}]$

For the first term:
By using T.1, T.2, and T.8, alongside T.12 to transform the e^{-s} term, we get:
$\boldsymbol{u_1(t) = [\frac{5}{2} - \frac{5}{2}e^{-2(t-1)} - 5e^{-2(t-1)}(t-1)]H(t-1)}$

For the second term:
By using T.1, T.2, and T.8 for the last term, we get:
$\boldsymbol{u_2(t) = \frac{5}{4} - \frac{5t}{2} + \frac{5t^2}{2} - \frac{5}{4}e^{-2t}}$

For the third term:
By using T.1 and T.8, we get:
$\boldsymbol{u_3(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}}$

By combining each of the reverse-transformed functions, we get $u(t)$:
$\boldsymbol{u(t) = u_1 - u_2 + u_3}$
Since u_1 has all of its terms multiplied by a Heaviside function, they will not easily simplify with the other terms from u_2 and u_3. But, there are a lot of terms which can be combined:
$u(t) = u_1 - [\frac{5}{4} - \frac{5t}{2} + \frac{5t^2}{2} - \frac{5}{4}e^{-2t}] + [\frac{1}{2} - \frac{1}{2}e^{-2t}]$
$u(t) = u_1 - \frac{5}{4} + \frac{5t}{2} - \frac{5t^2}{2} + \frac{5}{4}e^{-2t} + \frac{1}{2} - \frac{1}{2}e^{-2t}$
$u(t) = u_1 + \frac{3}{4}e^{-2t} - \frac{5}{4} + (\frac{5t}{2} - \frac{5t^2}{2} + \frac{1}{2})$
$u(t) = u_1 + \frac{3}{4}e^{-2t} - \frac{5}{4} + \frac{-5t^2+5t+1}{2}$

$\boldsymbol{u(t) = [\frac{5}{2} - \frac{5}{2}e^{-2(t-1)} - 5e^{-2(t-1)}(t-1)]H(t-1) + \frac{3}{4}e^{-2t} - \frac{5}{4} + \frac{-5t^2+5t+1}{2}}$