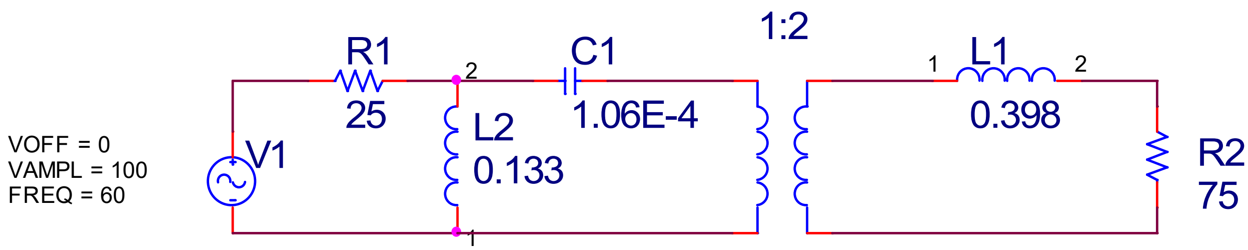
Problem 1) Ideal Transformers and Power



1.1: For the transformer circuit above, find the equivalent circuit (without a transformer with only a source impedance and load impedance). ***You may use either referral method though you should figure out which would best to use. (15 pts)***

The above circuit has a 100 V, 60 Hz source.

$V_{A1} := 100V$ $\quad R_1 := 25\Omega$ $\quad C_1 := 1.06 \cdot 10^{-4} F$ $\quad R_2 := 75\Omega$

$f := 60Hz$ $\quad L_2 := 0.133H$ $\quad L_1 := 0.398H$

$\omega := 2 \cdot \pi \cdot f$ $\quad N_1 := 2$

$\omega = 376.991 \frac{1}{s}$

$L_2 \cdot \omega = 50.14\,\Omega$ $\quad \frac{-1}{C_1 \cdot \omega} = -25.024\,\Omega$ $\quad L_1 \cdot \omega = 150.042\,\Omega$

$Z_{L2} := 50j\Omega$ $\quad Z_{C1} := -25j\Omega$ $\quad Z_{L1} := 150j\Omega$

Convert to thevenin using a voltage divider (for voltage across L2...)

$$V_{Th} := V_{A1} \cdot \frac{Z_{L2}}{R_1 + Z_{L2}}$$

$$V_{Th} = (80 + 40i)\ V$$

$$\sqrt{80^2 + 40^2} = 89.443$$

$$\operatorname{atan}\left(\frac{40}{80}\right) = 26.565 \cdot deg$$

$$89.4 < 26.6deg$$

Combine secondary impedances

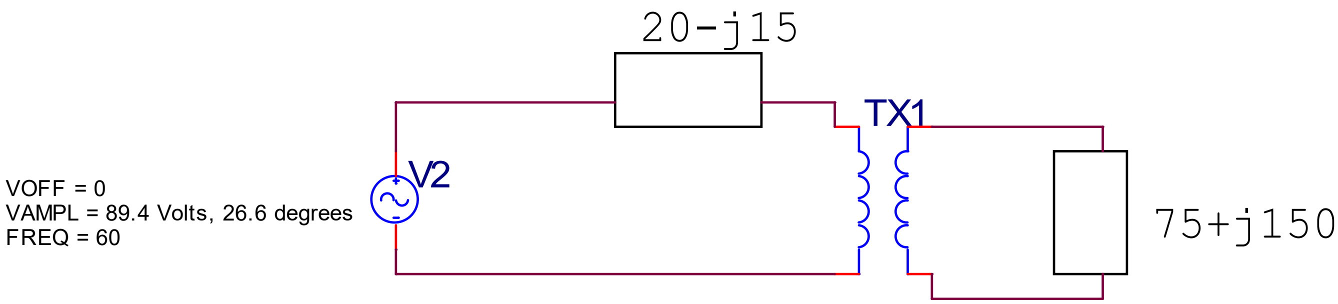
$$Z_{Load} := Z_{L1} + R_2$$

$$Z_{Load} = (75 + 150i)\ \Omega$$

(recognition and correct use of thevenin 7 pts)
They can leave any value in polar or rectangular form.

Find thevenin impedance by shorting source then combining like resistors

$$Z_{Th} := \frac{R_1 \cdot Z_{L2}}{R_1 + Z_{L2}} + Z_{C1} \qquad Z_{Th} = (20 - 15i)\ \Omega$$



Equivalent circuit:

1st equivalent circuit: Refer to secondary

$$Z_{sps} := N_1^{\,2} \cdot Z_{Th} \qquad Z_{LEQps} := Z_{Load}$$

$$Z_{LEQps} = (75 + 150i)\ \Omega$$

$$Z_{sps} = (80 - 60i) \cdot \Omega$$

$$V_{sps} := N_1 \cdot V_{Th}$$

This one should be preferred since you are looking for the voltage across R2.

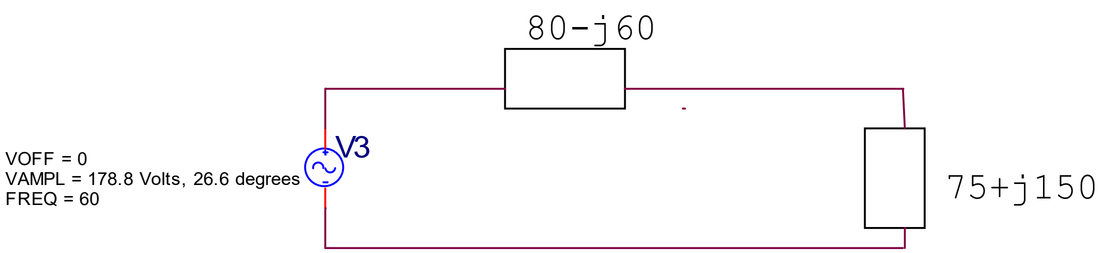
$$V_{sps} = (160 + 80i)\ V$$

$$\sqrt{160^2 + 80^2} = 178.885$$

$$\operatorname{atan}\left(\frac{80}{160}\right) = 26.565 \cdot deg$$

Correct referral process (either one) 7 pts
Correct values 1 pt

$$242.245 < 13.95deg$$



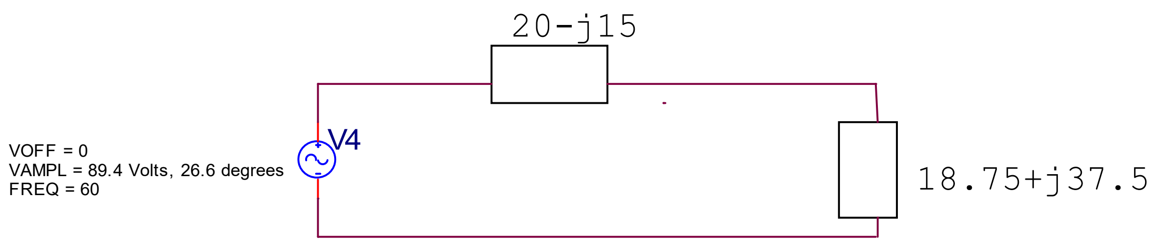
2nd equivalent circuit: Refer to primary

$$Z_{LEQsp} := \frac{Z_{Load}}{{N_1}^2} \qquad Z_{ssp} := Z_{Th} \qquad V_{ssp} := 96.9 < 14deg$$

$$Z_{ssp} = (20 - 15i)\ \Omega \qquad \sqrt{94^2 + 23.5^2} = 96.893$$

$$Z_{LEQsp} = (18.75 + 37.5i)\ \Omega \qquad \mathrm{atan}\left(\frac{23.5}{94}\right) = 14.036 \cdot deg$$

$89.4 < 26.6deg$ same as Vth circuit



1.2: Find the voltage across the R2. ***Put in phasor/polar form. (10 pts)***

Use refer to secondary. You can use voltage divider to find it.

$$V_{LEQ} := V_{sps} \cdot \frac{(75 + 150j)}{80 - 60j + 75 + 150j}$$

$$V_{LEQ} = (84.047 + 144.747i)\ V$$

$$V_{R2} := V_{LEQ} \cdot \frac{75}{75 + 150j} = (74.708 - 4.669i)\ V$$

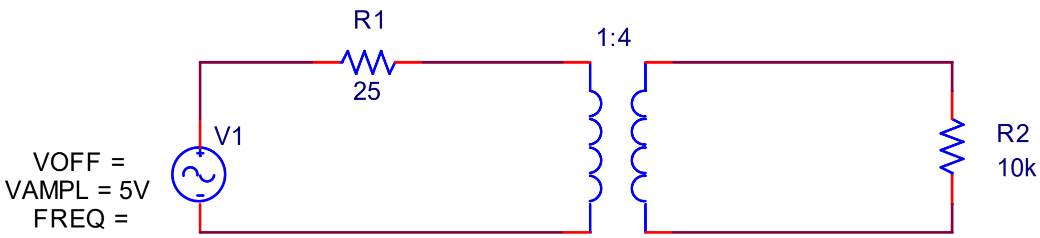
$$\sqrt{74.7^2 + \left(-4.67^2\right)} = 74.554$$

$$\mathrm{atan}\left(\frac{-4.97}{74.7}\right) = -3.806 \cdot deg$$

$74.5 < -3.8deg$

R2	

Problem 2) Real Transformers



2.1: Determine the voltage across R2 in phasor form

Refer the primary to secondary

$V_s := 5V$ $\quad R_{12} := 25\Omega$ $\quad R_{22} := 10k\Omega$

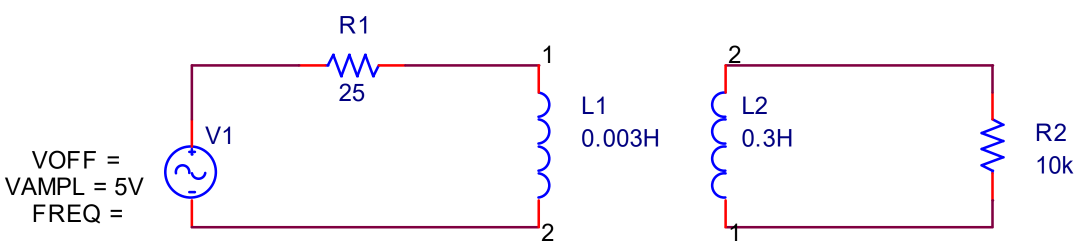
$N_2 := 4$

$V_{sps2} := N_2 \cdot V_s$

$V_{sps2} = 20\,V$

$Z_{sps2} := N_2^{\,2} \cdot R_{12}$

$Z_{sps2} = 400\,\Omega$



$M = k \cdot \sqrt{L_1 \cdot L_2}$

2.2: If the real transformer is constructed of two inductors (as show) and the coupling coefficient is k=0.8, determine the Tee model of the circuit. Draw your circuit.

$k := 0.8$

$L_{12} := 0.003H$

$L_{22} := 0.3H$

$$M := k \cdot \sqrt{L_{12} \cdot L_{22}}$$

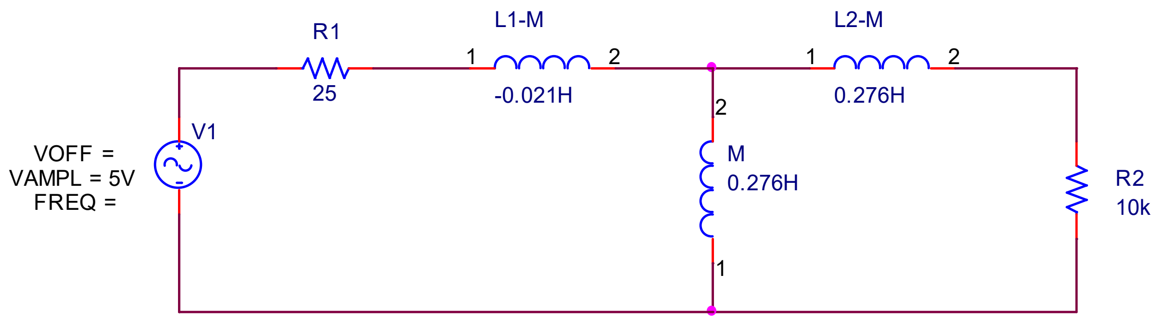
$$M = 0.024 \cdot H$$

In the Tee model, the inductor would be replaced with

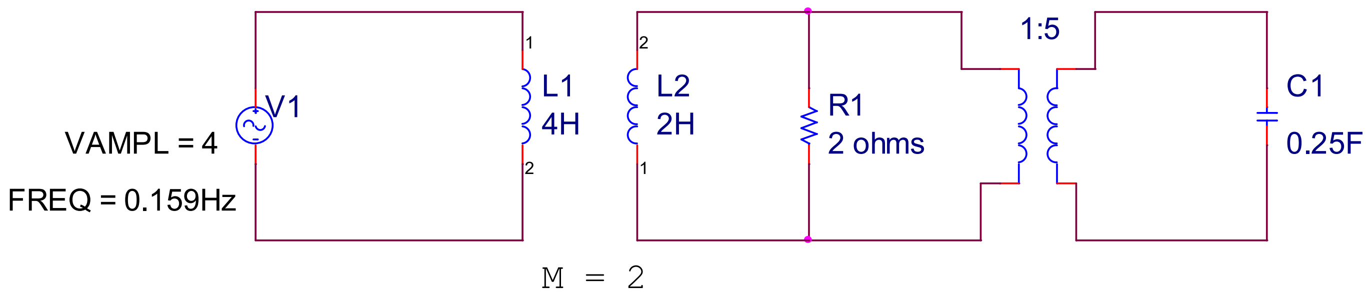
$$L_{12} - M = -0.021\,H$$

$$L_{22} - M = 0.276\,H$$

$$M = 0.024\,H$$



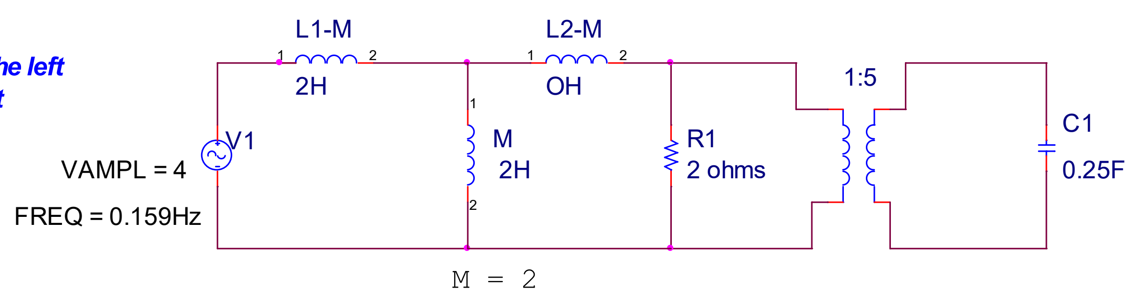
Problem 3) Real and Ideal Transformers



In the above circuit, L1 and L2 are coupled by the mutual inductance, M. The voltage source has a voltage V1(t) = 4 cos(t) (ω= 1 rad/s). The the transformer on the right is an ideal transformer.

3.1. Draw the equivalent circuit without transformers (should include only a source impedance and a load impedance with values) ***(15 pts)****. Note: Draw the equivalent circuit so you can easily calculate current through C1. Include all calculations and supporting diagrams along the way.*

Must use tee model on the left
Use Thevenin equivalent
Then refer to secondary



$$\omega := 2\cdot\pi\cdot 0.159 = 0.999$$

$$V_{Th} = \frac{\frac{4j}{2j+2}}{2j + \frac{4j}{2j+2}}\cdot 4 \qquad \frac{\frac{4j}{2j+2}}{2j + \frac{4j}{2j+2}}\cdot 4 = 1.6 - 0.8i$$

$$\sqrt{1.6^2 + (-0.8)^2} = 1.789$$

$$\operatorname{atan}\left(\frac{-0.8}{1.6}\right) = -26.565\cdot deg$$

$$V_{Th} := 1.789 < -26.6deg$$

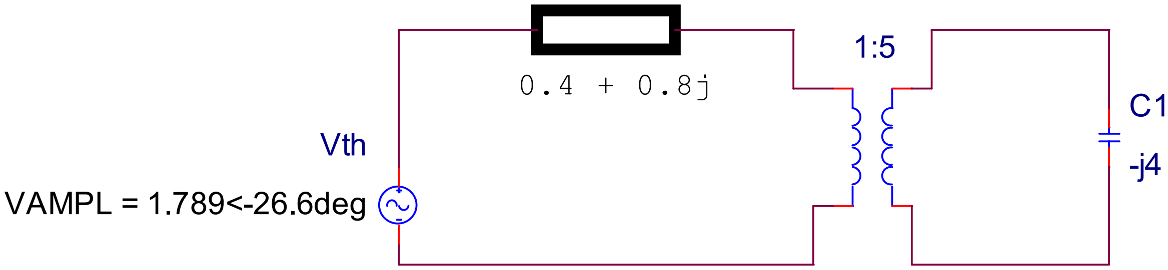
$$R_{Th} := \frac{\frac{2j\cdot 2j}{4j}\cdot 2}{2 + \frac{2j\cdot 2j}{4j}}$$

$$Z_{C1} = \frac{1}{j\cdot 0.25}$$

correct thevenin for primary side

$$Z_{C1} = -j4$$

$$R_{Th} = 0.4 + 0.8i$$



correct referral to secondary to determine current through load impedance, though they can also refer to primary then use the turns ratio in part b.

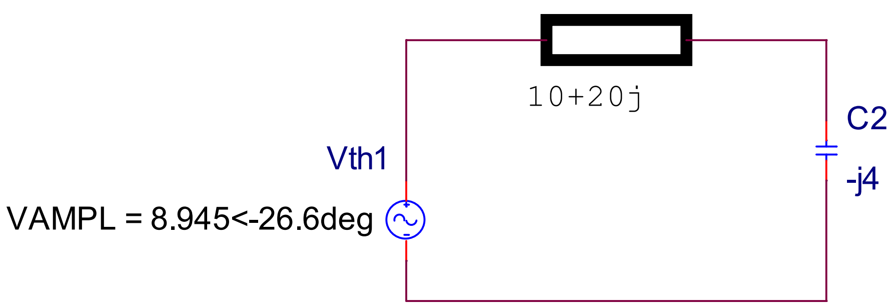
Must refer to secondary therefore

$N_1 := 5 \qquad 5 \cdot 1.789 = 8.945$

$N_1 \cdot V_{Th} = 8.945 < -26.6deg$

$5^2 \cdot R_{Th} = 10 + 20i$

Equivalent Circuit (you can add values in either polar or rectangular form)



3.2: Determine the current through C1. ***Put in time domain form.***

$$Z_{eq} := 10 + 20i - 4i = 10 + 16i$$

$$\sqrt{10^2 + 16^2} = 18.868$$

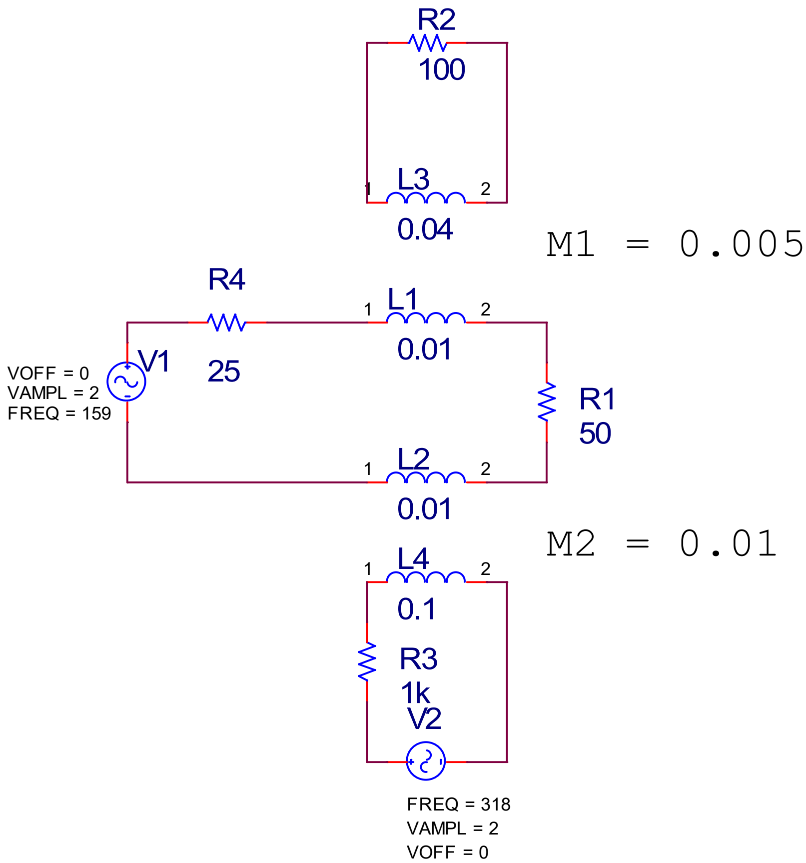
$$\operatorname{atan}\left(\frac{16}{10}\right) = 57.995 \cdot \text{deg}$$

$$I_{C1} = \frac{8.945 < -26.6\text{deg}}{18.868 < 58\text{deg}}$$

$$I_{C1} = 0.474 < -84.6$$

$$\boxed{I_{C1} = 0.474 \cdot \cos(t - 84.6\text{deg})}$$

Problem 4) Mutual Inductance



Find the current and voltage through R1 in the above couple circuits. There are two locations with inductive coupling and both sets have additive coupling. Additionally, there are two voltage sources with different excitation frequencies. Suggestion, use superposition in your analysis.

Use the T-model for mutual inductance. Note, one of the equivalent inductances is 0. This is equivalent to a short circuit.

$M_{41} := 0.005$

$M_{42} := 0.01$

$L_{43} := 0.04$

$L_{42} := 0.01$

$L_{41} := 0.01$

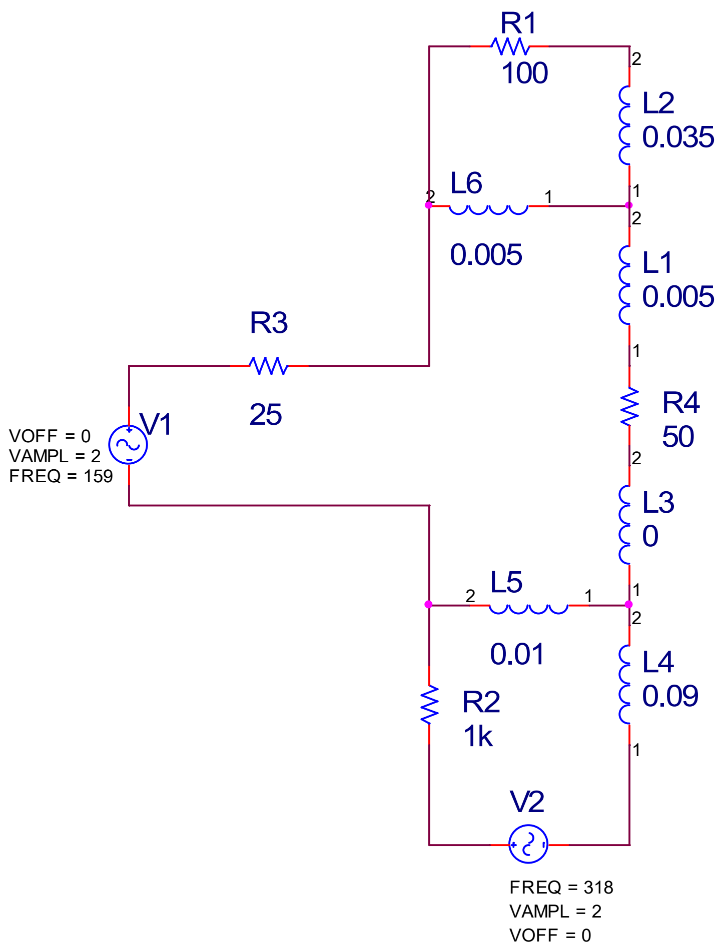
$L_{44} := 0.1$

$L_{43} - M_{41} = 0.035$

$L_{42} - M_{42} = 0$

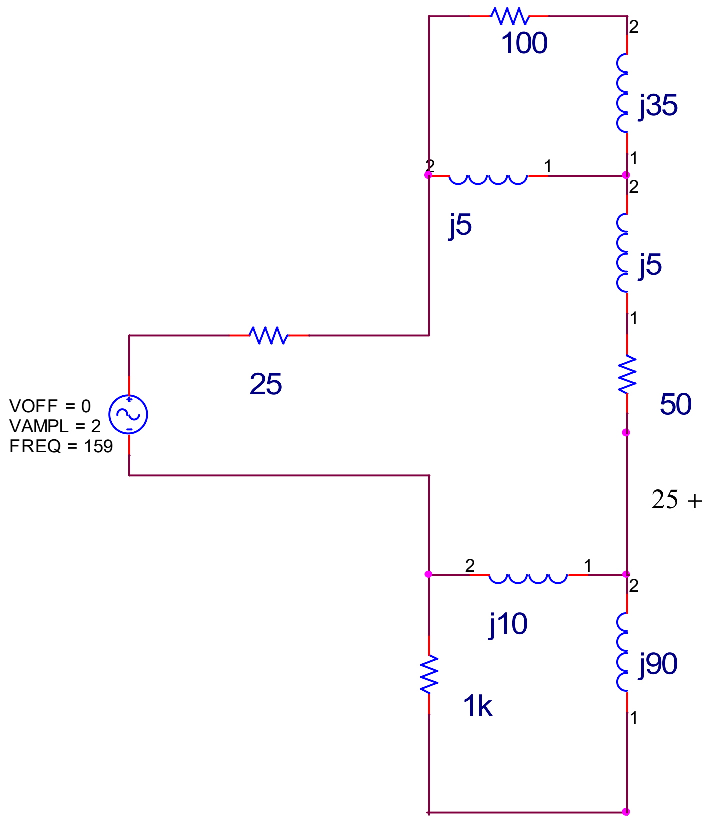
$L_{41} - M_{41} = 5 \times 10^{-3}$

$L_{44} - M_{42} = 0.09$



USE SUPERPOSITION!

Cosidering the 159 Hz source



Using current divider circuit analysis to find IR1

Top $\frac{(100+35j)\cdot 5j}{100+35j+5j} = 0.216+4.914i$

Bottom $\frac{(1000+90j)\cdot 10j}{1000+90j+10j} = 0.099+9.99i$

Add up impedances

$$25+0.216+4.914j+50+5j+0.099+9.99j = 75.315+19.904i$$

$$I_{R1} := \frac{2}{(75.315+19.904i)}$$

$$I_{R1} = 0.02482-0.00656i \quad \text{[A]}$$

$$\sqrt{0.02482^2+\left(-6.56\cdot 10^{-3}\right)^2} = 0.02567$$

$$\operatorname{atan}\left(\frac{-6.56\cdot 10^{-3}}{0.02482}\right) = -14.805\cdot \deg$$

$$V_{R1} := I_{R1}\cdot 50$$

$$V_{R1} = 1.241-0.328i \quad \text{[V]}$$ at 159 Hz

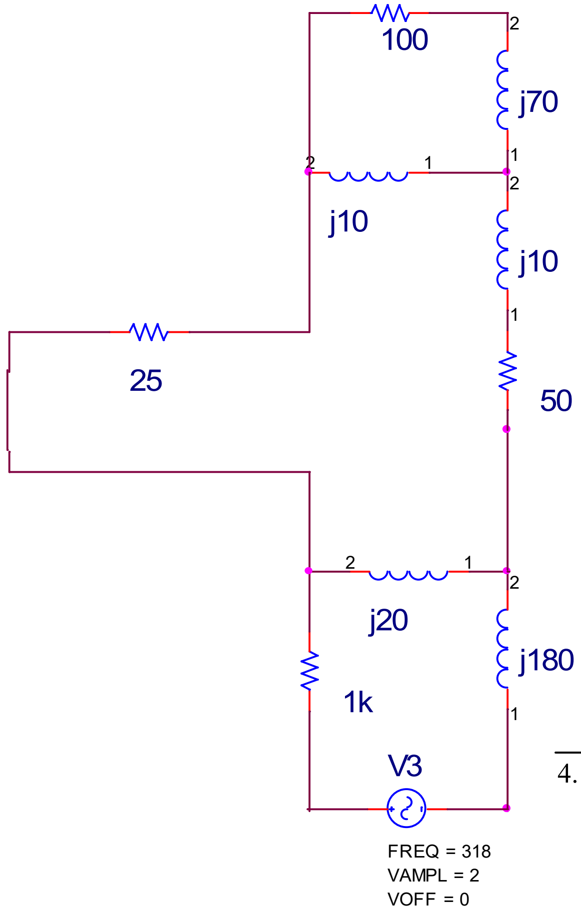
$$\sqrt{1.241^2+(-0.328)^2} = 1.284$$

VR1 in polar form for 159 Hz source

$$\operatorname{atan}\left(\frac{-6.56\cdot 10^{-3}}{0.025}\right) = -14.703\cdot \deg$$

Considering the 318 Hz Source

You can use current divider or maybe be mesh analysis...I show both below



Using current divider:

$$\text{Top}\quad \frac{(100+70j)\cdot 10j}{100+70j+10j} = 0.61 + 9.512i$$

$$0.61 + 9.512i + 10i + 50 + 25 = 75.61 + 19.512i$$

Add up impedances in series (furthest away from source)

$$\frac{(75.61+19.512i)\cdot 20j}{75.61+19.512i+20j} = 4.155 + 17.828i$$

$$\frac{2}{4.155+17.828i+180i+1000} = 1.917\times 10^{-3} - 3.777i\times 10^{-4}$$

To find IR1 in second loop away from source

$$\frac{20i}{(75.61+19.512i+20i)}\cdot\left(1.917\times 10^{-3} - 3.777i\times 10^{-4}\right) = 2.866\times 10^{-4} + 3.573i\times 10^{-4}$$ [A] at 318 Hz

OR: Using mesh analysis, YAAAY!

$$i_1 \cdot 100 + i_1 \cdot j70 + i_1 \cdot j10 - i_{2j10} = 0$$

$$i_1 \cdot (100 + j80) - i_2 \cdot j10 = 0$$

$$i_2 \cdot j10 - i_1 \cdot j10 + i_2 \cdot j10 + i_2 \cdot 50 + i_2 \cdot j20 - i_3 \cdot j20 + i_2 \cdot 25 = 0$$

$$-i_1 \cdot j10 + i_2 \cdot (j40 + 75) - i_3 \cdot j20 = 0$$

$$i_3 \cdot 1000 + i_3 \cdot j20 - i_2 \cdot j20 + i_3 \cdot j180 = -2$$

$$-i_2 \cdot j20 + i_3 \cdot (j200 + 1000) = -2$$

$$M_1 := \begin{bmatrix} (100 + 80i) & -10i & 0 \\ -10i & (75 + 40i) & -20i \\ 0 & -20i & 1000 + 200i \end{bmatrix}$$

$$C_M := \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \qquad M_1^{-1} C_M = \begin{pmatrix} -7.806 \times 10^{-6} + 3.491i \times 10^{-5} \\ 2.867 \times 10^{-4} + 3.574i \times 10^{-4} \\ 1.917 \times 10^{-3} - 3.777i \times 10^{-4} \end{pmatrix} \begin{matrix} IR1 \\ IR2 \\ IR3 \end{matrix}$$

$$I_{R1bmag} := \sqrt{\left(2.867 \cdot 10^{-4}\right)^2 + \left(3.574 \cdot 10^{-4}\right)^2} = 4.582 \times 10^{-4}$$

$$I_{R1b\phi} := \mathrm{atan}\left(\frac{3.574 \cdot 10^{-4}}{2.867 \cdot 10^{-4}}\right) = 51.264 \cdot \mathrm{deg}$$

$$V_{R1b} := \left(2.867 \times 10^{-4} + 3.574i \times 10^{-4}\right) \cdot 50 = 0.014335 + 0.01787i$$ [V]

$$V_{R1bmag} := \sqrt{(0.014335)^2 + (0.01787)^2} = 0.023$$

$$V_{R1b\phi} := \mathrm{atan}\left(\frac{0.01787}{0.014335}\right) = 51.264 \cdot \mathrm{deg}$$

$$I_{R1Total} = 0.0257 \cdot \cos(1000t - 14\mathrm{deg}) + 4.582 \cdot 10^{-4} \cdot \cos(2000t + 51.24\mathrm{deg})$$ [A]

$$V_{R1Total} = 1.28 \cdot \cos(1000t - 14\mathrm{deg}) + 0.023 \cdot \cos[(2000t + 51.24)\mathrm{deg}]$$ [V]