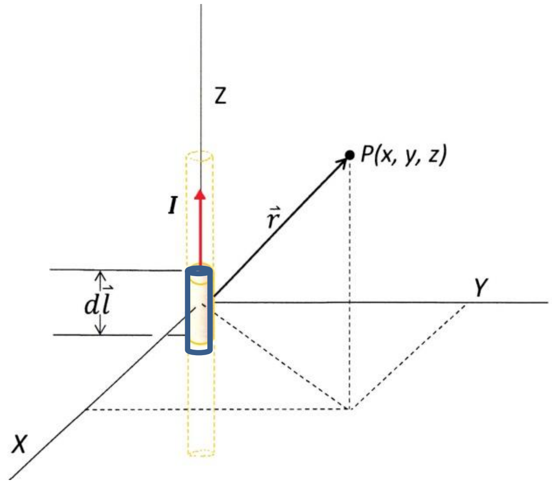
28A – Calculating the Field using Biot-Savart

Represented in the diagram to the right is an element of a conductor centered at the origin of a Cartesian coordinate system and pointing along the z axis. A current, ***I***, is passing along the length of this element dl of the conductor. The point *P = (x, y, z)*, is the position at which the increment of magnetic field $d\vec{B}$ due to this element of current is to be observed.



1) Write down the vector differential form of the Biot-Savart law from section 28.02 of the textbook. Describe what each variable (I, dl, r) in the relation means.

   $d\vec{B}$ = u_0/(4 pi) * (I dl x \vec r)/r^3

2) Write the vector $d\vec{l}$ in Cartesian coordinates using unit vectors $(\hat{\imath}, \hat{\jmath}, \hat{k})$. dxi + dyj + dzk

3) Write the vector $\vec{r}$ from the origin to the point *P = (x, y, z)*. \vec r=xi+yj+zk

4) Perform the cross product of $d\vec{l}$ with $\vec{r}$ using Cartesian unit vectors.

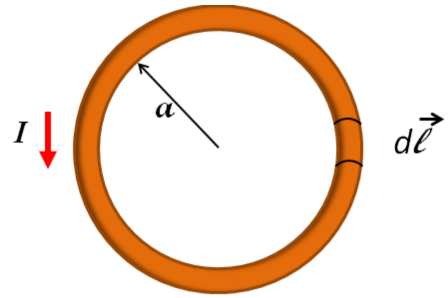
i	j	k
dx	dy	dz
x	y	z

i (dy z-dz y) + j (dz x-dx z) + k (dx y-dy x)

5) Now find $d\vec{B}$ produced at point *P = (x, y, z)* by the current carrying element *dl* centered at the origin.

u_0/(4 pi) * (I dl sin(\theta))/r^2

6) Integrating *dl* to find the field is usually difficult. A special case is the field at the center of a circular ring of current of radius *a*.



a) What is the direction of the magnetic field at the center of the circle for the element *dl* indicated by the arrow carrying current *I* in the widdershins direction as shown. (into paper, out of paper, up paper…)

out of the page

b) What is the magnitude of the field element dB due to the short wire element (the arrow) of length d*l* in terms of *I*, *dl*, and *a*?

dB=u/4pi * I dl/a^2

Note that the direction for dB is the same for every incremental length dl around the whole loop.

c) Integrate your expression above over the length of the wire around the loop. $\left(\vec{B} = \int d\vec{B}\right)$

B=u_0/4pi * I 2 pi/a=u_0*I/(2a)

(You should have shown that $|B| = \frac{\mu_0 I}{2a}$ at the center of the loop.)

d) Find in the textbook and write out the expression for the field as a function of distance along the axis of a circular current carrying wire. (Change the variable in the text to coincide with the variables in this activity.)

B=u_0/4pi * I dl /((a^2+x^2)^3/2)

e) What does the expression in part d reduce to at the center of the loop?

B=u_0 I / 2a

f) What does the expression in part d reduce to when the distance *x* from the loop is large enough that the radius *a* can be neglected in the denominator? (This is the field for a magnetic dipole.)

u_0 I a^2/(2x^3)

28B – Experiment: Measurement Magnetic Field on a Coil Axis

Objective: To observe how the magnitude of the magnetic field generated in a coil depends on distance along the axis of the coil.

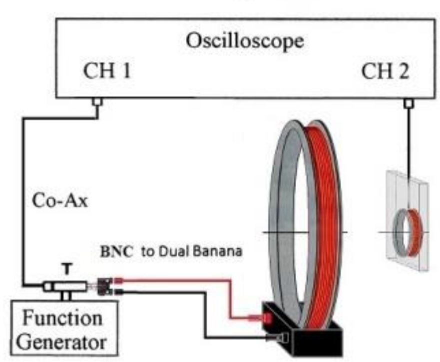
Equipment: Large 15-turn coil, small 2100-turn pick-up coil with RCA/BNC connector, Function Generator, one coaxial tee connector, one BNC cable, one male BNC to banana connector, one female BNC to banana connector, four banana wires (two long, two short), Oscilloscope, Digital Multimeter, Measuring stick.

The experimental measurement technique involves a concept, Faraday's Law, that will be addressed in a later module. Here are the essentials for now.

- The function generator drives sinusoidal current through the large coil and thereby generates a sinusoidally varying magnetic field.
- A sinusoidal EMF is generated across the small coil, which is proportional to the rate of change of the magnetic flux through the small coil. For a fixed frequency, the voltage amplitude produced in the small coil is proportional to the amplitude of the magnetic flux passing through the small coil.
- The oscilloscope is used to measure the sinusoidal voltage amplitude. (This ac technique has the advantage over dc techniques in that it does not pick-up the steady field due to the Earth.)

1) Use the DMM or a meter stick to measure and record the following:
   a) The radius a of the large coil. a = .075 m
   b) The resistance R of the large coil including its series resistor. R = 36.4 Ω
   c) The radius r of the small coil. r = .0125 m

Block Diagram of SETUP



- Wire the system as described below and as shown in the schematic to the right. A photographic image and detailed schematic can be found in the online LMS resources for this course. Instructions on setting up and using an oscilloscope are given in an appendix to this manual.
  - Plug the coaxial tee into the main output of the function generator and connect one side to channel 1 of the oscilloscope using a coaxial cable.
  - Plug the coax side of the banana-coax into the other side of the tee. Connect this to the large coil via the jacks, which have a ~40 ohm resistor (check it!) in series with the coil.
  - Set the frequency generator to produce a sinusoidal signal at a frequency of 20,000 Hz with the main amplitude dial set to about mid-range.

- Use the input of channel 1 for the trigger (auto mode). Set the oscilloscope to view several cycles of the signal on the screen by setting the time-base (to about 100 microseconds/division) and the volts/division for channel 1 (to about 2V/division). The current through coil 1 is thus: $I_1(t) = \frac{V_1}{R} \cos \omega t$.
- Connect the second coil to the channel 2 input of the oscilloscope and observe the amplitude of the EMF from coil 2 as you move coil 2 around.
- Record the amplitude of the voltage across coil 2 while coil 2 is centered and coaxial with coil 1.

2) Measure the ac voltage amplitude that the function generator is supplying across the large coil. Use the resistance you measured to calculate the ac current.

AC volts 5 V

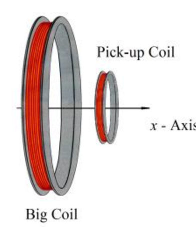
AC current .137 A

3) Estimate the strength of the magnetic field at the center of the large coil using the radius of the coil, the ac current, and the number of turns. (Write out the expression that you are using in the space below and then your value on the line.)

B=u_0I/2a=1.25*10^-6*.137/(2*.075)

1.14*10^-6 T

4. Hold the pick-up coil so that it is near the center of the big coil and in the same plane as the big coil as sketched. You should see an ac volts signal of between 0.3V and 1.5 V. Observe the ac voltage from the pick-up coil as you move it away from the big coil along the axis of the big coil.



Describe qualitatively in a sentence how the ac signal changes as you move the pick-up coil away from the center along the axis.

decreases in aplitude with distance

5. Hold the pick-up coil near the center and in the plane of the large coil again. Rotate (about the vertical axis) the small coil (pick-up coil) slowly through 180 degrees and describe in a sentence how the ac signal changes with rotation. As a comparison, when the two coils are parallel the angle is zero. (At what angle(s) do you observe the largest signal? At what angle(s) do you observe the smallest magnitude signal?)

Largest: 0 Smallest: 90

6. Hold the small coil parallel to the plane of the large coil and record the ac voltage amplitude as a function of the distance from the center of the large coil as you move it along the axis as sketched in the figure above.

Distance *x* (cm)	ac Volts (V)	Measured $V(x)/V(0)$	Theoretical $B(x)/B(0)$
0	2.5	1	1
5	1	.4	.57
10	.5	.2	.2
15	.2	.08	.1
20	.1	.04	.03
25	.05	.02	.01

7. Plot the ratio of V(x) to V(0) as a function of distance from the center of the coil on the grid below.
8. The theoretical prediction for the field on the axis is: $B(x) = \frac{N\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$. Write out what each variable in this relation means.

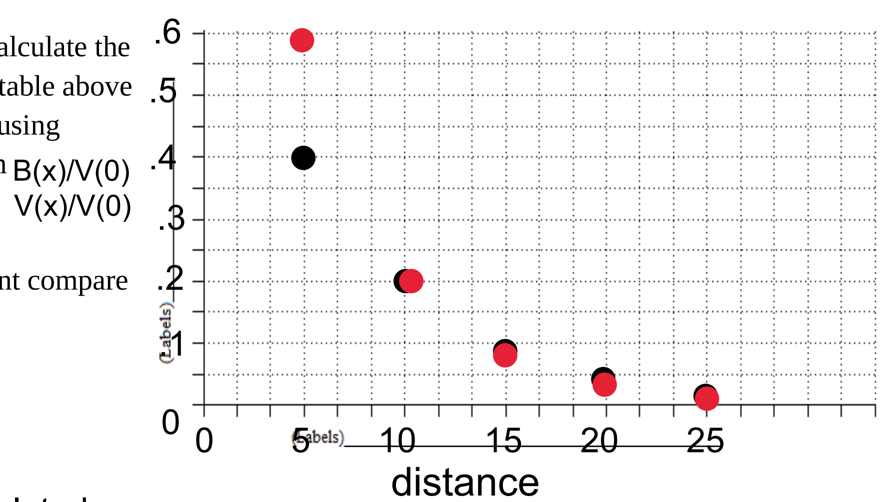
N Number of turns

I Current

a radius

x distance

9. Use the equation in (8) above to calculate the ratio of $B(x)/B(0)$. Add it to the table above and plot it on the grid to the right using markers to distinguish theory from experiment.



10. Discuss how theory and experiment compare to one another.

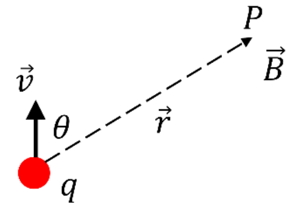
They're extremely closely correlated

 Name Hayden Fuller Section 6

28C – Making Magnetic Fields – Biot Savart Review

1) Write down the law of Biot and Savart for finding the magnetic field near a moving *charge*:

B=u_0/4pi * (qv x r)/r^2



2) A positive charge moves up the paper as shown on the right. What direction does the magnetic field point at P?
   A) Up B) Down C) Into the paper D) Out of the paper
   E) Along r Ans. C

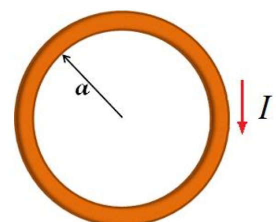
Explain: right hand rule. Velocity up, field to the right is into the paper

3) For a fixed distance *r*, for what angle θ is the magnetic field the largest?
   A) 0 B) 90° C) 45° D) None, it is the same at all angles. Ans, B

Explain: sin(90)=1=max(sin(x))

4) Write down the law of Biot and Savart for finding the magnetic field near a very short current carrying wire:

dB=u_0/4pi * IdLsin(\theta)/r^2



5) What is the direction of the magnetic field at the center of a coil of wire when a current passes clockwise around the coil? The coil is in the plane of the paper as shown to the right.
   A) Into paper B) Out of paper C) Left Down

Ans A

Explain: right hand rule.

6) Using the Biot-Savart expression for magnetic field at the center of a circular wire, find the magnetic field at the center of a half circle of radius *a* carrying a current of *I*.

B=u_0I2a

7) A current I = 1 A flows around a coil a = 0.1 m in radius and consisting of N = 20 loops of wire. The B field at the center of the coil is closest to:
   A) $5\mu_0$ T B) $10\mu_0$ T C) $50\mu_0$ T D) $100\mu_0$ T
   Ans. D