# Problem Set #1

Growth and Innovation

Michael A. Klein

Due 1/24/23, by 2:00PM

**Please submit your answers through LMS as a single pdf**

*Note - lower case letters represent per capita values.

1. This problem set asks you to analyze the basic Solow model using a particular functional form for the aggregate production function. Specifically, the production function has a "Cobb-Douglass" form:

$$Y = K^{\alpha} L^{1-\alpha}, \qquad 0 < \alpha < 1$$

There is no technological progress ($A = 1$ ), and labor grows at a constant exogenous rate $n > 0$ ($\gamma_L = n$) as in class. Assume a constant fraction $s > 0$ of income is saved/invested, and that capital depreciates at the constant rate of $\delta > 0$.

(a) Verify that the Cobb-Douglass form exhibits (i) constant returns to scale (CRS) in $K$, $L$ and (ii) positive and diminishing returns to each input

(b) Derive an expression for the rate of growth in capital per capita ($\gamma_k$)

(c) Solve for the steady state (equilibrium) level of capital per capita ($k^*$)

(d) Derive an expression for the rate of growth in output per capita ($\gamma_y$). Verify that $\gamma_y$ is decreasing in $k$.

(e) In words, describe how this finding relates to the notion of convergence

(f) Suppose that the economy experiences a permanent *decrease* in the savings rate $s_1 < s_0$. Find the new steady state $k$. Describe the behavior of economic growth along the transition path (tell an economics story about movement to the new equilibrium). Illustrate this in two sketched diagrams: 1) investment/depreciation in levels, 2) investment/depreciation in rates.

a)i)Y=K^a*L^{1-a}
multiply K by a constant C and multiply L by a constant D
(CK)^a*(DL)^{1-a}
=C^a*K^a*D^{1-a}*L^{1-a}
=C^a*D^{1-a}*(K^a*L^{1-a})
=C^a*D^{1-a}*Y
so as you increase K and L, Y increases at a constant scale
ii) 0<a<1
add a constant C to K and a constant D to L
(K+C)^a*(L+D)^{1-a}
so as you increase K and L, Y increases by a smaller and smaller ammount

b) divide by L to get per capita
y=... yeah I'm confused here

gamma k=s_k*y-(n+delta)k

c)

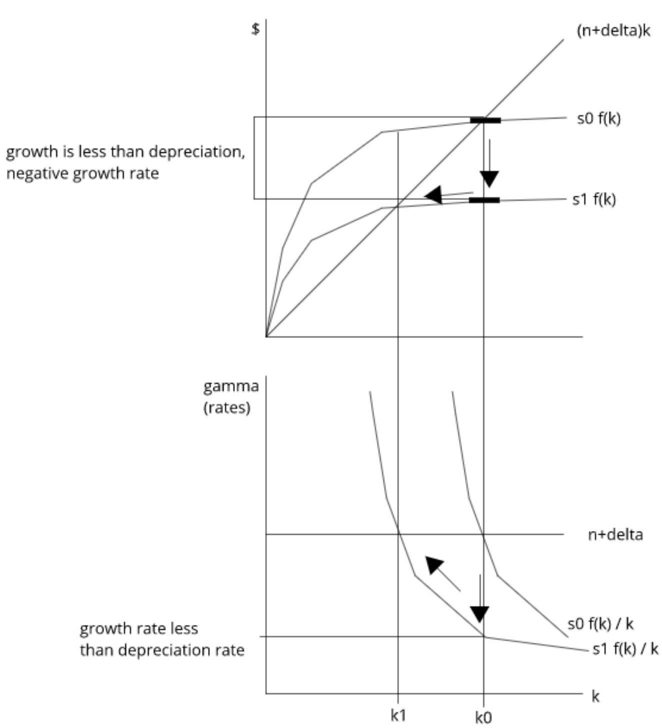
(k*)=(k*)(1-delta)+sf(k*)

d)

yeah I'm still really confused on these derivations

e)since the growth rate is decreasing with k, growth will slow as k increases towards an equilibrium point. At this point, the growth rate will be zero, and beyond it, it will be negative, leading to all starting points converging towards this equilibrium.

f) this lower s decreases investment. This new investment level is lower than depreciation, which leads to a negative growth rate, and the k value to decrease towards the new equilibrium.



2. Subsistence consumption: Consider the standard Solow model with positive population growth at rate $n > 0$, and constant technology $A = 1$. However, suppose that households must reach a threshold level of income prior to saving. That is, if income per capita is below a threshold $y \leq \bar{y}$, then $s = 0$ ($c = y$). Refer to this as the "poor" case. If per capita income is above this threshold $y > \bar{y}$, then agents save a constant fraction $s > 0$ of their *disposable* income $(y - \bar{y})$. Refer to this as the "rich" case. Assume the aggregate production function is given by

$$Y = K^{\alpha} L^{1-\alpha}, \qquad 0 < \alpha < 1$$

(a) Derive the law of motion for the per capita capital stock ($\dot{k}$) in both the rich and the poor case. ( Hint: Note that

$$\bar{y} = \bar{k}^{\alpha} \;\rightarrow\; \bar{k} = \bar{y}^{1/\alpha}$$

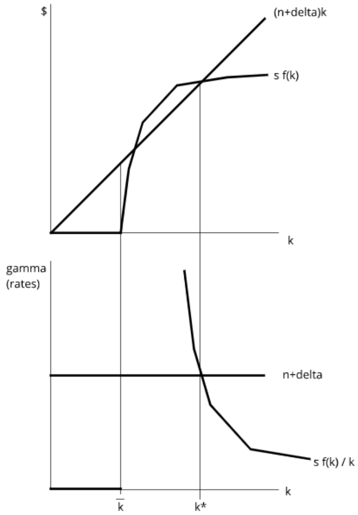
so the poor case includes $k \leq \bar{k}$ and the rich case includes $k > \bar{k}$.)

(b) Use these two equations to draw a diagram illustrating the investment curve per capita curve and the depreciation per capita curve (the levels diagram). Clearly indicate the level of $k$ where the switch from the rich to the poor case occurs.

(c) Can there be two steady states (equilibria) with non-zero levels of $k$? Explain using your diagram.

(d) How does this model change the convergence prediction associated with the standard Solow model?

a) in the poor case:
investment = sf(k) so when s=0, investment=0.
depreciation = (n+delta)k, which will remain positive.
since investment is always zero and depreciation is always positive, depreciation will always be (n+delta)k above investment, leading to a growth rate of d/dt k = -(n+delta), and a pit where being poor leadsto an equilibrium of 0.
d/dt k = -(n+delta)
x = s*disposable/k ; x = s*(k-kbar)/k
investment=s (k-kbar)/k*f(k) ; depreciation = (n+delta)k

b) 


c) yes, ish. There's a steady state where investment grows past, and at one point equals depreciation, but it's an unstable equilibrium since a small growth will lead to a positive growth rate and a small negative growth will lead to a negative growth rate, so it does not converge to this equilibrium.

d) this new model makes poor economies now converge to k=0 rather than grow towards the regular equilibrium, and it introduces a new equilibrium point that economies do not converge to just past kbar where investment surpasses depreciation.