Homework 8

Released: April 12th

Due: April 23rd

1. Electromagnetic Power Density

A wave traveling in a non-magnetic medium with $\epsilon_r = 16$ has an electric field:

$$\vec{E} = [2\cos(\pi 10^6 t + kx)\,\hat{y} - 3\cos(\pi 10^6 t + kx)\,\hat{z}]\ \text{[V/m]}$$

a) Determine the propagation direction of the wave.

b) Determine the magnetic field $\vec{H}$.

c) Calculate the average power density of the wave.

d) If we are dealing with a lossy medium, $\vec{E}$ and $\vec{H}$ acquire an extra exponential term $e^{-\alpha z}$. What additional term does the Poynting vector $\vec{S}$ then acquire in a lossy medium?

2. Electromagnetic Waves at Material Interfaces (Normal Incidence)

The light from an LED originates from holes and electrons recombining in a semiconductor material and emitting a photon, which then passes through another dielectric material before it is emitted into air. Consider the following system of material interfaces:

- for $z < 0$ the material is GaN with $\epsilon_{r,GaN} = 5.8$
- for $0 < z < d$, the material is SiO_2 with $\epsilon_{r,SiO2} = 3.9$
- for $d < z < \infty$, the material is air

An electromagnetic wave with $\tilde{E}(r) = 2e^{-jk_{GaN}z}\hat{x}$ [V/m] and a wavelength in GaN equal to 450nm is normally incident on the GaN/SiO_2 interface.

a) Draw a diagram of this system of materials, including axes, material properties and the direction of travel of the wave.

b) What is the frequency f of this electromagnetic wave? Also, calculate k in each material. Is the wavelength different in each material?

c) Calculate the reflection coefficient Γ and transmission coefficient τ at the GaN/SiO_2 interface and write the phasor expression for wave transmitted from the GaN into the SiO_2 $\widetilde{\boldsymbol{E}}_{T1}$.

d) Calculate the reflection coefficient Γ and transmission coefficient τ at the SiO_2/air interface and write the phasor expression for the portion of the incident wave $\widetilde{\boldsymbol{E}}_{T1}$ that is transmitted from the SiO_2 into air $\widetilde{\boldsymbol{E}}_{T2}$.

e) Is more power transmitted into air in this case than would be if there were only a GaN/air interface (i.e. no layer of SiO_2 in between GaN and air)? Ignore transmitted wave components from multiple reflections between the interfaces.

3. Electromagnetic Waves at Material Interfaces (Oblique Incidence – Snell's Law)

A plane wave in air has an electric field defined by $\widetilde{\boldsymbol{E}}^i = (3\hat{x} + 5\hat{y} - 4\hat{z})e^{-j(4x+3z)}$ [V/m] and is incident upon the planar surface of a dielectric material with $\epsilon_r = 4$ that occupies the half-space where $z \geq 0$. Determine:

a. The angle of incidence (θ_i), angle of refraction (θ_t), and the frequency of the wave.

b. The reflection and transmission coefficients Γ and τ.

c. The field $\widetilde{\boldsymbol{E}}^r$ of the reflected wave, and the field $\widetilde{\boldsymbol{E}}^t$ of the wave transmitted into the dielectric medium.

d. The average power density carried by the wave into the dielectric medium.

4. Electromagnetic Waves at Material Interfaces (Oblique Incidence – Brewster Angle)

A parallel-polarized plane wave is incident from air onto a (nonmagnetic) dielectric medium with $\epsilon_r = 9$.

a. What is the Brewster angle for this interface and polarization?

b. If the plane wave is incident on the interface at the Brewster angle, what is the refraction angle?

c. If the plane wave were perpendicular-polarized instead, what would the Brewster angle be?

d. If the plane wave were unpolarized (as light typically is) and incident on the interface at the Brewster angle, what would the polarization of the transmitted wave be?

5. Terms, Notation, Symbols, etc.

On the left-hand side of the table below, write the name of the Greek letter listed in the symbol column and the units that correspond to the symbol. In the right-hand side of the table, write the correct symbol next to the name of the quantity it represents.

Symbol	Name of Greek Letter	Units	Symbol for Quantity	Name of Quantity
α	***alpha***	***Np/m***	$\boldsymbol{\alpha}$	Attenuation Constant
β or k				Capacitance
γ				Characteristic Impedance
Γ				Conductivity
ϵ_0				Displacement Current Density
$\vec{D}$	N/A			Electric Field Intensity
$\vec{E}$	N/A			Electric Scalar Potential
$\vec{H}$	N/A			Electric Flux Density (Displacement Field)
$\vec{J}_s$	N/A			Index of Refraction
$\vec{J}_d$	N/A			Inductance
ρ_v				Intrinsic Impedance
λ				Magnetic Field Intensity
Λ				Magnetic Flux Density
n	N/A			Magnetic Flux Linkage
σ				Magnetic Vector Potential
C	N/A			Permeability of Free Space
L	N/A			Permittivity of Free Space
$\mathcal{R}$	N/A			Phase Constant or Wave Number
$\vec{S}_{av}$	N/A			Poynting Vector (avg. power density)
τ				Propagation Constant
μ_0				Reflection Coefficient
V	N/A			Reluctance
η				Surface Current Density
$\vec{B}$	N/A			Transmission Coefficient
$\vec{A}$	N/A			Volume Charge Density
Z_0	N/A			Wavelength