1) Laplace transforms/Transfer functions

<u>Use Laplace transform tables!!!!</u>

1.1: Find the Laplace transform of

$f(t) = \left(\cos(2t) + e^{-4t}\right)\cdot u(t)$ (simplify into one ratio)

(s(s+4)+(s^2+4))/((s+4)(s^2+4))

(2s^2+4s+4)/(s^3+4s^2+4s+16)

1.2: Find the poles and zeros of the following functions. Indicate any repearted poles and complex conjugate poles. Expand the transforms using partial fraction expansion.

1. $F(s) = \frac{20}{(s+3)\cdot\left(s^2+8s+25\right)}$

zeroes: None
poles: -3, -4+-3j (complex conjugates)

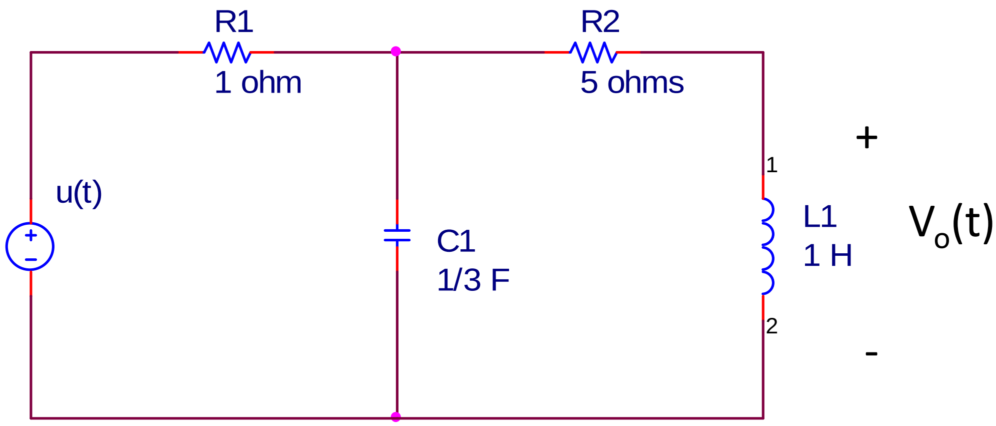
2. s^2+9s+6

$F(s) = \frac{2s^2+18s+12}{s^4+9\cdot s^3+34\cdot s^2+90\cdot s+100}$ =A1/(S+2)+A2/(s+5)+A3/(s+1-3j)+A3*/(s+1+3j)

(x+2)(x+5)(x^2+2x+10)

zeroes: -.725, -8.27
poles: -2, -5, -1+-3j (complex conjugates)

2) Circuits and Differential Equations



2.1: Draw the s-domain equivalent circuit. Assume all intial conditions are zero and the source is an arbitrary source.

1ohm 5ohm

1/s i1 1/sC=3/s ohm i2 sL=s ohm

i1 and i2 ccw
positive is up and right, current flow is down and left

2.2 Using impedances, determine the expression for Vo(t). Consider using mesh analysis then make one ratio.

-V(V1)-V(R1)+V(C)=0
1/s+i1+(i1-i2)*3/s=0
1/s+i1+i1*3/s+i2*-3/s=0
1/s+i1(1+3/s)+i2(-3/s)=0
i1(1+3/s)+i2(-3/s)=-1/s
i2(1/3)(s^2+5s+3)(1+3/s)+i2(-3/s)=-1/s
i2(s^2/3+5s/3+1+s+5+3/s-3/s)=-1/s
i2(s^2/3+8s/3+6)=-1/s
i2=-1/(s(s^2/3+8s/3+6))

-V(C)-V(R2)+V(L)=0
-(i1-i2)*3/s+i2*5+i2*s=0
-i1*3/s+i2*3/s+i2*5+i2*s=0
i1(-3/s)+i2(s+5+3/s)=0
i2(s+5+3/s)=i1(3/s)
i2(1/3)(s^2+5s+3)=i1

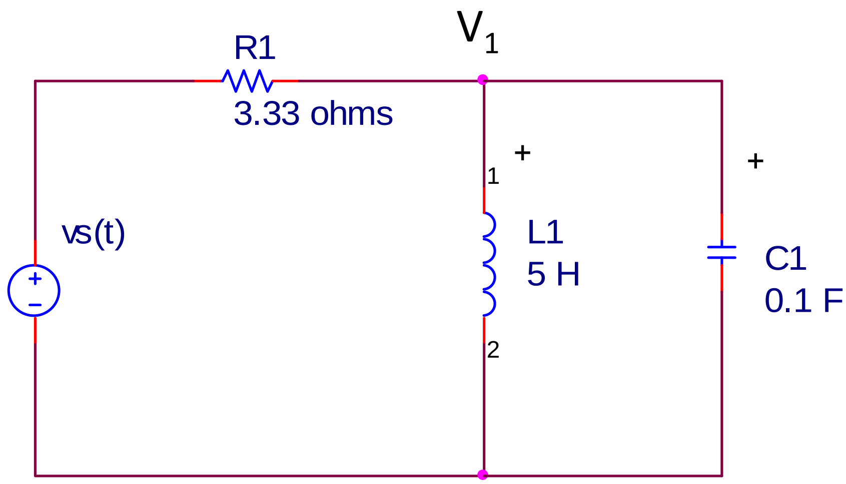
V0(s)=s*i2
V0(s)=-1/(s^2/3+8s/3+6)
V0(s)=-3/(s^2+8s+18)

2.3 Find Vo(t) which is the VL(t) for t>0 using Vs=1 u(t).

s^2+8s+18=0
s=-4+-jsqrt2

A1/(s+4+jsqrt2)+A1*/(s+4-jsqrt2)=-3/((s+4-jsqrt2)(s+4+jsqrt2))
A1=3/(2jsqrt2)
A1*=-3/(2jsqrt2)
3/(2jsqrt2)/(s+4+jsqrt2)+-3/(2jsqrt2)/(s+4-jsqrt2)
3/(2jsqrt2)*e^{(-4-jsqrt2)t}-3/(2jsqrt2)*e^{(-4+jsqrt2)t}

3) RLC and initial conditions



$v_S(t) = 10\,u(t)$

AND assume that -1A flows through the inductor and +5V is across the capacitor at t=0....i.e. vc(0)=5 and iL(0)= -1

3.1: Draw the s-domain equivalent with initial conditions.

10/3 ohm

5s ohm

10/s ohm

+
10/s V
-

-
-5V
+

+
5V
-

+
5/s V
-

3.2: Find the value of the voltage across the capacitor, vc(t), using nodal analysis (at node V1) and laplace.

$IR+IL+IC=0$

$VR/RR+VL/RL+VC/RC=0$

$(V1-10/s)/(10/3)+(V1-5)/(5s)+(V1-5/s)/(10/s)=0$

$(V1-10/s)(3/10)+(V1-5)(1/5s)+(V1-5/s)(s/10)=0$

$(3/10*V1-3/s)+(1/5s*V1-1/s)+(s/10*V1-.5)=0$

$V1(3/10+1/5s+s/10)+(-3/s-1/s-.5)=0$

$V1(s/10+3/10+1/5s)=(.5+4/s)$

$V1=(5s+40)/(s^2+3s+2)$

$V1=5*(s+8)/((s+1)(s+2))$

$V1/5=A/(s+1)+B/(s+2)$

$A(s+2)+B(s+1)=(s+8)$

$As+A2+Bs+B=s+8$

$s(A+B)+(2A+B)=s+8$

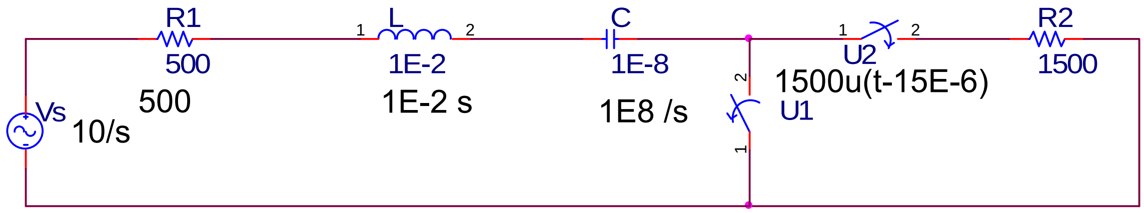
$A+B=1$, $A=1-B$, $2A+B=8$, $2(1-B)+B=8$, $2-B=8$, $-6=B$, $A=7$

$V1/5=7/(s+1)-6(s+2)$

$V1=35/(s+1)-30/(s+2)$

$V1(t)=35*e^{-1*t}-30*e^{-2*t}*u(t)$

4) RLC parallel circuits



In the above circuit, the source turns on at t=0 with a voltage of 10V. Additionally, switch U1 is closed and switch U2 is open. At 15E-6 s switch U1 opens and switch U2 closes. The source also turns off at 15E-6 s.

4.1: Use Laplace analysis to determine the voltage across the capacitor as a function of time for 0<t<15E-6 (s)

4.2: Use Laplace analysis to determine the voltage across the capacitor as a function of time for t>15E-6 s

4.1) VC(s)=Vs*ZC/(ZR1+ZL+ZC)
VC(s)=Vs*1/(sC)/(R1+sL+1/(sC))
VC(s)=Vs*1/(CL)/(sR1/L+s^2+1/LC)
VC(s)=10/s*1/(1E8*1E-2) / (s^2+s*(500/1E-2)+1/(1E8*1E-2)
VC(s)=10/s*1E-6 / (s^2+s5E4+1E-6)

VC(s)=1E-5 / s(s^2+s5E4+1E10)
VC(s)=1E-5 / s(s+25E3+96824j)(s+25E3-96824j)
VC(s)=A0/s+A1/(s+25E3+96824j)+A1*/(s+25E3-96824j)
A0=1E-5 /(s+25E3+96824j)(s+25E3-96824j)|s=p0=0
A0=1E-5 /(25E3+96824j)(25E3-96824j)
A0=1E-5/1E10=1E-15
A1=1E-5 /s(s+25E3-96824j)|s=p1=-25E3-96824j
A1=1E-5 /(-25E3-96824j)(-96824j-96824j)
A1=-5.333E-16+4.84E8

4.2) VC(s)=Vs*ZC/(ZR1+ZL+ZC+ZR2)
VC(s)=Vs*1/(sC)/(R1+sL+1/(sC)+R2)
VC(s)=Vs*1/(CL)/(sR1/L+s^2+1/LC+R2s/L)
VC(s)=Vs*1/(CL)/(s^2+s(R1+R2)/L+1/LC)
VC(s)=10/s*1/(1E8*1E-2) / (s^2+s*(2000/1E-2)+1/(1E8*1E-2)
VC(s)=10/s*1E-6 / (s^2+s*(2E5)+1/(1E6)