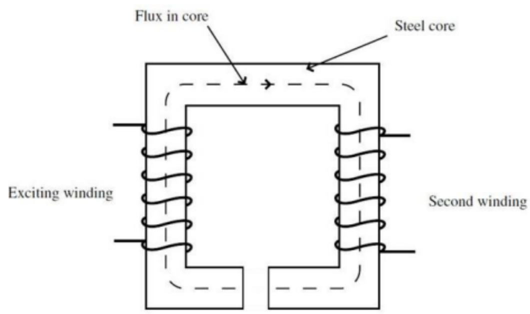
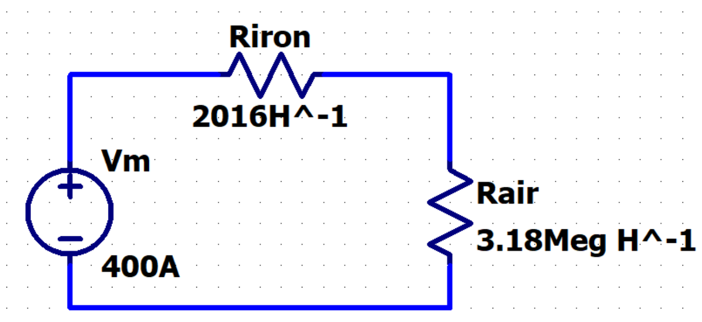
Hayden Fuller
Fields and Waves HW7

1. Transformer on an Iron Core with an Air Gap [12 Points]
   The transformer below consists of two coils on an iron core with an air gap. The iron core portion has a perimeter of 475cm and the air gap is 25cm wide, giving a total perimeter through the iron core region and air gap of 500cm. Additionally, the iron core has a square cross-sectional profile with side lengths of 25cm. The permeability of the iron core region is 3000μ0 and the permeability of the air gap region is μ0. The exciting winding has 1000 turns and the second winding has 3000 turns.



   a. Draw a magnetic circuit to represent the structure when the current through the coil is 400mA DC and calculate and label the following:



      i. the MMF

mmf=N*I=1000*400mA=400A

      ii. the reluctance of the iron core and air gap region

R=l/(u*A)
Riron=0.475/(3000 u0 * .25^2)=2016 H^-1
Rair=0.25/(u0 * .25^2)=3.18x10^6 H^-1

      iii. the total magnetic flux through the core. (For this part of the problem, ignore the second winding.)

phi=Vm/(Riron+Rair)=400A/(2016+3.18x10^6 H^-1)=1.26x10^-4Wb

   b. Calculate the magnitude of the H field in the air gap

H=B/u0=phim/u0A=1.26xx10^-4/(4pix10^-9 H/m * .25m^2)=1604 A/m

   c. Calculate the magnetic force felt by the two pieces of the core on either side of the air gap. Is this an attractive or repulsive force?

Fm=Bair^2/2u0=25.9N
Attractive, F=-\nabla(B^2/2u0)

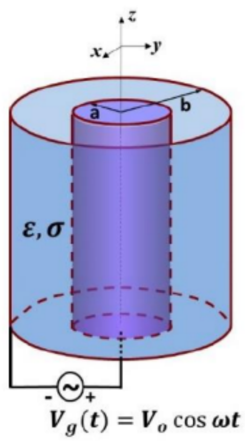
d. Suppose that the 400mA DC current is replaced with a 400mA 100Hz AC current. What is the resulting emf across the secondary coil?

emf=-d/dt (N2 phi(t))=-3000*1.26x10^-4 d/dt sin(200pi t)=-237.5 cos(200pi t) V

e. Is it preferable to use a hard or a soft ferromagnetic material for the core of this transformer? Why?

Soft, less losses to hysteresis

2. Displacement Current and the Quasi-static Approximation [16 Points]
For a coax-cable capacitor, the volume between the cylindrical copper conductors is filled with a lossy dielectric with permittivity εr and conductivity σ. The radius of the inner conductor is a and the effective inner radius for the outer conductor is b. The cable length $\boldsymbol{l}$ is much shorter than the wavelength, but $l \gg b$. The voltage applied across the coax-cable capacitor is $V(t)$ = V0cos (ωt) V.



a. Determine the displacement field $\vec{D}$ between the conductors, the displacement current Id and the conduction current Ic passing through the capacitor. What is the phase angle between Ic and Id? Which current leads (or which one lags)? Hint: you can use either time domain or phasor domain; you need to integrate the current passing through a cylindrical surface.

D=epsilon E
\int\int D ds=Qinner
qE l2pir=Q
E=Q/(l2pir e)
V(r)-V(b)=V(r)=-\int_b^r Q/(l2pir e) dr = - Q/(l2pi e) ln(r/b)
V(a)=V0=Q/(l2pi e) ln(a/b)
Winner=-l2pi e V0 cos(wt) /ln(a/b)
D=eE=e V0 cos(wt)/(r ln(a/b))
Id=d/dt \int\int D ds=j l2pi e w V0 sin(wt)/ln(a/b)
Ic-\int\int J ds=s \int_0^l \int_0^2pi V0 cos(wt)/(r ln(a/b)) dphi dz = l2pi s V0 cos(wt)/ln(a/b)

b. Evaluate Id, Ic and the ratio of their amplitudes when εr= 27, σ = 2 × 10−8 S/m, a = 0.45 mm, b = 1.57 mm, l = 100 m, V0 = 10V and f = 1MHz.

Id=l2pi e w V0 sin(wt)/ln(a/b)=7.54cos(2pi Meg t +pi/2) A
Ic=l2pi s cos(wt)/ln(a/b)=100 cos(wt) uA
Id/Ic=7.54A/100uA=75.4k, displacement dominates

c. Using your answer from part b, determine at which frequency the amplitudes of Id and Ic are equal.

Id=Ic
l2pi e w V0/ln(a/b)=l2pi s V0/ln(a/b)

ew=s
f=s/2pie=13.3Hz

d. Using your knowledge from circuit theory, calculate Id and Ic, then compare them with your results from part a. Hint: you will need the expressions for capacitance and conductance per unit length of a coaxial cable to calculate these quantities

Id=C dV/dt=l2pi e w V0 sin(wt)/ln(b/a)

3. Plane Wave in a Lossy Medium [13 Points]
   Today's computer microprocessors (and many other electronic and photonic devices) are built on a silicon (Si) substrate. For pure Si, $\varepsilon_r \approx 12$. Assume the Si substrate resistivity is $\rho = 10\ \Omega \cdot \text{cm}$.
   a. Determine the frequency range in which the Si substrate can be treated as a good insulator (dielectric), and the frequency range in which the Si substrate can be treated as a good conductor.

p=10 ohm cm * 1m/100cm=0.1 ohm m
Insulator: e''/e'=s/we'<<0.1
f >> s/2pi e' = 1.5 THz
Insulator: e''/e'>>100
f<<150GHz

   b. If a plane wave is traveling in the Si substrate at 100 MHz, find the attenuation constant and phase constant (α and β), wavelength, and intrinsic impedance (η).

Conductor
a=sqrt(pi f u s)=20pi
B=a=20pi rad/m
lambda=2pi/B=0.1m
eta=(1+j)a/s=2\sqrt(2) pi e^jpi/4 ohms

   c. Using your results from b, if the wave is traveling in the y-direction with an E-field amplitude of 10 V/m measured at y = 0, find E and H for the wave in the phasor domain. $\vec{E} = E\hat{x}$

E=10V/m e^-20piy e^-j20piy \hat x
H=kE/eta=5/(pi\sqrt(2)) e^-20piy e^-j(20piy-pi/4) \hat z

4. Wave Polarization [12 Points]
   Determine the polarization of the following waves (i.e., linear, circular or elliptical) and their propagation direction. If a wave is linearly polarized, also determine the inclination angle. For nonlinear polarization, determine the rotation direction (LH or RH). Draw a polarization diagram for each case with a few data points to show your work
   a. $\tilde{E}(z)$ = $(3\hat{x} - j3\hat{y})e\ j25\pi z$ [V/m]

-z
Ex -pi/2 of Ey
Circular polarization
Left hand circular

   b. $\tilde{E}(z,t)$ = 2 cos($106\pi t$ − 0.5z + 45°) $\hat{x}$ + sin($106\pi t$ − 0.5z − 45°) $\hat{y}$ [V/m]

+z
Ex pi of Ey
Linear polarization
arctan(-1/2)=-26.6deg
-26.6deg inclination linear

   c. $\tilde{E}(z)$ = $(3e\ j\ \pi\ 6\hat{x} - 3e\ j\ \pi\ 3\hat{y})\ e\ {-j3\pi z}$ [V/m]

+z
Ex 7/6pi of Ey
Elliptical polarization
Right hand eliptical