Fields and Waves I

Lecture 23

EM Waves at Normal Incidence

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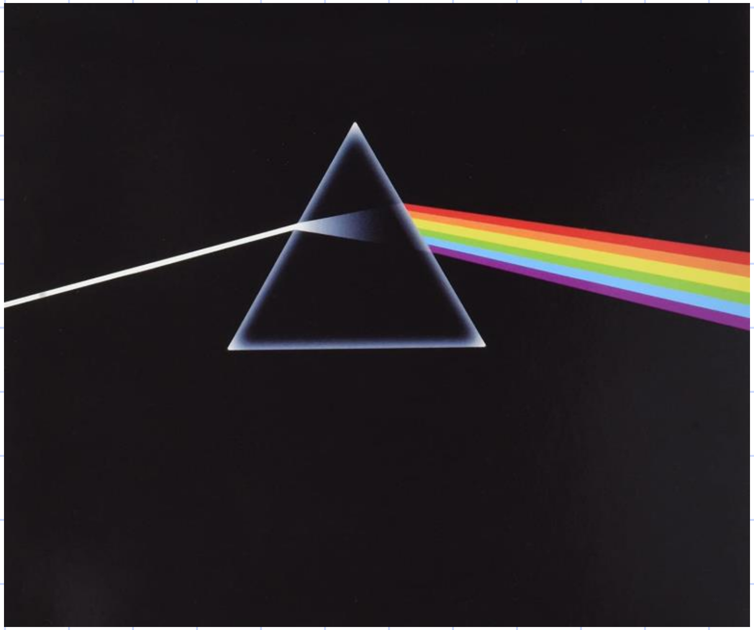
Materials from other sources are referenced where they are used.
Those listed as Ulaby are figures from Ulaby’s textbook.

Course Admin

- Oral exams this week and next week
- Homework 8 now out (due Apr 23rd)
- Unit 4 review material now online
- Final Exam - 11:30am-2:30pm Greene 120

Agenda

- Review
- EM Waves and Boundaries
- Multiple Boundaries
- Applications



EM Waves and Boundaries

- What are the three types of wave polarization? And what quantities do we need to specify the wave for each one?

EM Waves and Boundaries

Suppose we have:

$E_x = 4\cos(\omega t)$
$E_y = 5\cos(\omega t + 30)$

What kind of polarization does this represent?

Lissajous figures

Wave Polarization

Elliptical Polarization

$$\tan 2\gamma = (\tan 2\psi_0)\cos\delta \quad (-\pi/2 \le \gamma \le \pi/2), \qquad (7.59a)$$

$$\sin 2\chi = (\sin 2\psi_0)\sin\delta \quad (-\pi/4 \le \chi \le \pi/4), \qquad (7.59b)$$

where ψ_0 is an ***auxiliary angle*** defined by

$$\tan\psi_0 = \frac{a_y}{a_x} \qquad \left(0 \le \psi_0 \le \frac{\pi}{2}\right). \qquad (7.60)$$

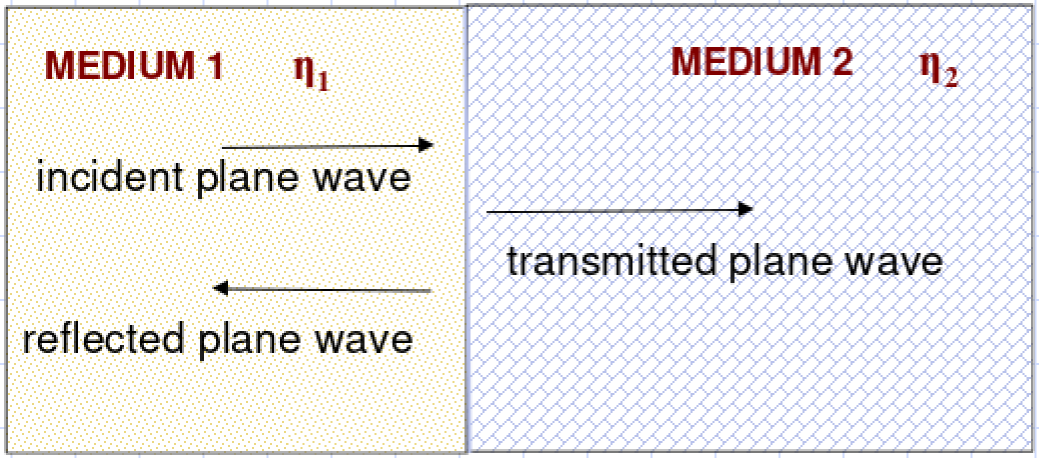
Ulaby pg. 329

EM Waves and Boundaries

Now we will consider the mathematics of EM waves hitting material boundaries at normal incidence.

- This means that there is no angle between the Poynting vector of the wave and the vector defining the boundary interface surface. The wave is hitting the boundary "head-on."
- This means that the Poynting vectors (incident, transmitted, reflected) are all in one dimension.
- This simplified the math and makes the treatment of these cases similar to a transmission line.

EM Waves and Boundaries



- wave is normally incident on an infinite interface separating two different media
- impedance discontinuity
- similar to the transmission lines
- EM waves represented with rays or wavefronts

EM Waves and Boundaries

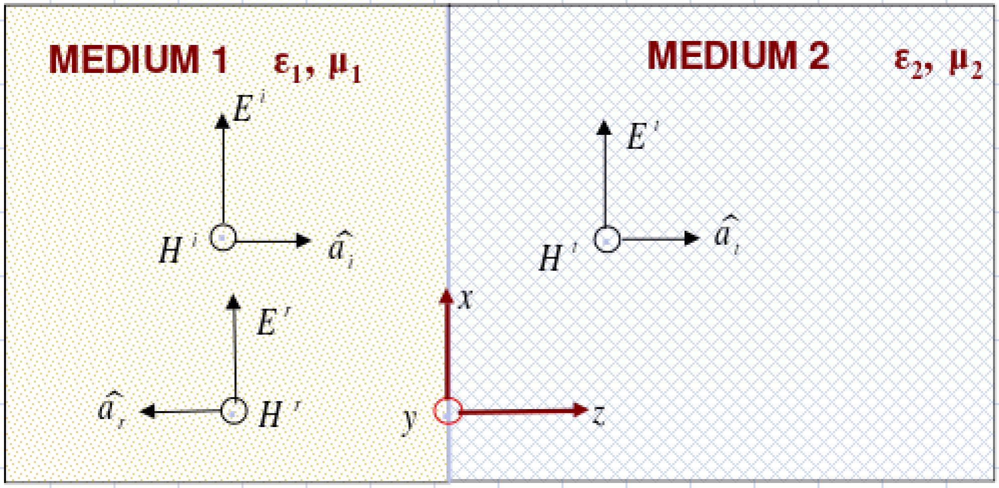
Solving EM wave reflection/transmission problems has some similarity to solving T-line reflection and transmission problems.

- Write down the complete electric and magnetic field expressions (including vector directions) in the area where the fields are known
- Calculate the reflection / transmission properties of the interface
- Calculate the previously unknown electric and magnetic fields

EM Waves and Boundaries

Lossless Media

- two lossless, homogenous, dielectric media



z=0

k (or wavenumber) is often used for EM waves but it is functionally the same as β (phase constant)

Incident wave $\tilde{E}^i(z) = \hat{a}_x E_0^i e^{-jk_1 z}$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

EM Waves and Boundaries

Lossless Media

transmitted wave $\tilde{E}^{t}(z) = \hat{a}_x E_0^t e^{-jk_2 z}$ $\qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$

$$\tilde{H}^{t}(z) = \hat{a}_z \times \frac{\tilde{E}^{t}(z)}{\eta_2} = \hat{a}_y \frac{E_0^t}{\eta_2} e^{-jk_2 z} \qquad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

reflected wave $\tilde{E}^{r}(z) = \hat{a}_x E_0^r e^{jk_1 z}$ $\qquad k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$

$$\tilde{H}^{t}(z) = (-\hat{a}_z) \times \frac{\tilde{E}^{r}(z)}{\eta_1} = -\hat{a}_y \frac{E_0^r}{\eta_1} e^{jk_1 z} \qquad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}$$

EM Waves and Boundaries

- To write boundary conditions, we note that if the EM wave hits a boundary head-on (i.e. not at an angle), it has only tangential field components to the boundary.
- For a conducting boundary, the field must be zero inside the conductor.
- For a dielectric boundary, both the electric and magnetic fields must be continuous
- In both cases, incident + reflected field on one side must equal transmitted field on the other.

EM Waves and Boundaries

Electric and Magnetic Boundary Conditions

Arguing from analogy with Electric Fields

$$\vec{D} = \epsilon \vec{E} \qquad \vec{B} = \mu \vec{H}$$

$$\oint \vec{D} \cdot \vec{ds} = Q_{encl} \Rightarrow D_{n1} - D_{n2} = \rho_s$$

$$\oint \vec{B} \cdot \vec{ds} = 0 \Rightarrow B_{n1} - B_{n2} = 0$$

$$\oint \vec{E} \cdot \vec{dl} = 0 \Rightarrow E_{t1} - E_{t2} = 0$$

$$\oint \vec{H} \cdot \vec{dl} = I_{inc} \Rightarrow H_{t1} - H_{t2} = J_s$$

EM Waves and Boundaries

Boundary Conditions

Tangential component of the electric field should be continuous across the boundary

Tangential component of magnetic field should be continuous (no current source)

Note that inc + ref = trans

not inc = ref + trans

At the boundary (z=0):

$$\tilde{E}_1(0) = \tilde{E}_2(0) \quad \text{or} \quad E_0^i + E_0^r = E_0^t$$

(comes from boundary conditions)

$$\tilde{H}_1(0) = \tilde{H}_2(0) \quad \text{or} \quad \frac{E_0^i}{\eta_1} + \frac{E_0^r}{\eta_1} = \frac{E_0^t}{\eta_2}$$

EM Waves and Boundaries

Example 1

A 10 GHz plane wave has an electric field magnitude of 100 V/m and propagates in the the $\mathbf{a}_z$ direction through a perfect dielectric with $\varepsilon_r = 9$. **E** is in the $\mathbf{a}_x$ direction.

a. What are the incident **E** and **H** phasors?

b. At z = 0, the wave strikes a perfect conductor. What are the reflected **E** and **H** phasors?

c. Use the boundary conditions to find the surface current density in the conductor.

d. Draw the standing wave pattern for **E** and **H** (include numbers for amplitude and position).

f. Calculate the total **E** and **H**. (phasor & time domain form).

EM Waves and Boundaries

Example 1

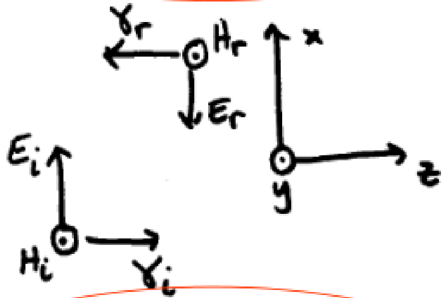
a. $\vec{E}_i = E_m e^{-\gamma_i z}\,\hat{a}_x = E_m e^{-j\beta_1 z}\,\hat{a}_x$ $\quad E_m = 100$

$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1} = 2\pi\times 10^{10}\sqrt{\mu_0\, 9\varepsilon_0} = 629$

$\eta_1 = \sqrt{\dfrac{\mu_0}{9\varepsilon_0}} = 125.6$ $\qquad H_m = \dfrac{E_m}{\eta} = 0.796$

$$\boxed{\vec{E}_i = 100\, e^{-j629z}\,\hat{a}_x \qquad \vec{H}_i = 0.796\, e^{-j629z}\,\hat{a}_y}$$

b.



$E_{\tan} = 0$ ➡ E changes sign at boundary

H_t does not change sign

$\vec{\gamma}$ also changes sign

so $-\vec{\gamma}\cdot\vec{r} = +|\gamma_1| z$

$$\boxed{\vec{E}_r = -100\, e^{+j629z}\,\hat{a}_x \qquad \vec{H}_r = 0.796\, e^{+j629z}\,\hat{a}_y}$$

EM Waves and Boundaries

Example 1

c. $\hat{a}_n \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$

$\vec{J}_s = \hat{a}_z \times \left(-(\vec{H}_i + \vec{H}_r)\Big|_{z=0} \right)$

$= \hat{a}_z \times (-1)(0.796 + 0.796)\hat{a}_y = \boxed{1.59\,\hat{a}_x \ \text{A/m}}$

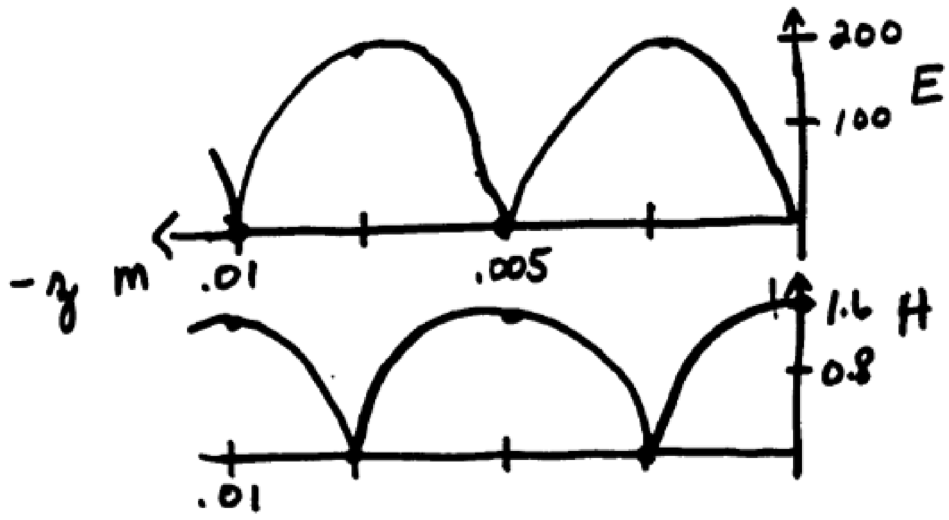
a_n Region 1 is dielectric $\vec{H}_1 = \vec{H}_i + \vec{H}_r$

Region 2 is conductor $\vec{H} = 0$

$\hat{a}_n$ points from 1 to 2 $\hat{a}_n = \hat{a}_z$

Why can we say that H=0 in a conductor?

d. $\lambda = \frac{2\pi}{\beta} = 0.01$ m



EM Waves and Boundaries

Example 1

$$\vec{E} = \left(100\, e^{-j629z} - 100\, e^{+j629z}\right)\hat{a}_x \quad \leftarrow \text{this form OK}$$

$$= 100\left(-2j \sin 629z\right)\hat{a}_x = \boxed{-j200 \sin(629z)\, \hat{a}_x}$$

$$\hat{\vec{H}} = 0.796\left(e^{-j629z} + e^{+j629z}\right)\hat{a}_y = \boxed{1.59 \cos(629z)\, \hat{a}_y}$$

TIME DOMAIN

$$\vec{E}(z,t) = \mathrm{Re}\left\{\hat{E}(z)\, e^{j\omega t}\right\} = \mathrm{Re}\left\{-j200 \sin(629z)\, e^{j\omega t}\hat{a}_x\right\}$$

$$= \boxed{200 \sin(629z) \sin\omega t\ \hat{a}_x} \qquad \omega = 2\pi \times 10^{10}$$

$$\vec{H}(z,t) = \mathrm{Re}\left\{\hat{H}(z)\, e^{j\omega t}\right\} = \boxed{1.59 \cos(629z) \cos\omega t\ \hat{a}_y}$$

EM Waves and Boundaries

Standardized Notation

We now use the standardized notation of reflection and transmission coefficients, just as we did with transmission lines.

Γ	Ratio of the reflected wave magnitude to the incident wave magnitude. (We use the electric field by convention.)
τ	Ratio of the transmitted wave magnitude to the incident wave magnitude. (We use the electric field by convention.)

EM Waves and Boundaries

Reflection and Transmission Coefficients

$$\Gamma = \frac{E_0^r}{E_0^i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$ Normally incident

$$\tau = \frac{E_0^t}{E_0^i} = \frac{2\eta_2}{\eta_2 + \eta_1}$$ Normally incident

Γ and τ are real for lossless dielectric media

$$\tau = 1 + \Gamma$$ Normally incident

<u>For nonmagnetic media</u>

$$\Gamma = \frac{E_0^r}{E_0^i} = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}}$$

EM Waves and Boundaries

Reflection and Transmission Coefficients

For nonmagnetic media, $\mu = \mu_0$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon}} \qquad \varepsilon = \varepsilon_r \varepsilon_0$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\frac{\mu_0}{\varepsilon_2}} - \sqrt{\frac{\mu_0}{\varepsilon_1}}}{\sqrt{\frac{\mu_0}{\varepsilon_2}} + \sqrt{\frac{\mu_0}{\varepsilon_1}}}$$

$$= \frac{\cancel{\left(\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)}\left(\frac{1}{\sqrt{\varepsilon_{r2}}} - \frac{1}{\sqrt{\varepsilon_{r1}}}\right)}{\cancel{\left(\sqrt{\frac{\mu_0}{\varepsilon_0}}\right)}\left(\frac{1}{\sqrt{\varepsilon_{r2}}} + \frac{1}{\sqrt{\varepsilon_{r1}}}\right)}$$

$$= \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}\varepsilon_{r2}}} \cdot \frac{\sqrt{\varepsilon_{r1}\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}}$$

$$= \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}}$$

EM Waves and Boundaries

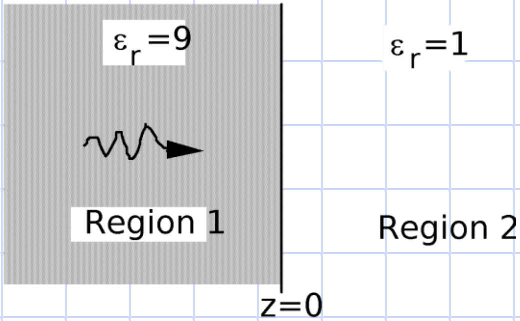
Do Lecture 23, Exercise 1 in groups of up to 4.

EM Waves and Boundaries

Example 2

The same wave as in example 1 (last lecture) strikes a dielectric-air boundary at z=0 as shown below.

a. Find the reflection and transmission coefficients.
b. What are the reflected and transmitted electric field phasors?
c. What are the reflected and transmitted H phasors? What is H_t/H_i?
d. What is the standing wave ratio in the dielectric? Sketch the standing wave pattern for **E** and **H**. Run sing_bnd.m for this problem.
e. What is the average power density of the incident, reflected, and transmitted waves?



EM Waves and Boundaries

Example 2

PROBLEM 2

a. $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$; $\eta_2 = \eta_0$ $\eta_1 = \sqrt{\frac{\mu_0}{9\epsilon_0}} = \frac{\eta_0}{3}$; $\Gamma = \frac{\eta_0 - \frac{\eta_0}{3}}{\eta_0 + \frac{\eta_0}{3}} = \boxed{0.5}$

$$\boxed{T = 1 + \Gamma = 1.5}$$

b. $\hat{\vec{E}}_r = \Gamma E_i \, e^{+j\beta_{01} z} \hat{a}_x = \boxed{50 \, e^{+j629z} \hat{a}_x}$

$\hat{\vec{E}}_t = T E_{i_0} e^{-j\beta_2 z} \hat{a}_x$; $\beta_2 = \omega\sqrt{\mu_0 \epsilon_0} = 209.6$

$$\boxed{\hat{\vec{E}}_t = 150 \, e^{-j210z} \hat{a}_x}$$

How is it that E_t is bigger than E_i? Does that make sense?

c. $\hat{\vec{H}}_r = \frac{-|\hat{\vec{E}}_r|}{\eta_1} \hat{a}_y = \frac{-50}{125.6} e^{+j629z} \hat{a}_y = \boxed{-0.398 \, e^{+j629z} \hat{a}_y}$

$\hat{\vec{H}}_t = \frac{|\hat{\vec{E}}_t|}{\eta_2} \hat{a}_y = \frac{150}{377} e^{-j210z} \hat{a}_y = \boxed{+0.398 \, e^{-j210z} \hat{a}_y}$

$\boxed{H_t / H_i = 0.5}$ $\leftarrow$ $E_t/E_i > 1$, but $H_t/H_i < 1$

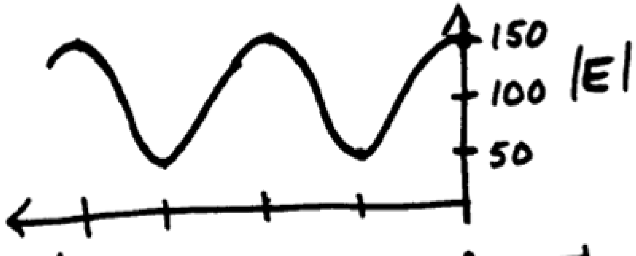
EM Waves and Boundaries

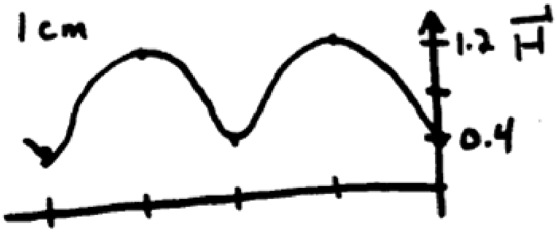
Example 2

d. $E_{max} = 100(1+|\Gamma|) = 150 \qquad E_{min} = 50$

$H_{max} = (.796)(1+|\Gamma|) = 1.19 \qquad H_{min} = .398$

$$SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 3$$





Note that there is a conservation of power here.

e. $|\vec{S}_{av,i}| = \frac{1}{2}\frac{|\vec{E}_i|^2}{\eta_1} = \frac{1}{2}\frac{(100)^2}{125.6} = 39.8$

$|\vec{S}_{av,r}| = \frac{1}{2}\frac{|\vec{E}_r|^2}{\eta_1} = \frac{1}{2}\frac{(50)^2}{125.6} = 9.95$

$|\vec{S}_{av,t}| = \frac{1}{2}\frac{|\vec{E}_t|^2}{\eta_2} = \frac{1}{2}\frac{(150)^2}{377} = 29.8$

$|\vec{S}_{av,i}| = |\vec{S}_{av,r}| + |\vec{S}_{av,t}|$

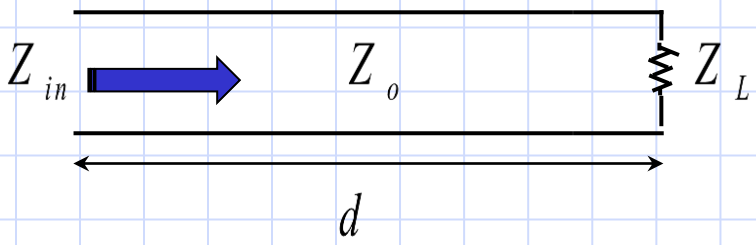
or

$\vec{S}_{av,i} + \vec{S}_{av,r} = \vec{S}_{av,t}$

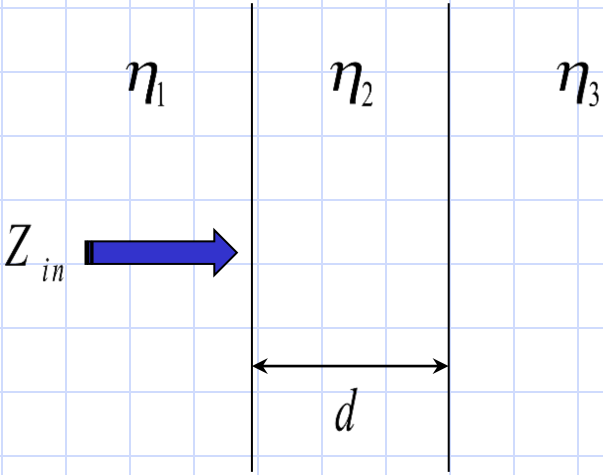
points in $-\hat{a}_z$

Multiple Boundaries

Lossless Transmission Lines



Uniform Plane Waves in Lossless Media



Input impedance:

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta d}{Z_o + jZ_L \tan \beta d}$$

$$Z_{input} = \eta_2 \frac{\eta_3 + j\eta_2 \tan \beta_2 d}{\eta_2 + j\eta_3 \tan \beta_2 d}$$

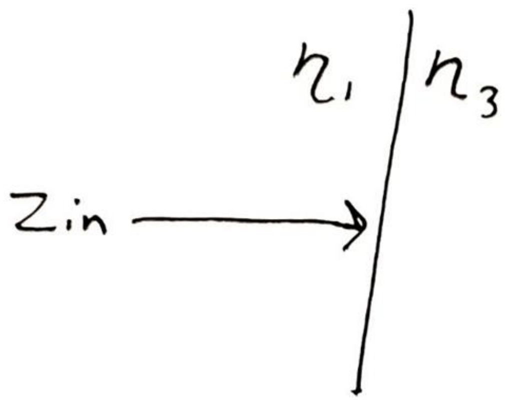
Note that both of these equations are defined by three regions.

Multiple Boundaries

Special Cases

When $d = 0$,

$$Z_{in} = \eta_2 \frac{\eta_3 + \cancelto{0}{j\eta_2 \tan\beta_2 d}}{\eta_2 + \cancelto{0}{j\eta_3 \tan\beta_2 d}} = \eta_3$$



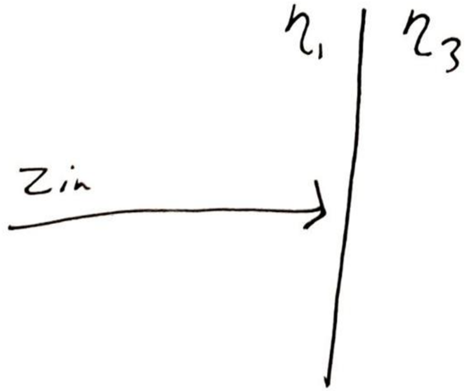
Second region effectively disappears because it is infinitely thin.

Multiple Boundaries

Special Cases

When $\eta_2 = \eta_3$,

$$Z_{in} = \eta_3 \frac{\eta_3 + j\eta_3 \tan \beta_2 d}{\eta_3 + j\eta_3 \tan \beta_2 d} = \eta_3$$



Second region effectively disappears because it is identical to the third region.

Multiple Boundaries

Special Cases

Half wavelength case:

$$\tan \beta d = \tan(\pi) = 0$$

$$Z_{in} = \eta_2 \frac{\eta_3 + \cancelto{0}{j\eta_2 \tan \beta_2 d}}{\eta_2 + \cancelto{0}{j\eta_3 \tan \beta_2 d}} = \cancel{\eta_2} \frac{\eta_3}{\cancel{\eta_2}} = \eta_3$$

Much like a half-wavelength transmission line acts like it isn't there as far as standing wave patterns are concerned, the same is true of a half-wavelength EM wave medium at normal incidence.

Multiple Boundaries

Special Cases

Quarter wavelength case:

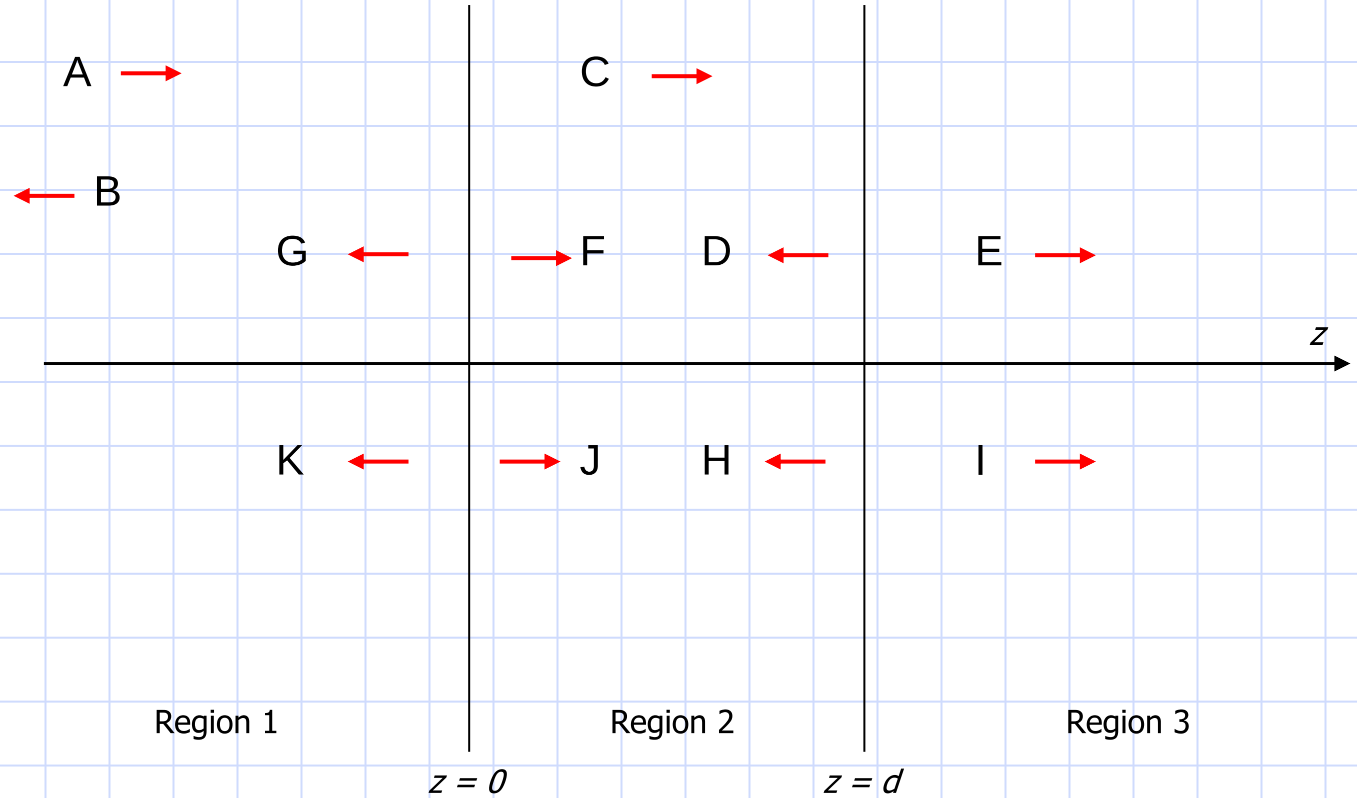
$$\tan \beta d \Rightarrow \tan\left(\frac{\pi}{2}\right) \rightarrow \infty$$

$$Z_{in} = \eta_2 \frac{\eta_3 + j\eta_2 \tan \beta_2 d}{\eta_2 + j\eta_3 \tan \beta_2 d} \rightarrow \eta_2 \frac{j\eta_2 \cancel{\tan \beta_2 d}}{j\eta_3 \cancel{\tan \beta_2 d}}$$

$$Z_{in} = \frac{\eta_2^{\,2}}{\eta_3}$$

Like the quarter wavelength transmission line, the quarter wavelength EM wave medium at normal incidence acts like a "transformer".

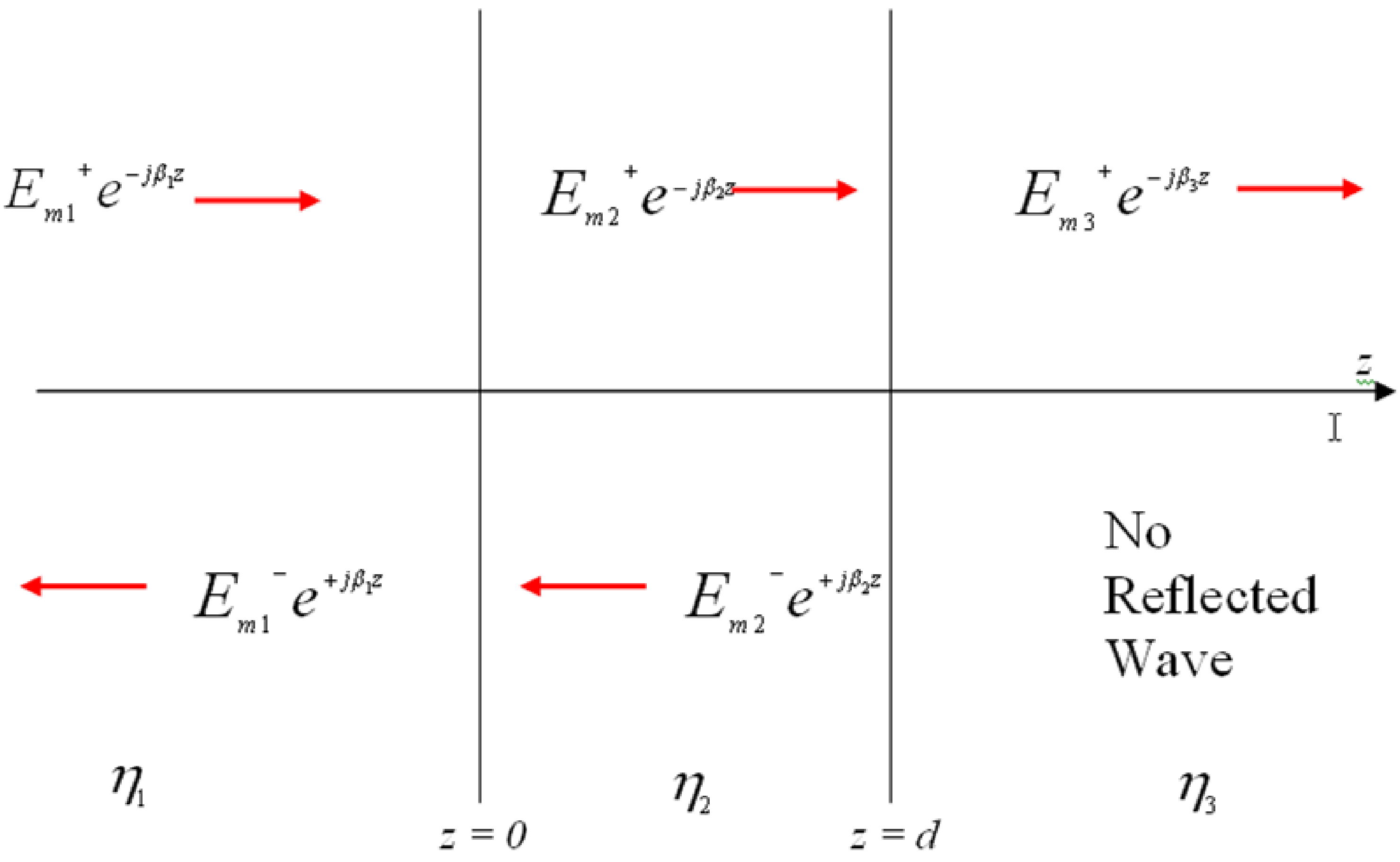
Multiple Boundaries



If A is incident, each reflection produces the additional waves shown.

Multiple Boundaries

The waves take the usual general form in each region.



Applications

Radomes

http://igscb.jpl.nasa.gov/network/site/areq.html

- Protect radio equipment
- Designed to have minimal attenuation at radio frequencies

http://www.cmmacs.ernet.in/nal/picts/

Applications

Radomes

http://www.air-and-space.com/2003%20Miramar%20Airshow%20statics%20page%202.htm

Applications

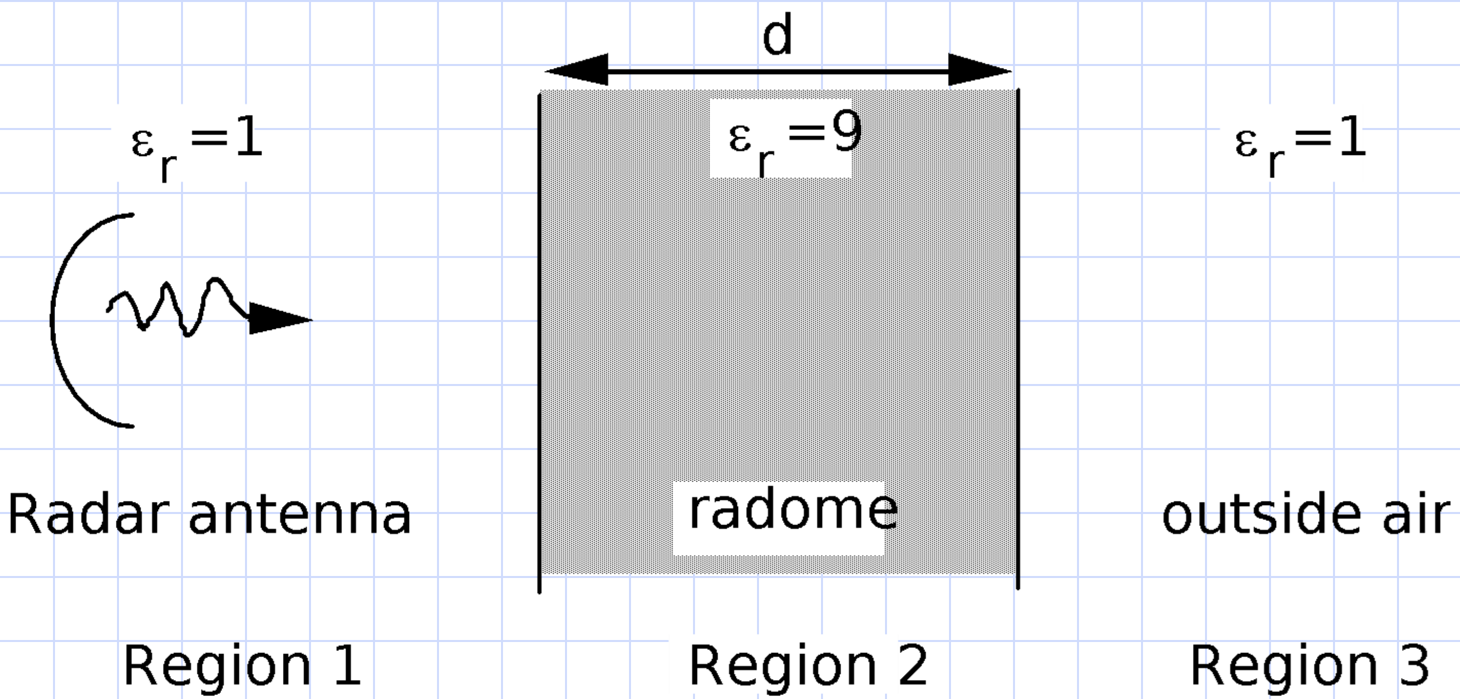
Radomes

http://en.wikipedia.org/wiki/File:Navy-Radome.jpg

Applications

Example 3

A 10 GHz radar transmitter is used in the configuration shown below. Note that the radome-outside air boundary is identical to the boundary examined in example 2.



Applications

Example 3

a. What is |**E**|/|**H**| at the z=0 boundary of example 2? (equivalent to the region 2-3 boundary in this problem). Compare it with the value in air.
b. Now refer to the full radome problem. Where can you put the left boundary (between regions 1 and 2) so that |**E**|/|**H**| in the radome matches that in the air on the left? For mechanical reasons, the radome must be more than 2 cm thick.
c. What is Γ for this value of d?
d. What is Γ if d is 0.2 mm thinner than designed?

Applications

Example 3

a. $\dfrac{\vec{E}}{|\vec{H}|} = \dfrac{150}{.398} = 377 \doteq \eta_0$

b. Any peak in standing wave, also has $\dfrac{|\vec{E}|}{|\vec{H}|} = \eta_0 \Rightarrow \boxed{d = 2.5\text{cm}}$

c. $\dfrac{\vec{E}}{\vec{H}}$ in dielectric matches air $\Rightarrow \therefore$ boundary conditions satisfied w/o reflectn

$\boxed{\Gamma = 0}$ ~~can~~ alternate view $Z_{in}(z) = \dfrac{|\vec{E}|}{|\vec{H}|}$ $\Gamma = \dfrac{Z_{in} - \eta_1}{Z_{in} + \eta_1}$

d. $Z_{in}(z) = \eta_2 \dfrac{\eta_3 + j\eta_2 \tan\beta_2 d}{\eta_2 + j\eta_3 \tan\beta_2 d}$ with $\eta_2 = \dfrac{\eta_0}{3}$, $\eta_3 = \eta_0$, $\beta_2 = 629$, $d = .0248$

get $Z_{in}(z) = 341 + j104$

$\Gamma = \dfrac{Z_{in}(z) - \eta_1}{Z_{in}(z) + \eta_1} = \boxed{0.151\, e^{j1.76}}$

Applications

Example 3

$$Z_{input} = \eta_2 \frac{\eta_3 + j\eta_2 \tan \beta_2 d}{\eta_2 + j\eta_3 \tan \beta_2 d} \qquad \beta_2 d = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$$

$$Z_{input} = \eta_2 \frac{\eta_3 + j\eta_2 \tan \pi}{\eta_2 + j\eta_3 \tan \pi} = \eta_2 \frac{\eta_3}{\eta_2} = \eta_3$$

Thus, since regions 1 and 3 are the same, the input impedance gives a perfect match and no reflection occurs.

Applications

Example 3

The radome is a half wavelength thick. We can also use a layer one quarter wavelength thick to eliminate reflections from a lens (at least at a particular frequency).

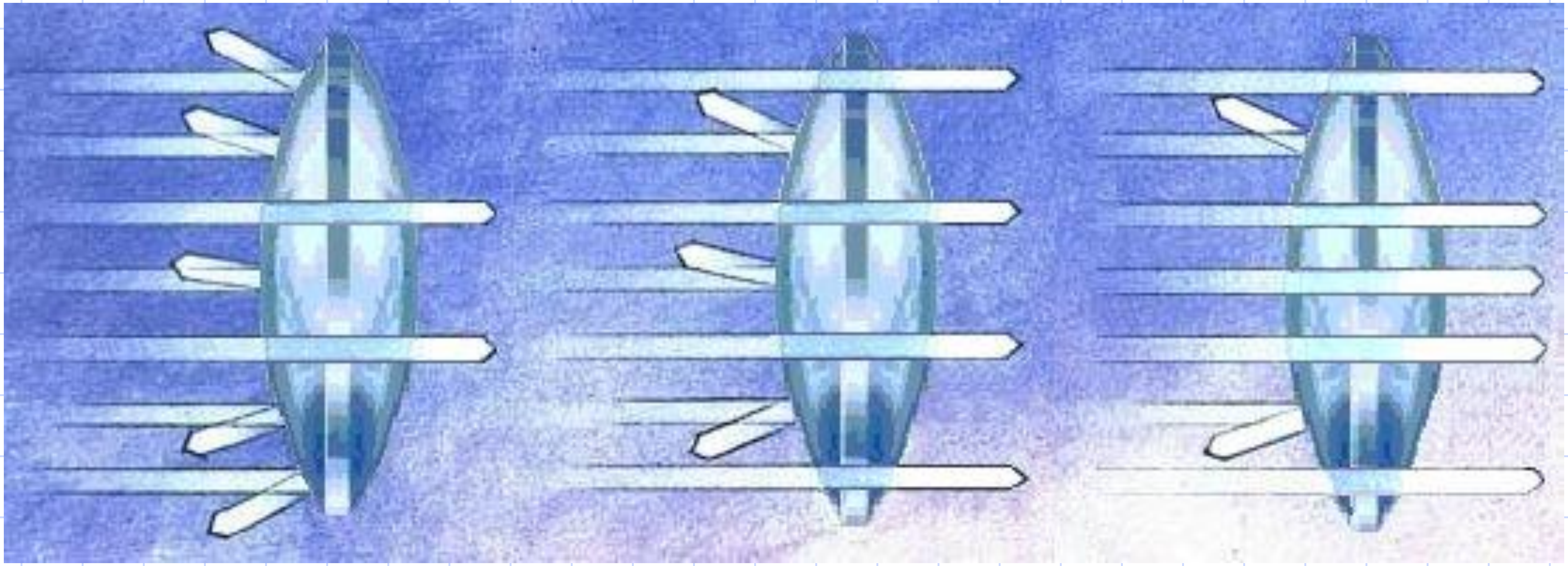
$$Z_{input} = \eta_2 \frac{\eta_3 + j\eta_2 \tan \beta_2 d}{\eta_2 + j\eta_3 \tan \beta_2 d} \qquad \beta_2 d = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$$

$$\eta_2 = \sqrt{\eta_1 \eta_3} \qquad Z_{input} = \eta_2 \frac{\eta_3 + j\eta_2 \tan {}^{\pi}\!/_{2}}{\eta_2 + j\eta_3 \tan {}^{\pi}\!/_{2}} = \frac{\eta_2^{\,2}}{\eta_3} = \eta_1$$

Thus, since the input impedance is the same as the intrinsic impedance of region 1, we again have a perfect match and no reflection occurs.

Applications

Anti-Reflection Coatings



From left to right a lens without coating, single coated and multi coated. From the first to the third image the light transmission improved from 96 to 99.5%

http://www.astrosurf.org/lombry/reports-coating.htm