37B - Units in Special Relativity, Atomic Physics, and Nuclear Physics

Physicists and engineers who do regular calculation in atomic, solid state, and nuclear physics find it easier to use units that are not MKSA.[1]

The electron-volt (eV) is the energy gained when an elementary charge of one electron is accelerated through a potential difference of one volt. In electron-volt units, the charge on the electron is "1 electron charge", (not 1.6 x 10^{-19} Coulombs.) Similarly in special relativity, we can refer to the speed of light as "1 speed of light, i.e., 1*c*", rather than 3 x 10^8 m/s.

1) Calculate the energy gained by one electron accelerating through one volt in MKSA units. Do not just look this up, show the calculation.

1eV = 1.602x10^-2 J

The energy gained by an elementary charge accelerating through 10^6 volts is 1 million electron volts or 1 MeV, whereas 1 meV is one milli-electron volts, which is frequently used in molecular physics.

2) In special relativity, the relationship between mass and energy, $E = mc^2$, leads to a convenient unit for mass, MeV/c^2.
   a) What is the mass of a carbon-12 atom ($^{12}_{6}C$) in kilograms?

   .012 kg

   b) What is the rest energy of $^{12}_{6}C$ in Joules?

   1.08*10^15 J

   c) What is the rest energy of $^{12}_{6}C$ in MeV?

   1.08*10^9 MeV

   d) What is the rest mass of $^{12}_{6}C$ in MeV/c^2. (Just use the relationship $m = E/c^2$ with $c = 1$ and E in MeV.)

   1.08*10^9 MeV/c^2

[1] MKSA stands for "meters, kilograms, seconds, and amperes;" it is another name for the SI system of units.

37C – Special Relativity: Relativistic Momentum and Energy

RELATIVISTIC MOMENTUM

The relativistic momentum is given by: $\vec{p} = \gamma m \vec{v}$ where m in this equation is referred to as the rest mass. (The product γm is sometimes referred to as the "relativistic mass" – this term is no longer in common use among physicists. Issues related to the use of the term relativistic mass are mentioned in the text.)

1) A muon has a mass $m_\mu = 1.88 \times 10^{-28}$ kg and is travelling at 0.99889*c*.
   a) Find the numerical value of the relativistic momentum (kg-m/s) of the muon.

   1.196*10^-18 kg*m/s

   b) If you were to use the classical momentum relation $p = mv$ instead of the (correct) relativistic one, what v would produce the same momentum?

   6.362*10^9 m/s

RELATIVISTIC ENERGY

You are already familiar with the famous equation between mass and energy, "$E = mc^2$", where *E* is the rest energy and as a consequence *m* is the *rest mass*. If the mass is NOT at rest, one must use:

$$E = \gamma mc^2$$

E here corresponds to the total energy of a particle, including both the kinetic energy *K* and energy stored in the form of *rest mass energy*.

The kinetic energy *K* is the total energy minus the *rest mass energy*:

$$K = E - \mathrm{m}c^2 = (\gamma - 1)\mathrm{m}c^2$$

In the low velocity (v < < c) limit, the kinetic energy reduces to the classical expression:

$$K = \frac{1}{2} mv^2$$

Also, the total energy and momentum are related by:

$$E^2 = (mc^2)^2 + (pc)^2$$

2) Find the rest energy in Joules for the following particles.
   a) Rest energy for an electron. The mass of the electron is $m_e = 9.11 \times 10^{-31}$ kg.

   8.199*10^-14 J

   b) Rest energy for a muon. The mass of the muon is $m_\mu = 1.88 \times 10^{-28}$ kg.

   1.692*10^-11 J

3) The mass of a particle is frequently expressed in units of MeV/c^2. You can do this for the cases above by just converting Joules to MeV to get the energy in MeV. The mass is just stated as that energy "over c-squared" rather than actually doing the division by (3 x 10^8 m/s)^2.

a) Rest mass for an electron in MeV/c^2.

0.51 MeV/c^2

b) Find the relativistic mass of the muon from the previous section in units of MeV/c^2.

1.057 MeV/c^2

4) A photon is a massless particle that has energy and momentum.

a) Using the relations above, what is the relation between total energy and momentum for the photon?

E=p*c

b) Find the numerical value of the momentum in kg m/s for a 1 MeV photon.

5.33*10^-22 kg m/s

c) Find the numerical value of the momentum in kg m/s of an electron of kinetic energy 1 MeV

5.4*10^-22

5) Show that the kinetic energy

$$K = E - \mathrm{m}c^2 = (\gamma - 1)\mathrm{m}c^2$$

for speeds $v << c$ yields

$$K = \frac{1}{2}mv^2$$

Hint: Use the following expression obtained from the binomial expansion.

$$(1 - x^2)^{-(1/2)} = 1 + \frac{1}{2}x^2 - \cdots$$

k=(sqrt(v^2/c^2)-1)mc
k=(1+q/2(v^2/c^2)-1)mc^2
k=1/2mv^2

NUCLEAR ENERGY FROM THE SUN

The sun is powered through a nuclear fusion process by which four protons are combined to produce one alpha particle (He nucleus) and two positrons. (Positrons are antimatter particles and have the same mass as electrons but opposite electrical charge sign.)

6a) What is the numerical value of the rest energy of a proton in Joules? 15*10^-11 J

b) What is the numerical value of the rest energy of a positron in Joules? 81.9/10^-15 J

c) What is the numerical value of the rest energy of an alpha particle in Joules?

59.8*10^-11 J

d) Numerically what is the mass difference (in kg) between four protons and one alpha particle plus two positrons?

.046*10^-27 kg

e) What is the percentage mass difference between four protons and an alpha plus two positrons?

.68%

f) Numerically, how much energy in Joules is released through the conversion of one kilogram of protons (N protons) to N/4 alpha particles and N/2 positrons? (It is a good approximation to multiply 1 kg by the fractional mass difference from part e [= percentage mass difference/100] to find the mass converted to energy.)

.412*10^-11 J

By comparison, burning 1 gallon of gasoline (which has a mass $\simeq$ 3.2 kg) releases $\simeq 1.3 \times 10^8$ J. How does the energy released from burning a kilogram of gasoline compare the energy released by conversion of 1 kg of protons into alpha particles?

7) Nucleons are bound to one another via the strong nuclear force. We can find the binding energy per nucleon for an atom by taking the difference between the rest mass energy of the constituents and of the nucleus itself. In the following, we compare the binding energy per nucleon for a common element (iron54) and for a fissionable element (uranium-235). Fill in the blanks in the tables below. Note: 1 unified mass unit (u) = 931.50 MeV/c^2. (You can find masses of atoms and their isotopes [in u] at

https://pubchem.ncbi.nlm.nih.gov/periodic-table/ ; click on the element in the periodic table that shows up at the link, and in the menu that pops up choose the option "element page," then scroll down to section 8 of that page to find the isotopes and their measured masses).

Iron-54 has 26 protons and 28 neutrons

	Constituent mass (u)	Constituent mass energy (MeV)	Number	Total mass energy (MeV)
Proton	1.00782	968.78	26	24408.28
Neutron	1.00867	939.58	28	26308.8
Electron		0.511	26	13.286
			Sum of Constituents	50715
Fe-54	55.84	5.2*10^4	1	5.2*10^4
			Difference	1299

Binding energy per nucleon = Difference in total mass energy/ Number of nucleons.

Binding energy/nucleon for Fe-54 is 24 MeV.

Uranium-235 has 92 protons and 143 neutrons

	Constituent mass (u)	Constituent mass energy (MeV)	Number of particles	Total mass energy (MeV)
Proton	1.00782	938.78	92	86367
Neutron	1.00867	939.58	143	134362
Electron		0.511	92	47
			Sum of Constituents	220777.6
U-235	235	218902.5	1	218902.5
			Difference	1875

Binding energy/nucleon for U-235 is 7.97 MeV.

8) One nuclear fission reaction involving uranium, rubidium, cesium, and neutrons is

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{236}_{92}\mathrm{U} \rightarrow ^{96}_{37}\mathrm{Rb} + ^{137}_{55}\mathrm{Cs} + 3\ ^{1}_{0}\mathrm{n}$$

The three neutrons can then go on to induce fission in three more U-235 nuclei. Fill in the table below to find the energy released when 1 mole of U-235 fissions to the products above.

	Number	Mass (u)	Mass energy (MeV)
U-235	1	235	218939
Neutron	1	1	939.6
	Total in	236	219878.6
Cs-137	1	136.91	127531.5
Rb-96	1	95.93	89358.8
Neutron	3	3	2818.6
	Total out	236	219709.06

Energy released per input U-235: 173.54 MeV

Energy released by conversion of 1 mole of U-235: 1.67*10^-13 Joules

Energy released by conversion of 1 kg of U-235: 7*10^13 Joules

For comparison, TNT releases about 1.4×10^{7} J/kg. The factor by which U-235 is greater than this amount per kilogram of reaction is:

5*10^6