Fields and Waves I

Lecture 4

Pulses on Transmission Lines
Lossy Transmission Lines

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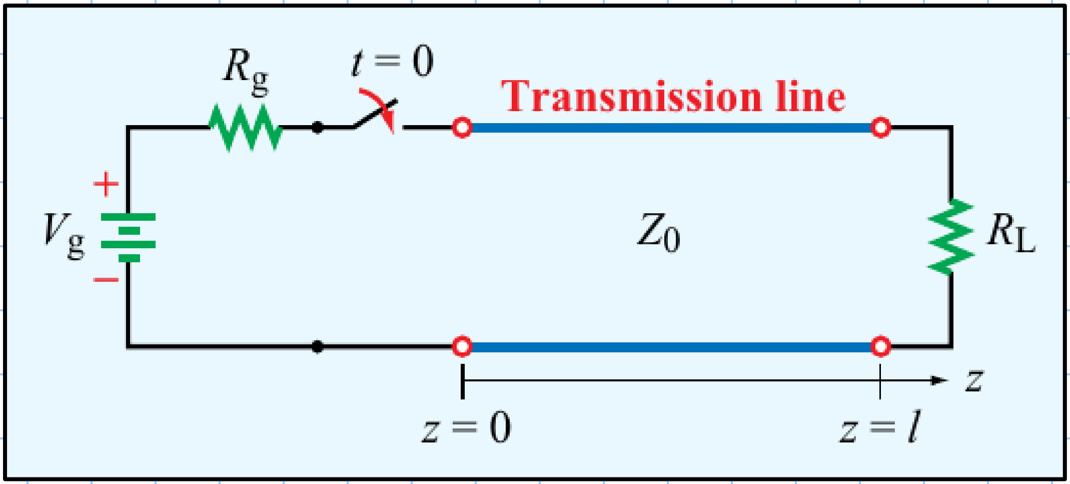
Wrap-Up

- Exam 1 coming up a week from Monday (2/5).
  - Covers lectures 1-6 and the Unit 1 Core Skills (which will be released ASAP)
  - I will release practice problems next week.
  - Exams use standardized crib sheets

Transients On T-Lines

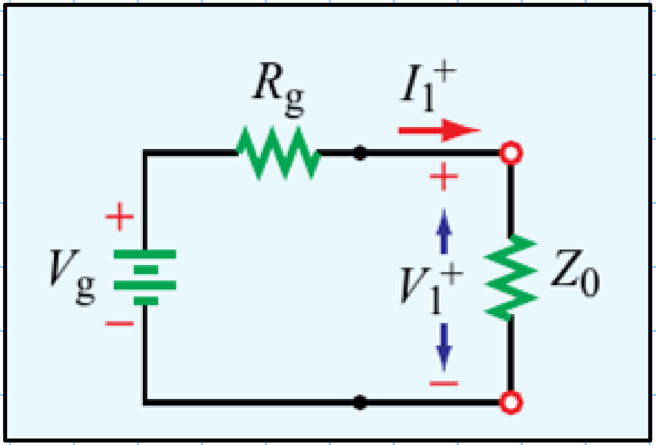


Ulaby pg. 115

Consider the circuit above. At t=0, the switch closes and a voltage pulse begins moving forward down the line.

- Is there a backwards traveling wave at this point?
- Are there any standing waves?
- What is the ratio of current and voltage on the line at this moment?

Transients On T-Lines

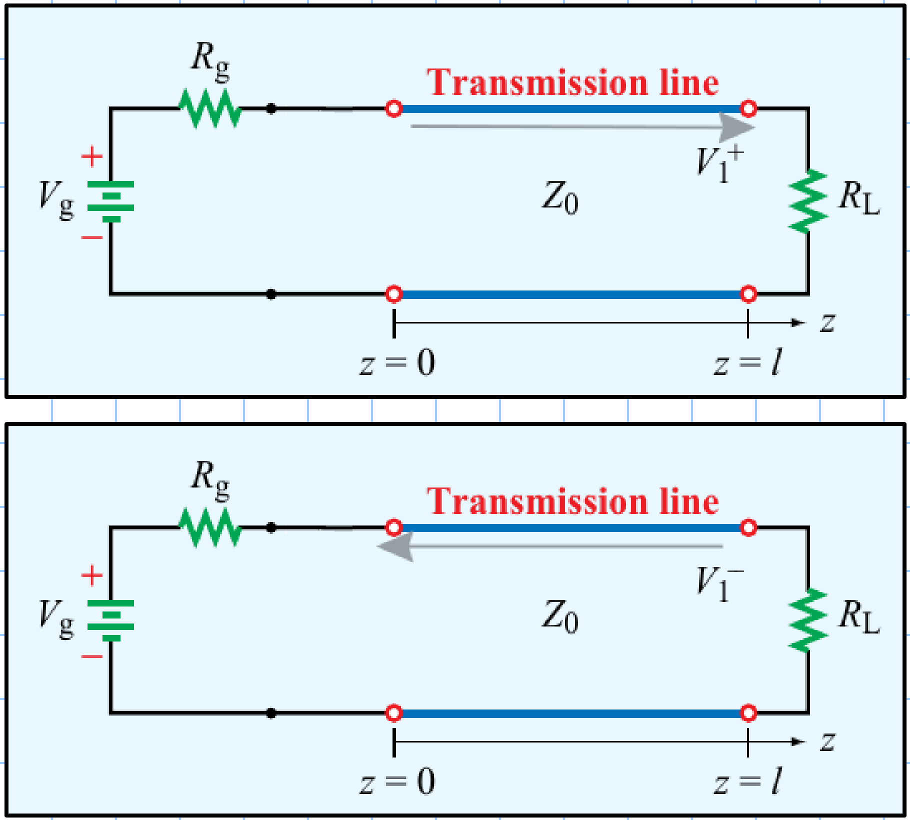


Ulaby pg. 115

At t=0, the t-line circuit therefore behaves equivalently to this one. So what are the current and voltage expressions for the load?

$$V_1^+ = \frac{V_g Z_0}{Z_g + Z_0} \qquad I_1^+ = \frac{V_g}{Z_g + Z_0}$$

Transients On T-Lines



$$V_1^- = \Gamma V_1^+$$

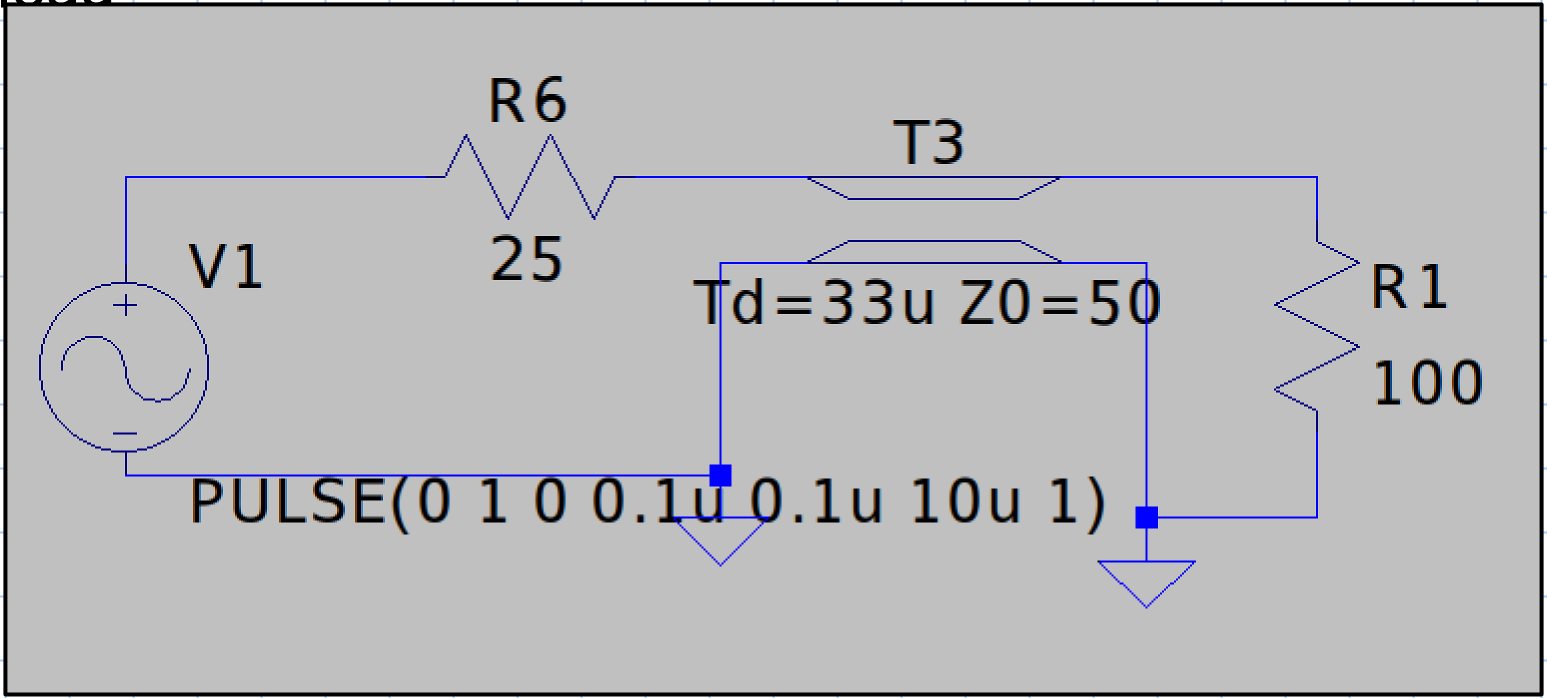
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{V_0^-}{V_0^+} = -\frac{I_0^-}{I_0^+}$$

This voltage pulse still reflects at the load as governed by the reflection coefficient.

Transients On T-Lines

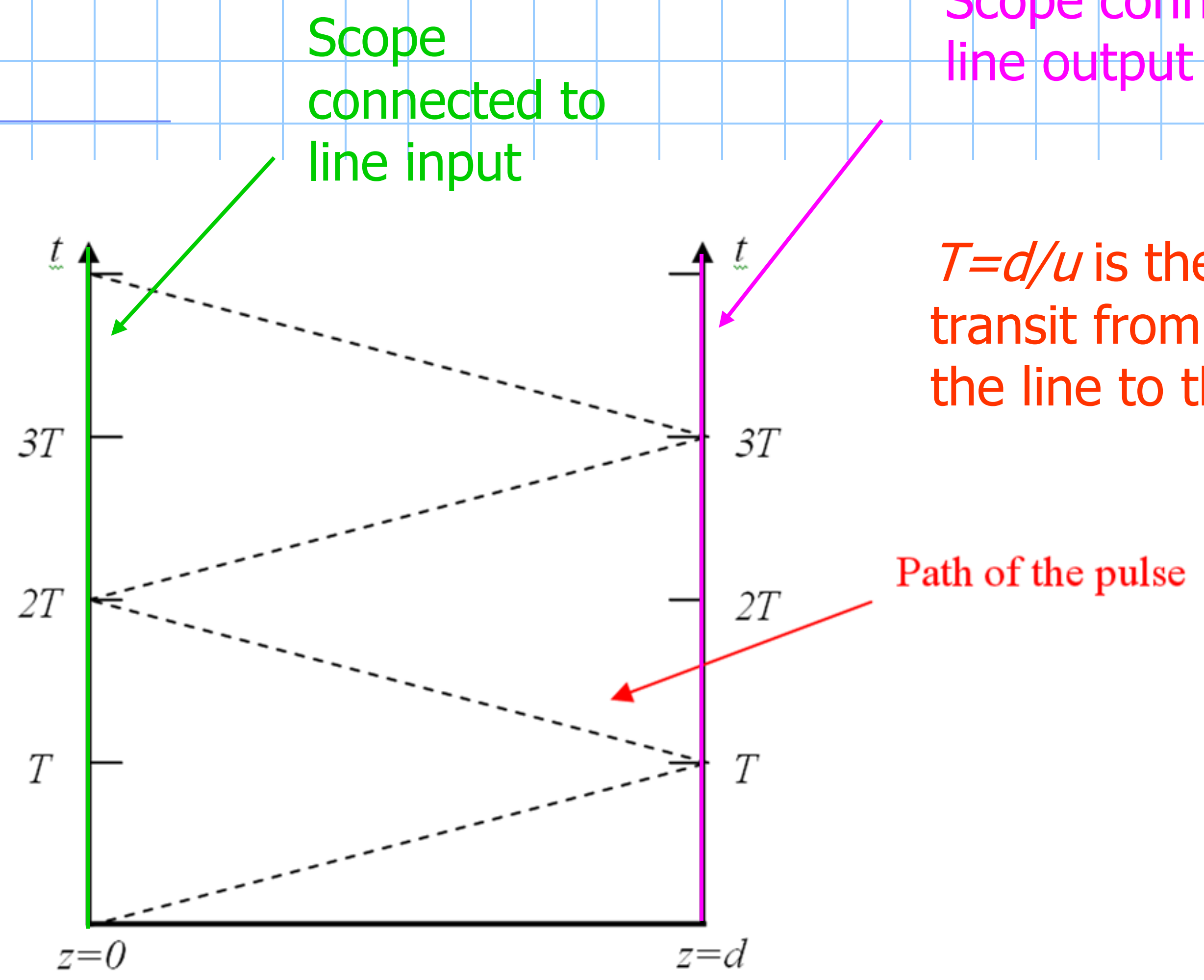
Next example - Square pulse, mismatched source and load



Bounce Diagram

- There is a systematic method for applying this information using what is called a bounce diagram or lattice diagram
- Each step of the process is included
- Space and time information are included

Bounce Diagram

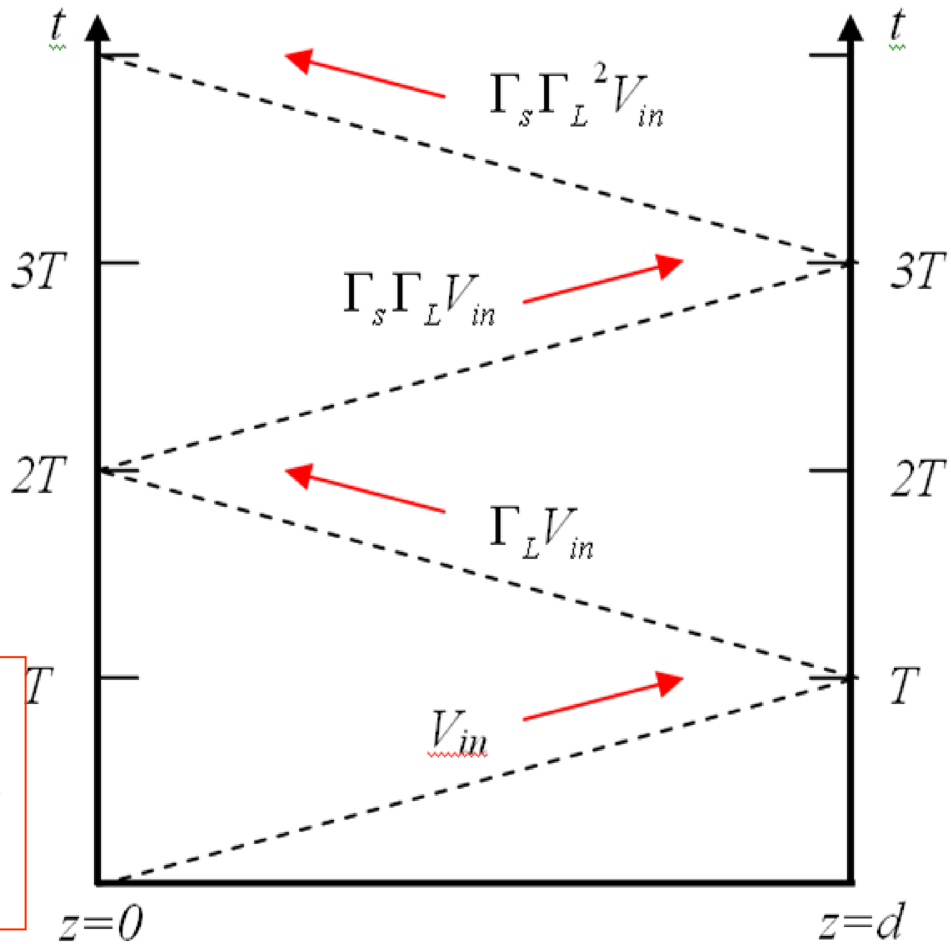


T=d/u is the time to transit from one end of the line to the other

Bounce Diagram

$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o}$$

$$V_{in} = V_s \frac{Z_{in}}{Z_s + Z_{in}}$$



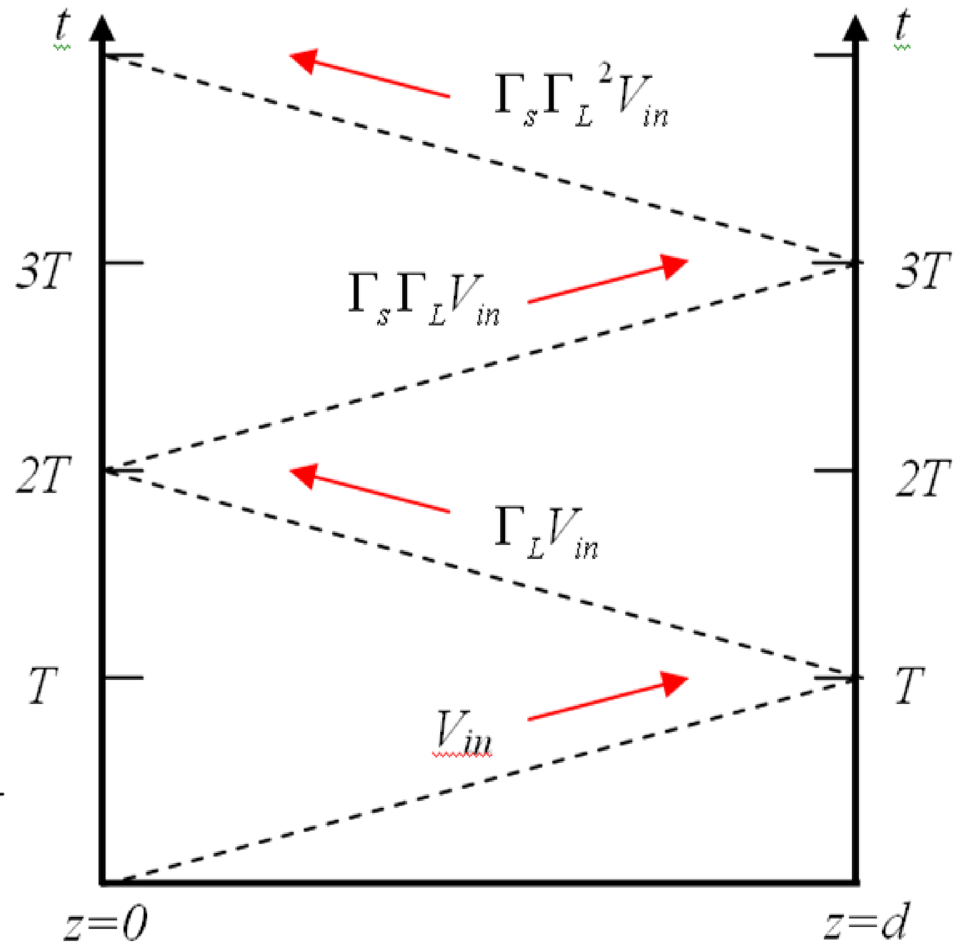
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$T = d/u$$

Bounce Diagram

What about when the source and load impedances are matched?

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}$$



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Bounce Diagram

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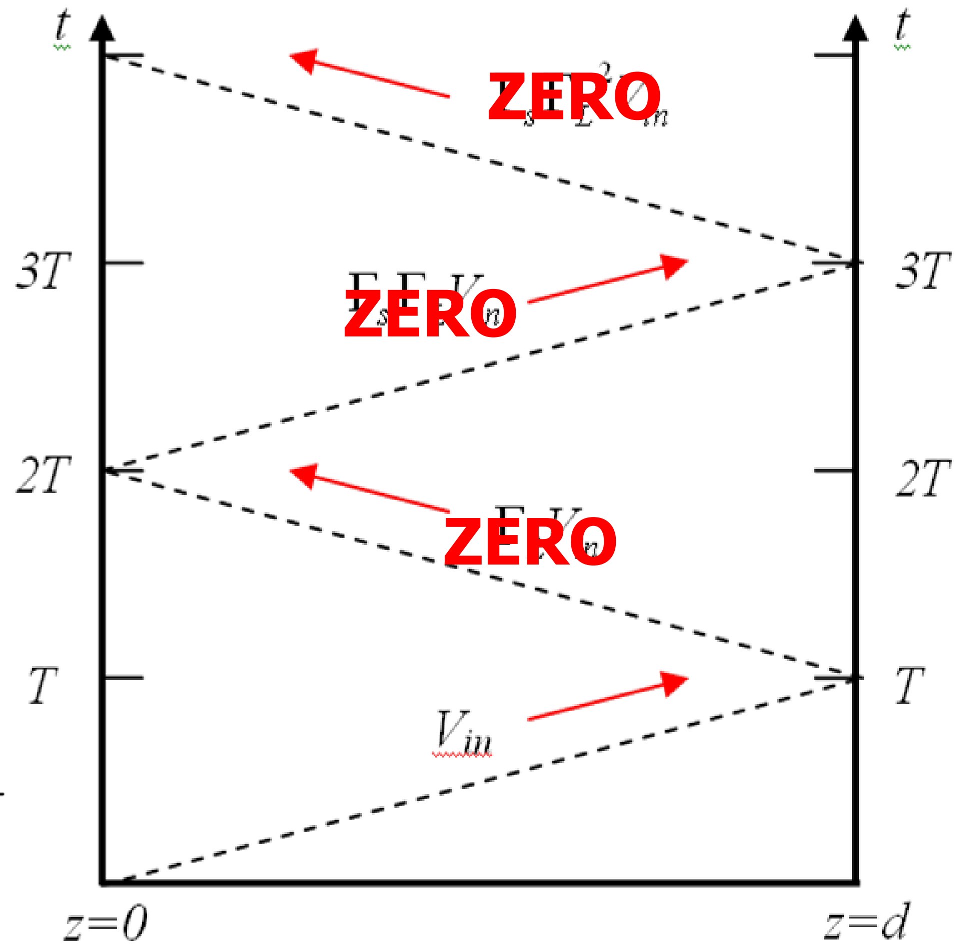
ZERO



$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

ZERO

$$V_{in} = V_s \frac{Z_{in}}{Z_s + Z_{in}}$$

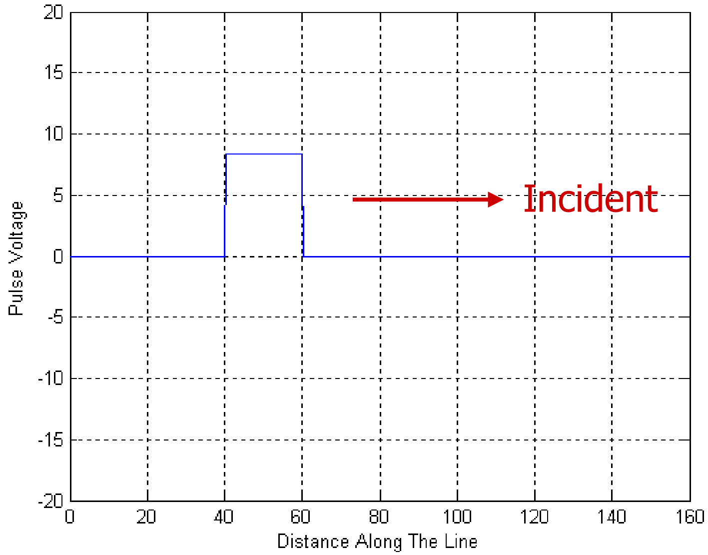
$$T = d/u$$

Bounce Diagram

- If the pulse width is much less than the transit time *T*, then only a single incident and reflected pulse will occur at the load or source end while reflection occurs.
- This is much simpler to consider.

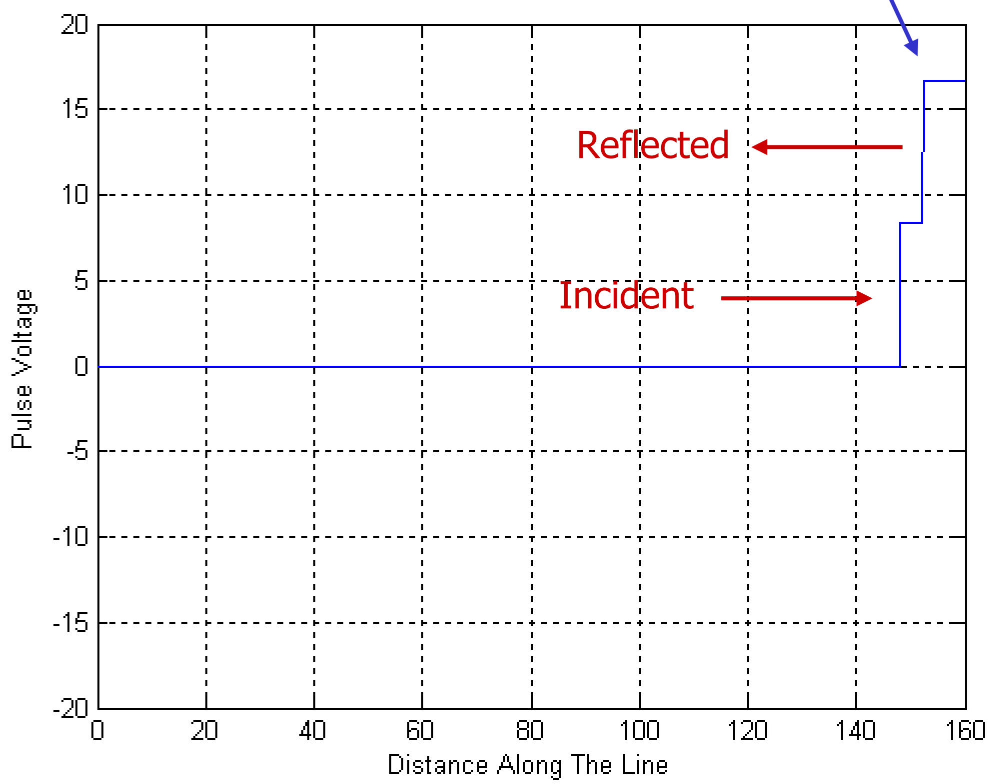
Bounce Diagram

- Zooming in allows us to see how reflection actually occurs.

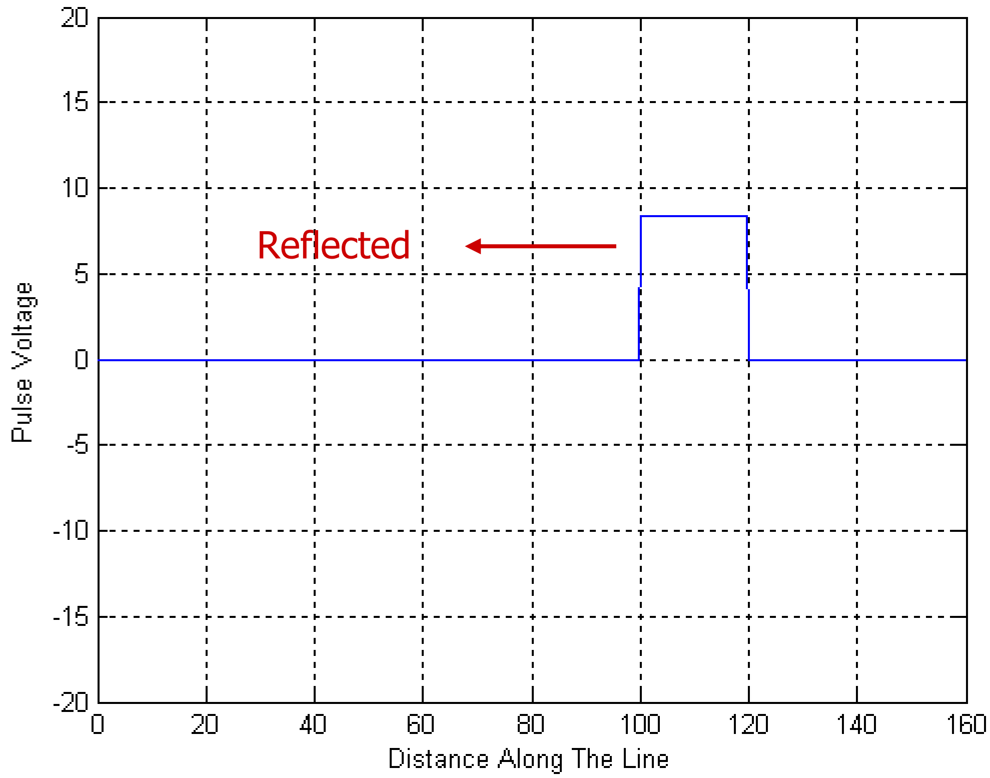


Bounce Diagram

Both Incident and Reflected Pulses Must Exist Simultaneously



Bounce Diagram



Bounce Diagram

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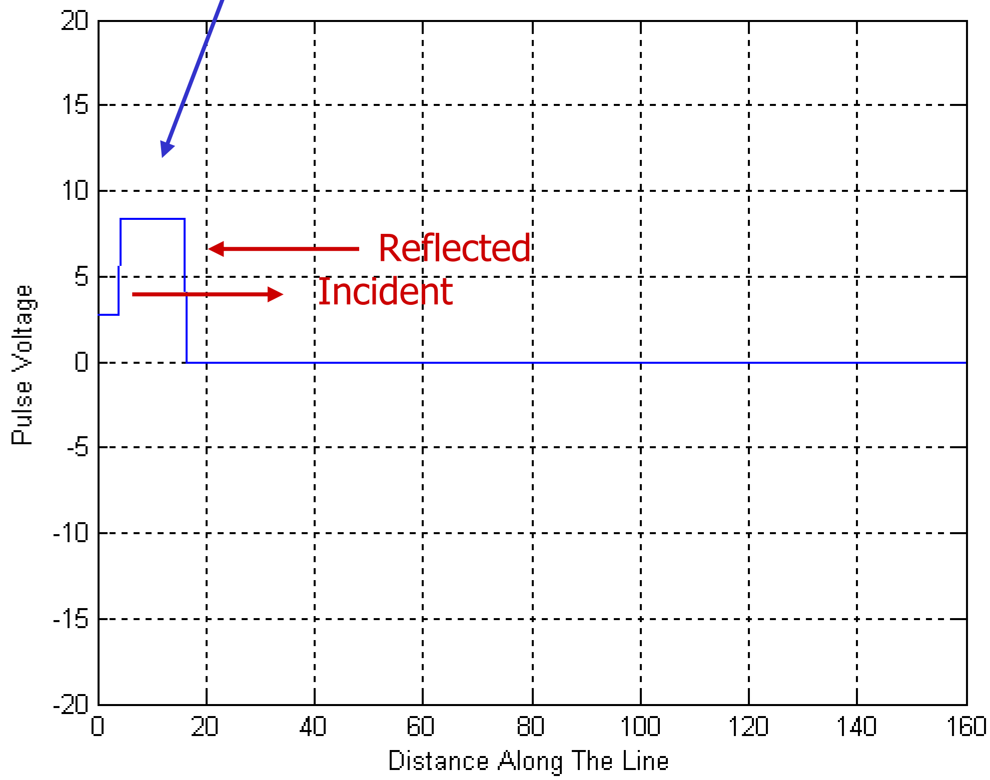
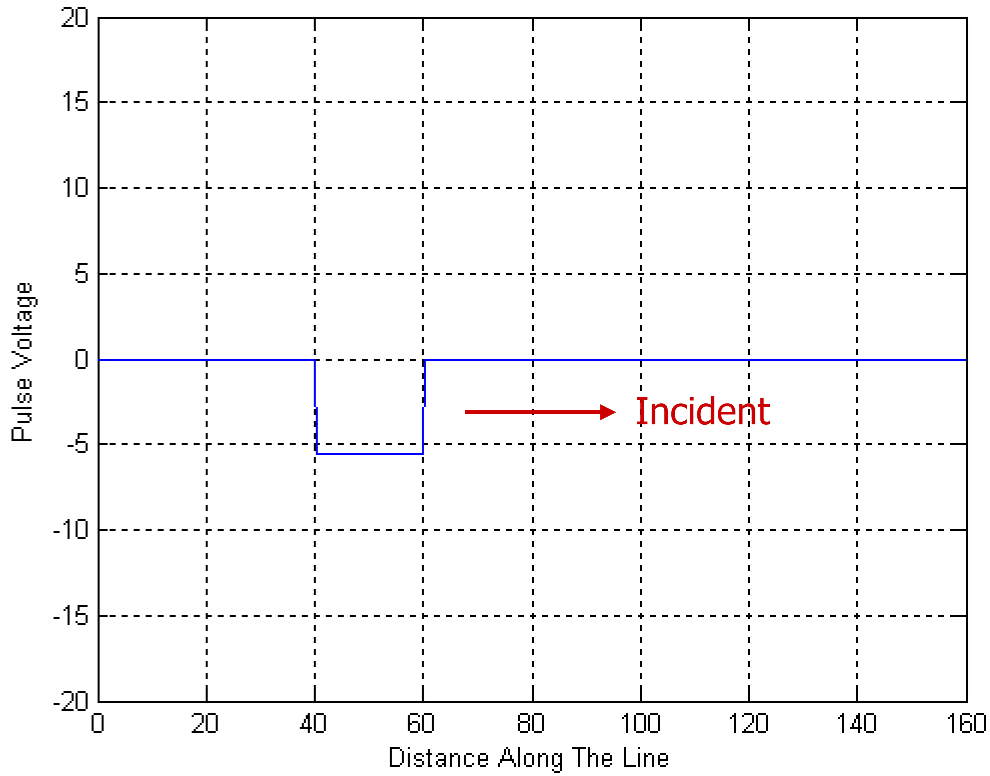
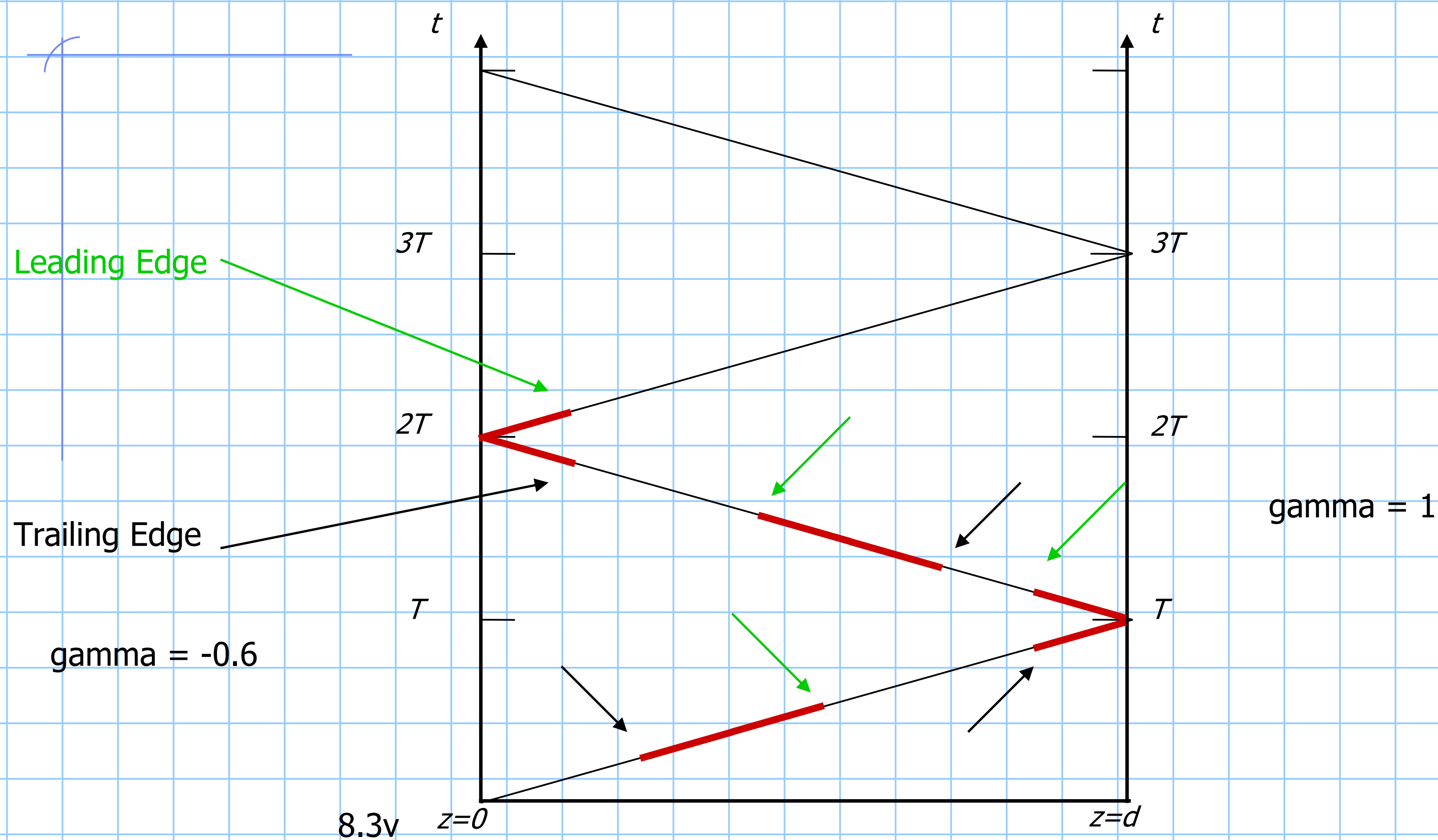


Bounce Diagram

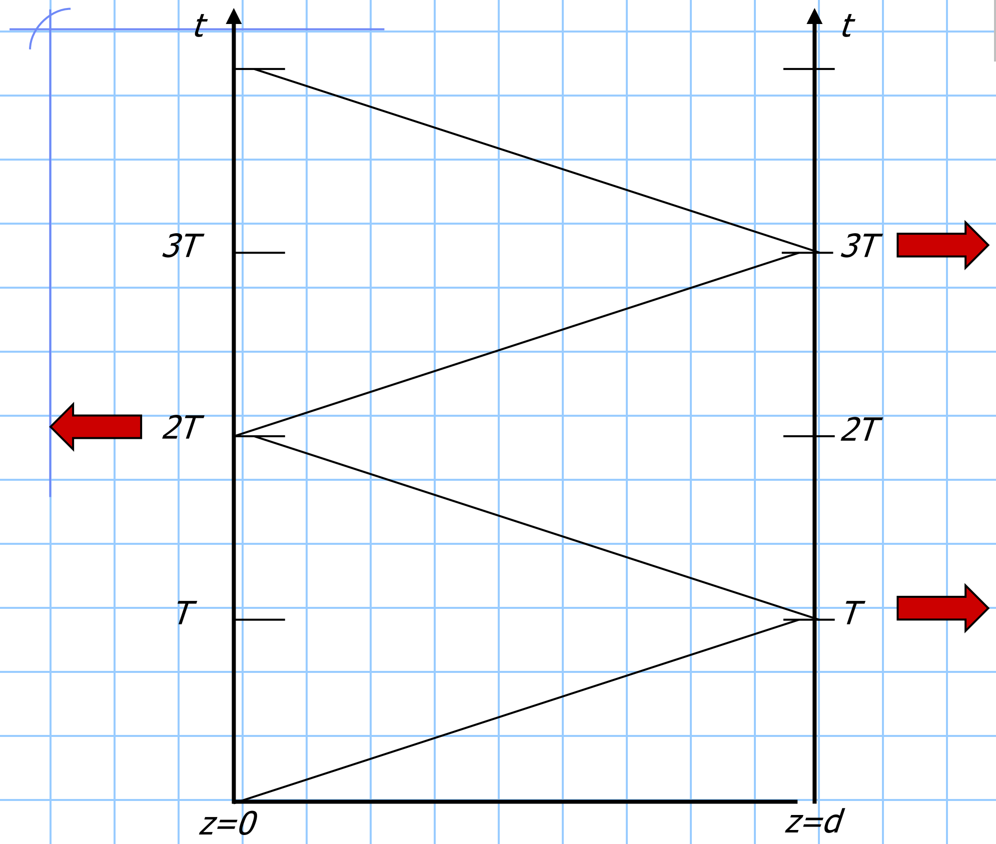


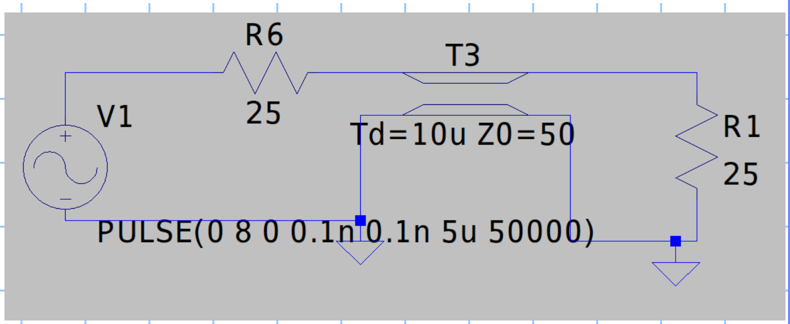
Bounce Diagram

Again, expanding the pulse for clarity, we see that incident and reflected pulses exist simultaneously



Bounce Diagram Example 1 (short pulse)





- 8V short input pulse
- 25Ω source and load impedance
- 50Ω characteristic impedance
- 10 microsecond time delay

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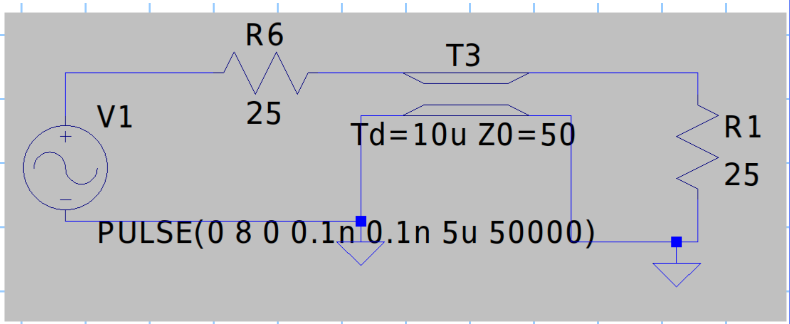

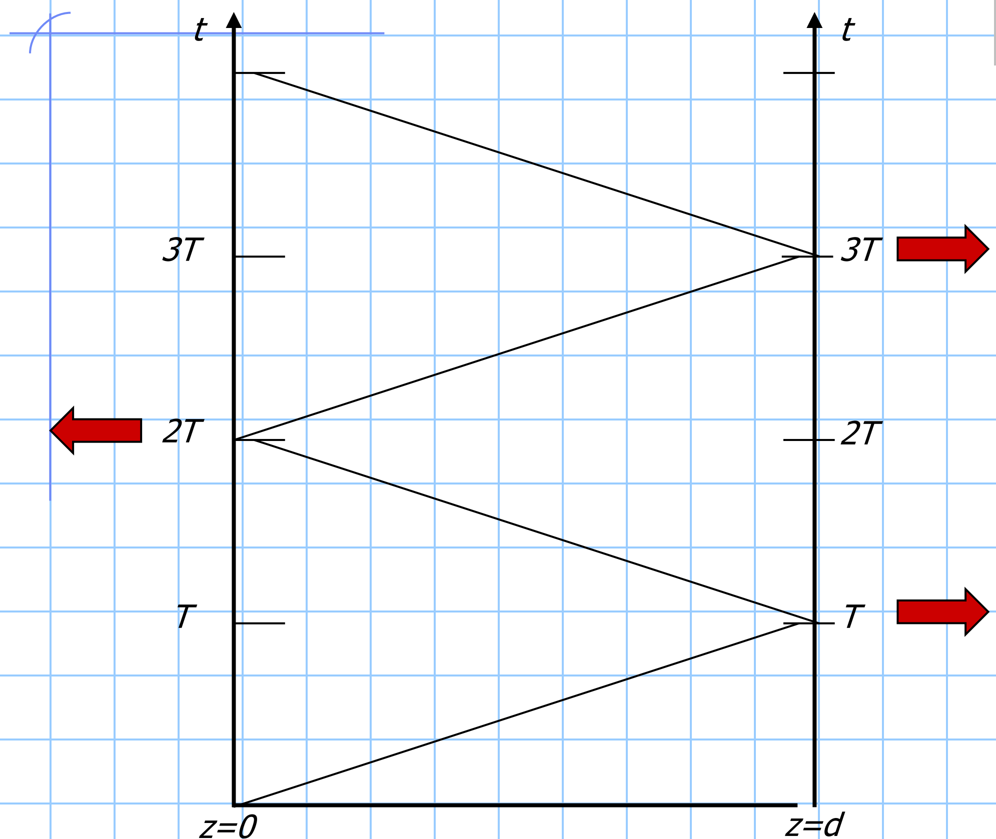

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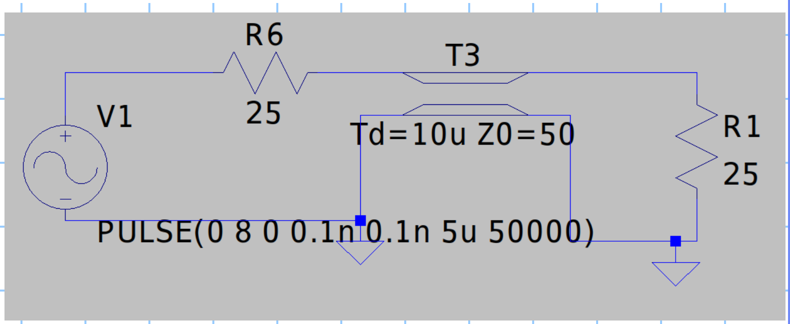

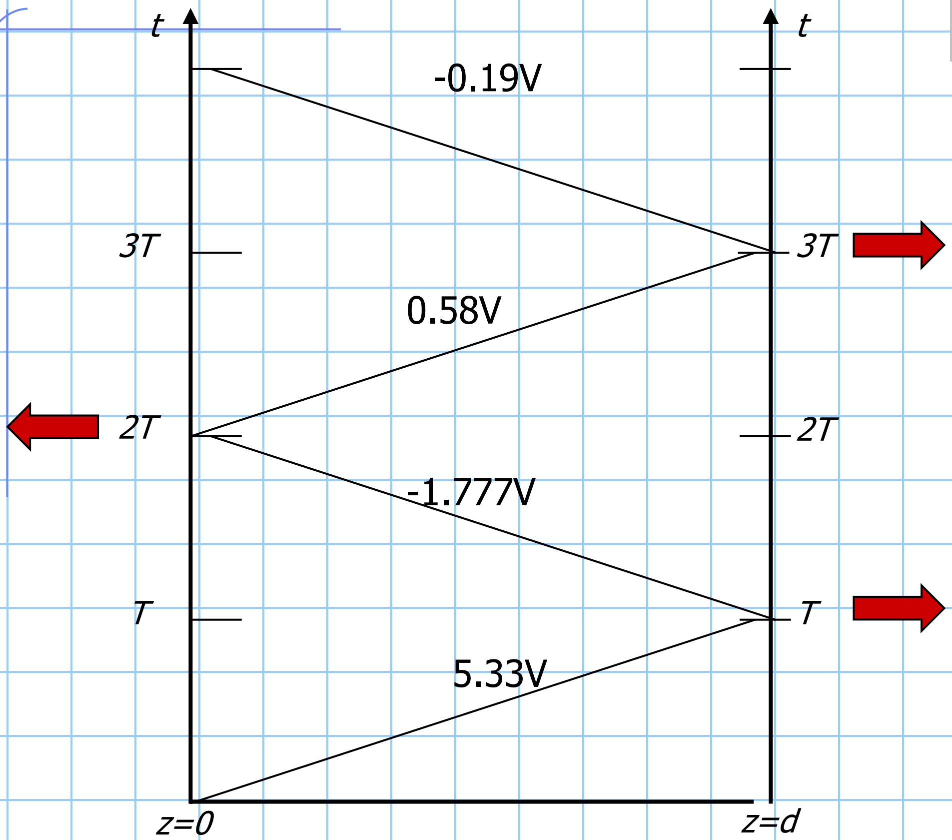

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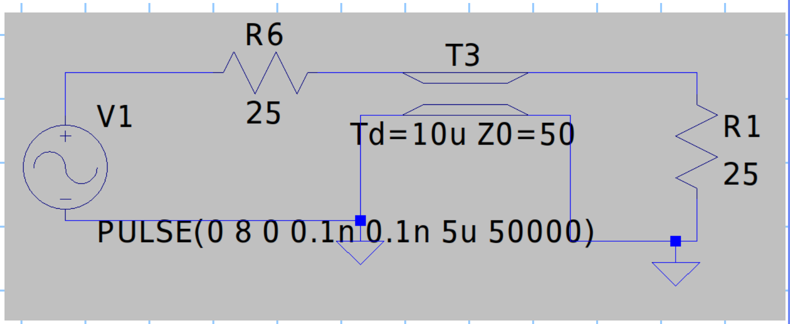

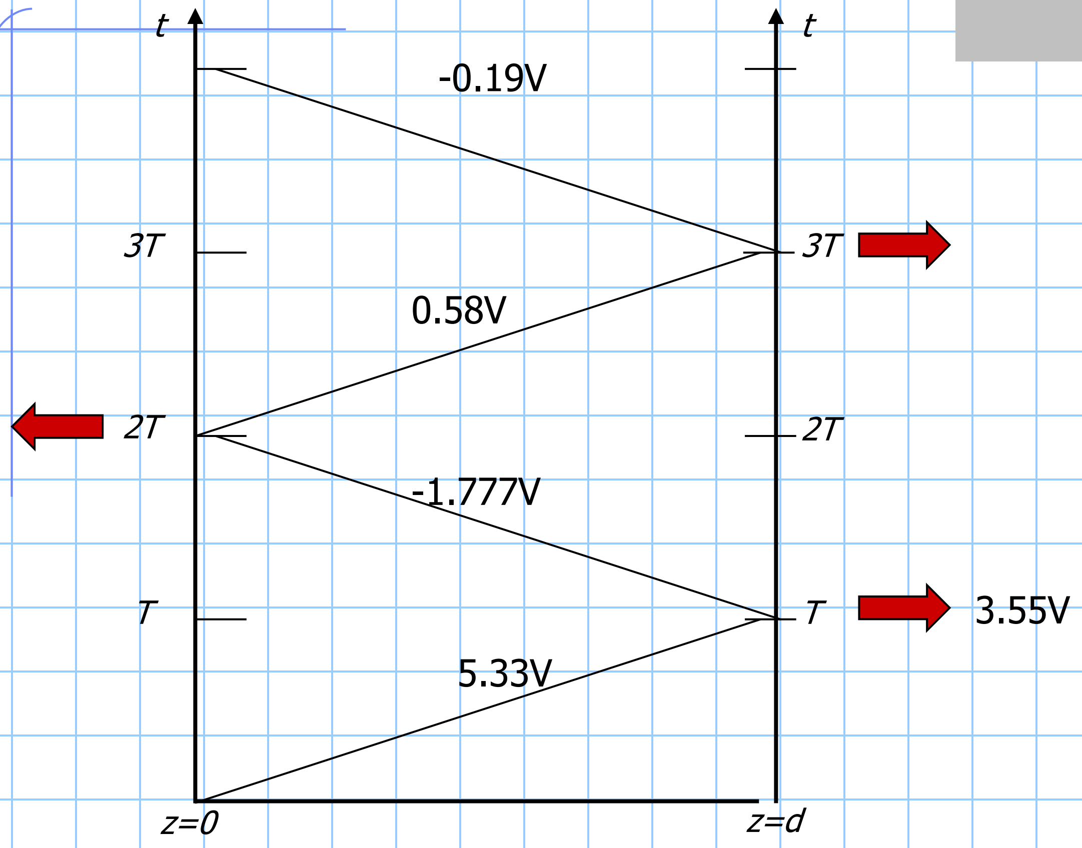

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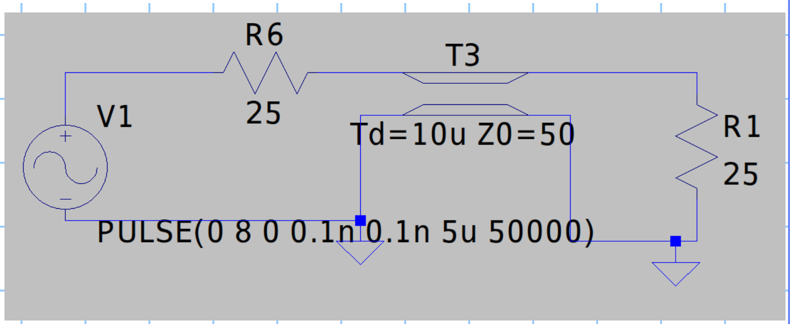

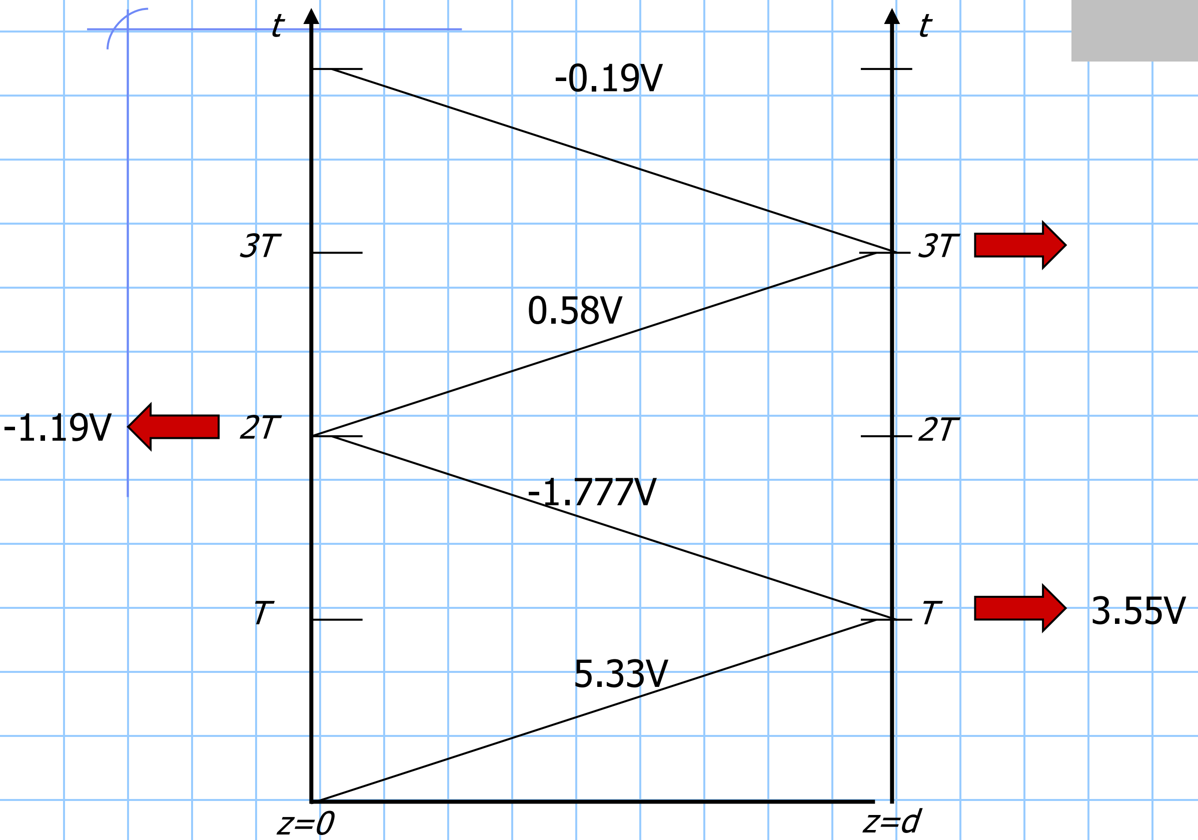

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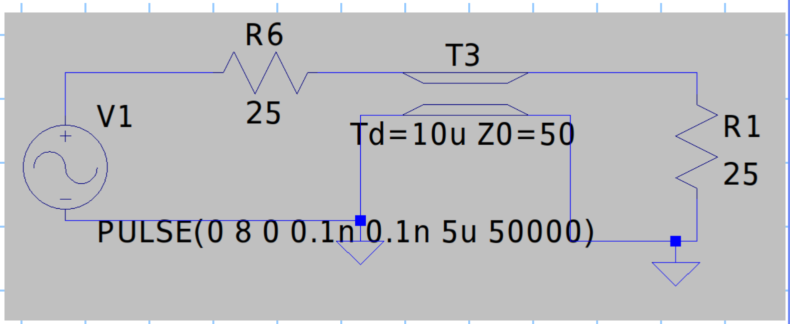

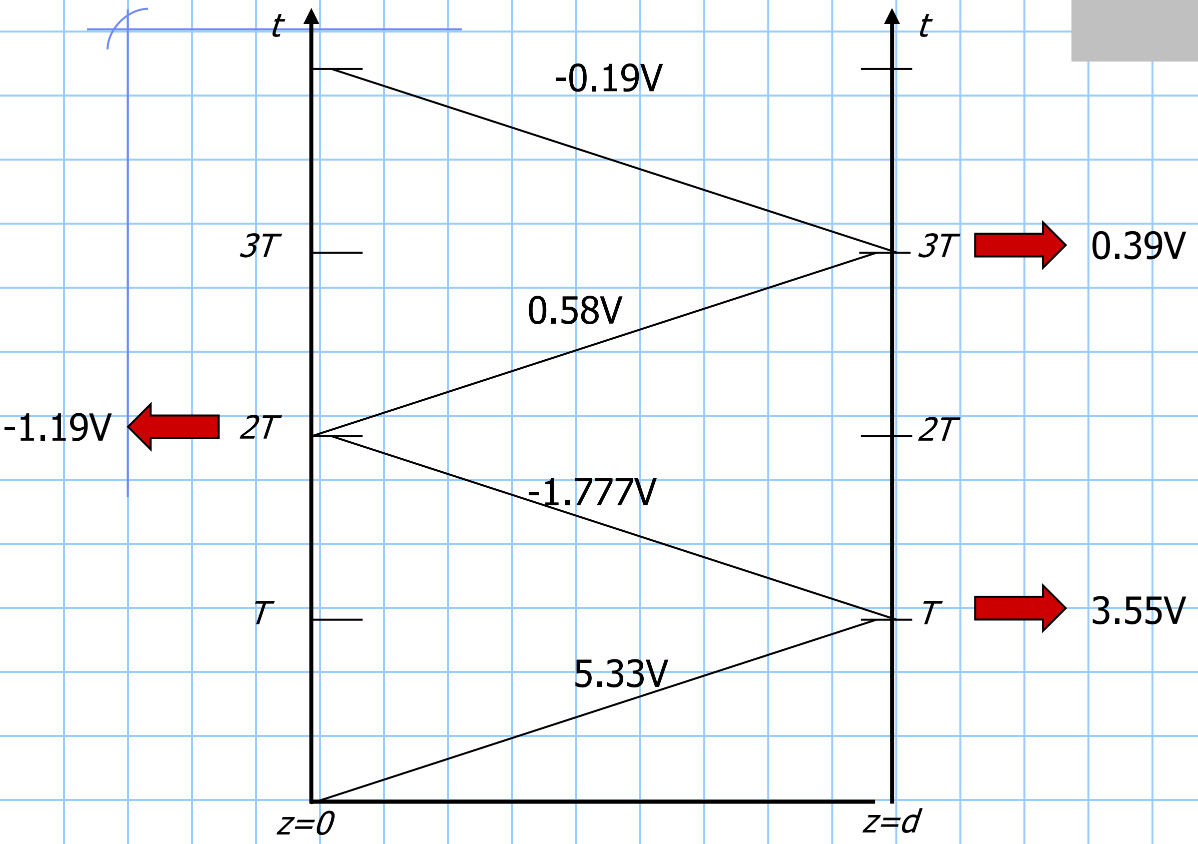

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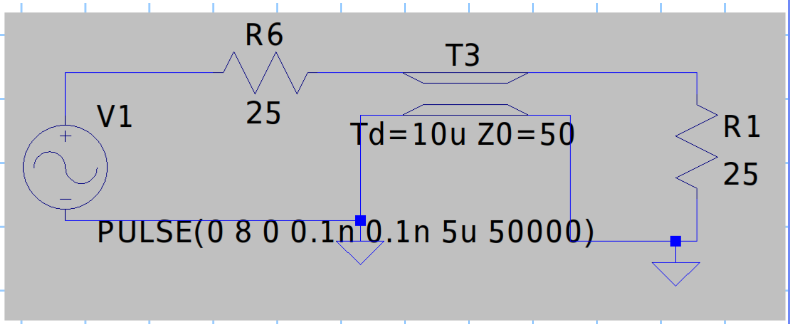
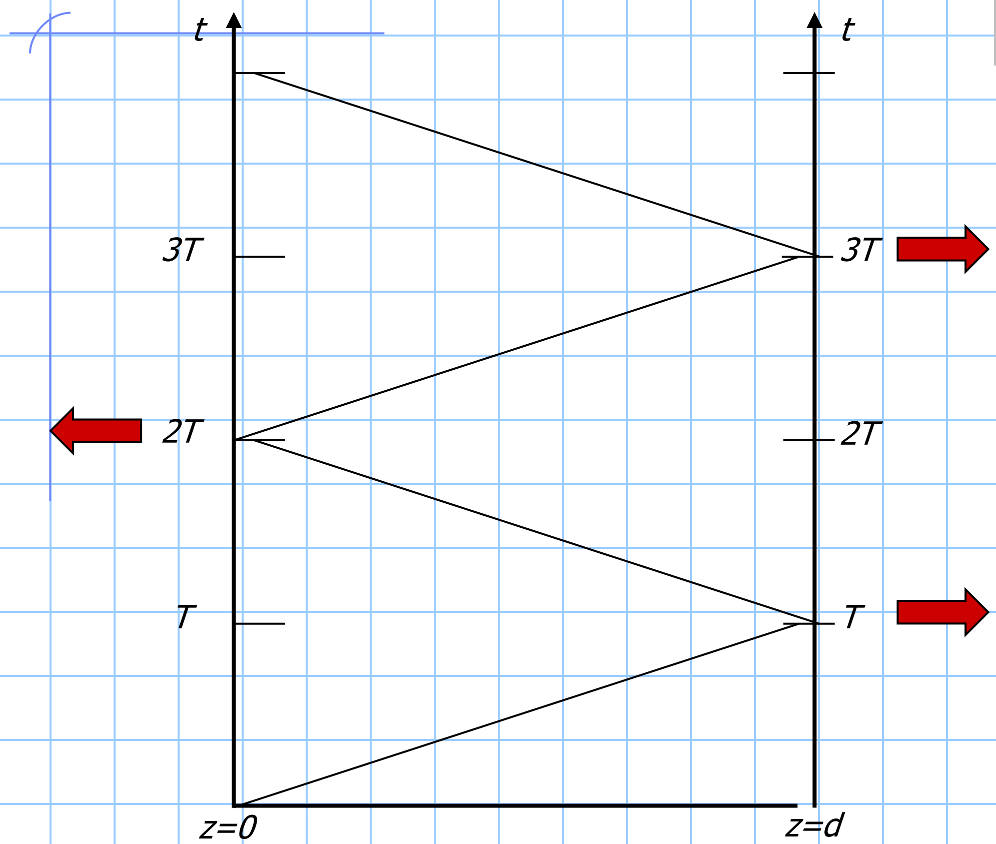
Bounce Diagram Example 2 (DC input)





- **8V switches on at t=0**
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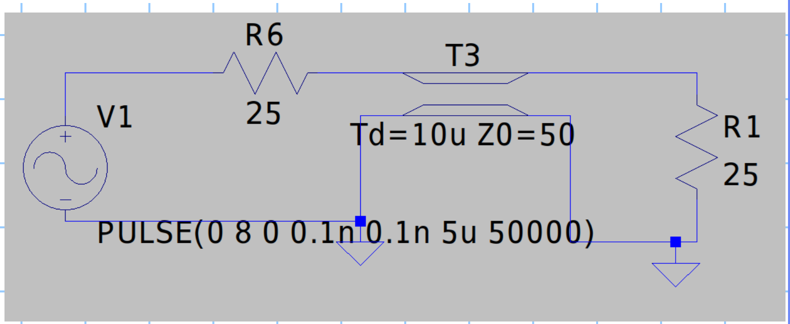

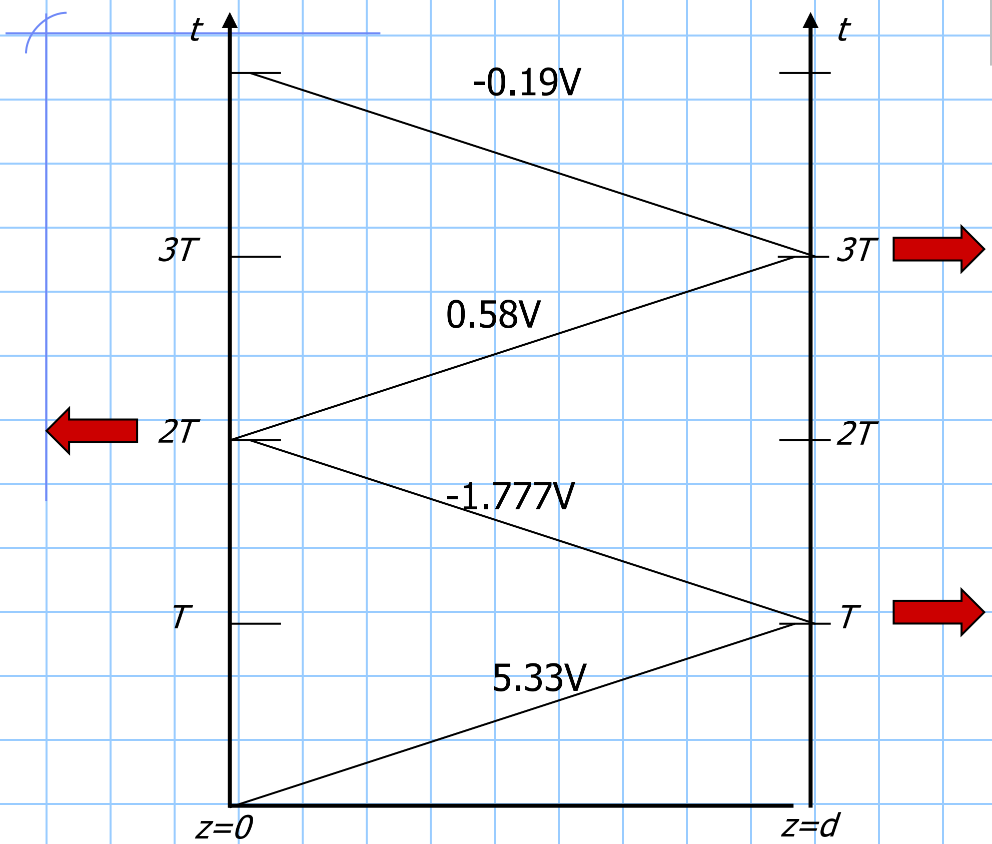

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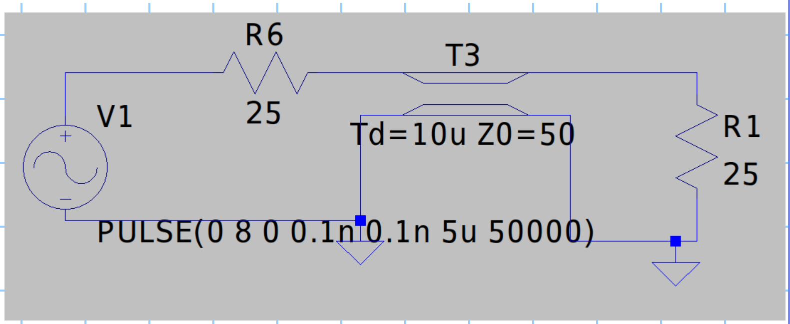

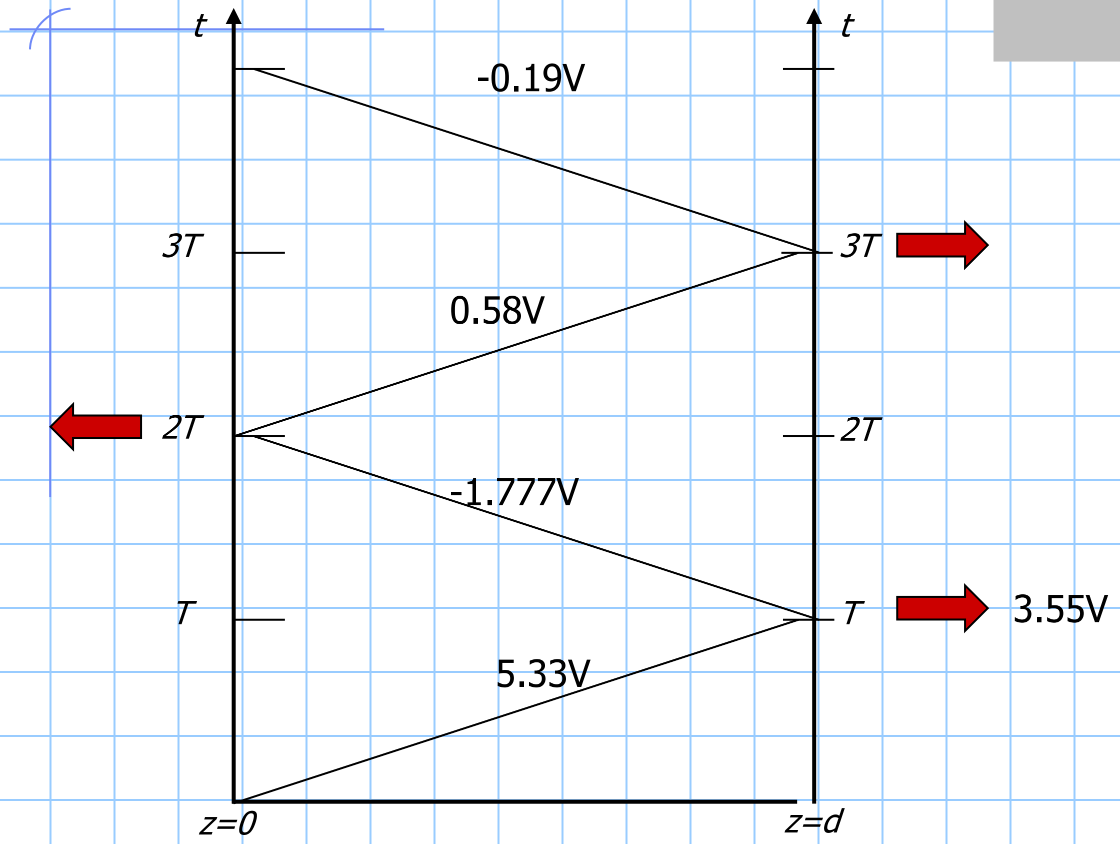

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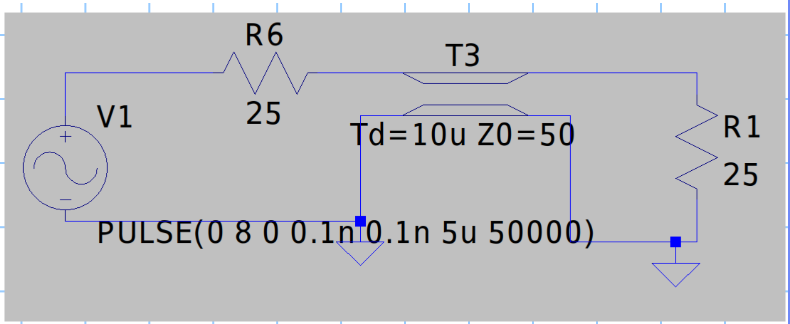

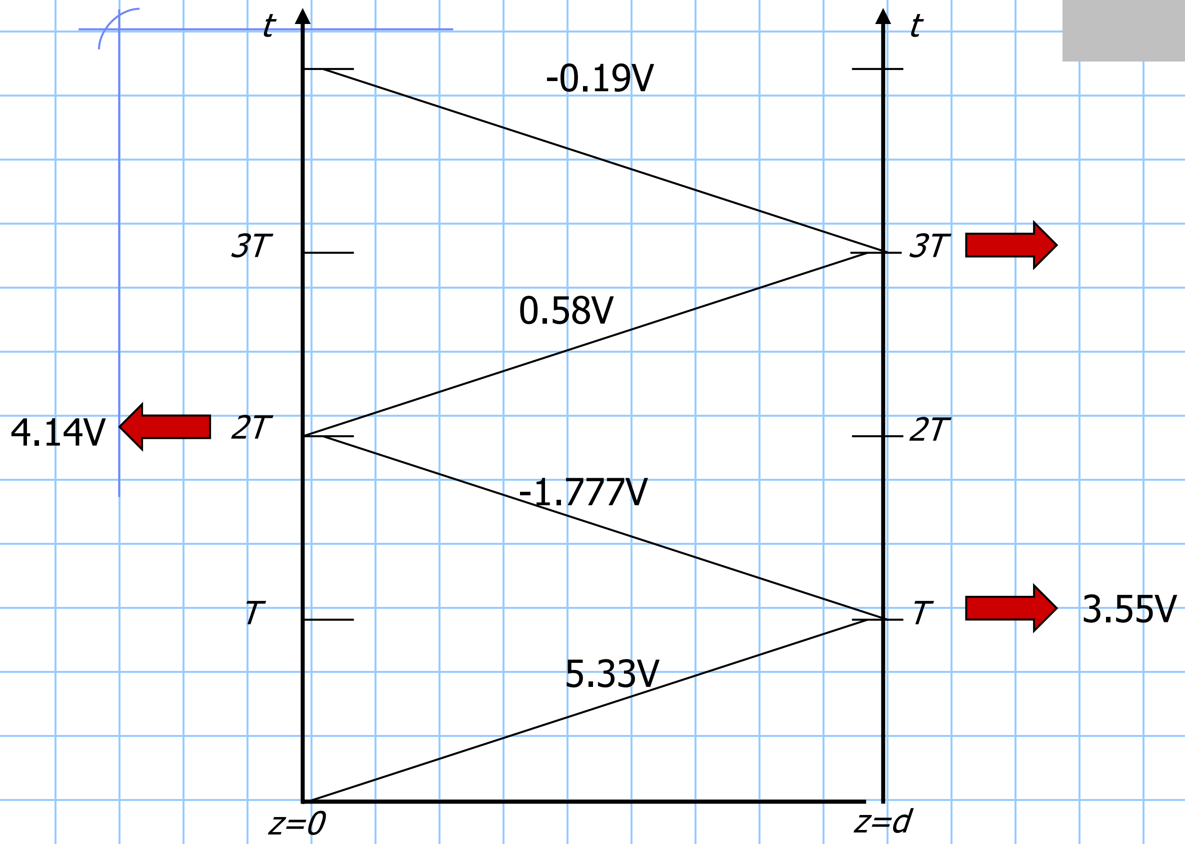

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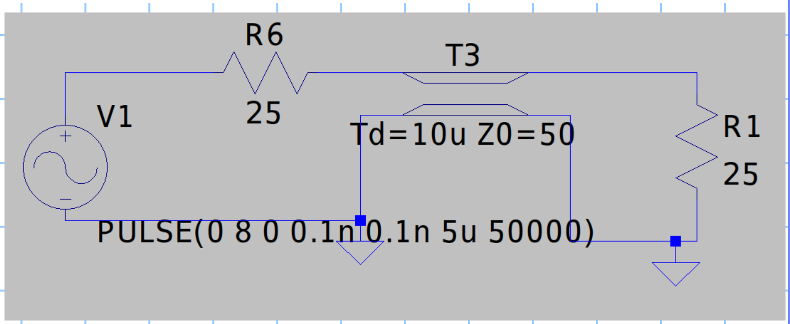

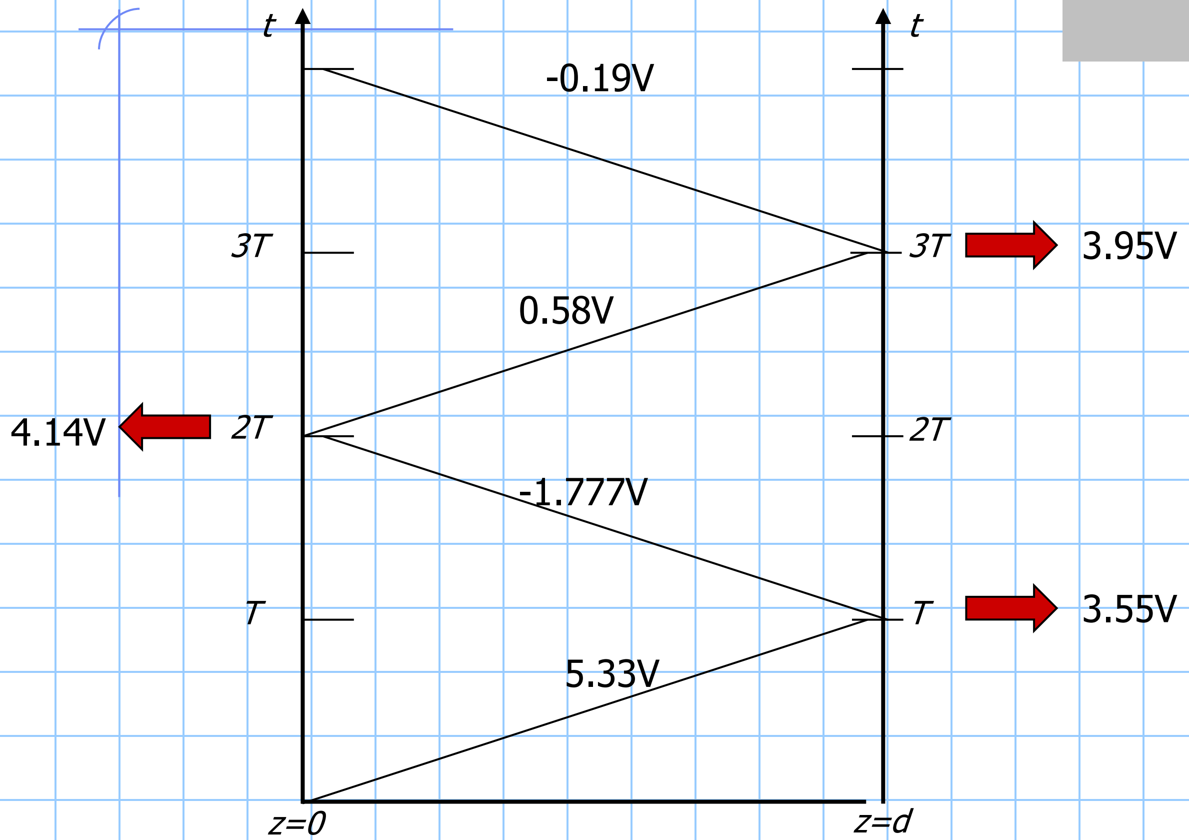

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5.33v - 1.77 V + 0.58 V - 0.19 V = 3.95 V

Review

Bounce Diagram

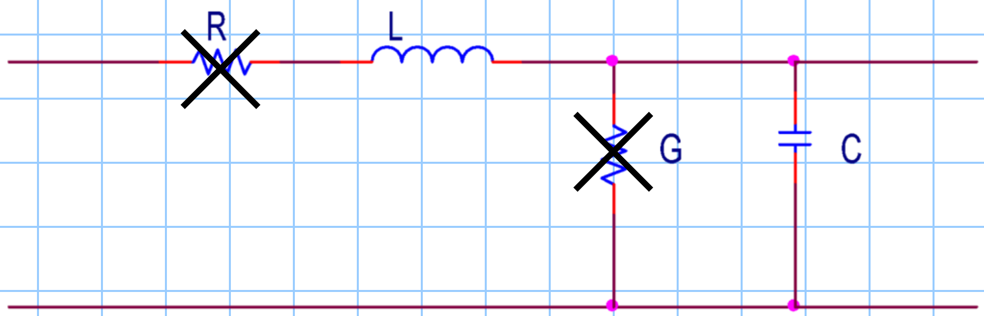
- Do Lecture 5 Exercise 1 on Gradescope. You may work in groups of up to 4.

Lossy T-Lines

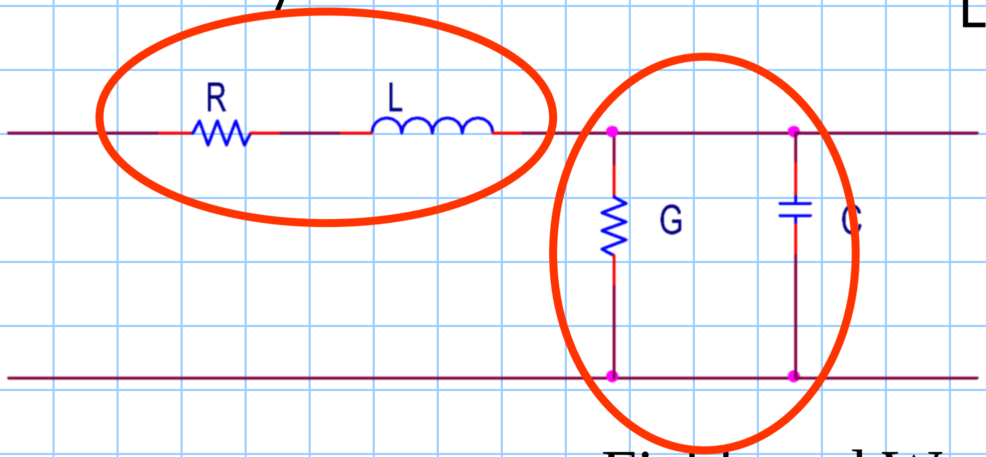
- In our discussion of transmission lines so far, we have assumed that the lines are lossless.
- Real transmission lines are never lossless. So how do we generalize what we've learned to "lossy" lines?

Lossy T-Lines

Lossless Model of TL has no R or G (R′=G′=0):



Lossy Model of TL:



Loss effects due to Resistances:

R - resistance of conductors

G - conductivity of insulators

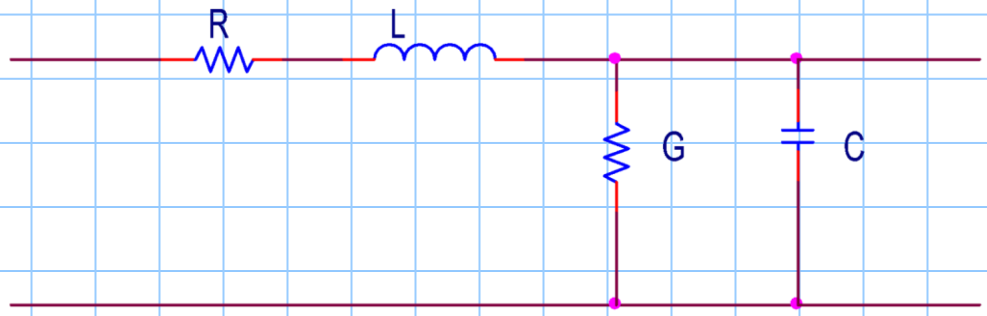
- both are ideally small

Lossy T-Lines

- What is the difference between impedance and admittance?

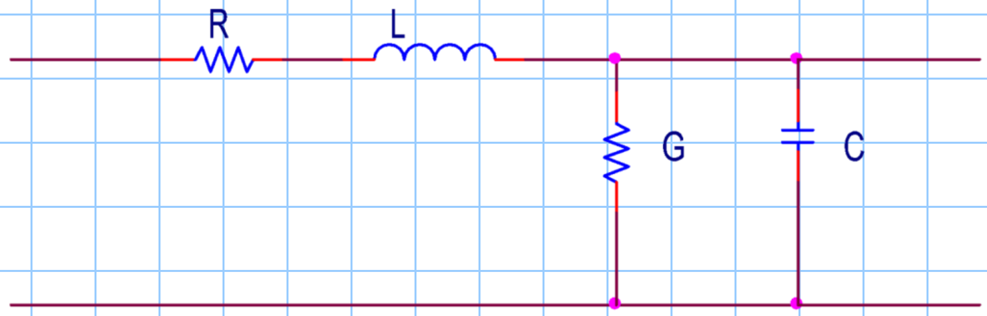
Lossy T-Lines

- What is the difference between impedance and admittance?
  - Admittance is the inverse of impedance
- When we say that R’=G’=0 for a lossless transmission line, what do we mean?



Lossy T-Lines

- What is the difference between impedance and admittance?
  - Admittance is the inverse of impedance
- When we say that R′=G′=0 for a lossless transmission line, what do we mean?
  - We mean that the series per unit length resistance R′ is 0 (short circuit) and the parallel per unit length admittance G′ is 0 (open circuit)



Lossy T-Lines

Lossless Telegrapher Equations

$$\frac{\partial i}{\partial z} = -c\frac{\partial v}{\partial t}$$

$$\frac{\partial v}{\partial z} = -l\frac{\partial i}{\partial t}$$

We have the following phasor property:

$$\frac{d}{dt}\left(z(t)\right) \Leftrightarrow j\omega\tilde{Z}$$

Therefore:

$$-\frac{d\tilde{V}(z)}{dz} = j\omega l\,\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = j\omega c\,\tilde{V}(z)$$

Lossy T-Lines

Lossy Telegrapher Equations

$$-\frac{d\tilde{V}(z)}{dz} = (r + j\omega l)\tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = (g + j\omega c)\tilde{V}(z)$$

Lossy T-Lines

Lossless Telegrapher Equations

Combining the two equations, we get two wave equations:

$$\boxed{\frac{\partial^2 \widetilde{V}(z)}{\partial z^2} - (j\omega l)(j\omega c)\widetilde{V}(z) = 0}$$

$$\boxed{\frac{\partial^2 \widetilde{I}(z)}{\partial z^2} - (j\omega l)(j\omega c)\widetilde{I}(z) = 0}$$

We make a substitution:

$$\boxed{u^2 = (j\omega l)(j\omega c)} \qquad \boxed{u = \sqrt{(j\omega l)(j\omega c)}}$$

Lossy T-Lines

Lossy Telegrapher Equations

$$\frac{d^2\tilde{V}(z)}{dz^2} - (r + j\omega l)(g + j\omega c)\tilde{V}(z) = 0$$

$$\frac{d^2\tilde{I}(z)}{dz^2} - (r + j\omega l)(g + j\omega c)\tilde{I}(z) = 0$$

Alternatively:

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2\tilde{V}(z) = 0 \qquad \frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2\tilde{I}(z) = 0$$

where $\gamma = \sqrt{(r + j\omega l)(g + j\omega c)}$

Lossy T-Lines

Lossy Telegrapher Equations

$$\gamma = \sqrt{(r + j\omega l)(g + j\omega c)}$$

$$\gamma = \alpha + j\beta$$

$$\alpha = \Re\left\{ \sqrt{(r + j\omega l)(g + j\omega c)} \right\}$$

$$\beta = \Im\left\{ \sqrt{(r + j\omega l)(g + j\omega c)} \right\}$$

For a lossless line, α=0 because r=g=0.

Lossy T-Lines

Lossy Wave Equation Forms

$$v(z) = V^{+}e^{-\gamma z} + V^{-}e^{+\gamma z} \qquad v(z) = V^{+}e^{-\gamma z} + V^{-}e^{+\gamma z}$$

$$i(z) = I^{+}e^{-\gamma z} + I^{-}e^{+\gamma z} \qquad i(z) = \frac{V^{+}}{Z_o}e^{-\gamma z} - \frac{V^{-}}{Z_o}e^{+\gamma z}$$

$$v(z) = V^{+}\left(e^{-\gamma z} + \Gamma_L e^{+\gamma z}\right)$$

$$\boxed{\gamma = \alpha + j\beta}$$

$$i(z) = \frac{V^{+}}{Z_o}\left(e^{-\gamma z} - \Gamma_L e^{+\gamma z}\right)$$

Lossy T-Lines

For lossless systems: $$\beta = \omega\sqrt{lc}$$

For lossy systems:

$$\gamma = \alpha + j\beta = \sqrt{(r + j\omega l)(g + j\omega c)}$$

The phasors have the factor:

$$e^{-\gamma z} = e^{-\alpha z} \cdot e^{-j\beta z}$$

Attenuation/loss factor due to resistance

Lossy T-Lines

Lossless Characteristic Impedance

Our two phasor expressions are now related....

$$\boxed{\tilde{V}(z) = V_o^+ e^{-uz} + V_o^- e^{uz}}$$

$$\boxed{\tilde{I}(z) = \frac{u}{j\omega l}\left(V_o^+ e^{-uz} - V_o^- e^{uz}\right)}$$

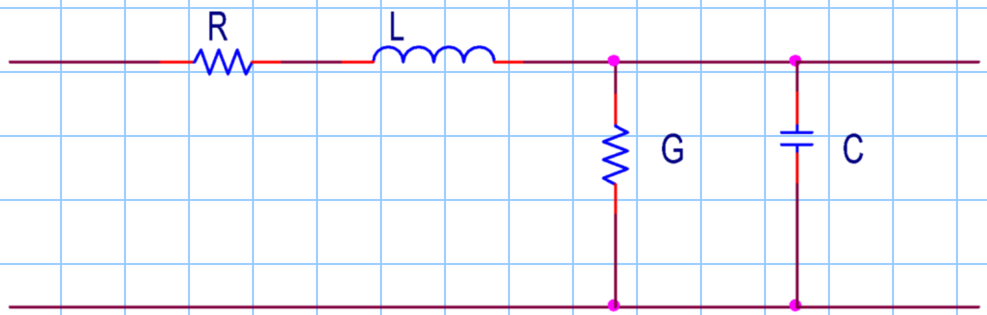
... such that the voltage/current ratio is a constant. (In other words, we have an expression for **impedance!**)

$$\frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \frac{j\omega l}{u} = \frac{j\omega l}{\sqrt{(j\omega l)(j\omega c)}} = \sqrt{\frac{l}{c}}$$

Lossy T-Lines

Lossy Characteristic Impedance

For a lossless system, Z_o represents $= \frac{\hat{V}}{\hat{I}}$ $\qquad Z_o = \sqrt{\frac{l}{c}}$



Replace ***jωl*** with ***r + jωl*** Replace ***jωc*** with ***g + jωc***

Characteristic Impedance

$$Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}}$$

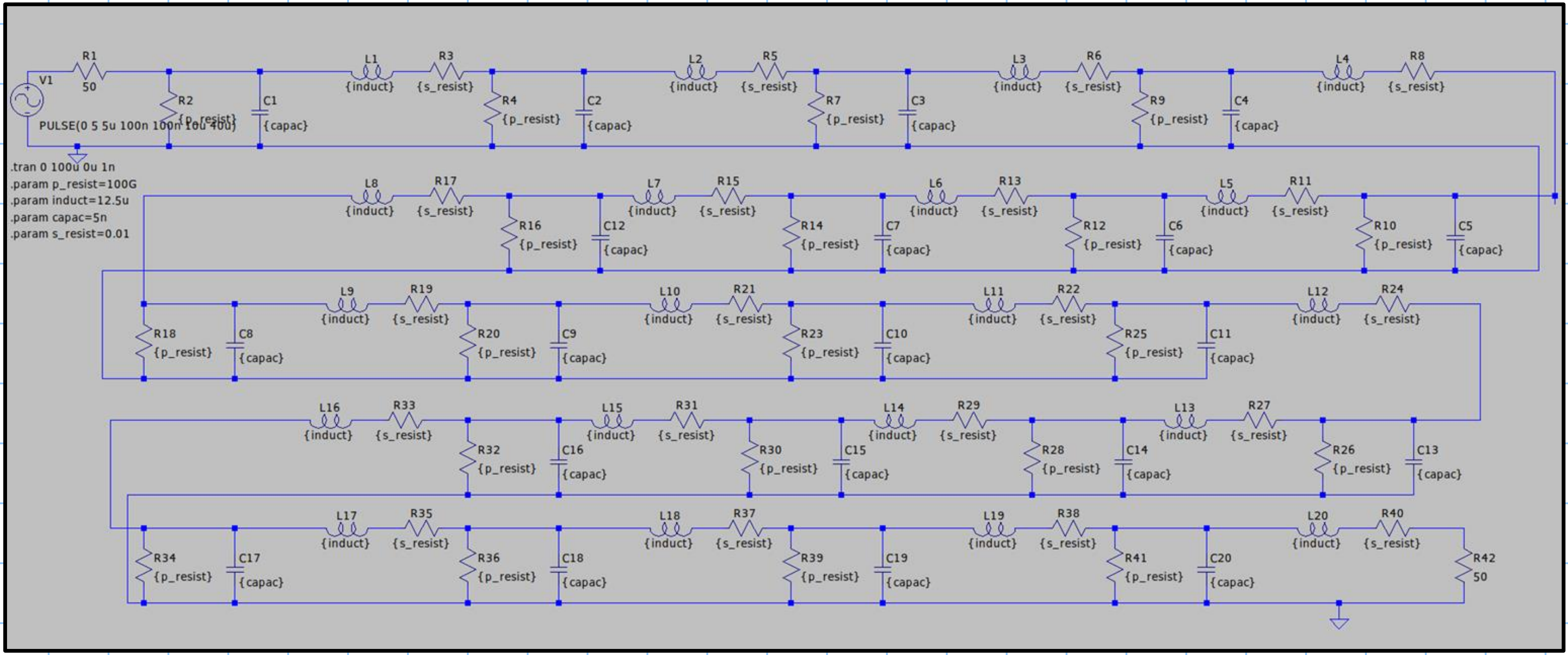
The lossy Z_0 can be complex (and therefore have a phase)!

Lossy T-Lines

It's simulation time.

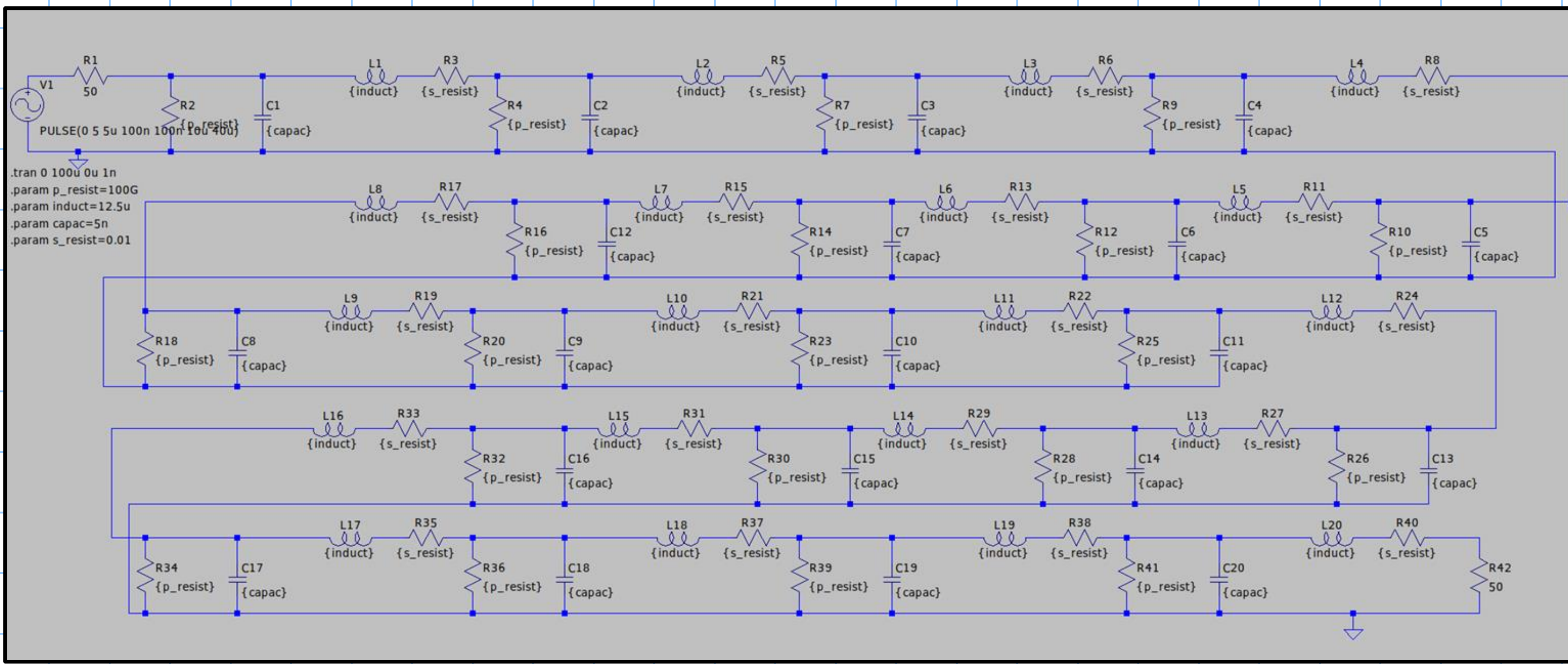
Lossy T-Lines

Example 1 - 1 km of RG-58 cable
With 20 segments, how much length does each segment represent?



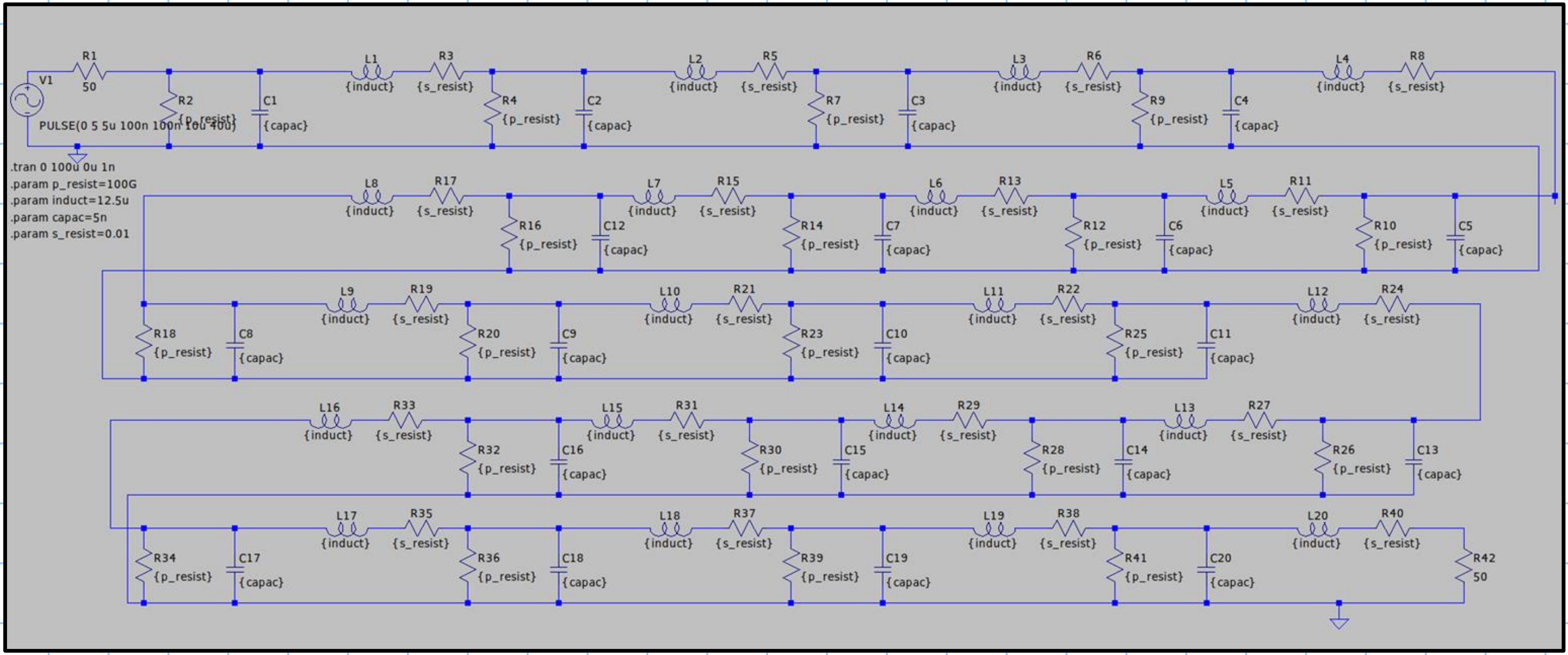
Lossy T-Lines

Velocity factor of RG-58 cable is ⅔ the speed of light (L’s and C’s have been scaled accordingly.) 1 km of cable will therefore have a time delay of 5μs.



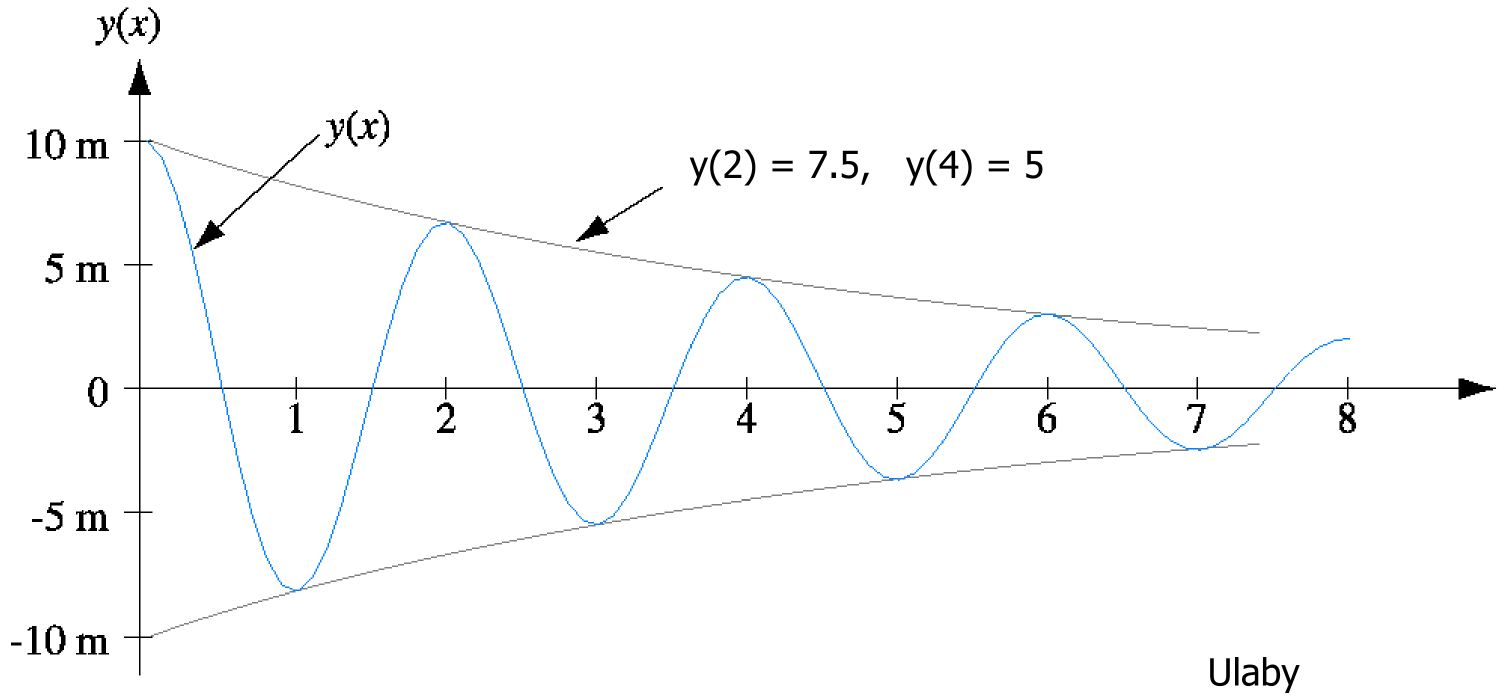
Lossy T-Lines

Let’s change the series resistance to 10 per segment.
What changes do you observe?



Lossy T-Lines

- The attenuation factor α is due to ohmic losses on the line.
- Looking at a plot like the one below, how do you find α and β?



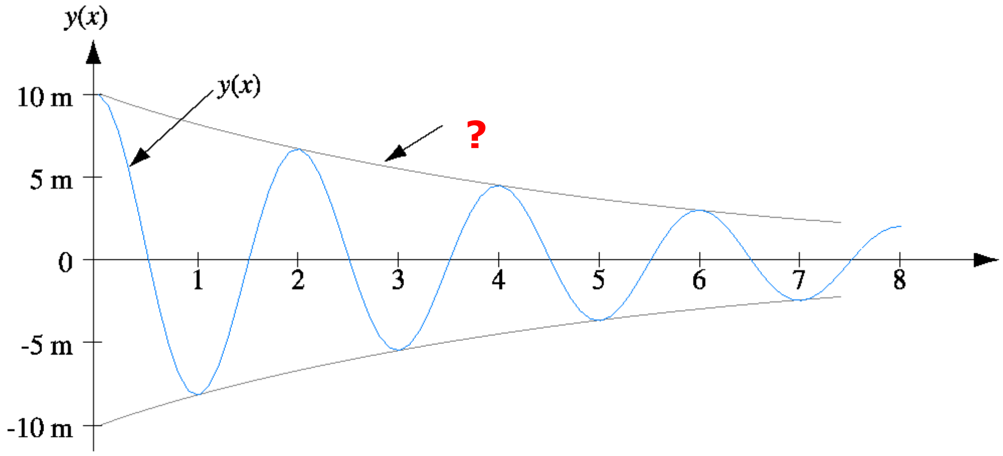
Lossy T-Lines

$$v(z) = V^{+}\left(e^{-\gamma z} + \Gamma_L e^{+\gamma z}\right)$$

Attenuation factor (ohmic losses)

$$e^{-\gamma z} = e^{-(\alpha + j\beta)z} = e^{-\alpha z} e^{-j\beta z}$$

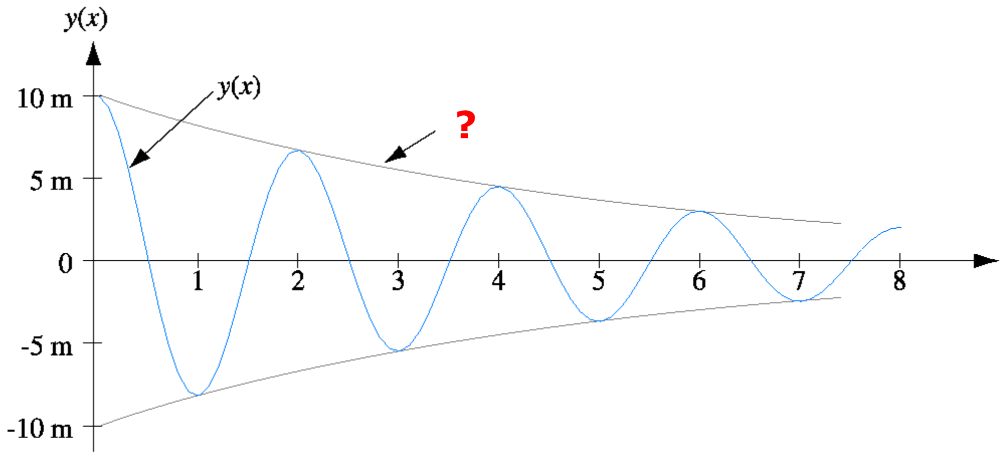
Phase factor (wave propagation)



Lossy T-Lines

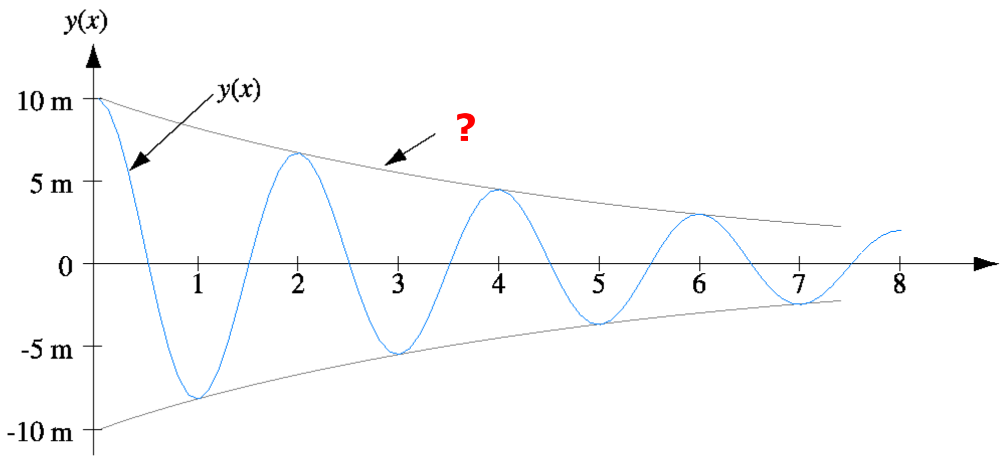
$$10e^{-4\alpha} = 5 \qquad e^{-4\alpha} = 0.5$$

$$-4\alpha = ln(0.5) \qquad \alpha = \frac{-ln(0.5)}{4} = 0.173$$



Lossy T-Lines

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{2} = \pi$$



Lossy T-Lines

- Do Lecture 5 Exercise 2 on Gradescope in groups of up to 4.

Low-Loss Lines

- Although real transmission lines are not lossless, in practice the loss of a well-engineered t-line is quite low.
- **Example**: Assume the following: *f = 1MHz* & standard RG58 cable parameters & r per unit length of 0.1 Ohm per meter, the wave is seen to attenuate markedly in *2000 meters.*

Low-Loss Lines

- Using the Binomial Theorem $\sqrt{1+x} \approx 1 + \frac{x}{2}$ for *x<<1*.

$$Z_o = \sqrt{\frac{r + j\omega l}{g + j\omega c}} \approx \sqrt{\frac{r + j\omega l}{j\omega c}} \approx \sqrt{\frac{j\omega l}{j\omega c}}\sqrt{1 + \frac{r}{j\omega l}} \approx \sqrt{\frac{l}{c}}\left(1 - j\frac{r}{2\omega l}\right)$$

$$\gamma = \sqrt{(r + j\omega l)(g + j\omega c)} \approx \sqrt{(r + j\omega l)(j\omega c)} \approx \sqrt{(j\omega l)(j\omega c)}\sqrt{1 + \frac{r}{j\omega l}}$$

$$\gamma = \alpha + j\beta \approx j\omega\sqrt{lc}\left(1 - j\frac{r}{2\omega l}\right)$$

Low-Loss Lines

$$Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}} \approx \sqrt{\frac{r + j\omega l}{j\omega c}} \approx \sqrt{\frac{j\omega l}{j\omega c}}\sqrt{1 + \frac{r}{j\omega l}} \approx \sqrt{\frac{l}{c}}\left(1 - j\frac{r}{2\omega l}\right)$$

Note that in the low-loss case, the following will be true:

$$|Z_0| \approx |\sqrt{\frac{l}{c}}(1 - j\frac{r}{2\omega l})| \approx \sqrt{\frac{l}{c}}$$

This equation is sufficient when you do not care about the phase of Z_0. (This angle is small and may often be ignored in practice.)

Low-Loss Lines

$$\gamma = \alpha + j\beta \approx j\omega\sqrt{lc}\left(1 - j\frac{r}{2\omega l}\right)$$

- The propagation and attenuation constants become

$$j\beta \approx j\omega\sqrt{lc} \qquad \alpha \approx \omega\sqrt{lc}\left(\frac{r}{2\omega l}\right) = \frac{r}{2\sqrt{\frac{l}{c}}} = \frac{r}{2Z_0}$$

Low-Loss Lines

- Assume the following: *f = 1MHz* & standard RG58 cable parameters & r per unit length of 0.1 Ohm per meter, the wave is seen to attenuate markedly in *2000 meters.*

$$\alpha = \frac{r}{2Z_0} = \frac{0.1}{(2)(50)} = 0.001$$

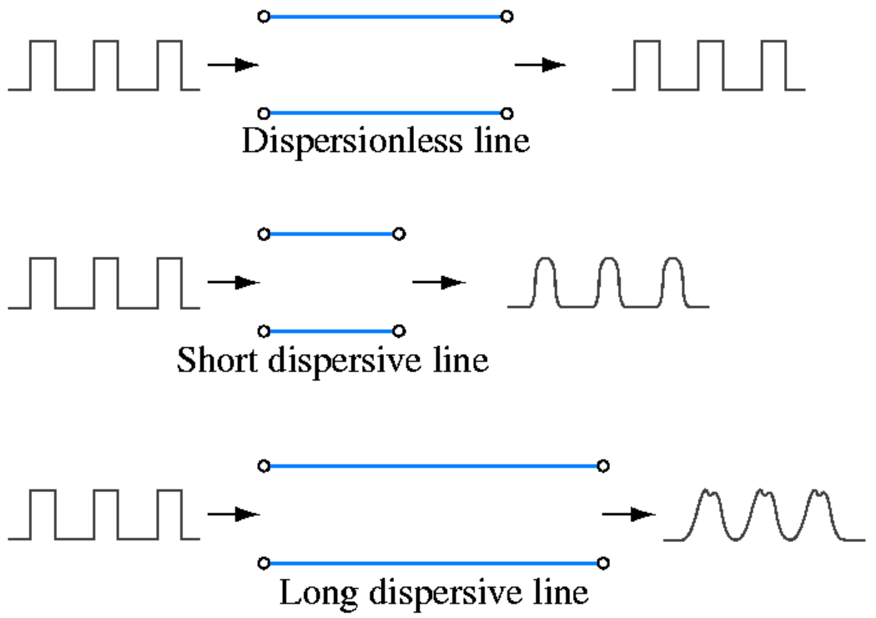
$$\beta = \omega\sqrt{lc} = (10^6)\sqrt{(0.25 * 10^{-6})(100 * 10^-12)} =$$
$$0.005$$

T-Line Parameters

- Aside from attenuation (loss of amplitude), what other problems could you have with your signal after running it through a very long transmission line with losses?

T-Line Parameters

Dispersion: A dispersive transmission line will have frequency-dependent impedance behavior, leading to distortion of signals. (Keep in mind that a square pulse is composed of a series of harmonic frequencies)



T-Line Parameters

- Note that the propagation constant varies with frequency

$$\gamma = \alpha + j\beta = \sqrt{(r + j\omega l)(g + j\omega c)}$$

- Z_o is also frequency dependent and not purely resistive

$$Z_o = \sqrt{\frac{r + j\omega l}{g + j\omega c}}$$

Distortionless Lines

- Distortion in a transmission line limits its useful length. Attenuation can be addressed by adding amplification. However, distorted signals cannot generally be undistorted, so a method needed to be found to eliminate it.
- Remarkably, lines can be made distortionless by adding loss. That is, we can trade additional attenuation for clarity of signal.

Distortionless Lines

For practical lines, the conductance per unit length g is negligible. Thus, we will add loss between the conductors so that

$$\frac{r}{l} = \frac{g}{c}$$

This is called the **Heaviside condition** and it can be achieved with periodic lumped shunt resistors.

Distortionless Lines

- For this combination of parameters

$$\gamma = \alpha + j\beta = \sqrt{(r + j\omega l)(\frac{rc}{l} + j\omega c)} = \sqrt{\frac{c}{l}}\left(r + j\omega l\right)$$

$$\alpha = r\sqrt{\frac{c}{l}} \qquad \beta = \omega\sqrt{lc}$$

Distortionless Lines

- The characteristic impedance also simplifies

$$Z_o = \sqrt{\frac{r + j\omega l}{g + j\omega c}} = \sqrt{\frac{r + j\omega l}{\frac{rc}{l} + j\omega c}} = \sqrt{\frac{l}{c}}\sqrt{\frac{\frac{r}{l} + j\omega}{\frac{r}{l} + j\omega}} = \sqrt{\frac{l}{c}}$$

Distortionless Lines

- Consider our simulated "bad RG-58" cable. What g is required to make it distortionless?

$$\frac{\frac{10\Omega}{50m}}{0.25\mu H/m} = \frac{g}{100pF/m}$$

Distortionless Lines

- Consider our simulated “bad RG-58” cable. What g is required to make it distortionless?

$$\frac{\frac{10\Omega}{50m}}{0.25\mu H/m} = \frac{g}{100pF/m}$$

$$g = 0.2mS/m$$

What resistance would this be per unit length?

Distortionless Lines

What resistance would this be per unit length?

$$g = 0.2mS/m$$

$$(0.2mS/m)(50m) = 10mS$$

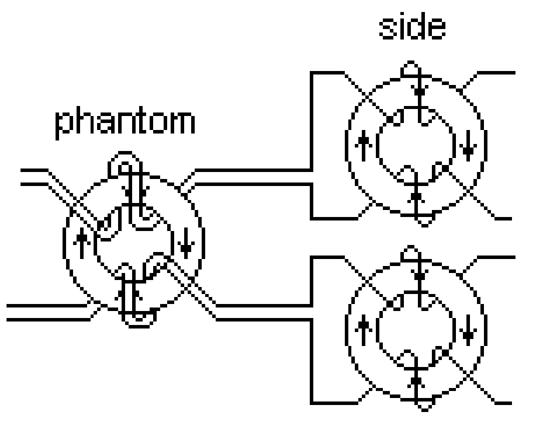
$$\frac{1}{10mS} = 100\Omega$$

(value of each parallel resistor in the t-line simulation, equivalent to 50m)

$$r = \frac{100\Omega}{50m} = 2\Omega/m$$

Distortionless Lines

- In the early days of telephony, Heaviside proposed making lines distortionless. This was done by adding inductance rather than conductance since the losses were not increased significantly.



Campbell-Shaw Loading

http://www.du.edu/~jcalvert/tech/cable.htm

Distortionless Lines

- Adding these components made it possible for phone calls to go from NY to Chicago.
- Then in the 1850s an even more impressive engineering feat was achieved: the first transatlantic undersea cable. This would not have been possible without knowledge of the Heaviside Condition.

Distortionless Lines

Wrap-Up

Modern submarine cable with repeaters

prog.world