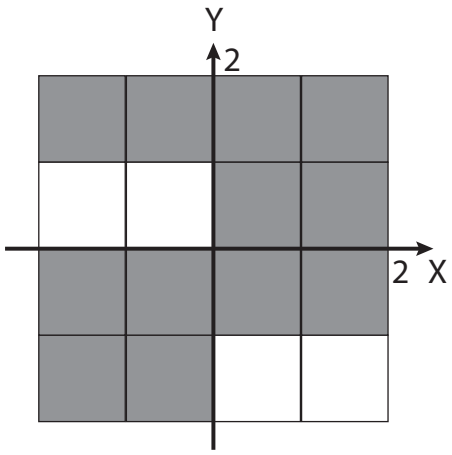
Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 2500: Engineering Probability, Spring 2023

Homework #2: due Wednesday, Feb. 8th.
Show all work for full credit!
Submit your work as a single PDF on Gradescope, labeling each problem number with a page.

1. We randomly pick a point (X, Y) in the shaded region below (that is, we have an equal probability of $\frac{1}{12}$ of landing in any of the shaded squares).



Consider the following events:

$A = \{X > 0\}$
$B = \{Y > 0\}$
$C = \{Y > 1\}$

In each subproblem below, determine whether or not the pair of events is independent. You must provide a computation and/or a diagram of the events and their intersection for full credit, not just a yes or no answer.

(a) (5 points.) A and B.

(b) (5 points.) A and C.

(c) (5 points.) B and C.

2. Jennifer Walters, Attorney at Law, defends her clients in a series of cases. She wins each case with probability $\frac{1}{3}$, independent of the results of other cases. Let C be the number of cases she requires to obtain her first win.

(a) (5 points.) What type of random variable is C? (i.e., one of uniform, geometric, binomial, Poisson...)

(b) (10 points.) Determine the probability mass function of C. Give your answer as a closed-form equation, and sketch the PMF indicating the first 4 non-zero numerical values to 4 decimal places.

(c) (15 points.) Compute $P(C \leq 8)$ using the formula for a finite geometric sum.

More fun on the next page ⟶

3. Using the same setup as above, Jennifer defends 7 cases. Let W be the number of wins.

   (a) (5 points.) What type of random variable is W? (i.e., one of uniform, geometric, binomial, Poisson...)

   (b) (10 points.) Compute $P(W = 4)$.

   (c) (10 points.) Compute $P(W \geq 5)$.

   (d) (10 points.) Determine the probability that her second win was on case 4, given that she won exactly 4 cases.

4. (10 points.) Now we consider Jennifer's entire career, in which she defended 6000 cases. Use Bernoulli's theorem to put a lower bound on the probability that she won between 1800 and 2200 of them.

5. The number of Dad Jokes that Ted Lasso makes is modeled as a Poisson random variable with an average of 1 joke every 30 minutes.

   (a) (5 points.) What is the probability that there are no jokes within a given 2-hour practice?

   (b) (5 points.) What is the probability that there are at least 3 jokes in the same 90-minute game?