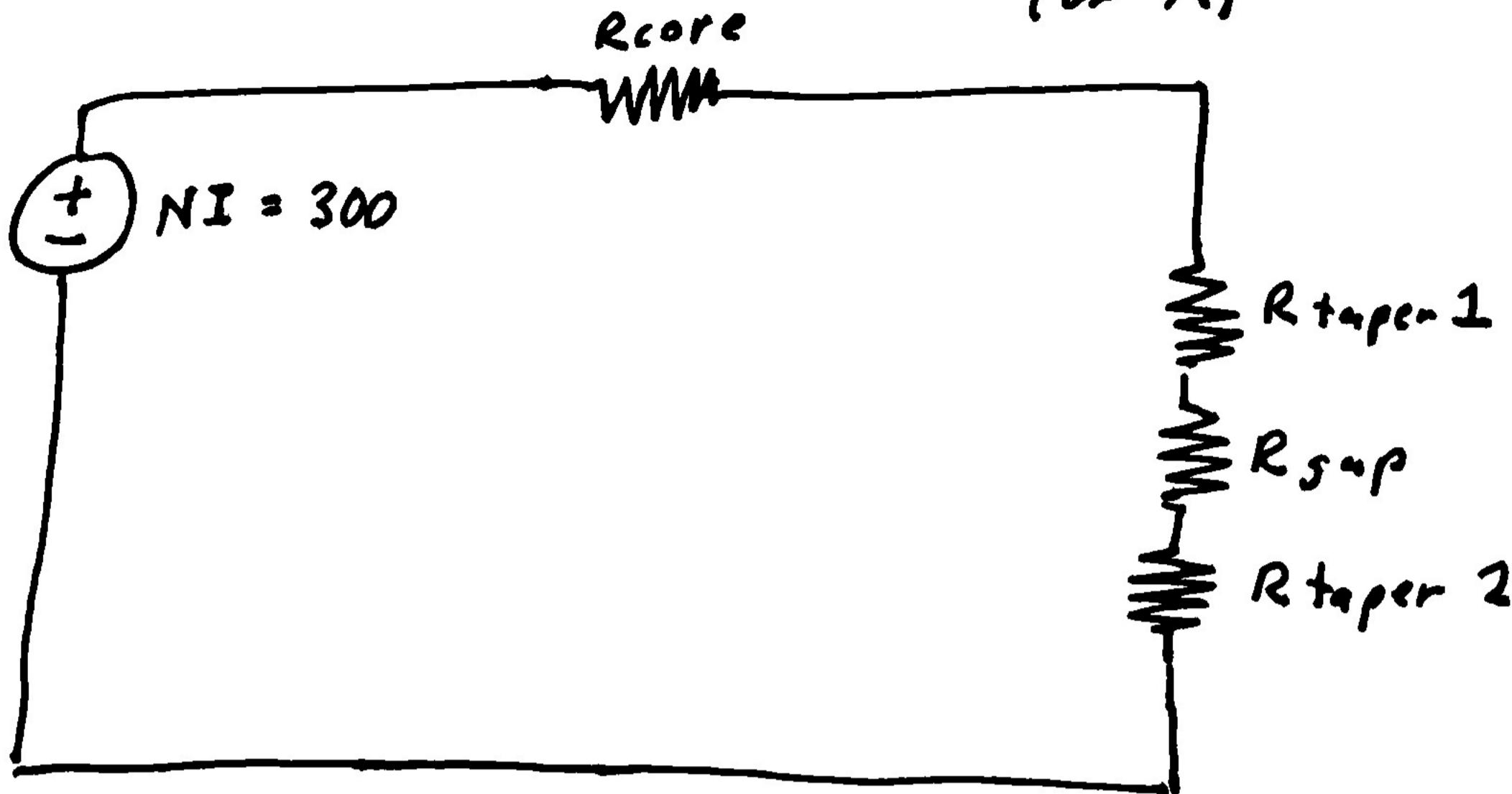


① a.) $NI = (100\text{mA})(3000) = 300 \text{ ampere-turns (MMF)}$



$$R_{core} = \frac{l}{NA} = \frac{500\text{cm} - 80\text{cm}}{(5000\mu_0)(5\text{cm})^2} = 2.67 \times 10^5 \text{ H}^{-1}$$

$$R_{taper\ 1} = R_{taper\ 2} = \frac{(100\text{cm} - 20\text{cm})/2}{(5000\mu_0)(3.5\text{cm})^2} = 5.2 \times 10^4 \text{ H}^{-1}$$

$$R_{gap} = \frac{20\text{cm}}{(500\mu_0)(2\text{cm})^2} = 7.95 \times 10^5 \text{ H}^{-1}$$

$$\Psi = \frac{NI}{R_{core} + R_{taper\ 1} + R_{taper\ 2} + R_{gap}}$$

$$\Psi = \frac{300 \text{ A}}{1.165 \times 10^6 \text{ H}^{-1}} = 2.59 \times 10^{-4} \text{ Wb}$$

$$b.) \Delta (\text{flux linkage}) = N\Psi = (3000)(2.57 \times 10^{-4} \text{ Wb}) \\ = 0.771 \text{ Wb}$$

$$L = \frac{\Delta}{I} = 7.71 \text{ H}$$

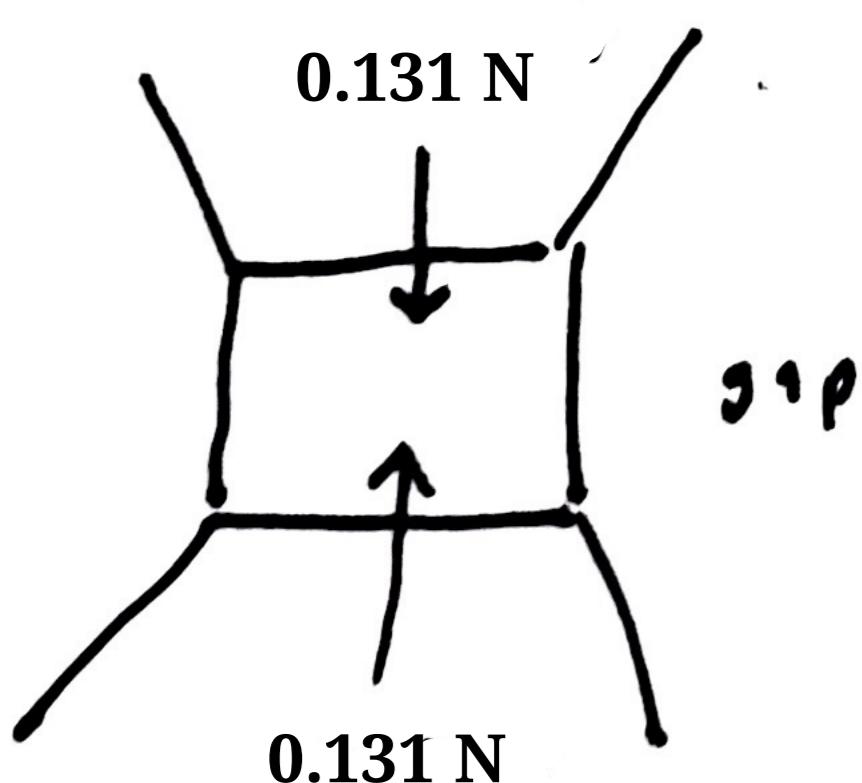
$$\therefore w_m = \frac{1}{2} \frac{|B|^2}{\mu} \quad \begin{matrix} \text{recall that } \vec{B} \text{ represents} \\ \text{flux density.} \end{matrix}$$

$$B = \frac{\Psi}{A} = \frac{2.57 \times 10^{-4} \text{ Wb}}{(2\text{cm})^2} = 0.6425 \text{ T}$$

$$w_m = \frac{1}{2} \frac{(0.6425 \text{ T})^2}{500 \mu_0} = 328.5 \text{ J/m}^3$$

$$328.5 \text{ J/m}^3 \cdot (2\text{cm})^2 = 0.131 \text{ N}$$

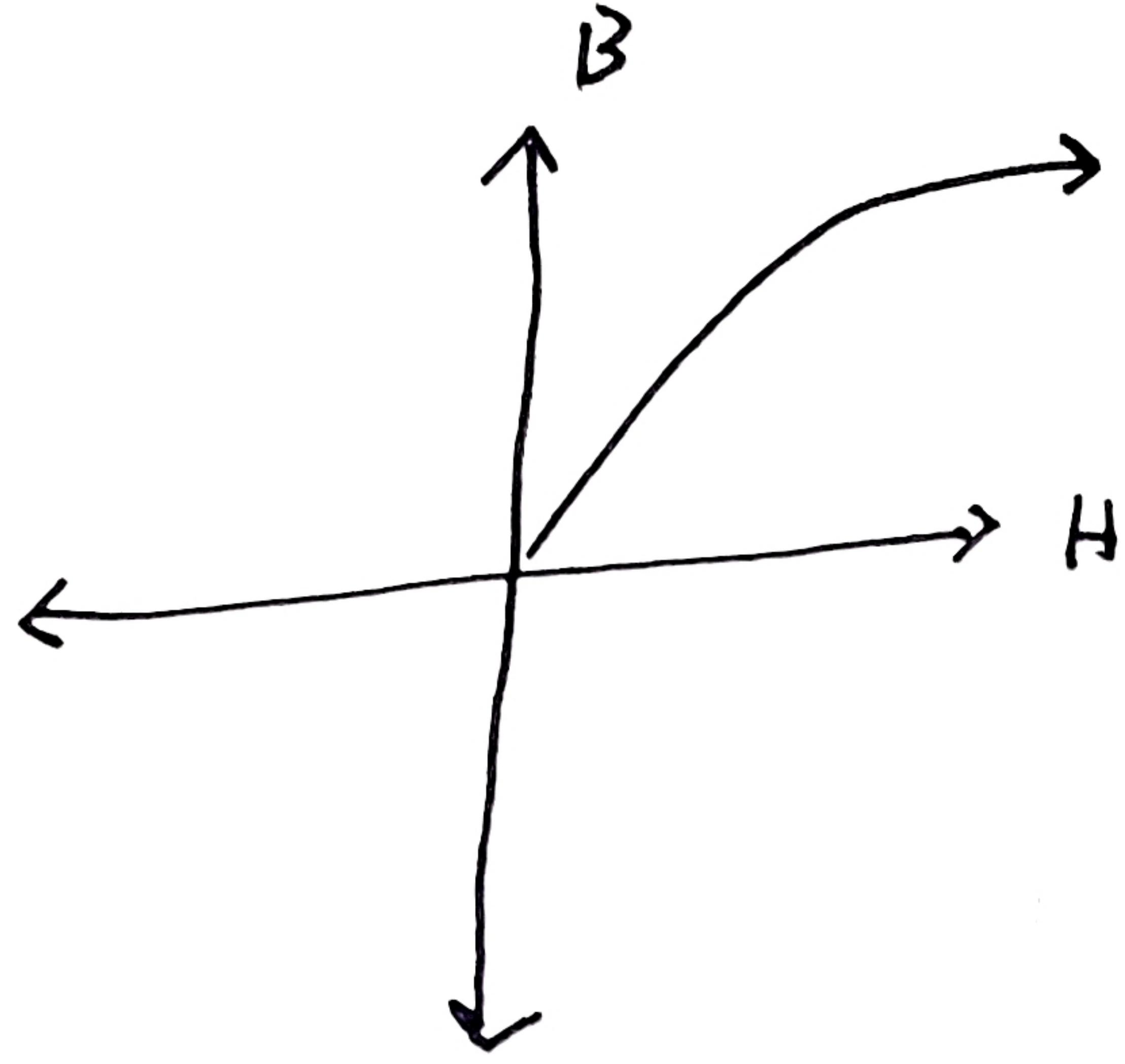
(pushing the tapered
region into the gap
on both sides)



Note that there is also a 0.0258 N force pushing against the 0.258 N force in both directions due to the energy density inside the tapered regions.

(We have neglected to show the force here since it is small.)

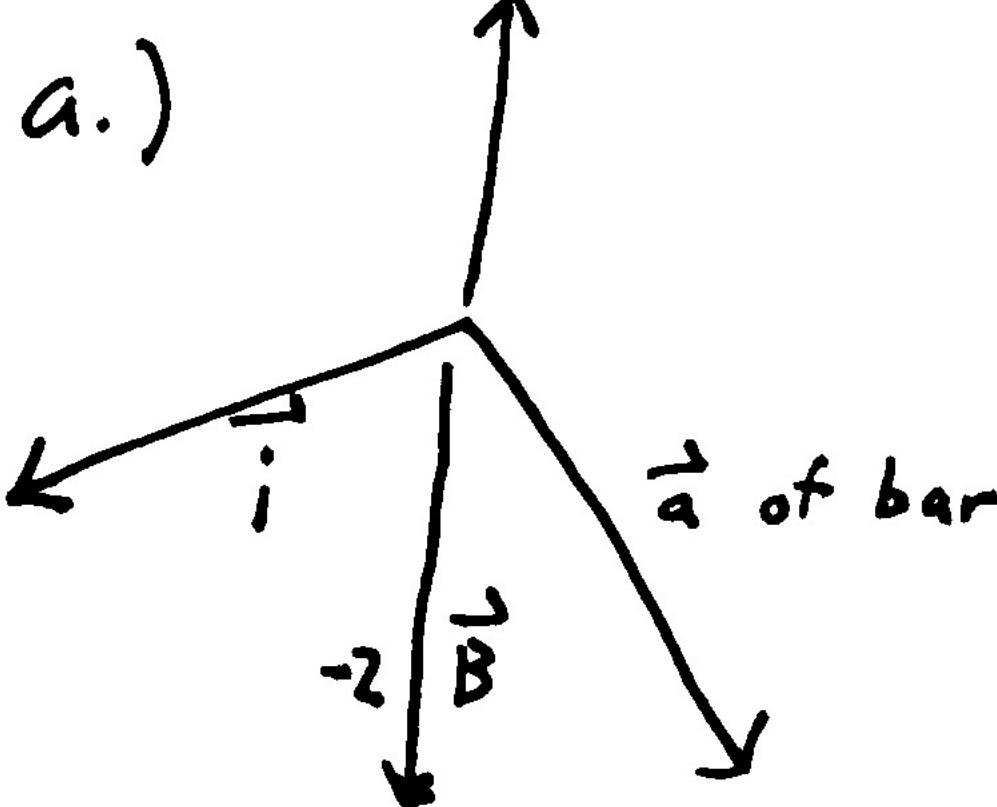
d.)



As shown above, the $H-B$ curve begins to flatten at higher H (caused by high current).

This means that N is effectively lower. So for additional current added beyond this point. L will effectively be lower.

(2.)



$$\vec{F} = \vec{I}\vec{l} \times \vec{B}$$

Magnetic field must point into the page.
If the rails are laying flat on the ground, this means into the ground.

x = projectile distance

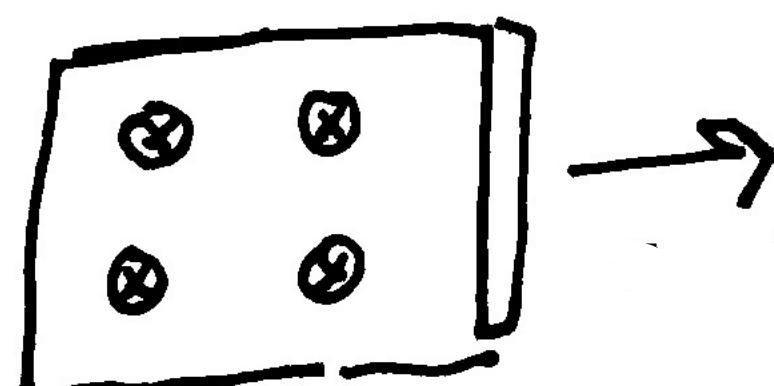
$$b.) \frac{V}{0.2 + \eta} = \frac{5}{x} A$$

$$|F| = IlB. \quad \text{Assume } l = 20\text{cm for the bar.}$$

$$|F| = \left(\frac{5}{x} A\right)(20\text{cm})(0.2\text{T}) = \frac{0.2}{x} N \quad (\text{points to the right})$$

(This answer does not take into account back emf, which will be addressed in the next question.)

c.)



$$\frac{d\Psi}{dt} = (0.2\text{T})(0.2\text{m}) V$$

The back emf reduces the effective voltage of the battery (and therefore the current), reducing the force on the bar.

The bar will reach its top speed when the force is zero, which occurs when the back emf reaches the voltage of the battery (1V).

$$(1\text{V}) = (0.2\text{T})(0.2\text{m})v$$

$$v = 25 \text{ m/s}$$

$$③ \text{ a.) } I = 50 \text{ mA} \quad \text{so} \quad J = \frac{50 \text{ mA}}{\pi (1 \text{ cm})^2} = \frac{500}{\pi} \text{ A/m}^2$$

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enc}}$$

\vec{B} will point in ϕ direction.

For $0 \leq r \leq 1 \text{ cm}$

$$H(2\pi r) = J \cdot \pi r^2$$

$$\vec{H} = \frac{\frac{500}{\pi} \pi r^2}{2\pi r} \hat{\phi} = \frac{250r}{\pi} \hat{\phi} \text{ A/m}$$

$$\vec{B} = N \vec{H} = 5000 N_o \vec{H} = \frac{(1.25 \times 10^6) N_o r}{\pi} \hat{\phi} \text{ T}$$

Let outer conductor be located at 3cm.

For $1 \text{ cm} \leq r \leq 3 \text{ cm}$,

$$H(2\pi r) = 50 \text{ mA}$$

$$\vec{H} = \frac{(25 \times 10^{-3})}{\pi r} \hat{\phi} \text{ A/m}$$

$$\vec{B} = N_o \vec{H} = \frac{(25 \times 10^{-3}) N_o}{\pi r} \hat{\phi} \text{ T}$$

For $3 \text{ cm} \leq r \leq 4 \text{ cm}$,

$$\vec{H} = \vec{B} = 0 \quad \text{because} \quad I_{\text{enc}} = 0.$$

$$b.) \vec{B} = \nabla \times \vec{A}$$

\vec{A} has only a \hat{z} component
since this is the direction
of current.

$$\frac{(25 \times 10^{-3}) N_0}{\pi r} \hat{\phi} = \left[\frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \hat{r} + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right] \hat{\phi} + \underbrace{\frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right] \hat{z}}$$

zero because \vec{B} has no \hat{r} component

zero because \vec{A} has no \hat{r} component

zero because \vec{B} has no \hat{z} component

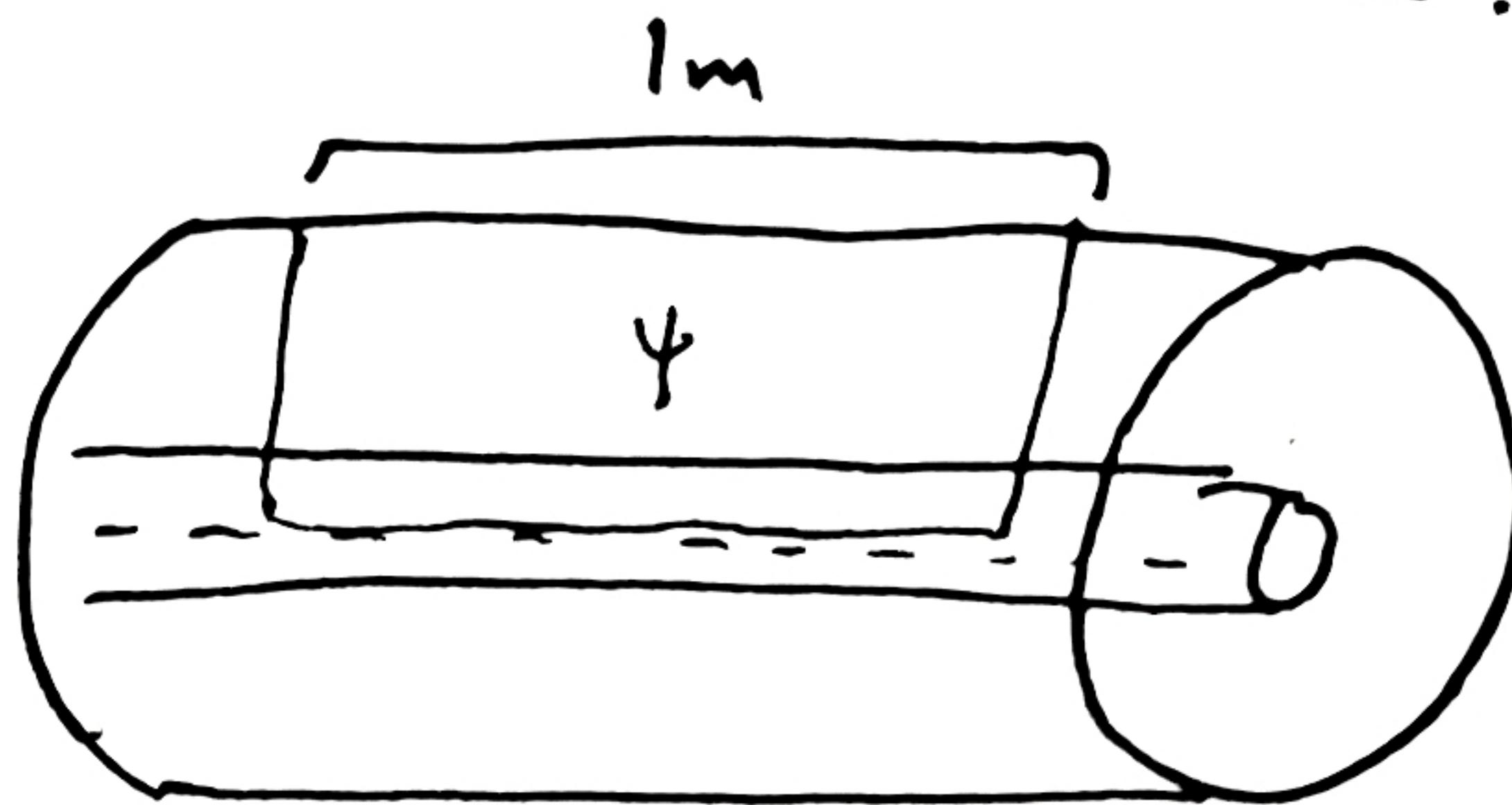
$$\frac{(25 \times 10^{-3}) N_0}{\pi r} \hat{\phi} = - \frac{\partial A_z}{\partial r} \hat{\phi}$$

$$A(r) = - \frac{(25 \times 10^{-3}) N_0}{\pi} \ln(r) \hat{z}$$

$$c.) L = \frac{\Delta}{I}$$

Since there are no coils/turns, $L = \frac{\psi}{I}$

$$\psi = \int \vec{B} \cdot \vec{dS}$$



We will find the flux through the rectangle above which is a surface in $\hat{\phi}$.

It goes from $r = 0\text{cm}$ to $r = 3\text{cm}$.

Its length is 1m so finding the flux through it will allow us to determine the inductance per meter.

$$\begin{aligned} \int \vec{B} \cdot \vec{dS} &= \iint_{\substack{0 \\ z \\ r}}^{1 \quad 0.01} \frac{(1.25 \times 10^6) N_0 r}{\pi} dr dz + \iint_{\substack{0 \\ z \\ r}}^{1 \quad 0.03} \frac{(25 \times 10^{-3}) N_0}{\pi r} dr dz \\ &= \left| \frac{(1.25 \times 10^6) N_0 r^2}{2\pi} \right|_0^{0.01} + \left| \frac{(25 \times 10^{-3}) N_0}{\pi} \ln(r) \right|_{0.01}^{0.03} \end{aligned}$$

$$\psi = 2.5 \times 10^{-5} \text{ Wb} + 1.099 \times 10^{-8} \text{ Wb}$$

$$L = \frac{2.5 \times 10^{-5} \text{ Wb}}{50 \text{ mA}} = 0.5 \text{ mH}$$

$$④ \text{ a.) } \vec{B}_2 = 0.2\hat{x} + 0.4\hat{y}$$

$$\vec{B}_{1n} = \vec{B}_{2n}$$

$$\vec{B}_{1n} = 0.4\hat{y}$$

$$\vec{B}_{2t} = 0.2\hat{x}$$

$$\vec{H}_{2t} = \vec{B}_{2t}/5000\mu_0 \hat{x} = \frac{4 \times 10^{-5}}{\mu_0} \hat{x}$$

$$\vec{H}_{1t} - \vec{H}_{2t} = \vec{J}_s$$

$$\vec{J}_s = 0 \text{ so } \vec{H}_{1t} = \vec{H}_{2t}$$

$$\vec{H}_{1t} = \frac{4 \times 10^{-5}}{\mu_0} \hat{x}$$

$$\vec{B}_{1t} = 200\mu_0 \vec{H}_{1t} = 0.008\hat{x}$$

$$\vec{B}_1 = 0.008\hat{x} + 0.4\hat{y}$$

$$\text{b.) } \vec{H}_{1t} - \vec{H}_{2t} = \vec{J}_s .$$

If \vec{H}_{2t} remains fixed while \vec{J}_s changes, \vec{H}_{1t} will change.

This will change the x component of \vec{B}_1 .