Sit-Down Rework Exam

Name: ________________________ Solutions

RIN: __________________________

Mark the skills you wish to test today:

1a	1b	1c	1d	1e	1f	1g	1h	1i	1j

21	2b	2c	2d	2e	2f	2g	2h	2i	2j	2k	2l

<u>Instructions:</u>

1) Read ALL directions carefully
2) You have **1 hour and 50 minutes** to complete this rework exam
3) You are not required to complete all material on this rework; you may complete as much or as little as you wish. Whenever your answer demonstrates improved mastery of Core Skills, your previous score for those skills will be replaced by the new sources from this exam.
4) If you choose to skip a part of a problem in accordance with direction 3 above and a subsequent part on the problem depends on the numerical answer from the part of the problem, you may assume or guess a numerical value for the part of the problem you skipped and proceed from there accordingly. **If you do this, make VERY clear what values you are assuming and list them clearly.**
5) Show your work in enough detail to allow the graders to completely follow your thought process.
6) Make sure your calculator is set to perform trigonometric functions on radian and not degrees. Use at least 2 significant digits.
7) Make sure to write your answer legibly. You may use the back of the exam or ask for scratch pages.

1. Signal through a Transmission Line (Skills 1b, 1e, 1f, 1j)
Consider the following input signal:

V(t)=10sin(2π(10000)t-0.0003z)

a. Calculate the reflection coefficient and the SWR of the system (where Z_0 = 50Ω and Z_L = 375Ω).

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{375-50}{375+50} = 0.765$$

b. Find the amplitude of the reflected wave.

$$|V_0^-| = \Gamma |V_0^+| = 10 \cdot 0.765 = 7.65\ V$$

c. Determine the average power of the original incident wave, the power of the reflected wave, and the power transmitted to the load.

$$P^i_{av} = \frac{|V_0|^2}{2Z_0} = \frac{10^2}{100} = 1\ W$$

$$P^r_{av} = -|\Gamma|^2 P^i_{av} = -0.765^2 = 0.585\ W$$

$$P_{av} = P^i_{av} + P^r_{av} = 0.414\ W$$

d. What is the input impedance at the transmission line source given a line length of 20km?

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta L}{Z_0 + jZ_L \tan \beta L}$$

$$Z_{in} = 50 \frac{375 + j50 \tan(0.0003 \times 20000)}{50 + j375 \tan(0.0003 \times 20000)}$$

$$Z_{in} = 8.84 - j28.2$$

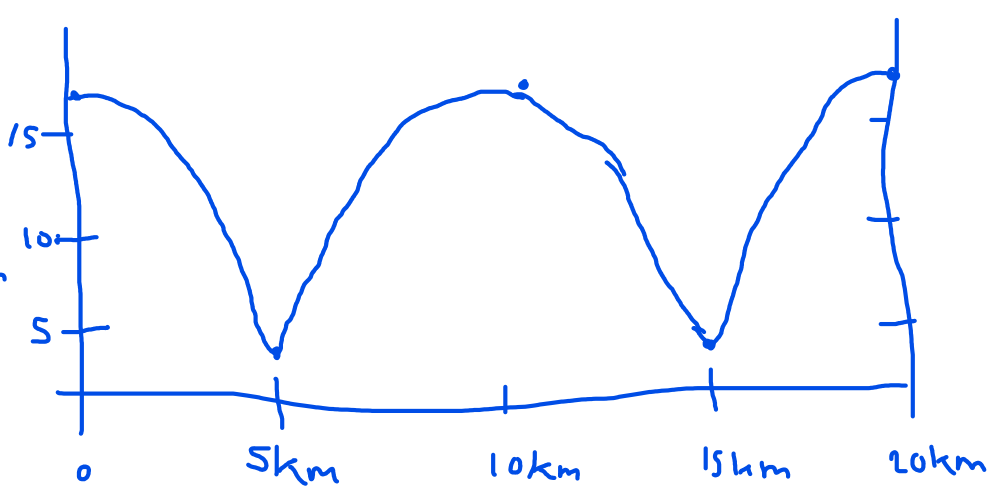
e. Sketch the standing wave amplitude on the 20km line. Indicate the source and load end and label meters on the x axis and volts on the y axis. The plot should clearly show voltage maxima and minima and correctly indicate where on the line they will occur.

$\lambda = \frac{2\pi}{\beta} = 20944\text{m}$

standing wave pattern repeats every $\frac{\lambda}{2}$ or ~10km

$10(1 + |\Gamma|) = 17.65$

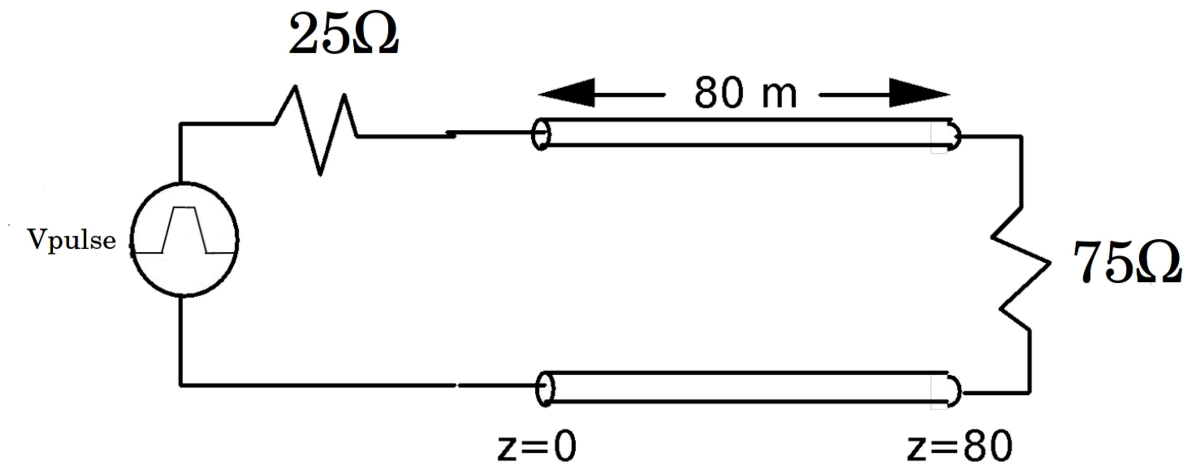
$10(1 - |\Gamma|) = 2.35$



2. Pulses on a Transmission Line (Skill 1i)

Consider the circuit below. Assume that the characteristic impedance of the transmission line is 50Ω. Assume that the source emits a single voltage pulse that is very short in duration compared to the time delay of the transmission line.

Choose an input pulse amplitude that is at least 5V and at most 15V.



a. What is the amplitude of the pulse as it first enters the transmission line?

Let Vpulse = 10V.
Vin = 10*50/(25+50) = 6.67V

b. Assume that the velocity of the pulse on the transmission line is 80% of the speed of light. What is the time delay T of the transmission line?

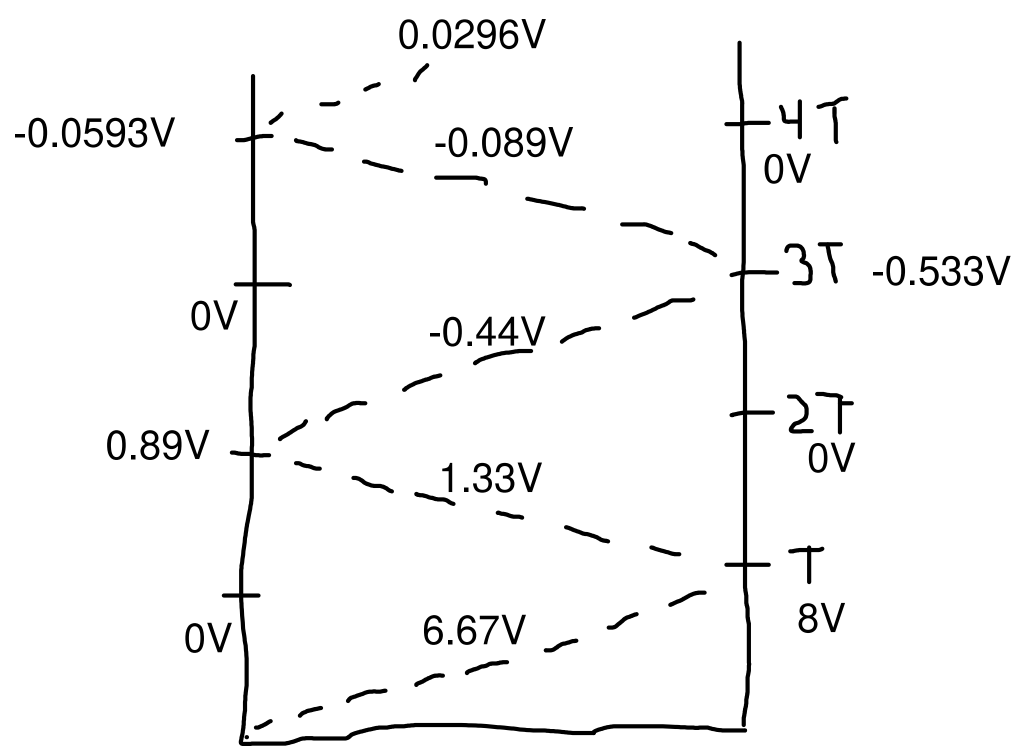
(80m) / (0.8c) = 333 nanoseconds

c. What is the reflection coefficient at the source and at the load?

$$\Gamma_L = \frac{75-50}{75+50} = \frac{1}{5}$$

$$\Gamma_S = \frac{25-50}{25+50} = \frac{-1}{3}$$

d. Create a bounce diagram showing the propagation of the pulse on the transmission line for a period of time equal to 4T. The bounce diagram should include the amplitude of the input pulse, the amplitude of each reflected pulse, and the transmission line input and load voltage at each interval of T.



3. Properties of a Transmission Line (1g, 1h) Skill 1b was mistakenly listed in problem 1 but was actually in this problem.

Given a transmission line with L' = 50 μH/m, Z_0 = 50 Ω, R' = 0.05 Ω/m, Length = 3000m, and an input frequency of 100kHz:

a. State whether the transmission line is low-loss at this frequency, then calculate C', μ_p, T, ω, β, λ. (Consider the transmission to be low loss if the resistive impedance is less than 1% as high as its reactive impedance.)

$Z_0 = 50\,\Omega$ $\qquad 0.05\,\Omega/m \ll 2\pi \cdot 10^5 \cdot 50\times10^{-6}$

so line is low loss.

$Z_0 = \sqrt{\frac{L'}{C'}} \rightarrow 50 = \sqrt{\frac{50\times10^{-6}}{C'}} \rightarrow C' = 20\,pF/m$

$u = \frac{1}{\sqrt{L'C'}} = 10^6\ m/s \qquad T = \frac{1}{f} = 10^{-5}\,s$

$\lambda = u/f = 10\,m \qquad \beta = \frac{2\pi}{\lambda} = 0.628\ rad/m$

b. What is the difference between L and L'? (Explain this conceptually using words.)

L is inductance of something. For a transmission line, it increases proportionally with length.

L' is inductance per unit length.

c. Determine the attenuation constant for the line. If a 10V 100kHz sinusoidal signal enters the line at the source end, determine what its amplitude will be once it reaches the load end.

since the line is low-loss,

$\alpha = \frac{r'}{2Z_0} = 5\times10^{-4}$

$10e^{-(3000)\alpha} = 2.23\ V$

d. What value of G' would make this line dispersionless?

$$\frac{r'}{l'} = \frac{g'}{c'} \qquad g' = 2\times 10^{-5}\ S/m$$

4. Coaxial Cable Capacitor (Skills 2b, 2c, 2d, 2g, 2h)

Consider a coaxial cable transmission line. It contains the following layers:

- From r=0mm to 1mm, a cylindrical conductor that is grounded (and connected to an external circuit)
- From r=1mm to 2mm, a layer of dielectric with permittivity $10\varepsilon_0$
- From r=2mm to 3mm, layer of conductor which is not connected to any external circuit
- From r=3mm to 4mm, a layer of dielectric with permittivity $10\varepsilon_0$
- From r=4mm to 5mm, a layer of conductor that is connected to an external circuit

At any given moment of operation, the capacitor has some charge +Q on the outer conductor and charge -Q on the inner conductor, which we will represent as the charge magnitude Q. Because the middle conductor layer is not connected to an external circuit, the net charge on this layer must be zero. The dielectric layers also have no net charge.

a.) Write an expression for the both the D-field and the E-field for 0mm< r < 6mm as a function of Q. Be sure to specify the direction of the field.

for $0 \le r < 1\text{mm}$: $\vec{D} = \vec{E} = 0$ ($Q_{enc} = 0$, conductor)

for $1 \le r < 2\text{mm}$: (units of $\vec{E}$ are V/m)

The Gaussian surface is a cylinder that encloses the inner conductor. Let ℓ be the cable's length.

$$|\vec{D}|\, 2\pi r \ell = -Q \longrightarrow \vec{D} = \frac{-Q}{2\pi r \ell}\hat{r} \rightarrow \vec{E} = \frac{-Q}{20\pi\varepsilon_0 r \ell}$$

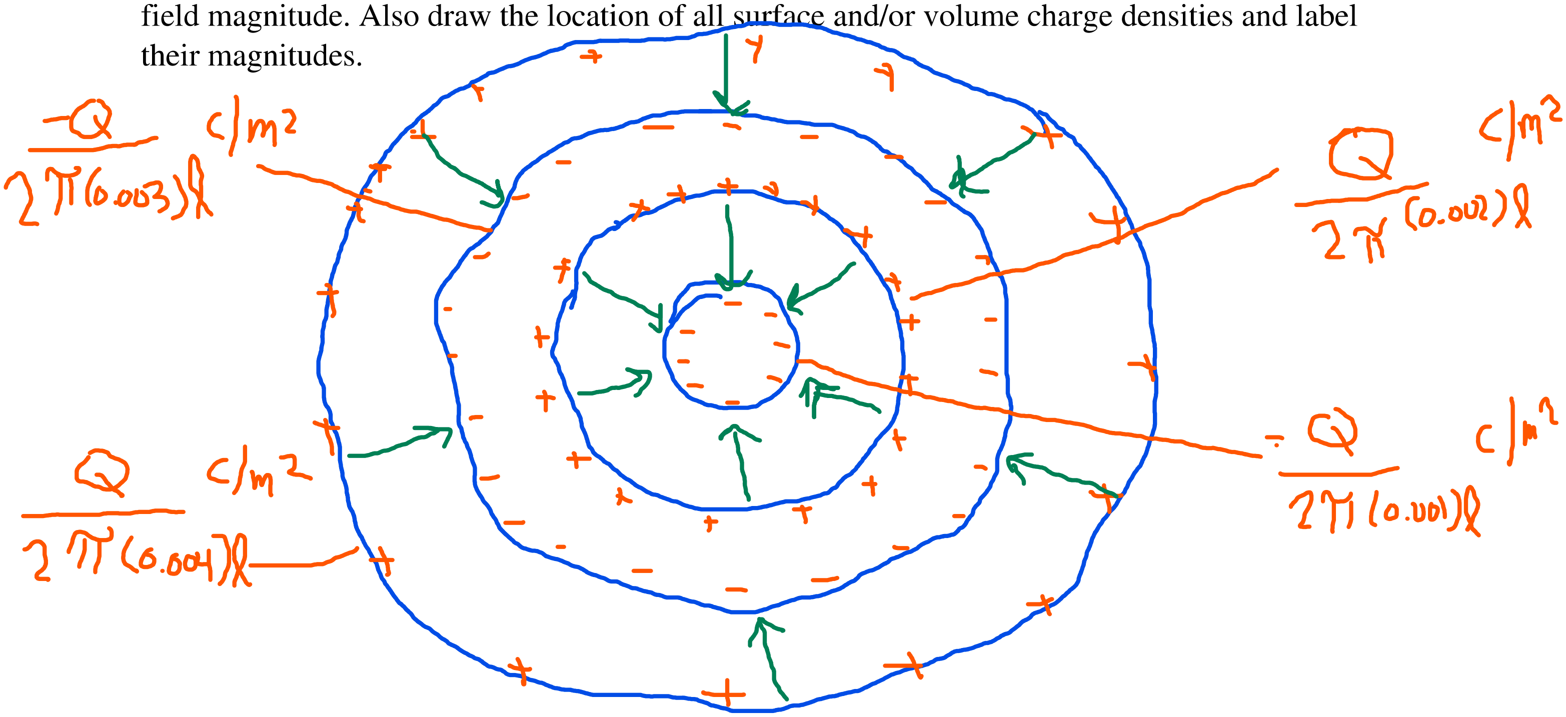
for $2\text{mm} \le r < 3\text{mm}$: $\vec{D} = \vec{E} = 0$ (conductor, $Q_{enc} = 0$)

for $3 \le r < 4\text{mm}$: $\vec{E} = \frac{-Q}{20\pi\varepsilon_0 r \ell}$

for $4\text{mm} \le r < 5\text{mm}$: $\vec{E} = \vec{D} = 0$ (conductor, $Q_{enc} = 0$)

for $5\text{mm} \le r < 6\text{mm}$: $\vec{E} = \vec{D} = 0$ ($Q_{enc} = 0$)

b.) Draw a cross-section of this coaxial cable and sketch the E-field inside. Be sure to show the direction of the field and do your best to draw the field line density as being proportional to the field magnitude. Also draw the location of all surface and/or volume charge densities and label their magnitudes.



c.) Write an expression relating the voltage of this capacitor to Q, and calculate its capacitance per unit length. (Hint: calculate the capacitance for a 1 meter-long segment of this coaxial cable.

$$V = -\int_a^b \vec{E}\cdot d\vec{\ell} = -\int_{0.001}^{0.002} \frac{-Q}{20\pi\varepsilon_0 r\ell} - \int_{0.003}^{0.004} \frac{-Q}{20\pi\varepsilon_0 r\ell}$$

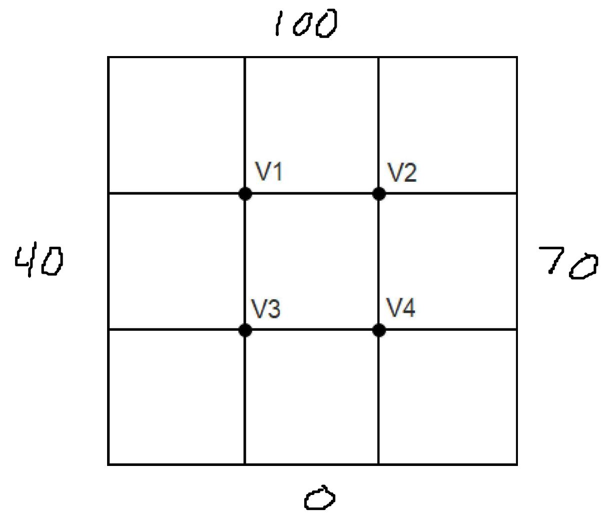
$$= \frac{Q}{20\pi\varepsilon_0 \ell}\left[\ln r \Big|_{0.001}^{0.002} + \ln r \Big|_{0.003}^{0.004} \right]$$

$$= \frac{(0.98)\,Q}{20\pi\varepsilon_0 \ell}$$

$$C' = \frac{Q}{V\ell} = \frac{20\pi\varepsilon_0}{0.98} = 567\,pF/m$$

Finite Difference (Skill 2k, 2i)

Suppose that a square region is bounded by 100V on its top side, 40V on its left side, 70V on its right side, and is grounded on the bottom. Inside the region is free space. The length of one side of the square region is 3m.



a.) Calculate the voltages at V1-V4 using one iteration of the finite difference method.

Initial guesses: $V_1 = V_2 = V_3 = V_4 = 0$

$$V_1 = \frac{1}{4}(100 + 40 + 0 + 0) = 35$$

$$V_2 = \frac{1}{4}(100 + 35 + 0 + 70) = 51.25$$

$$V_3 = \frac{1}{4}(35 + 40 + 0 + 0) = 18.75$$

$$V_4 = \frac{1}{4}(51.25 + 18.75 + 0 + 70) = 35$$

b.) Approximate the magnitude of the electric field between V1 and V2 by finding the voltage difference between these two points and dividing it by the distance between the two points. What is the magnitude of this field? What is the energy density in this field?

Entire 3x3 grid is 3m wide, so the distance between V_1 and V_2 is 1m.

$$|\vec{E}| = \left|\frac{V_1 - V_2}{d}\right| = \left|\frac{(35 - 51.25)\,V}{1\,m}\right| = 16.25\ V/m$$

$$W_m = \frac{1}{2}\varepsilon|\vec{E}|^2 = \frac{1}{2}\varepsilon_0(16.25)^2 = 1.17\,nJ/m^3$$

Voltage Expression (1a, 1c, 1d)

a.) Write an expression for a sinusoidal voltage traveling wave V(z,t), choosing all relevant parameters. The wave cannot have an amplitude of zero. Explain how it is possible to tell from the expression that it does not describe a standing wave.

$$V(z,t) = 20\cos(10^6\pi t - 500z)$$

the cosine contains a t term and a z term with one subtracted from the other.

b.) Now write your expression in part a in phasor notation.

$$\tilde{V}(z) = 20e^{-j500z}$$

c.) Calculate the velocity of the wave from part a. Then suppose that this signal is traveling on a lossless transmission line. Come up with two possible values of inductance and capacitance per unit length for this transmission line that result in the velocity you calculated. Determine what the characteristic impedance of this line will be.

$$\beta = \frac{\omega}{u} \rightarrow u = \frac{\omega}{\beta}$$

$$u = \frac{10^6\pi}{500} = 6283\ m/s$$

$$u = \frac{1}{\sqrt{L'C'}}$$

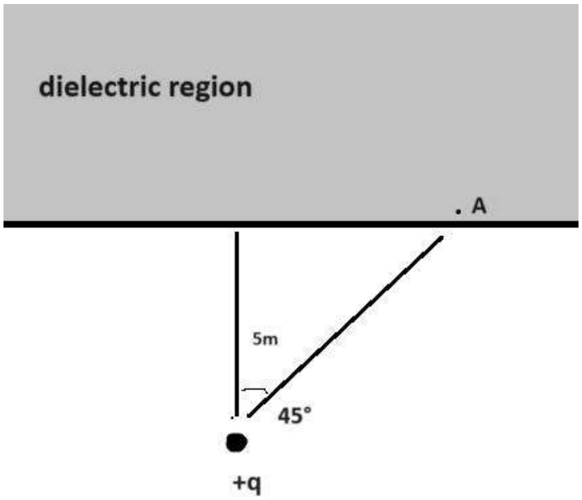
if $C' = 1\ mF/m$

then $L' = \frac{1}{C'u^2} = 25.33\ \mu H/m$

$$Z_0 = \sqrt{\frac{L'}{C'}} = 0.159\ \Omega$$

7. Dielectric and Charge (Skill 2e, 2f, 2j, 2l)

A positively charged particle (+q) has a net charge of one microcoulomb. It is located 5 meters away from a wall of dielectric material with a relative permittivity of 7.



a.) Determine the magnitude and direction of the electric field at point A. Note that point A lies just inside the dielectric 45 degrees up and to the right from the charged particle.

5m * sqrt(2) = 7.07m

To find E in the air right below A, use Coulomb's Law:

E = (1*10^-6 C)/4**(0)*(7.07 m)^2
E = 179.8 V/m (pointing 45° northeast)

This can be split into tangent and normal components:

E1t = 179.8*sin(45°) = 127.1 V/m
E1n = 179.8*cos(45°) = 127.1 V/m

Now we calculate E1t and E1n at point A inside the dielectric.

E2t = E1t = 127.1 V/m
E2n = E1n/7 = 18.16 V/m

Magnitude = sqrt (E2t^2 + E2n^2) = 128.4 V/m

Direction up from horizontal:

arctan(E2n/E2t) = 8.1°

b.) If a one microcoulomb positive charge were placed at point A, what force would it experience? (Do not consider this additional charge for parts c and d.)

(128.4 V/m)*(1*10^-6 C) = 0.128 mN

c.) If the conductivity of the dielectric is 2 microsiemens per meter, what is the current density at point a?

$$\vec{J} = \sigma \vec{E} = (2 \times 10^{-6}\ S/m)(128.4\ V/m)$$

$$= 256\ nA/m^2$$

d.) Assume that the +q point charge begins moving vertically toward the dielectric, which has a dielectric breakdown strength of 2 megavolts per meter. How close must the charge get to the dielectric before it causes dielectric breakdown on the surface of the dielectric just above it?

Use Coulomb's law to find the E-field and set it equal to the breakdown strength.

E = (1*10^-6 C)/4*7*pi*e0*r^2 = 2*20^6 V/m

r = 2.5 cm

8. Impedance Matching (Skill 21)

You are attempting to match an antenna to a 75Ω transmission line with a velocity of 0.7c. The system is designed to operate at 2 MHz.

a.) Initially, the impedance of the antenna is 100+j 150Ω. Calculate what this impedance is in normalized form, and then plot it on the Smith Chart.

$$\frac{100+j150}{75} = 1.33+j2$$

Smith chart on back

b.) The antenna impedance then changes to 300+50Ω. Calculate what this impedance is in normalized form, and then plot it on the Smith Chart on the next page.

$$\frac{300+j50}{75} = 4+j0.66$$

c.) Choose one of the two load impedances from a or b and design a stub that matches the load to the transmission line. You may choose to use either an open-circuit or a short-circuit stub. You must report the distance (in meters) from the load at which the stub is located and the length of the stub. On the Smith Chart you should label the following:

1.) the input admittance of the transmission line at the place where the stub is located before the stub is added

2.) the distance in wavelengths between the load and the place where the stub is added

3.) the load admittance and input admittance of the stub, and the distance between the beginning and end of the stub

We choose $z_L = 1.33+j2$

$y_L = 0.23-j0.34$ $\quad y'_L \approx 1+j1.5$

distance between y_L and y'_L is $0.06\lambda + 0.176\lambda = 0.136\lambda$

$\lambda = \frac{u}{f} = \frac{0.7 \times c}{2\times10^6} = 104.9\,m$

location of stub: $0.136 \cdot 104.9 = 14.3\,m$ from load

cont. on next page

The Complete Smith Chart

Black Magic Design