Fields and Waves I

Lecture 15

Intro to Magnetic Fields
Exam 2 Review

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Materials from other sources are referenced where they are used.
Those listed as Ulaby are figures from Ulaby's textbook.

Laplace and Poisson Equations

- Laplace's Equation: $$\nabla^2 V = 0$$

- Poisson's Equation $$\nabla^2 V = -\frac{\rho}{\varepsilon}$$

$$\nabla^2 = \begin{bmatrix} \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z} \end{bmatrix} \bullet \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

(in cartesian coordinates)

Exam 2

- Will take place during class **next Monday** (if you have extra time, I will reach out)
- As with Exam 1, there will be a prepared crib sheet
- Unit 1 crib sheets will be available as well
- Exam 2 crib sheets and practice exam + solutions are on the shared drive

Exam 2

2a	Correctly represent a normalized load impedance on a Smith chart, identifying the magnitude and phase of the corresponding reflection coefficient, and reading the standing wave ratio from the chart.
2b	Calculate a single stub match for an open-circuit or short-circuit load using a Smith chart.

- This means graphing on a printed Smith Chart (which will be provided.)
- Should be able to transform an impedance by rotating towards the generator or load.

Skill 2a/2b

Suppose that we have a 50 ohm transmission line with a 100+j100 ohm load. Match this load to the transmission line with an open circuit stub.

https://em8e.eecs.umich.edu/jsmodules/ch2/mod2_6.html

Exam 2

2c	Understand the geometry of all surface and volume integrals in Cartesian, cylindrical, and polar coordinates, and be able to correctly specify the integral limits and differential elements.

- Practice visualizing the integrals and knowing when to use each one.
- Review the crib sheet to see what information will be provided.

Field Math

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of $\mathbf{A}$ $\quad |\mathbf{A}| =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\quad \overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P(x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P(r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P(R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}} \times \hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}} \cdot \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}} \cdot \hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}} \cdot \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} \cdot \hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}} \cdot \hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}} \times \hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\quad \mathbf{A} \cdot \mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\quad \mathbf{A} \times \mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $\quad d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $\quad dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

Field Math

Differential Surfaces and Volumes

How do you describe the shapes of all the ds surfaces in the following coordinate systems?

- Cartesian coordinates:

https://mathinsight.org/cartesian_coordinates

- Cylindrical coordinates:

https://mathinsight.org/cylindrical_coordinates

- Spherical coordinates:

https://mathinsight.org/spherical_coordinates

Exam 2

2d	Be able to successfully calculate the voltage between two points based on the electric field between them.
2e	Use Gauss's Law to calculate an electric field from the geometries of a region's materials and charge distributions, or vice versa.
2f	Demonstrate an understanding of the geometry of electric fields and the difference between the D-field and E-field.
2j	Calculate the capacitance of a given distribution of conductors and/or dielectrics with simple geometries.

- For voltage, make sure you know what your ground / reference point is!
- Crib sheet does not have parallel capacitance field / infinite plane field, etc. Memorize or know how to derive these.

Exam 2

Consider a coaxial cable transmission line. Its innermost layer is a grounded cylindrical conductor of radius 2mm. Around the inner conductor is a 2mm thick layer of dielectric with permittivity $10\varepsilon_0$. Around this is an outer conductor consisting of a very thin shell. At any given moment of operation, the capacitor has some charge +Q on the outer conductor and charge -Q on the inner conductor, which we will represent as the charge magnitude Q.

a.) Write an expression for the both the D-field and the E-field for 0mm< r < 5mm as a function of Q. Be sure to specify the direction of the field.

Exam 2

$$\oint \vec{D}\cdot d\vec{S} = Q_{enc} \qquad \vec{D} = \varepsilon \vec{E}$$

for $0 \le r < 2mm$: $\vec{D} = \vec{E} = 0$

(this is a conducting region)

for $2mm \le r < 4mm$:

Let Q be a charge per unit length and let's consider a cylindrical Gaussian surface with $l = 1m$.

$$|\vec{D}| 2\pi r l = |\vec{D}| 2\pi r = -Ql = -Q$$

$$\vec{D} = \frac{-Q}{2\pi r}\hat{r} \qquad \vec{E} = \frac{-Q}{20\varepsilon_0 \pi r}\hat{r}$$

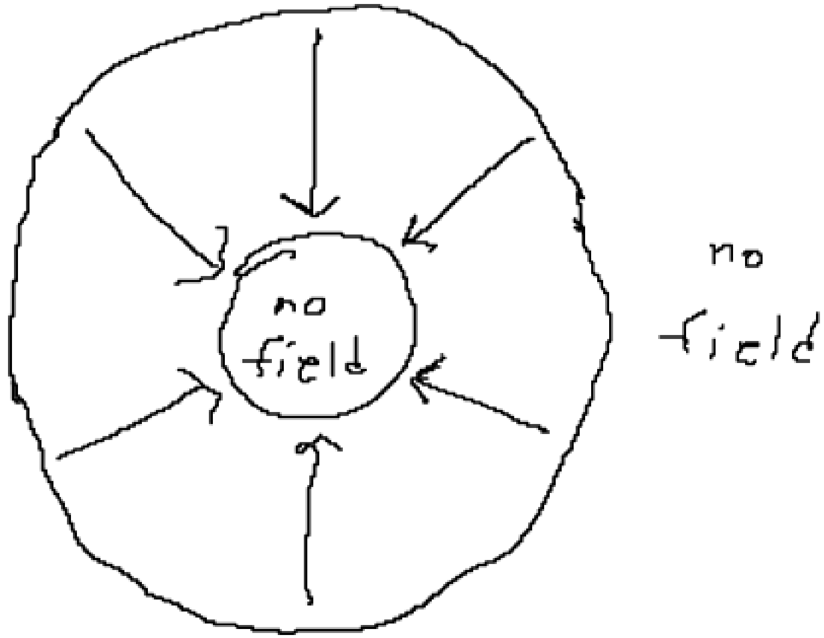
Exam 2

For $r \geq 4mm$,

$\vec{D} = \vec{E} = 0$ because $Q_{enc} = 0$.

b.) Draw a cross-section of this coaxial cable and sketch either the D-field or the E-field inside. Be sure to show the direction of the field and do your best to draw the field line density as being proportional to the field magnitude

Exam 2



Exam 2

c.) Write an expression relating the voltage of this capacitor to Q, and calculate its capacitance per unit length. (Hint: calculate the capacitance for a 1 meter-long segment of this coaxial cable.)

$$V = -\int_a^b \vec{E} \cdot \vec{dl} = -\int_{0.002}^{0.004} \frac{-Q}{20\varepsilon_0 \pi r}\, dr$$

$$= \frac{Q}{20\varepsilon_0 \pi} \Big| \ln r \Big|_{0.002}^{0.004} = (0.693)\frac{Q}{20\varepsilon_0 \pi}$$

$$C' = \frac{Q}{V} = \frac{20\varepsilon_0 \pi}{0.693} = 803 \text{ pF/m}$$

Exam 2

2g	Calculate electric force from electric charge using Coulomb's Law.
2h	Evaluate a static electric field at a boundary between two materials with different permittivities.
2i	Demonstrate an understanding of the effect of perfect conductors on the electric field both inside of them and outside their surfaces.

- 2i includes boundary conditions for conductors, and Method of Images.

Electrostatics

Coulomb's Law

$$\vec{E} \text{ ,of } Q_1 \text{ is } = \frac{Q_1}{4\pi\varepsilon_0 R^2} \cdot \hat{a}_R$$

Unit vector pointing away from Q_1

Then,

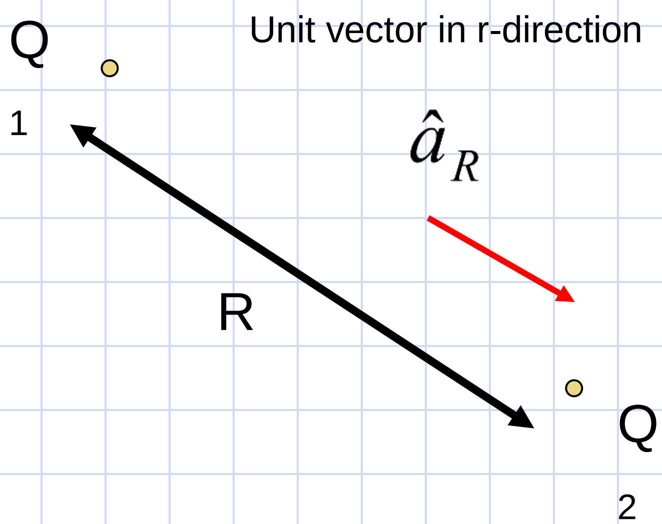
$$\vec{F}_{12} = Q_2 \cdot \vec{E}$$

- we work with <u>E-Field</u> because Maxwell's equations written in those terms

Electrostatics

Coulomb's Law

$\vec{F}$ (force), between point charges



$$\vec{F}_{12} = \frac{Q_1 \cdot Q_2}{4\pi\varepsilon_0 R^2} \cdot \hat{a}_R$$

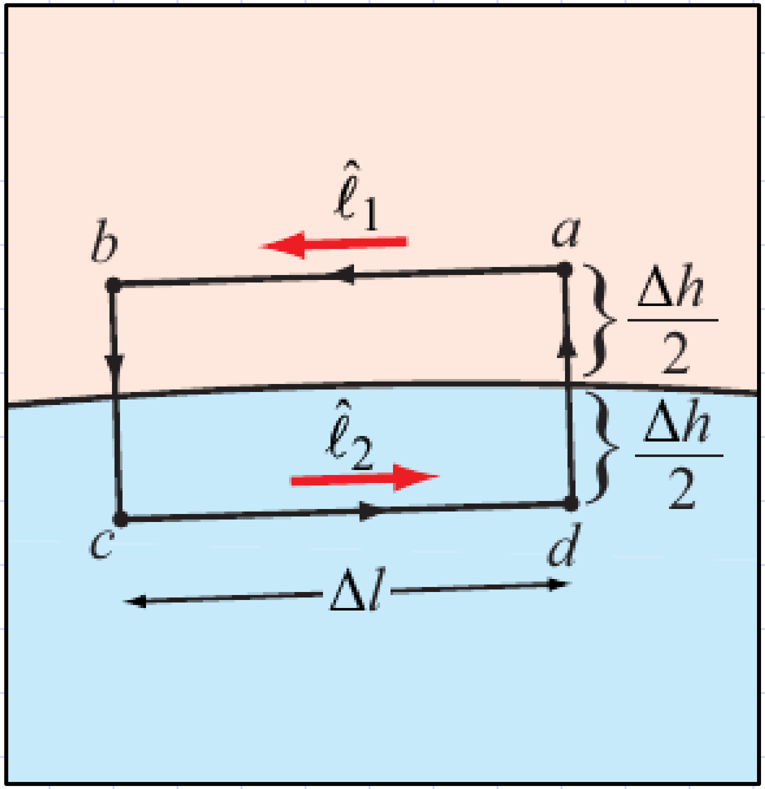
Force on Charge 2 by Charge 1

Exam 2

2j	Calculate the capacitance of a given distribution of conductors and/or dielectrics with simple geometries.
2k	Calculate the energy density in an electric field.
2l	Calculate the dielectric breakdown of a given dielectric medium.

- Know both the charge/voltage method and energy method of calculating capacitance
- For dielectric breakdown problems, know how to identify the place where dielectric breakdown will start.

Boundary Conditions

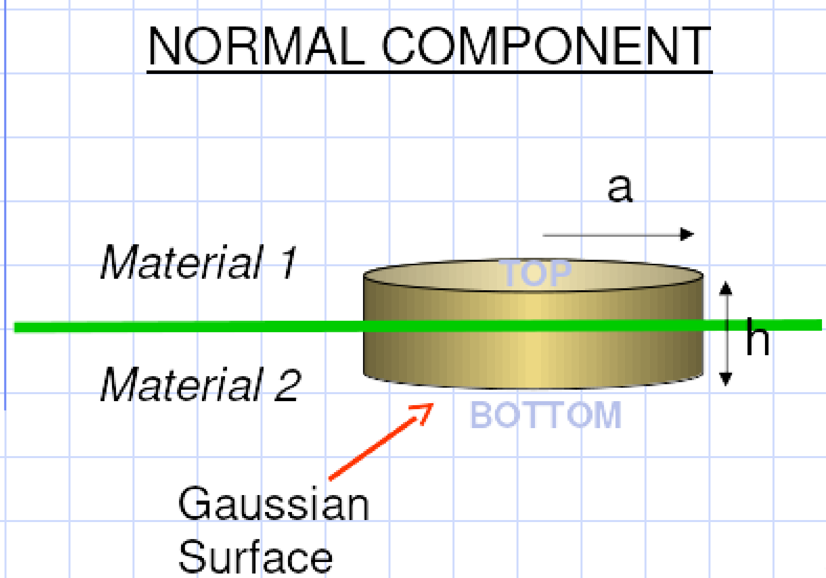


Ulaby

$$\vec{E}_{1t} = \vec{E}_{2t}$$

- So component of the E-field that is tangent to a media boundary is continuous across it.
- What about normal to the boundary?

Boundary Conditions



$$\oint D \bullet ds = Q_{enclosed}$$

Take h << a **(a thin disc)**

$$Q_{enclosed} = \rho_s \cdot A$$

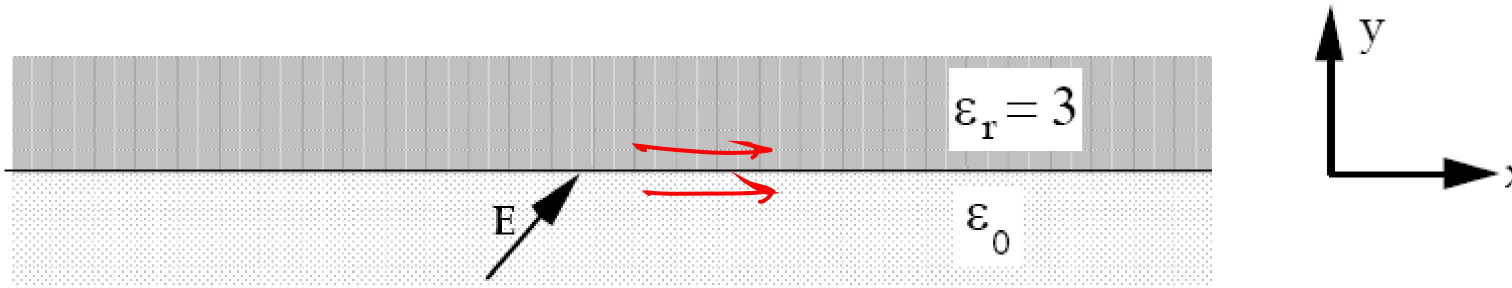
$$\oint D \bullet ds = \int_{TOP} D \bullet ds + \int_{BOTTOM} D \bullet ds$$

$$= (D_{1n} - D_{2n}) \cdot A$$

$$\therefore D_{1n} - D_{2n} = \rho_s$$

Boundary Conditions

The **E** field on the air side of a dielectric-dielectric boundary is **E** = 100 $\mathbf{a}_x$ + 100 $\mathbf{a}_y$. What is **E** on the dielectric side?



Boundary Conditions

$E_{1t} = E_{2t} \Rightarrow E_{1x} = E_{2x} \Rightarrow \therefore E_{2x} = 100$

Air = Region 1

Diel = Region 2

$D_{1n} = D_{2n} \Rightarrow \varepsilon_o E_{1y} = 3\varepsilon_o E_{2y} \Rightarrow E_{2y} = \frac{E_{1y}}{3} = \frac{100}{3} = 33\tfrac{1}{3}$

$$\boxed{\vec{E}_2 = 100\,\hat{a}_x + 33\tfrac{1}{3}\,\hat{a}_y}$$

Exam 2

2m	Use Laplace and/or Poisson's equations via the Finite Difference Method to solve a simple voltage field.
2n	Know the relationship between conductivity, current density, and electric field. Given appropriate information, be able to calculate these or related quantities (such as resistance or current).

- Any finite difference problems will be hand calculations and therefore simple.

Review

$$j = \sigma \cdot E$$ = Ohm's Law

Conductivity - units of S/m or 1/ohm-m

- varies from 10^7 to 10^{-15}

Good conductor eg. Cu

Good insulator

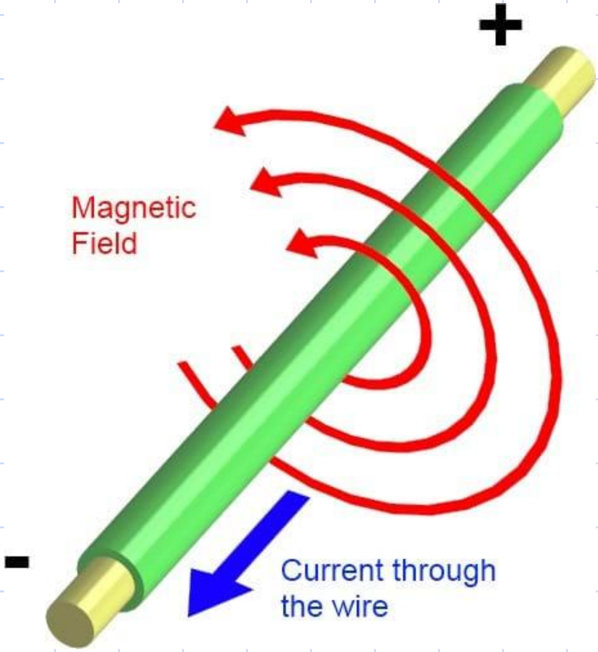
$j = \sigma \cdot E$, is Fields and Waves version of Ohm's Law

Note that this violates one of our electrostatic assumptions: that the e-field will have no curl.

Magnetostatics

Do Lecture 15, Exercise 1 in groups of up to 4.

Magnetostatics



TeachEngineering.org

- In magnetostatics, we have current. All currents are constant.
- Thus, all magnetic fields are constant

Magnetostatics

Electrostatic Version of Maxwell's Equations

Integral Form

$$\oint \vec{D} \cdot d\vec{S} = \int \rho dV$$

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Differential Form

$$\nabla \cdot \vec{D} = \rho$$

$$\nabla \times \vec{E} = 0$$

Magnetostatics

Magnetostatic Version of Maxwell's Equations

Integral Form

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s} = I_{enc}$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

Differential Form

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

- Electrostatics are based on two Maxwell's equations, but simplified.
- Electrostatics are based on the other two Maxwell's equations, but simplified.
- In these simplified Maxwell's equations, electric and magnetic fields are separate. In the full Maxwell's equations, they are coupled.

Magnetostatics

Magnetostatic Version of Maxwell's Equations

Integral Form

Differential Form

$$\boxed{\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s} = I_{enc} \qquad \nabla \times \vec{H} = \vec{J}}$$

$$\oint \vec{B} \cdot d\vec{S} = 0 \qquad \nabla \cdot \vec{B} = 0$$

These equations tell you that magnetic field arises from current (and circulates around the current).

Magnetostatics

Magnetostatic Version of Maxwell's Equations

Integral Form

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s} = I_{enc}$$

Differential Form

$$\nabla \times \vec{H} = \vec{J}$$

$$\boxed{\oint \vec{B} \cdot d\vec{S} = 0 \qquad \nabla \cdot \vec{B} = 0}$$

These equations tell you that magnetic field doesn't have any sources or sinks (i.e. no "magnetic charge".) It just circulates.

Magnetostatics

There are 2 different ways of writing the magnetic field.

$\vec{H}$ (often called just called "magnetic field" or "H-field") has units of amperes per meter (A/m). Thus it has a direct relationship with current.

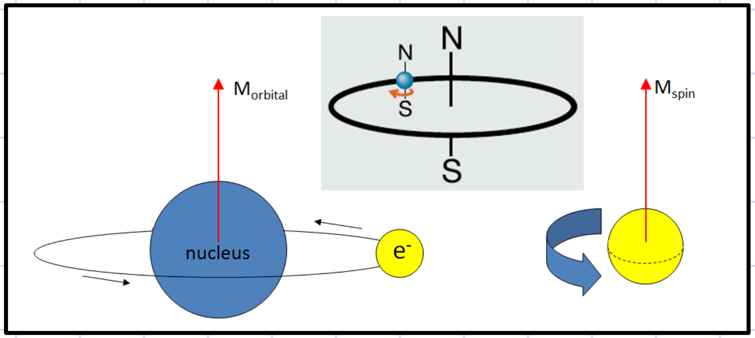
$\vec{B}$ (often called "magnetic flux density"), has units of teslas (T) or webers per square meter (Wb/m^2).

Magnetostatics

The two quantities are related by: $\vec{B} = \mu\vec{H}$

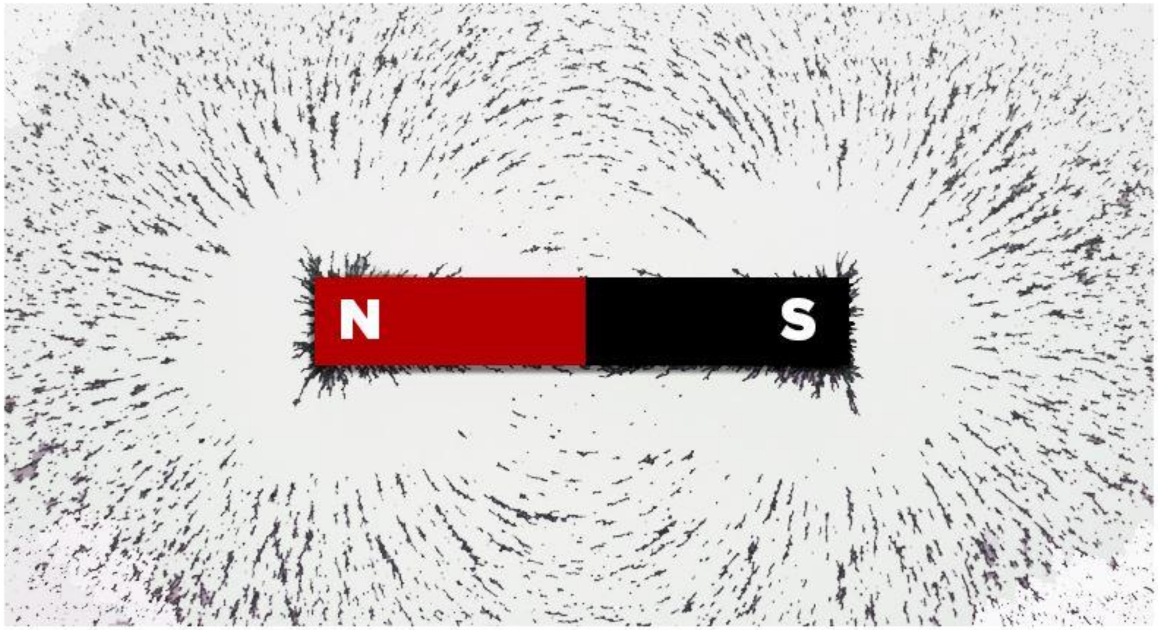
μ represents *permeability*, a property of the medium that the magnetic field is propagating through.

In essence: atoms are tiny magnets pointed in random directions. In some materials, this intrinsic magnetism is stronger than in others, leading to higher μ. The higher μ is, the more an externally-applied H-field leads to a stronger B-field due to alignment of the atoms.



starfishmedical.com

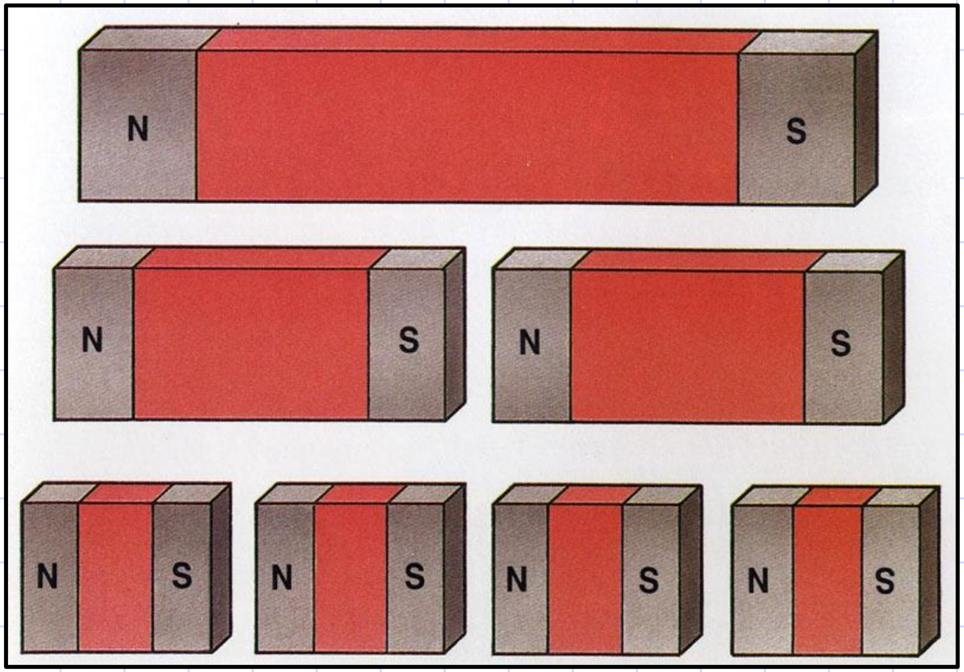
Magnetostatics



byjus.com

Permanent magnets exhibit electric field due to these atomic magnetic moments (which are essentially the magnetic field of the "spinning" electrons in atomic orbitals.

Magnetostatics



$$\oint \vec{B} \cdot d\vec{S} = 0$$

$$\nabla \cdot \vec{B} = 0$$

ency123.com

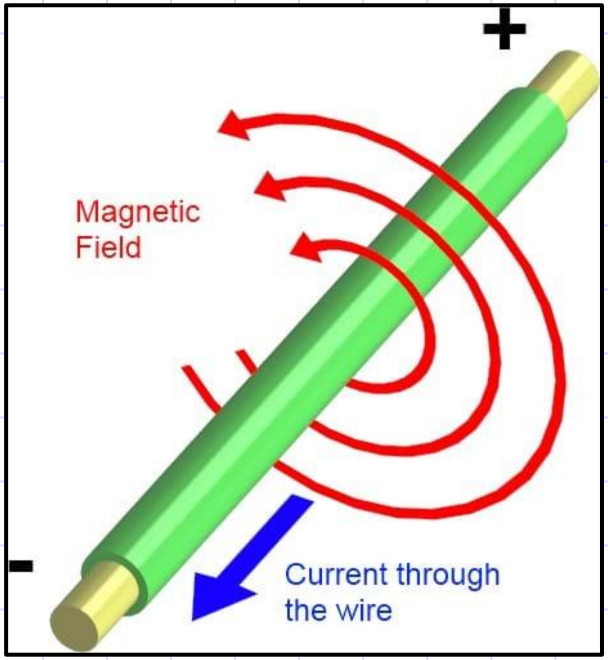
A permanent magnet that is cut in two will not separate into "north" and "south" magnetic charges, it will just become two magnets with two new north and south poles. This is due to Maxwell's Equations.

Magnetostatics

In Lecture 8 we said the following:

"Circulation of a vector field in practice is often due to a *distribution* of curl - that is, the vector field act less like it has one whirlpool and more like it has a distribution of infinitesimally-small whirlpools."

Ampere's Law



In magnetostatics, current is the cause of magnetic field circulation or "whirlpooling". The more current passes through a loop, the more magnetic field circulates around the loop.

We solved electrostatic problems using Gauss's law and drawing a Gaussian surface. We will solve magnetostatic problems by using Ampere's Law and drawing an Amperian loop.

$$\boxed{\oint \vec{H} \cdot \vec{dl} = \int \vec{J} \cdot \vec{ds} = I_{enc}}$$

Ampere's Law

$$\nabla \times \vec{H} = \vec{J}$$

Ampere's Law

$$\nabla \times H = j$$

Ampere's law

$$\oint H \bullet dl = \int j \bullet ds = I_{net}$$

$$B = \mu_0 \cdot H$$

Examples of Amperian loops

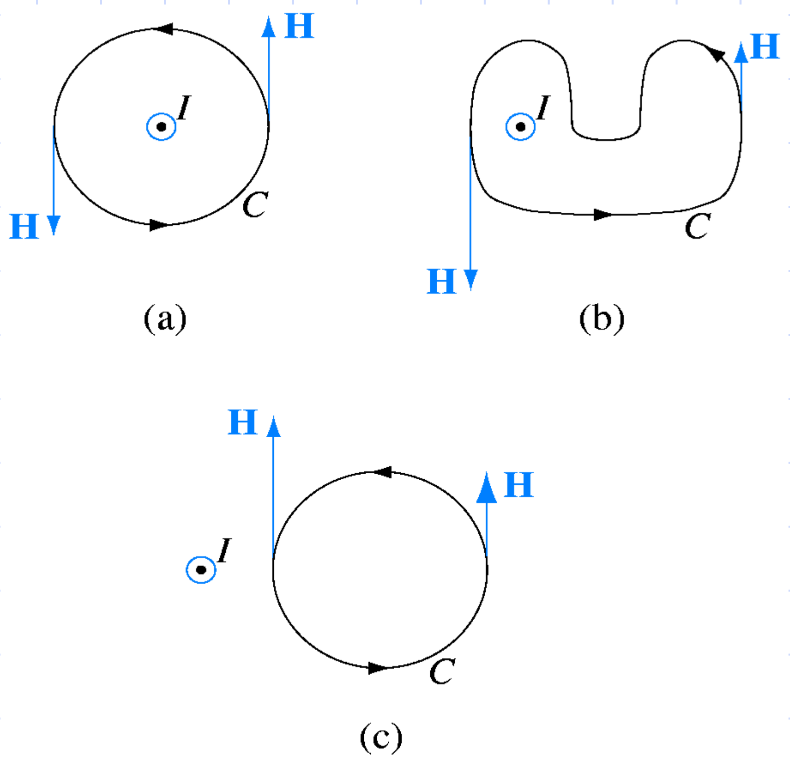
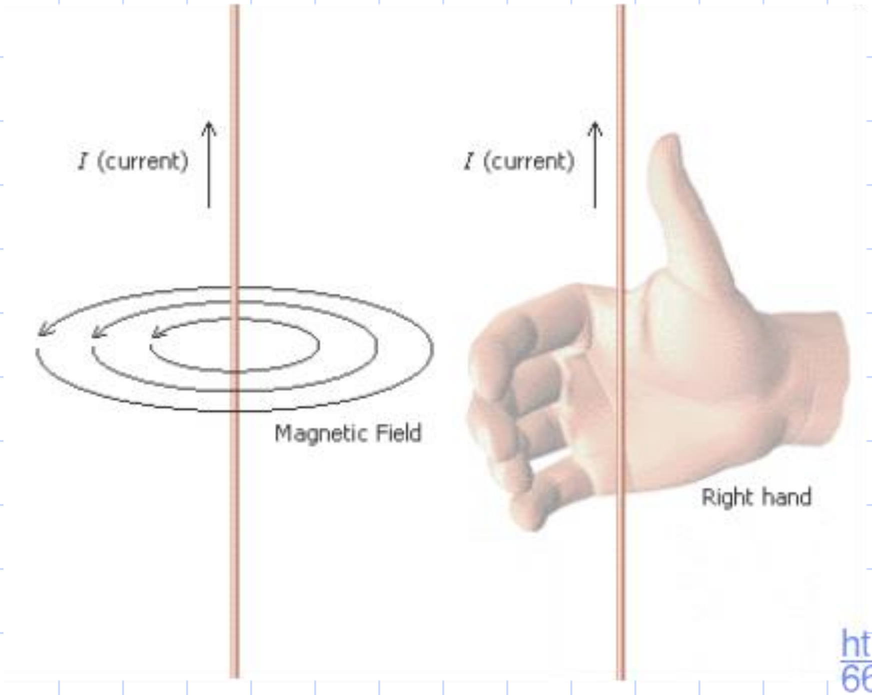


Figure 5-16

Ampere's Law

Direction of B $\longrightarrow$ B wraps around I & j



Use right-hand rule

- thumb along I & j
- fingers are in B

http://encarta.msn.com/media_701504656_761566543_-1_1/Right-Hand_Rule.html

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Ampere's Law

- In the case of multiple current channels, use field superposition as you would with point charges in electrostatics.

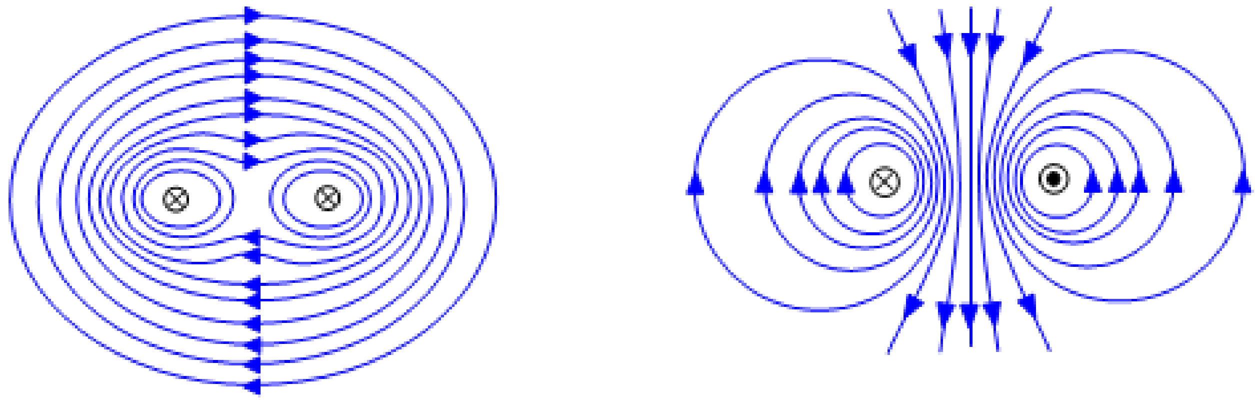
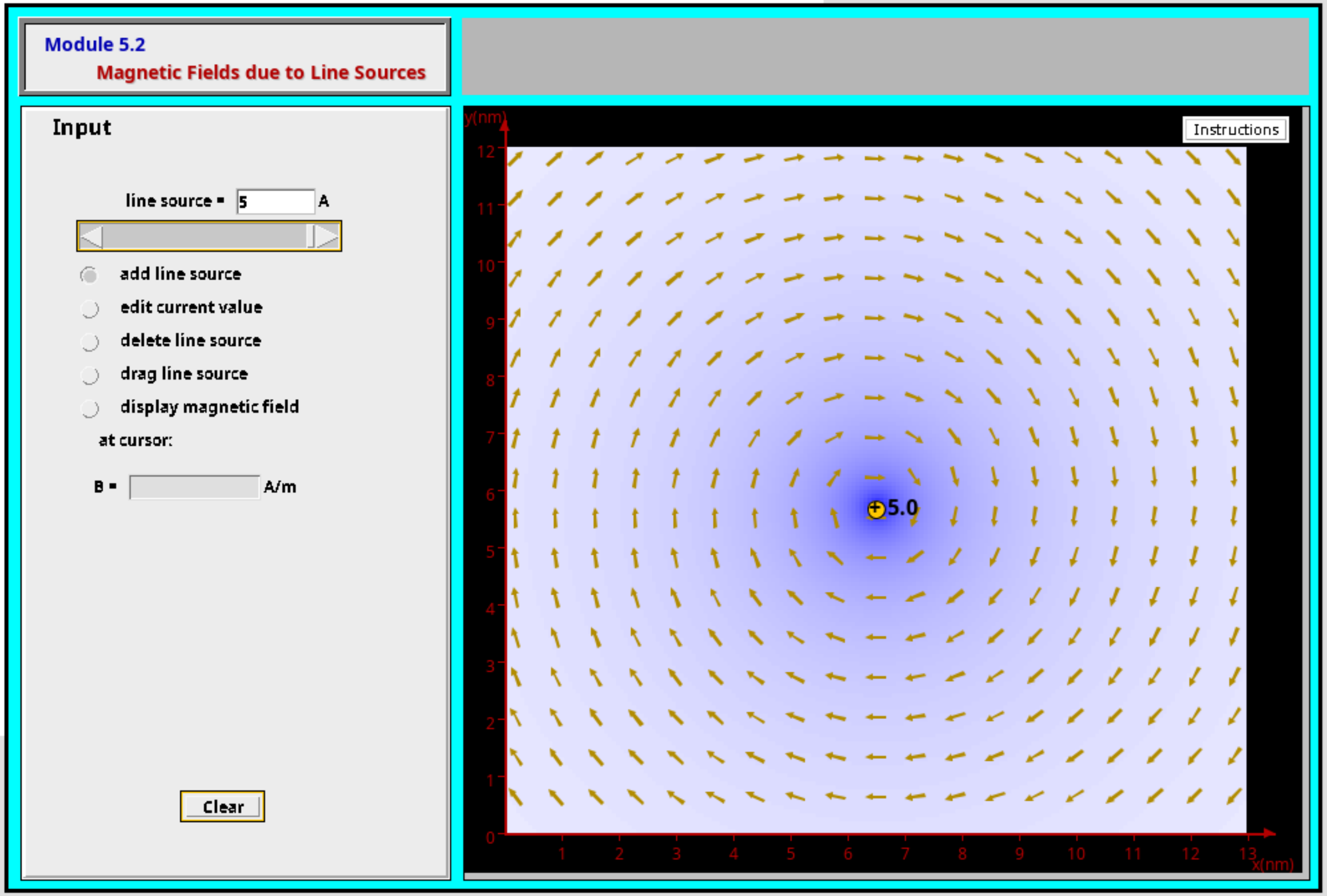


Figure 2(a) Figure 2(b)

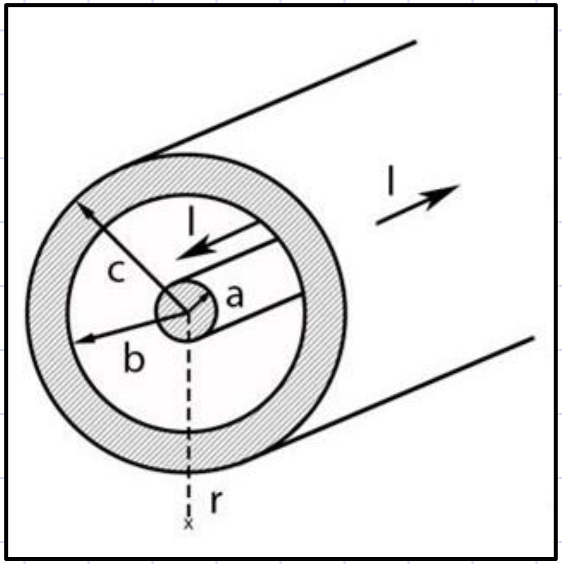
scienceforums.net

Ampere’s Law



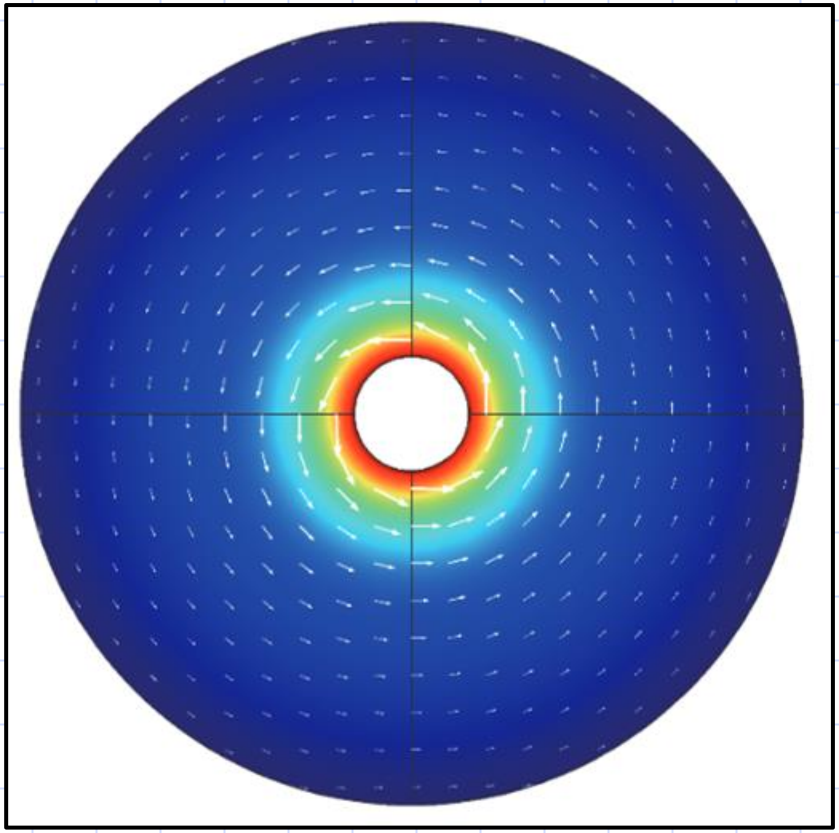
Module 5.2
Magnetic Fields due to Line Sources
Input
line source = 5 A
add line source
edit current value
delete line source
drag line source
display magnetic field
at cursor:
B = A/m
Clear
Instructions
y(nm)
x(nm)
5.0

Ampere’s Law



What does the magnetic field look like for this conductive coaxial cable?

Ampere's Law



comsol.com

The field between the conductors looks something like this. (you can check this using the right hand rule)

A quick point: conductors do not shield magnetic fields in the same way they shield electric fields. Why is this?

Ampere's Law

$$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s} = I_{net}$$

$$\vec{B} = \mu_0 \vec{H}$$

for $r < a$: $\vec{J} = \frac{I}{\pi a^2} \hat{z}$

$$H(2\pi r) = \left(\frac{I}{\pi a^2}\right) \pi r^2$$

$$\vec{H} = \frac{I}{2\pi a^2} r \, \hat{\phi}$$

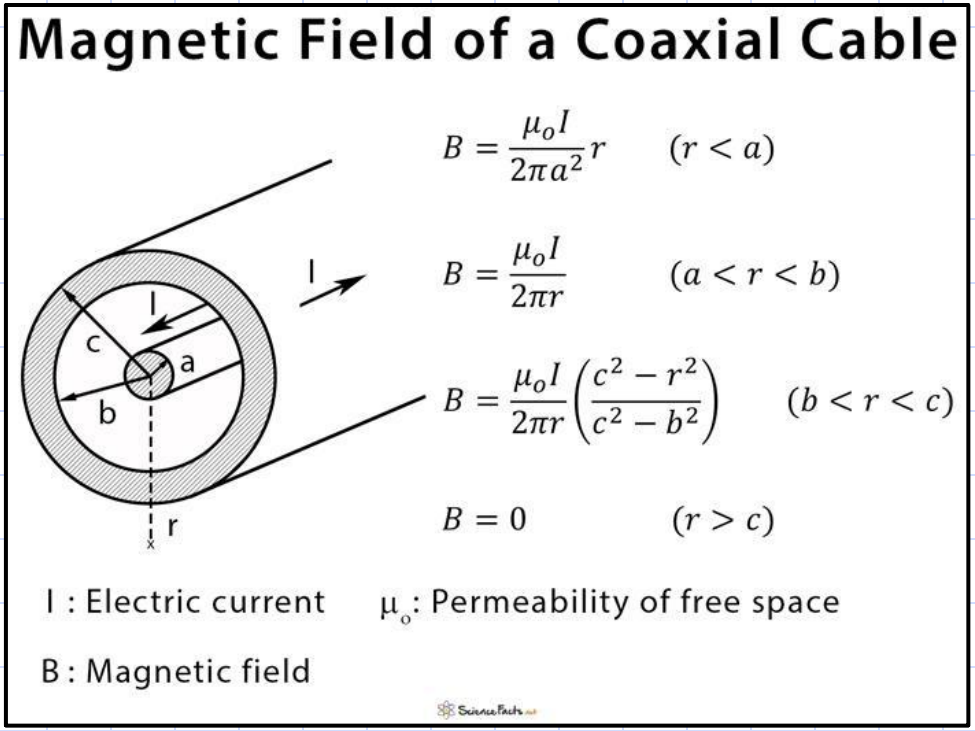
$$\vec{B} = \frac{\mu_0 I}{2\pi a^2} r \, \hat{\phi}$$

for $a < r < b$:

$$H(2\pi r) = I$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

Ampere’s Law



Ampere's law

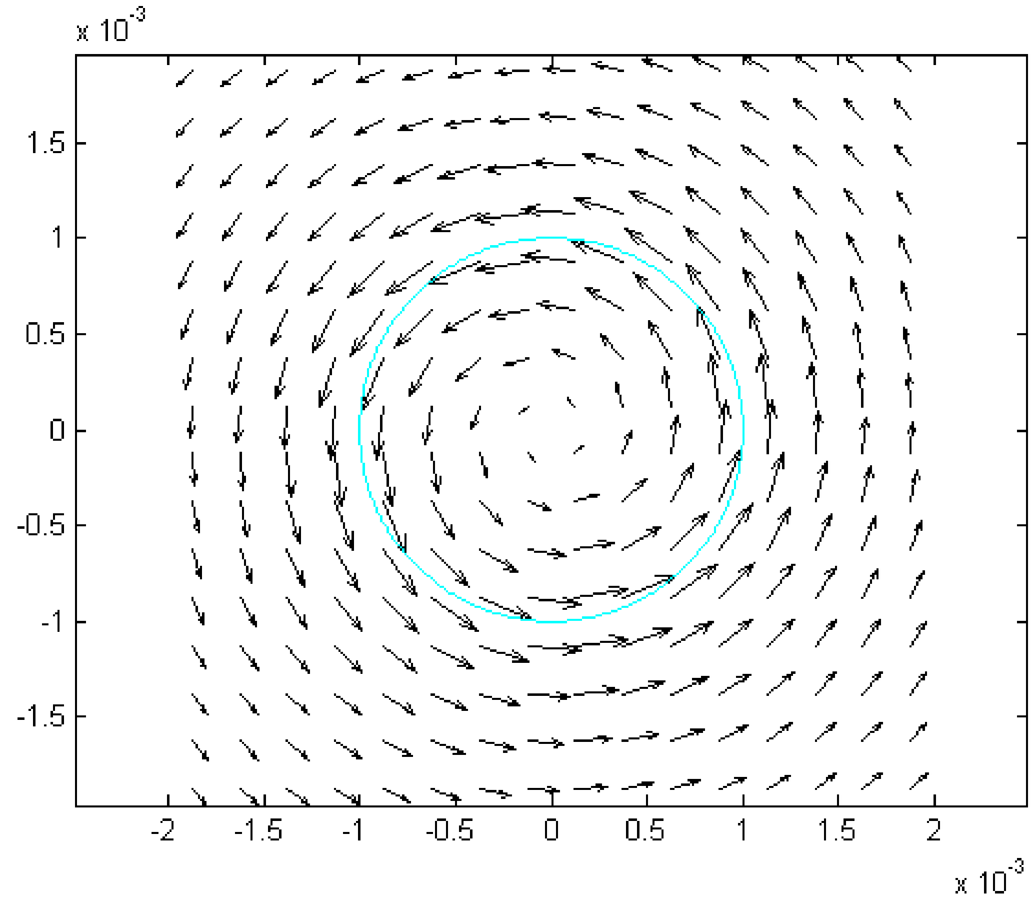
$$\oint H \bullet dl = \int j \bullet ds = I_{net}$$

$$B = \mu_0 \cdot H$$

sciencefacts.net

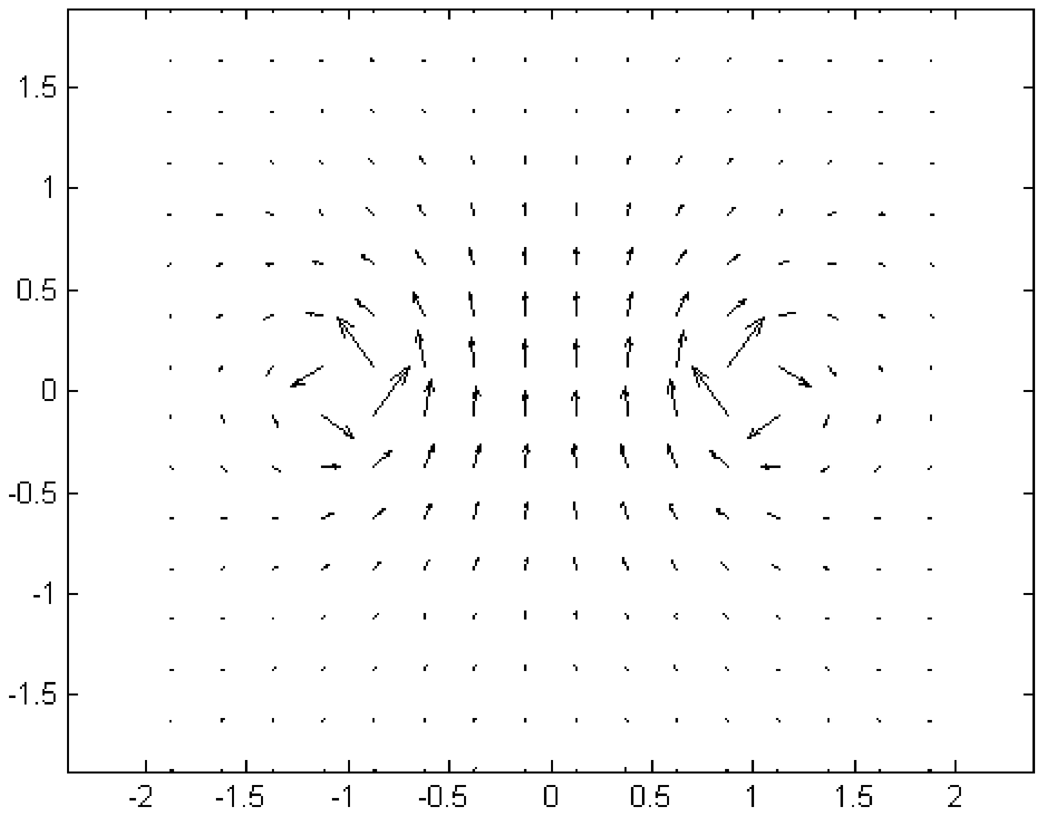
Magnetostatics

Is this field electrostatic or magnetostatic?



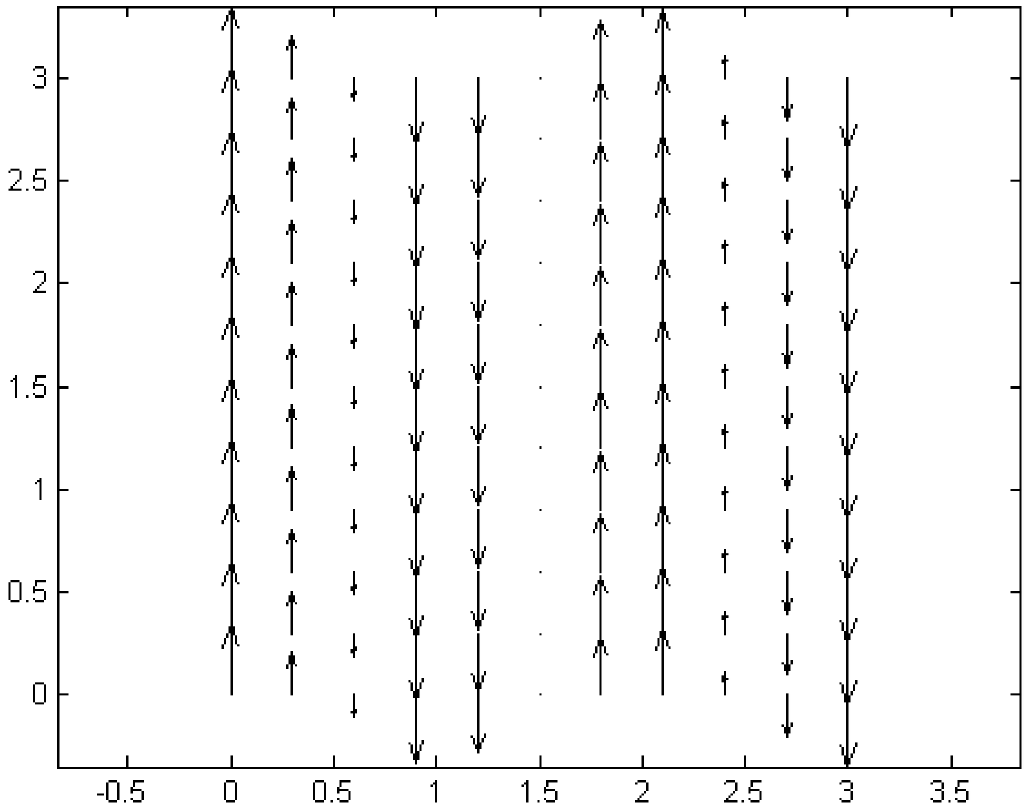
Magnetostatics

Is this field electrostatic or magnetostatic?



Magnetostatics

Is this field electrostatic or magnetostatic?



Magnetostatics

Do Lecture 12, Exercise 2 in groups of up to 4.