CSCI 2300 Final Exam
Fall 2021.

Open notes and textbook use are permitted. You may use electronic devices to compose your answers, but you may not use them to solve problems or check your work. There are 4 questions adding to 80 points + 1 OPTIONAL BONUS question which you do not have to do.

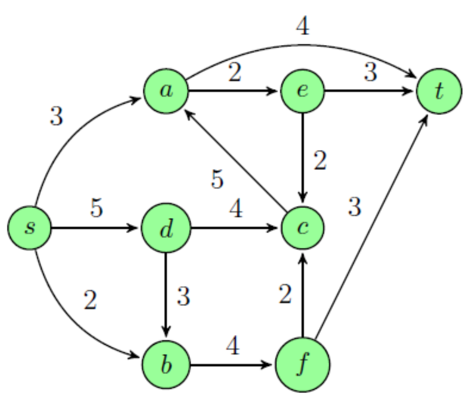
120mins exam time

LAST NAME -----------------

FIRST NAME -------------------

RIN NUMBER ----------------------

Question 1 [20 points]:



1.1. [3 points]:
Consider the graph in the figure with edge capacities as shown. Assume that the Ford–Fulkerson algorithm would stop with max flow: 3+4+2=9 on this graph.

List the edges that form a minimum capacity s-t cut for this graph.

1.2. [3 points] What is the time complexity of the Ford–Fulkerson algorithm? Why is it considered to be pseudo-polynomial time?

1.3. [2 points]: What are the two elements of dynamic programming?

1.4. [2 points]: Is dynamic programming a bottom-up or top-down approach? Why?

1.5. [2 points]: Is a greedy algorithm a bottom-up or top-down approach? Why?

1.6. [1 point]: What is memoization?

1.7. [3 points]: Given an undirected graph G=(V,E), the Shortest Path Problem (SHP) asks you to find a path from vertex u to vertex v with the fewest edges. Prove or disprove that SHP has optimal substructure.

1.8. [4 points]: Given an undirected graph G=(V,E), the Longest Simple Path problem (LSP) asks you to find a simple path (one with no cycles) from u to v with most edges. Prove or disprove that LSP has optimal substructure.

Question 2 [20 points]: The classical Traveling Salesman Problem asks for the most efficient way to visit every city on a map exactly once, starting and ending at the same city.

Consider a variation where the salesman does not have to visit *every* city. Additionally, the salesman is paid a variable amount for each city he visits. He pays for his own travel, so his net profit is the sum of his payments and his expenditures. He still must return home at the end.

This can be formulated as a graph problem. Given an undirected graph G=(V,E,D,P), where D is a matrix of distances between vertices and P is a list of payouts per vertex, find a cycle that maximizes the profit of the salesman.

2.1. [5 points] What is the appropriate subproblem? Hint: It should include the *last* city being visited for that subproblem.

2.2. [10 points] Formulate this problem as a Dynamic Programming problem and give your algorithm.

2.3. [5 points] What is the total running time of your DP algorithm?

Question 3 [20 points]: Linear Programming.

3.1. [10 points]: Suppose you are making a weekly schedule. Your schedule has 20 weekday daytime (**WD**) hours, 10 weekday nighttime (**WN**) hours, and 10 weekend (**WE**) hours.

You can do *only one* of the following activities at a time:

1. Solve puzzles 2. Hang out with your friends 3. Do part-time programming work.

You have 3 goals to achieve:

1. You must solve exactly 50 puzzles. For each **WD** hour or **WE** hour you can solve 2 puzzles, but for each **WN** hour, you can only solve 1 puzzle (because you are tired :))
2. You must spend *at least* 10 hours with your friends, but they are not available during **WD** hours.
3. You must make money by working. $10/h for **WD** hours, $15/h for **WN** hours, $20/h for **WE** hours.

Model this problem as Linear Program to ensure that you make as much money as possible whilst solving all the puzzles and spending enough time with your friends.

You are NOT required to solve the linear program. You only need to formulate it.

3.2. [10 points]: What is the dual of the following LP formulation?

Max 3x1 +x2 +2x3
Subject to
x1+x2+3x3 <=30
2x1+2x2+5x3 <= 24
4x1+x2+2x3 <=36
x1,x2,x3 >=0

Question 4 [20 points]: Reductions and NP completeness.

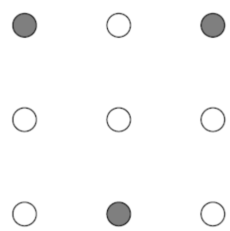
4.1. [10 points]: Show how we can solve any instance of a 2-SAT problem (i.e., each clause has at most 2 literals), in O(N) time, using a reduction to a problem solved via DFS. Hint: what should there be *N* of? Also, do you remember about strongly connected components?

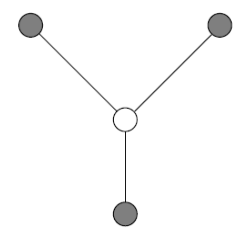
4.2. [10 points]: Consider the Minimum Leaf Spanning Tree problem: Given a graph G = (V, E) and an integer k, is there a spanning tree T in G that contains at most k leaves?

Show that MLST problem is NP-Complete by describing a reduction (hint: use the Hamiltonian Path problem, which asks if we can find a path that visits every vertex once).

Bonus Question [16 points]: You do not have to solve this problem unless you like to get extra credit. Partial credit may be awarded, but it will be less generous than on other problems.

In the MINIMUM STEINER TREE problem, the input consists of: a complete graph $G = (V, E)$ with distances d_{uv} between all pairs of nodes; and a distinguished set of *terminal nodes* $V' \subseteq V$. The goal is to find a minimum-cost tree that includes the vertices V'. This tree may or may not include nodes in $V - V'$.





Suppose the distances in the input are a metric (recall the definition on page 277). Show that an efficient ratio-2 approximation algorithm for MINIMUM STEINER TREE can be obtained by ignoring the nonterminal nodes and simply returning the minimum spanning tree on V'. (*Hint:* Recall our approximation algorithm for the TSP.)