Hayden Fuller
Fields and Waves HW2

1) Standing Wave Patterns on Lossless Transmission Lines
   All parts of Problem 1 refer to the standing wave pattern below. The characteristic impedance of the transmission line is Z0 = 60Ω.
   a) Simply by inspecting the features of the standing wave pattern, determine if ZL is purely real (resistive), purely imaginary (reactive), or a combination of both. How can you tell?

Real, the voltage at the load is at a peak of the standing wave, an imaginary component would shift it to be between the peaks.

   b) Again, simply by inspecting features of the standing wave pattern, determine the sign of ΓL , the reflection coefficient at the load.

The standing wave is at a minimum at the load, so the reflection coefficient is negative.
G = (ZL-Z0)/(ZL+Z0)
ZL<Z0
GammaL < 0, negative

   c) Calculate the standing wave ratio (SWR) for this standing wave pattern.

Vmax/Vmin=6/2=3

   d) Keeping in mind your result from b), calculate ΓL , the reflection coefficient at the load.

3=(1+|G|)/(1-|G|)
2=|4G|
|G|=1/2
G=-1/2

   e) Calculate ZL, the load impedance.

-1/2=(ZL-60)/(ZL+60)
(ZL+60)=-2(ZL-60)
ZL+60=-2ZL+120
3ZL=60
ZL=20 Ohms

   f) What is the value of $V0$ +, the amplitude of the incident voltage wave?

VL=V0+ + V0-
VL=V0+ + (G*V0+)
VL=(1 + G) * V0+
2=(1 + -1/2) * V0+
2=1/2 * V0+
V0+=4V

2) Input Impedance of Lossless Transmission Line

a) What is the wavelength of a 60 MHz voltage signal on a transmission line with a characteristic impedance Z0 = 75Ω and velocity factor vf = 0.7?

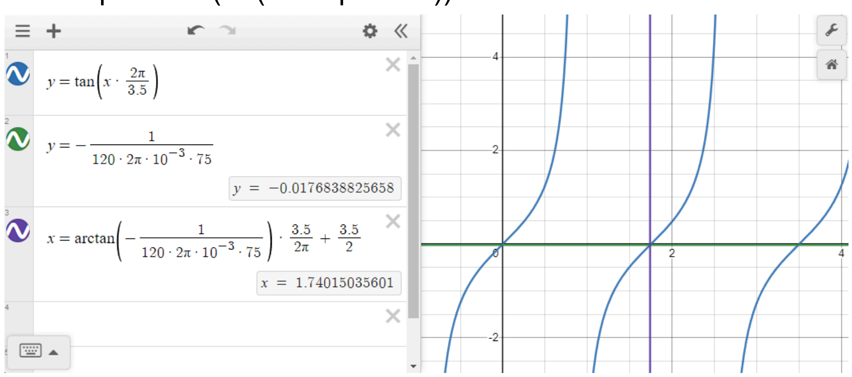
u=0.7c
fl=u
l=u/f
l=0.7*c/60M (m/s / s^-1 = m)
l=3.5m

b) You are tasked with replacing the capacitor in circuit A below with a short-circuited transmission line, resulting in circuit B.

… answered in c)

c) Using the same transmission line properties as in (a) and a source voltage frequency of 60MHz, what is the minimum length of the transmission line that would present the same input impedance to the generator circuit as the capacitor in circuit A?

Zin=j Z0 tan(L2pi/l)
Zc=1/jwC
j Z0 tan(L2pi/l)=1/jwC
tan(L2pi/l)=-1/wCZ0
L2pi/l=arctan(-1/wCZ0)
L=l/2pi arctan(-1/wCZ0)
L=3.5/2pi arctan(-1/(120Mpi*n*75)) (negative)
L=3.5/2pi arctan(-1/(120Mpi*n*75))+3.5/2



L=1.7401m

d) Suppose instead that you were given an open-circuited transmission line with which to replace the capacitor in circuit A of part (b). What is the minimum length of the transmission line that would present the same input impedance as the capacitor in circuit A?

Zin=-j Z0 cot(L2pi/l)
Repeat with cot, L=arccot(-1/(120*2pi*10^-3*75))*3.5/2pi
L=0.8848m

e) Given the circuit below, what is the minimum length of the transmission line T1 (same properties as in part a) that can be used to prevent reflection from occurring between the generator resistance R1 and the input of the transmission line? Will reflections still occur on the transmission line at the load?

l/2=1.75m, there will still be reflections at the load end because of the Z0 ZL mismatch

f) If R2 in the circuit above is instead 150Ω and transmission line T1 is a quarter wave transformer, what characteristic impedance Z0 of T1 will ensure that no reflection occurs between the generator impedance R1 and the transmission line? Will reflections still occur on the transmission line at the load?

l/4=.875m
Zin=Z0^2/ZL
Z0=sqrt(ZinZL)
Z0=sqrt(100*150)
Z0=122 Ohms

g) If the frequency of the source voltage in the circuit you designed in part f is changed, will the quarter wave transformer still ensure that no reflections occur between R1 and the input to the transmission line? Why or why not?

No, changing frequency changes wavelength, meaning our transmission line is no longer a quarter wavelength, and there will be reflections

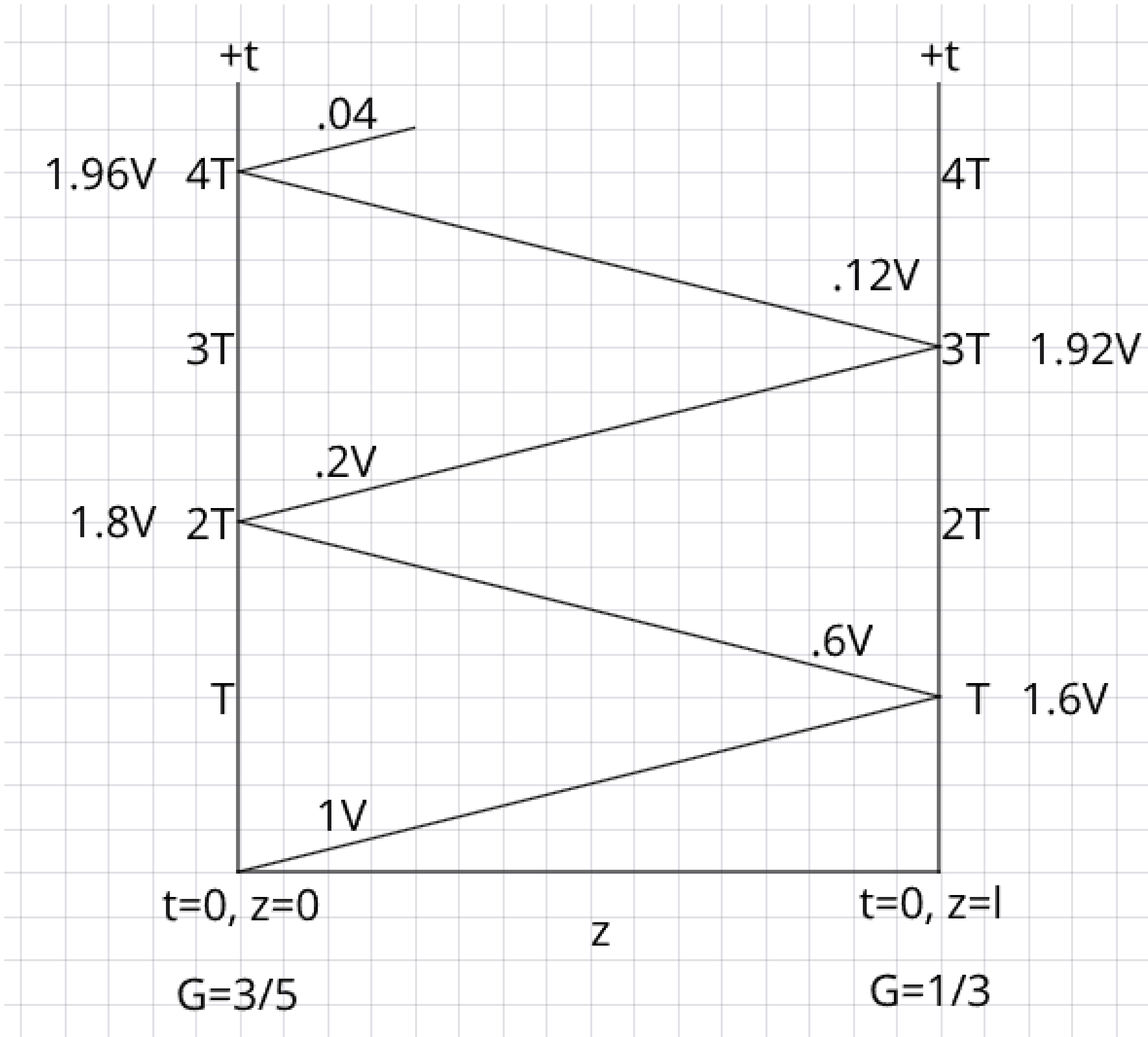
3) Transient Signals on Lossless Transmission Lines
   All parts of Problem 3 refer to the circuit below. At time t=0, the switch S1 closes and a 3V DC source supplies voltage to the circuit. The transmission line has a characteristic impedance of Z0 = 50Ω.
   a) What is the amplitude of the forward-traveling voltage wave $V0$ + that enters the transmission line at t = 0?

Nothing happening yet, so Zin=Z0=50.
Voltage divider, 3*50/150=1V

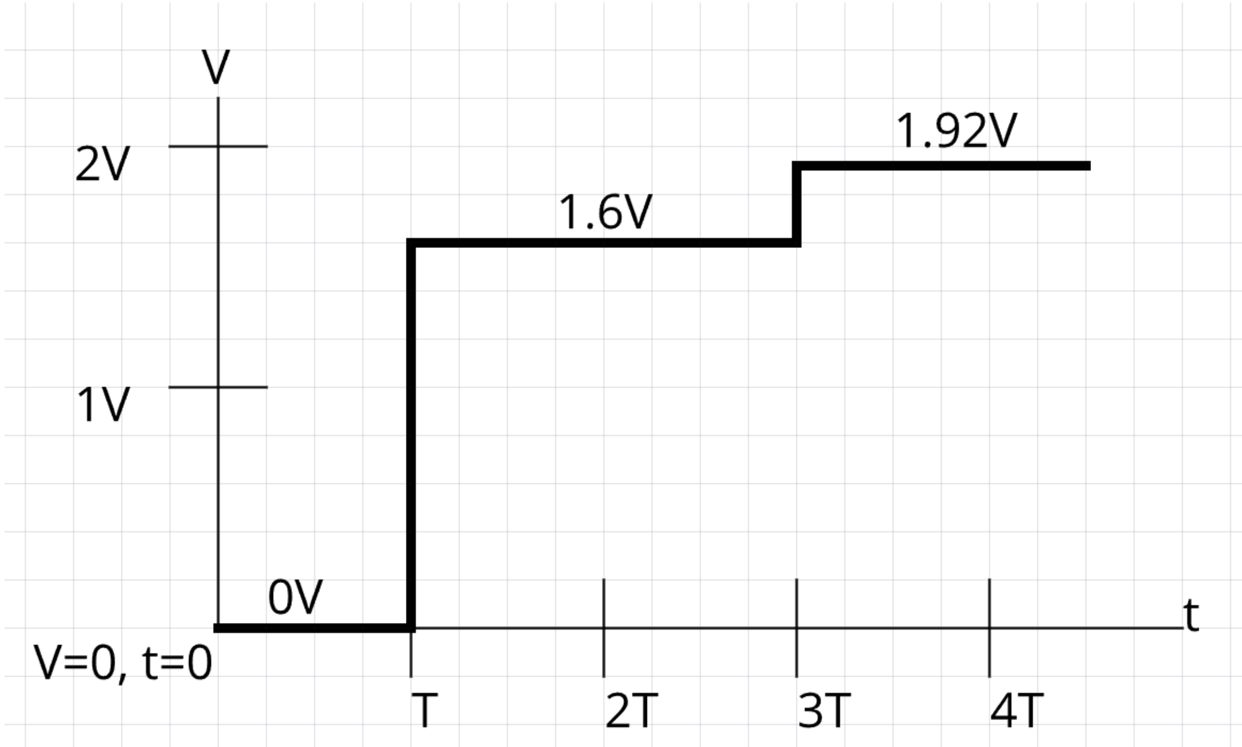
   b) What are ΓL , the reflection coefficient at the load, and Γg, the reflection coefficient at the input to the transmission line?

GL=(ZL-Z0)/(ZL+Z0)=(200-50)/(200+50)=150/250=3/5=.6
Gg=(Zg-Z0)/(Zg+Z0)=(100-50)/(100+50)=50/150=1/3=.333

   c) Draw a bounce diagram for the circuit above from t = 0 to t = 4T, where T is the time delay on the transmission line. Be sure to label both the time axis and distance axis, voltage wave amplitudes and directions of travel, and the voltages at the load and generator during reflection.


+t
+t
.04
1.96V 4T
4T
.12V
3T
3T 1.92V
.2V
1.8V 2T
2T
.6V
T
T 1.6V
1V
t=0, z=0
t=0, z=l
z
G=3/5
G=1/3

d) Sketch the voltage amplitude at the load vs. time for the timespan t = 0 to t = 4T. Be sure to label voltage amplitudes and critical times.



e) Assuming enough time has elapsed, what are the steady-state voltage and current on the line? Does this agree with the result you expect from DC circuit theory?

Vinf=Vg ZL/(Zg+ZL)=2V
Iinf=Vg/(Zg+ZL)=10mA
Yes, with DC the transmission line acts like a normal resistor, in this case, in a simple voltage divider.

f) If you were to replace RL with a capacitor instead, what would you expect the steady-state voltage and current to be? Why?

The capacitor would charge to 3V and act as an open circuit.
GL=(inf-50)(inf+50)=1
Vinf=3V inf/(50+inf) = 3V
Iinf = 3/(50+inf) = 0A
Again, same as a resistor with DC

4) Conceptual Questions

a) The standing wave ratio can be found using the absolute value of reflection coefficient at the load. Why is the absolute value used? Why doesn't the sign of the reflection coefficient matter?

The constructive and destructive interference will have the same peaks and valleys whether the ratio is positive or negative.
The equation is SWR = (1+|G|)/(1-|G|). With a positive G, we have (1+G)/(1-G). If we negate this G we have (1-G)/(1+G)=((1+G)/(1-G))^-1=SWR^-1.
You'll get the same ratio, just backwards, because when G is negative the locations of peaks and valleys swap, and a ratio greater than one is basically saying the valleys are greater than the peaks, but that just changes what's a peak and what's a valley, inverting it back to less than one.

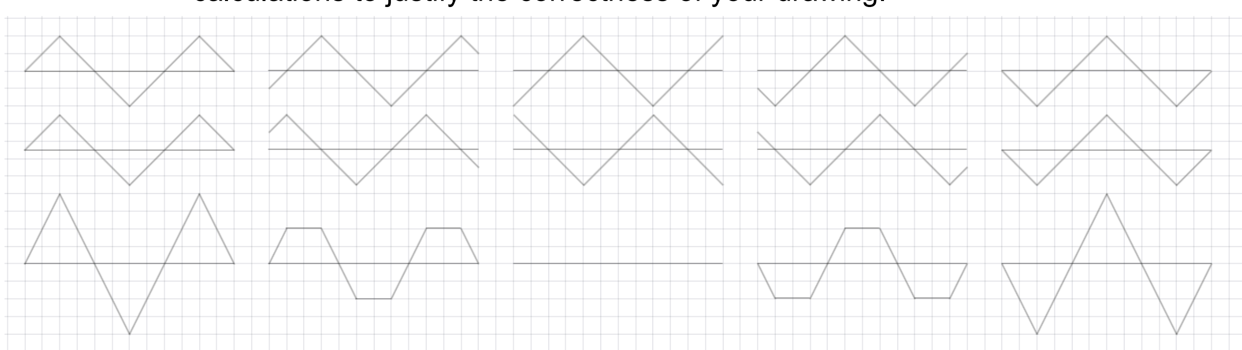
b) Suppose that in Problem 1, the frequency of the sinusoidal input voltage signal were increased by 50%. Would this change the standing wave pattern? If so, describe how.

It would add another standing wave, but otherwise remain the same. The wire is currently 2 standing wavelengths long, 2*1.5=3, so we still have an integer multiple.

c) Why don't standing wave patterns ever have values below 0?

The forward and backward traveling waves can at most cancel each other out and give a 0. Alternatively, they can at most add to double. Therefore the standing wave will be in the range of 0% to 200% of the original wave, and will therefore always be positive.

d) In principle, a standing wave pattern could exist for a non-sinusoidal input signal as well. Choose some non-sinusoidal input signal, length of line, Z0, and load impedance and draw the standing wave pattern that will result from it. Show calculations to justify the correctness of your drawing.



A triangle wave becomes a triangle wave with peaks and valleys cut off every oscillation.