Midterm. CS2300. October 14 2021

Open book, notes. No electronic devices (laptops, tablets, calculators, smartphones, supercomputers)

Question 1 [15 points]:

1. [5 pnts] Is $gcd(F_{n+1},F_n)=1$ for $n\geq 1$, where F_n is the n-th Fibonacci element? Justify your answer for full credit.

2. [5 pnts] Is $2^{2^n} = O(2^n)$? Justify your answer for full credit.

3. [5 pnts] is $(\sqrt{2})^{\log n} = \sqrt{n}$? Justify you answer for full credit.

Question 2 [15 points]: Compute

a) $17^{30} \bmod 31 =$

b) $3^{201} \bmod 11 =$

c) $53^{1069} \bmod 54 =$

Question 3 [15 points]: Give the Big Oh asymptotic bounds for the following Recurrence Relations:

1. $P(n) = 27P(n/3) +2$; $P(1) =1$.

2. $Q(n) = Q(n-1)+n$; $Q(1) = 1$.

3. $R(n) = R(n/2)+ ½$; $R(1)=1$.

4. $S(n) = rS(n/4)+n$; $S(1) =1$.

5. $T(n) = T(n/2)+n$; $T(1)$.

Question 4 [15 points]:

1. [10pnts] Prove or disprove that for any two nodes *s* and *t* in a directed graph, their strongly connected components are either identical or disjoint.

2. [5pnts] How many different topological orderings does the directed graph G=(V,E) with 5 nodes V={A,B,C,D,E} have with the following edges: E={(A→B),(A→C),(B→E),(D→E),(C→D)}?

Question 5 [20 points]: *Pick either 5.1 or 5.2.* ***Do not solve both of them.***

1. [20pnts] Given a set S of n integer elements write and algorithm to find whether any three elements (not necessarily distinct) add up to 0. *A pseudo code would suffice.* What is time complexity of your algorithm?

   (e.g.:for n=6 and S={-2,9,-2,12,4,-15,-13}, the numbers -2,-2,4 add up to 0. Note that you cannot use a single element multiple times. Thus, -2,-2,-2,-2,-2,-2,12 is **NOT** correct).

   Answer:

2. [20 pnts] You are given two sorted list of N integers in ascending order e.g., A= {-1,3,4,7 ,8, 12}, and B={5, 9, 10, 13, 14, 15}. Give an algorithm to merge these two lists to a single list in ascending order. *A pseudo code would suffice*. What is time complexity of your algorithm?

Question 6 [20 points]: Consider a simple hash function ($H(X_i) = (aX_i + c) \bmod N$.

i=1..M, so X contains M elements. M is larger than N. *a* and *c* are constants.

Consider the case where X= {0,1,...,8,9} and N=4.

1. [5pnts] Suppose we choose a=6, c=1. Is H(X) a universal hash function? Justify your answer.

2. [5 pnts] How would you change the parameters to make this a universal hash function?

3. [5 pnts] What is the probability that two distinct inputs will collide with the original hash function?

4. [5 pnts] What is the expected number of collisions? (Hint: use indicator random variables)