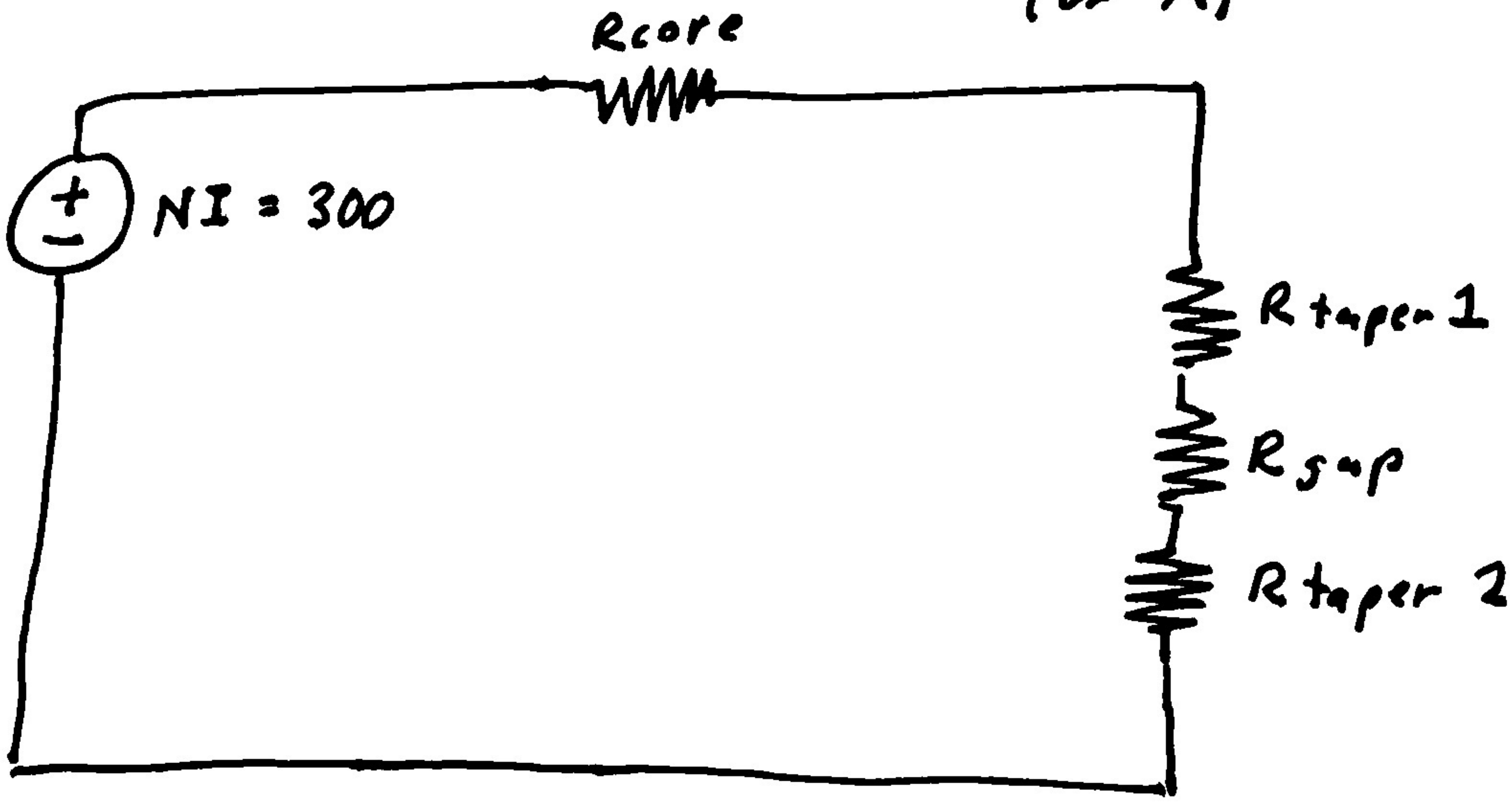
1. a.) $NI = (100\,mA)(3000) = 300$ ampere-turns (MMF) (or A)



$$R_{core} = \frac{l}{\mu A} = \frac{500\,cm - 80\,cm}{(5000\mu_0)(5\,cm)^2} = 2.67 \times 10^5\ H^{-1}$$

$$R_{taper\,1} = R_{taper\,2} = \frac{(100\,cm - 20\,cm)/2}{(5000\mu_0)(3.5\,cm)^2} = 5.2 \times 10^4\ H^{-1}$$

$$R_{gap} = \frac{20\,cm}{(500\mu_0)(2\,cm)^2} = 7.95 \times 10^5\ H^{-1}$$

$$\Psi = \frac{NI}{R_{core} + R_{taper\,1} + R_{taper\,2} + R_{gap}}$$

$$\Psi = \frac{300\ A}{1.165 \times 10^6\ H^{-1}} = 2.59 \times 10^{-4}\ Wb$$

b.) Λ (flux linkage) $= N\Psi = (3000)(2.59 \times 10^{-4}\ Wb)$

$= 0.777\ Wb$

$$L = \frac{\Lambda}{I} = 7.77\ H$$

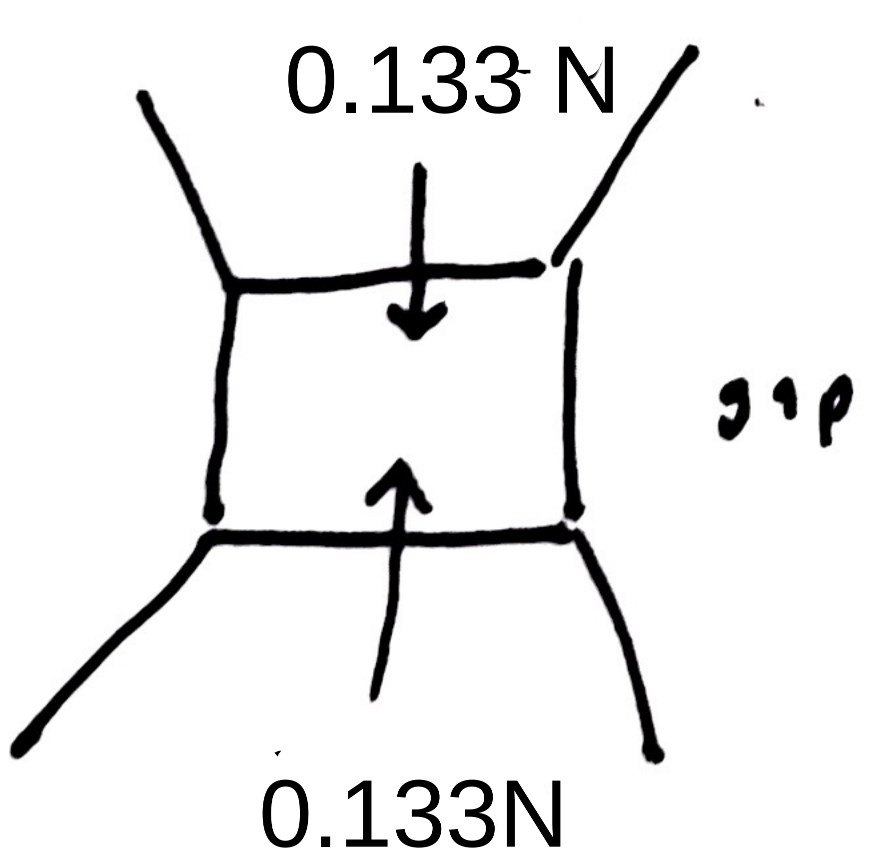
.) $$w_m = \frac{1}{2}\frac{|B|^2}{\mu}$$ recall that $\vec{B}$ represents flux density.

$$B = \frac{\Psi}{A} = \frac{2.59 \times 10^{-4}\ Wb}{(2cm)^2} = 0.647\ T$$

$$w_m = \frac{1}{2}\frac{(0.647\ T)^2}{500\mu_0} = 333.1\ J/m^3$$

$$333.1\ J/m^3 \cdot (2cm)^2 = 0.133\ N$$

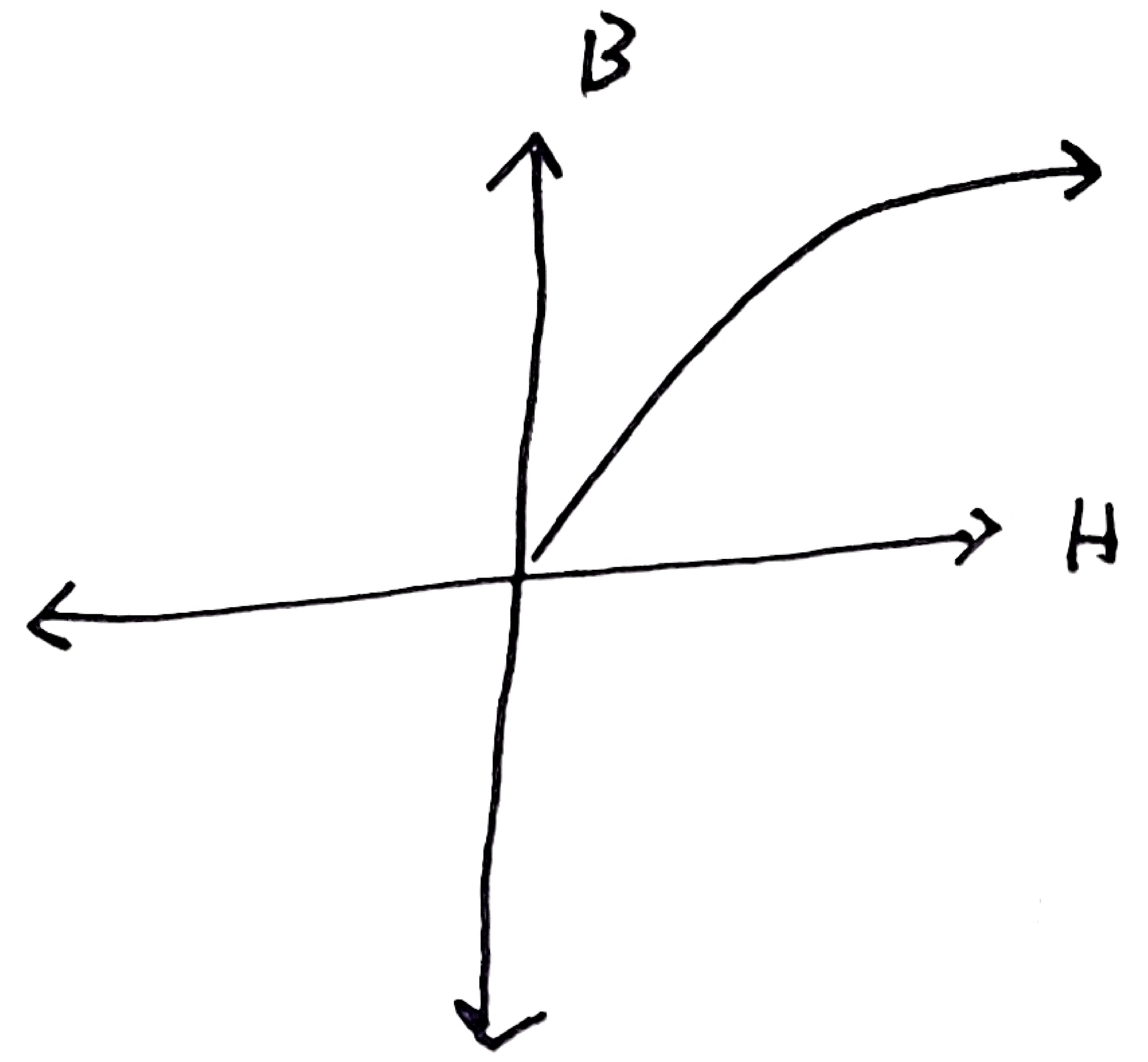
(pushing the tapered region into the gap on both sides)



Note that there is also a 0.0133N force pushing against the 0.133N force in both direction due to the energy density inside the tapered regions.

(We have neglected to show the force here since it is small.)

d.)



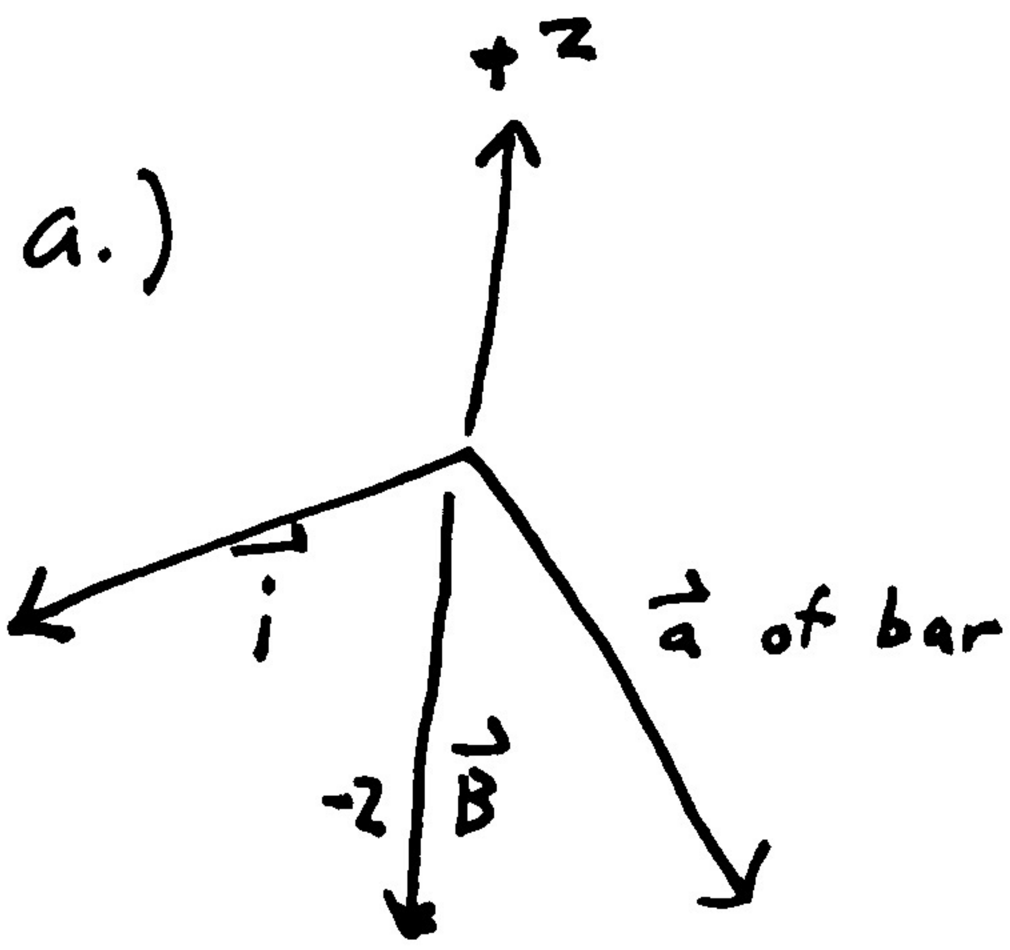
As shown above, the H-B curve begins to flatten at higher H (caused by high current).

This means that μ is effectively lower. So for additional current added beyond this point. L will effectively be lower.

(2.)

$\vec{F} = I\vec{l} \times \vec{B}$

a.)



Magnetic field must point into the page. If the rails are laying flat on the ground, this means into the ground.

x = projectile distance

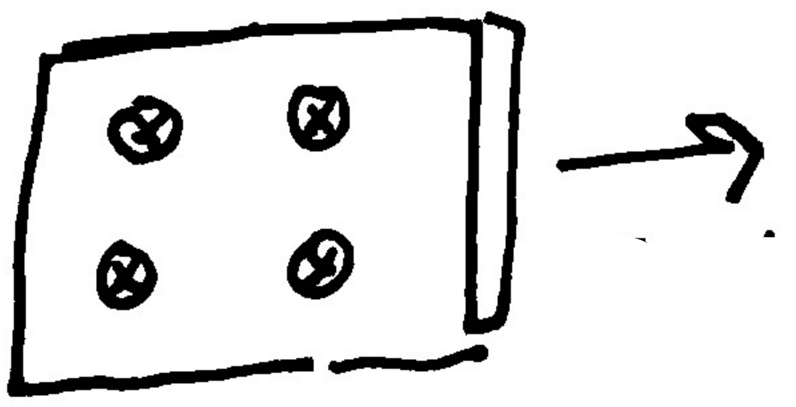
b.) $\frac{1V}{0.2x\,\Omega} = \frac{5}{x}A$

$|F| = I l B.$ Assume $l = 20cm$ for the bar.

$|F| = (\frac{5}{x}a)(20cm)(0.2\ T) = \frac{0.2}{x}N$ (points to the right)

(This answer does not take into account back emf, which will be addressed in the next question.)

c.)



$$\frac{d\Psi}{dt} = (0.2T)(0.2m)v$$

The back emf reduces the effective voltage of the battery (and therefore the current), reducing the force on the bar.
The bar will reach its top speed when the force is zero, which occurs when the back emf reaches the voltage
of the battery (1V).

(1V) = (0.2 T)(0.2m)v

v = 25 m/s

3. a.) $I = 50\,mA$ so $J = \frac{50\,mA}{\pi (1\,cm)^2} = \frac{500}{\pi}\ A/m^2$

$$\oint \vec{H} \cdot \vec{dl} = I_{enc}$$

$\vec{B}$ will point in ϕ direction.

For $0 \le r \le 1cm$

$$H(2\pi r) = J \cdot \pi r^2$$

$$\vec{H} = \frac{\frac{500}{\pi} \pi r^2}{2\pi r} \hat{\phi} = \frac{250\,r}{\pi} \hat{\phi} \quad A/m$$

$$\vec{B} = \mu \vec{H} = 5000 \mu_0 \vec{H} = \frac{(1.25 \times 10^6) \mu_0 r}{\pi} \hat{\phi} \quad T$$

Let outer conductor be located at 3cm.

For $1cm \le r \le 3cm$,

$$H(2\pi r) = 50\,mA$$

$$\vec{H} = \frac{(25 \times 10^{-3})}{\pi \cdot r} \hat{\phi} \quad A/m$$

$$\vec{B} = \mu_0 \vec{H} = \frac{(25 \times 10^{-3}) \mu_0}{\pi r} \hat{\phi} \quad T$$

For $3cm \le r \le 4cm$,

$\vec{H} = \vec{B} = 0$ because $I_{enc} = 0$.

b.) $\vec{B} = \nabla \times \vec{A}$

$\vec{A}$ has only a $\hat{z}$ component since this is the direction of current.

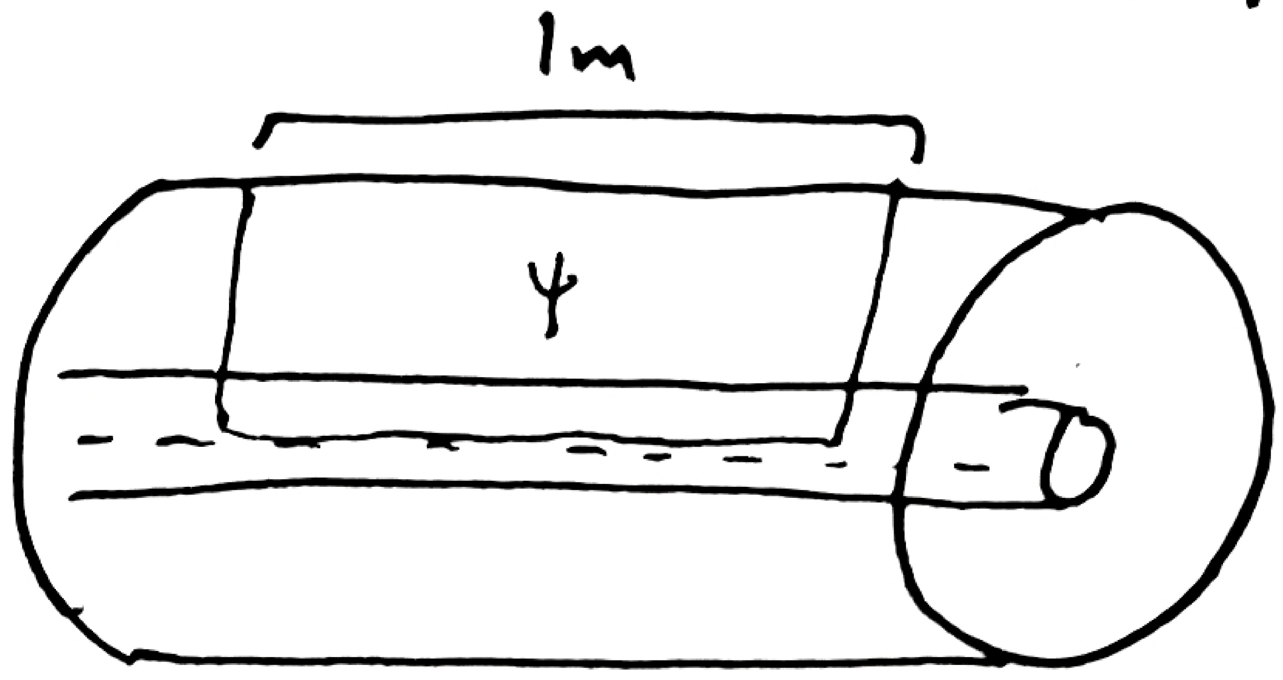
$$\frac{(25\times10^{-3})\mu_0}{\pi r}\hat{\phi} = \underbrace{\left[\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{r}}_{\text{zero because } \vec{B} \text{ has no } \hat{r} \text{ component}} + \left[\underbrace{\frac{\partial A_r}{\partial z}}_{\text{zero because } \vec{A} \text{ has no } \hat{r} \text{ component}} - \frac{\partial A_z}{\partial r}\right]\hat{\phi} + \underbrace{\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]\hat{z}}_{\text{zero because } \vec{B} \text{ has no } \hat{z} \text{ component}}$$

$$\frac{(25\times10^{-3})\mu_0}{\pi r}\hat{\phi} = -\frac{\partial A_z}{\partial r}\hat{\phi}$$

$$A(r) = \frac{-(25\times10^{-3})\mu_0}{\pi}\ln(r)\,\hat{z}$$

c.) $L = \frac{\Lambda}{I}$.

Since there are no coils/turns, $L = \frac{\Psi}{I}$.



$$\Psi = \int \vec{B} \cdot \vec{dS}$$

We will find the flux through the rectangle above which is a surface in $\hat{\phi}$.

It goes from $r = 0cm$ to $r = 3cm$.
Its length is 1m so finding the flux through it will allow us to determine the inductance per meter.

$$\int \vec{B} \cdot \vec{dS} = \underbrace{\int_0^1}_{z} \underbrace{\int_0^{0.01}}_{r} \frac{(1.25 \times 10^6)\mu_0 r}{\pi} \, dr \, dz + \underbrace{\int_0^1}_{z} \underbrace{\int_{0.01}^{0.03}}_{r} \frac{(25 \times 10^{-3})\mu_0}{\pi r} \, dr \, dz$$

$$= \left| \frac{(1.25 \times 10^6)\mu_0 r^2}{2\pi} \right|_0^{0.01} + \left| \frac{(25 \times 10^{-3})\mu_0}{\pi} \ln(r) \right|_{0.01}^{0.03}$$

$$\downarrow$$

$$\Psi = 2.5 \times 10^{-5} \text{ Wb} + 1.099 \times 10^{-8} \text{ Wb}$$

$$L = \frac{2.5 \times 10^{-5} \text{ Wb}}{50 \text{ mA}} = 0.5 \text{ mH}$$

(4.) a.) $\vec{B}_2 = 0.2\hat{x} + 0.4\hat{y}$

$$\vec{B}_{1n} = \vec{B}_{2n}$$

$$\vec{B}_{1n} = 0.4\hat{y}$$

$$\vec{B}_{2t} = 0.2x$$

$$\vec{H}_{2t} = \vec{B}_{2t}/5000\mu_o\,\hat{x} = \frac{4\times10^{-5}}{\mu_o}\,\hat{x}$$

$$\vec{H}_{1t} - \vec{H}_{2t} = \vec{J}_s$$

$$\vec{J}_s = 0 \quad \text{so} \quad \vec{H}_{1t} = \vec{H}_{2t}$$

$$\vec{H}_{1t} = \frac{4\times10^{-5}}{\mu_o}\,\hat{x}$$

$$\vec{B}_{1t} = 200\mu_o\vec{H}_{1t} = 0.008\hat{x}$$

$$\vec{B}_1 = 0.008\hat{x} + 0.4\hat{y}$$

b.) $\vec{H}_{1t} - \vec{H}_{2t} = \vec{J}_s$.

If $\vec{H}_{2t}$ remains fixed while $\vec{J}_s$ changes, $\vec{H}_{1t}$ will change.

This will change the x component of $\vec{B}_1$.