Chapter 7. Small-signal admittance

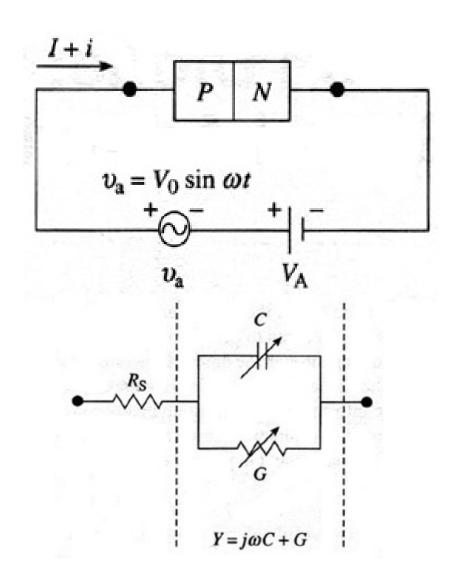
We will study the small signal response of the pn junction diode. A small ac signal (v_a) is superimposed on the DC bias. This results in ac current (i). Then, admittance Y is given by:

$$Y = i/v_a = G + j\omega C$$

Specifically, the following parameters will be studied:

- Reverse bias junction or depletion layer capacitance
- Forward bias diffusion or charge storage capacitance
- Forward and reverse bias conductance.

Capacitance measurements



$$I = DC$$

$$i = ac$$

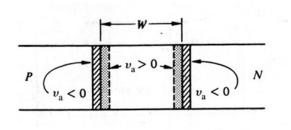
$$Y = admittance$$

$$= \frac{i}{v_{ac}} = G + j\omega C$$

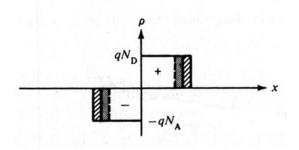
i and v_a depend on the applied DC bias

Reverse bias junction capacitance

A pn junction under reverse bias behaves like a capacitor. Such capacitors are used in ICs as voltage-controlled capacitors.



Depletion layer width under small ac superimposed on DC bias voltage.



Looks similar to a parallel plate capacitor.

$$C_{j} = \frac{\varepsilon_{Si}A}{W}$$

where W is the depletion-layer width under DC bias.

Reverse bias junction capacitance

$$W = \left[\frac{2\varepsilon_{\text{Si}}}{q} \left(\frac{N_{\text{A}} + N_{\text{D}}}{N_{\text{A}} N_{\text{D}}} \right) (V_{\text{bi}} - V_{\text{A}}) \right]^{1/2}$$
 For pn junction

$$= \left[\frac{2\varepsilon_{\text{Si}}}{qN_{\text{B}}} \left(V_{\text{bi}} - V_{\text{A}} \right) \right]^{1/2}$$

For p⁺n or pn⁺ junction where $N_{\rm B}$ is the doping on the lightly doped side

$$C_{\rm J} = \frac{\varepsilon_{\rm Si} A}{W} = A \left(\frac{\varepsilon_{\rm Si} q N_{\rm B}}{2(V_{\rm bi} - V_{\Delta})} \right)^{1/2}$$
 For asymmetrically doped junction

 $C_{\rm J}$ increases with $N_{\rm B}^{1/2}$

 $C_{\rm J}$ decreases with applied reverse bias

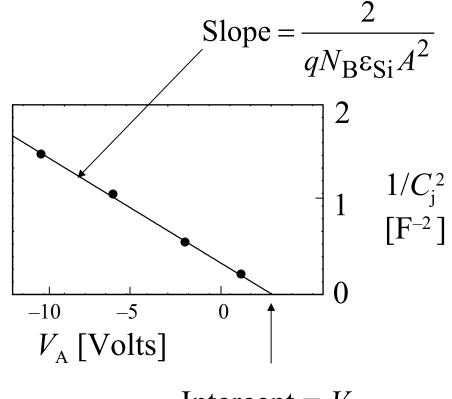
Parameter extraction/profiling

C-V data from a pn junction is routinely used to determine the doping profile on the lightly doped side of the junction.

$$C_{\rm J} = \frac{\varepsilon_{\rm Si} A}{W} = A \left(\frac{\varepsilon_{\rm Si} q N_{\rm B}}{2(V_{\rm bi} - V_{\rm A})} \right)^{1/2}$$

$$\frac{1}{C_{\rm J}^2} = \frac{2}{A^2 q N_{\rm B} \varepsilon_{\rm Si}} (V_{\rm bi} - V_{\rm A})$$

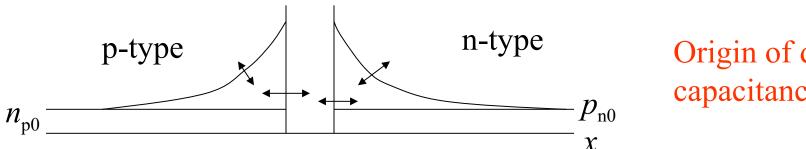
If the doping on the lightly doped side is uniform, a plot of $1/C_J^2$ versus V_A should be a straight line with a slope inversely proportional to N_B and an extrapolated $1/C_J^2 = 0$ intercept equal to V_{bi} .



Intercept = $V_{\rm bi}$

Forward bias diffusion capacitance, $C_{\rm D}$

 $C_{\rm D}$ is also called the charge storage capacitance. The variation of the injected minority-carrier charge, which is a function of the applied bias, results in the diffusion capacitance. Both $C_{\rm I}$ and $C_{\rm D}$ are always present, but for the forward-bias case, C_D becomes dominant.



Origin of diffusion capacitance

For a p⁺n junction, $I = Q_p/\tau_p$ where Q_p is total excess charge in n-side

$$Q_{p} = I\tau_{p} = qA \frac{D_{p}\tau_{p}}{L_{p}} p_{n0} \left[\exp\left(\frac{qV_{A}}{kT}\right) - 1 \right] \approx qAL_{p} p_{n0} e^{\frac{qV_{A}}{kT}}$$

$$C_{D} = \frac{dQ_{p}}{dV} = \frac{q}{kT} qAL_{p} p_{n0} \exp\left(\frac{qV_{A}}{kT}\right) = \frac{q}{kT} I \tau_{p}$$

Forward bias conductance

$$G_{\rm D} = \frac{\mathrm{d}I}{\mathrm{d}V} = \frac{qAD_{\rm p}p_{\rm n0}}{L_{\rm p}} \frac{\mathrm{d}}{\mathrm{d}V} \left(\mathrm{e}^{\frac{qV_{\rm A}}{kT}} \right) = \frac{q}{kT}I$$

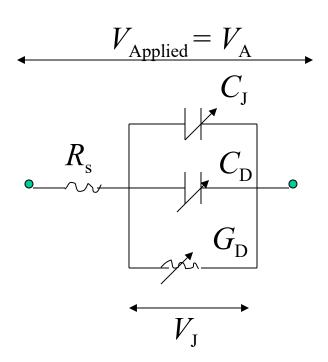
$$G_{\rm D} = \frac{q}{kT} I$$

$$G_{\rm D} = \frac{q}{kT} I$$
 $C_{\rm D} = \frac{q}{kT} I \tau_{\rm p}$



$$\omega \tau_{p} \ll 1$$

Complicated at higher frequencies.



Equivalent circuit for a diode

Example

Problem: Consider a p⁺n junction forward biased such that the forward current is 1 mA. Assume the lifetime of holes is 10⁻⁷ s. Calculate the diffusion capacitance and the diffusion resistance.

Solution:
$$C_D = 3.86 \text{ nF}$$
 $r_d = 1/G_D = 25.9 \Omega$

The current through the depletion layer will mostly be carried by (holes, electrons: choose one)?

Plot the current carried by the holes and electrons through the n-type region, assuming that the diffusion length of holes is 1 μ m.