

Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 2500: Engineering Probability, Spring 2023

Homework #4: due Monday, March 13th, at 11:59PM.

Show all work for full credit!

Submit your work as a single PDF on Gradescope, labeling each problem number with a page.

1. (25 points.) Dr. Michael Morbius is an awesome character that everyone loves unironically. He regularly declares that “it’s Morbin’ time”, which lasts a random length of time X . The CDF for the random variable X is described by the formula

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{4}x^3 & x \in [0, 1] \\ \frac{1}{4}x + \frac{1}{2} & x \in [1, 2] \\ 1 & x > 2 \end{cases}$$

- (a) (5 points.) Sketch the CDF (by hand). Make sure to include important points on the x and y axes.
- (b) (5 points.) Use the CDF to compute $P\left(X < \frac{3}{2}\right)$.
- (c) (5 points.) Use the CDF to compute $P\left(X \in \left[\frac{1}{2}, \frac{9}{8}\right]\right)$.
- (d) (10 points.) Determine the formula for the PDF corresponding to this CDF, and sketch it (by hand). Make sure to include important points on the x and y axes.

2. (25 points.) The length X of a champion trombone solo has the CDF given by the formula

$$F_X(x) = \frac{1}{2} \left(\frac{e^{2x} - 1}{e^{2x} + 1} \right) + \frac{1}{2}$$

- (a) (5 points.) Show that the function satisfies the “rules” $F_X(-\infty) = 0$ and $F_X(\infty) = 1$.
- (b) (5 points.) Sketch the CDF (by hand). Make sure to include important points on the x and y axes.
- (c) (5 points.) Use the CDF to compute $P(-2 < X < 1)$.
- (d) (5 points.) Use the CDF to compute $P(X = 2)$.
- (e) (5 points.) Determine the PDF corresponding to this CDF.

3. (25 points.) The PDF for a continuous random variable X is described by the formula

$$f_X(x) = \begin{cases} cx^2 & x \in [-2, 3] \\ 0 & \text{otherwise} \end{cases}$$

- (a) (10 points.) Determine the value of c that makes this a valid PDF.
 - (b) (5 points.) Sketch the PDF (by hand) for the random variable X . Make sure to include important points on the x and y axes.
 - (c) (5 points.) Use the PDF to compute $P(X \in [-1, 2])$.
 - (d) (5 points.) Determine the CDF corresponding to this PDF.
4. (25 points.) The time between deaths in a Kaizo Mario World speedrun is modeled by an exponential random variable X (measured in minutes) with parameter $\lambda = \frac{1}{3}$.
- (a) (8 points) Compute the probability that there are no deaths in a 2-minute period.
 - (b) (8 points) Compute the probability that there are exactly 4 deaths in a 5-minute span. Hint: remember the connection between the exponential and Poisson random variables.
 - (c) (9 points) What would λ need to be so that the events {less than 10 minutes between deaths} and {greater than 10 minutes between deaths} are equally probable? That is, half of the PDF's mass is on one side of 10 minutes and half is on the other (the median, not the mean).