Addition rules:

1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are mutually exclusive events, $P(A \cap B) = \emptyset$

2) $P(A \cup B) = P(A) + P(B)$

Total probability:

1) $P(B) = P(A \cap B) + P(A' \cap B) = P(B|A)P(A) + P(B|A')P(A')$

If E_1, E_2 and E_3 are mutually exclusive events, $P(E_1 \cap E_2 \cap E_3) = \emptyset$

2) $P(B) = P(E_1 \cap B) + P(E_2 \cap B) + P(E_3 \cap B) = P(B|E_1)P(E_1) + P(B|E_2)P(E_2) + P(B|E_3)P(E_3)$

Conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

Bayes' Theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

If E_1, E_2 and E_3 are mutually exclusive events, $i.e.$ $P(E_1 \cap E_2 \cap E_3) = \emptyset$

$$P(E_1|B) = \frac{P(B|E_1)P(E_1)}{P(B|E_1)P(E_1) + P(B|E_2)P(E_2) + P(B|E_3)P(E_3)}$$

Mean and Variance of a discrete random variable

$$\mu = E\{X\} = \sum_{\text{all } x} x\, f(x) \qquad \sigma^2 = V\{X\} = E\{(X-\mu)^2\} = \sum_{\text{all } x} (x-\mu)^2\, f(x)$$

Discrete uniform distribution

A special case: if all outcomes, x_i, are integers with spacing 1: $a, a+1, a+2, \ldots, b$, then:

$$\mu = \frac{a+b}{2}, \qquad \sigma^2 = \frac{(b-a+1)^2 - 1}{12}$$

Binomial distribution:

$$f(x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & \text{for } x = 0, \ldots, n \\ 0 & \text{else} \end{cases} \qquad \text{where, } \binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

$$\mu = np, \qquad \sigma^2 = np(1-p)$$

Poisson distribution:

$$f(x) = \begin{cases} \frac{\lambda^x \exp(-\lambda)}{x!} & \text{for } x = 0,1,2,3,\ldots \\ 0 & \text{else} \end{cases}$$

$$\mu = \lambda, \qquad \sigma^2 = \lambda$$

Mean and Variance of a continuous random variable

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) \mathrm{d}x, \qquad \sigma^2 = V(X) = E((X-\mu)^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) \mathrm{d}x$$

Continuous uniform distribution

The probability density function is $f(x) = 1/(b-a)$ for $a \le x \le b$

$$E\{X\} = \mu = \frac{b+a}{2}, \qquad V\{X\} = \sigma^2 = \frac{(b-a)^2}{12}$$

Normal distribution

General form $X \sim N\{\mu, \sigma^2\}$:

$$f(x) = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sqrt{2\,\pi}\,\sigma} \qquad -\infty < x < \infty$$

$$E\{X\} = \mu, \qquad V\{X\} = \sigma^2$$

Standard normal distribution $Z \sim N\{0,1\}$:

$$Z = \frac{X-\mu}{\sigma} \qquad f(z) = \frac{e^{-\frac{1}{2}z^2}}{\sqrt{2\,\pi}} \qquad -\infty < z < \infty$$

Exponential distribution

The probability density function is $f(x) = \lambda\, e^{-\lambda x}$ for $0 \le x < \infty$

$$E\{X\} = \frac{1}{\lambda}, \qquad V\{X\} = \frac{1}{\lambda^2}$$