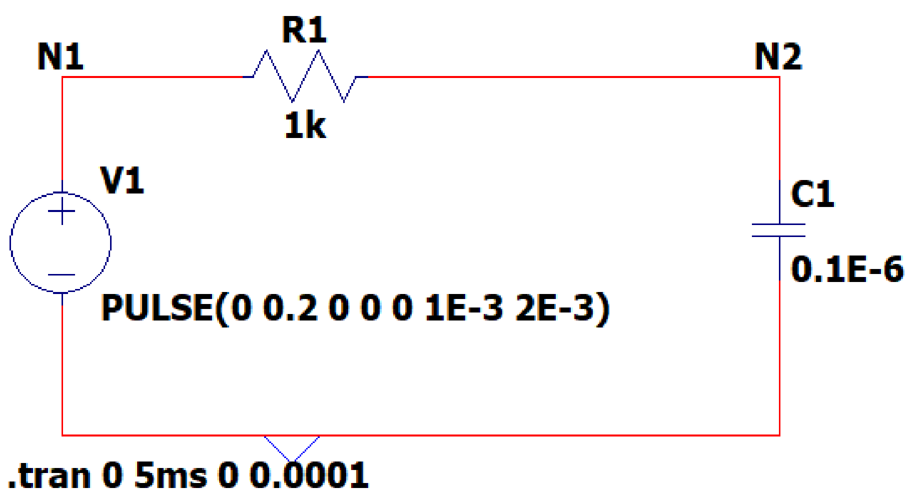
Proof of Concepts

A: RC Circuit-Prove that the time constant changes with different component values.

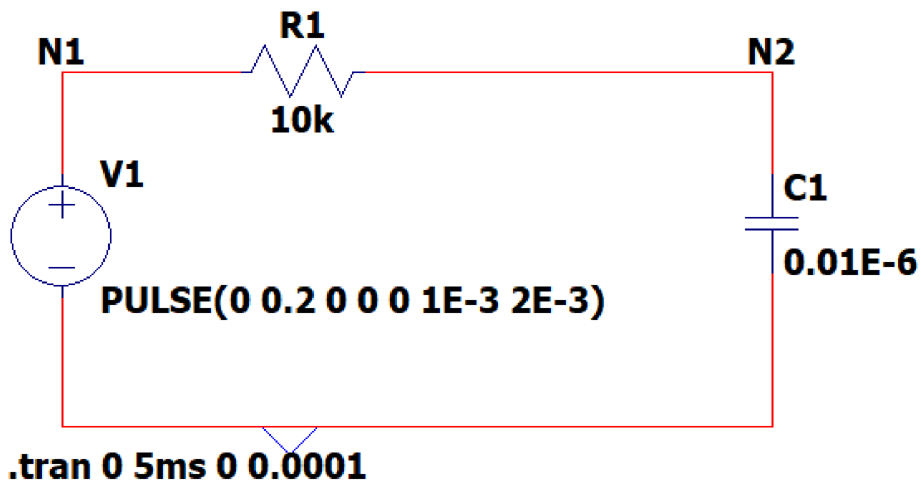
1) build circuit

Building Block: Simple RC circuit: voltage source provides voltage into a resistor (1k in first image, 10k in second image) which is in series with a capacitor (0.1E-6 in the first image, 0.01E-6 in the second image).

$R = 1k\Omega \,\&\, C = 0.1 \times 10^{-6}F$:



$R = 10k\Omega \,\&\, C = 0.01 \times 10^{-6}F$:



Measurement:

a) calibrate to see it

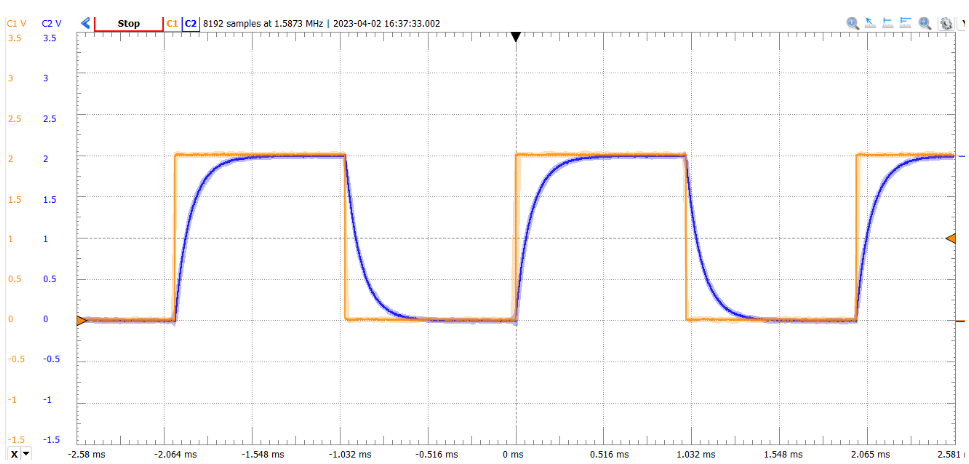
$R = 1k\Omega$ & $C = 0.1 \times 10^{-6}F$: 500Hz

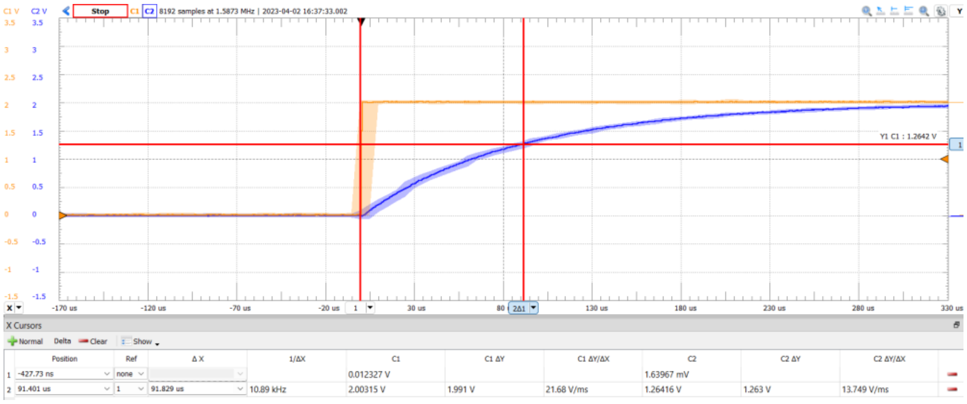
$R = 10k\Omega$ & $C = 0.01 \times 10^{-6}F$: 50Hz

b) take time constant measurement

Find how long it takes to go from 0V to 2V*1/e=1.2642V

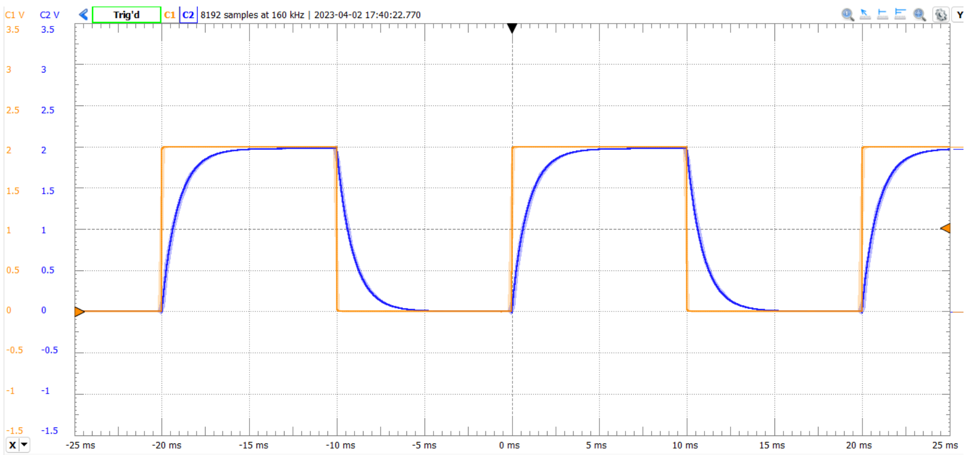
$$R = 1k\Omega \;\&\; C = 0.1 \times 10^{-6}F:$$

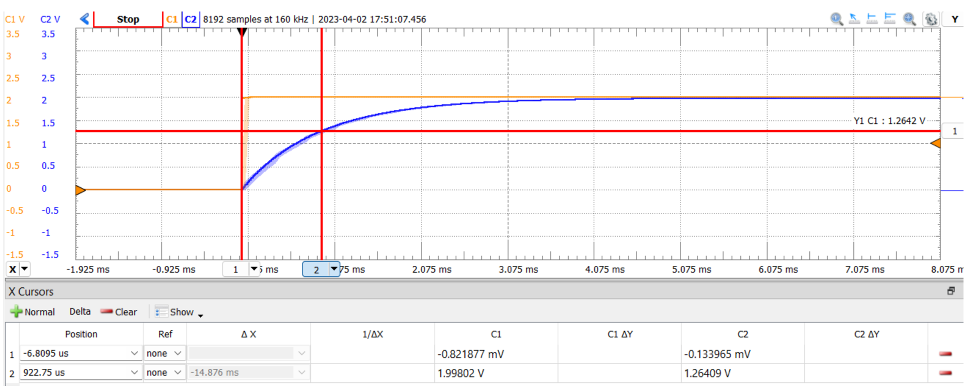




τ = 91.829 µs

$$R = 10k\Omega \,\&\, C = 0.01 \times 10^{-6}F:$$





τ = 922.75 µs

Measurement Analysis:

c) compare experimental results with expected

$$R = 1k\Omega \,\&\, C = 0.1 \times 10^{-6}F:$$

τ = R*C=1k*0.1µ=0.1m

τ =0.1ms

Percent error (100-91.829)/100=8.17%

Two components with 5% error each gives about 10% error, so 8.17% is acceptable.

$$R = 10k\Omega \;\&\; C = 0.01 \times 10^{-6}F:$$

τ = R*C=10k*0.1µ=1m

τ =1ms

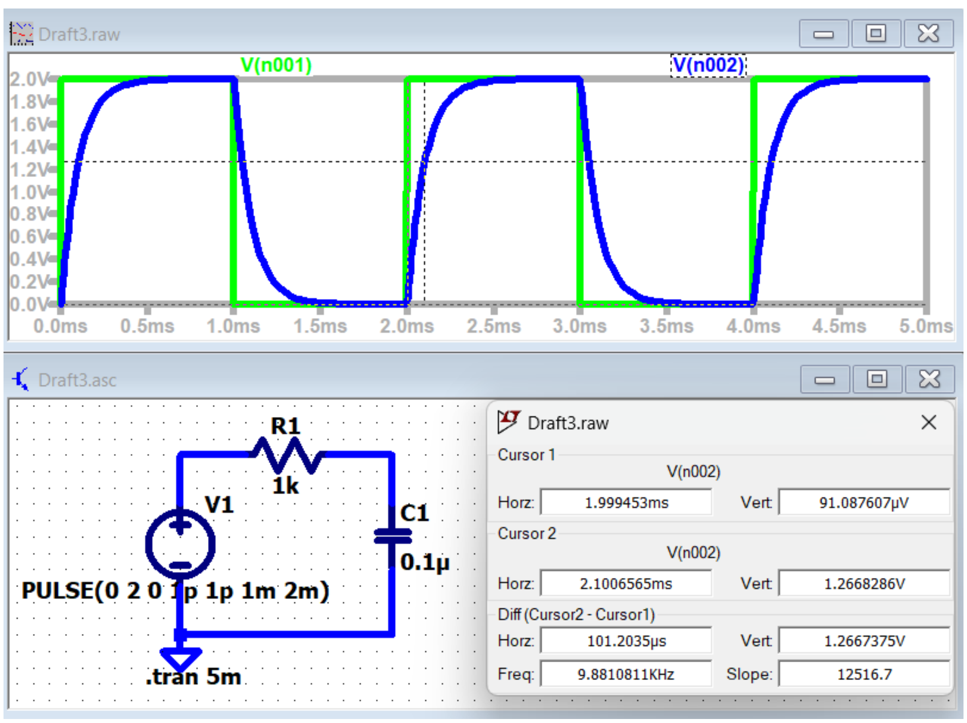
Percent error (1000- 922.75)/1000=7.73%

Two components with 5% error each gives about 10% error, so 7.73% is acceptable.

Simulation:

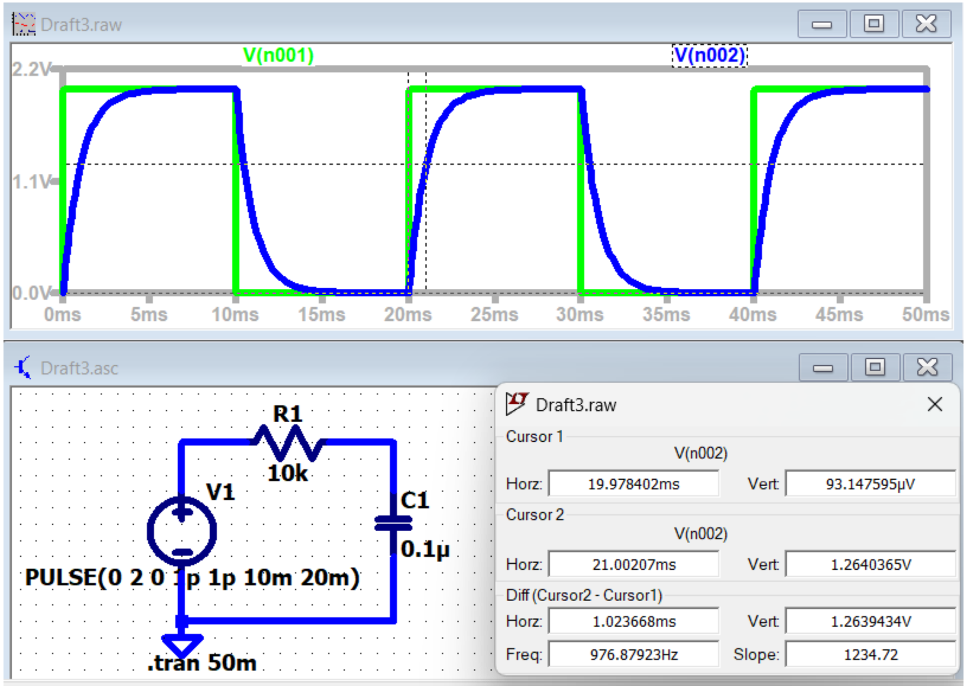
d) implement in LT spice and compare

$$R = 1k\Omega \;\&\; C = 0.1 \times 10^{-6}F:$$



τ = 101.2035 µs

$$R = 10k\Omega \;\&\; C = 0.01 \times 10^{-6}F:$$



τ = 1023.668 µs

Simulation Analysis:

$$R = 1k\Omega \,\&\, C = 0.1 \times 10^{-6}F:$$

τ = R*C=1k*0.1µ=0.1m

τ =0.1ms

Percent error (100- 101.2035)/100=1.2%

$$R = 10k\Omega \,\&\, C = 0.01 \times 10^{-6}F:$$

τ = R*C=10k*0.1µ=1m

τ =1ms

Percent error (1000-1023.668)/100=2.37%

Overall Analysis and Discussion:

$$\tau = RC$$

	R=1k	R=10k	R=1k %error	R=10k %error
Calculated	0.1ms	1ms	0	0
Experimental	0.091829ms	0.92275ms	8.17%	7.73%
Simulated	0.1012035	1.023668	1.2%	2.37%

This gave us expected values of 0.1ms and 1ms. Our experimental values gave a percent error of 8.17% and 7.73%, and with two components with 5% error each, we could expect up to about 10% error, so our values are acceptable. Our simulation values gave a percent errors of 1.2% and 2.37%. This is much closer to calculated values.

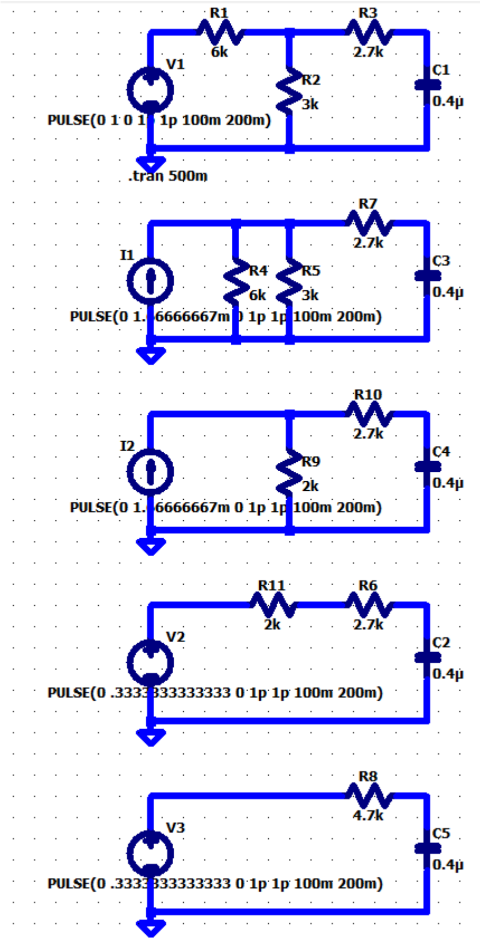
I believe part of this error is due to rise time. LTspice has a very large rise time by default that gave us data with 50% error, so I set it to 1ps to essentially remove it, but it still has more than the calculation accounts for and less than the actual circuit.

This shows that the time constant of a circuit changes according to the equation listed above.

B: RC Circuit: Prove the RC time constant using Thevenin calculations for 1 u(t).

a) Analytically determine time constant

In the image below are all the steps to convert to Thevenin equivalent, and all the simulations align



I=V/R=1/6k=.167m

Rp=(R1^-1+R2^-1)^-1=(6k^-1+3k^-1)^-1=2k

V=I*R=.167m*2k=0.333

Rs=R1+R2=2k+2.7k=4.7k

Vth=0.333V, Rth=4.7k ohm

τ = R*C=4.7k*0.4µ=1.88m

τ = 1.88ms

b) differential expression for voltage across capacitor, solve for V=u(t), plot

$V1 = VR + VC$

$u(t) = I \cdot R + \frac{1}{C} \int (I)\, dt$

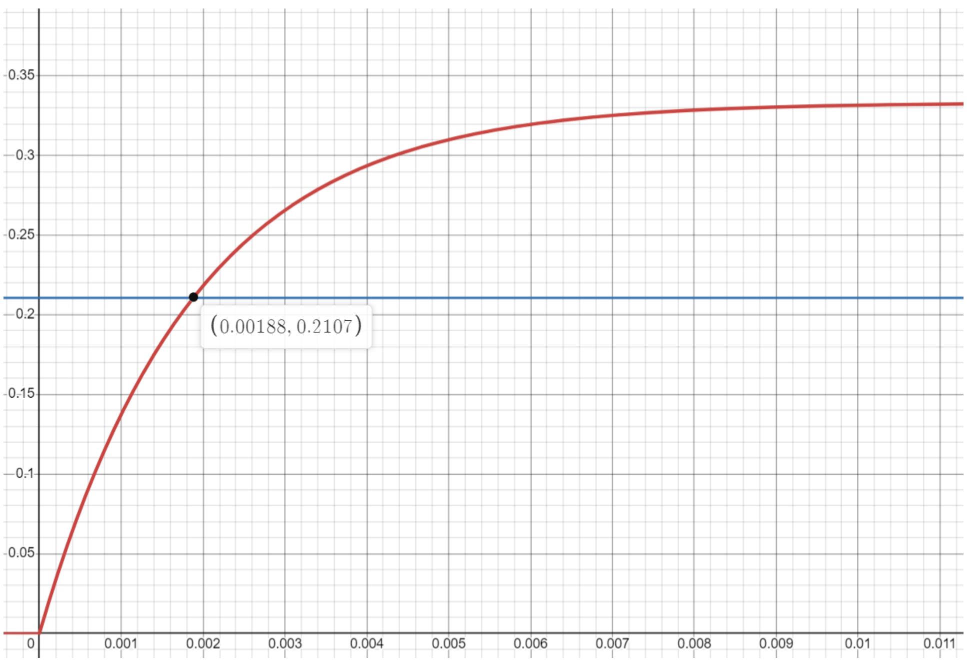
$I = C \cdot \frac{d}{dt}(VC)$

$u(t) = R \cdot C \cdot \frac{d}{dt}(VC) + VC$

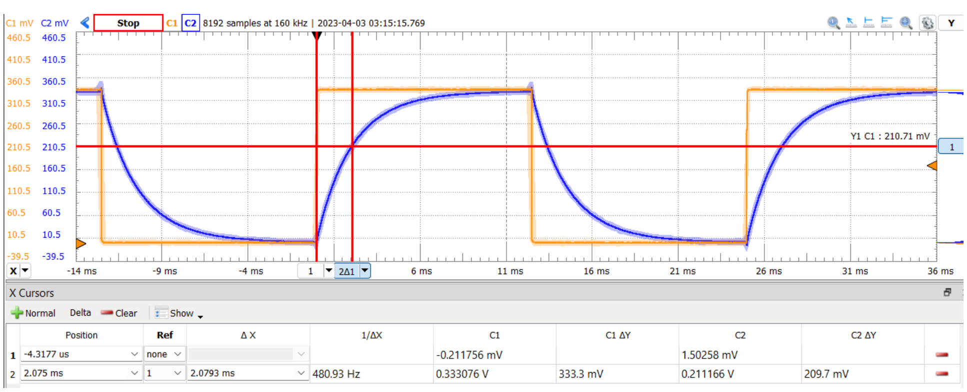
$\frac{d}{dt}(VC) + \frac{1}{RC} \cdot VC = \frac{1}{RC} \cdot u(t)$

$\frac{d}{dt}(VC) + \frac{1}{0.00188} \cdot VC = \frac{1}{0.00188} \cdot u(t)$

$VC(t) = (1 - e^{-t/(0.00188)}) \cdot u(t)$



c) experimental capacitor voltage



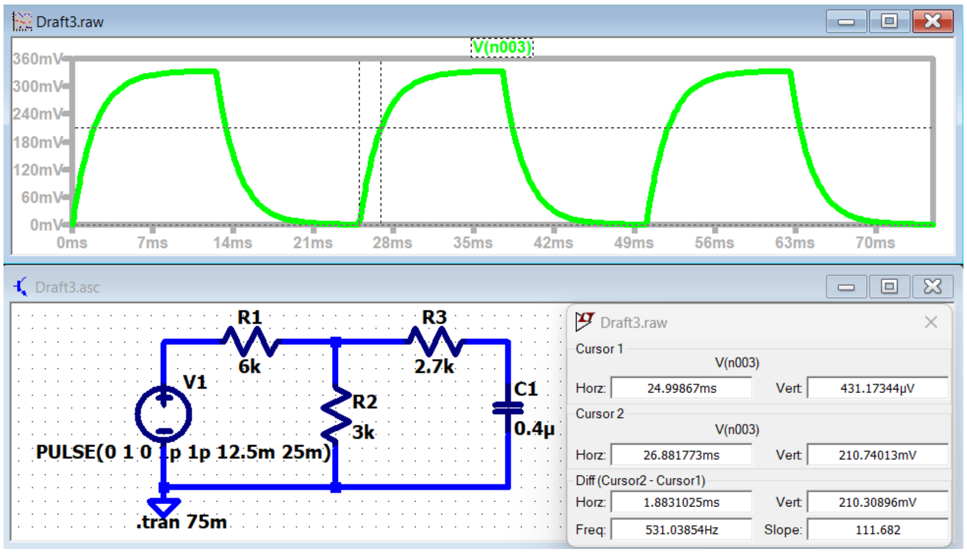
d) estimate time constant

τ = 2.08ms

e) compare results to expected

The results are very similar, but we have a longer time constant here, 2.08ms being 10.6% greater than the expected 1.88ms. I measured my capacitors and they actually added up to 0.5uF, and the resistor actually measured 4.54k ohm. This results in a time constant of 2.27, which brings our error down to 8.8%, but that's still surprisingly high considering component error is accounted for.

f) Simulation and comparison



Simulation show a time constant of 1.883ms, extremely close to the expected value, down to error in my ability to move cursors accurately.

C.1: RLC Series Circuit: Prove how adjusting the resistance changes the circuit from overdamped to underdamped.

1) built circuit

a) set to overdamped

i) measure resistance

2k ohm

ii) find Vc(t)

$\alpha=R/(2L)=2k/(2*22m)=45.454k$

$\omega0=1/sqrt(LC)=1/sqrt(22m*0.1u)=21.320k$

$\alpha1, \alpha2 = \alpha$ +- sqrt(α^2- ω0^2) = 45.454k +- sqrt(45.454k^2+ 21.320k^2) = 85599,5310

Vc=A1e^{-85599 t} + A2e^{-5310 t} + 1

Vc(infty)=1=A1e^{-α1 t} + A2e^{-α2 t} + A3 = A3

A3=1

Vc(0)=0=A1e^{-α1 t} + A2e^{-α2 t} + 1 = A1+A2+1

A1+A2=-1, A1=-A2-1

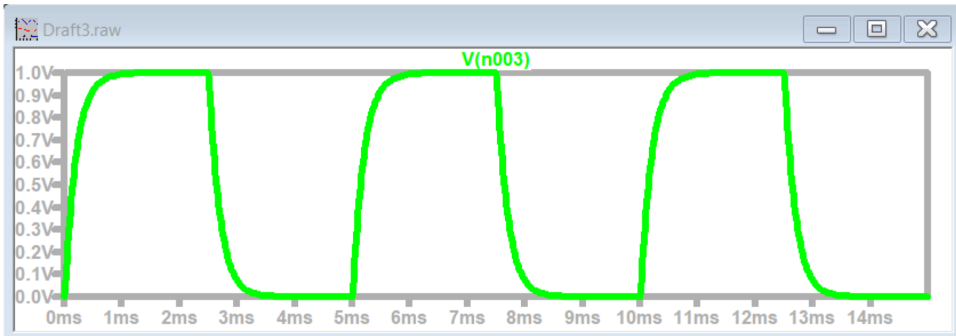
d/dt Vc(0)=I=0= -α1*A1e^{-α1 t} -α2*A2e^{-α2 t} = -α1*A1 -α2*A2

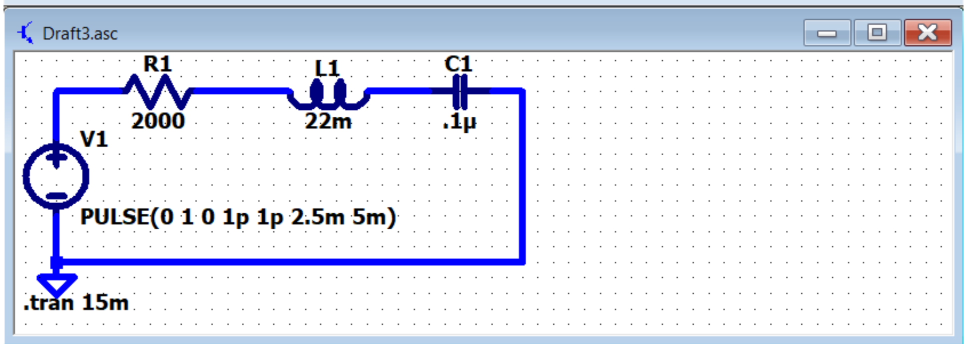
α2*A2= -α1*A1, 5310*A2 = -85599*A1, 5310*A2 = -85599*(-A2-1), 5310*A2 = 85599*A2 + 85599, 5310*A2 = 85599*A2 + 85599, 80289*A2 = -85599, A2 = -85599/80289, A2=-1.066, A1=0.066

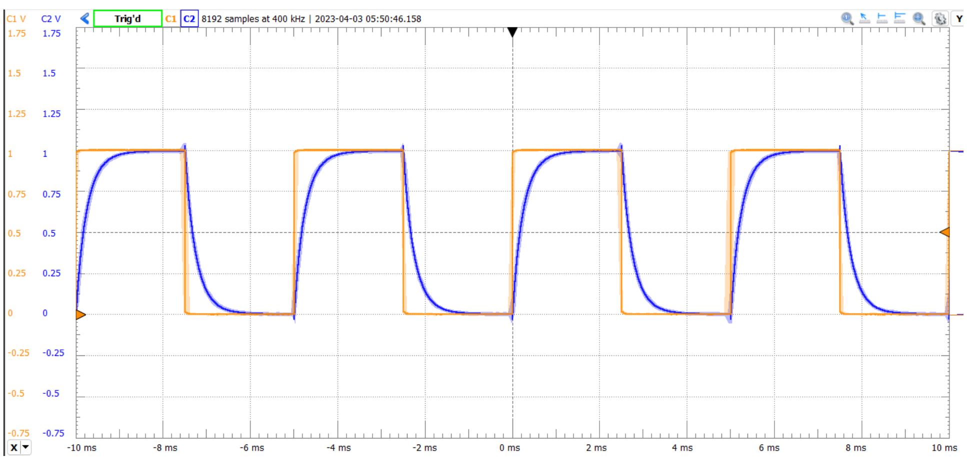
Vc(t) = 0.066*e^{-85599 t} -1.066*e^{-5310 t} + 1

Vc(t) ~= -e^{-5310 t} + 1

iii) simulate in LT spice







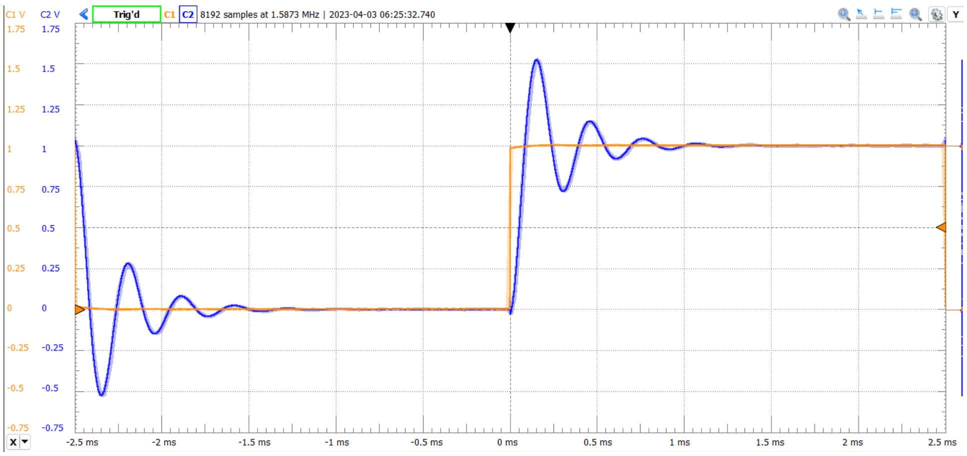
iv) ignored resistance?

The inductors have resistance that I measured to be 58 ohms, which isn't much compared to the 2k, but the 3% increase should create a measurable difference in output

2) set to underdamped (50 ohm)

i) compare experimental, analytic, and LT spice

Experimental:



I couldn't find a 2.2mH inductor, so I made one with 2x 1mH and 2x 0.1mH, this comes up later as it more than doubles the resistance of the circuit.

Analytic:

$\alpha=R/(2L)=50/(2*22m)=1136$

$\beta=\mathrm{sqrt}(\omega0\text{^}2 - \alpha\text{^}2)=\mathrm{sqrt}(21320\text{^}2-1136\text{^}2)=21290$

$Vc=e\text{^}\{-\alpha\ t\}(A1\cos(\beta\ t)+A2\sin(\beta\ t))+A3$

$Vc(\mathrm{infty})=1=e\text{^}\{-\alpha\ t\}(A1\cos(\beta\ t)+A2\sin(\beta\ t))+A3=A3$

$A3=1$

$Vc(0)=0=e\text{^}\{-\alpha\ t\}(A1\cos(\beta\ t)+A2\sin(\beta\ t))+1=A1+1$

$A1=-1$

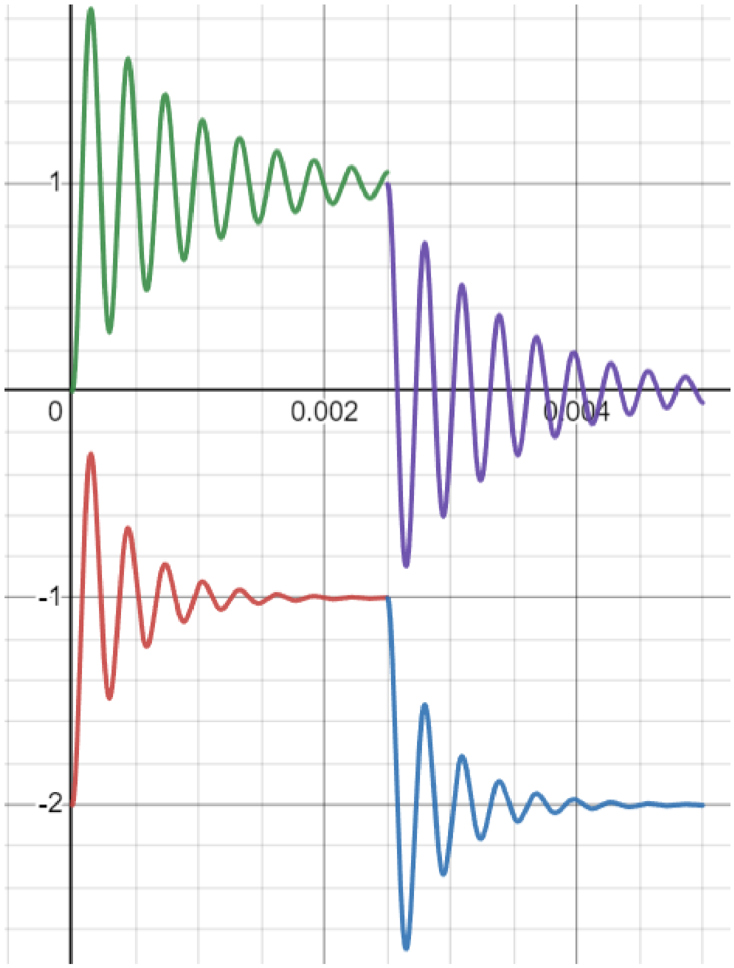
$Vc(t)=e\text{^}\{-\alpha\ t\}(-\cos(\beta\ t)+A2\sin(\beta\ t))+1$

$d/dt\ Vc(t)=e\text{^}\{-\alpha\ t\}((A2\alpha-\beta)\sin(\beta\ t) + (-\alpha-A2*\beta)*\cos(\beta\ t))$

$d/dt\ Vc(0)=0=(-\alpha - A2*\beta)$

$-\alpha=A2*\beta$, $A2=-\alpha/\beta$, $A2=-.05$

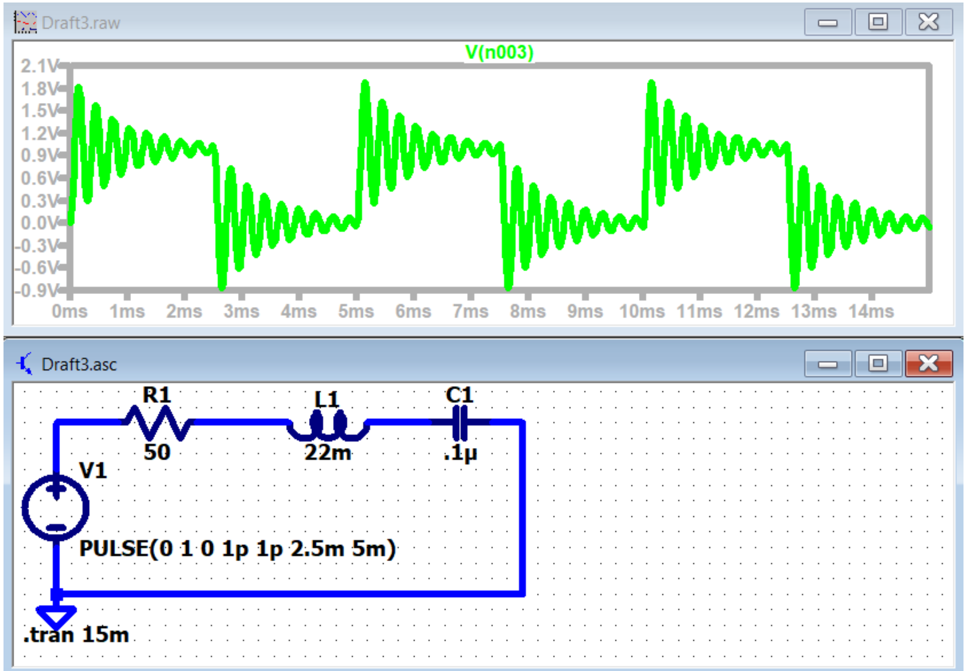
$Vc(t)=e\text{^}\{-1136\ t\}(-\cos(21290\ t) -.05\sin(21290\ t))+1$



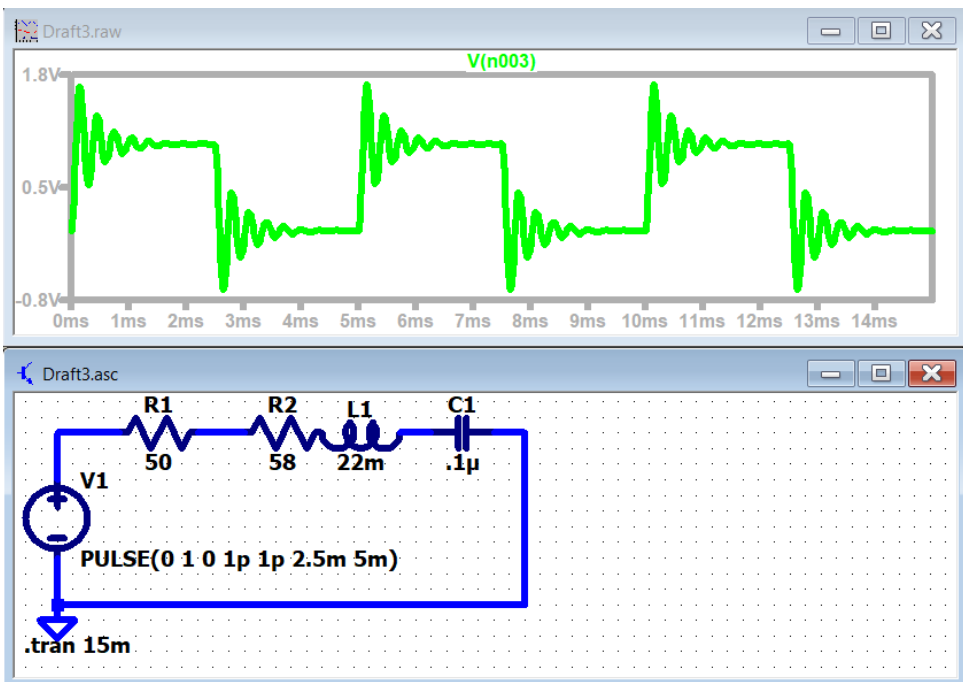
It's clear that these functions didn't decay nearly as fast at the experimental functions, so I added another set that accounts for R=R1+R(L1),

Vc(t)=e^{-2455 t}(-cos(21178 t) -.11 sin(21178 t))+1

Simulation:



Same as the last one, it clearly doesn't decay as fast as it should, so I did it again with added resistance for results closer to the experiment



The rest is coming soon!

It's 7am. This is due in an hour. I need sleep. And breakfast.

C.2: RLC Parallel Circuit: Prove how the attenuation constant and oscillation constant relates to an underdamped, overdamped, or critically damped circuit. (Discuss why/whether you can ignore R2 used to calculate the current through the inductor.

Building Block: Short description and schematic

Clearly label all nodes you will reference

Analysis:

Equation and short description.

Describe clearly how you are applying the concept

Simulation:

Screenshot of simulation

Clearly labeled with nodes and/or input/output that matches with schematic above. Any important portions of output are identified (i.e., the output of the point at which the bridge is balanced in a parameter sweep).

Measurement:

Screenshot of Waveforms output from circuit above.

Remember to clearly show all axes in a measurement plot. Also identify any important portions of the output.

Discussion:

Comparison of Analysis, Simulation and Measurement results. Both a simple summary of results (like a numerical chart of values) and a simple description that details if the results are as you expect. Also include any speculation as to why they may be different from one another if they are different. How different is too much for example…explore this.

C.3: Using s-domain analysis (Laplace Transforms), prove that you can adjust the circuit from overdamped to underdamped.

Building Block: Short description and schematic

Clearly label all nodes you will reference

Analysis:

Equation and short description.

Describe clearly how you are applying the concept

Simulation:

Screenshot of simulation

Clearly labeled with nodes and/or input/output that matches with schematic above. Any important portions of output are identified (i.e., the output of the point at which the bridge is balanced in a parameter sweep).

Measurement:

Screenshot of Waveforms output from circuit above.

Remember to clearly show all axes in a measurement plot. Also identify any important portions of the output.

Discussion:

Comparison of Analysis, Simulation and Measurement results. Both a simple summary of results (like a numerical chart of values) and a simple description that details if the results are as you expect. Also include any speculation as to why they may be different from one another if they are different. How different is too much for example…explore this.

D: Prove that your chosen timing circuit functions as you expected.

Building Block: Short description and schematic

Clearly label all nodes you will reference

Analysis:

Equation and short description.

Describe clearly how you are applying the concept

Simulation:

Screenshot of simulation

Clearly labeled with nodes and/or input/output that matches with schematic above. Any important portions of output are identified (i.e., the output of the point at which the bridge is balanced in a parameter sweep).

Measurement:

Screenshot of Waveforms output from circuit above.

Remember to clearly show all axes in a measurement plot. Also identify any important portions of the output.

Discussion:

Comparison of Analysis, Simulation and Measurement results. Both a simple summary of results (like a numerical chart of values) and a simple description that details if the results are as you expect. Also include any speculation as to why they may be different from one another if they are different. How different is too much for example…explore this.