ECSE 2500
Lec 2
Jan. 12

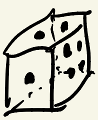
Topic: Definition of probability (problems)
Axioms / Rules

☐ Key Probability Terminology

Experiments, Sample space, Events, Event class/space

An experiment specifies a procedure and a resulting measurement. Example:

E1: Roll a 6-sided Die and count the number of dots



E2: Roll two 6-sided Dice and count the sum of dots

E3: Roll two 6-sided Dice and report the dots on each die and concatenate into a 2-dimensional vector

E4: Flip a coin 5 times and count the number of heads

E5: Keep flipping coin and repeat until we see the first head

E6: Flip coin 3 times and record all head/tail patterns

Continous measurement → E7: Pick a number at random between 0 and 1

E8: Measure the lifetime of hard drive in terms of days/mins.

E9: Pick a number X at random between 0 and 1, then pick another Y at random between X and 1.

Note: - Outcomes can be discrete or continuous
- Can be finitely or infinitely many outcomes
- Can have same procedure but different measurements
- Can involve a sequence of repeated experiments

☐ The **sample space** is the set of all possible outcomes (measurements). Exactly one outcome from the sample space occurs on each experiment. They cannot occur simultaneously.

E1: $\{1, 2, 3, 4, 5, 6\}$

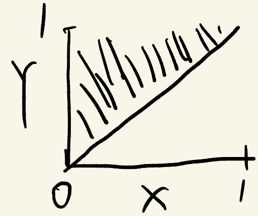
E2: $\{2, 3, 4, 5, \cdots, 12\}$

E6: $\{HHH, HHT, HTH, HTT, THH, TTH, THT, TTT\}$

Head Tail

E7: $\{x \mid 0 \leq x \leq 1\}$ or $[0, 1]$

E9: $\{(x,y) \mid 0 \le x \le y \le 1\}$



E5: $\{1, 2, 3, \cdots\}$

We use notation S to denote sample space and an outcome by $\delta \in S$.

(delta)

□ An **Event** is a set of outcomes, i.e., a subset of sample space S: $A \subseteq S$.

In the case where we have a discrete sample space and a finite # of events, we can list all possible events.

$\boxed{\text{Example}}$ $S := \{1, 2, 3\}$ then

what are all possible events ?

= what are all subsets of sample space S ?

$S = \{1, 2, 3\}$

Possible events: $\left\{ \begin{array}{llll} \phi & \{1\} & \{1,2\} & \{1,2,3\} \\ & \{2\} & \{2,3\} & \\ & \{3\} & \{1,3\} & \end{array} \right\}$

This is called the power set of S (a.k.a. 2^S)

a set of sets

Often we can specify the events in word:

A1: "The die roll was an even number" $= \{2, 4, 6\}$

A4: "We got more than 3 heads in 5 coin flips."
$= \{4, 5\}$

A8: "The device lasts between 3 to 5 years."

A9: "X and Y differ by less than 0.1."

We use the notation F to denote the event class that is the set of events we will associate probability to.

Note: - For discrete sample space, F is all subsets of sample space S.

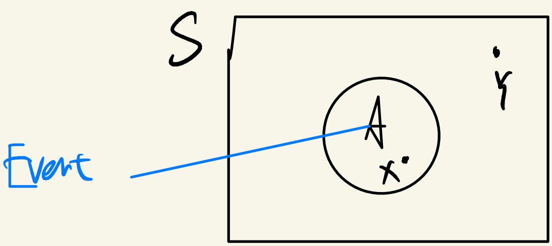
- For continuous sample space, F is all unions, complements and intersections of intervals such as

$$[0, 0.5),\ (-\infty, 12],\ [1,2]\cup[3,5]$$

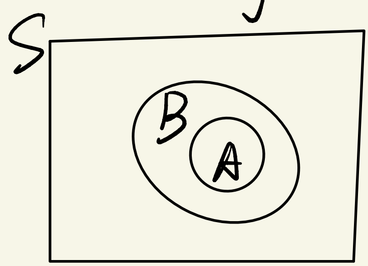
Need more complex math to rigorously define it

Borel Field

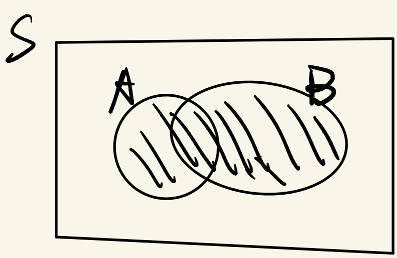
☐ A quick review of set theory



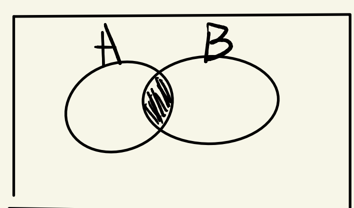
$x \in A$, $y \notin A$



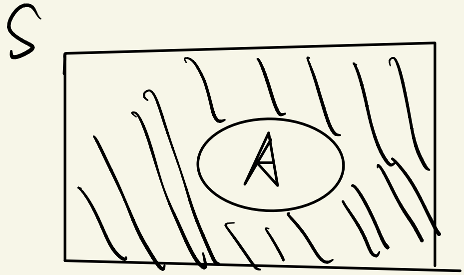
$A \subset B$



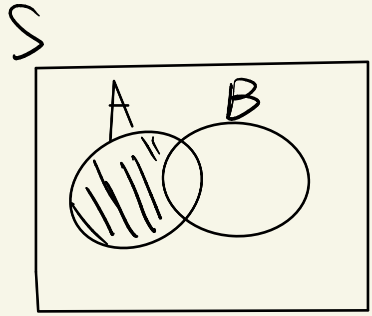
$A \cup B$



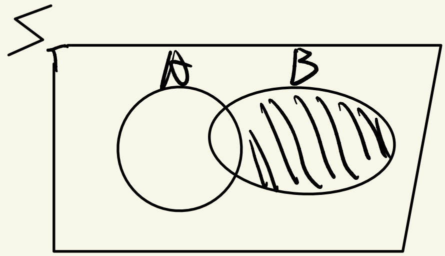
$A \cap B$



A^c



$A - B$



$B - A$