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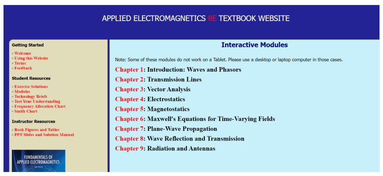
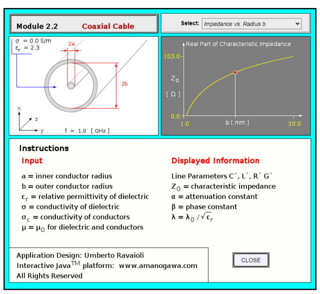
Fields and Waves HW1

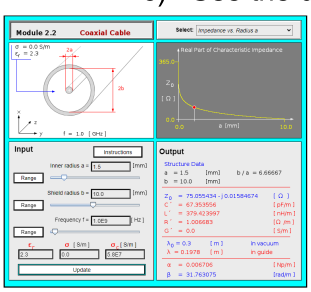
1) Coaxial Cables
    a) The Ulaby textbook's companion website (https://em8e.eecs.umich.edu/) has many resources that we'll make use of during the course this semester. One of these resources is a set of applications that allow you to interact with fundamental concepts in electricity and magnetism. Navigate to the site and click on the Modules tab.



    b) Once on the Interactive Modules page, click on "Chapter 2: Transmission Lines", then "Coaxial Cable". Coaxial cables are one of the main types of transmission line, which you'll find in many different applications. It may take some time to load (and you may need to install Java).



    c) Use the drop-down menu next to "Select" to select "Impedance vs. Radius a".



    d) Let's consider the case of a coaxial cable with an inner diameter (2a) of 3mm, an outer diameter (2b) of 9mm, and a dielectric constant (εr) of 2.1. What is the characteristic impedance of this coaxial cable to the nearest ohm? Additionally, we will be considering a lossless coaxial cable here, so ignore imaginary components of the characteristic impedance Z0 (and thus any frequency-dependence of Z0).

Charazteristic impedance Z_0 = 75 ohm

    e) What is the velocity of the signal on this coaxial cable in m/s?

u=1/sqrt(LC) = 1/sqrt(67.35x10^-12 F/m * 379.4x10^-9 H/m) =197x10^6 m/s (about c/2)

    f) If the wavelength of the signal being transmitted on this coaxial cable is 2m, what are the approximate phase constant β in rad/m and the frequency f in Hz?

f=u/l s^-1=m/s /m 197x10^6 / 2 = 98.9x10^6 Hz = f = 98.9 MHz
2pi rads / 2m   rad/m = β = pi

g) Now consider the case of a lossy transmission line, which the app calculates by default. How does Z0 change as frequency increases? For a given frequency, what happens to Z0 as the conductivity of the conductor (σc) decreases (as the line resistance increases)?

Z0 decreases as frequency increases, dropping to 45+j0 ohms, while 1Hz is 170-j163 ohms.
Z0 increases as conductivity decreases.

2) Phasor Notation
   Note: Remember when you convert from the z-t domain to phasor notation, that you first have to convert to a cosine base, and that when converting from phasor notation back to the z-t domain, you obtain a cosine. e^jt = cos(t)+j sin(t)
   a) a) Convert the following voltage from the time domain into phasor notation:
      $V(t)$ = 5j sin(ωt)

V = 5j sin(wt)
V = -5j cos(wt+pi/2)
~V = -5j e^(j(pi/2))
~V = -5 e^(j(pi))

   b) b) Convert the following current phasor into the time domain:
      ~I=0.10 e^(-j(Bz+pi/4)).

~I = 0.10 e^(-j(Bz+pi/4))
~I = 0.10 e^(j(-Bz-pi/4))
I = 0.10 cos(wt-Bz-pi/4)

   c) c) Is the current wave in b) a traveling wave or standing wave? If it's a traveling wave, which direction is it traveling?

Traveling, +z

3) 3. Traveling Waves and Standing Waves Part 1
   The voltage and current on a transmission line are given by
   $v(z, t)$ = 9 cos(2π × 10^7 t + 1.5πz)
   $i(z, t)$ = 0.09 cos(2π × 10^7 t + 1.5πz)
   a) Are the voltage and current waves traveling waves or standing waves or neither? If they are traveling waves, what is their frequency f in Hz and which direction are they traveling (+z or -z)?

Traveling, 10MHz, -z

   b) What are the period T and wavelength λ of the voltage and current waves?

cos(2π × 10^7 t), T=10^-7 seconds=100 ns
cos(1.5πz), λ=4/3m

c) What is the characteristic impedance of the transmission line?

V=IR, R=V/I, 9/0.09=100 ohms

d) What is the velocity of the signal on the transmission line?

u=λ/T=4/3*10^7=13.33x10^6 m/s = 13.33Mm/s

e) What are the capacitance and inductance per unit length of the transmission line?

u=1/sqrt(LC)=13.33x10^6

V_0^+/I_0^+=sqrt(L/C)=9/0.09=100

C=750 pF/m

L=7.5 uH/m

Z_0=sqrt(L/C)=100 ohms

f) Express both the voltage and current waves in phasor form.

j5=5e^(j pi/2)=5 cos(wt+pi/2)

9 cos(2π × 10^7 t + $1.5\pi z$)

~V=9e^(j10^7t+3/4z)

~I=0.09e^(j10^7t+3/4z)

4) Traveling Waves and Standing Waves Part 2

Consider the following system consisting of a voltage source Vs, source resistance Rs = 50Ω, transmission line with characteristic impedance Z0 = 50Ω, and load resistance RL, as shown below. At time t = 0, the voltage source Vs is switched on and sends a voltage wave of the form ~V = 2V^+ e^-jBz towards the transmission line.

a) If RL = 50Ω, write the expression for ~V_L, the voltage across the load resistor R_L. How much of the incident wave ~V is reflected between the end of the transmission line and RL?

None.

b) For the rest of the problem, suppose instead that RL = 0. What is the reflection coefficient Γ? How much of the incident wave is reflected when the load is a short circuit?

Γ=-1. At 0. the full wave is reflected out of phase (ends are 0). At open circuit, it would be in phase (ends are peak to peak).

c) Write the expression for the total voltage on the transmission line, including both incident and reflected waves. Recall that the general solution for the Telegrapher's Equations in phasor form is ~V=V^+ e^-jBz + V^- e^+jBz and that V^- = ΓV^+. Leave your answer in phasor form, in terms of V^+.

~V=V^+ e^-jBz + V^- e^+jBz

~V=V^+ e^-jBz + ΓV^+ e^+jBz

d) Convert your answer to the z-t domain. Your answer should be in the form of a sum of two cosines.

~$V=V^+ e^{-jBz} + \Gamma V^+ e^{+jBz}$

$V=V^+ \cos(wt-Bz) + \Gamma V^+ \cos(wt+Bz)$

e) Using the trigonometric identity $\cos(\theta) - \cos(\phi) = -2 \sin((\theta+\phi)/2) \sin((\theta-\phi)/2)$, convert your answer into a product of sine functions. What kind of wave is this?

$V=V^+ \cos(wt-Bz) + \Gamma V^+ \cos(wt+Bz)$

$\cos(\theta) - \cos(\phi) = -2 \sin((\theta+\phi)/2) \sin((\theta-\phi)/2)$

f) What is the reflection coefficient when the load resistance RL is an open circuit (RL=∞)? Would you expect a standing wave to form in this case? Why or why not?

$\Gamma=1$, full reflection, but this time in phase, leading to a standing wave still, but with the max/min points and the constant 0 points swapped.