# Problem Set #2

Innovation and Growth

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Due Friday 2/03/23 2:00PM

1. Consider Solow model with both human capital ($H$) and positive technology growth. Assume the production function is

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta} \qquad \alpha, \beta > 0, \quad \alpha + \beta < 1$$

Assume the population ($L$) grows at rate $n > 0$, technology ($A$) grows at rate $g > 0$, and constant fractions of income are invested in $K$ and $H$ given by $s_k > 0$ and $s_h > 0$. Both forms of capital depreciate at constant rate $\delta > 0$.

   (a) Starting from the law of motion for aggregate capital stocks, derive the law of motion for both capital stocks per effective unit of labor. Use a "hat" to denote units per effective labor unit. (i.e. $\hat{k} = K/AL$)

   (b) Solve for the equilibrium levels of $\hat{k}$ and $\hat{h}$

   (c) Comparative statics #1) Starting from an initial equilibrium, suppose that the fraction of income invested in physical capital decreases, while the fraction invested in human capital remains unchanged. How will the equilibrium values of $\hat{k}$ and $\hat{h}$ change? Illustrate in two "rate" diagrams and interpret your findings.

   (d) Comparative statics #2) Starting from an initial equilibrium, suppose that the population growth rate decreases and the fraction of income invested in human capital decreases (perhaps due to an aging population). Assume the fraction of income invested in physical capital remains unchanged. How will the equilibrium values of $\hat{k}$ and $\hat{h}$ change? Illustrate in two "rate" diagrams and interpret your findings.

2. Consider the Solow model with perfectly competitive factor markets (labor and loanable funds). Assume technology is constant ($A$ normalized to 1). Suppose that the economy experiences a permanent increase in the rate of immigration so that the rate of population growth in the economy is now permanently higher ($n_1 > n_0$).

   (a) Using the "rates" illustration of the steady state equilibrium of the Solow model, determine how the increased immigration impacts the equilibrium level of capital per capita, and describe the economy's transition to its new steady state.

   (b) Using the fact that $y = Rk + w$ in equilibrium, illustrate how factor prices ($w$ and $R$) adjust along the path to the new steady state

   (c) Illustrate this factor price adjustment using our depiction of factor markets (supply and demand for labor and loanable funds)

   (d) Interpret the policy conclusions of this analysis. Do you think that the Solow model is accurately capturing aspects of the current controversy surrounding immigration in the U.S.? Can you think of anything that the model is missing that would change the conclusions of the analysis?