$$C = \frac{1\,nC}{78.9\,V} = 12.67\,pF$$

$$E = \frac{1}{2}(12.67\,pF)(78.9\,V)^2 = 39.4\,nJ$$

f.) Voltage is $-\int_a^b \vec{E} \cdot \vec{dl}$

In dielectric, going from $z=0$ in bottom plate to z somewhere in the dielectric,

$$V = -\int_0^z (-2.25\,kV/m)\,dz = z(2.25 \times 10^3)\,V$$

$$\nabla^2 V = \frac{\partial^2}{\partial z^2}\left(z(2.25 \times 10^3)\right) = 0$$

In the air gap, going from $z=0$ at the point between the air and the dielectric to z somewhere in the air gap

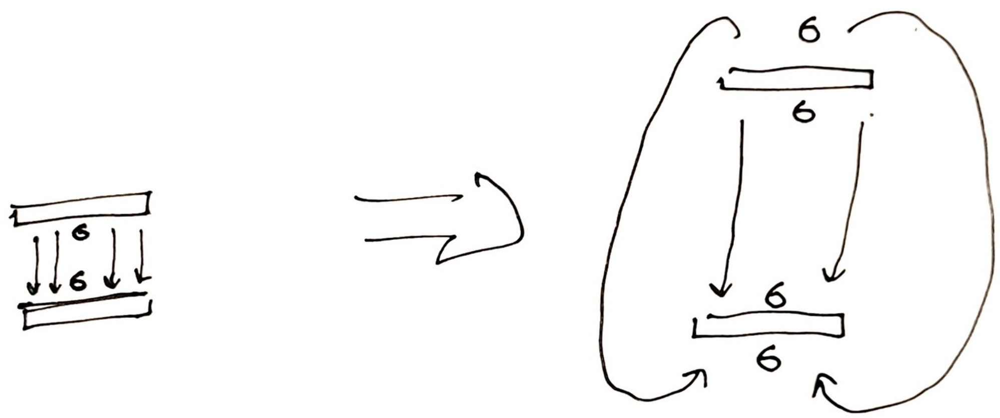
$$V = -\int_0^z (-11.29\,kV/m)\,dz + (\text{voltage across dielectric})$$

$$= (11.29 \times 10^3)z + (0.01)(2.25 \times 10^3)$$

$$= 22.5 + z(11290)$$

$$\nabla^2 V = \frac{\partial^2}{\partial z^2}\left(22.5 + z(11290)\right) = 0$$

g.) The e-field and capacitance expressions will be different, as fringing fields will now begin to dominate. The plate surface charge will no longer be confined to the region between plates.



a.) $\vec{E}_{2t} = \underline{\vec{E}_{1t} = 20\hat{x}}$

$\vec{D}_{1n} - \vec{D}_{2n} = \rho_s = 0$

$\vec{D}_{2n} = \vec{D}_{1n}$

$\varepsilon_2 \vec{E}_{2n} = \varepsilon_1 \vec{E}_{1n}$

$\varepsilon_0 40\hat{y} = 6\varepsilon_0 \vec{E}_{1n}$

$\vec{E}_{1n} = \frac{1}{6} 40\hat{y} = 6.67\hat{y}$

$\vec{E}_1 = 20\hat{x} + 6.67\hat{y}$

(E on top)

6.) $\vec{E}_{1t} = \vec{E}_{2t} = 100\hat{x}$

$\vec{D}_{1n} - \vec{D}_{2n} = \rho_s$

$\varepsilon_0 \vec{E}_{1n} - 2\varepsilon_0(100\hat{y}) = (10^{-9}\ c/m_2)$

$\vec{E}_{1n} = \frac{(10^{-9}\ c/m_2)}{\varepsilon_0} + 200\hat{y} = 312\hat{y}$

$\vec{E}_1 = 100\hat{x} + 312\hat{y}$

4.

Initial guess : $V_1 = V_2 = V_3 = V_4 = 0$

a.) 1st iteration:

$V_1 = (80\,V + 20\,V + 0\,V + 0\,V)/4 = 25\,V$

$V_2 = (80\,V + 25\,V + 0\,V + 0\,V)/4 = 26.25\,V$

$V_3 = (25\,V + 20\,V + 50\,V + 0\,V)/4 = 23.75\,V$

$V_4 = (26.25\,V + 23.75\,V + 50\,V + 0\,V)/4 = 25\,V$

2nd iteration:

$V_1 = (80\,V + 20\,V + 23.75\,V + 26.25\,V)/4 = 37.5\,V$

$V_2 = (80\,V + 37.625\,V + 25\,V + 0\,V)/4 = 35.66\,V$

$V_3 = (37.625\,V + 20\,V + 50\,V + 25\,V)/4 = 33.16\,V$

$V_4 = (35.66\,V + 33.16\,V + 50\,V + 0\,V) = 29.7\,V$

b.) $E_y \approx \frac{-\Delta V}{\Delta x} = -\frac{(80 - 37.625)}{0.1\,m} = -423.7\,V/m$ in $\hat{y}$

c.) 1. Smaller size of calculation elements.

2. More iterations.

5. Capacitnace part 2

a.) $J = \dfrac{I}{\text{Area}} = \dfrac{1\,mA}{\pi(0.5\,mm)^2} = 1270\ A/m^2$

$J = \sigma E$

$1270\ A/m^2 = (5.9 \times 10^7) E$

$|E| = 21.53 \times 10^{-6}\ V/m$

b.) Electric field magnitude expression:

$\oint \vec{D} \cdot d\vec{S} = Q_{enc}$

$2\pi r l \cdot D = Q_{enc}$, $l = 5\,mm$

$\vec{D} = \dfrac{Q_{enc}}{2\pi r l}\,\hat{r}$

$\vec{D} = \varepsilon \vec{E}$ and Q on the outer conductor will equal Q_{enc} on the inner conductor.

The dielectric will fail first at its smallest radius, r = 1mm. So we calculate the charge that will cause the capacitor to fail in the following manner:

$$E_{breakdown} = \frac{Q_{outer}}{2\pi\varepsilon r \ell} = \frac{Q_{outer}}{2\pi(3\varepsilon_0)(1mm)(5mm)} = 300kV/cm$$

$$Q_{outer} = (300kV/cm) \cdot 2\pi \cdot 3\varepsilon_0 \cdot 1mm \cdot 5mm = 25.03\ nC$$

$$1\ mA = 1 \times 10^{-3}\ C/s$$

$$(1 \times 10^{-3} C/s) \cdot (x\ \ s) = (25.03 \times 10^{-9}\ C)$$

$$x = 25.03\ \mu s.$$

6. Conducting Plane

(6.) The $-2nC$ charge causes positive charge to accumulate on the planes surface in such a way that the $-2nC$ charge sees a $+2nC$ "image charge" 1cm below the surface.

$$\vec{F} = \frac{\overset{-2nC}{\overset{\downarrow}{Q_1}}\ \overset{+2nC}{\overset{\downarrow}{Q_2}}}{4\pi\varepsilon_0 \underset{1cm}{\underset{\uparrow}{R^2}}} = 3.59 \times 10^{-4}\,N \text{ pointing down.}$$