

Culombs Law, conductors, insulators, polarization, induced charges, adding vector fields and forces

$$\vec{F}_{1on2} = \vec{F}_{12} = -\vec{F}_{21} = q_2 \vec{E}_1 = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} = k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12}; \quad \vec{F}_{tot} = q_0 \vec{E}_{tot}; \quad \vec{E}_{tot}(X_0, y_0, z_0) = \int d\vec{E}(x', y', z') = \int k \frac{dq'(x', y', z')}{r_0^2} \frac{\vec{r}_0}{r_0}, \quad \vec{r}_0 = \vec{r}_0 - \vec{r}' = (x_0 - x')\hat{i} + \dots, \quad \vec{r}' = x'\hat{i} + \dots$$

distance away from line charge linearly, line starts at 0, at $x=-D$, $\vec{E} = -k \int_0^L \frac{\lambda dx'}{(D+x')^2} \hat{i}$, $V = k\lambda \ln(\frac{D+L}{D})$ with θ up from x axis, $r_x = x \cos \theta$, $r_y = y \sin \theta$, $r = \sqrt{r_x^2 + r_y^2}$, $k = 9 * 10^9 = \frac{1}{4\pi\epsilon_0}$, $\epsilon_0 = 8.85 * 10^{-12}$

Electric field for point charges, electric field for a continuous distribution of charge

$$\vec{F}_E = q\vec{E}; \quad \vec{E}_s = k \frac{q_s}{r^2} \frac{\vec{r}}{r} = k \frac{q_s}{r^2} \hat{r}$$

Gauss's law and electric flux through a surface, Use of Gauss's law to find field

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \int E \cdot dA \cos \phi = \frac{Q_{encl}}{\epsilon_0}, \quad \phi = \angle \vec{E} - d\vec{A}, \quad d\vec{A} = dA \hat{n} \quad \text{net elec field } \vec{E} = 0, \quad V = c \quad \text{within a cond.}$$

gauss sphere: $\Phi_E = \oint \vec{E}(r) \cdot d\vec{A} = E(r) 4\pi r^2$, $E(r) = k \frac{q}{r^2}$,

sphere radius R: outside or point charge: $V = k \frac{q}{r}$, $E = k \frac{q}{r^2}$ inside: cond: $V = k \frac{q}{R}$, $E = 0$, insulating: $E = k \frac{qr}{R^3}$

long thin wire: $E(r) = \lambda/(2\pi r \epsilon_0)$ thin flat sheet: $E = \sigma/(2\epsilon_0)$, stepped: go from in to out matching net $Q_i n$

infinite plane w/ cylinder in it, $E = \sigma/\epsilon_0$

Electric potential for point charge, distribution. Electric field vs potential, equipotential. Potential for group of points, conservation of energy.

$$\text{Change Elec Pot Enrgy } \Delta U = - \int_{\vec{r}_A}^{\vec{r}_B} q \vec{E} \cdot d\vec{s} = -W_{AB}; \quad \text{Change Elec Pot } \Delta V = \frac{\Delta U_E}{q} = - \int_{\vec{r}_A}^{\vec{r}_B} \vec{E} \cdot d\vec{s} \quad \text{so } \Delta U_E = q \Delta V$$

Point charge, Σ for system $V(r) = \frac{kq}{r}$, $U_E = k \frac{q_1 q_2}{R_{12}} + \dots$; Field from pot: $E_x = -\Delta V = -\frac{\delta V}{\delta x} - \dots$

work on closed path = 0;

Caps, Dielectrics, steady state, equiv, energy storage, electric field energy density

$$C = Q/V = \frac{\epsilon_0 A}{d} = kC_0, \quad \text{ElcPotEnrInCap } U_E = .5QV = .5Q^2/C = .5CV^2, \quad \text{EnrFieldDen } u_E = .5\epsilon_0 E^2, \quad E = \frac{\sigma}{k\epsilon_0},$$

$$V_1 = V \frac{C_{equiv}}{C_1}$$

Current and density J , Resistance and itivity, Power relations and dissipation, DC steady state, KCVL Ohms

$$I = \frac{dQ}{dt}, \quad I = \vec{J} d\vec{A}, \quad \vec{J} = qn\vec{v}_d = I/A. \quad E = \rho J, \quad V = IR, \quad R = \rho L/A, \quad P = IV = I^2 R = V^2/R; \quad V_{bat} = \text{EMF} - Ir$$

Temp: conductor: $\rho(T) = \rho_0 + \rho_0 \alpha(T - T_0)$ semi: $\rho(T) = \rho_0 e^{(\frac{E_a}{kT})}$, $E_a = \text{actiEngr}$, $k = 1.38e - 23 = \text{bolt const.}$

Magnetic forces and fields

$$\vec{F} = q\vec{v} \times \vec{B}, \quad \text{finger velocity, curl field, thumb force, flip for negative. } \vec{F}_B = I\vec{L} \times \vec{B}, \quad r = \frac{mv}{|q|B}$$

misc

$$W = q\Delta V, \quad \text{Centripital force } F = mv^2/r, \quad E = -\Delta V/d, \quad V = kq/r, \quad V = \Delta KE = -\Delta PE, \quad KE = 0.5 * mv^2$$

$F = ma$, earth south is north, use conventional, $\vec{c} = \vec{a} \times \vec{b}$, $|\vec{c}| = |\vec{a}||\vec{b}| \sin \theta_{ab}$, cross is det, dot is sum

$$\text{RMS} = \sqrt{\sum(x^2)}, \quad \% \text{error} = (\text{act-exp})/\text{exp}$$

Force	F	$kg * m/s^2$	Newton	N
Energy/Work	$U, KE W$	$N * m, W * s$	Joule	J
Charge	Q	$A * s$	Coulomb	C
Chg den linear	λ	C/m	—	C/m
Chg den surface	σ	C/m^2	—	C/m^2
Chg den volume	ρ	C/m^3	—	C/m^3
Elec Field	E	N/C	—	N/C
Elec Flux	Φ	$N * m^2/C$	—	Nm^2/C
Elec Potential	V	$J/C, W/A$	Volt	V
Current	I	C/s	Amp	A
Current density	J	I/m^2	—	I/m^2
Resistance	R	V/A	Ohm	Ω
Resistivity	ρ	$E/J, RA/L$	—	Ωm
Power	P	$VA, J/s$	Watt	W
Capacitance	C	Q/V	Farad	F
Magnetic field	B	$Ns/Cm, N/mA$	Tesla	T
Magnetic field	Φ	Tm^2	Weber	Wb

Sources of magnetic fields, law of Biot-Savart for moving charges and current elements, Magnetic fields of current carrying wires and loops, Magnetic forces between conductors
 field from point charge moving $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$, $B = \frac{\mu_0}{4\pi} \frac{qv \sin \theta}{r^2}$, velocity, radius to measurement, from current element Biot-Savart swap $q\vec{v} > \int Id\vec{l}$, right hand, thumb conventional current/positive charge. axis of loop: $B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} = \frac{\mu_0 \mu}{2\pi(s^2 + a^2)^{3/2}}$, $\mu = IA$, $x = \text{far}$ - $B = \frac{\mu}{x^3}$

Current same direction, fields oppose, attract. $F = L \frac{\mu_0 I_1 I_2}{2\pi r}$

Straight wire: $B = \frac{\mu_0 I}{2\pi r}$, Center of a loop: $B = \frac{\mu_0 I}{2r}$, inside: $\frac{\mu_0 I}{2\pi R^2} r$

Solenoid: inductance: $L = \frac{\Phi_B}{i} = \frac{N\Phi_{B,loop}}{i} = \frac{NBA_{loop}}{i} = \frac{N\mu_0 n i A_{loop}}{i} = \mu_0 N \frac{N}{l} A_{loop} = \frac{\mu_0 N^2 \pi r_s^2}{l} = \pi \mu_0 n^2 r_s^2 l$

inside a Solenoid: $B = \mu_0 n I$, voltage $\int_a^b \vec{E}_{nc} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -L \frac{di}{dt}$, i from + to - increase, EMF

Ampere's law, calculating magnetic fields from ampere's law. Magnetic moments and magnetism, magnetic force and torque on a current loop/magnetic moment

Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ total field in a circular path around a wire is equal to μ_0 times current enclosed
 current density \vec{J} , $I_{enc} = \int \vec{J}_{net} \cdot d\vec{A} = J \cdot A \cos \theta$

Magnetic moment: $\vec{\mu} = I\vec{A}$, current in loop times area of loop, right hand direction. Torque $\tau_{B,net} = \vec{\mu} \times \vec{B}$, right hand rule for spin direction

Magnetic flux, Faraday's law, Lenz's law, Electromagnetic Induction.

Magnetic flux $\Phi_B = \int \vec{V} \cdot d\vec{A} = \int B dA \cos \theta$, Faraday's law: EMF from changing Mflux $\epsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \Phi_B$, for N loops, $\cdot N$. Lenz's law, this EMF induces opposing (attracting) magnetic field. B increase up, EMF and i cw, induced B down, net small B up

Displacement current, Maxwell's equations: "displacement current" is built up charge, $I_d = \epsilon_0 \frac{d}{dt} \Phi_E$, fixed

Ampere's $\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_c + I_d)_{enc} = \mu_0 I_{c,enc} + \mu_0 \epsilon_0 \frac{d}{dt} \Phi_{E,enc}$

Maxwell's: Gauss's for \vec{E} : $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ for \vec{B} : $\oint \vec{B} \cdot d\vec{A} = 0$

Faraday stationary: $\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \Phi_B$, Ampere stationary: $\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_c + \epsilon_0 \frac{d}{dt} \Phi_E)_{enc}$

Self and mutual Inductance, EMF and current in circuits, Magnetic field energy and energy density

self inductance: $\Phi_B = Li$, $L = \frac{\Phi_B}{i}$, Mutual: $M = M_{12} = \frac{N_1 \Phi_{B1}}{i_2} = M_{21} = \frac{N_2 \Phi_{B2}}{i_1}$, $\frac{d\Phi_B}{dt} = \frac{d}{dt} Li = L \frac{di}{dt}$, $\epsilon_L = -L \frac{di}{dt}$, $\epsilon_1 = -M \frac{di_2}{dt}$

magnetic energy in an inductor $U_B = 0.5 Li^2$, region in field \vec{B} has energy density $u_B = \frac{U_B}{v} = \frac{B^2}{2\mu_0}$

Circuit Transients, RC, RL, LC, and RLC. Characteristic decay times and oscillation frequencies

$I(C) = C \frac{dV_C}{dt}$, $V(L) = L \frac{dI_L}{dt}$

RC: charge $q(t) = C\epsilon(1 - e^{-t/RC})$, $i = \frac{dq}{dt}$, $i(t) = \frac{\epsilon}{R} e^{-t/RC}$ discharge: $q(t) = Q_0 e^{-t/RC}$, $i(t) = -\frac{Q_0}{RC} e^{-t/RC}$

RL: charge $i(t) = \frac{\epsilon}{R}(1 - e^{-tR/L})$, discharge $i(t) = i_0 e^{-tR/L}$

LC: Q(C) $q(t) = Q \cos(\omega t + \phi)$, I(L) $i(t) = \frac{dq}{dt} = -\omega Q \sin(\omega t + \phi)$, $\omega = 1/\sqrt{LC}$ $T = \frac{2\pi}{\omega}$, $\omega = 2\pi * \text{freq}$,

$U_E = \frac{(q(t))^2}{2C}$, $U_B = 0.5 L (i(t))^2$, $U_{tot} = U_E + U_B = \frac{Q^2}{2C}$, $L \frac{di}{dt} = -\frac{q}{C}$, $\frac{d^2 q}{dt^2} = -\frac{1}{LC} q$

Alternating current circuits, phasors, reactance, impedance, resonance, power, transformers

AC: $RMS = \frac{1}{\sqrt{2}} \text{max}$, $X_L = \omega L$, V_L is 90 ahead, $X_C = \frac{1}{\omega C}$, V_C is 90 behind

$i(t) = I \cos(\omega t)$, L: $V_L(t) = \omega L I \cos(\omega t + \pi/2) = V_L \cos(\omega t + \pi/2)$

series LRC AC: $V = \sqrt{V_R^2 + (V_L - V_C)^2} = I \sqrt{R^2 + (X_L - X_C)^2}$, *net* impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

current phasor is shared, V_R matches, V_L leads 90, V_C lags 90, $V_S = V_R + V_L + V_C$, some phase inbetween
 ϕ , $\tan \phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$, resonance: at ω_0 , $X_L = X_C$, $Z = R$,

$q(t) = Q e^{-t/\tau_d} \cos(\omega' t + \phi)$, $\tau_d = 2L/R$, $\omega' = \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$

Power: $P_{average} = 0.5 V_{amp} I_{amp} \cos \phi_{V-I} = V_{RMS} I_{RMS} \cos \phi_{V-I}$, $\cos \phi = R/Z$ for series LRC

Transformer: $\frac{V_2}{V_1} = \frac{N_2}{N_1}$