Homework # 9

Due: Wednesday, August 9th

Problem 1. (20 points.)By assuming that $\mathcal{L}\{\delta(t)\} = 1$ and $\mathcal{L}\{u(t)\} = 1/s$, and by only using time-shift and frequency shift properties of Laplace transform, determine the Laplace transform of each of the following signals

a) $x(t) = e^{-2t}u(t+1)$

b) $x(t) = e^{-2t}u(t) + e^{-4t}u(t-2)$

Problem 2. (15 points.) Signal $x(t)$ satisfies

$$\frac{d}{dt}x(t) + 2x(t) = e^{-4t}u(t) + 2u(t-1)$$

What is the Laplace transform of $x(t)$?

Problem 3. (15 points.) Let $x(t)$ be a signal specified as

$$x(t) = \sum_{n=0}^{\infty} e^{-nT}\delta(t-nT)$$

where $T > 0$. Determine the Laplace transform of $x(t)$

Problem 4. (50 points.) Determine the bilateral Laplace transform of the following signals.

a) $e^{-3t}u(t) + e^{-2t}u(-t)$

b) $e^{2t}u(-t) + e^{-3t}u(-t)$

c) $e^{2t}u(t-5)$

d) $e^{-3t}\cos(3t)u(t)$

e) $x(t) \begin{cases} e^{3t}\cos(2t) & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$

1) L{d(t)}=1 L{u(t)}=1/s
a) e^-2t u(t+1)
L{e^-at u(t)}=1/(s+a) ROC Re{s}>-a
L{e^-2t u(t)}=1/(s+2) ROC Re{s}>-2
L{e^-2(t+1) u(t+1)}=e^-s(-1)/(s+2) ROC Re{s}>-2
L{e^-2t u(t+1)}=e^(s+2)/(s+2) ROC Re{s}>-2
X(s) = e^(s+2)/(s+2) ROC Re{s}>-2

b) e^-2t u(t) + e^-4t u(t-2)
L{e^-2t u(t)}=1/(s+2) ROC Re{s}>-2
L{e^-4t u(t)}=1/(s+4) ROC Re{s}>-4
L{e^-4(t-2) u(t-2)}=e^-s(2)/(s+4) ROC Re{s}>-4
L{e^-4t u(t-2)}=e^-2(s+4)/(s+4) ROC Re{s}>-4
L{e^-2t u(t) + e^-4t u(t-2)} = 1/(s+2) + e^(-2s-8)/(s+4) ROC Re{s}>-2
X(s) = 1/(s+2) + e^(-2s-8)/(s+4) ROC Re{s}>-2

2) d/dt x + 2x = e^-4t u(t) + 2u(t-1)
L{ }=
sX + 2X = 1/(s+4) + 2e^(-s)/s
(s+2) X = (s+2e^(-s)(s+4))/s(s+4)
X(s) = (s+2e^(-s)(s+4))/s(s+4)(s+2)

3)
x=sum_n=0^inf e^-nT d(t-nT)
X=sum_n=0^inf e^nT L{d(t-nT)}
X=sum_n=0^inf e^nT e^-nts
X=sum_n=0^inf e^nT(s+1)
Re{s}<-1 => X(s) -> inf
Re{s}>-1 => X(s) = a^n   a=e^-T(s+1)
a<1 so   X(s) = 1/(1-a)
X(s) = 1/(1-e^(-T(s+1)))   ROC Re{s}>-1

4)
a) e^-3t u(t) + e^-2t u(-t)
L{e^-3t u(t)} = 1/(s+3) Re{s}>-3
L{e^-2t u(-t)} = -1/(s+3) Re{s}<-2
X(s) = 1/(s+3) - 1/(s+2)

b) e^2t u(-t) + e^-3t u(-t)
L{e^-3t u(-t)} = -1/(s+3) Re{s}<-3
L{e^2t u(-t)} = -1/(s-2) Re{s}<2
X(s) = -1/(s+3) - 1/(s-2)

c) e^2t u(t-5)
e^2t u(t-5) = e^(2(t-5)+10) u(t-5) = e^10 e^2(t-5) u(t-5)
L{e^2(t-5) u(t-5)} = e^-5s 1/(s-2) Re{s}>2
X(s) = e^10 e^-5s 1/(s-2) Re{s}>2

d) e^-3t cos(3t) u(t)
e^-3t cos(3t) u(t) = e^-3t u(t) .5[e^j3t + e^-j3t] = .5 [e^-3(1-j)t + e^-3(1+j)t]
X(s) = .5[1/(s+3(1-j)) + 1/(s+3(1+j))] Re{s}>-3

e) gate(t) e^3t cos(2t)
X=int_0^1 .5 e^3t [e^j2t +e^-j2t] e^-st dt
X=int_0^1 .5 e^t(3+2j-s) dt + int_0^1 e^t(3-2j-2) dt
X=1/(2(3+2j-s)) e^t(3+2j-s) |_0^1 + 1/(2(3-2j-s)) e^t(3-2j-s) |_0^1
X=1/(2(3+2j-s)) [e^(3+2j-s) - 1] + 1/(2(3-2j-s)) [e^(3-2j-s) - 1]