Fields and Waves I

Lecture 7

Smith Charts
Matching Stubs
Exam 1 Review

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Announcements

- HW 3 **not for a grade**
- HW 2 solutions going out **ASAP**

Power Analysis (rms)

To find time-averaged incident power:

$$P^i_{av} = \frac{1}{T}\int_0^T P^i(d,t)dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} P^i(d,t)dt$$

$$= \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{|V_0^+|^2}{Z_0} cos^2(\omega t + \beta d + \phi^+)dt$$

$$P^i_{av} = \frac{\omega}{2\pi}\frac{\pi}{\omega}\frac{|V_0^+|^2}{Z_0} = \frac{|V_0^+|^2}{2Z_0}$$

Power Analysis (rms)

T-line Intuition

Lossy T-Lines

Bounce Diagrams

Phasors

Standing Wave Patterns

Input Impedance

Input Impedance

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 tan(\beta L)}{Z_0 + jZ_L tan(\beta L)}$$

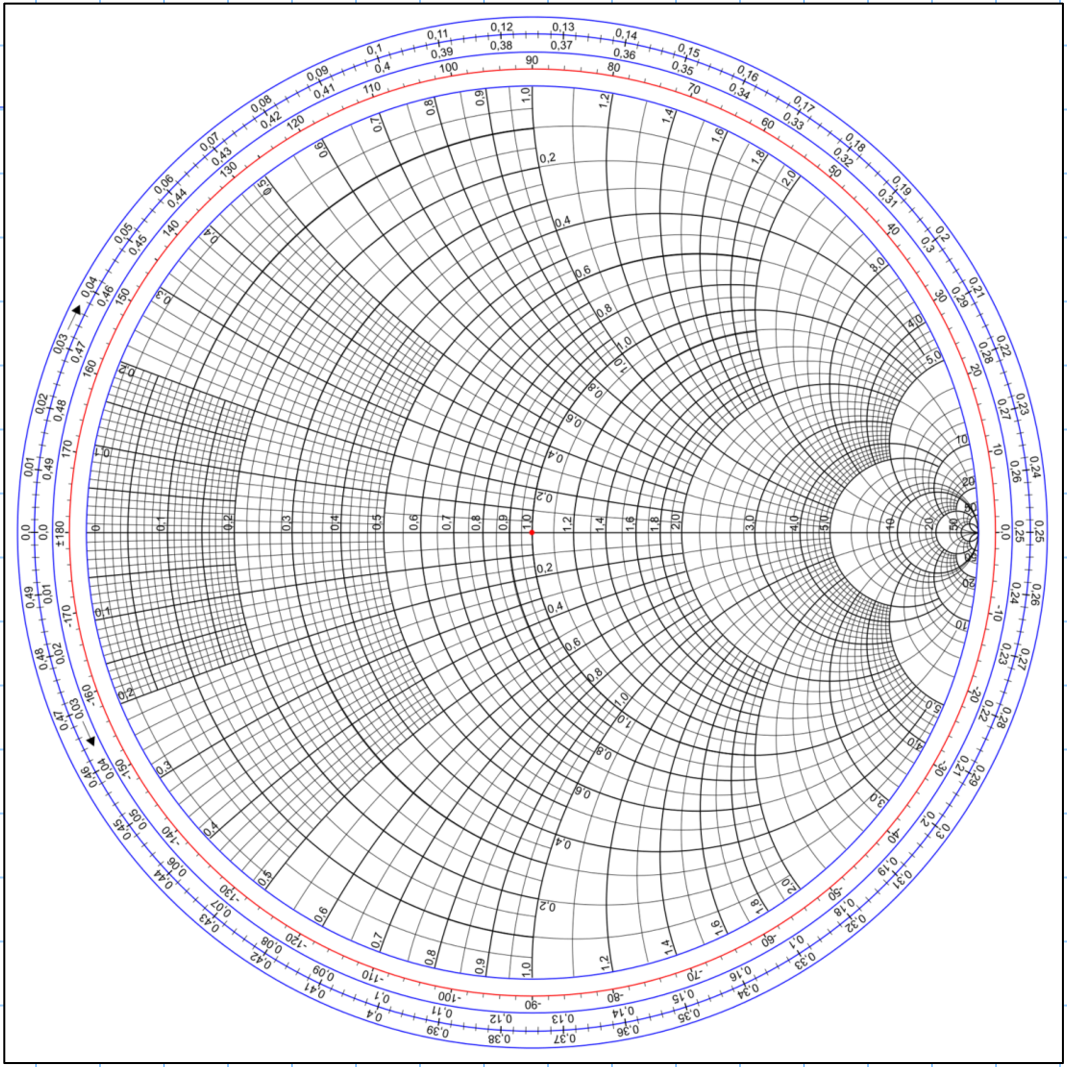
Basic Wave Properties

$\beta = \frac{\omega}{u}$	$\lambda = \frac{2\pi}{\beta} = \frac{u}{f}$

Input Impedance

- Much of the following slides were written by Prof. Nick Shuley from the University of Queensland (UQ). The books he uses the most are:
  - Fundamentals of Engineering Electromagnetics (Cheng)
  - Fundamentals of Applied Electromagnetics (Ulaby)
  - Microwave Engineering (David Pozar)

Smith Chart



Complex Γ-plane

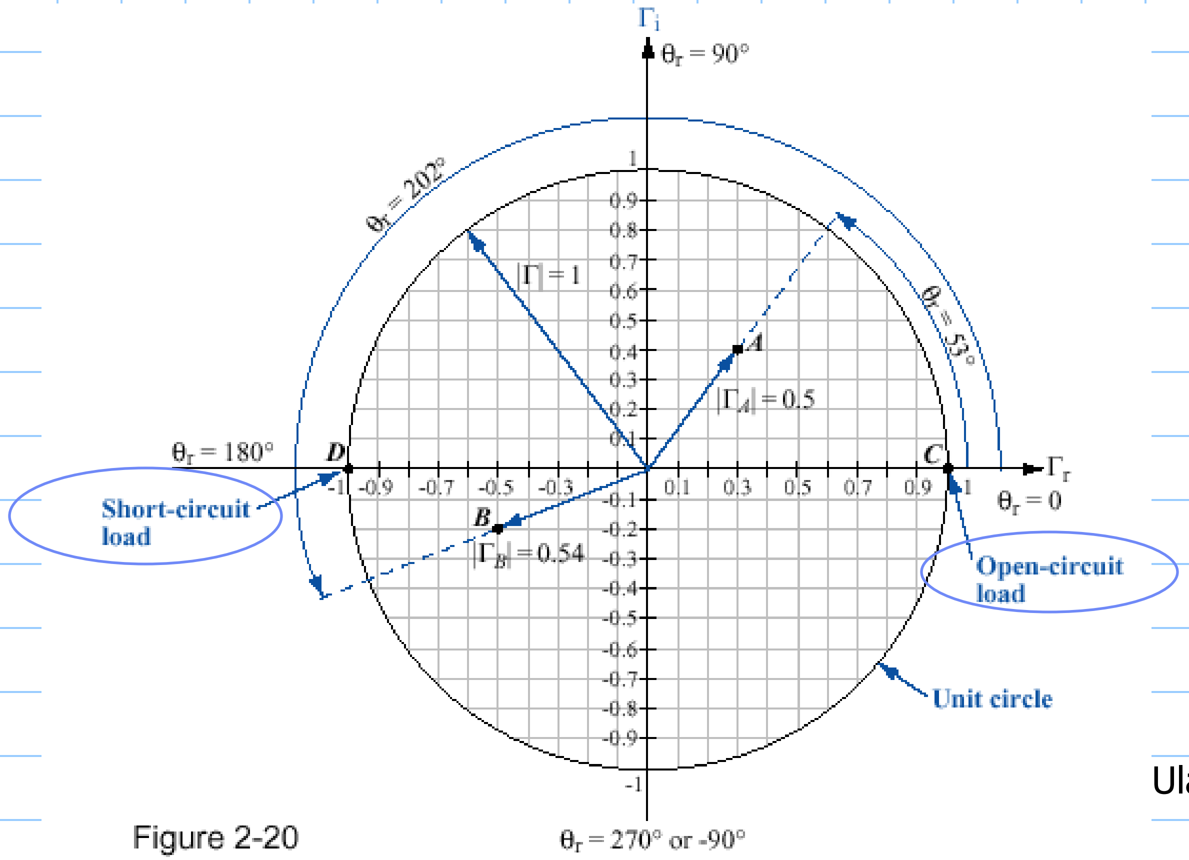


Figure 2-20

Ulaby

Smith Chart

We need to relate impedances to reflection coefficients:

First, we normalize all impedances with respect to the characteristic impedance of the line:

$$z = \frac{Z}{Z_0} \qquad \text{e.g.} \quad z_L = \frac{Z_L}{Z_0}$$

For an impedance of Z_R becomes:

$$\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_R/Z_0 - 1}{Z_R/Z_0 + 1} = \boxed{\frac{z_R - 1}{z_R + 1}} \Leftrightarrow z_R = \boxed{\frac{1+\Gamma}{1-\Gamma}} \quad ..(6.2)$$

Smith Chart

Now since the normalized impedance can be written as:

$$z_R = r_R + jx_R \qquad ..(6.3)$$

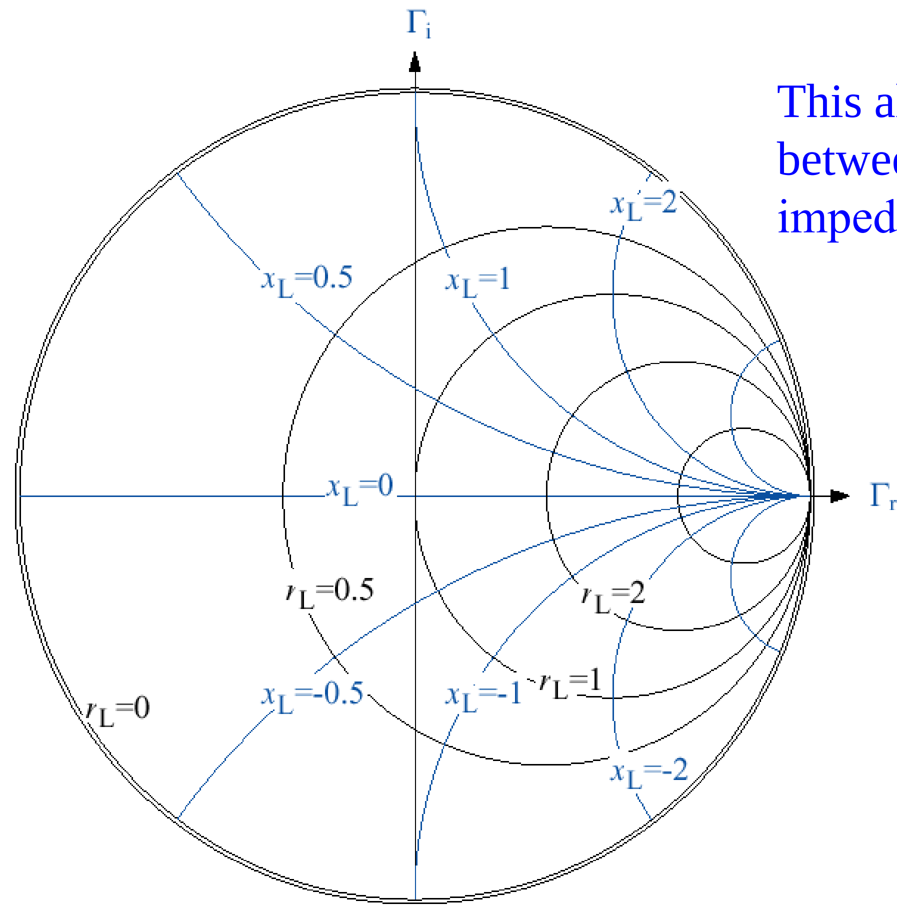
we set (6.3) equal to (6.2) using the real and imaginary parts of (6.1). This gives:

$$r_R + jx_R = \frac{1+\Gamma}{1-\Gamma}$$

We can then solve for the r_R and x_R in terms of Γ. Graphical families of all possible solutions to this equation constitute the Smith Chart.

Smith Chart

A Smith chart is therefore a polar plot of Γ, with contours of real and imaginary parts of z superimposed on top.



This allows easy conversion between normalized impedance z and Γ

Ulaby

Smith Chart

- A Smith chart printout is available on the shared drive
- This chart is designed to be printed out and done by hand with a ruler
- However, you can also use it on your computer using digital tools
- ImageJ is a useful tool for this

Smith Chart

Frequency Sweep @ 4:00

Smith Chart

- Example:
  - Let's say that a t-line has a characteristic impedance of 40Ω and a load impedance of 20+40jΩ.
  - What is the magnitude and phase of the reflection coefficient?

Smith Chart

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Normalized load impedance is 0.5+j

Smith Chart

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  - Let's say that a t-line has a characteristic impedance of 40Ω and a load impedance of 20+40jΩ.
  - What is the magnitude and phase of the reflection coefficient?

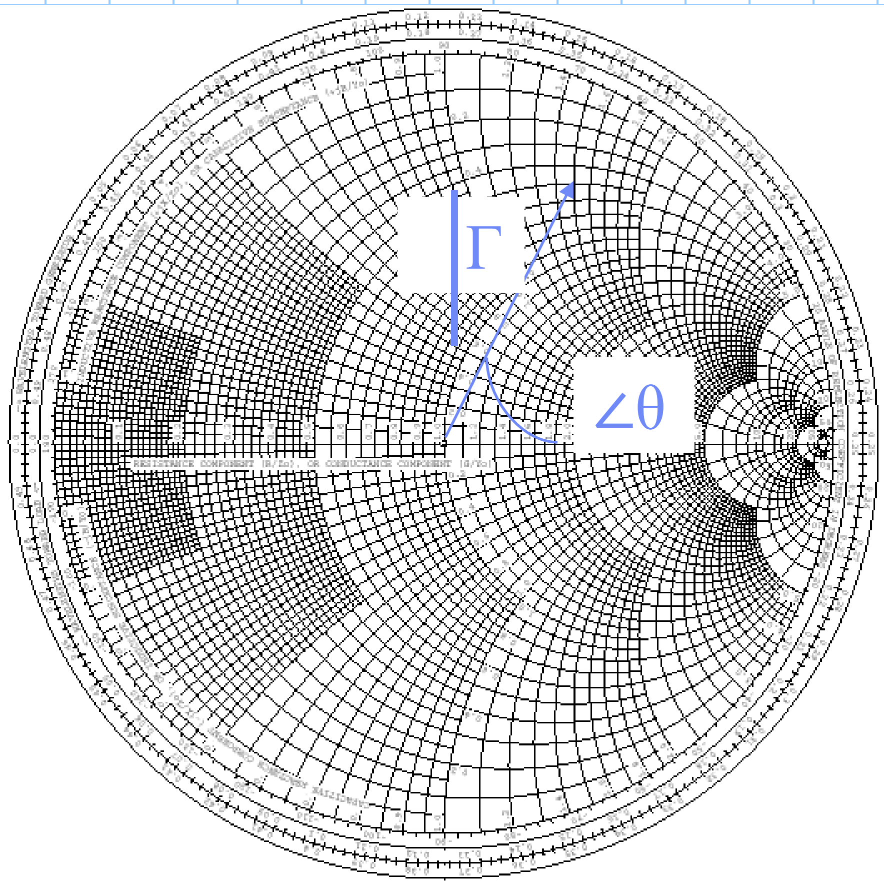
Normalized load impedance is 0.5+j

$\Gamma \approx 0.63 \angle 85°$

Smith Chart

The reflection coefficient is proportional to the length of the radial vector on the chart. The length of the vector to the periphery corresponds to $|\Gamma| = 1$.

The phase angle of the reflection coefficient is measured from the positive
direction of the horizontal axis



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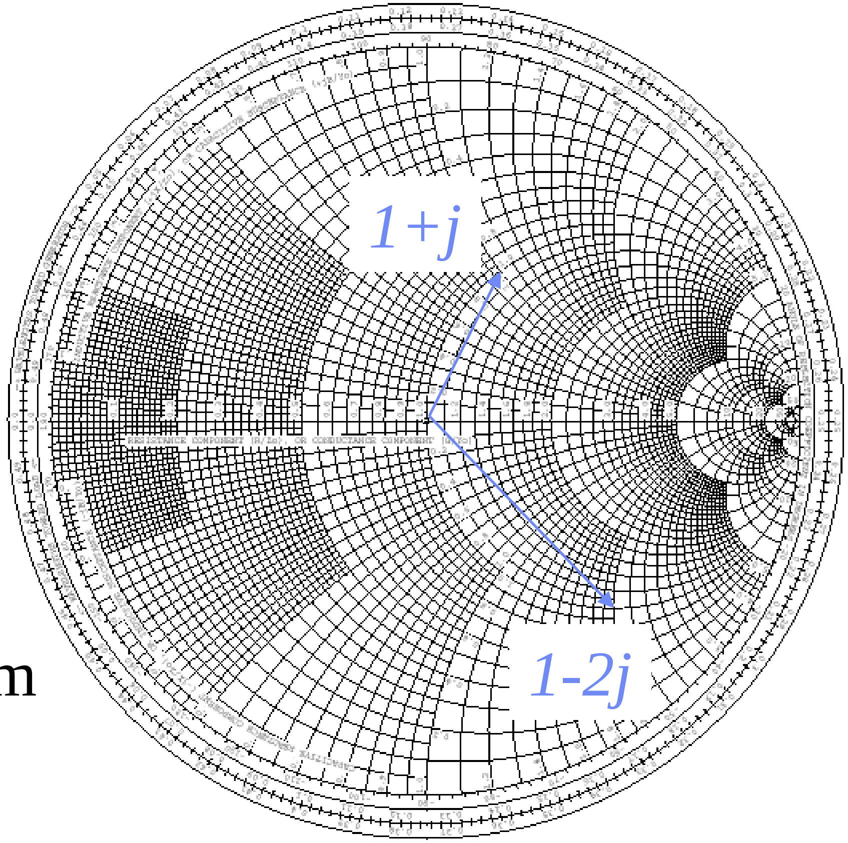
Smith Chart

All impedances in the top half are inductive e.g. 1+j

All impedances in the bottom half are capacitive e.g. 1-2j



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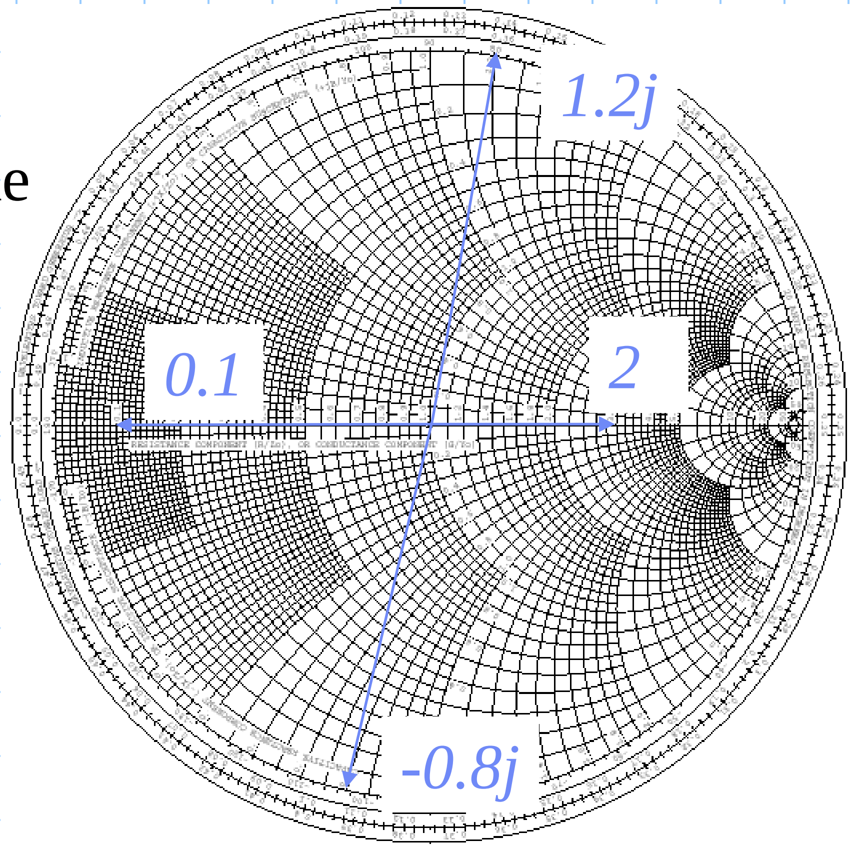
Smith Chart

purely real impedances are along the horizontal center line

purely imaginary impedances are along the periphery



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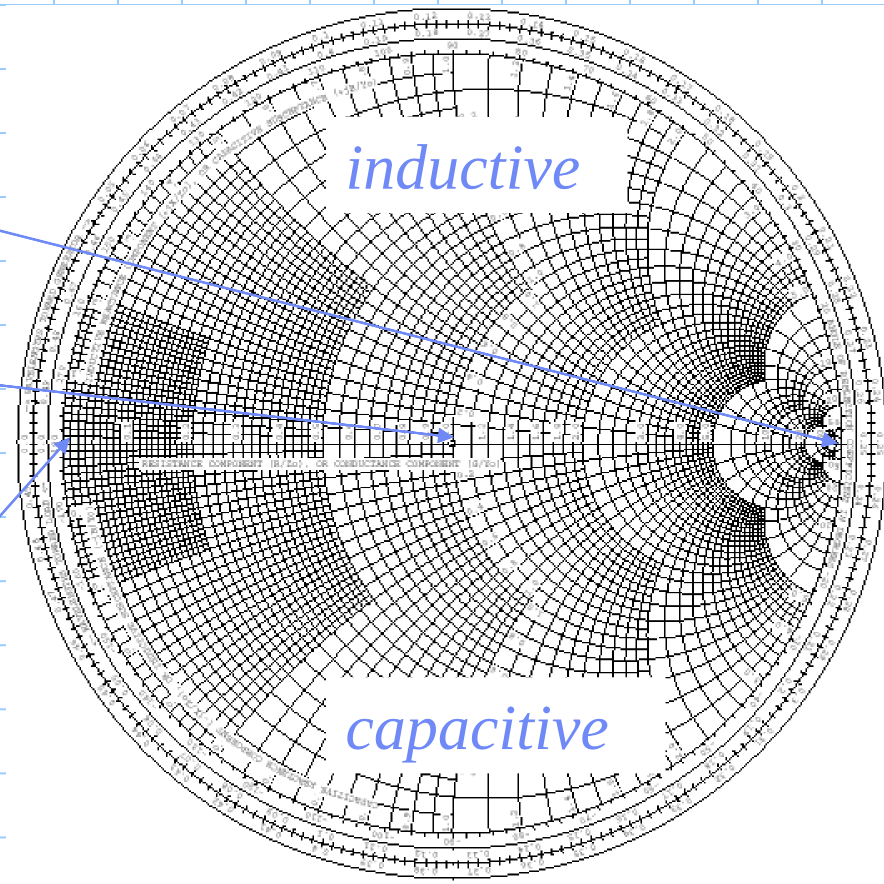
Smith Chart

open circuit point
(infinite impedance)

unity impedance z =1
(match point)

short circuit point
(zero impedance)



Ulaby

Smith Chart

What do you notice about the angle between the open and short circuit on the Smith Chart?

Smith Chart

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They are 180 degrees away from one another.

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What length of transmission line is required to make an open circuit look like a short circuit or vice versa?

Smith Chart

What do you notice about the angle between the open and short circuit on the Smith Chart?

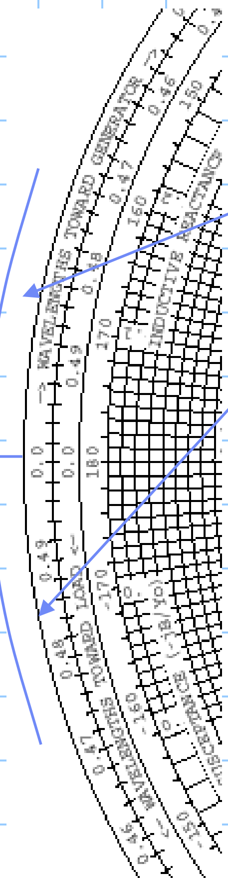
They are 180 degrees away from one another.

What length of transmission line is required to make an open circuit look like a short circuit of vice versa?

A quarter wavelength.

Thus, the angle on a Smith chart is also measured in wavelength (of the AC input signal).

Smith Chart



two scales on the periphery (in wavelengths)
1 towards generator (clockwise)
1 towards load (counterclockwise)

Note also that once around the whole chart is a total length of $\lambda/2$

Ulaby

Smith Chart

If the normalized impedance of a load is +j, what length of transmission line (in wavelengths) is required to make the load look like a real impedance?

Smith Chart

If the normalized impedance of a load is +j, what length of transmission line (in wavelengths) is required to make the load look like a real impedance?

An eighth of a wavelength. (which will cause it to look like an open circuit)

Smith Chart

If the normalized impedance of a load is 3+3j, what length of transmission line (in wavelengths) is required to make the load look like a real impedance?

Smith Chart

If the normalized impedance of a load is 3+3j, what length of transmission line (in wavelengths) is required to make the load look like a real impedance?

Reflection coefficient is **$\approx 0.7 \angle 20°$**

20/360 = 5.5% of a half wavelength = 2.7% of a wavelength

Smith Chart

For the impedance we found in our first example (0.5 + j), what is the standing wave ratio?

(Read from the bottom of the chart)

Smith Chart

SUMMARY

- The Smith Chart allows the graphical solution of the transmission line equation for *Z*.
- The Chart gives direct conversion between Γ and *Z*.