CSCI 2200 — Foundations of Computer Science (FoCS)
Homework 5 (document version 1.0)

Overview

- This homework is due by 11:59PM on Thursday, December 8
- You may work on this homework in a group of no more than four students; unlike recitation problem sets, **your teammates may be in any section**
- You may use at most **three** late days on this assignment
- Please start this homework early and ask questions during office hours and at your December 7 recitation section; also ask (and answer) questions on the Discussion Forum
- Please be concise in your answers; even if your solution is correct, if it is not well-presented, you may still lose points
- You can type or hand-write (or both) your solutions to the required graded problems below; **all work must be organized in one PDF that lists all teammate names**
- You are strongly encouraged to use LaTeX, in particular for mathematical symbols; see references in Course Materials
- **EARNING LATE DAYS:** for each homework that you complete using LaTeX (including any tables, graphs, etc., i.e., no hand-written anything), you earn one additional late day; you can draw graphs and other diagrams in another application and include them as image files
- To earn a late day, you must submit your LaTeX files (i.e., `*.tex`) along with your one required PDF file—please name the PDF file `hw5.pdf`
- Also note that the earned late day can be used retroactively, even back to the first homework assignment!

Warm-up exercises

The problems below are good practice problems to work on. Do not submit these as part of your homework submission. **These are ungraded problems.**

- **Problem 13.46.**
- **Problem 13.55.**
- **Problem 13.58.**
- **Problem 14.1.**
- **Problem 14.2.**
- **Problem 14.18.**
- **Problem 14.19.**
- **Problem 14.48.**
- **Problem 15.6.**
- **Problem 15.18.**
- **Problem 15.32.**
- **Problem 15.33.**
- **Problem 15.35.**
- **Problem 15.46.**
- **Problem 16.26.**
- **Problem 24.2.**
- **Problem 24.3.**
- **Problem 24.9.**
- **Problem 24.11(a-e,g,i-v,x-z).**

Graded problems

The problems below are required and will be graded.

- ***Problem 13.42.**
- ***Problem 13.50.**
- ***Problem 14.15(b-c).**
- ***Problem 14.34.**
- ***Problem 14.63(g).**
- ***Problem 15.12.**
- ***Problem 24.11(f,h,w).**
- ***Problem 25.7.**

Some of the above problems (graded and ungraded) are transcribed in the pages that follow.

Graded problems are noted with an asterisk (*).

If any typos exist below, please use the textbook description.

- ***Problem 13.42.** To determine if a graph G with 50 vertices is 3-colorable, you test all possible 3-colorings. Your computer checks a million 3-colorings per second. Estimate how long it is going to take, in the worst case.
  There are 3^{50} possible ways a graph with 50 verticies can be 3 colorable
  Because we can check 1 million 3-colorings a second, it would take us:
  $\frac{3^{50}}{10^6} = 7.1789798769 * 10^{17}$ seconds,

- ***Problem 13.50.** How many 7-digit phone-numbers are non-decreasing (each digit is not less than the previous one.)
  With each new digit comes one less possible number in its place, resulting in the decrease every time we multiply. There are 10^7 total possible number combinations in general.
  $10^7 - 10 * 9 * 8 * 7 * 6 * 5 * 4 = 9,395,200$

- ***Problem 14.15(b-c).** Consider the binary strings consisting of 10 bits.

  (b) How many contain (i) 5 or more consecutive 1's (ii) 5 or more consecutive 0's?
  The answer to i and ii are the same, therefore only work to i will be shown since it applies to both (but using ones instead of zeroes and vice versa)
  If there must be 5+ consecutive 1's, then at the least we have something like 1111100000, but any of the zeroes can be a one or zero.
  This makes an equation of $1^5 * 2^5 = 32$

  (c) How many contain 5 or more consecutive 0's or 5 or more consecutive 1's?
  This will be the addition of 2 32's as the magnitude of each of these subsets is 32: $32 + 32 = 64$

- ***Problem 14.34.** Consider all permutations of $\{1, 2, 3, 4, 5, 6\}$. A permutation is good if any of the sub-sequences 12, 23, or 56 appear. How many good permutations are there? We will use the permutation formula and then multiply it by 3 because there are 3 sub sequences which we are accounting for. $\frac{n!}{(n-r)!} * 3 = \frac{6!}{(6-2)!} * 3 = 90$

- ***Problem 14.63(g).** Here are some counting problems on graphs to challenge you.

  (g) How many Hamiltonian cycles are in $K_{n,n}$? [Hint: a Hamiltonian cycle is a cycle on graph $G = (V, E)$ that starts and ends at vertex $v_0 \in V$, visiting each vertex in set $V - \{v_0\}$ (i.e., all other vertices) exactly once.]

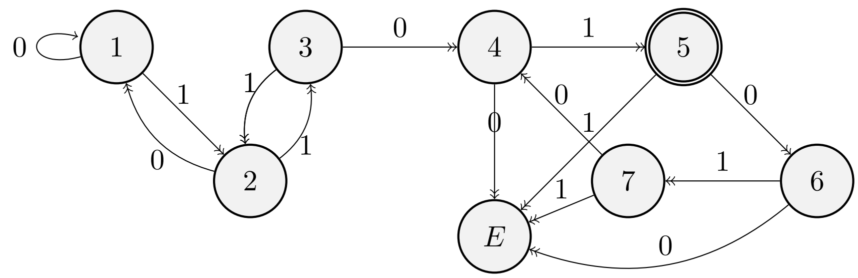
  $(\Pi_{i=1}^{n} i^2)/n$ you have n options going to the other side, then n-1 options going back, and n-1 options going back to the other side again, then n-2...

- ***Problem 15.12.** Roll a 6-sided die 5 times. What is the probability: (a) some number repeats (b) you get no sixes?
  a:
  $(\frac{1}{6})^2 * (\frac{5}{6})^3 * 100\% = 1.6\%$
  $\frac{1}{6}$ comes from the fact that the first roll can be any of the 6 numbers. It is squared because it

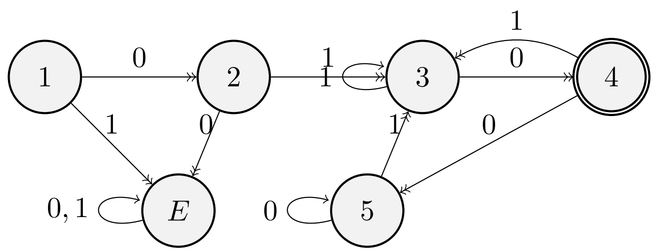
must happen again to be able to account for a repeated roll. The $\frac{5}{6}$ comes from the remaining possible numbers on the die in the roll.
b:
$(\frac{5}{6})^5 * 100\% = 40\%$
The $\frac{5}{6}$ comes from the 5 out of 6 possible numbers which can be rolled each time in any combo. It is raised to the power of 5 because the die is rolled 5 times.

- ***Problem 24.11(f,h,w).** Give DFAs for the following languages, a.k.a., computing problems.

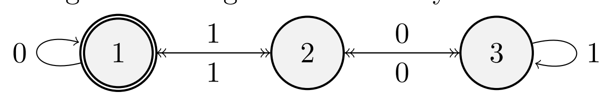
  (f) $\mathcal{L} = \{1^{\bullet 2n}01^{\bullet 2k+1} \mid n, k \geq 0\}$.



  (h) Strings which begin with 10 and end with 01.

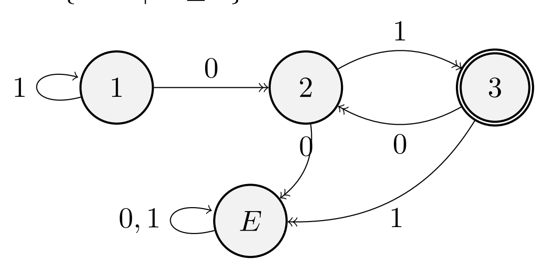


  (w) Strings whose length is divisible by 3.



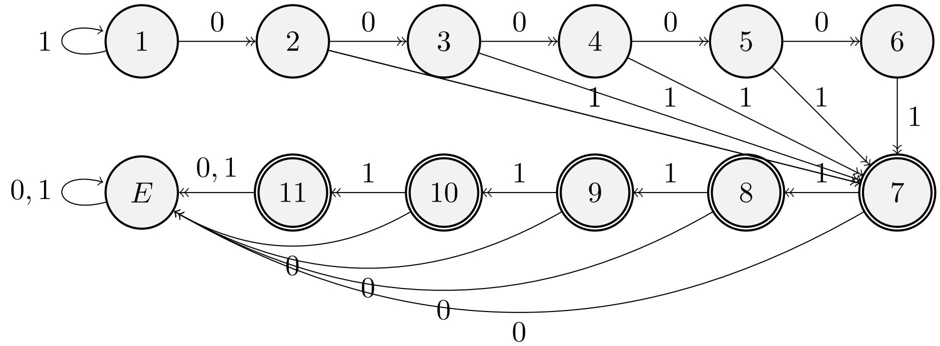
- ***Problem 25.7.** Give a DFA and a CFG for each problem.

  (a) $\mathcal{L} = \{01^{\bullet n} \mid n \geq 0\}$



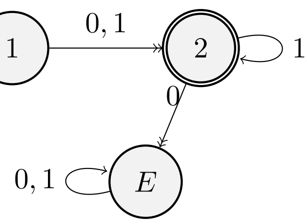
CFG: s $\rightarrow \epsilon$ |01S

(b) $\mathcal{L} = \{0^{\bullet n}1^{\bullet n} \mid 0 \leq n \leq 5\}$



CFG: S $\rightarrow \epsilon$ |0S1

(c) $\mathcal{L}$ = { strings which end in a 1 }.



CFG: S $\rightarrow$ #1