Exam 1

Instructions

1.) Unless otherwise specified, you have one class period to complete the questions below.

2.) Read all directions carefully.

3.) Show your work in enough detail to allow the graders to completely follow your thought process.

4.) Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use at least 2 significant digits.

5.) Make sure to write your answers legibly. You can write on the back of the exam pages or ask for scratch paper

6.) **Make sure to report units with all answers for full credit!**

Solutions

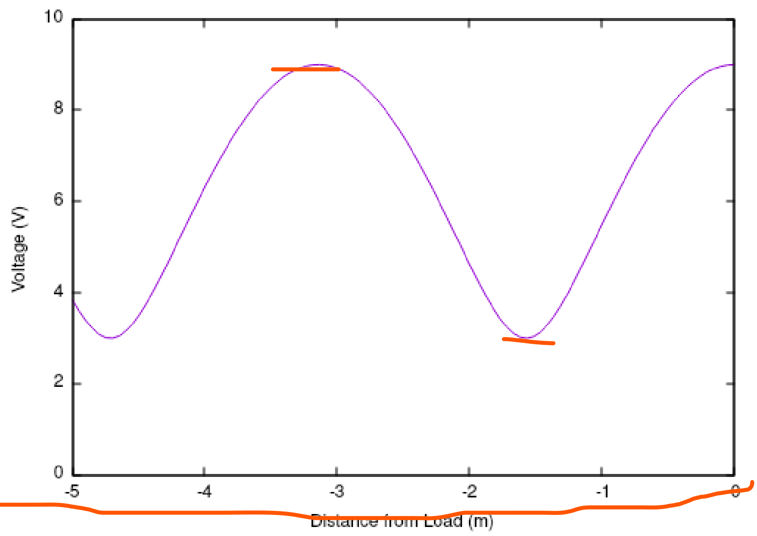
1. Standing Waves

The following plots show the magnitude of the sinusoidal voltage on a $Z_0 = 60\Omega$ transmission line with respect to position on the line. Suppose that the rightmost side of the plot represents the position of a load at the end of a transmission line.

a.) State whether the graph shown depicts a transmission line matched to the load, a transmission line not matched to the load, or neither (a pattern that couldn't physically occur for a sinusoidal input voltage).

b.) If the plot represents a non-matched load and sinusoidal input signal:
- ❖ Find the standing wave ratio,
- ❖ Find the reflection coefficient,
- ❖ Find the load impedance,
- ❖ Find $|V_0^+|$, the magnitude of the initial forward-traveling voltage wave.

a.)



$SWR = \frac{9}{3} = 3$

$SWR = 3 = \frac{1+|\Gamma|}{1-|\Gamma|}$

$|\Gamma| = \frac{1}{2}$.

Since V_{max} is at load,

$\Gamma = \frac{1}{2}$

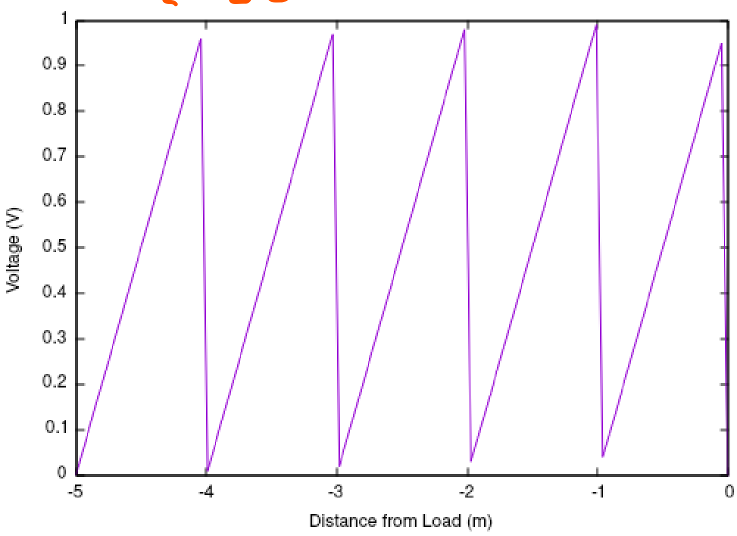
$|V_0^+|(1+|\Gamma|) = V_{max}$

$|V_0^+|(1.5) = 9$

$|V_0^+| = 6$

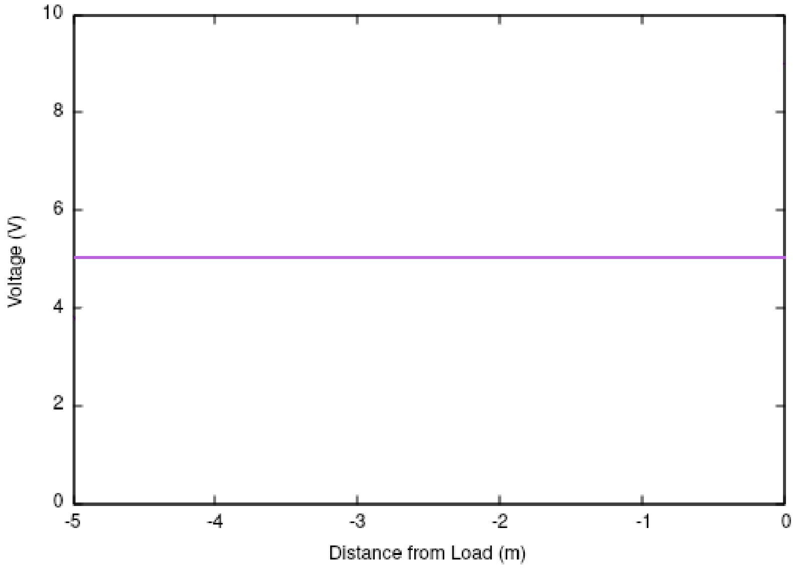
b.)

$\Gamma = \frac{1}{2} = \frac{Z_L - 60}{Z_L + 60}$ $\quad Z_L = 180\ \Omega$



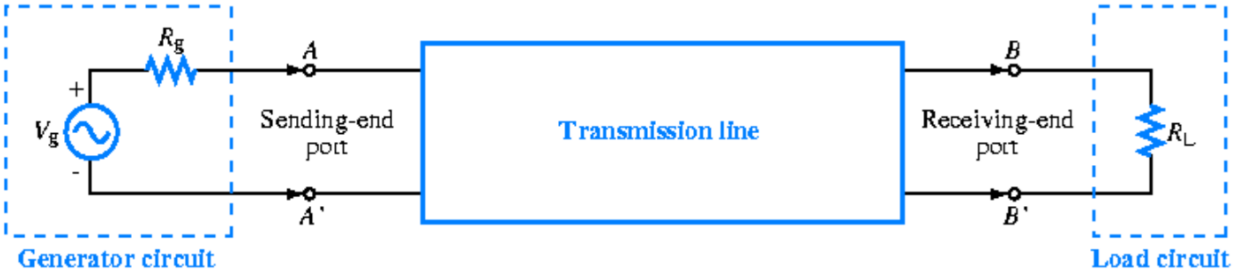
not a sinusoidal standing wave pattern.

c.)



matched to load.

2. Lossless Transmission Lines



Assume a sinusoidal source is connected to a lossless transmission line, as shown.

a.) Assume that the transmission line load is an open circuit. For what line lengths will the input impedance observed at the sending point end also be a ~~short~~ open circuit? Select your answers from the options below.

$\boxed{0}$ $\lambda/8$ $\lambda/4$ $3\lambda/8$ $\boxed{\lambda/2}$ $5\lambda/8$ $3\lambda/4$ $7\lambda/8$ $\boxed{\lambda}$ $9\lambda/8$ $5\lambda/4$ $11\lambda/8$ $\boxed{3\lambda/2}$

b.) Suppose that this line has characteristic impedance $Z_0 = 30\Omega$. If the transmission line length 300m and the signal wavelength is 40m, what is the input impedance? *(Simplify this expression and represent it as a real, imaginary or complex number.)*

Line length $L = \dfrac{300m}{40m} = 7.5\lambda$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta l)}$$

$$\beta L = \frac{2\pi}{\lambda} \cdot 7.5\lambda = 15\pi$$

$$\tan(15\pi) = 0$$

$$Z_{in} = Z_0 \frac{Z_L}{Z_0} = Z_L = \infty\,\Omega$$

(open circuit)

3. Properties of Waves

Consider the following two voltage expressions. For each one:

a.) state whether it represents a traveling or a standing wave.

b.) If it does represent a traveling wave, find the frequency *f*, the velocity *u* and its direction (+z or -z), the phase constant β, and represent it in phasor notation.

a.) $V(z,t) = 6 \sin(200\pi t - \underbrace{0.00002\pi}_{\beta} z)$

Traveling wave in +z direction.

$u = \frac{\omega}{\beta} = \frac{200\pi}{0.00002\pi} = 10^7 \text{ m/s}$ $\lambda = \frac{2\pi}{\beta} = 10^5 \text{ m}$

$f = \frac{u}{\lambda} = \frac{10^7}{10^5} = 100 \text{ Hz}$

$\tilde{V}(z) = 6e^{-j0.00002\pi z - j\frac{\pi}{2}}$

b.) $V(z,t) = 3 \sin(300\pi t)*\cos(\pi z)$

Standing wave.

c.) If you found that one of the two voltage expressions above was a traveling wave, assume Z_0 = 50Ω and determine the average power it delivers. What average power will be delivered to the load by this wave if Z_L = 200Ω ?

part a.) is a traveling wave.

$P_{i\,avg} = \frac{|V_0^+|^2}{2Z_0} = \frac{6^2}{100} = 0.36 \text{ W}$

$\Gamma_L = \frac{200-50}{200+50} = 0.6$

$P_{avg} = \frac{|V_0^+|^2}{2Z_0}(1-|\Gamma_L|^2) = 0.36(1-0.6^2)$

$= 0.23 \text{ W}$

4. Pulses On Transmission Lines

Suppose that a transmission line has reflection coefficient **Γ** = ½ at the load and **Γ** = -0.3 at the source. The time delay T of the line is 200 microseconds.

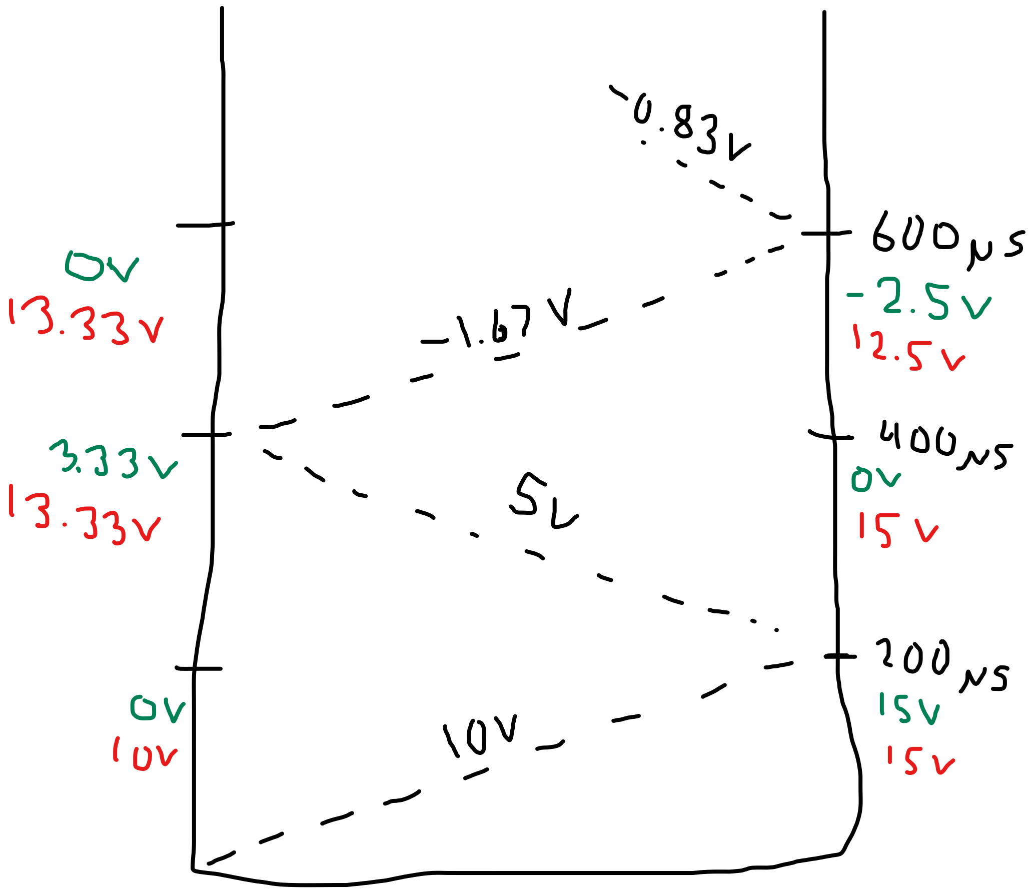
a.) Suppose that you input a pulse of duration 30 microseconds into the transmission line. Choose an amplitude that for the pulse to have as it enters the transmission line. This amplitude can be in the range 5V to 10V. Create a bounce diagram showing the propagation of the pulse on the transmission line for a period of time equal to 3T. The bounce diagram should include the amplitude of the input pulse, the amplitude of each reflected pulse, and the voltage measured at both the input and load of the line at each interval of T.

short pulse voltages in green

b.) Now suppose the short pulse is replaced by switching on a DC source. The amplitude of the voltage entering the line is the same one you chose in part a. What are the new voltages that you would measure at time 1T, 2T and 3T at the source and load? Justify your answer with either a second bounce diagram or a short explanation.

DC source voltages in red

Let $V_0^+ = 10V$



5. Coaxial Cable

The RG-11 coaxial cable type has a resistance of 4 mΩ per meter. It has a capacitance of 50 nanofarads per meter. Its characteristic impedance is 75 Ω. Assume that G'=0.

a.) Is this a low-loss transmission line at 10 MHz? Justify your answer. (Consider the transmission to be low loss if its resistive impedance is less than 1% as high as its reactive impedance.)

$$Z_0 = \sqrt{\frac{r' + j\omega L'}{g + j\omega C'}} \rightarrow 75\,\Omega = \sqrt{\frac{4\times10^{-3} + j\omega L'}{j\omega(50\times10^{-9})}} \qquad \omega = 2\pi \cdot 10\times10^{6}$$

$$L' = 281\ \mu H/m \qquad |j\omega L'| = 17655$$

$|r'| \ll |j\omega L'|$ so line is low loss

b.) What is the inductance per meter? (If you are using the low-loss assumption to make this calculation, say so.)

See part a

c.) Calculate this transmission line's attenuation constant? (If you are using the low-loss assumption to make this calculation, say so.)

Using low-loss assumption,

$$\alpha = \frac{r'}{2Z_0} = 2.67\times10^{-5}\ m^{-1}$$

c.) Suppose that you add parallel resistance to increase G'. What value of G' would make this line dispersionless?

$$\frac{r'}{L'} = \frac{g'}{c'} \qquad g' = 2.5\times10^{-6}\ S/m$$