ECSE 2110 Elec. Energy Systems

Final Examination, Spring 2018

May 8, 2018, LOW 3051, 3-6 PM

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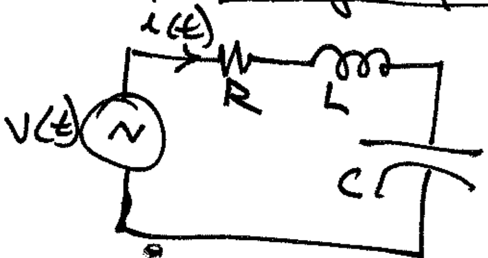
(1) (a) Single-phase AC Circuit



$V(t) = 100\sin(\omega t)$

$R = 1\,\Omega$, $L = 1$ mH, $C = 10\,\mu F$

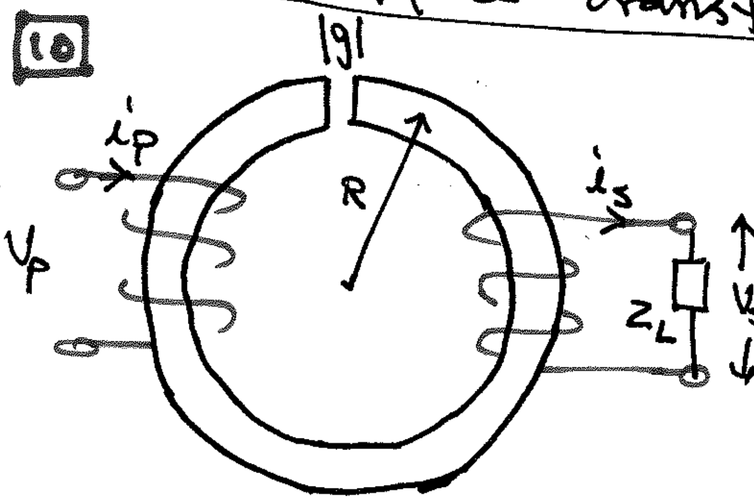
[10] (i) $\omega = 100$ rad/sec, Calculate $i(t)$, $S = P + jQ$

(ii) $\omega = 10000$ " , " " "

Explain the differences in the two results.

(1) (b) Gapped Transformer

[10]



$R = 10$ cm, $g = 1$ mm

core cross-section $= 2\ cm^2$

$\mu_r = 1000$, $\omega = 300$ rad/sec.

- Neglect leakage flux.
- " flux fringing at gap.
- " core & copper losses

$V_p(t) = 100\sin(\omega t)$ $\quad N_p/N_s = 0.2$, $N_p = 1000$, $N_s = 5000$

- (i) Calculate magnetizing current ($i_{p(\text{no load})}$ i.e. open secondary wdg)

(ii) For $Z_L = 10\,\Omega$, calculate i_s, i_p

(2) Synchronous Motor

300 kW, 480 V, Y-connected, $X_S = 1\Omega$, $R_A \simeq 0$, 8 Pole, 60 Hz

Assume P_{F+W}, P_{core}, P_{SLL} are negligible

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(a) Motor is operating at rated load and 0.9 lagging pf.
Calculate $E_A\angle\delta$, I_A, Torque, Max Torque

(b) If $|E_A|$ is increased by 20%, how do $E_A\angle\delta$, I_A, pf, Torque, Torque(max) change (?)

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(3) Transmission Lines $T = 40°C$

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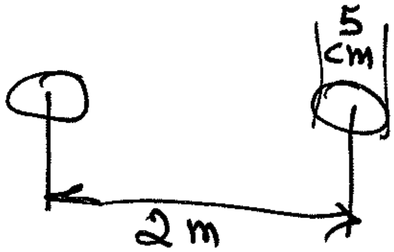


$\rho_{20°C} = 2\times10^{-8}\ \Omega\text{-m}$, $M = 250$

$R_{AC} = (1.05)R_{DC}$, $f = 60$ Hz, $V_S = 8$ kV

Length $= 100$ km

(a) Calculate R, L, C for the overhead single-phase line

(b) Calculate the A, B, C, D constants and then calculate the line charging current for short, medium and long lines.

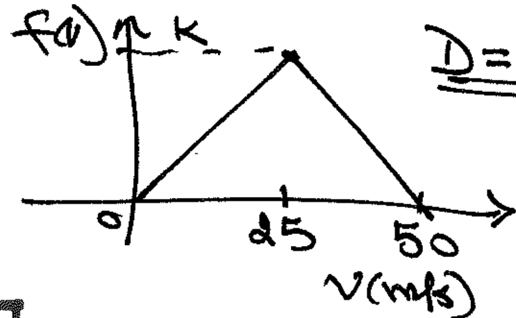
RLC → 5

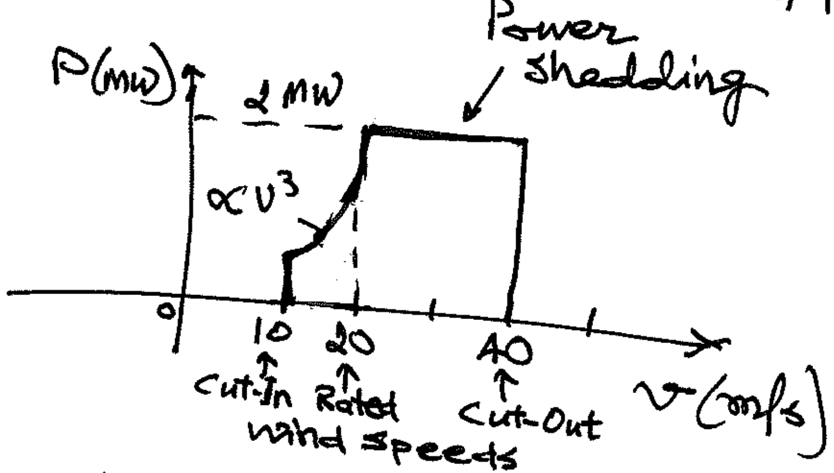
ABCD — S: 2.5
M: 5
L: 7.5

} $I_{charging}$] 5

④ Wind power

$D = 50\,m$





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(a) Calculate K

(b) " $(V_{av})^3$ and $(V^3)_{av}$. Compare.

(c) P_{av} in wind assuming 15°C and 1 ATM

(d) Energy delivered by wind turbine in a year assuming efficiency @ V=20 m/s valid from V=10 to 20 m/sec.

(e) Maximum energy wind turbine can deliver in a year.

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⑤ Solar Energy

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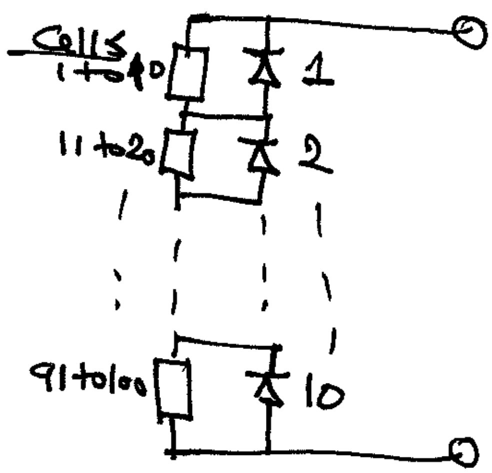
100 solar cells are connected in series to form a module. A bypass diode is connected to a sub-module consisting of 10 cells in series (to save cost).



$T = 50^\circ C$

$I_{sc} = 4\,A$, $I_0 = 10^{-9}\,A$, $V_d = 0.5\,V$, $R_p = 10\,\Omega$, $R_s = 0.01\,\Omega$

$V_{d\,bypass} = 0.6\,V$

Calculate

I, V_{module} and Power for

(a) Healthy system

(b) Two shaded cells covered by the same bypass diode

(i) without bypass diode

(ii) with " "

5(c) Two shaded cells, each belonging to different sub-modules i.e. they do not share the same bypass diode

(i) Without bypass diode

(ii) with " "

Hint: Answers to b(i) and c(i) are the same,

$V = \frac{100}{\sqrt{2}}\angle 0$

(1) $V = I\left(R + j\omega L - \frac{j}{\omega C}\right)$ $\quad$ $R = 1\Omega,\ L = 10^{-3} H,\ C = 10^{-5} F$

$\therefore$ (i) $\omega = 100$ rad/sec

$$I = \frac{\frac{100}{\sqrt{2}}\angle 0}{1 + j\,100\times 10^{-3} - \frac{j}{100\times 10^{-5}}} = \frac{70.71\angle 0}{1 + j(0.1 - 10^{3})} \left(= \frac{70.71\angle 0}{1 - j\,999.9} \right)$$

$\therefore$ $I = 7.0725\times 10^{-5} + j\,7.071768\times 10^{-2} = 0.070718\angle 89.943^{\circ}$ | $i(t) = .10001\sin(\omega t + 89.943^{\circ})$

$S = VI^{*} = (70.71\angle 0)(7.0725\times 10^{-5} + j\,7.072\times 10^{-2}) = 5.001\times 10^{-3} - j\,5.0005$ $(P + jQ)$

$P \uparrow$ $\quad$ $\uparrow Q$

(ii) $\omega = 10000$ rad/sec

$$I = \frac{70.71\angle 0}{1 + j\,10^{4}\times 10^{-3} - \frac{j}{10^{4}\times 10^{-5}}} = \frac{70.71\angle 0}{1 + j10 - j10} = \frac{70.71\angle 0}{1}$$

$\therefore$ $I = 70.71\angle 0$ Amps $\Rightarrow i(t) = 100\sin(\omega t)$ A

$S = VI^{*} = 70.71\angle 0 * 70.71\angle 0 = 5\times 10^{3} + j0 = P + jQ$

The low frequency ($\omega = 100$) increased the capacitive reactance to reduce the current magnitude with virtually a 90° leading current. As frequency increased ($\omega = 10000$), it coincided with L-C series resonance and reduced impedance down to its resistance value enabling source to deliver real power at unity p.f.

9/10

1(b) $\mathcal{R} = \dfrac{g}{\mu_0 A} + \dfrac{l_{Fe}}{\mu_0 \mu_r A}$

$g = 1\text{ mm} = 10^{-3}\text{ m}$

$A = 2\text{ cm}^2 = 2\times10^{-4}\text{ m}^2$

$l_{Fe} = (2\times\pi\times10 - 0.1)\times10^{-2}\text{ m}$

$\mu_r = 1000$

$\mu_0 = 4\pi\times10^{-7}\text{ H/m}$

$\omega = 300\text{ rad/sec}$

$= \dfrac{10^{-3}}{\mu_0\times2\times10^{-4}} + \dfrac{0.2\pi - 10^{-3}}{\mu_0\times10^{3}\times2\times10^{-4}}$

$\therefore \mathcal{R} = \dfrac{1}{\mu_0}\left[5 + \pi - 5\times10^{-3}\right] = \left(\dfrac{4.995+\pi}{\mu_0}\right)$

$L = N^2 P = (1000)^2 \times \dfrac{4\pi\times10^{-7}}{(4.995+\pi)} = \dfrac{0.4\pi}{4.995+\pi} = \underline{0.15444\text{ H}}$

$X_m = \omega L = 300 * L = \dfrac{120\pi}{4.995+\pi} = \underline{46.3328\ \Omega}$

$V_p = \frac{100}{\sqrt2}\angle 0 = 70.71\angle 0\text{ Volts}$

(i) $\therefore$ $I_{magnetizing} = \dfrac{70.71}{j46.3328} = \underline{-j1.5261473\text{ Amps}}$ $\boxed{i_m = 2.1583\sin(\omega t - 90^\circ)}$ primary side

(ii) $V_p = \frac{100}{\sqrt2} = 70.7\text{ Volts}$, $\dfrac{N_p}{N_s} = 0.2 \Rightarrow V_s = \frac{500}{\sqrt2} = \underline{353.55\text{ Volts}}$ secondary side

$\underline{Z_L = R_L = 10\ \Omega}$ $\Rightarrow$ $\underline{i_s = \dfrac{500/\sqrt2}{10} = \dfrac{50}{\sqrt2}\text{ Amps}} = \underline{\underline{35.35534\text{ A}}}$ $\boxed{i_s(t) = 50\sin(\omega t)}$

Then $i_s' = \dfrac{i_s}{a} = \dfrac{50/\sqrt2}{0.2} = 250/\sqrt2\text{ Amps} = \underline{176.7767}\text{ Amps}$

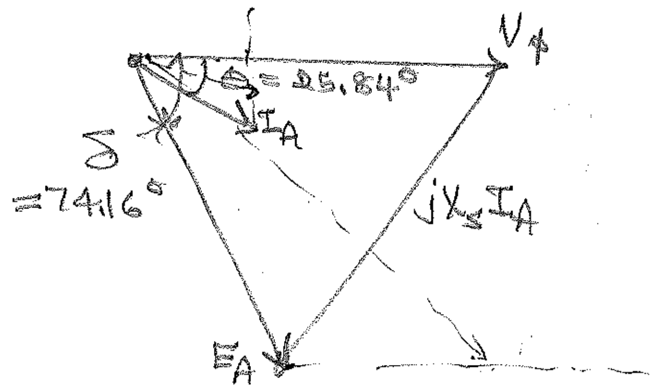
$\therefore$ $i_p = \left(\dfrac{250}{\sqrt2} - j1.52447\right) = \underline{176.78833\ \angle{-0.4946^\circ}}$

$\boxed{i_p(t) = \underline{250.00932}\sin(\omega t - 0.4946^\circ)}$

primary side

② 300 kW, 480 V, Y-connected, $X_S = 1\,\Omega$, $R_A \simeq 0$, 8 Pole, 60 Hz
(Negligible losses) Syn. motor

(a) Rated load @ 0.9 lagging pf



$\bar{E}_A = \bar{V}_\phi - jX_S \bar{I}_A$
$= 160\sqrt{3}\angle 0 - \frac{1 \times 1000}{1.44\sqrt{3}}\angle 90 - \cos^{-1}(.9)$
$= 375.08\angle -74.1625$

$V_\phi = \frac{480}{\sqrt{3}}\angle 0 = 277.1\angle 0$

$I_A = \frac{300 \times 10^3}{\sqrt{3} \times 480 \times 0.9}\angle -\cos^{-1}(.9) = \frac{1000}{1.44\sqrt{3}}\angle -\cos^{-1}(.9) = 400.94\angle -25.84°$

Torque $= \frac{P}{\omega_m}$, $\omega_m = \frac{\pi}{30} * \frac{f_e \times 120}{P} = \frac{\pi}{30} \times \frac{60 \times 120}{8} = 30\pi$ rad/sec

$= \frac{300 \times 10^3}{30\pi} = \frac{10^4}{\pi} = 3183.1$ N-m

$T_{max} = \frac{\text{Torque}}{\sin(74.16)} = 3308.7$ N-m

(b) $|E_A|$ increased by 20%

Since $P = \frac{3V_\phi \cdot E_A}{X_S} \cdot \sin\delta$ is a constant, $E_{A_1}\sin\delta_1 = E_{A_2}\sin\delta_2$

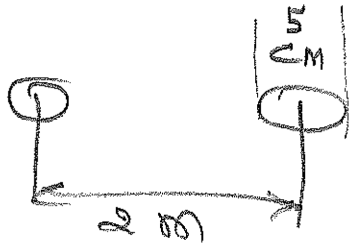
$\therefore \sin\delta_2 = \left(\frac{E_{A1}}{E_{A2}}\right)\sin\delta_1 = \frac{\sin(74.16)}{1.2} = 0.8017 \Rightarrow \delta_2 = 53.293°$

$\bar{I}_A = \frac{\bar{V}_\phi - \bar{E}_A}{jX_S} = \frac{277.13\angle 0 - 375.08 \times 1.2\angle -53.293°}{1\angle 90} = 360.9346\angle -1.285°$

Torque remains the same $= 3183.1$ N-m

$T_{max} = \frac{\text{Torque}}{\sin(53.293)} = 3970.44$ N-m $\rightarrow$ an improvement as δ reduced!

(3)



$\rho_{20^\circ C} = 2\times10^{-8}\ \Omega\text{-m}$, $M = 250$

$R_{AC} = (1.05)R_{DC}$, $f = 60\text{ Hz}$, $V_s = 8\text{ kV}$

Length $= 100\text{ km}$, $T = 40^\circ C$

$= \underline{10^5\text{ m}}$

(a) R, L, C

Resistance

$\rho_{40^\circ C} = \dfrac{250+40}{250+20} \times (2\times10^{-8})\ \Omega\text{-m}$ ← $\rho_{20^\circ C}$

$= \dfrac{58}{27}\times10^{-8}\ \Omega\text{-m}$

$$R_{DC} = \frac{\frac{58}{27}\times10^{-8}\times100\times10^{3}}{\frac{\pi}{4}(5\times10^{-2})^2}$$

$$= \frac{58\times1.6}{27\pi}\ \Omega = 1.094\ \Omega \Rightarrow R_{AC} = (1.05)R_{DC} = \underline{1.15\ \Omega}$$

Inductance

$$L = \frac{\mu}{\pi}\left[\frac{1}{4} + \ln\left(\frac{D}{r}\right)\right]\times10^5\ \text{H}$$

$\left\{\begin{array}{l} D = 200\text{ cm} \\ r = 5/2 = 2.5\text{ cm}\end{array}\right.$

$$= \frac{4\cancel{\pi}\times10^{-7}\times10^5}{\cancel{\pi}}\left[0.25 + \ln\left(\frac{2}{.025}\right)\right]$$

$$= 0.04[0.25+\ln(80)] = \underline{0.185281\ \text{H}}$$

Capacitance

$$C = \frac{\pi\epsilon}{\ln(D/r)}\times l = \frac{\pi\times8.854\times10^{-12}}{\ln(80)}\times10^5 = \underline{6.35\times10^{-7}\ \text{F}}$$

(b) Short line $\underline{A=1}$, $B = Z = R + jX_L$, $\underline{C=0}$, $\underline{D=1}$

$= 1.15 + j120\pi\times0.185281$

$= 1.15 + j69.85 = \underline{69.86\angle 89.06^\circ}$

Medium line $A = D = \dfrac{ZY}{2} + 1 = \dfrac{69.86\angle 89.06 \times 2.392\times10^{-4}\angle 90}{2} + 1$

$= \underline{0.991642\angle 0.008^\circ}$

$Y = j\omega C$

$= j120\pi\times6.35\times10^{-7}$

$= j2.393\times10^{-4}$

$$B = Z = 69.86\angle 89.06^\circ$$

$$C = Y\left(\frac{ZY}{4}+1\right) = j2.393\times10^{-4}\left[\frac{69.86\angle 89.06^\circ * 2.393\times10^{-4}\angle 90}{4}+1\right]$$

$$= 2.383\times10^{-4}\angle 90.004^\circ$$

Long line

$$A = D = \cosh(\gamma \ell) \qquad \gamma = \sqrt{z\cdot y}$$

$$B = Z_c \sinh(\gamma \ell) \qquad Z_c = \sqrt{\frac{z}{y}}$$

$$C = \frac{1}{Z_c}\sinh(\gamma \ell) \qquad \ell = 100 \text{ km}$$

$$z = 0.6986\angle 89.06^\circ \ \Omega/\text{km} \qquad \gamma = \sqrt{0.6986\angle 89.06 * 2.393\times10^{-6}\angle 90}$$

$$y = 2.393\times10^{-6}\angle 90 \ \text{S/km} \qquad = 1.0631\times10^{-5} + j1.29291\times10^{-3}$$

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{291888.283 - j4800.4019} \qquad = 1.292954\times10^{-3}\angle 89.5285^\circ$$

$$= \sqrt{291927.7542}\angle -0.9422^\circ/2 = 540.3034\angle -0.4711^\circ$$

$$A = D = \cosh(1.0631\times10^{-3} + j1.29291\times10^{-1})$$

$$\cosh(\gamma \ell) = \frac{e^{\gamma \ell}+e^{-\gamma \ell}}{2} = \frac{e^{(\alpha+j\beta)}+e^{-(\alpha+j\beta)}}{2} \qquad \alpha = 1.0631\times10^{-3}, \ \beta = 1.29291\times10^{-1}$$

$$= \frac{(e^{\alpha}+e^{-\alpha})}{2}\cos\beta + j\frac{(e^{\alpha}-e^{-\alpha})}{2}\sin\beta = 0.991654 + j1.371\times10^{-4}$$

$$\& \ \sinh(\gamma \ell) = 1.05422\times10^{-3} + j0.12893$$

$$B = Z_c\sinh(\gamma \ell) = Z_c\left[\frac{e^{\alpha}-e^{-\alpha}}{2}\cos(\beta) + j\frac{e^{\alpha}+e^{-\alpha}}{2}\sin\beta\right] = 1.14235 + j69.655$$

$$C = \frac{1}{Z_c}\sinh(\gamma \ell) = -1.09455\times10^{-8} + j2.3864\times10^{-4}$$

Line charging current (I_S) with $I_R = 0$, $V_S = 8\,kV$

$$V_S = A \cdot V_R + B \cancel{I_R}^{0}$$
$$I_S = C \cdot V_R + D \cdot \cancel{I_R}^{0} \Rightarrow \boxed{I_S = C \cdot V_R = \frac{C}{A} \cdot V_S}$$

(i) Short Line $C = 0$, $A = 1$ $\Rightarrow$ $I_S = 0$

(ii) Medium Line $C = 2.383 \times 10^{-4} \angle 90.004^\circ$, $A = 0.991642 \angle 0.008^\circ$

$$\therefore I_S = \frac{2.383 \times 10^{-4} \angle 90.004^\circ}{0.991642 \angle 0.008^\circ} \times 8000 = 1.338 \times 10^{-4} + j1.92248$$
$$= 1.92248 \angle 89.996^\circ \text{ A}$$

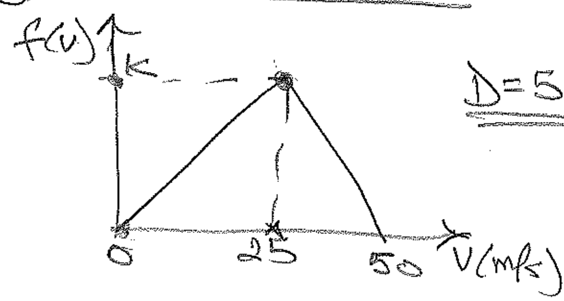
(iii) Long Line

$$C = -1.09455 \times 10^{-8} + j2.3864 \times 10^{-4}$$
$$A = 0.991654 + j\,1.371 \times 10^{-4}$$

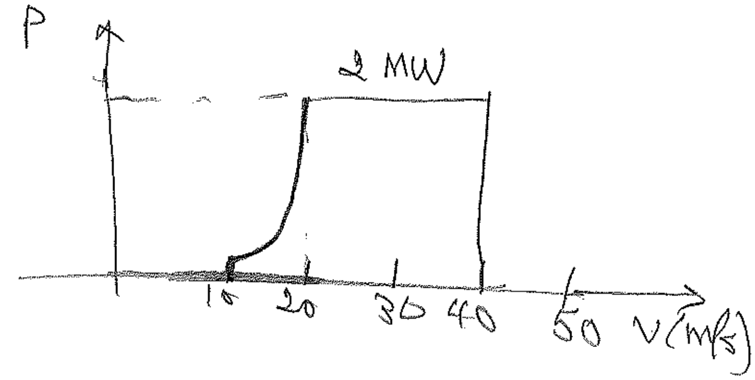
$$\therefore I_S = \frac{C}{A} V_S = 1.778 \times 10^{-4} + j1.92515 = 1.92515 \angle 89.9947^\circ \text{ A}$$

Note: Medium & Long Line models give similar results.

(4) Wind power



$D = 50\,m$



(a) K

$$f(V) = \frac{K}{25}V \quad 0 \le V \le 25$$
$$= \frac{K(50-V)}{25} \quad 25 \le V \le 50$$

$$1 = \int_0^\infty f(v)\,dv = \int_0^{25} \frac{K}{25} V\,dV + \int_{25}^{50} \frac{K}{25}(50-V)\,dV$$

$$\therefore 1 = \frac{K}{25}\left(\frac{25^2}{2}\right) + \frac{K}{25}\left[50\{50-25\} - \frac{1}{2}(50^2 - 25^2)\right]$$

$$\therefore 1 = 12.5K + K[50 - 37.5] = 25K$$

$$\therefore K = 0.04 \;(= 1/25)$$

(b) $(V_{av})^3$ and $(V^3)_{av}$

$$V_{av} = \int_0^\infty V f(v)\,dv = \int_0^{25} \frac{K}{25} V^2\,dV + \int_{25}^{50} \frac{K}{25}(50V - V^2)\,dv$$

$$= \frac{K}{25}\left[\frac{25^3}{3} + \frac{50(50^2 - 25^2)}{2} - \frac{50^3 - 25^3}{3}\right] = 25 \text{ m/sec}$$

$$\therefore (V_{av})^3 = (25)^3 = 15{,}625 \;(\text{m/sec})^3$$

$$(V^3)_{av} = \int_0^\infty V^3 f(v)\,dv = \frac{K}{25}\left[\int_0^{25} V^4\,dV + \int_{25}^{50}(50V^3 - V^4)\,dV\right]$$

$$= \frac{K}{25}\left[\frac{25^5}{5} + \frac{50(50^4 - 25^4)}{4} - \frac{50^5 - 25^5}{5}\right] = 1.5 \times 25^3 \;(\text{m/s})^3$$

$$23{,}437.5 \;(\text{m/s})^3$$

$(V^3)_{av}$ is greater by 1.5 compared to $(V_{av})^3$!

(c) P_{av} wind

$$P_{av} = \frac{1}{2}\rho A (V^3)_{av} = \frac{1}{2} \times 1.225 \times \frac{\pi}{4}(50)^2 \times 1.5 \times 25^3 \times 10^{-6} \text{ MW}$$

$$= 28.187 \text{ MW!!}$$

(d) Energy delivered by wind turbine in a year

$$P_{wind\ @20m/s} = \frac{1}{2}\rho A V^3 = \frac{1}{2}\times 1.225 \times \frac{\pi}{4}(50)^2 \times 20^3 \times 10^{-6}$$
$$= 9.621 \text{ MW}$$

$\therefore$ Efficiency @ 20 m/s $= \frac{2}{9.621}\times 100 = 20.7876\,\%$

$$P_{wind\,(10\le V\le 20)} = \frac{1}{2}\rho A (V^3)_{av\,(10\le V\le 20)}$$
$$= \frac{1}{2}\times 1.225 \times \frac{\pi}{4}(50)^2 \times 992 \times 10^{-6}$$
$$= 1.19302 \text{ MW}$$

$$(V^3)_{av\,(10,20)} = \int_{10}^{20} V^3 f(v)\,dv = \frac{k}{25}\int_{10}^{20} V^4\,dV$$

$$= \frac{k}{25}\left(\frac{20^5 - 10^5}{5}\right) = 992$$

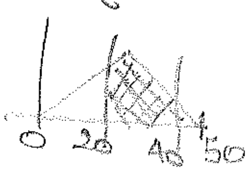
Efficiency $= 20.7876\,\%$

$\therefore$ $P_{WT} = 0.248$ MW $(10 \le V \le 20)$ ← (av. P_{WT} from 10 to 20 m/s wind)

$$\text{Prob.}(10 \le V \le 20) = \int_{10}^{20} f(v)\,dv = \frac{k}{25}\left(\frac{20^2 - 10^2}{2}\right) = 0.24$$

and

$$\text{prob.}(20 \le V \le 40) = \int_{20}^{40} f(v)\,dv = \int_{20}^{25}\frac{k}{25}V\,dv + \int_{25}^{40}\frac{k}{25}(50-V)\,dv$$



$$= \frac{k}{25}\left[\frac{25^2 - 20^2}{2} + 50(40-25) - \frac{(40^2 - 25^2)}{2}\right]$$
$$= 0.6$$

$\therefore$ $P_{av\,(WT)} = 0.248 \times 0.24 + 2 \times 0.6 = 1.25952$ MW

$\therefore$ Energy delivered by wind turbine in a year

$$= P_{av\,(WT)} \times 8760 = 11{,}033.4 \text{ MW-hr}$$

(e) Maximum energy wind turbine can deliver in a year $= 2 \text{ MW} \times 8760 = 17{,}520$ MW-hr

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(5) Solar Energy

100 solar cells in series
1 bypass diode every 10 cells

$T = 50^{\circ}C$

$I_{sc} = 4$ A
$I_o = 10^{-9}$ A
$V_d = 0.5$ V
$R_p = 10\,\Omega$
$R_s = 0.01\,\Omega$

(a) I, V_{module}, Power – Healthy system

$$I = I_{sc} - I_d - \frac{V_d}{R_p}$$
$$= 4 - .062381 - \frac{0.5}{10}$$
$$= \underline{3.88762 \text{ A}}$$

$$I_d = I_0\left[e^{\frac{qV_d}{kT}} - 1\right]$$

$$\frac{qV_d}{kT} = \frac{1.602\times10^{-19}\times0.5}{1.381\times10^{-23}\times323.15} = 17.949$$

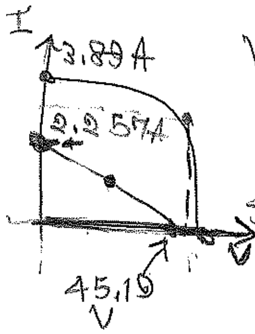
$\therefore I_d = 0.062381$

$$V_{module} = 100\left[0.5 - 3.88762\times0.01\right] = \underline{46.11238 \text{ V}}$$

$P = I \cdot V_{module} = \underline{179.27}$ Watts (Ideal value $4\times50 = 200$ W)

(b) Two shaded cells covered by the same bypass diode ($V_d = 0.6$V)

(i) without bypass diode, if $I = 3.88762$ were drawn



$$V = \frac{98}{100}\times46.11238 - 3.88762\left[2(10+.01)\right]$$
$$= 45.19 - 77.83 = -32.64\text{V} \leftarrow \text{infeasible!}$$

Since this is not possible as V can at most go to zero, the current will reduce to 2.257 A $\leftarrow$ to $\left(\frac{45.19}{2(10+.01)}\right)$ render V=0. Max Power $= \frac{2.257\times45.19}{4} = \underline{25.5 \text{ W}}$
($I = 1.1285$A, $V = 22.595$V)

(ii) with bypass diode, only 90/100 cells are in operation & the V_d bypass drop.

$$V = \frac{90}{100}\times46.11238 - 0.6 = \underline{40.90\text{V}}$$

$I = \underline{3.88762 \text{ A}}$ $\Rightarrow$ $\underline{P = 159.01 \text{ W}}$

(c) (i) $\rightarrow$ Same as b(i)

(ii) With two bypass diodes in action only 80/100 cells are in operation as two sets of 10 cells are bypassed.

$$V = \frac{80}{100} \times 46.11238 - 2 \times 0.6 = 35.69\ V$$

$$I = 3.88762\ A \Rightarrow P = 138.75\ W$$