Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 2500: Engineering Probability, Spring 2023

Homework #6: due Wednesday, March 29th, at 11:59PM.
Show all work for full credit!
Submit your work as a single PDF on Gradescope, labeling each problem number with a page.
While you can check your answers with Wolfram Alpha, remember that you're going to have to do integrals by hand on the exam!

1. (25 points.) Consider the following PDF from Homework 4:

$$f_X(x) = \begin{cases} \frac{3}{35}x^2 & x \in [-2,3] \\ 0 & \text{otherwise} \end{cases}$$

   (a) (8 points.) Compute $E(X)$.

   (b) (8 points.) Compute $\text{Var}(X)$.

   (c) (9 points.) Compute $E\left(e^{X^3}\right)$.

2. (30 points.)

   Let $Y = 7X^2 - 1$, where X is the same random variable from Problem 1.

   (a) (10 points.) Compute $E(Y)$, based only on your answers from Problem 1.

   (b) (20 points.) Compute the CDF of Y. Remember we computed the CDF of X in HW4, which you can use. Note this is a little tricky; think about how the values of X map to values of Y. Don't forget to specify the ranges of values where each "piece" of the CDF is valid.

3. (20 points.) Let X be a random variable representing the length of an episode of Andor, in minutes. Initially we don't know anything about the PDF of X, other than that it is symmetric about its mean.

   (a) (7 points.) Suppose we learn that $E(X) = 50$. Compute the Markov bound on $P(X \geq 55)$.

   (b) (8 points.) We now further learn that $\text{Var}(X) = 16$. Compute the Chebyshev bound on $P(|X - 50| \geq 5)$, and then divide this by two to get an estimate of $P(X \geq 55)$ since we know the distribution is symmetric.

   (c) (5 points.) Finally, we learn the complete distribution of X: it's actually a Gaussian with the mean and variance given above. Compute the actual value of $P(X \geq 55)$. How does it compare to the bounds above?

More fun on the next page ⟶

4. (25 points.) A sandworm on Arrakis has a circular cross-section with radius R, where R is modeled as a uniform random variable on $[15,25]$ meters.

   (a) (15 points.) The length of the sandworm is $L = 400$ meters. Let V be the (random) volume of the sandworm (remember, this is equal to its length times its cross-sectional area). Determine the PDF of V.

   (b) (10 points.) The amount of spice S in kilograms generated by the sandworm is related to its volume V by the equation $S = \left(\frac{V}{\pi}\right)^{\left(\frac{1}{2}\right)}$. Compute the PDF of S.

   As above, don't forget to specify the ranges of values where each piece of the PDF is valid.