

Homework # 6**Due: Tuesday, August 1st**

Question 1. (10 points) Explain how power system equipment and buildings are protected against lightning strokes.

Question 2. (10 points) Why the power flow analysis involves with non-linear equations?

Question 3. (40 points) Using Gauss-Seidel method, find the root of the following equation. Use initial guess of the root as 0. Stop iterations, if the solution does not seem to converge in 5 iterations.

$$f(x) = 2x^3 - 3x^2 + 5x - 2 = 0$$

Question 4. (40point) Use Newton-Raphson method and hand calculations to find the solution of the following equations.

$$x_1^2 - 2x_1 - x_2 = 3$$

$$x_1^2 + x_2^2 = 41$$

Start with the initial estimates of $x_1^{(0)} = 2$ and $x_2^{(0)} = 3$. Perform three iterations

1)

Transmission lines are often protected by a grounded shield wire. Buildings are often protected by lightning arresters. Power stations can be protected with a "net" of shield wires, arresters, or both.

2)

Because $P = V_1 V_2 / X_{12} \sin(\theta_1 - \theta_2)$ and $Q = V_1 V_2 / X_{12} \cos(\theta_1 - \theta_2) - V_2^2 / X_{12} \cos(\phi)$, and because of the trig functions, we know $YV = (P + jQ) / V^*$ is a nonlinear function. Power depends on sin and cos of voltage angle and shunts depend on V_i^2 . These are extremely difficult to solve purely analytically, so we instead use iterative methods.

3) $2x^3 - 3x^2 + 5x - 2 = 0$

$-5x = 2x^3 - 3x^2 - 2$

$x = -2/5x^3 + 3/5x^2 + 2/5$

$x = -.4x^3 + .6x^2 + .4$

$x(0) = 0$

$x(1) = -.4x(0)^3 + .6x(0)^2 + .4 = .4$

$x(2) = -.4x(1)^3 + .6x(1)^2 + .4 = .4704$

$x(3) = -.4x(2)^3 + .6x(2)^2 + .4 = .4911$

$x(4) = -.4x(3)^3 + .6x(3)^2 + .4 = .4973$

$x(5) = -.4x(4)^3 + .6x(4)^2 + .4 = .4992$

$x(6) = -.4x(5)^3 + .6x(5)^2 + .4 = .4997$

$x(7) = -.4x(6)^3 + .6x(6)^2 + .4 = .4999$

$x = .5$

$-.4(.5)^3 + .6(.5)^2 + .4 = .5$

4)

I used Matlab notation because that's how I learned how to do this using an example from class, but I did these calculations on TI-84 (I assume that counts as "hand calculations" because I can't do $[6.7857, -1; 8.7857, 9.8571]^T - 1 [-2.5829; -2.5880]$ without a calculator)

$f = [x_1^2 - 2x_1 - x_2 = 3; \quad \text{upwards opening parabola}$

$x_1^2 + x_2^2 = 41] \quad \text{circle}$

$J = [2x_1 - 2, -1;$

$2x_1, 2x_2]$

$x_0 = [2; 3]$

$Dx_0 = [1; 1]$

$C = [3; 41]$

$k = 0$

$f_0 = [x_1^2 - 2x_1 - x_2; x_1^2 + x_2^2] = [-3; 13]$

$Dc_0 = C - f_0 = [6; 28]$

$J_0 = [2x_1 - 2, -1; 2x_1, 2x_2] = [2, -1; 4, 6]$

$Dx_0 = J_0 \backslash Dc_0 = [4; 2]$

$x_1 = x_0 + Dx_0 = [6; 5]$

$k = 1$

$f_1 = [x_1^2 - 2x_1 - x_2; x_1^2 + x_2^2] = [19; 61]$

$Dc_1 = C - f_1 = [-16; -20]$

$J_1 = [2x_1 - 2, -1; 2x_1, 2x_2] = [10, -1; 12, 10]$

$Dx_1 = J_1 \backslash Dc_1 = [-1.6071; -0.0714]$

$x_2 = x_1 + Dx_1 = [4.3929; 4.9286]$

$k = 2$

$f_2 = [x_1^2 - 2x_1 - x_2; x_1^2 + x_2^2] = [5.5829; 43.5880]$

$Dc_2 = C - f_2 = [-2.5829; -2.5880]$

$J_2 = [2x_1 - 2, -1; 2x_1, 2x_2] = [6.7857, -1; 8.7857, 9.8571]$

$Dx_2 = J_2 \backslash Dc_2 = [-0.3706; 0.0678]$

$x_3 = x_2 + Dx_2 = [4.0222; 4.9964]$

$k = 3$

$f_3 = [x_1^2 - 2x_1 - x_2; x_1^2 + x_2^2] = [3.13374; 41.1420]$

$Dc_3 = C - f_3 = [-0.13374; -0.1420]$

$J_3 = [2x_1 - 2, -1; 2x_1, 2x_2] = [6.0444, -1; 8.0444, 9.9928]$

$Dx_3 = J_3 \backslash Dc_3 = [-0.0221; 0.0036]$

$x_4 = x_3 + Dx_3 = [4.0001; 5.0000]$

$k = 4$

$f_4 = [x_1^2 - 2x_1 - x_2; x_1^2 + x_2^2] = [3.0005; 41.0005]$

that's converged enough for me to say $x_1 = 4$ and $x_2 = 5$