Microelectronics Technology S 2024 Crib Sheet Exam 1+2 Hayden Fuller

Basic:

n-type, majority e minority h, donors, 5 electron, P, As, Sb, p-type, majority h minority e, acceptors, 3 electron, B, Al, Ga, In

$p_n$ holes in n side, minority

$1eV = 1.6 \times 10^{-19} J$

$k = 1.38 \times 10^{-23} J/K = 8.6 \times 10^{-5} eV/K$ , $kT = 0.025eV$

$E_{G,Si} = 1.12eV$

$n_i = 10^{10}$ , $n_i^2 = np$ , $n = n_i e^{(E_F - E_i)/kT}$ , $p = n_i e^{(E_i - E_F)/kT}$

$p - n + N_D - N_A = 0$ , $n^2 - n(N_D - N_A) - n_i^2 = 0$

$N_D > N_A \;\; => \;\; n = N_D - N_A$ ; $p = n_i^2/n \;\; N_D \approx N_A \;\; => \;\; n = p = n_i$

Band diagrams: n-type: $E_C, E_F, E_i, E_v$ , p-type: $E_C, E_i, E_F, E_v$

Point defect: one atom missing. Electron generation: one electron missing

Electron moving: breaks off and moves. Hole moving: electron line rotates into hole.

effective mass:

Fermi function $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$ , Steps from 1 to 0 at $E_F$ at 0K, smoothe at temp.

$E_F = 1 - e^{\frac{E-E_F}{kT}} = \frac{E_C + E_V}{2}$ in intrinsic

Distribution of carriers = distribution of states * probability of occupancy = $g(E)f(E)$

Conduction band electrons: $n_0 = \int_{E_C}^{E_top} g_C(E) f(E) dE$ , holes in VB: $p_0 = \int_{E_bottom}^{E_v} g_V(E)(1 - f(E))dE$

total free electron concentration $3kT$ away from edges (non-degenerate): $n = N_c e^{-\frac{E_C - E_F}{kT}}$ , hole: $p = N_v e^{-\frac{E_F - E_V}{kT}}$

where effective density of states $N_C = 2.8 \times 10^{19} cm^{-3}$ and $N_C = 1 \times 10^{19} cm^{-3}$ , $3kT$ around $N_{AorD} = 2 \times 10^{17}$

Drift: caused by electric field, drift velocity $v_d = \mu_p E \;\;\; cm/sec = cm^2/Vs * V/cm$

$I = Q/T$ , $J_{P|drift} = I/A = qp\mu_p E = \frac{E}{\rho}$

resistivity: $\rho = 1/(1p\mu_p + qn\mu_n)$

resistivity measurement: 4 point probe, eddy current apparatus

Diffusion: random thermal mothion, high to low concentration, must be a concentration gradient

Flux $F = -D\frac{d\eta}{dx}$ , $\eta =$ particle concentration, $D =$ diffusion coefficient

holes/electrons go high to low,that's flux, but diffusion current is negative for electrons

$J_{p|diff} = -qD_p \frac{dp}{dx}$ , $J_{n|diff} = qD_n \frac{dn}{dx}$

$J_p = J_{p|drift} + J_{p|diff} = q\mu_p pE + -qD_p \frac{dp}{dx}$ , $J_n = J_{n|drift} + J_{n|diff} = q\mu_n nE + qD_n \frac{dn}{dx}$ , $J = J_n + J_p$

Band bending: electric field bends the band diagram

$KE = E - E_C$ , $PE = E_C - E_{ref} = -qV$ (for electrons), $V = -(E_C - E_{ref})/q$ , $E = -\frac{dV}{dx} = \frac{dE_{C,V,i}}{dx}/q$

Hot point measurement: Hot end makes particles move away.

p-type: holes move away, current goes out hot probe. n-type: electrons move away, current goes into hot probe

in thermal equilibrium: $E_F$ is constant, net current $J_{p|drift} + J_{p|diff} = 0$ , recombination and generation cancel

Einstein: $J_{n|drift} + J_{n|diff} = q\mu_n nE + qD_n \frac{dn}{dx} = 0$ , $E = \frac{dE_i}{dx}/q$ , $n = n_i e^{(E_F - E_i)/kT}$

electrons: $\frac{D_n}{\mu_n} = \frac{kT}{q}$ , holes: $\frac{D_p}{\mu_p} = \frac{kT}{q}$

recombination:

band to band recombination gives off light, band to band generation through thermal and light absorption, RG center is indirect-middle step

auger recombination, electron drops, and gives another electron KE. Impact ionization, on a slope, electron moves and falls

SI is mostly RG recombination due to impurities

direct semiconductors: k is matched so with less energy there's a photon. With a difference in k, more energy, phonon.

RG statistics:

if photon energy $hv$ is greater than band gap $E_G$ , iti's absorbed and an electron is moved up.

absorption: $I = I_0 e^{-\alpha x}$ , each photon creates an e-h pair. $\frac{dn}{dt}|_{light} = \frac{dp}{dt}|_{light} = G_L(x, \lambda) = G_{L0} e^{-\alpha x}$

$\alpha$ drops off with wavelength. Higher wavelength, lower frequency, lower energy, doesn't get absorbed

indirect thermal recombination-generation, $n_0, p_0$ under thermal equilibrium, $n, p$ as functions of t.

$\Delta n = n - n_0$ , $\Delta p = p - p_0$ , $\Delta$'s are deviations from equilibrium. $N_t$ is number of RG centers/cm $^3$

low level injection condition assumed, change in majority carrier concentration negligable, $\Delta p << n_0$ , $n \approx n_0$

$\frac{dp}{dt} = \frac{dp}{dt}|_R + \frac{dp}{dt}|_G + G_L(x, \lambda)$ , hole build up = recomb loss + gen gain + external light

$\frac{dp}{dt}|_R = -C_p N_t p$

thermal equilibrium: $\frac{dp}{dt}|_G = -\frac{dp}{dt}|R = C_p N_t p_0$

generally when $G_L = 0$ , $\frac{dp}{dt} = -\frac{\Delta p}{\tau_p}$ , minority carrier lifetime $\tau_p = \frac{1}{C_p N_t}$

$\frac{\delta \Delta p}{\delta p} = -\frac{\Delta p}{\tau_p}$

perturbation removed at $t = 0$: $\Delta p = \Delta p(0) e^{-t/\tau_p}$

$\frac{dp}{dt} = fracdpdt|_{drift} + \frac{dp}{dt}|_{diff} + \frac{dp}{dt}|_{thermalRG} + \frac{dp}{dt}|_{light/other}$

current input: holes: $\frac{dp}{dt} = \frac{1}{q}\frac{dJ_p}{dx} + \frac{dp}{dt}|_{thermalRG} + \frac{dp}{dt}|_{light/other}$ , electrons: first term is positive

Minority carrier diffusion equiations: electrons for p type, simplifications

$J_n = q\mu_n nE + qD_n \frac{dn}{dx} \approx qD_n \frac{dn}{dx}$

$\frac{dn}{dx} = \frac{d}{dx}(n_0 + \Delta n) = \frac{d\Delta n}{dx}$

$\frac{dn}{dt}|_{thermalRG} = -\frac{\Delta n}{\tau_n}$ , $\frac{dn}{dt}|_{light} = G_L$
$\frac{dn}{dt} = \frac{d}{dt}(n_0 + \Delta n) = \frac{d\Delta n}{dt}$
$\frac{d\Delta n_p}{dt} = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$
$\frac{d\Delta p_n}{dt} = D_p \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} + G_L$
Minority carrier diffusion length: $L_p = (D_p \tau_p)^{1/2}$ , average distance minority carriers can diffuse
misc
low level injection assumption, majority carriers don't change significantly
p+ n- is forward biased
$L_p$ is minority p, so n side NEW PAGE Microelectronics Technology S 2024 Crib Sheet Exam 1+2 Hayden Fuller
Equilibrium energy band diagram for pn junction $kT/q = .0256V$
$n = n_i e^{(E_F - E_i)/kT}$ , $p = n_i e^{(E_i - E_F)/kT}$ , $E_F$ low for $p$ , high for $n$
$V = (E_{ref} - E_C)/q$ , $E_{ref} - E_C = qV$ , $E = 1/q dE_C/dx = 1/q dE_i/dx$ , $\rho/\epsilon = dE/dx$ , $\epsilon = K_s \epsilon_0$
conceptual pn junction formation
p gives some positive to n and n gives some electrons to p, creating negative region in p and positive region in n
Built in voltage $V_{bi}$ , after formation net drift and diffusion currents sum to zero
E field from $nN_D$ to $pN_A$ , $V_{bi} = 1/q[(E_i - E_F)_p + (E_F - E_i)_n] = kT/q \ln(p_p n_n / n_i^2)$
$(E_i - E_F)_p = kT \ln(p/n_i)$ , $(E_F - E_i)_n = kT \ln(n/n_i)$ , $p_p/p_n = n_n/n_p = e^{V_{bi} q/kT}$
Depletion approximation
Poisson $dE/dx = \rho/(K_s \epsilon_0) = q/(K_s \epsilon_0)(N_D - N_A)$ for $-x_p < x < x_n$ , $0$ elsewhere
Quantitative analysis: E field
$dE/dx = \rho/\epsilon = -qN_A/\epsilon = qN_D/\epsilon$
$E(x) = \{-qN_A(x_p + x)/\epsilon\} - x_p < x < 0, \{-qN_D(x_n - x)/\epsilon\} 0 < x < x_n, 0 x < - - x_p, x > x_n$
Relationship between $x_n$ and $x_p$
$E_{max} = -qN_A x_p/\epsilon = -qN_D x_n/\epsilon$ , $N_A x_p = N_D x_n$ (equal net charge)
$W = x_n + x_p$ , $x_n = WN_A/(N_A + N_D)$ , $x_p = WN_D/(N_A + N_D)$ , if $N_A >> N_D$ then $W \approx x_n$ , viceversa
$E = -dV/dx$ , $V_{bi} = -\int_{-xp}^{xn} E(x)dx = N_D x_n W q/(2\epsilon) = W^2 q N_A N_D/(2\epsilon(N_A + N_D))$
$W = \sqrt{V_{bi} 2\epsilon(N_A + N_D)/(qN_A N_D)} = \sqrt{2\epsilon(N_A + N_D)(V_{bi} - V_A)/(qN_A N_D)}$
$dV/dx = \{qN_A(x_p + x)/\epsilon\}$ , $-x_p < x < 0$ , $\{qN_D(x_n - x)/\epsilon\}$ , $0 < x < x_n$
$V(x) = \{qN_A(x_p + x)^2/2\epsilon\}$ , $-x_p < x < 0$ , $\{V_{bi} - qN_D(x_n - x)^2/2\epsilon\}$ , $0 < x < x_n$
Drift due to E field n to p, holes to p, constant. Diffusion due to added minority carriers, holes to n. E
$V_A = 0$ , med E field, med diffusion currents. $V_A > 0$ , small E, large diff. $V_A < 0$ , large E, small diff
$V_A$ breaks $E_F$ , + to p, smaller gap, p side lowers, n side raises
$V_A$ up linear, $E_i$ gap down linear, carrier concentration exp dec, diffusion current incr exp with $V_A$
drift constant because limited by how often, not how fast
net $= I_{diff} - I_{drift}$ . $V_A = 0$ $I_{diff} = I_{drift} = I_0$ . $I = I_0 e^{V_A/V_{ref}} - I_{drift} = I_0(e^{V_A/V_{ref}} - 1)$
carrier concentrations under equilibrium, $carrier_{side}$ . p side minority electron $n_p$
$p_p/p_n = e^{(V_{bi} - V_A)q/kT}$ , low level injection $p_n = p_{n0} e^{V_A q/kT}$ , $n_p = n_{p0} e^{V_A q/kT}$
minority carrier concentration under bias graph
p side has $n_{p0}$ , slopes up into $n_p = n_{p0} + \Delta n_p(x'')$ for total $\Delta n_p(0)$ , $\Delta n_p(x'') = \Delta n_p(0) e^{-x''/L_n}$
$\Delta p_n(x_n) = p_n(x_n - p_{n0} = p_{n0}(e^{V_A q/kT} - 1)$ , $\Delta n_p(-x_p) = n_{p0}(e^{V_A q/kT} - 1)$
carrier injection under forward bias
x" axis $\Delta n_p(0) = n_{p0}(e^{V_A q/kT} - 1)$ , $\Delta n_p(x'') = \Delta n_p(0) e^{-x''/L_n}$
x' axis $\Delta p_n(0) = p_{n0}(e^{V_A q/kT} - 1)$ , $\Delta p_n(x') = \Delta p_n(0) e^{-x'/L_p}$
Current and minority carrier diffusion
$J_p(x) = qp\mu_p E - qD_p dp/dx$ , $J_n(x) = qn\mu_n E - qD_n dn/dx$ , simplified $J_p(x) = -qD_p dp/dx$
$\delta\Delta p/\delta t = D_p \delta^2 \Delta p/\delta x^2 - \Delta p/\tau_p + G_L$ , $\delta \Delta n/\delta t = D_n \delta^2 \Delta n/\delta x^2 - \Delta n/\tau_n + G_L$ , simplified $0 = D_p \delta^2 \Delta p/\delta x^2 - \Delta p/\tau_p$
diode: $J_p(x' = 0) = \Delta p_n(0) q D_p/L_p = p_{n0} q D_p/L_p (e^{V_A q/kT} - 1)$ and $J_n(x'' = 0) = -n_{p0} q D_n/L_n (e^{V_A q/kT} - 1)$
for total current $J = J_0(e^{V_A q/kT} - 1) = (p_{n0} q D_p/L_p + n_{p0} q D_n/L_n)(e^{V_A q/kT} - 1)$
large forward $V_A >> kT/q$ , $J = J_0 e^{V_A q/kT}$ . Large reverse $V_A << -kT/q$ , $J = -J_0$
Avalanching, Zener, RG current, if $V_A$ approaches $V_b i$ , high current. Series current, high level injection
IV Reverse- Breakdown to G-R part
IV Forward- G-R part(1/2kT) to Ideal(q/kT) to High Level Injection to Series Resistance Effect
reverse breakdown: $V_{BR} \propto 1/N_B$ , $V_{BR}$ is breakdown voltage, $N_B$ is bulk doping on lightly doped side
Avalanching: lightly doped diodes, diff current flips direction, impact ionization, one e from p to n creates more
Electric field must hit critical $E_{CR}$ . steep fall, multiplication factor $M = 1/[1 - (|V_A|/V_{BR})^m]$ , m 3 to 6
$E(x = 0) = -qN_D x_n/\epsilon_{Si} = -\sqrt{(V_{bi} - V_A) 2qN_A N_D/[\epsilon_{Si}(N_A + N_D)]}$
Breakdown when $E(0) = E_{CR}$ , $\sqrt{V_{BR} 2qN_A N_D/[\epsilon_{Si}(N_A + N_D)]}$
Zener: tunneling, wall becomes thin when tall,
$I_{R-G}$ increases with depletion layer volume $W$ increases with reverse voltage.
$I_{R-G} = -qAn_i W/2\tau_0$ where $\tau = (\tau_p + \tau_n)/2$
in forward bias: $I_{R-G} = I_0'(e^{V_A q/2kT} - 1)$ , total forward current $= I_{diff} + I_{R-G}$ , $I_{diff} = I_0(e^{V_A q/kt} - 1)$ where $I_0 = qA(D_n +$

$n_i^2/L_nN_A + D_pn_i^2/L_pN_D)$
since $I_{diff} \propto n_i^2$ grows faster than $I_{R-G} \propto n_i$ , RG is negligable in forward bias, more ideal in Ge and high temp
$V_A$ approaches $V_{bi}$ , $I \approx I_0e^{(V_A - IR_s)q/kt}$
$log(I)$ vs $V_A$ is slope $q/kT$ but veers right by $\Delta V$ . $\Delta V$ vs $I$ gives linear slope $R_s$
High level injection: when $V_A$ within $0.2V$ ish of $V_{bi}$ , $I = e^{V_Aq/2kT}$ , minority hits majority and they increase linearly together
$log(I)$ vs $V_A$ shikanes with Avalanch/Zener breakdown, thermal gen in depletion, origin, thermal recombonation in depletion, ideal q/kT in middle, high level injection q/2kT above, serries resistance above
Small signal admittance $Y = i/v_a = G + j\omega C$ , res RS to cap CD+ cap DJ + res GD
$C_j = \epsilon_{Si}A/W = A\sqrt{\epsilon_{Si}qN_B/2(V_{bi} - V_A)}$ , up with $\sqrt{N_B}$ , down with reverse bias
$W = \sqrt{2\epsilon_{Si}(N_A + N_D)(V_{bi} - V_A)/(qN_AN_D)} = \sqrt{2\epsilon_{Si}(V_{bi} - V_A)/(qN_B)}$
$1/C_J^2 = 2(V_{bi} - V_A)/(A^2qN_B\epsilon_{Si})$ , vs $V_A$ , slope first part, $= 0$ at $V_{bi}$
$C_D$ charge storage cap dominant in forward bias. $p + n$ has $I = Q_p/\tau_p$ where $Q_p$ total excess charge n side
$Q_p = I\tau_p = qAD_p\tau_pp_{n0}/L_p * [e^{V_Aq/kT} - 1] \approx qAL_pp_{n0}e^{V_Aq/kT}$
$C_D = dQ_p/dV = I\tau_pq/kT$ , $G_D = Iq/kT$
Transient response, charge $Q_p$ goes zero when turned off from current flow and recomb, $dQ_p/dt = i(t) - Q_p/\tau_p$
$Q_p = qAL_p\Delta p_n(0)$ , to maintain charge, current $I = qAL_p\Delta p_n(0)/\tau_p$ must be supplied at $x' = 0$
$Q_p(t) = I\tau_pe^{-t/\tau_p}$ , $I_F = V_F - V_{on}/R_F \approx V_F/R_F$ , $I_R = V_R + v_A(t)/R_R \approx V_R/R_R$
charge between reverse and forward curves needs to be moved, drop over time is pulled from axis
storage delay time: $dQ_p/dt = i - Q_p/\tau_p = -I_R - Q_p/\tau_p$ for $0 < t < t_s$ , $t_s = \tau_p \ln(1 + I_F/I_R)$
applications: rectifiers, low R in forward, p+ n n+ prefered, reduce parasitic resistance, low $I_0$ in reverse, High voltage breakdown, p+nn+high band gap materials
switching, fast, dope with gold to reduce lifetimes, narrow base for small stored charge
Zener, heavy dope p+ and n+ for low breakdown, reference voltage
Varactor, variable resistance, V controlled C for tuning radio or TV, $C_J \propto V_A^-1/2$ (abrupt, dope to linear)
Opto-elect, photodetect, solar cells, LED, laser diodes. PhotoD: $I_L = -qAG_L(L_N + W + L_P)$ , $I = I_{dark} + I_L$
BJT: pnp: IE in IB+IC out. npn: IB+IC in, IE out
biasing modes: B is expected to be - for pnp, Mode, EB polarity, CB polarity. Saturation, F, F. ACTIVE, F, R. Inverted, R, F. Cutoff, R, R. A S
n C I. Vert+ VEB pnp VBE npn. Horiz+ VCB pnp VBC npn
electrostatic equilibrium p+ n p EBC,
$V = -1/q(E_C - E_{ref})$ , up and flatens in B, drops to flat in C
$E = 1/qdE_C/dx = 1/qdE_i/dx$ , sharp negative triangle left B, smaller positive left C
$dE/dx = \rho/\epsilon$
forward, p+ thinB n, small E to p+, big h/e and small e/h, same thinB small E, minority e lower than minority h, both going up
reverse, n wideB p, large E to p, e/h and h/e, minorities drop off to 0
combine for p+ n p, curve up to thin, curve down and drop to wide, up to e
make B very thin, curve up to thin, drop to zero for rev bias, back up a bit, D and CS $I = \alpha I_E$ , B has $I = (1 - \alpha)I_E$
emitter efficiency $\gamma = I_{EP}/(I_{EP} + I_{EN}) = I_{EP}/I_E$
base transport factor $\alpha_T = I_C/I_{Ep}$
$I_C = \alpha_TI_{EP} = \alpha_T\gamma I_E = \alpha_{dc}I_E$ , $\alpha_{dc} = \alpha_T\gamma$
$I_C = \beta_{dc}I_B$ , $\beta_{dc} = \alpha_{dc}/(1 - \alpha_{dc}) = \alpha_T\gamma/(1 - \alpha_T\gamma)$
detailed quantitative analysis, assume pnp, steady state, low level, only drift and diff, no gen, one dimension, etc.
solve minority carrier diffusion equations for each of the three regions
$\delta\Delta p/\delta t = D_p\delta^2\Delta p/\delta x^2 - \Delta p/\tau_p + G_L$ , $\delta\Delta n/\delta t = D_n\delta^2\Delta n/\delta x^2 - \Delta n/\tau_n + G_L$
under steady state $G_L = 0$ , $0 = D_p\delta^2\Delta p/\delta x^2 - \Delta p/\tau_p$ , $0 = D_n\delta^2\Delta n/\delta x^2 - \Delta n/\tau_n$
for pnp base, only interested in holes (current in E and split)
$\Delta n = n - n_0$ excess carriers above equilibrium, area of excess carriers $= Q_n$ .
$X_B$ and $X_E$ flow awayfrom BE junction, $I_E = I_P - I_N \approx (qAp_{B0}D_B/L_B * e^{V_{EB}q/kT}) + (qAn_{E0}D_E/L_E * e^{V_{EB}q/kT})$
$I_P = Q_p/\tau_B$ , $I_P = Q_n/\tau_E$ . $n_E$ curve up, $p_B$ linear down, $n_C$ collecter curve up
$I_E$ broken down into $I_n = qaD_ndn/dx$ and $I_p = -qAD_pdp/dx$
$I_C = qAD_Bp_B(0)/W_B = qAp_{B0}D_B/W_B * e^{V_{EB}q/kT}$
$I_E$ made up of $I_{EP}$ and $I_{EN}$
$I_{EP} = I_c + qAW_B\Delta p_B(0)/2\tau_B \approx qAp_{B0}D_B/W_Be^{V_{EB}q/kT} + qAp_{B0}W_B/2\tau_Be^{V_{EB}q/kT}$
$I_B = qAp_{B0}W_B/2\tau_Be^{V_{EB}q/kT} + qAn_{E0}D_E/L_Ee^{V_{EB}q/kT}$ (recombination + e injection to E)
$\alpha_T = 1/[1 + (W_B/L_B)^2/2]$ , $\gamma = 1/[1 + D_En_{E0}W_B/D_Bp_{B0}L_E] = 1/[1 + D_EW_BN_B/D_BL_EN_E]$
BJT in cutoff, minority carriers drop off on E and C, zero in B.
BJT in saturation, E and C curve up, $p_{B0}$ is linear down but still high, above E below C.
Base width modulation $I_C \approx qAD_B\Delta p_B(0)/W_Be^{V_{EB}q/kT}$ , B drops to 0 at C
Early effect, CB reverse bias up, depletion width up, W down, $I_C$ up
punch through, $W$ approaches $0$ . for high reverse CB, EB barrier lowers, and large $I_C$ at high $V_{CE0}$ due to either punchthrough or avalanch