Fields and Waves I

Lecture 17

Faraday's Law
Inductance

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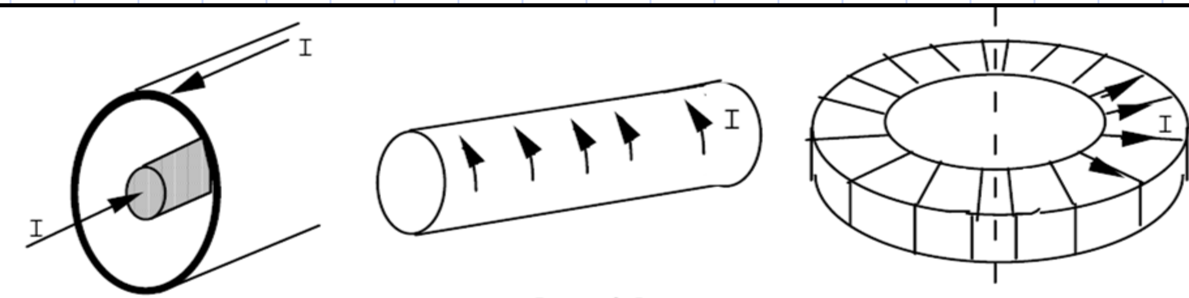
Materials from other sources are referenced where they are used.
Those listed as Ulaby are figures from Ulaby's textbook.

Overview

- Review
- Faraday's Law
- Inductance
- Applications
- Boundary Conditions
- Wrap-Up

Review



Coaxial cable I is in z direction | **Solenoid** I is in ϕ direction | **Toroid** I wraps around core

coax $\vec{B} = B_\phi(r)\,\hat{a}_\phi$
solenoid $\vec{B} = B_z(r)\,\hat{a}_z$ } ignoring end effects
torus $\vec{B} = B_\phi(r,z)\,\hat{a}_\phi$ ← assume tightly wound

Review

- In a magnetostatics problem, we start by finding a current distribution and using a geometric argument to state what direction B or H will point in, and what coordinate variables it depends on.
- What is our next step?

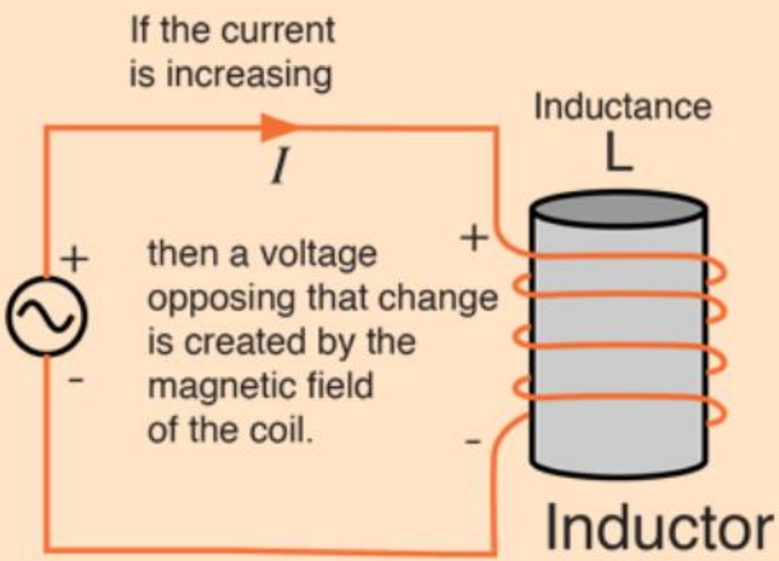
Review

Magnetic vector potential:

$$\vec{B} = \nabla \times \vec{A}$$

Vector potential will point in the same direction as current. Use the definition of curl for the appropriate coordinate system to relate B and A. Then use what you know about the direction of A to simplify the expression.

Review



Inductance is typified by the behavior of a coil of wire in resisting any change of electric current through the coil.

Arising from Faraday's law, the inductance L may be defined in terms of the emf generated to oppose a given change in current:

$$Emf = -L\frac{\Delta I}{\Delta t}$$

Unit for L: $\frac{\text{volt second}}{\text{ampere}} = \text{Henry}$

Time variation in current means that we can no longer use the electrostatic or magnetostatic version of Maxwell's Equations.

http://hyperphysics.phy-astr.gsu.edu/hbase/electric/induct.html#c1

Faraday's Law

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$$

Source: Wikimedia Commons

Faraday's Law: Changing magnetic field gives rise to an electric field.

Faraday's Law

- The Full Version of Maxwell's Equations

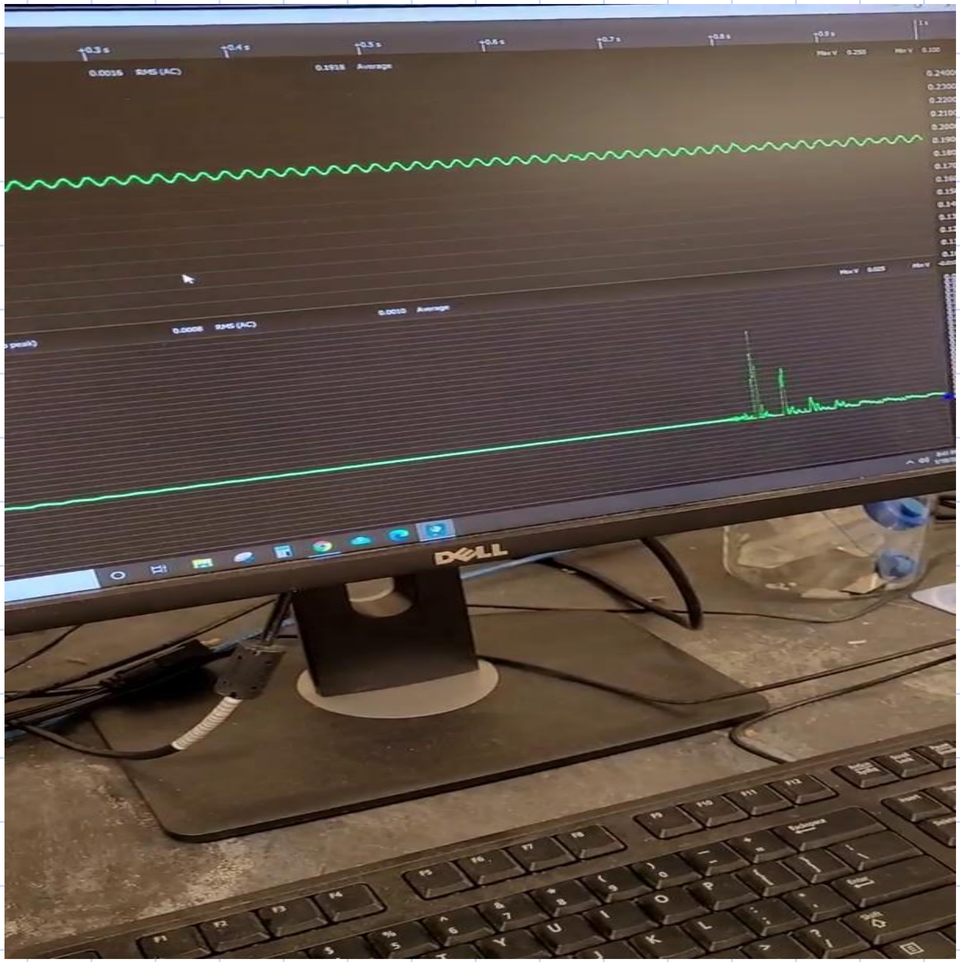
Integral Form	Differential Form
$\oint B \cdot d S = 0$	$\nabla \cdot B = 0$
$\oint D \cdot d S = \int \rho \, d v$	$\nabla \cdot D = \rho$
$\oint \vec{H} \cdot \vec{dl} = \int \vec{J} \cdot \vec{ds} + \frac{d}{dt} \int \vec{D} \cdot ds$	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
$\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int \vec{B} \cdot \vec{ds}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Faraday's Law

- The Full Version of Maxwell's Equations

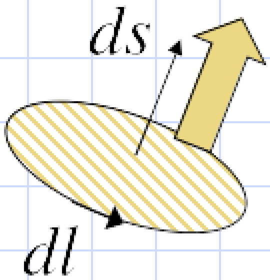
Integral Form	Differential Form
$\oint B \cdot dS = 0$	$\nabla \cdot B = 0$
$\oint D \cdot dS = \int \rho \, dv$	$\nabla \cdot D = \rho$
$\oint \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s} + \frac{d}{dt} \int \vec{D} \cdot ds$	$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{s}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Faraday’s Law



Faraday's Law

Electromotive Force (EMF)



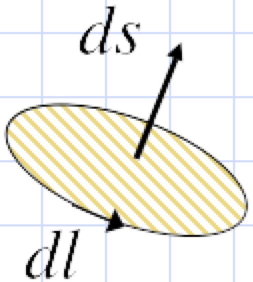
Time varying flux through a coil

$$V_{emf} = \oint \vec{E} \bullet d\vec{l} = -\frac{d}{dt}\Psi = -\frac{d}{dt}\int \vec{B} \bullet d\vec{s}$$ is the electromotive force

The emf is similar to a VOLTAGE

Orientation issues :

$$\oint \vec{E} \bullet d\vec{l} = -\frac{d}{dt}\int \vec{B} \bullet d\vec{s}$$



Use right hand rule for $d\vec{s}$ and $d\vec{l}$

Faraday's Law

Electromotive Force (EMF)

What does the flux derivative mean ?

$$V_{emf} = -\frac{d}{dt}\Psi = -\frac{d}{dt}\int \vec{B} \bullet d\vec{s} = -\left(\int \frac{\partial B}{\partial t} \bullet d\vec{s} + \int \vec{B} \bullet \frac{ds}{dt} \right)$$

The emf may come from:

- A dynamic field and a stationary loop
- A moving loop in a static field
- Both moving loop and dynamic field

Faraday’s Law

Electromotive Force (EMF)

- Suppose that we a circular loop of copper wire with radius 1cm. We use an electromagnet to produce a uniform magnetic field that points directly through the loop with magnitude 0.1 T and frequency 60Hz. What voltage will be induced in the magnet?

Faraday's Law

Electromotive Force (EMF)

$$\vec{B} = 0.1 \sin(120\pi t)\,\hat{z}$$

$$\Psi = \oint \vec{B} \cdot \vec{dS} = 0.1 \sin(120\pi t) \cdot \pi (0.01)^2$$

$$\Psi = (1 \times 10^{-5}) \sin(120\pi t) \quad Wb$$

$$V_{emf} = -\frac{d}{dt}\Psi = -(1 \times 10^{-5})(120\pi) \cos(120\pi t) \quad V$$

$$\tilde{V}_{emf} = -3.76 \text{ mV} \quad (@\ 60 \text{ Hz})$$

Faraday's Law

Electromotive Force (EMF)

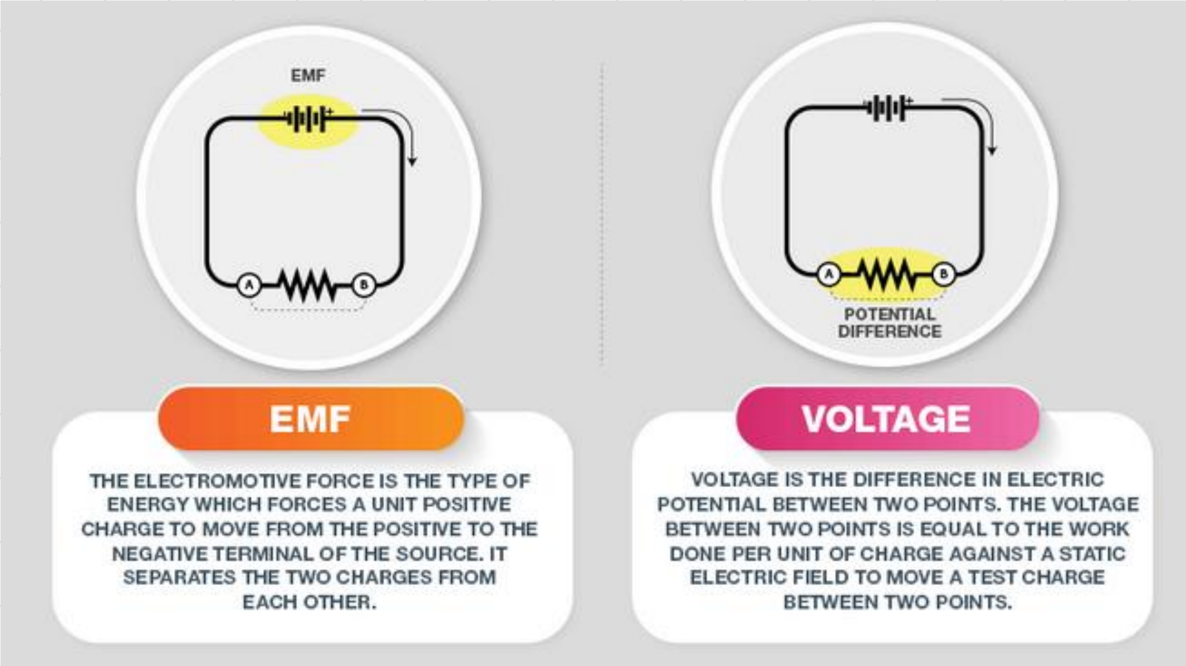
Is emf the same as voltage? Not quite.

- 1 volt = 1 joule per coulumb
- Electrical work: $W = q\Delta V$
- Therefore a closed-loop path through a voltage field, which starts and ends on the same voltage, does not do any work.
- Circuits are closed loops…. do they not do any work?

Faraday’s Law

Electromotive Force (EMF)

- Emf is *active*, while voltage is *passive.* They both have units of volts, but one can do work on a closed loop and the other can’t.
- Batteries and Faraday’s Law can both give us emf. In electrostatics, there can’t be emf.



byjus.com

Faraday's Law

Electromotive Force (EMF)

In other words...

Voltage

Gfycat

Electromotive Force

Gfycat

Faraday's Law

Electromotive Force (EMF)

- Suppose that the 0.1T field is no longer oscillating at 60Hz but instead remains constant. It still points directly through the loop.

a.) Is there any emf?

b.) What emf is generated if we increase the radius of the loop at a rate of 1cm per second?

Faraday's Law

Electromotive Force (EMF)

$\vec{B} = 0.1\,\hat{z}$ radius $r = t$

$\psi = \oint \vec{B} \cdot \vec{dS} = 0.1 \cdot \pi (t)^2 = 0.1\,\pi t^2$

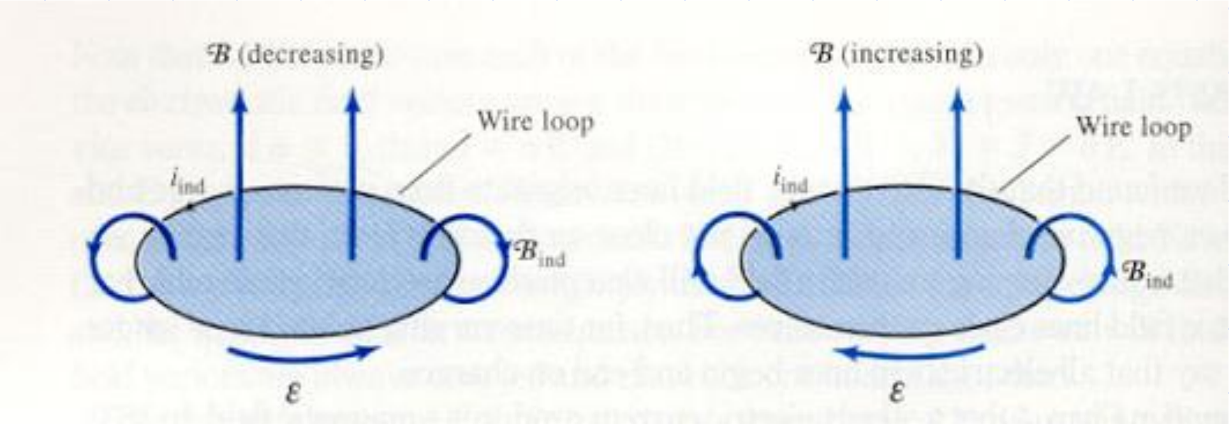
$V_{emf} = -\frac{d}{dt}\psi = -0.2\,\pi t$

Voltage drop is defined in the direction of the wire loop.

Faraday's Law

Lenz's Law

Lenz's law : *"The current in the loop is always in such a direction as to oppose the change of magnetic flux that produced it."*



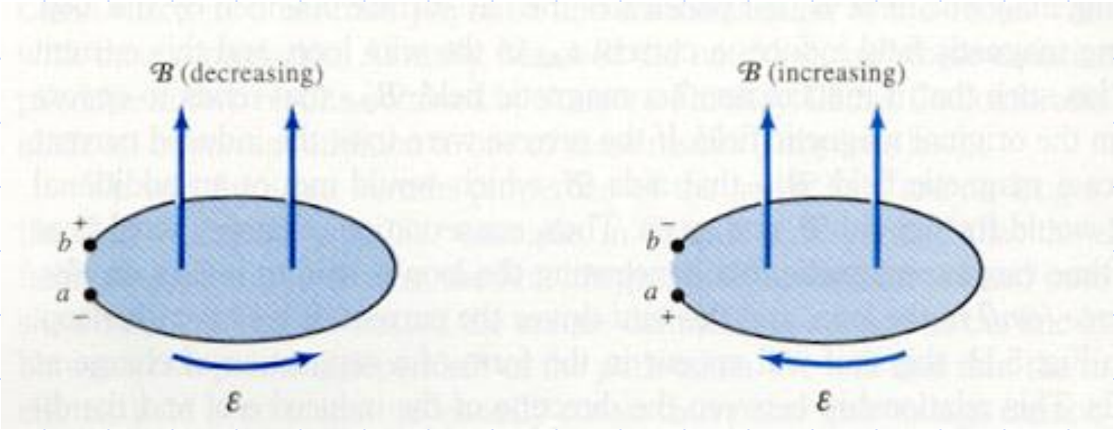
Ulaby

Low impedance Output

Faraday's Law

Lenz's Law

Lenz's law : *"The current in the loop is always in such a direction as to oppose the change of magnetic flux that produced it."*



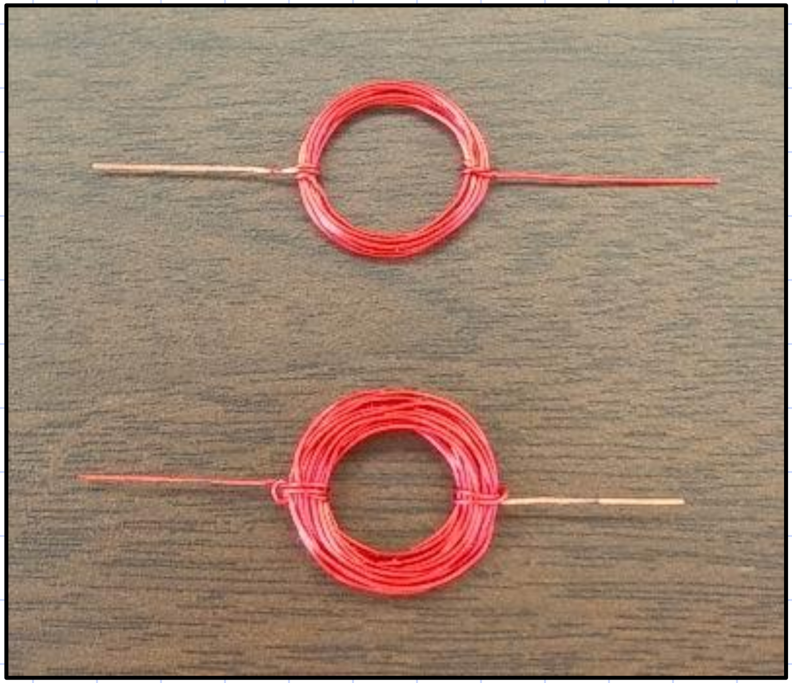
Ulaby

High impedance Output

Faraday's Law

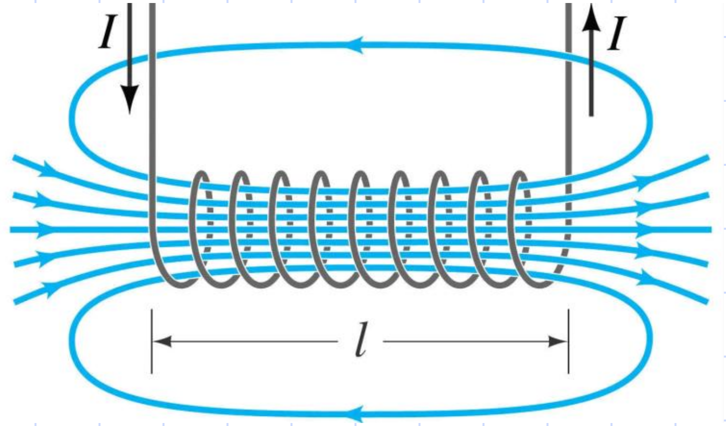
Lenz's Law (skip to 1:00)

Faraday’s Law



What if we turn our wire loop into a proper solenoid by increasing the number of turns of wire? How will the emf increase with the number of turns?

Faraday's Law

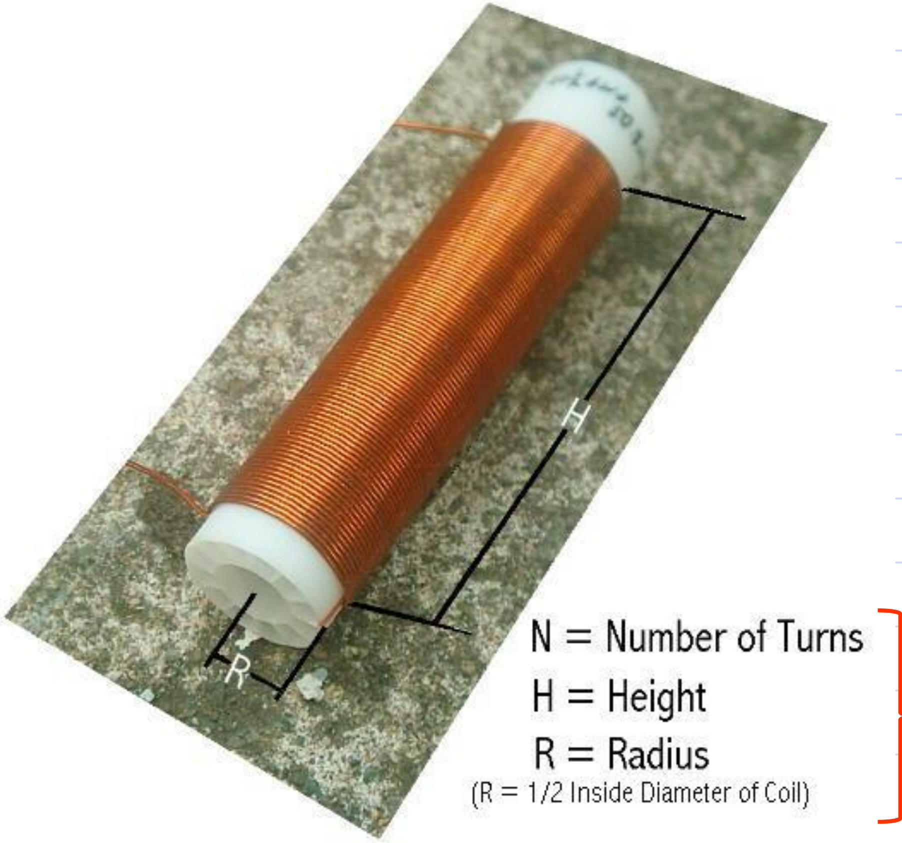


- In a solenoid, magnetic field lines pass through a series of coils. We say that the associated flux *links* all of the coils.
- We therefore specify flux linkage **Λ** as the product of the flux Ψ and the number of turns it passes through, N.
- If not all the magnetic field lines pass through all the turns of the coil, the flux linkage will be decreased.

Faraday's Law

Do Lecture 17, Exercise 1 in groups of up to 4.

Inductance



Geometric parameters for a solenoidal inductor

http://www3.telus.net/chemelec/Calculators/Helical-Coil-Calc.htm

Inductance

What is inductance?

- Inductance measures the tendency of an electrical conductor to oppose changes in current by way of an electromotive force
- Any conductor will have an inductance per unit length, but inductance can be greatly enhanced through geometry

$$v(t) = L\frac{di}{dt} = -N\frac{d\psi}{dt} = -\frac{d\Lambda}{dt}$$

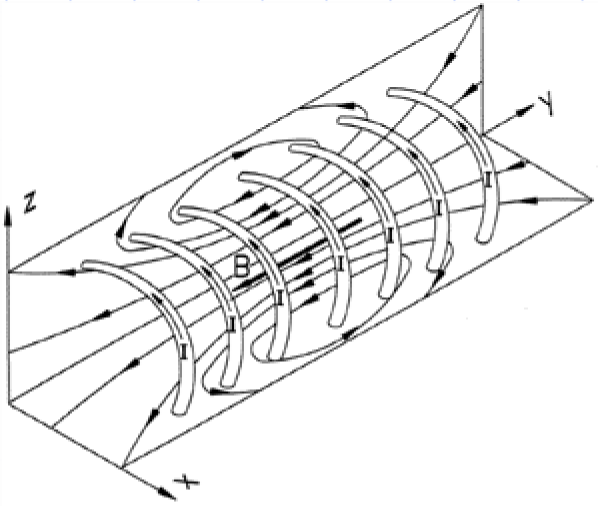
Inductance

Two types of Inductances:

- self inductance - e.g. inductors
- mutual inductance - e.g. transformers

Self Inductance:

- coil of wire with I_1 , creates B
- wire loop intersects $\int \vec{B} \bullet d\vec{s}$
- this creates $e.m.f. = -\frac{d}{dt} \int \vec{B} \bullet d\vec{s}$



http://www.gaussbusters.com/ppm93.html

Inductance

Self-Inductance

Two ways to calculate the inductance:

- Calculate the emf then use = L dI/dt.

or

- Calculate the total flux linkage and use L = Total Flux / I

Things to remember :

The flux linkage, $\Lambda = N \cdot \Psi$

- only if all loops intersect same flux
- not true for finite solenoid and will need: $\Lambda = \sum \Psi$

Inductance

Consider a solenoid with N turns, length l, and radius a. Assume the current is sinusoidal with a frequency f and ignore fringing effects.

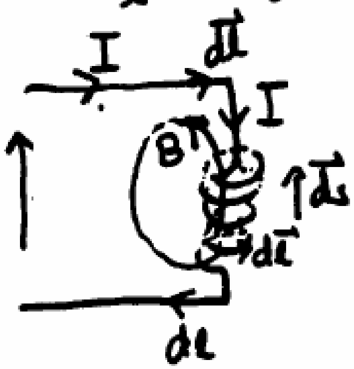
a. What emf, $\int \mathbf{E} \cdot d\mathbf{l}$ is induced around the solenoid (include all turns)?
b. The "voltage" across an inductor is the emf (with care taken about signs). Find the solenoid inductance by substituting the absolute value of the emf in part b. for the voltage in V = L dI/dt.
c. What is the flux linkage through all N turns?
d. Calculate L = Flux/I and compare with your answer to part c.

Inductance

a. $\text{emf} = \oint \vec{E}\cdot d\vec{l} = -\sum_i \frac{d}{dt}\int \vec{B}\cdot d\vec{s} = \boxed{-\frac{\mu_o N^2 \pi a^2}{l}\frac{dI}{dt}}$

b. $V = -\text{emf} = \frac{\mu_o N^2 \pi a^2}{l}\frac{dI}{dt} = L\frac{dI}{dt}$; $\boxed{L = \frac{\mu_o N^2 \pi a^2}{l}}$

inductor



for $I>0$ B points up → choose $\vec{ds}$ in same direction

RIGHT HAND RULE FORCES $d\vec{l}$ as shown

emf >0 means $\vec{E} \parallel d\vec{l}$ which pushes + charge to lower connection

c. $\Lambda = \sum \Psi_i = N\Psi_i = \boxed{\mu_o N^2 \pi a^2 I_o \cos \omega t / l}$

d. $L = \frac{\Lambda}{I} = \frac{\mu_o N^2 \pi a^2 I_o \cos \omega t / l}{I_o \cos \omega t} = \boxed{\mu_o \frac{N^2}{l}\pi a^2}$ = part a.

Inductance

Calculate numerical values for $\vec{B}$, ψ_m, emf, and L when N = 26, I = 100 mA, l = 16 mm, and a = 2.5 mm.

$$\vec{B} = \frac{\mu_0 N I}{l} = \mu_0 \frac{(26)(0.1)}{0.016} = 162.5\mu_0$$

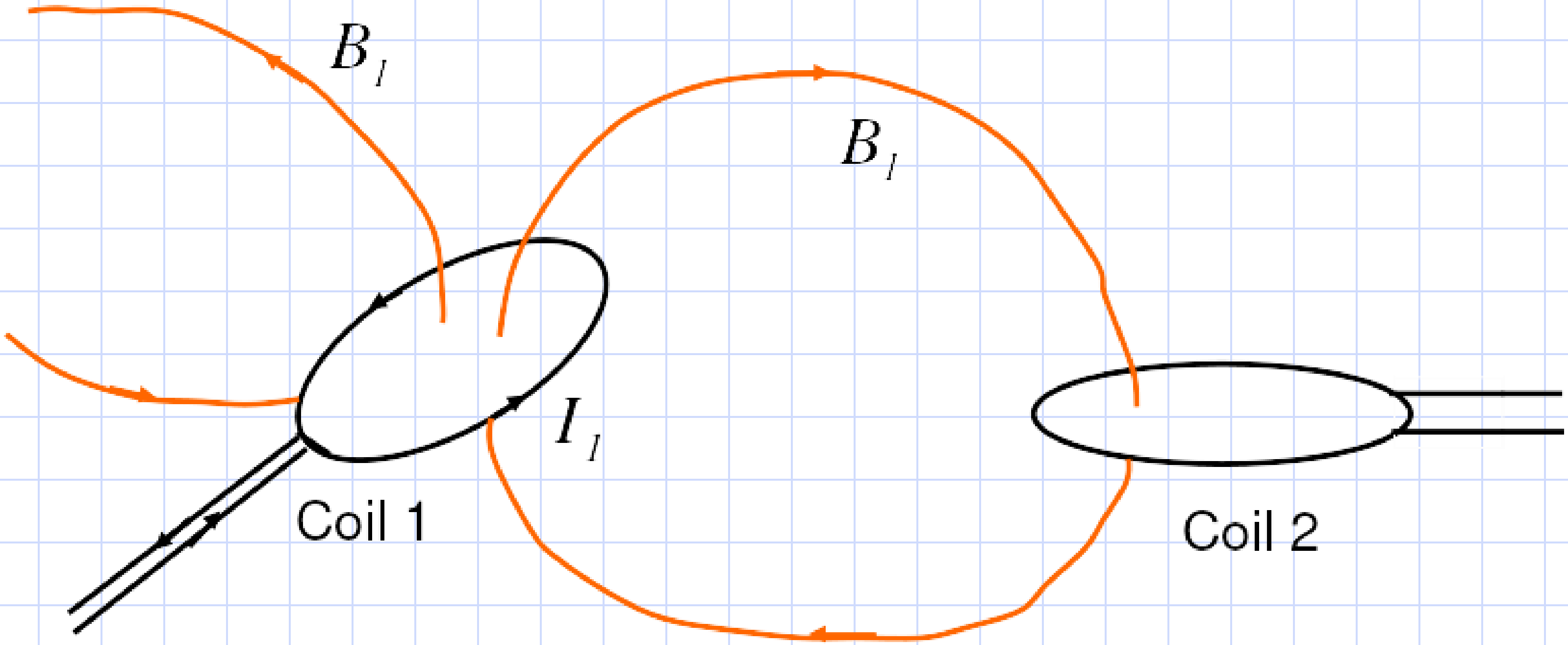
$$\psi_m = \int \vec{B} \cdot d\vec{S} = 162.5\mu_0 \pi (0.0025)^2 = 0.0032\mu_0$$

$$L = \frac{N\psi_m}{I} = \frac{(26)(0.0032)}{0.1}\mu_0 = 0.7977\mu_0 = 1\mu H$$

Does the ideal calculation (no fringing) over or under estimate the inductance value?

Inductance

Mutual Inductance



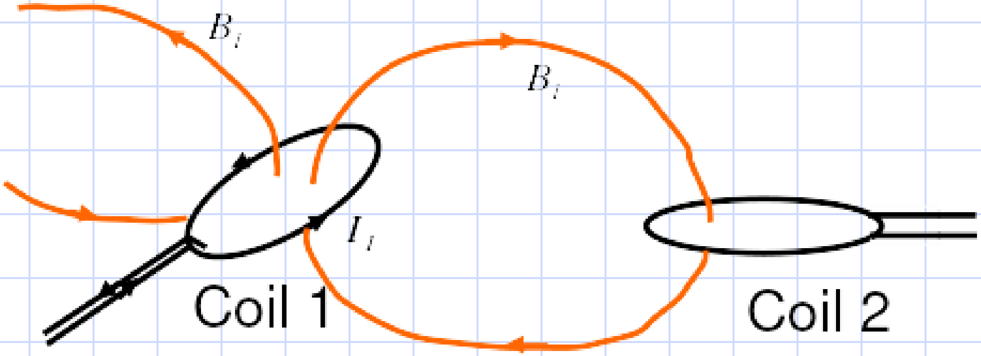
Mutual Inductance: Current through Coil 1 induces e.m.f. in Coil 2

Inductance

Mutual Inductance

$$L_{21} = \frac{\Lambda_{21}}{I_1}$$ where, $$\Lambda_{21} = \int B_1 \bullet ds_2$$

Mutual Inductance



Also, $$emf_2 = L_{21} \cdot \frac{dI_1}{dt}$$

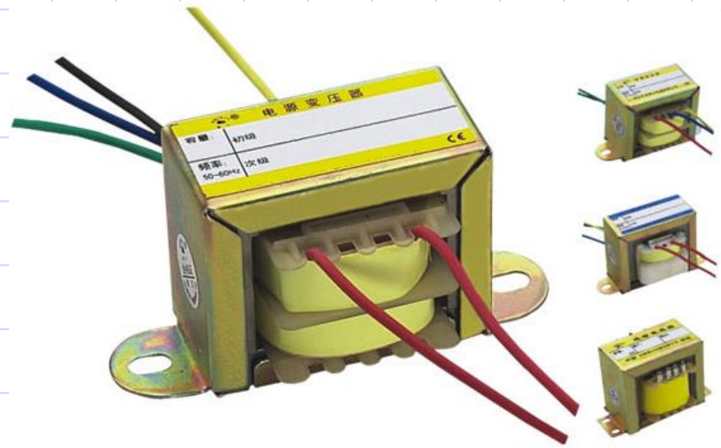
And, $$L_{12} = L_{21}$$

Applications

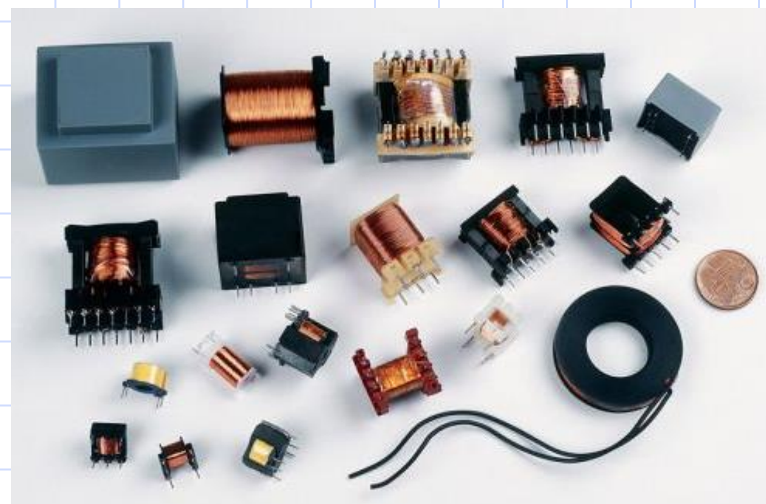
Transformers

- A huge range in sizes

http://www.meppi.com/Products/Transformers/Power/Pages/Core-formTransformers.aspx



http://www.transformerfactory.com/e1-model-small-power-transformer-1va-70a.html



http://en.ferilex.eu/transformers.html

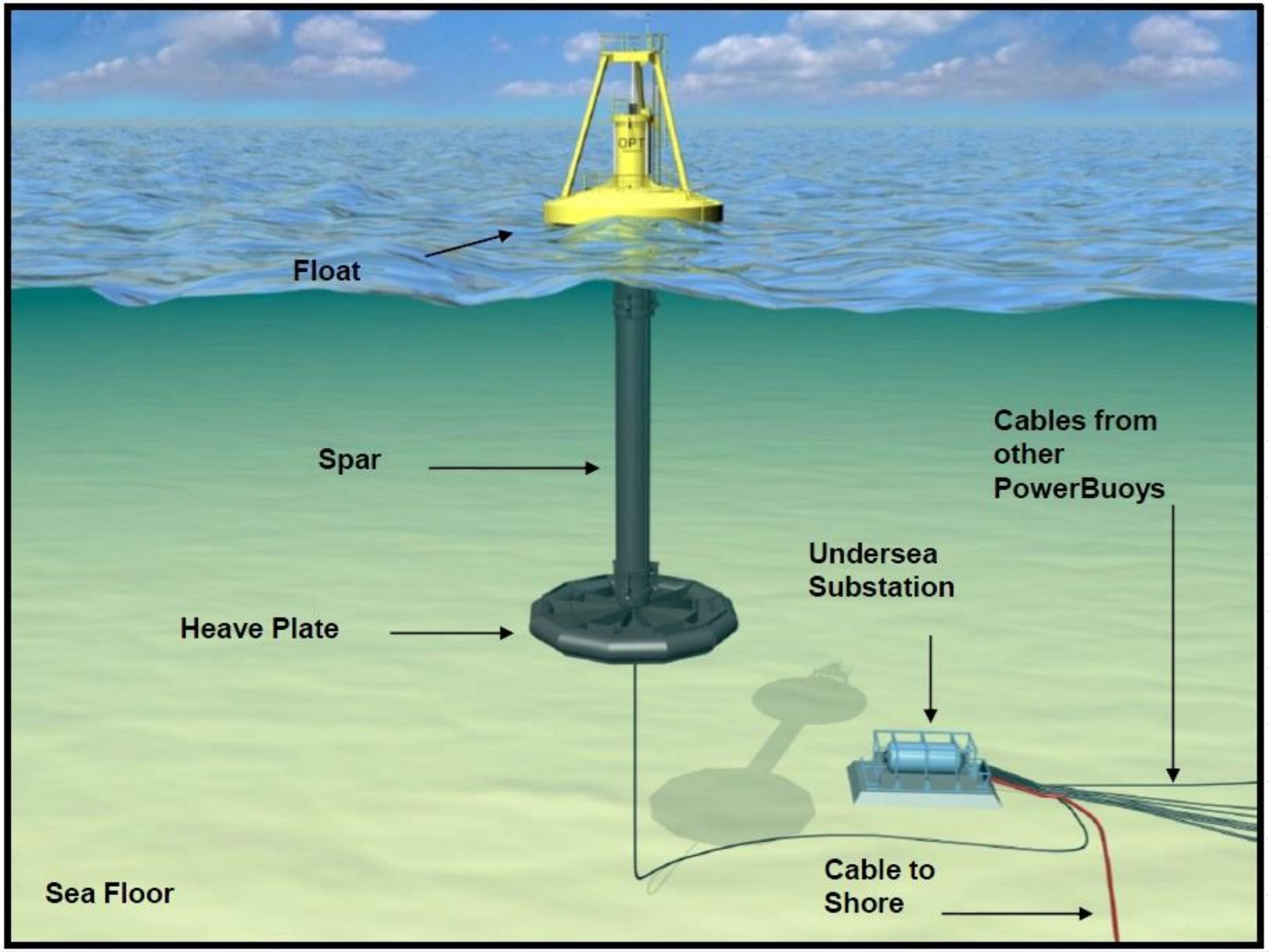
Applications

Hoover Dam

http://isu.indstate.edu/jspeer/conservation/

Applications

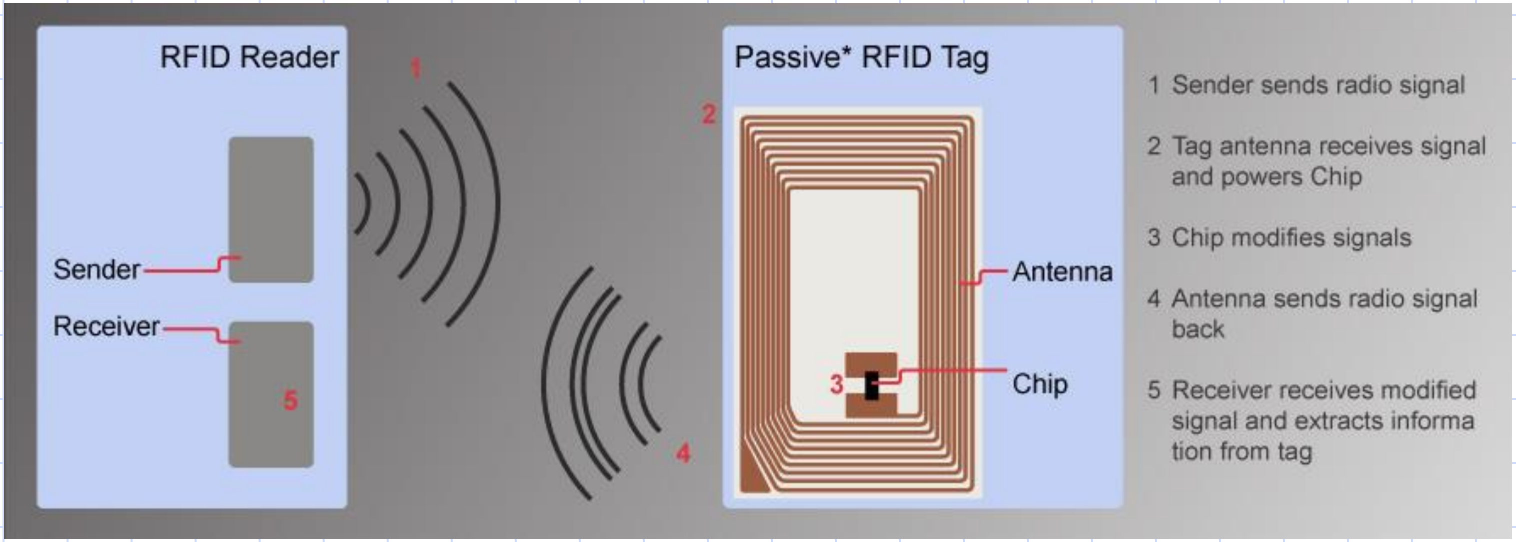
Wave Energy



https://openei.org/wiki/Main_Page

Applications

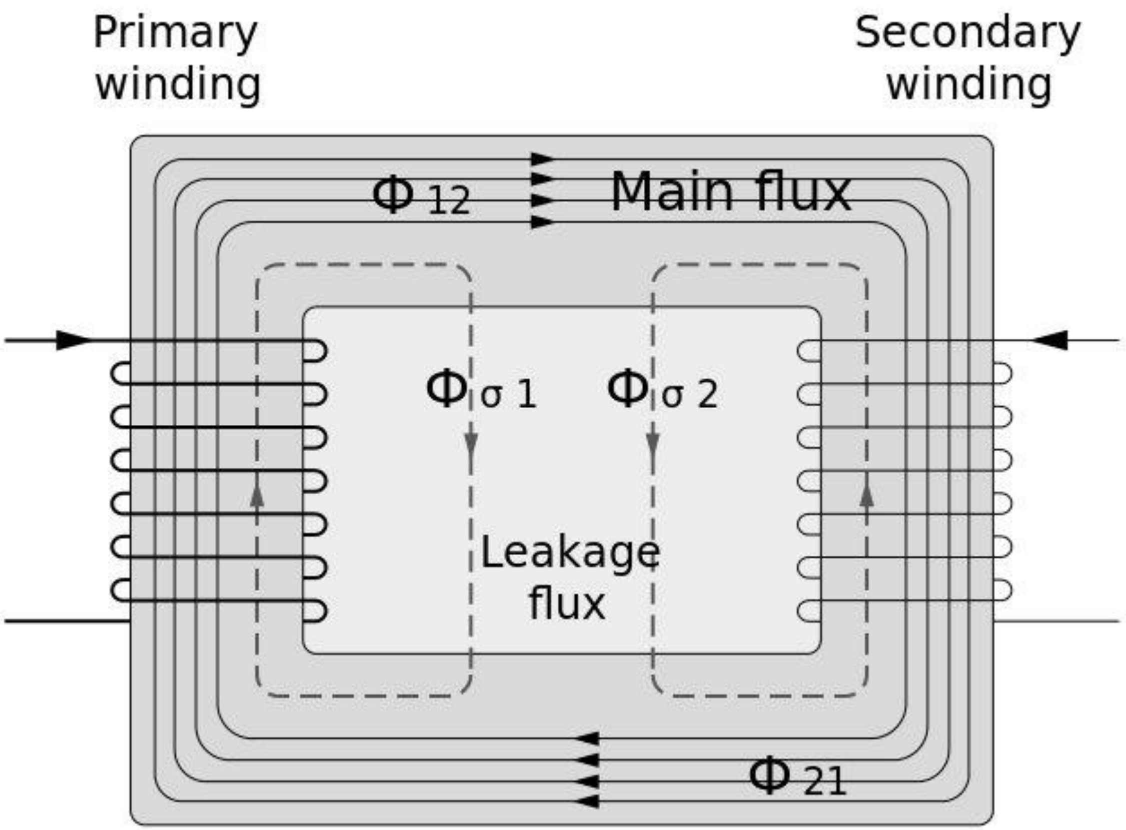
- An emf generated by Faraday's Law can power a whole circuit directly.



cxjrfidfactory.com

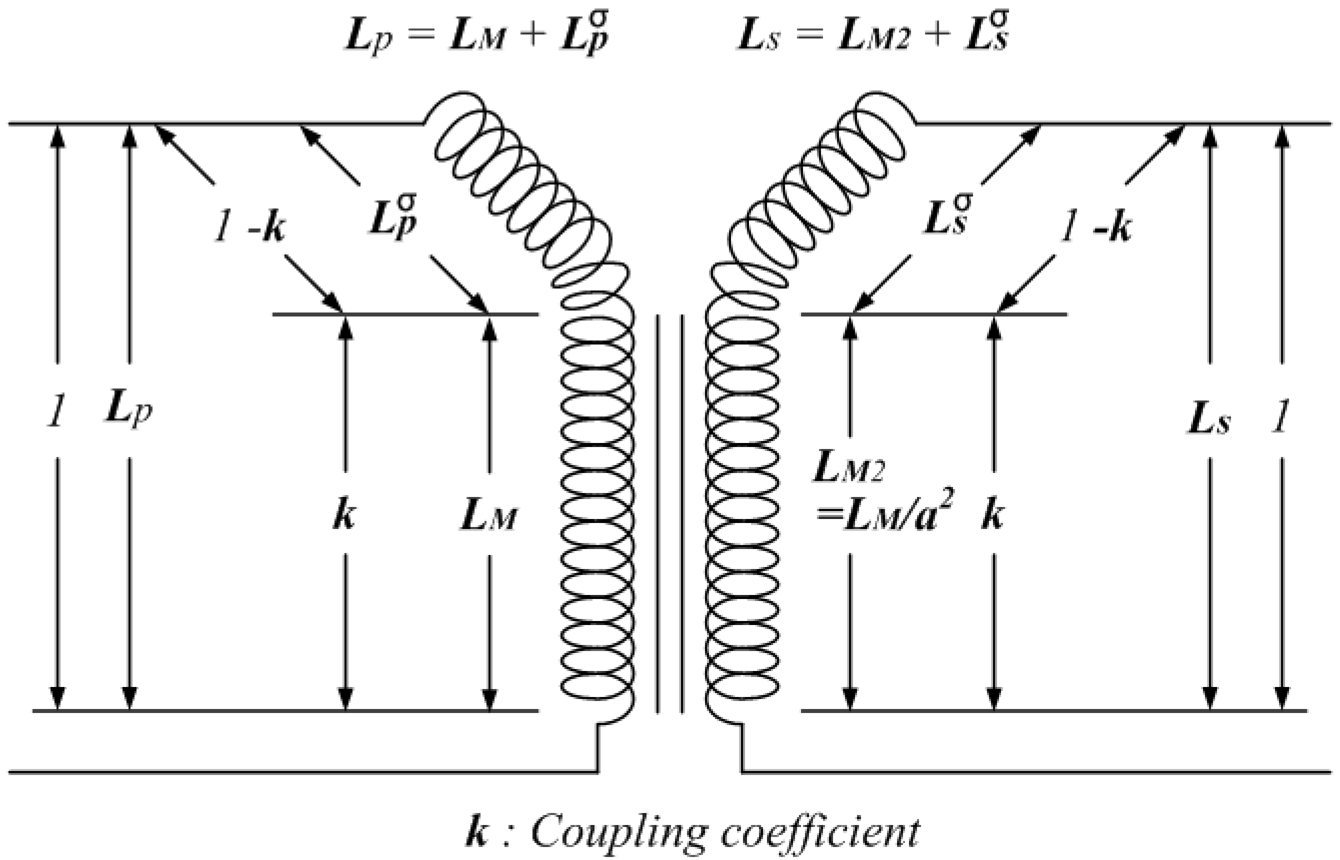
Applications

Leakage Inductance in Transformers



Applications

Leakage Inductance in Transformers



Wikipedia

Boundary Conditions

Arguing from analogy with Electric Fields

$$\vec{D} = \epsilon \vec{E} \qquad \vec{B} = \mu \vec{H}$$

$$\oint \vec{D} \cdot \vec{ds} = Q_{encl} \Rightarrow D_{n1} - D_{n2} = \rho_s$$

$$\oint \vec{B} \cdot \vec{ds} = 0 \Rightarrow B_{n1} - B_{n2} = 0$$

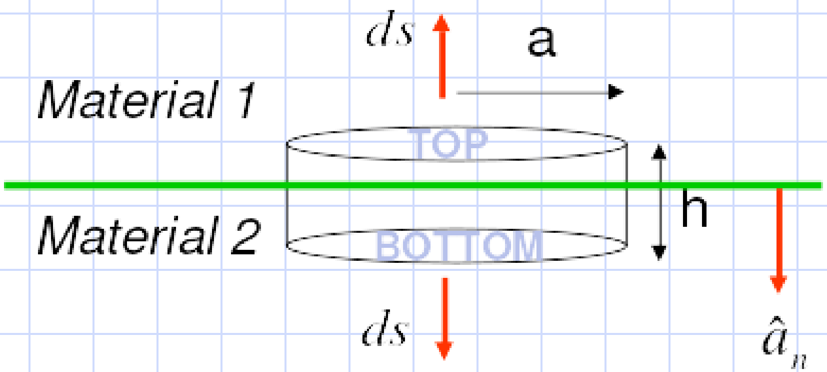
$$\oint \vec{E} \cdot \vec{dl} = 0 \Rightarrow E_{t1} - E_{t2} = 0$$

$$\oint \vec{H} \cdot \vec{dl} = I_{inc} \Rightarrow H_{t1} - H_{t2} = J_s$$

Boundary Conditions

Normal component

$$\oint B \bullet ds = 0 = \int_{TOP} B \bullet ds + \int_{BOTTOM} B \bullet ds$$



$$B_{n1} \cdot AREA - B_{n2} \cdot AREA = 0$$

$$\therefore B_{n1} = B_{n2}$$

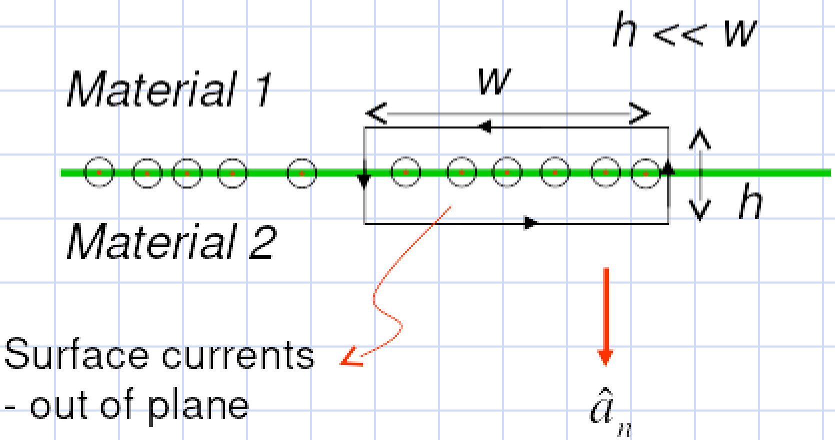
Take h << a (**a thin disc**)

- ignore contribution from the sides

Boundary Conditions

Tangential component

$$\oint H \bullet dl = H_{t2} \cdot w - H_{t1} \cdot w = I_{net}$$



I_{net} *can only be due to surface currents = J_s.w*

$$\therefore H_{t2} - H_{t1} = J_s$$

or

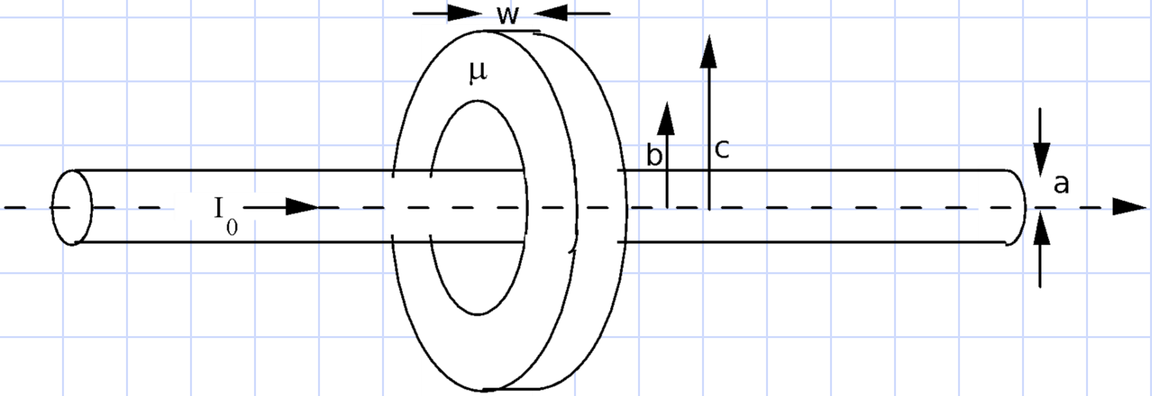
$$\hat{a}_n \times (H_2 - H_1) = J_s$$

In low frequency cases $J_s = 0$, then, $H_{t1} = H_{t2}$

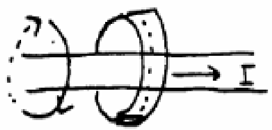
Boundary Conditions

For the configuration shown, calculate the magnetic fields.

Show that the boundary conditions are satisfied.



Boundary Conditions

a. 

Path for $\oint \vec{H}\cdot d\vec{l}$ is a circular loop
- One path inside toroid + 1 outside

$$\oint \vec{H}\cdot d\vec{l} = I_{enc}$$

For both cases $\vec{H}$ does not vary along loop, so $\oint \vec{H}\cdot d\vec{l} = H_\varphi 2\pi r$

" " " , $I_{enc} = I$

$\therefore$ $\boxed{\vec{H} = \frac{I}{2\pi r}\hat{a}_\varphi}$ For both regions

$$\boxed{\begin{aligned}\vec{B} &= \frac{\mu_0 I}{2\pi r}\hat{a}_\varphi \quad \text{in air}\\ &= \frac{\mu I}{2\pi r}\hat{a}_\varphi \quad \text{in toroid}\end{aligned}}$$

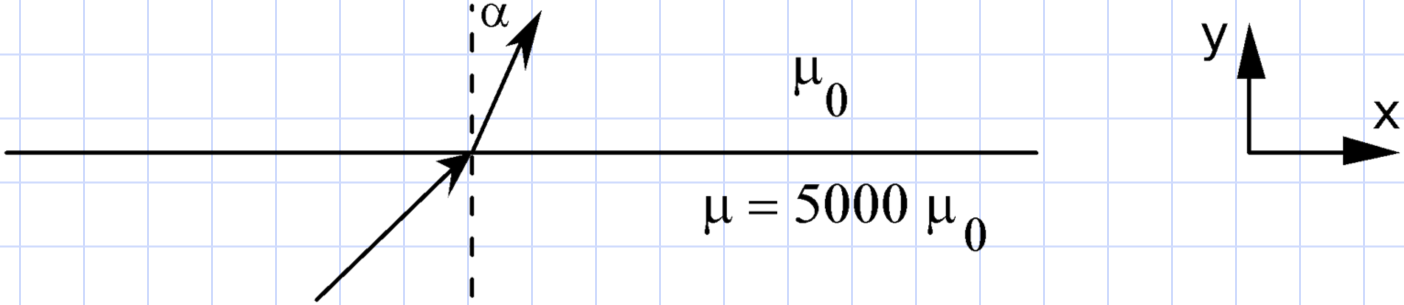
b. $\vec{H}$ + $\vec{B}$ are tangential to air-toroid boundary

$\boxed{H_{1t} = H_{2t}}$ satisfied because same expression

$\boxed{B_{1n} = B_{2n} = 0}$ also satisfied

Boundary Conditions

On the iron side of the iron-air boundary below,
B = 0.1 **a**x + 0.1 **a**y Tesla.
What is **H** on the iron side?
What is **B** on the air side?
Approximately, what direction is **B** outside a ferromagnet?



Boundary Conditions

a. $\vec{B} = \mu \vec{H} \Rightarrow \therefore \vec{H} = \dfrac{\vec{B}}{5000\mu_o} = \dfrac{0.1(\hat{a}_x + \hat{a}_y)}{5000\mu_o} = \boxed{15.9\,\hat{a}_x + 15.9\,\hat{a}_y = \vec{H}_{iron}}$

b. Boundary conditions
$$\begin{cases} B_{1n} = B_{2n} \Rightarrow B_{y,iron} = B_{y,air} = 0.1\ \text{T} \\ H_{1t} = H_{2t} \Rightarrow H_{x,iron} = H_{x,air} = 15.9\ \text{A/m} \\ B_{x,air} = \mu_o H_{x,air} = 15.9\mu_o = 2\times 10^{-5}\ \text{T} \end{cases}$$

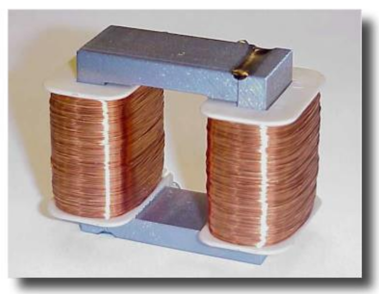
$$\boxed{\vec{B}_{air} = 2\times 10^{-5}\,\hat{a}_x + 0.1\,\hat{a}_y}$$

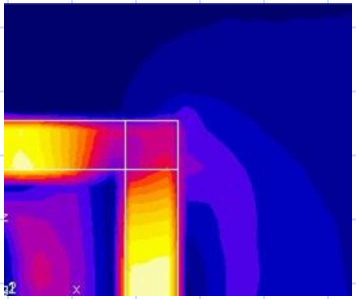
c. $\vec{B} \approx$ normal to surface on air side of air-ferromagnet boundary

Boundary Conditions

Field lines exit normal to the surface in high μ materials

Also, field lines like to travel through high μ materials





http://www.cedrat.com

Transformers :

- Use IRON to direct flux
- Primary and secondary windings intercept almost all the flux - no need for "smoothly-wound" coils
- Increase Inductance with increasing μ