CSCI 2300 — Algo
Homework 2
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- **Q1**
  Is $4^{1536} \equiv 9^{4824} \mod 35$
  $4^4 \mod 35 = 256 \mod 35 = 11$
  $9^2 \mod 35 = 81 \mod 35 = 11$
  $4^{1536} \equiv 4^{4*384} \equiv 11^384 \mod 35$
  $9^{4824} \equiv = 9^{2*2412} \equiv 11^{2412} \equiv 11^{6*384+108} \equiv 11^{6*384} * 11^{108} \mod 35$
  $11^{108} \equiv 121^{54} \equiv 6^{54} \equiv 36^{27} \equiv 1^27 \mod 35 = 1$
  $11^{6*384} \equiv 11^{384} \mod 35$

  Yes, $4^{1536} \equiv 9^{4824} \mod 35$

- **Q2**
  $x^{86} \equiv 6 \mod 29$
  $x^{28} \equiv 1 \mod 29$
  $x^{86} \equiv x^2 \mod 29$
  $x^2 \equiv 6 \mod 29$
  $x^2 \equiv 64 \mod 29$
  $x = 8$

- **Q3**
  Prove that GCD$(F_{n+1}, F_n) = 1$, for $n \geq 1$, where F_n is the n-th Fibonacci element.
  We prove this with induction
  Base: $n = 1$, $F_2 = 1$, $F_1 = 1$, GCD$(F_2, F_1) =$GCD$(1, 1) = 1$.
  Induction: Assume GCD$(F_{n+1}, F_n) = 1$
  $F_{n+2} = F_{n+1} + F_n$, $F_n = F_{n+2} - F_{n+1}$
  GCD$(F_{n+1}, F_n) =$GCD$(F_{n+1}, F_{n+2} - F_{n+1}) =GCD(F_{n+1}, F_{n+2}) =GCD(F_{n+2}, F_{n+1}) = 1$
  GCD$(F_{n+1}, F_n) = 1 \rightarrow$GCD$(F_{n+2}, F_{n+1}) = 1$
  therefore, GCD$(F_{n+1}, F_n) = 1$, for $n \geq 1$ $\blacksquare$