Chapter 2-3. States and carrier distributions

So far, we have concentrated on carrier properties of qualitative nature. We also need to:

Determine the carrier distribution with respect to energy in different bands.

Determine the quantitative information of carrier concentrations in different bands.

Two concepts will be introduced to determine this:

- **Density of states**
- **Fermi-Dirac distribution and Fermi-Level**

Density of states

There are 4 states per atom or $4 \times 5 \times 10^{22}$ / cm^3 states in each of conduction and valence bands of Si.

The distribution of these states in the bands are not uniform, but follows a distribution function given by the following equations.

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{\pi^2 \hbar^3}, \qquad E \geq E_c \quad \text{(2.6a)}$$

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3}, \qquad E \leq E_v \quad \text{(2.6b)}$$

Dependence of DOS near band edges

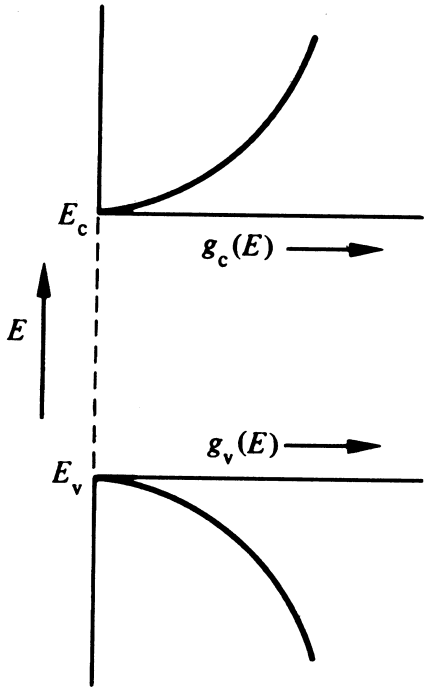


Figure 2.14

More on density of states (DOS)

$g_c(E)\,dE$ represents the # of conduction band states/cm^3 lying in the energy range between E and $E + dE$

$g_v(E)\,dE$ represents the # of valence band states/cm^3 lying in the energy range between E and $E + dE$

More states are available away from the band edges, similar to a seating arrangement in a football field

Units for $g_c(E)$ and $g_v(E)$: # per unit volume per unit energy, i.e., # / (cm^3 eV)

Energy bands are drawn with respect to electron energies

Fermi-Dirac distribution and the Fermi-level

Density of states tells us how many states exist at a given energy E. The Fermi function $f(E)$ specifies how many of the existing states at the energy E will be filled with electrons. The function $f(E)$ specifies, under equilibrium conditions, the probability that an available state at an energy E will be occupied by an electron. It is a probability distribution function.

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}} \qquad (2.7)$$

E_F = Fermi energy or Fermi level

k = Boltzmann constant = 1.38×10^{-23} J/K
= 8.6×10^{-5} eV/K

T = absolute temperature in K

Distribution function for gas molecules

Example: Gas molecules follow a different distribution function: The Maxwell-Boltzmann distribution

$$\frac{n_2}{n_1} = \frac{e^{\frac{-E_2}{kT}}}{e^{\frac{-E_1}{kT}}} = e^{\frac{-(E_2-E_1)}{kT}}$$

Let us look at the Fermi-Dirac distribution more closely.

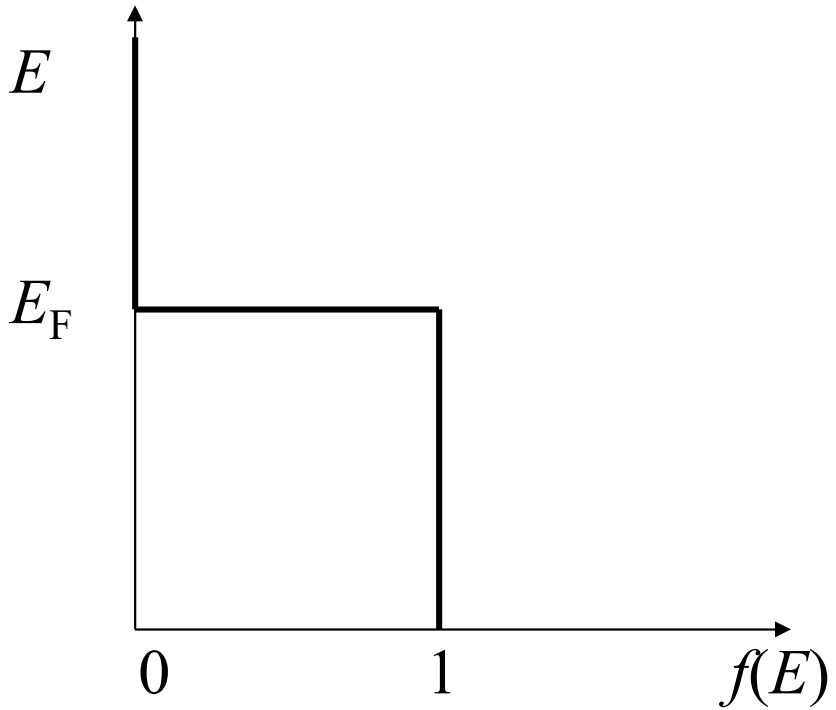
Fermi-Dirac distribution: **Consider $T \rightarrow 0$ K**

For $E > E_F$:

$$f(E > E_F) = \frac{1}{1 + \exp(+\infty)} = 0$$

For $E < E_F$:

$$f(E < E_F) = \frac{1}{1 + \exp(-\infty)} = 1$$



Fermi-Dirac distribution: **Consider $T > 0$ K**

If $E = E_F$ then $f(E_F) = ½$

If $E \geq E_F + 3kT$ then $\exp\left(\frac{E - E_F}{kT}\right) >> 1$

Thus, the following approximation is valid: $f(E) = \exp\left(\frac{-(E - E_F)}{kT}\right)$

i.e., most states at energies $3kT$ above E_F are empty.

If $E \leq E_F - 3kT$ then $\exp\left(\frac{E - E_F}{kT}\right) << 1$

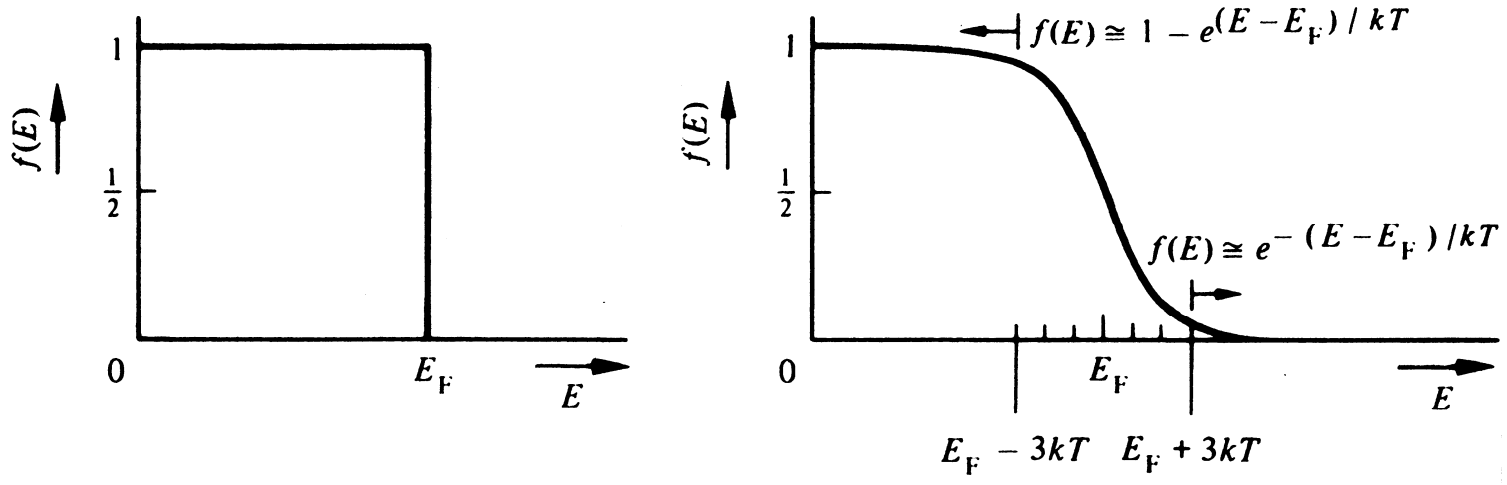
Thus the following approximation is valid: $f(E) = 1 - \exp\left(\frac{E - E_F}{kT}\right)$

So, $1 - f(E)$ = Probability that a state is empty, decays to zero.

So, most states will be filled.

kT (at 300 K) = 0.025eV, E_g(Si) = 1.1eV, so $3kT$ is very small in comparison.

Temperature dependence of Fermi-Dirac distribution

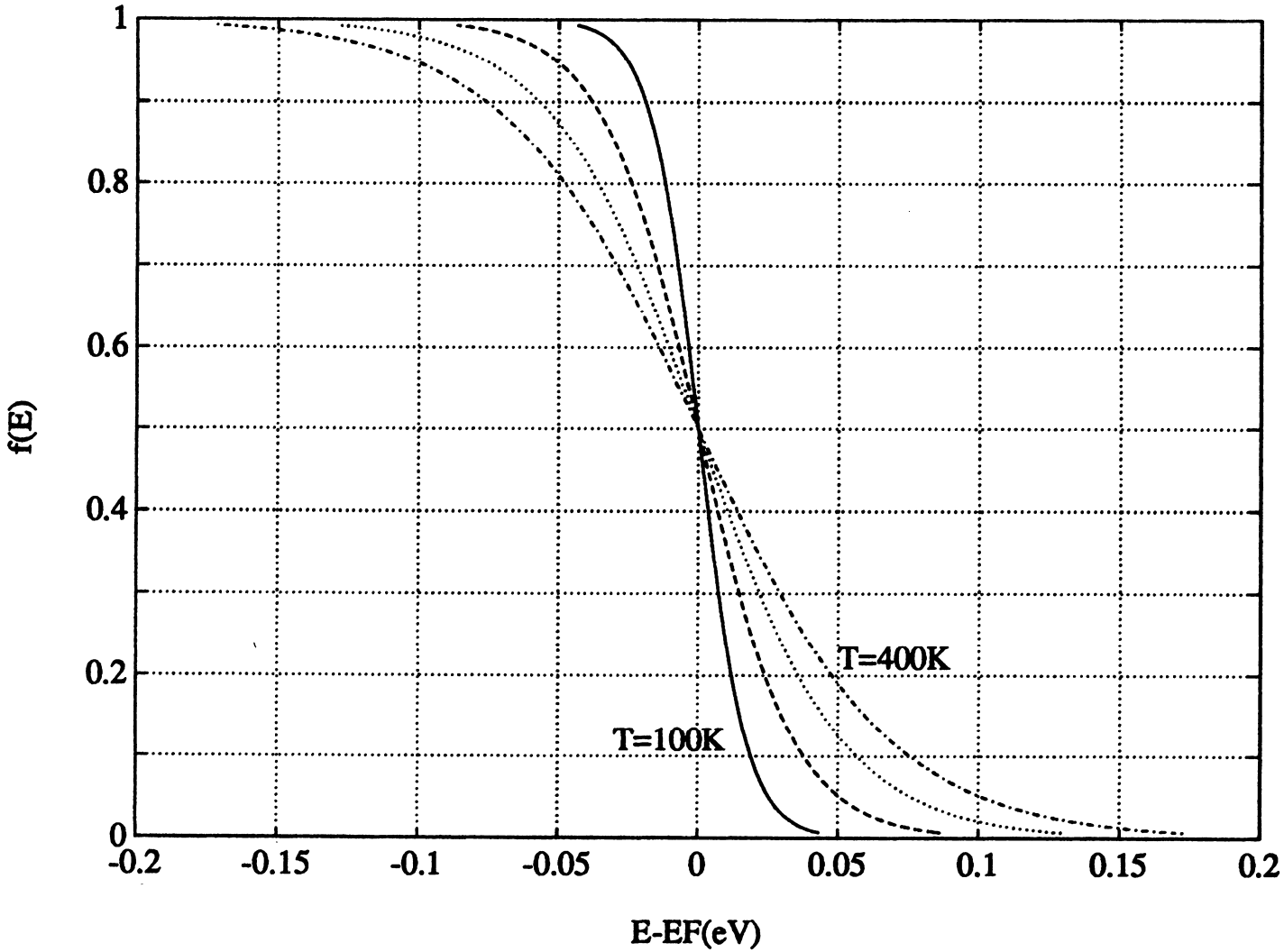


(a) $T \to 0$ K

(b) $T > 0$ K

Figure 2.15

Exercise 2.3



Example

Assume that the density of states is the same in the conduction band (CB) and valence band (VB). Then, the probability that a state is filled at the conduction band edge (E_C) is equal to the probability that a state is empty at the valence band edge.

Where is the Fermi level located?

$$f(E_C) = 1 - f(E_V) \longrightarrow \frac{1}{1+e^{\frac{E_C - E_F}{kT}}} = 1 - \frac{1}{1+e^{\frac{E_V - E_F}{kT}}} = \frac{1}{1+e^{\frac{E_F - E_V}{kT}}}$$

$$E_C - E_F = E_F - E_V \longrightarrow E_F = \frac{E_C + E_V}{2}$$

This corresponds to intrinsic material, where the # of electrons at E_C = # of holes (empty states) at E_V. Note that the probability within the band gap is finite, but there are no states available, so electrons *cannot* be found there.

Equilibrium distribution of carriers

Distribution of carriers = DOS × probability of occupancy

= $g(E)\ f(E)$

(where DOS = Density of states)

Total number of electrons in CB (conduction band) =

$$n_0 = \int_{E_C}^{E_{top}} g_C(E)\, f(E)\, dE$$

Total number of holes in VB (valence band) =

$$p_0 = \int_{E_{Bottom}}^{E_V} g_V(E)\,(1 - f(E))\, dE$$

Fermi-level positioning and carrier distributions

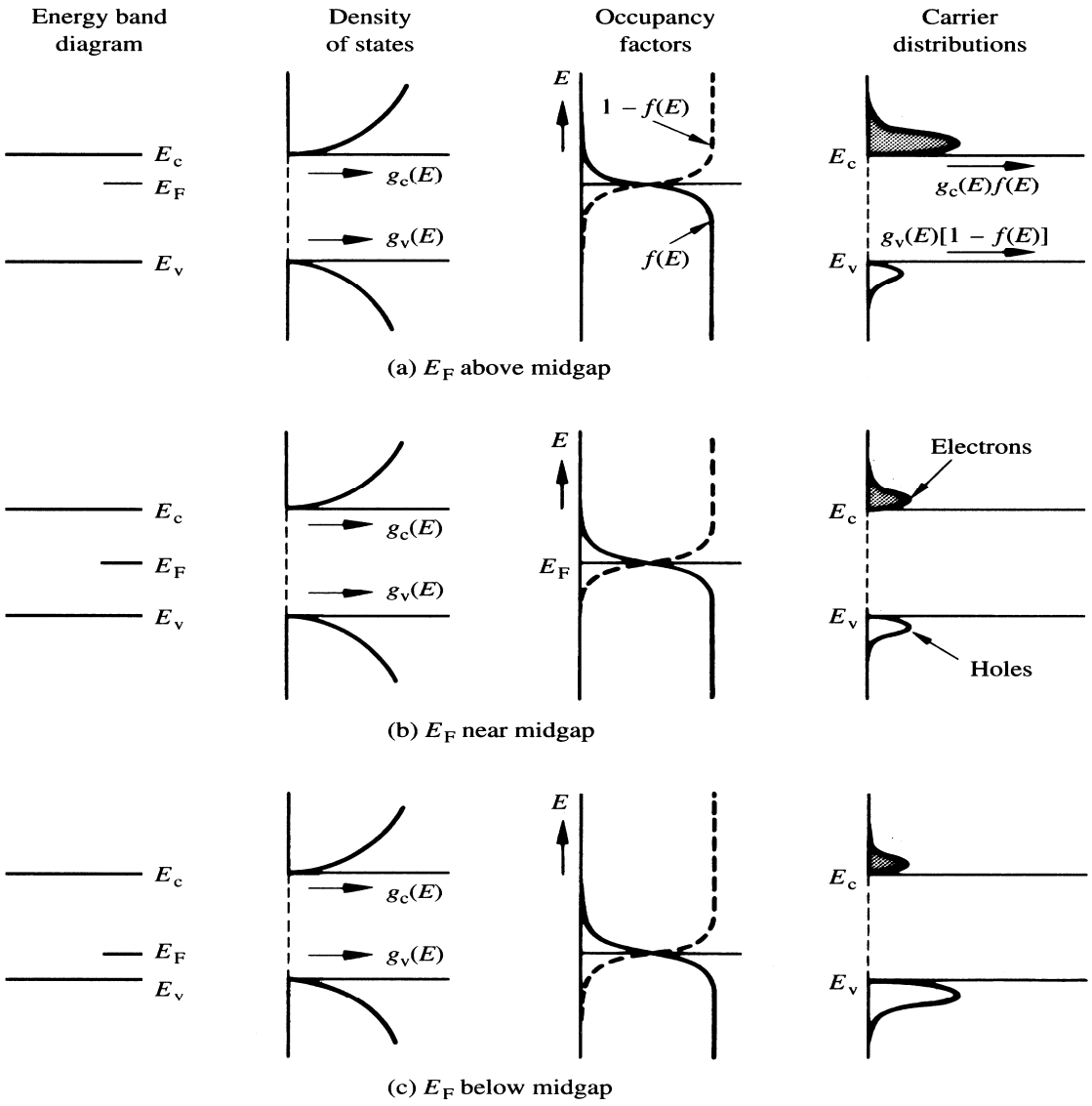


Figure 2.16

Visualization of carrier distribution

One way to convey the carrier distribution is to draw the following diagram. This diagram represents n-type material since there are more electrons than holes.

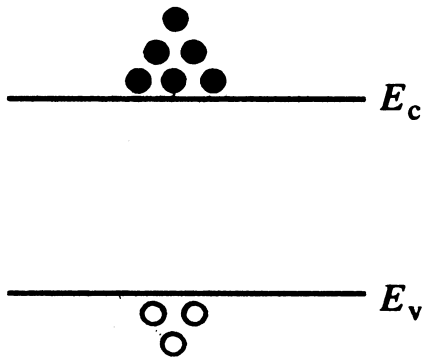


Figure 2.17

Visualization of carrier distribution (continued)

Another more useful way to convey the carrier distribution is to draw the following band diagrams. The position of E_F with respect to E_i is used to indicate whether is n-type, p-type or intrinsic.

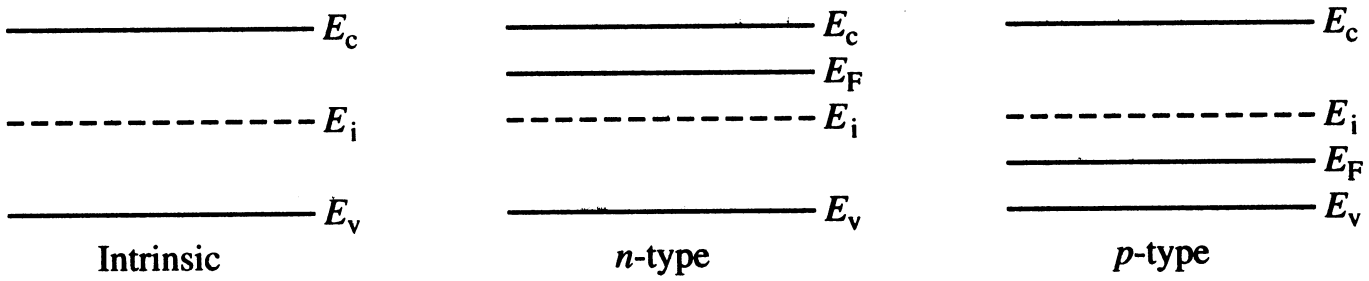


Figure 2.18