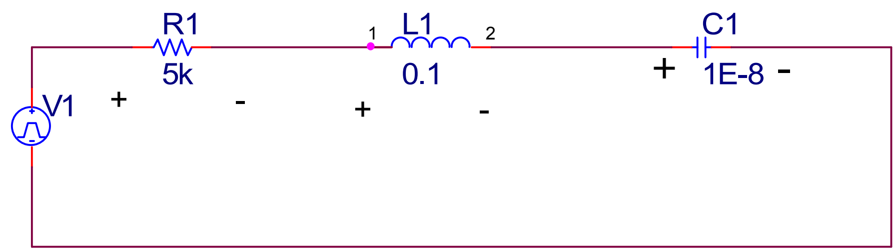
1) Initial Values, Final Values



At t = 0^-, the voltage across the capacitor is 5V (polarity shown), the current through the inductor is 2mA to the 'right' and the source is 10V. At t = 0^+, the voltage source becomes 5V and doesn't change for t > 0.

V 10V 2mA
R 10V 2mA
L -5V 2mA
C 5V 2mA

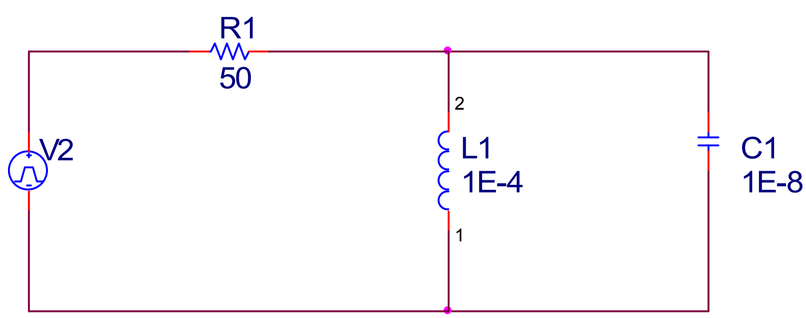
a. Determine the voltage across each component for t=0-. Determine the current through each component for t = 0-.

V 5V 2mA
R 10V 2mA
L -10V 2mA
C 5V 2mA

b. Determine the voltage across each component for t = 0+. Determine the current across each component for t = 0+.

c. Determine the voltage across each component for t goes to ∞. Determine the current across each component for to goes to ∞.

V 5V 0mA
R 0V 0mA
L 0V 0mA
C 5V 0mA



At t = 0^-, the voltage across the capacitor is 8V, the current through the inductor is 10mA 'downward' and the source is 10V. At t = 0^+, the voltage source becomes 3V and doesn't change for t > 0.

V 10V 40mA
R 2V 40mA
L 8V 10mA
C 8V 30mA

d. Determine the voltage across each component for t = 0-. Determine the current through each component for t = 0-. Determine the source voltage at t = 0-.

V 3V -100mA
R -5V -100mA
L 8V 10mA
C 8V -110mA

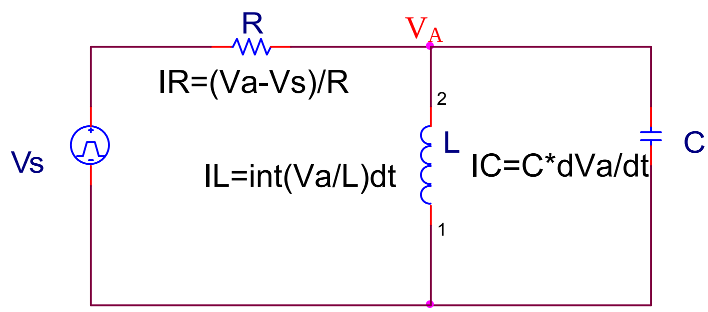
e. Determine the voltage across each component for t = 0+. Determine the current across each component the t = 0 +.

f. Determine the voltage across each componenet for t goes to ∞. Determine the current across each component for t goes to ∞.

V 3V 60mA
R 3V 60mA
L 0V 60mA
C 0V 0mA

IR+IC+IC=0
(Va-Vs)/R + int(Va/L)dt + C*dVa/dt = 0
Va/R + int(Va/L)dt + C*dVa/dt = Vs/R
C*dVa/dt + Va/R + int(Va/L)dt = Vs/R
C*d^2Va/dt^2 + 1/R*dVa/dt + Va/L = 1/R*dVs/dt
d^2Va/dt^2 + 1/RC*dVa/dt + Va/LC = 1/RC*dVs/dt

2) Circuits and Differential Equations



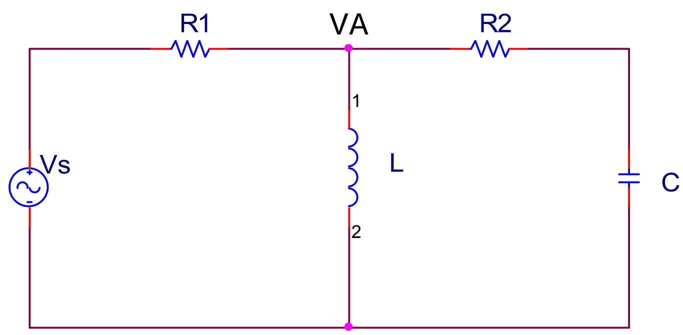
a. In the above the circuit, find the differential equation for the voltage across the capacitor C, VC(t). The source is an arbitrary source.

d^2Va/dt^2 + 1/RC*dVa/dt + Va/LC = 1/RC*dVs/dt

b. For the differential equation, determine the expression for the attenuation constant α, and the resonant frequency, ω_o.

a=1/(2RC)   w_0=1/sqrt(LC)

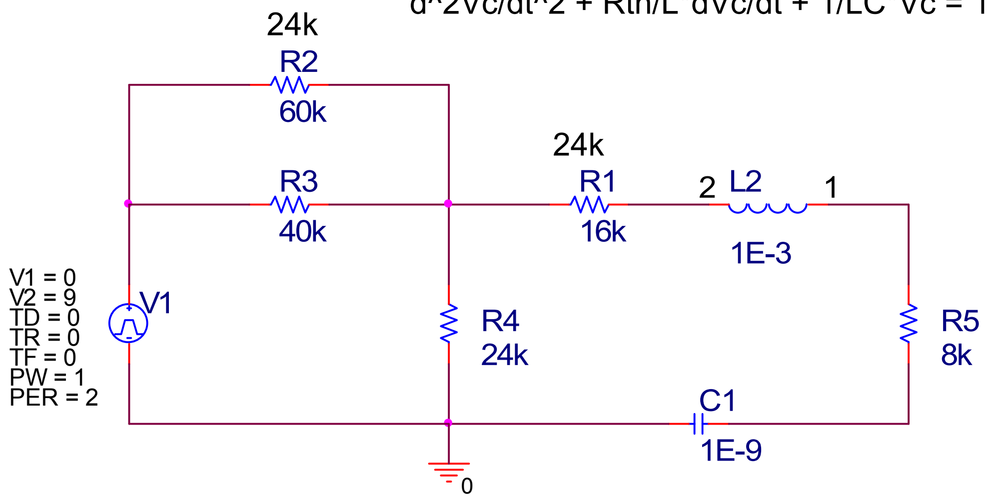
c. In the circuit below, find a differential equation for the voltage across C, Vc(t). The source is an arbitrary source.



d. For the differential equation, determine symbolic expressions for the attenuation constant, α, and the resonant frequency, ω_o, in terms of R1, R2, L and C.

3) RLC Series Circuits

LCdVc/dt + CRth*Vc + int(Vc)dt = int(Vth)
LCd^2Vc/dt^2 + CRth*dVc/dt + Vc = Vth
d^2Vc/dt^2 + Rth/L*dVc/dt + 1/LC*Vc = 1/LC*Vth



In the above circuit, the initial conditions are zero and the source can be considered a step function, 9u(t).

a. Determine the simplified circuit schematic. (Hint: Thevenin equivalent with ***inductor and capacitor*** as a load...and yes, two components can be a load!). VTH=4.5, ITH=.125mA, RTH=36k, Load: L and C

b. What is the intial (t= 0+) current through the capacitor? What is the initial (t= 0+) voltage through the capacitor? I=0, V=0

c. What is the DC steady state current though the capacitor as t approaches ∞?

I=0

d. What is the differential equation defining the current through the capacitor?

d^2Vc/dt^2+Rth/L*dVc/dt+1/LC*Vc=1/LC*Vth

e. Based on the differential equation, determine the s-polynomial for the circuit.

s^2+Rth/L s+1/LC=0 s^2+36*10^6 s+10^12

f. Determine the roots of the polynomial

-2.7799*10^4, -3.5972201*10^7

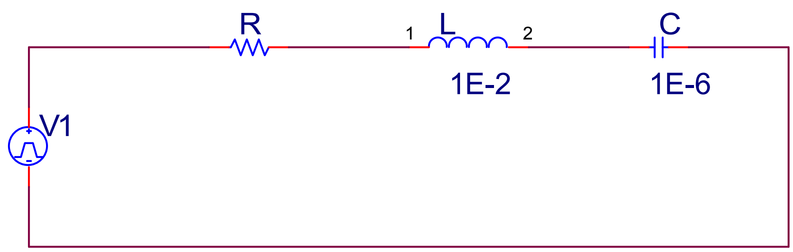
g. Is the system underdamped, overdamped or critically damped?

overdamped

h. Determine the general expression for the current through the capacitor (You do not need to determine the coefficients).

A1e^{-2.7799*10^4 t}+A2e^{-3.5972201*10^7 t}+A3

Problem 4) RLC series circuits



In the above circuit, the source voltage is 5V for t<0 and 10 V for t>0

a. What is the intial (t=0+) voltage across the inductor? What is the intial(t=0+) current through the inductor?

5V, 0A

b. What is the DC steady state current through the inductor at t approaches ∞.

0A

c. Symbollically (no values, just R, L, C etc.), what is the differential equation defining the voltage across the inductor? VL+VR+VC=VS, IL=IC=IR=IS dd/dtt(IL) + d/dt(IL) R/L + 1/LC*IL = 1/L*d/dt(VS)

LdIL/dt + IL R + 1/C*int(IL) = VS

d. For R = 4kΩ, determine the voltage across the inductor as a function f time for t>0. (Hint: Use differential equation for current through the inductor. Then use the differential relationship between inductor current and inductor voltage.) a=R/2L=2E5, w=1/rt(LC)=1E4, a>w, overD

IL=y, y''+y' 4E5 + 1E8*y = 1E2*VS' a1=-250, a2=-4E5, y(t)=A1e^{250t}+A2e^{4E5t}

e. For R = 200Ω, determine the voltage across the inductor as a function of time for t >0.

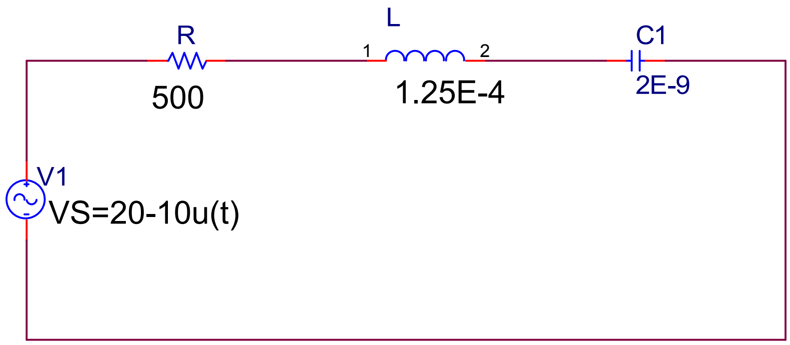
f. For R = 50Ω, determine the voltage across the inductor as a functin of time for t>0.

d) y(0)=A1e^0+A2e^0=0Amps, A1+A2=0, A1=-A2 1599A2=2, A2=2/1599~=1.25E-3, A1~-1.25E-3
d/dt(IL)*L=VL, d/dt(IL)=VL/L=5E2 y'(0)=250A1e^0+4E5A2e^0=500, 250A1+4E5A2=500, A1+1600A2=2
IL(t)=-1.25E-3*e^{250t}+1.25E-3*e^{4E5t}, V
VL(t)=-1.25E-3*250E-2*e^{250t}+1.25E-3*4E5E-2*e^{4E5t}=-3.125E-3*e^{250t}+5*e^{4E5t}
VL(t)=-3.125E-3*e^{250t}+5*e^{4E5t}

e) a=R/2L=1E4, w=1/rt(LC)=1E4, a=w, critD a1=a2=1E4 IL(t)=A1e^{-1E4t}+A2te^{-1E4t}
IL(0)=0=A1, A1=0, IL(t)=A2te^{-1E4t}, IL'(t)=A2e^{-1E4t}-1E4A2te^{-1E4t}, IL'(0)=A2=5E2, A2=500
IL(t)=500te^{-1E4t}, VL(t)=5*e^{-1E4t}-5E4*te^{-1E4t}

f) a=R/2L=2.5E3, w=1E4, a<w, underD, B=9682.5, IL(t)=e^{-2.5E3t}[A1cos(9682t)+A2sin(9682t)]
IL(0)=0=A1, A1=0, IL(t)=e^{-2.5E3t}[A2sin(9682t)],
IL'(t)=[-2.5E3e^{-2.5E3t}A2sin(9682t)]+[e^{-2.5E3t}9682A2cos(9682t)]
IL'(0)=9682A2=5E2, A2=.0516, IL(t)=e^{-2.5E3t}[0.0516sin(9682t)]
VL(t)=[.01*-2.5E3e^{-2.5E3t}0.0516sin(9682t)]+[.01*e^{-2.5E3t}0.0516*9682sin(9682t)]
VL(t)=[-1.29e^{-2.5E3t}sin(9682t)]+[5e^{-2.5E3t}sin(9682t)]

4) RLC series design problem



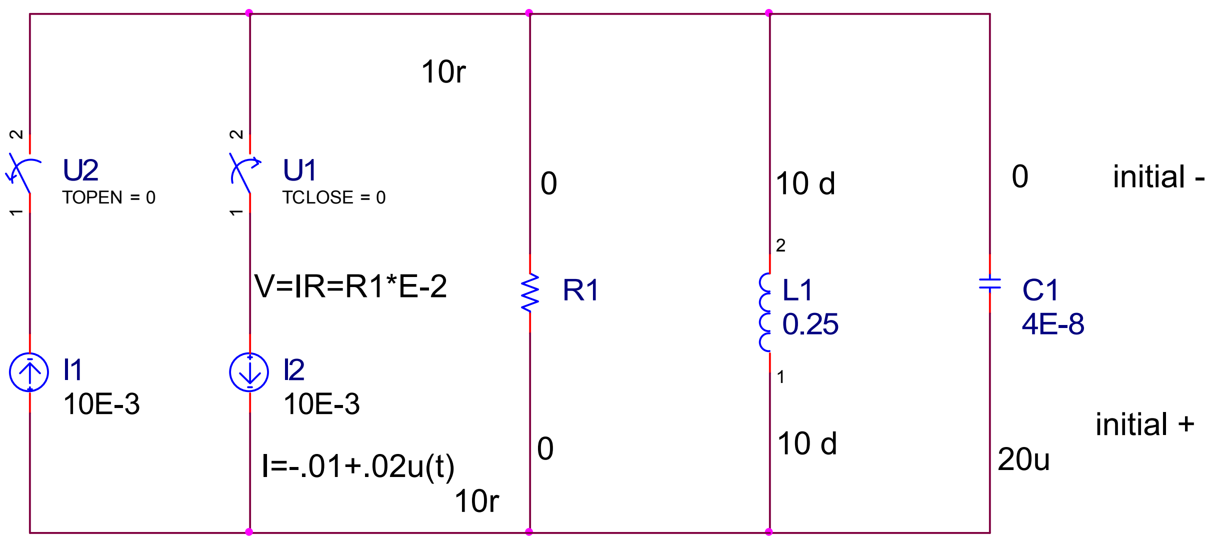
The above circuit has a capacitor voltage defined as

$$V_C(t) = 2E7t \cdot \exp(-2E6t) + 10 \cdot \exp(-2E6t) + 10$$

Determine the resistance, inductance, and source expression (the souce is a step function of some kind).

critically damped: VC(0)=20, VC(infty)=10, VS=20-10u(t)
a=w=1/sqrt(LC)=R/2L=2E6, 5E-7=sqrt(LC), 2.5E-13=LC, L=1.25E-4
R/2L=2E6, R=4E6*1.25E-4=500

5) RLC Parallel Circuits



At t =0, U1 closes and U2 opens.

a. What is the intial (t=0+) current through the capacitor? What is the initial (t=0+) voltage across the capacitor?

.02A, 0V

b. What is the DC steady state current though the capacitor as t goes to infinity?

0A

c. Find the current through the **CAPACITOR** as a function of time for R = 12.5k. (***Hint: Find the voltage across the capacitor equation first then use the current-voltage relationship of a capacitor to get the current! The reason why is because we know the intial conditions for a capacitor voltage not for capacitor current which is necessary to solve the problem)***

a=1E3, w=1E4, a<w, underD, B=9950, VC(t)=[e^{-1E3t}A1cos(9950t)]+[e^{-1E3t}A2sin(9950)]

d. Find the current through the **CAPACITOR** as a function of time for R = 0.25k.

c) VC(0)=0=A1, A1=0, VC(t)=e^{-1E3t}A2sin(9950t),
IC(t)=C*d/dt(VC(t))=[-1E3*4E-8*e^{-1E3t}A2sin(9950t)]+[9950*4E-8*e^{-1E3t}A2cos(9950t)]
IC(t)=[-4E-5*e^{-1E3t}A2sin(9950t)]+[3.97E-4*e^{-1E3t}A2cos(9950t)]
IC(0)=0.02=3.97E-4*A2, A2=50.25, VC(t)=50.25*e^{-1E3t}*sin(9950t),
IC(t)=4E-8*50.25*[[-1E3*e^{-1E3t}*sin(9950t)]+[9950*e^{-1E3t}*cos(9950t)]]
IC(t)=[-2.01E-3*e^{-1E3t}*sin(9950t)]+[2E-2*e^{-1E3t}*cos(9950t)]

d) a=5E4, w=1E4, a>w, overD, a1=-1010. a2=-98990, VC(t)=A1e^{1010t}+A2e^{98990t}
VC(0)=0=A1+A2, A1=-A2, IC(t)=4E-8[1010A1e^{1010t}+98990A2e^{98990t}]
IC(t)=4.04E-5A1e^{1010t}+3.95E-3A2e^{98990t}
IC(0)=2E-2=-4.04E-5A2+3.95E-3A2=3.92E-3A2, 3.92E-3A2=2E-2, A2=5.103, A1=-5.103
VC(t)=-5.103*e^{1010t}+5.103*e^{98990t}
IC(t)=-2.06E-4*e^{1010t}+2.02E-2*e^{98990t}