Electricity

$\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}\hat{r}$ (Coulomb's Law)

$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\hat{r}$ (in N/C or V/m)

$E = \frac{qz}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$ (charged ring)

z = axial distance R = radius of ring

$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{z^2}$ (charged ring @ large distance)

$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$ (charged disk)

$E = \frac{\sigma}{2\epsilon_0}$ (infinite sheet) σ = uniform charge density

$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ (in $V\cdot m$ or $\frac{N\cdot m^2}{C}$)

$\epsilon_0 \Phi_E = q_{enc}$ (Gauss' Law)

$E = \frac{\sigma}{\epsilon_0}$ (conducting surface)

$E = \frac{\lambda}{2\pi\epsilon_0 r}$ (∞-long line of charge)

λ = linear charge density r = radial distance

$E = \frac{\sigma}{2\epsilon_0}$ (sheet of charge)

$\frac{(\text{charge enclosed by sphere of radius } r)}{(\text{volume enclosed by sphere of radius } r)} = \frac{\text{full charge}}{\text{full volume}}$

$E = \left(\frac{q}{4\pi\epsilon_0 R^3}\right) r$ (uniform charge, field at $r \le R$)

$\Delta V = V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$

$\Delta V = \frac{\Delta U}{q}$ $V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$

$V = \int dV = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$ (due to continuous charge distribution)

$U = q_2 V_1 = \left(\frac{q_1}{4\pi\epsilon_0 r}\right) q_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r}$

$V = \frac{\lambda}{4\pi\epsilon_0} \ln\left[\frac{L + (L^2+d^2)^{1/2}}{d}\right]$ (line of charge)

λ = charge density L = length of rod

d = $\perp$ distance from left end of rod

$V = \frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+R^2} - z\right)$ (charged disk)

$E_x = -\frac{\partial V}{\partial x}$ $E_y = -\frac{\partial V}{\partial y}$ $E_z = -\frac{\partial V}{\partial z}$ $\vec{E} = -\vec{\nabla}V$

Capacitors

$q = CV$ $C = \frac{\epsilon_0 A}{d}$ (parallel-plate)

$C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}$ (cylindrical)

$C = 4\pi\epsilon_0 \frac{ab}{b-a}$ (spherical)

$C = 4\pi\epsilon_0 R$ (isolated sphere)

L = length of capacitor

a = inner radius b = outer radius

$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

$u = \frac{1}{2}\epsilon_0 E^2$ (energy density of parallel plate capacitor) (in J/m^3)

Capacitors in Parallel

$C = \Sigma C_i$ $q_T = q_1 + q_2 + \cdots$

$V_T = V_1 = V_2 = \cdots$

Capacitors in Series

$\frac{1}{C_T} = \Sigma \frac{1}{C_i}$ $q_T = q_1 = q_2 = \cdots$

$V_T = V_1 + V_2 + \cdots$

Current

$i = \frac{dq}{dt} = \int \vec{J}\cdot d\vec{A}$ (in $A = C/s$)

$J = I/A$ (in A/m^2) (current density)

$\vec{J} = (ne)\vec{v}_d$ ne = carrier charge density

$V = IR$ or $I = \frac{V}{R}$ or $R = \frac{V}{I}$

Resistors

$\rho = \frac{E}{J}$ (in $\Omega\cdot m$) (resistivity)

$\sigma = \frac{1}{\rho}$ (conductivity)

$\vec{J} = \sigma\vec{E}$

$R = \rho\frac{L}{A}$ L = length A = cross-sectional area

$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$ (resistivity changes with temp.)

$-\alpha$ is for semiconductor

Dielectric present: $\kappa\epsilon_0$ replaces ϵ_0

$P = iV = \frac{dU}{dt}$ (in W)

$P = iV = i^2 R = \frac{V^2}{R}$ (resistive dissipation)

Resistors in Parallel

$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$ $\Delta V_T = V_1 = V_2 = \cdots$

$i_T = i_1 + i_2 + \cdots$

Resistors in Series

$R_T = R_1 + R_2 + \cdots$ $V_T = V_1 + V_2 + \cdots$

$i_T = i_1 = i_2 = \cdots$

Charging a Capacitor

$q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$

$i = \frac{dq}{dt} = \left(\frac{\mathcal{E}}{R}\right)e^{-t/RC}$

$V_C = \frac{q}{C} = \mathcal{E}\left(1 - e^{-t/RC}\right)$

Discharging a Capacitor

$q = q_0 e^{-t/RC}$ $\tau = RC$ (time constant)

$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right)e^{-t/RC}$

Magnetism

$\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{q\vec{v}\times\vec{r}}{r^3}$ (due to a moving charge)

$B = \frac{\mu_0 i}{2\pi r}$ (long straight wire)

$B = \frac{\mu_0 i}{4\pi r}$ (semi∞ straight wire)

$B = \frac{\mu_0 i \phi}{4\pi r}$ (center of circular arc)

$B = \frac{\mu_0 i}{2r}$ (center of full circle)

$B = \left(\frac{\mu_0 i}{2\pi R^2}\right) r$ (inside straight wire) R = radius of wire r = radius of loop

$i_{enc} = i\frac{\pi r^2}{\pi R^2}$ (inside straight wire)

$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc}$ (Ampere's Law) (in $T\cdot m$)

$\vec{F}_B = q\vec{v}\times\vec{B}$ (on moving charge)

$\vec{F}_B = i\vec{L}\times\vec{B}$ (on wire with current)

$B = \mu_0 n i_0$ (inside a solenoid)

$B = 0$ (outside a solenoid)

$B = \frac{\mu_0 i N}{2\pi} \cdot \frac{1}{r}$ (in a toroid) $r \leftarrow$ radius of toroid circle

$\Phi_B = \oint \vec{B}\cdot d\vec{A}$ (in $T\cdot m^2 = Wb$)

$\mathcal{E}_{ind} = -N\frac{d\Phi_B}{dt}$ (coil of N turns)

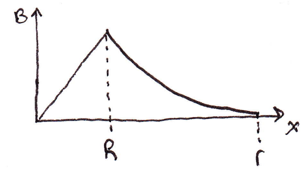
$\Phi_B = BA$ (for uniform magnetic field $\vec{B}$)

$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}BLx = BL\frac{dx}{dt} = BLv$

$i = \frac{BLv}{R}$ (induced current) from $\mathcal{E} = IR$

$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ (Faraday's Law)

Magnetic Field Inside/Outside Long Wire:



Inductance

$\Phi_B = \int \vec{B}\cdot d\vec{A}$ (magnetic flux through area A)

$\Phi_B = BA$ (uniform $\vec{B}$, $\vec{B}\perp\vec{A}$)

$\mathcal{E} = -\frac{d\Phi_B}{dt}$ (Faraday's Law)

$\mathcal{E} = -N\frac{d\Phi_B}{dt}$ (coil of N turns)

$L = \frac{N\Phi_B}{i}$ or $\Phi_B = Li$ (Inductance)

$\frac{L}{l} = \mu_0 n^2 A$ (Inductance of Solenoid per unit length)

L = inductance l = length of solenoid

n = # turns/unit length A = cross-sectional area

(inductance, L, in H, henries)

RL Circuits

$L\frac{di}{dt} + Ri = \mathcal{E}$

$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right)$ (Rise of current)

$\tau_L = \frac{L}{R}$ (time constant)

$i = \frac{\mathcal{E}}{R}e^{-t/\tau_L}$ (decay of current)

Transformers

$V_s = V_p\frac{N_s}{N_p}$ (transformation of voltage)

$I_s = I_p\frac{N_p}{N_s}$ (transformation of current)

$R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R$ (equivalent resistance)

p: primary s: secondary

LC Circuits

$\omega = \frac{1}{\sqrt{LC}}$

$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$

$q = Q\cos(\omega t + \phi)$ (charge)

$i = \frac{dq}{dt} = -\omega Q\sin(\omega t + \phi)$ (current)

$I = -\omega Q$ (current amplitude)

U_E and U_B: replace "q" with proper eqn.

RLC Circuits

("damped")

$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$

$q = Q^{-Rt/2L}\cos(\omega' t + \phi)$ (charge)

$\omega' = \omega_d = \sqrt{\omega^2 - \left(\frac{R}{2L}\right)^2}$ (driving frequency)

$\omega = \frac{1}{\sqrt{LC}}$ (natural frequency)

Reactances

$X_R = R$ $X_C = \frac{1}{\omega C}$ $X_L = \omega L$

$X_{total} = \sqrt{R^2 + (X_C - X_L)^2}$

$V = IX$ (for R, C, L, or total)

$V_0 = \sqrt{V_R^2 + (V_C - V_L)^2}$

$V_s(t) = V_0\sin(\omega t + \phi_0)$ (driving voltage)

$i(t) = I\sin(\omega t)$ (results from V_s)

$I = \frac{\mathcal{E}_m}{\sqrt{R^2 + \left(\omega_d L - \frac{1}{\omega_d C}\right)^2}}$ (current amplitude)

$\tan\phi = \frac{X_L - X_C}{R}$ (phase constant)

if $\omega_d = \omega = \frac{1}{\sqrt{LC}}$ (resonance)

$I_{rms} = \frac{I_{max}}{\sqrt{2}}$ $V_{rms} = \frac{V_{max}}{\sqrt{2}}$

$\mathcal{E}_{rms} = \frac{\mathcal{E}_{max}}{\sqrt{2}}$ (root mean square)

$P_{avg} = I_{rms}^2 R = \mathcal{E}_{rms} I_{rms}\cos\phi$ (average power)

Maxwell's Equations

$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0} = \Phi_E$ (Gauss' Law; Electric Fields)

$\oint \vec{B}\cdot d\vec{A} = 0$ (Gauss' Law; Magnetic Fields)

$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ (Faraday's Law)

$\oint \vec{B}\cdot d\vec{s} = \mu_0\epsilon_0\frac{d\Phi_E}{dt} + \mu_0 i_{enc}$ (Ampere-Maxwell Law)

$i_d = \epsilon_0\frac{d\Phi}{dt}$ (displacement current)

$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r$ (magnetic field inside a circular capacitor)

$B = \frac{\mu_0 i_d}{2\pi r}$ (magnetic field outside a circular capacitor)

Mutual Inductance

$M_{21} = \frac{N_2\Phi_{21}}{i_1} = M_{12} = M$

$\mathcal{E}_2 = -M\frac{di_1}{dt}$ $\mathcal{E}_1 = -M\frac{di_2}{dt}$

Simple Harmonic Motion

$T = \frac{1}{f}$ $\quad \omega = 2\pi f$ $\quad k$ = wave constant

$k = \frac{2\pi}{\lambda} \Longleftrightarrow \lambda = \frac{2\pi}{k}$

$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$ (wave speed)

$y(x,t) = y_m \sin(kx - \omega t)$

$y(x,t) = y_m \sin[k(x - vt)]$ (format)

$B = \frac{-\Delta p}{\Delta V / V}$ (Bulk modulus)

$V = \sqrt{\frac{B}{\rho}}$ ρ = density (wave speed)

$\Delta p(x,t) = \Delta p_m \sin(kx - \omega t)$ (change in pressure)

$\Delta p_m = (v\rho\omega)s_m$ (pressure amplitude)

$kx \pm \omega t$: phase → if $+ \Rightarrow -x$ direction
if $- \Rightarrow +x$ direction

Potential Energy

$U_E = \frac{q^2}{2C}$ (electric potential energy)

$U_B = \frac{Li^2}{2}$ (magnetic potential energy)

$u_B = \frac{B^2}{2\mu_0}$ (magnetic energy density)

Electromagnetic Waves

$E = E_m \sin(kx - \omega t)$ (electric field)

$B = B_m \sin(kx - \omega t)$ (magnetic field)

$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3.0 \cdot 10^8 \text{ m/s}$

$\frac{E}{B} = c \quad \frac{E_m}{B_m} = c$ (magnitude and amplitude ratio)

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$ (Poynting Vector)

$S = \frac{1}{\mu_0} EB = \frac{1}{c\mu_0} E^2$ (instantaneous energy flow rate)

$S_{avg} = I = \frac{1}{c\mu_0} E_{rms}^2$ (intensity)

(S and I in W/m^2 or $\frac{\text{watts}}{(\text{meter})^2}$)

$E_{rms} = \frac{E_{max}}{\sqrt{2}}$

$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{B^2}{2\mu_0}$ (electric energy density)

$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2}$ (intensity from an isotropic point source)

P_s = source power $\quad 4\pi r^2$ = SA of sphere

$\Delta p = \frac{\Delta U}{c}$ (momentum change, total absorption)

$\Delta p = \frac{2\Delta U}{c}$ (total reflection)

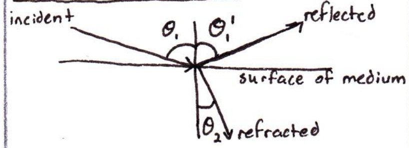
$p_r = \frac{I}{c}$ (total absorption)

$p_r = \frac{2I}{c}$ (total reflection)

$I = \frac{1}{2} I_0$ (intensity of polarized light that was polarized from randomized light)

$I = I_0 \cos^2\theta$ (intensity of polarized light that was polarized from polarized light)

Reflection/Refraction



$\theta_1' = \theta_1$ (angle of reflection)

$n_2 \sin\theta_2 = n_1 \sin\theta_1$ (angle of refraction)

$\theta_c = \sin^{-1}\frac{n_2}{n_1}$ (critical angle)

$\theta_1 > \theta_c \Rightarrow$ (total internal reflection)

$\theta_B = \tan^{-1}\frac{n_2}{n_1}$ (Brewster Angle; the angle at which reflected light is polarized)

Interference

$n = \frac{c}{v}$ (index of refraction)

$\lambda_n = \frac{\lambda}{n} \qquad N = \frac{L}{\lambda}$ $\quad N$ = # of wavelengths, L = length of medium

$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$ $\quad \lambda$ = wavelength in medium

$\Delta\phi = \# \cdot 2\pi = 2\pi \frac{L}{\lambda_i}(n_i - 1)$

= # of the fringe

$\Delta L = d\sin\theta$ (path length difference)

$d\sin\theta = m\lambda$, $m = 0,1,2\ldots$ (bright fringes)

$d\sin\theta = (m + \frac{1}{2})\lambda$, $m = 0,1,2\ldots$ (dark fringes)

$E_1 = E_0 \sin\omega t$

$E_2 = E_0 \sin(\omega t + \phi)$

$I = 4I_0\cos^2(\frac{1}{2}\phi)$ (intensity at P in double-slit interference)

$\phi = \frac{2\pi d}{\lambda}\sin\theta$

$I = \frac{E_m^2}{2\mu_0 c}$ (interference)

$\Delta\phi_1 = L_1 \frac{2\pi}{\lambda} \qquad \Delta\phi_2 = L_2 \frac{2\pi}{\lambda}$ (path length)

$\Delta\phi_{12} = (L_2 - L_1)\frac{2\pi}{\lambda}$ (path difference)

Diffraction

$a\sin\theta = \lambda$ (first minimum)

$a\sin\theta = m\lambda$ for $m = 1,2,3\ldots$

$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \left(\frac{2\pi}{\lambda}\right)\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix}$

$\Delta\phi = \left(\frac{2\pi}{\lambda}\right)(\Delta x \sin\theta)$

$I(\theta) = I_m\left(\frac{\sin\alpha}{\alpha}\right)^2$ (intensity in single-slit diffraction)

$\alpha = \frac{1}{2}\phi = \frac{\pi a}{\lambda}\sin\theta$

$\sin\theta = 1.22\frac{\lambda}{d}$ (first minimum, circular aperture)

d = diameter

$\theta_R = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) \approx \frac{1.22\lambda}{d}$ (Rayleigh's criterion)

if $\theta > \theta_R$ → resolvable

if $\theta < \theta_R$ → not resolvable

$I(\theta) = I_m(\cos^2\beta)\left(\frac{\sin\alpha}{\alpha}\right)^2$ (double-slit, interference and diffraction)

$\beta = \frac{\pi d}{\lambda}\sin\theta \qquad \alpha = \frac{\pi a}{\lambda}\sin\theta$

$\cos^2\beta$ → interference factor

$\left(\frac{\sin\alpha}{\alpha}\right)^2$ → diffraction factor

$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta}$ (half width of line at θ)

Relativity

Special Theory of Relativity Postulates:

① laws of physics are same for observers in all inertial frames

② speed of light in vacuum c has the same value c in all directions and in all inertial reference frames

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(v/c)^2}}$ (Lorentz Factor)

$\Delta t = \gamma\Delta t_0$ (time dilation)

Δt = proper time

Observers moving relative to inertial reference frame measure a longer time Δt between the events.

$L = L_0\sqrt{1-(v/c)^2} = \frac{L_0}{\gamma}$ (length contraction)

$E_0 = mc^2$ (mass energy/rest energy)

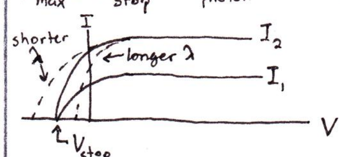
$E_{total} = E_0 + K = mc^2 + K$

$E_{total} = \gamma mc^2$ (total energy)

$K = mc^2(\gamma - 1)$ (kinetic energy)

Photons

$K_{max} = eV_{stop} \qquad E_{photon} = hf = \frac{hc}{\lambda}$



$p = \frac{hf}{c} = \frac{h}{\lambda}$ or $\lambda = \frac{h}{p}$

$K_{max} = E_{photon} - \phi$ $\quad \phi$ = work function

$I = \frac{\text{Total } E}{\text{Time} \cdot \text{Area}}$ (intensity of light)

$I = \frac{\#\text{ photons}}{\text{Time}\cdot\text{Area}} \times E_{photon} = \text{Flux}_{photons} \cdot E_{ph}$

Matter Waves

$\lambda = \frac{h}{p}$ (de Broglie wavelength)

$f = \frac{E}{h} \qquad \Delta x \cdot \Delta p \cong h$ (Heisenberg's Uncertainty Principle)

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

(Schrödinger's Equation)

$-\frac{h^2}{8\pi^2 m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$

ψ = wave function $\quad \psi^2$ = probability density

$P(x_1, x_2) = \int_{x_1}^{x_2} \psi^2\, dx$ (probability)

Potential Wells

$n\left(\frac{\lambda_n}{2}\right) = L$, $n = 1,2,3\ldots$ (energy levels for an infinite well)

$\lambda_n = \frac{2L}{n}$ for $n = 1,2,3\ldots$ (discrete set of de Broglie wavelengths for an infinite well)

$p_n = \frac{h}{\lambda_n} = \frac{nh}{2L}$, $n = 1,2,3\ldots$ (momenta, infinite well)

$E_n = \frac{p_n^2}{2m} = n^2\frac{h^2}{8mL^2}$, $n = 1,2,3\ldots$

(energies of allowed quantum states, infinite well)

$\psi_n(x) = A\sin\left(\frac{n\pi x}{L}\right)$ (wave function)

$\psi_n^2(x) = A^2\sin^2\left(\frac{n\pi x}{L}\right)$ (probability density)

$A = \sqrt{\frac{2}{L}}$

$\lambda = \frac{h}{\sqrt{2mE_n}}$ (finite well)

Electrical Conduction in Solids

$\rho = \frac{m}{ne^2\tau}$ (resistivity)

m = mass of e^-

n = mobile charge density

e = charge of e^-

τ = average time between collisions of e^- w/lattice

$E_F = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3}\frac{h^2}{m}n^{2/3} = \frac{0.121h^2}{m}n^{2/3}$

n = mobile e^- density $\quad$ (Fermi Energy of metals)

$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$ $\quad T$ = temp.

e = natural e $\quad k$ = Boltzmann constant

p-n junctions: $\left.\begin{matrix} p \longleftrightarrow + \\ n \longleftrightarrow - \end{matrix}\right\}$ current flows

$I = I_s\left(e^{eV_0/kT} - 1\right)$ (current)

I_s = saturation current

Units

1 joule = 1 J = 1 N·m = 1 C·V

1 henry = 1 H = 1 $\frac{T\cdot m^2}{A}$ = 1 Wb/A

1 weber = 1 Wb = 1 T·m² = 1 H·A

1 tesla = 1 T = 1 $\frac{N}{A\cdot m}$ = 10^4 gauss

Constants

$\epsilon_0 = 8.85 \cdot 10^{-12}$ F/m or $\frac{C}{N\cdot m^2}$

$\mu_0 = 4\pi \cdot 10^{-7}$ T·m/A or H/m

$k = \frac{1}{4\pi\epsilon_0} = 8.9875518 \cdot 10^9\ \frac{N\cdot m^2}{C^2}$

$m_p = 1.67 \cdot 10^{-27}$ kg

$m_e = 9.11 \cdot 10^{-31}$ kg

$e = 1.602 \cdot 10^{-19}$ C

$k_B = 1.38 \cdot 10^{-23}$ J/K

$k_B = 8.6173 \cdot 10^{-5}$ eV/K