Fields and Waves I

Lecture 18

Magnetic Force and Energy

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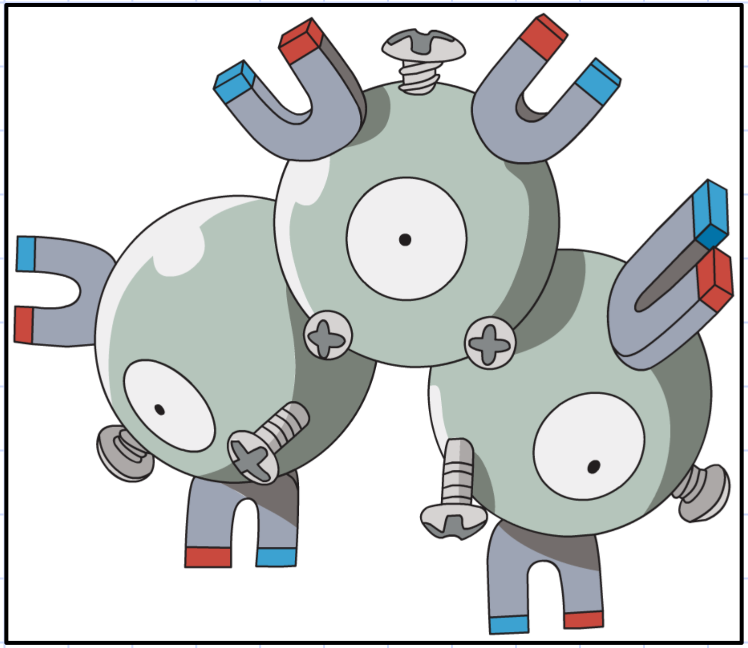
Materials from other sources are referenced where they are used.
Those listed as Ulaby are figures from Ulaby's textbook.

Overview

- Review
- Magnetic Energy
- Magnetic Force
- DC Motors
- Motors vs. Generators
- Wrap-Up



Review

Boundary Conditions

Arguing from analogy with Electric Fields

$$\vec{D} = \epsilon \vec{E} \qquad \vec{B} = \mu \vec{H}$$

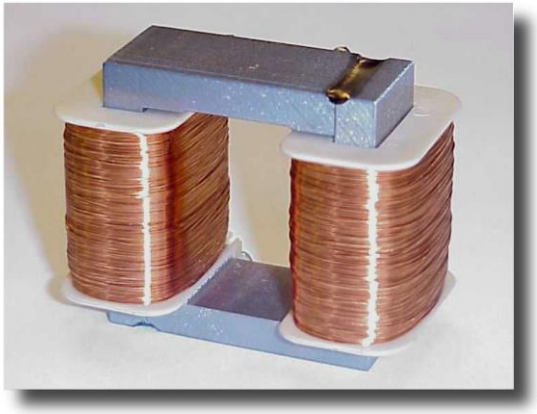
$$\oint \vec{D} \cdot \vec{ds} = Q_{encl} \Rightarrow D_{n1} - D_{n2} = \rho_s$$

$$\oint \vec{B} \cdot \vec{ds} = 0 \Rightarrow B_{n1} - B_{n2} = 0$$

$$\oint \vec{E} \cdot \vec{dl} = 0 \Rightarrow E_{t1} - E_{t2} = 0$$

$$\oint \vec{H} \cdot \vec{dl} = I_{inc} \Rightarrow H_{t1} - H_{t2} = J_s$$

Magnetic Circuits



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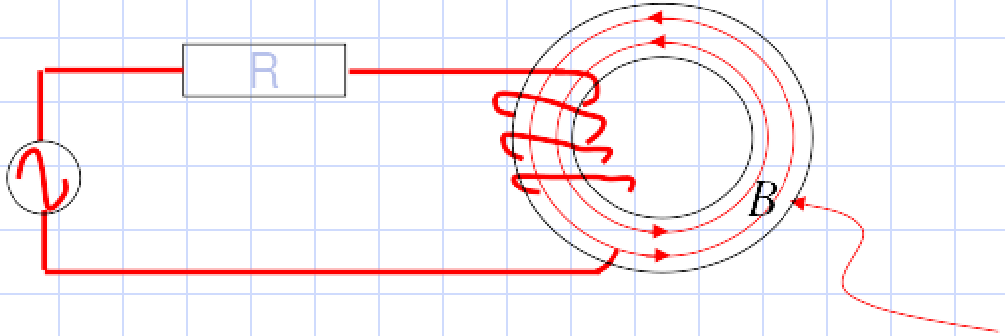
MAGNETIC CIRCUITS used to analyze relays, switches, speakers...

In a simple experiment:



Flux stays in TOROID

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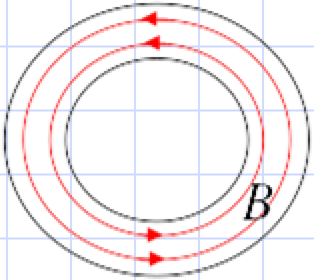
Magnetic Circuits

Flux is a constant = $\int B \bullet ds$

- Flux stays in toroid - so area is a constant

B and H are constant along the path



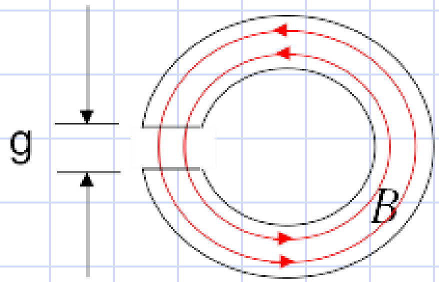
$$\therefore \oint H \bullet dl = 2 \cdot \pi \cdot r \cdot H_{\varphi} = N \cdot I$$

$$\Rightarrow \vec{H} = \frac{N \cdot I}{2 \cdot \pi \cdot r} \hat{a}_{\varphi}$$

$$\Rightarrow L \approx \frac{\mu_0 \cdot \mu_r \cdot N \cdot Area}{2 \cdot \pi \cdot r_{average}}$$

Magnetic Circuits

Introduce an air gap to toroïd:



$$\oint H \bullet d\bar{l} = (2 \cdot \pi \cdot r - g) \cdot H_{iron} + g \cdot H_{gap}$$

Apply boundary conditions across gap:

$$B_{1n} = B_{2n} \Rightarrow \mu_{iron} \cdot H_{n,iron} = \mu_0 \cdot H_{n,gap}$$

Can get very large <u>H</u> in gap $\Rightarrow$ $H_{n,gap} \approx 5000 \cdot H_{n,iron}$

$$\oint H \bullet d\bar{l} = NI \approx (2 \cdot \pi \cdot r + 5000 g) \cdot H_{iron}$$

Gap has very strong effect on H and on energy consumption

Magnetic Circuits

$$\oint H \bullet d\bar{l} = (2\cdot\pi\cdot r - g)\cdot H_{iron} + g\cdot H_{gap}$$

$$\oint \vec{H} \bullet d\vec{l} = N\cdot I = \frac{(2\pi\cdot r - g)}{AREA\cdot\mu}\cdot\underbrace{AREA\cdot B_{iron}}_{\Psi} + \frac{g}{AREA\cdot\mu_0}\cdot\underbrace{AREA\cdot B_{gap}}_{\Psi}$$

Magneto-motive force

Reluctance of iron

Reluctance of gap

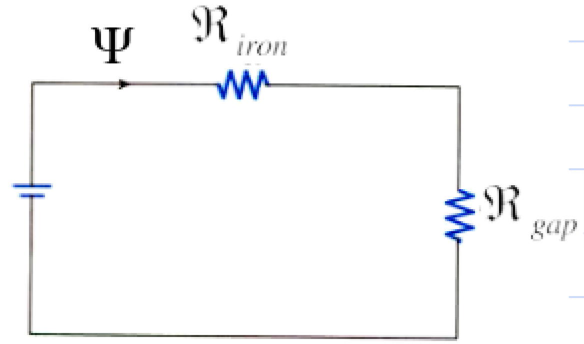
$$= (\Re_{iron} + \Re_{gap})\cdot\Psi$$

- enables us to draw analogy to electric circuits

Magnetic Circuits

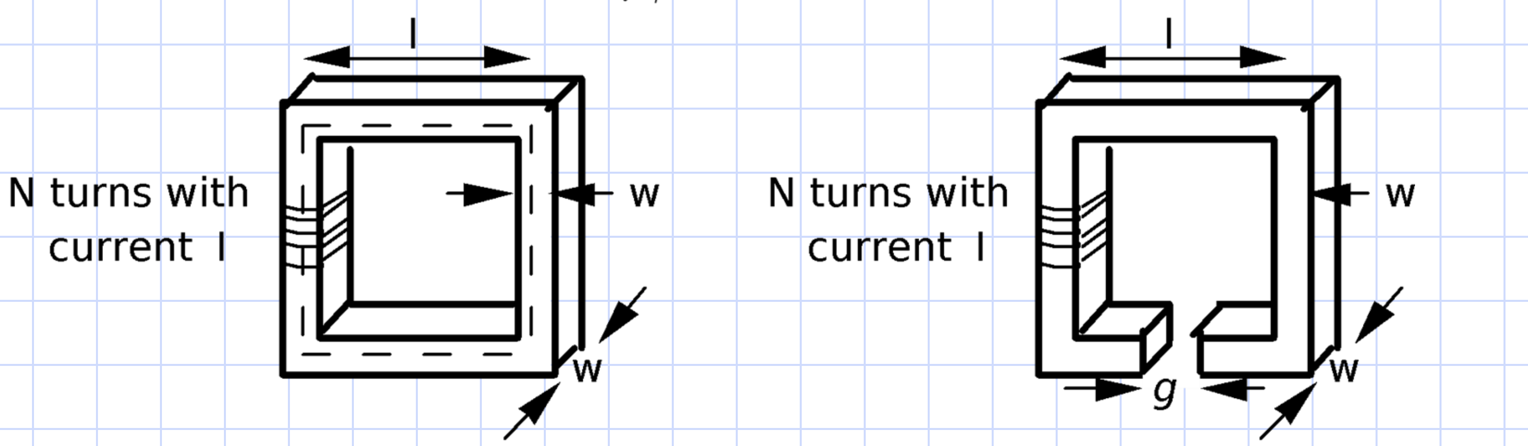
Electric Circuits	Magnetic Circuits	
V or e.m.f	NI or m.m.f	Magneto motive force
I	$\Psi = \int B \bullet ds$	Flux
$R = \dfrac{l}{\sigma \cdot A}$	$\Re = \dfrac{l}{\mu \cdot A}$	High μ - low reluctance path

Magnetic circuits model: $V = NI$



Magnetic Circuits

a. Evaluate $\int \mathbf{H} \cdot d\mathbf{l}$ around the dashed line in the figure on the left below. Then, determine $|\mathbf{H}|$ and $|\mathbf{B}|$ in the iron core. Make reasonable approximations.
b. What is the inductance, L?
c. For the figure on the left, what are the reluctance and magnetomotive force? Draw a magnetic circuit equivalent and show how to solve for the inductance using the circuit.
d. Analyze the situation on the right using magnetic circuits. Determine the flux through the iron core. What is the inductance? What is **H** in the core and in the gap?
e. Calculate numerical values for L, $|\mathbf{H}|_{gap}$ and $|\mathbf{H}|_{core}$ when N = 1000, I = 1 A, w = 5 cm, g = 1 cm, l = 20 cm, and $\mu_r = 5000$



Magnetic Circuits

- $\oint \vec{H} \cdot d\vec{l} = I_{net} = NI$

$|\vec{H}|$ is $\approx$ constant along path since flux stays in iron + area is $\approx$ constant

$\therefore \boxed{|\vec{H}| = \frac{NI}{4l}}$ $\quad \boxed{|\vec{B}| = \mu|\vec{H}| = \frac{\mu NI}{4l}}$

b. $L = \frac{\Lambda}{I} = \frac{N\Psi}{I} = \frac{N|\vec{B}|w^2}{I} = \boxed{\frac{\mu N^2 w^2}{4l} = L}$

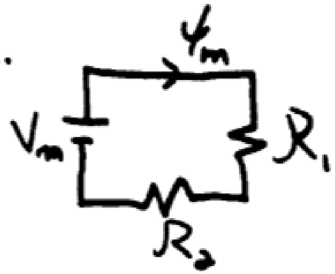
c. reluctance $\boxed{\mathcal{R} = \frac{4l}{\mu w^2}}$ $\quad \boxed{mmf = V_m = NI}$

$\Psi_m = \frac{V_m}{\mathcal{R}} = \frac{NI\mu w^2}{4l}$.

$\boxed{L = \frac{N\Psi_m}{I} = \frac{\mu N^2 w^2}{4l}}$

Magnetic Circuits

d.



$V_m = NI$; $\mathcal{R}_1 = \dfrac{4l - g}{\mu w^2}$ iron reluctance $\mathcal{R}_2 = \dfrac{g}{\mu_0 w^2}$

$$\Psi_m = \frac{V_m}{\mathcal{R}_1 + \mathcal{R}_2} = \frac{NI}{\frac{4l-g}{\mu w^2} + \frac{g}{\mu_0 w^2}} = \boxed{\frac{\mu w^2 NI}{4l - g + \mu_r g}} \approx \frac{\mu w^2 NI}{4l + \mu_r g}$$

$$\boxed{L = \frac{N\Psi_m}{I} = \frac{\mu w^2 N^2}{4l + (\mu_r - 1)g}}$$

$$\Psi_m = B w^2 = \mu H_{core} w^2 = \mu_0 H_{gap} w^2$$

$$H_{core} = \frac{\Psi_m}{\mu w^2} = \frac{NI}{4l + (\mu_r - 1)g} \qquad H_{gap} = \frac{\mu}{\mu_0} H_{core} = \frac{\mu_r NI}{4l + (\mu_r - 1)g}$$

e. $\boxed{L = .309\ \text{H}, \quad H_{core} = 19.7\ \text{A/m} \quad H_{gap} = 9.84 \times 10^4\ \text{A/m}}$

Magnetic Circuits

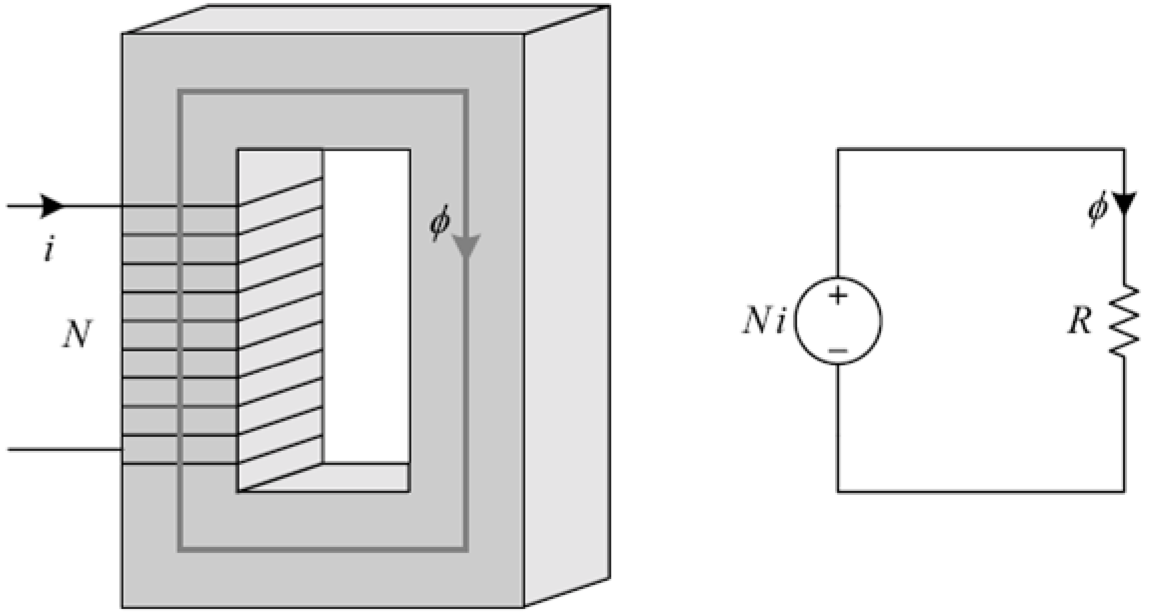
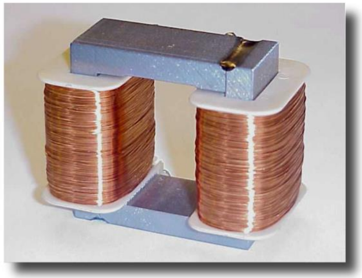


Figure 3-1. Single excited magnetic structure and its magnetic circuit model.

Brushless Permanent Magnet Motor Design, © Duane Hanselman

Magnetic Circuits



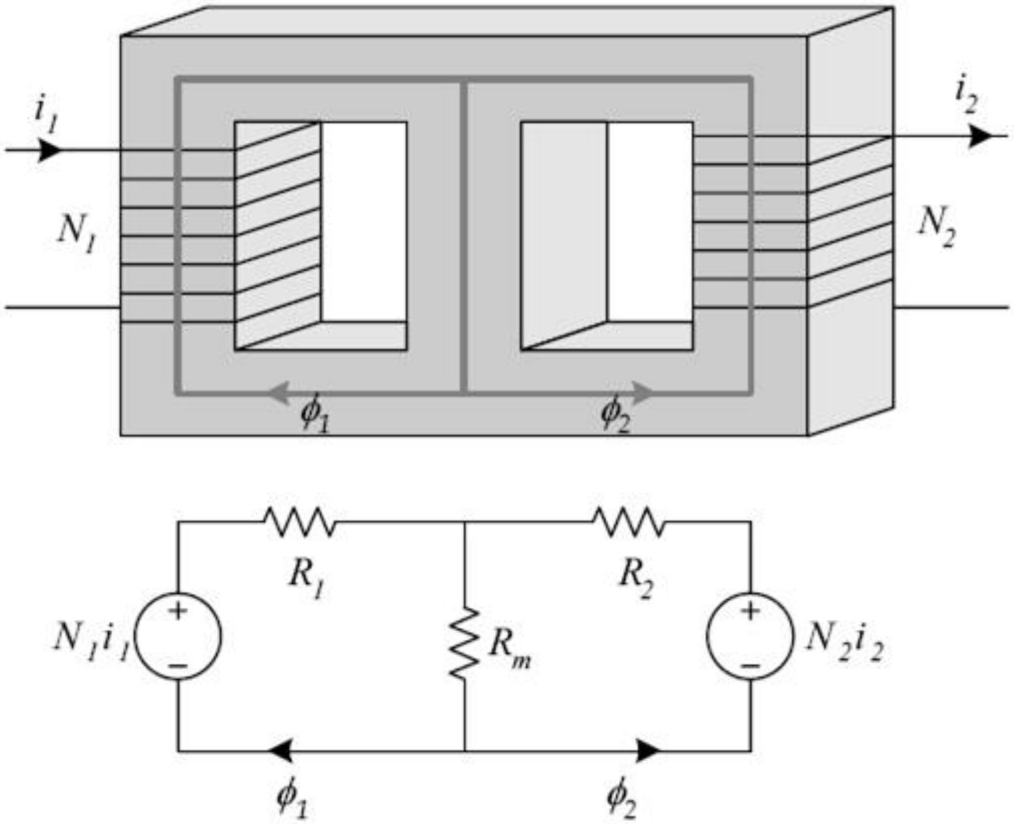


Figure 3-2. Doubly excited magnetic structure and its magnetic circuit model.

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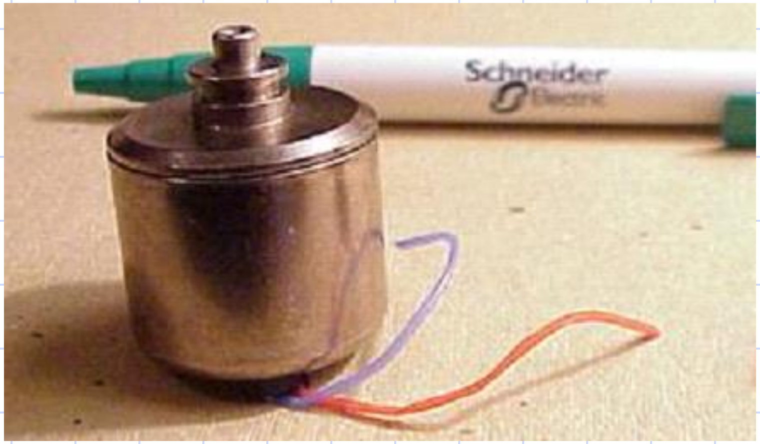
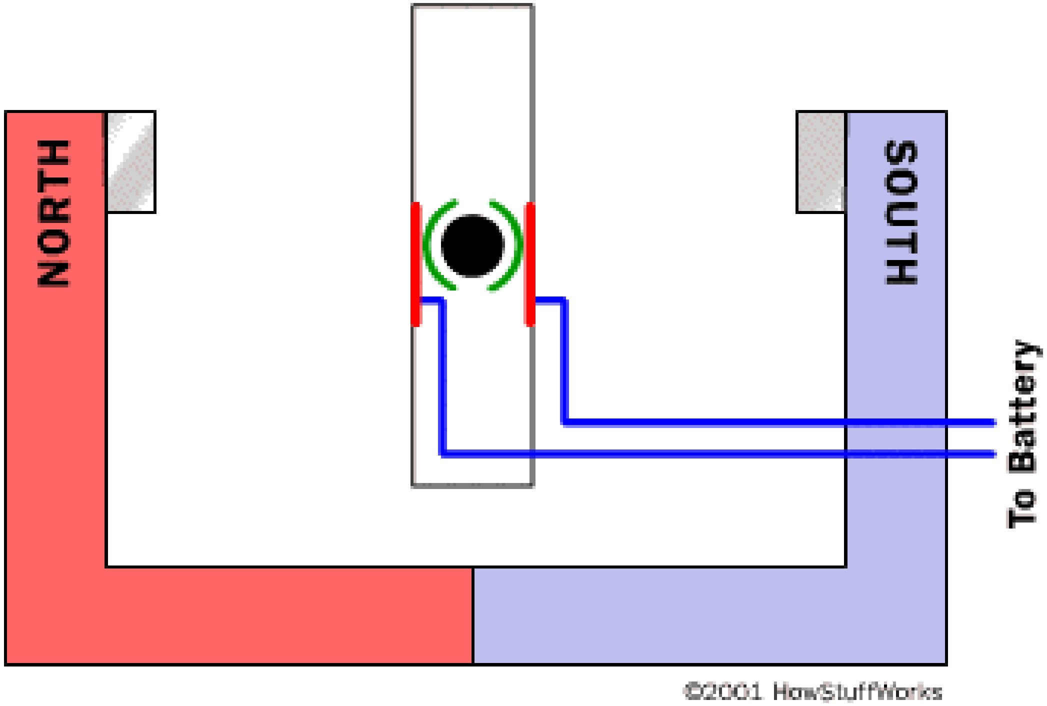
Magnetic Circuits

Do Lecture 18 Exercise 1 with in groups of up to 4.

Applications

DC Motor

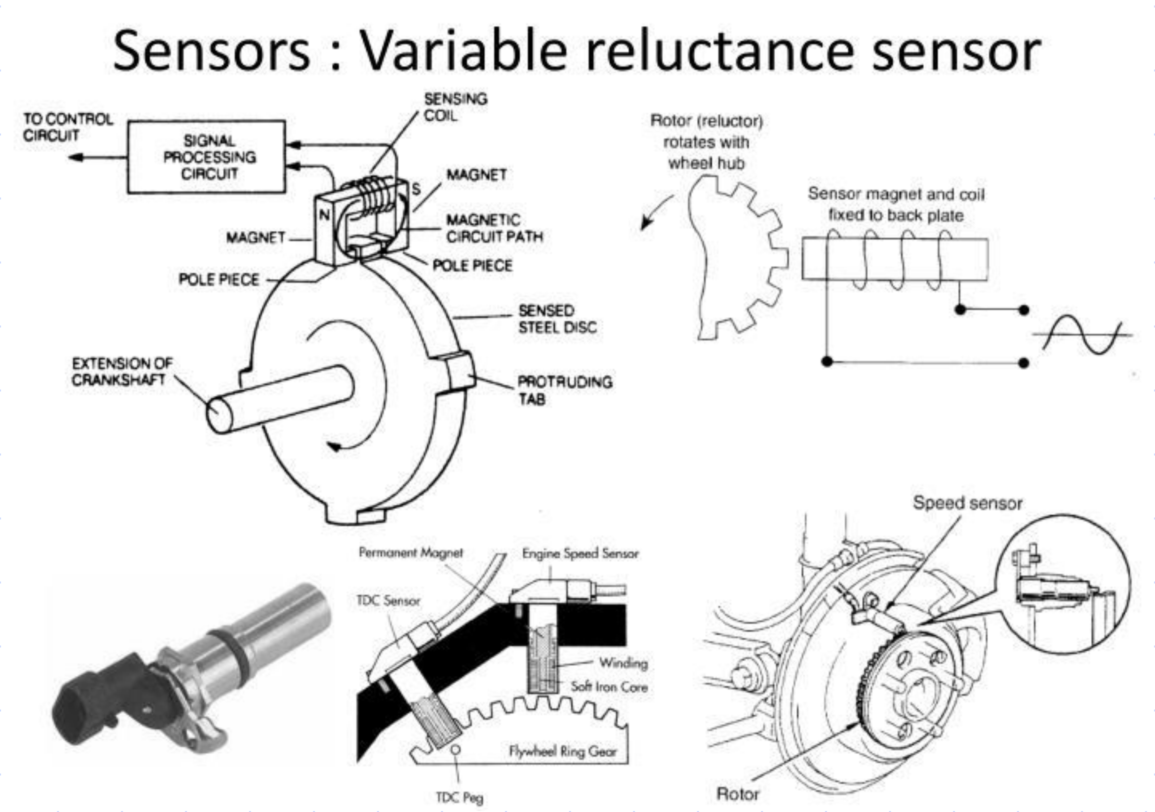




"Variable reluctance circuit"

Applications

Variable Reluctance Sensor



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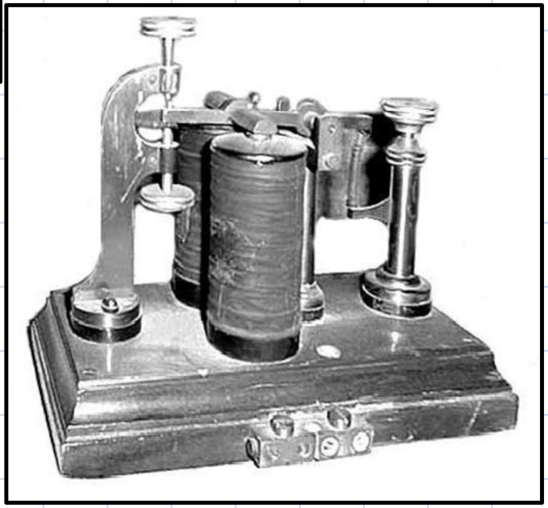
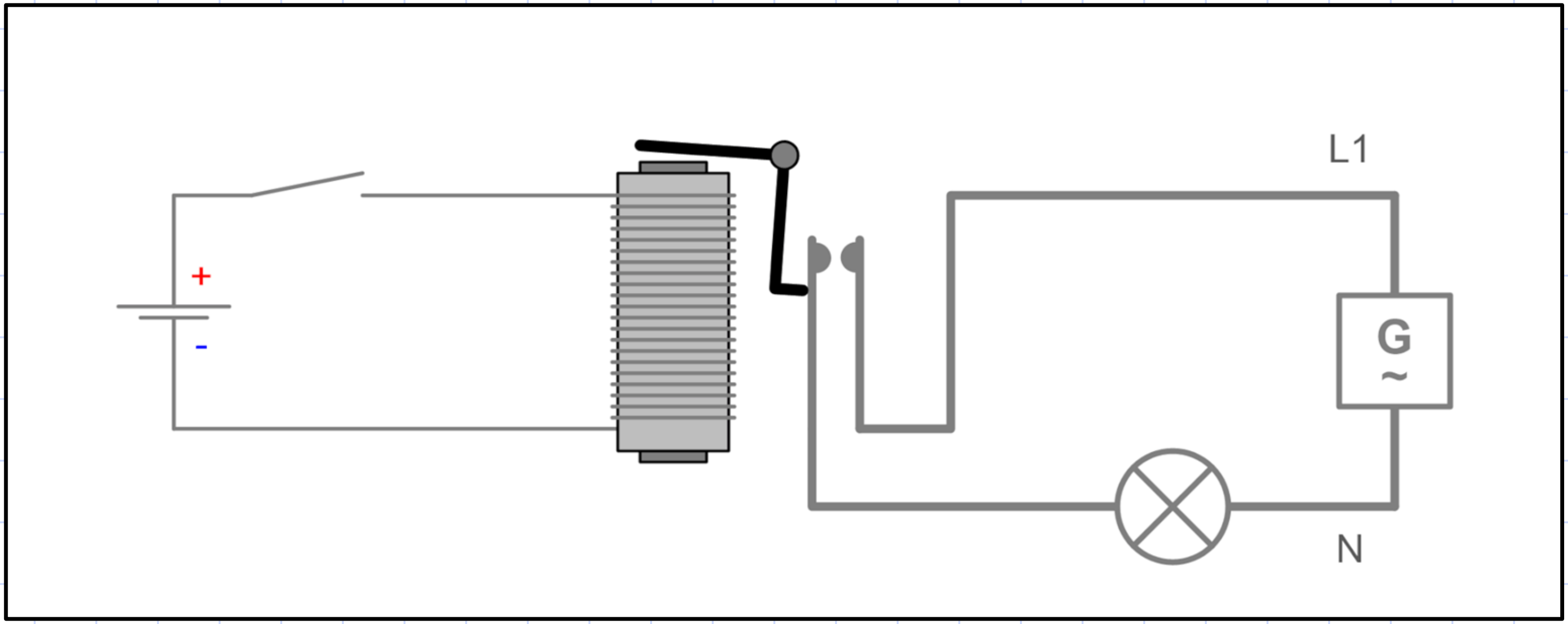
Applications

Relays



Applications

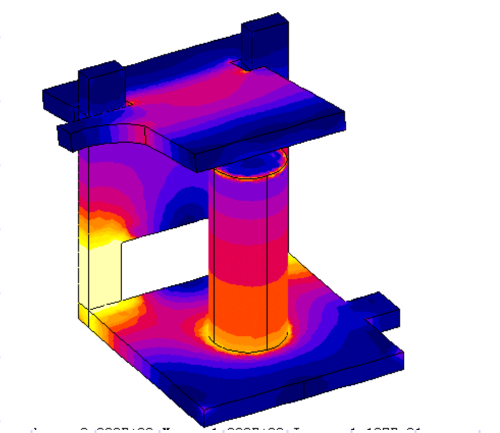
Relays

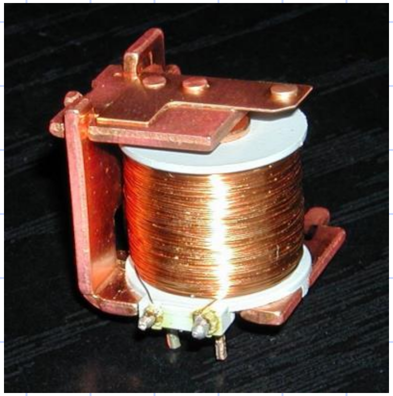


Wikipedia

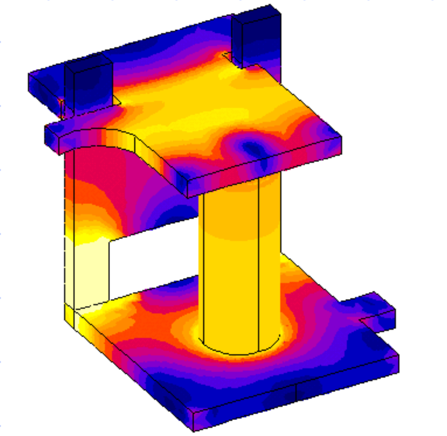
Flux distribution in a relay

High reluctance, low flux density





Low reluctance, High flux density



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MAGSOFT CORPORATION

Electric Energy

- The energy stored in capacitors is stored in the E-field

Define stored energy: $W_e = \frac{1}{2} \cdot CV^2$

Substitute values of C and V for parallel plate capacitor

$$W_e = \frac{1}{2} \cdot CV^2 = \frac{1}{2} \cdot \left(\varepsilon \frac{A}{d} \right) \cdot \left(\left| \vec{E} \right| \cdot d \right)^2 = \underbrace{\frac{1}{2} \cdot \varepsilon \left| \vec{E} \right|^2}_{\text{Energy Density}} \cdot \underbrace{Ad}_{\text{Volume}}$$

Magnetic Energy

Power in inductor:

$$P = I \cdot V = I \cdot L \cdot \frac{dI}{dt} = \frac{d}{dt}\underbrace{\left(\frac{1}{2} \cdot L \cdot I^2\right)}_{\text{energy in Inductor}}$$

Can we obtain energy in terms of <u>*B*</u> *and* <u>*H*</u> *fields ?*

Flux linkage: $L \cdot I = \Lambda = B \times Area \times N$

Also, $I = \frac{\oint H \bullet d\bar{l}}{N} = \frac{H \cdot length}{N}$

Magnetic Energy

Energy = $\frac{1}{2} \cdot L \cdot I^2 = \frac{1}{2} \cdot (L \cdot I) \cdot I = \frac{1}{2} \cdot (B \times Area \times N) \cdot \left(\frac{H \times Length}{N} \right)$

$$= \frac{1}{2} \cdot \int \vec{B} \bullet \vec{H} \cdot dv = W_m$$

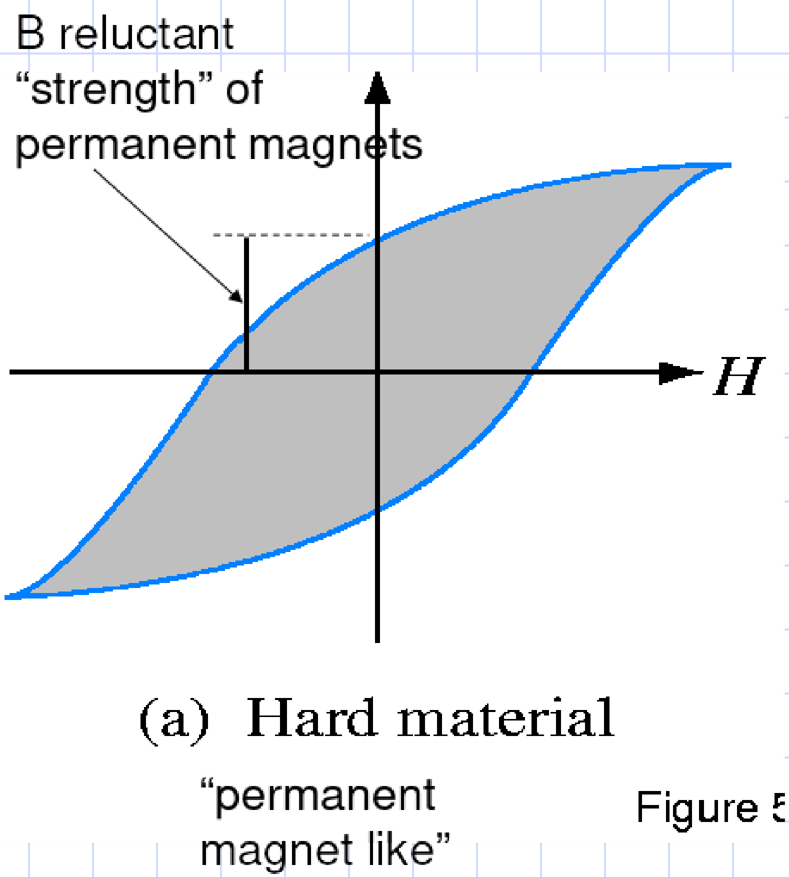
VOLUME

Energy stored in Magnetic field

Energy Density: (per unit volume)

$$\boxed{w_m = \frac{1}{2} \cdot \vec{B} \bullet \vec{H}} = \frac{1}{2} \cdot \frac{B^2}{\mu} = \frac{1}{2} \cdot \mu \cdot H^2$$

Magnetic Energy



- H-field has units of amperes per meter, B-field has units of teslas

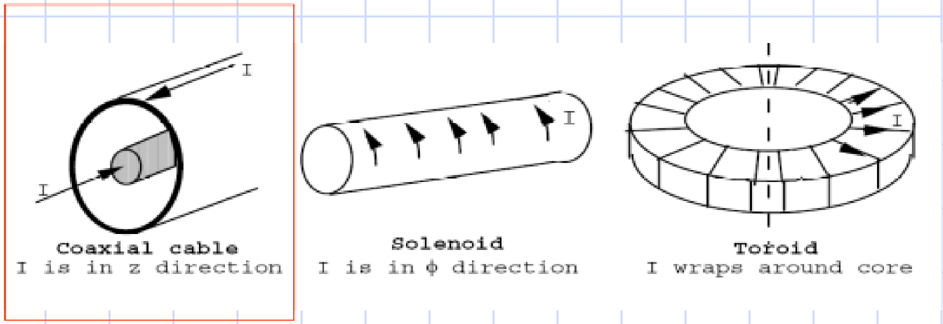
$$\mathrm{T} = \frac{\mathrm{V{\cdot}s}}{\mathrm{m^2}} = \frac{\mathrm{N}}{\mathrm{A{\cdot}m}} = \frac{\mathrm{J}}{\mathrm{A{\cdot}m^2}} = \frac{\mathrm{H{\cdot}A}}{\mathrm{m^2}} = \frac{\mathrm{Wb}}{\mathrm{m^2}} = \frac{\mathrm{kg}}{\mathrm{C{\cdot}s}} = \frac{\mathrm{N{\cdot}s}}{\mathrm{C{\cdot}m}} = \frac{\mathrm{kg}}{\mathrm{A{\cdot}s^2}}$$

- Product of H-field and B-field has units of energy density (J/m^3)
- Area inside the hysteresis curve also has units of J/m^3 and represents hysteresis losses

Magnetic Energy

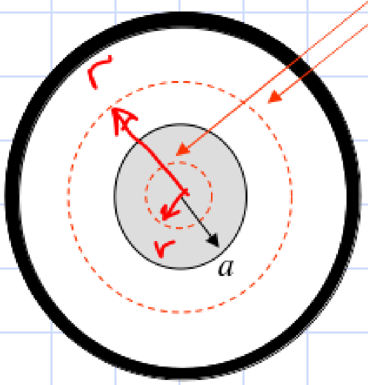
Coaxial Cable

One of the three standard configurations



Detailed Solution for Coax:

Ampere's Law Contours



Ampere's Law $\oint \vec{H} \cdot \vec{dl} = I_{enc}$

Left Hand Side: $\vec{H}_{\phi} 2\pi r$

Right Hand Side: $\begin{cases} I\dfrac{r^2}{a^2} & 0 \leq r \leq a \\ I & a \leq r \leq b \end{cases}$

Magnetic Energy

Coaxial Cable

$$\vec{H} = \hat{\phi}\frac{Ir}{2\pi a^2} \qquad 0 \le r \le a$$

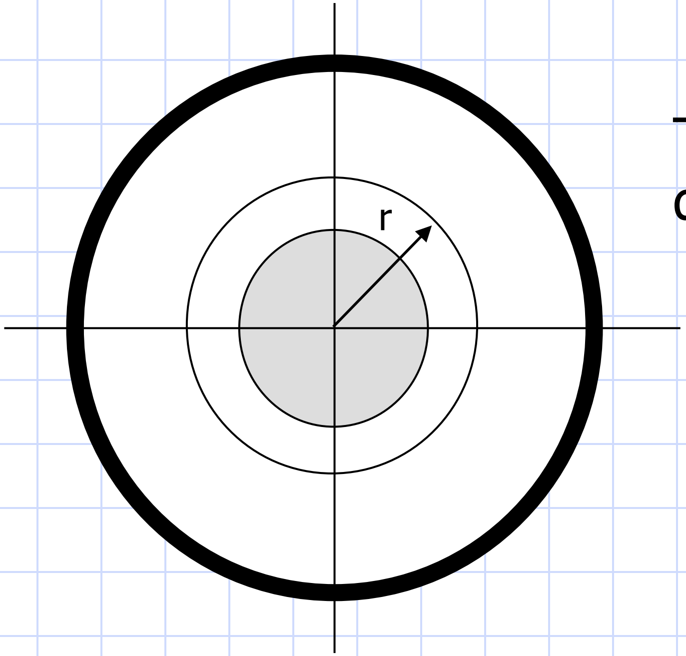
$$\vec{H} = \hat{\phi}\frac{I}{2\pi r} \qquad a \le r \le b$$

$$\vec{H} = 0 \qquad r \ge b$$

Assume the outer conductor is very thin

Magnetic Energy

Coaxial Cable



The energy in the magnetic field can be divided into two terms:

$$W_m = \frac{1}{2}\int (\vec{B}\cdot\vec{H})dv$$

$$W_m = \frac{1}{2} l (2\pi)\left(\int_0^a \mu\left(\frac{Ir}{2\pi a^2}\right)^2 r\,dr + \int_a^b \mu\left(\frac{I}{2\pi r}\right)^2 r\,dr\right)$$

Magnetic Energy

Coaxial Cable

$$W_m = \frac{\mu I^2}{4\pi} l \left(\int_0^a \left(\frac{r}{a^2} \right)^2 r\,dr + \int_a^b \left(\frac{1}{r} \right)^2 r\,dr \right)$$

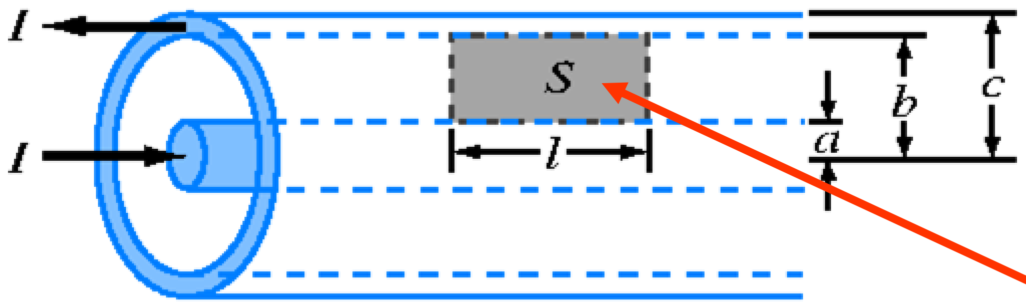
$$W_m = \frac{\mu I^2}{4\pi} l \left(\frac{1}{4} + \ln \frac{b}{a} \right) = \frac{1}{2} L I^2$$

$$L = \frac{\mu_o}{8\pi} l + \frac{\mu_o}{2\pi} l \ln \frac{b}{a} = L_i + L_e$$

$$\boxed{\frac{L}{l} = \frac{\mu_o}{8\pi} + \frac{\mu_o}{2\pi} \ln \frac{b}{a}}$$

Magnetic Energy

Coaxial Cable



Ulaby

To compute the inductance per unit length, we need to determine the magnetic flux through the area S between the conductors

(which dS surface element would we use to do this integral?)

Magnetic Energy

Coaxial Cable

The flux through the surface S:

$$\psi_m = \int \vec{B} \cdot d\vec{S} = l \int_a^b \frac{\mu_o I}{2\pi r} dr = l \frac{\mu_o I}{2\pi} \ln \frac{b}{a}$$

Note that the flux is linked only once since there is only one turn. Thus, the inductance is given by:

$$L = \frac{\Psi_m}{I} = l \frac{\mu_o}{2\pi} \ln \frac{b}{a} \quad \text{or} \quad \frac{L}{l} = \frac{\mu_o}{2\pi} \ln \frac{b}{a}$$

Magnetic Energy

Coaxial Cable

Using the flux:

$$\frac{L}{l} = \frac{\mu_o}{2\pi} \ln \frac{b}{a}$$

External inductance

???

Using the energy:

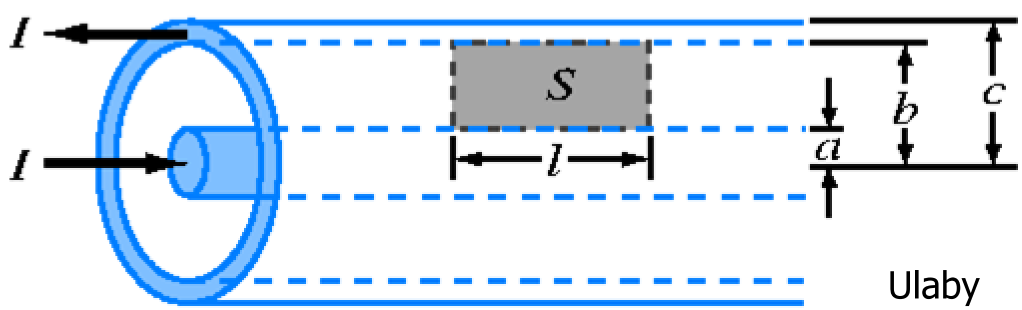
$$\frac{L}{l} = \frac{\mu_o}{8\pi} + \frac{\mu_o}{2\pi} \ln \frac{b}{a}$$

Total inductance

Additional term

Magnetic Energy

Coaxial Cable



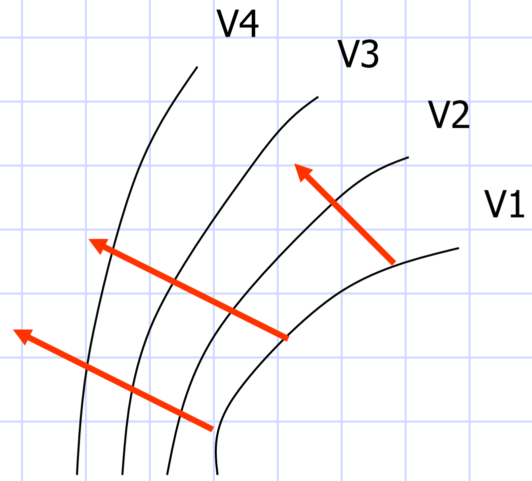
Note that this analysis does not incorporate the flux inside the center conductor so it does not give us the total inductance. However, figuring out the flux linking this current is difficult. Thus we leave this to our method based on energy.

External Inductance: What we have determined is called the external inductance, since it is inductance due to the magnetic field external to the current-carrying wires.

Internal Inductance: What we have neglected is the contribution to the inductance from the field inside the wires.

Magnetic Force

$$\vec{E} = -\nabla V$$



- Gradient points in the direction of largest change
- Therefore, E-field lines are perpendicular (normal) to constant V surfaces
- **We know that electric fields are capable of doing work. Magnetic fields can as well!**

Magnetic Force

Energy Approach

First approach - F does work and changes energy

$$W_m = \int \vec{F} \cdot dl \quad \Rightarrow \quad \vec{F} = -\nabla W_m$$

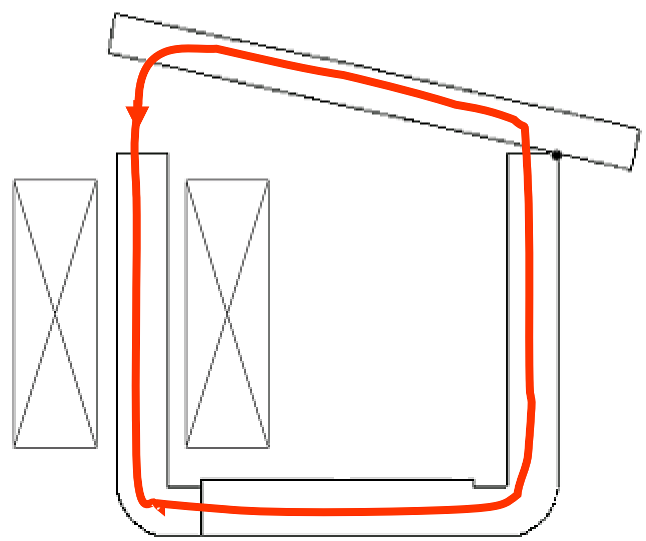
with the energy stored being :

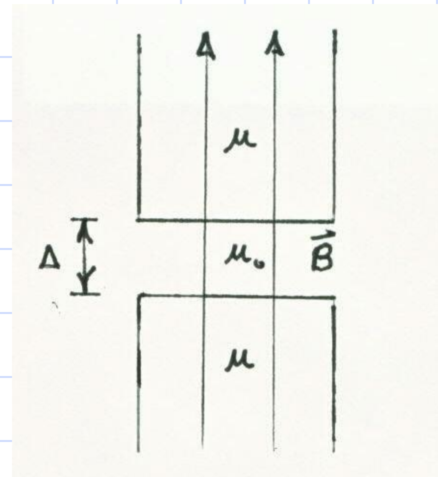
$$W_m = \frac{1}{2}\int (\vec{B} \cdot \vec{H}) dv = \frac{1}{2}\int LI^2$$

Magnetic Force

Relay Example

Consider a simple electromagnetic relay consisting of a solenoid and a moveable arm.





In the region of the gap, the normal component of the magnetic field will be continuous.

$$B_{gap} = B_{core} = B$$

Magnetic Force

Magnetic Pressure

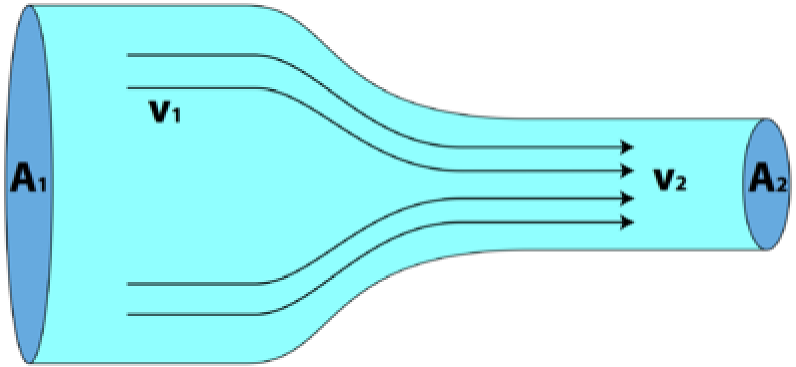
The magnetic field intensity H is very different in the gap and the core, since B is the same.

$$H_{gap} = \frac{B}{\mu_0} \qquad H_{core} = \frac{B}{\mu}$$

The magnetic energy density is also very different.

$$W_m = \frac{1}{2}\vec{B}\cdot\vec{H} \qquad w_{m\,gap} = \frac{1}{2}\frac{B^2}{\mu_o} \qquad w_{m\,core} = \frac{1}{2}\frac{B^2}{\mu}$$

Fluid Pressure



$$\frac{Kinetic\ \ energy}{Volume} = \frac{\frac{1}{2}mv^2}{V} = \frac{1}{2}\rho v^2$$

$$P = \frac{Force}{Area} = \frac{F}{A} = \frac{F \cdot d}{A \cdot d} = \frac{W}{V} = \frac{Energy}{Volume}$$

Magnetic Force

Magnetic Pressure

The difference in the magnetic field energy density on the two sides produces a pressure difference.

$$\frac{Joules}{m^3} = \frac{Joules/m}{m^2} = \frac{Newtons}{m^2}$$

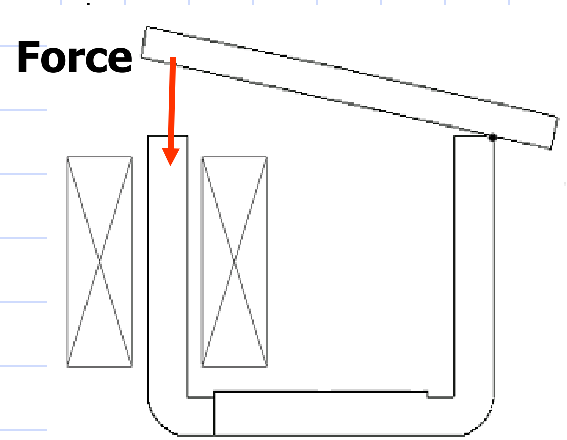
Energy Density

Pressure

High Pressure

Low Pressure

Net Force



Magnetic Force

Magnetic Pressure

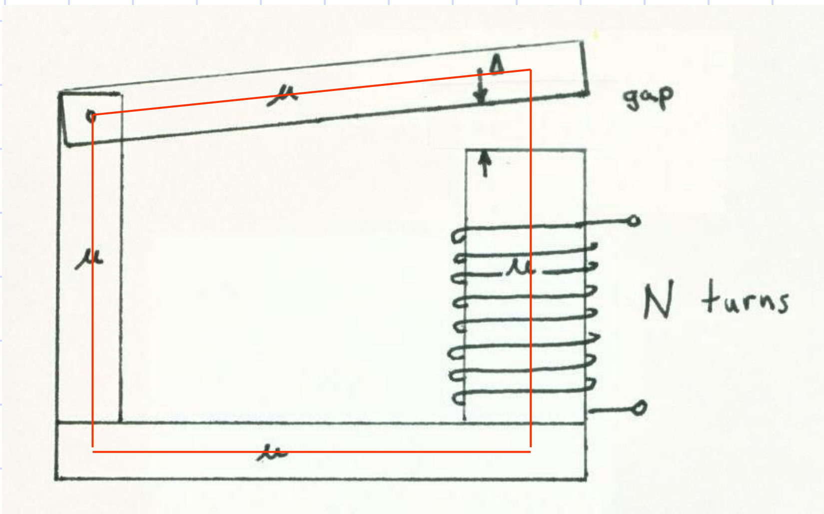
Since the pressure is so much higher on the gap side than in the core we only need to evaluate the pressure on the gap side to figure out the force. *S* is the area of the gap and core.

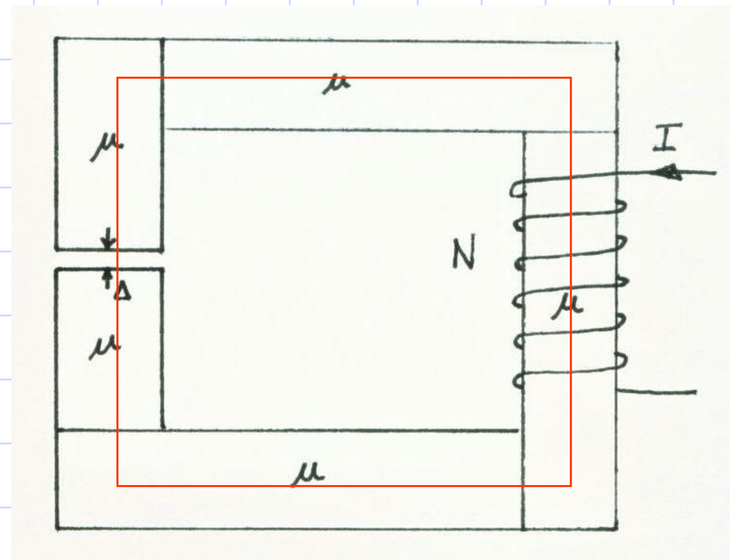
$$F = S w_{mgap} = \frac{1}{2} \frac{B^2}{\mu_o} S$$

To figure out the force, we first need to find the magnetic field, which we can do using the magnetic circuits technique.

Magnetic Force

Magnetic Pressure





To analyze this configuration, we will use the idealized version at the right. Assume that each leg has a length l_o and the area of each leg is S. The gap length is Δ. The reluctances are

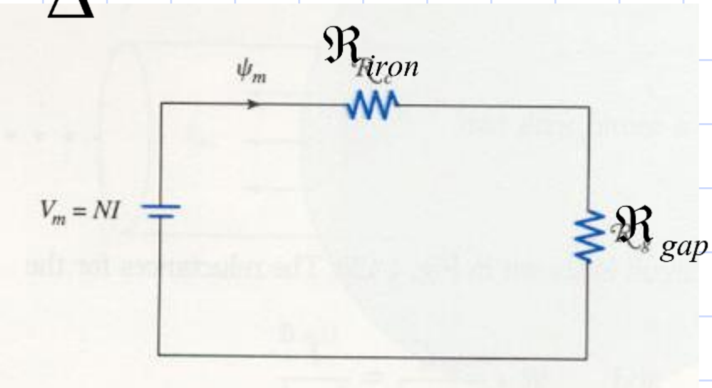
(R is reluctance, not resistance)

$$R_{gap} = \frac{\Delta}{\mu_0 S} \qquad R_{core} = \frac{4 l_0}{\mu S}$$

Magnetic Force

Magnetic Pressure

$$\psi_m = \frac{NI}{R_{gap} + R_{core}} \approx \frac{NI}{R_{gap}} = \frac{\mu_o NIS}{\Delta}$$



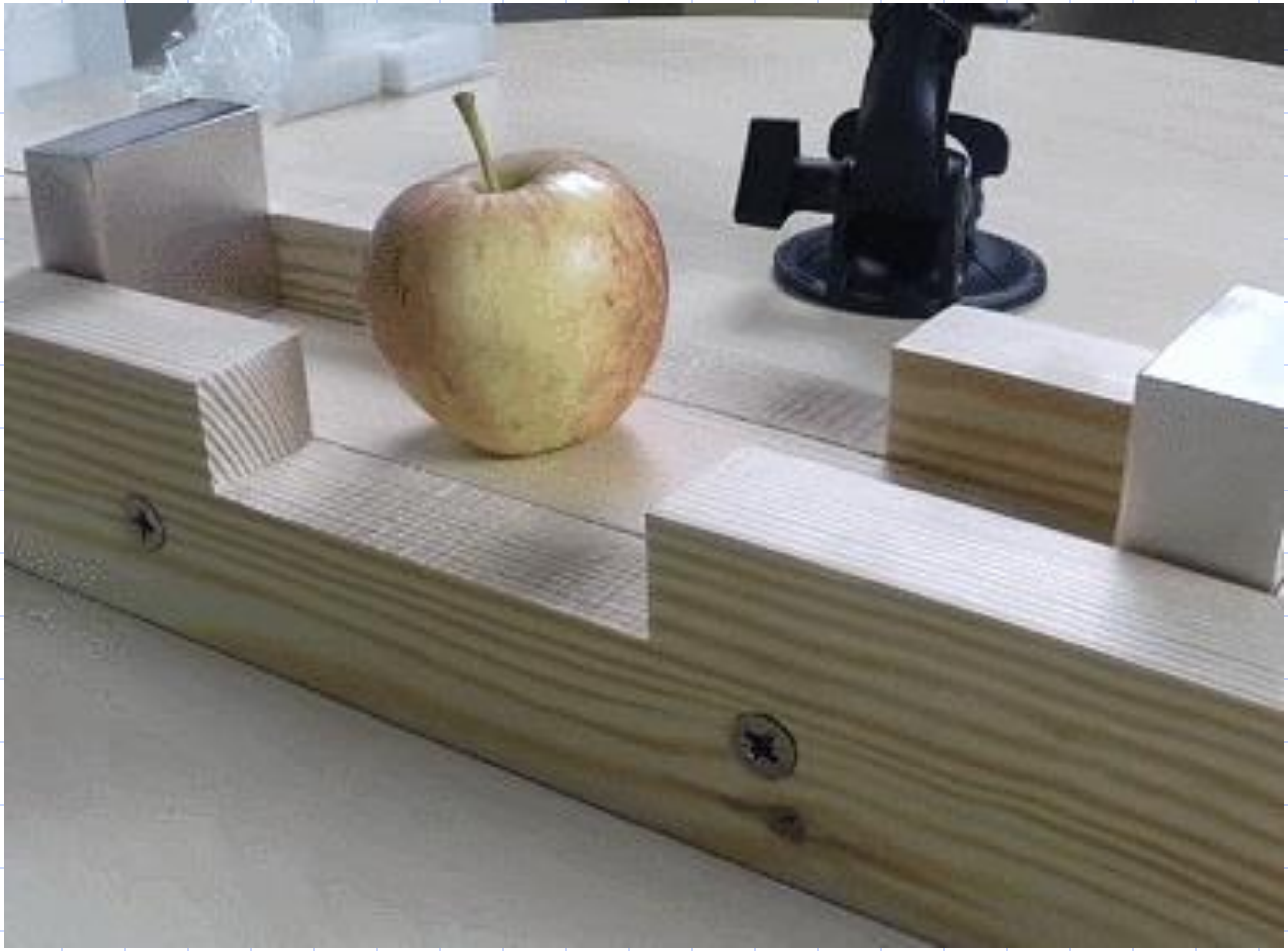
$$B = \frac{\psi_m}{S} \approx \frac{\mu_o NI}{\Delta}$$

$$F = Sw_{mgap} = \frac{1}{2}\frac{B^2}{\mu_o}S$$

$$F = \frac{1}{2}\frac{\left(\frac{\mu_o NI}{\Delta}\right)^2}{\mu_o}S = \frac{1}{2}\frac{\mu_o N^2 I^2}{\Delta^2}S$$

Magnetic Force

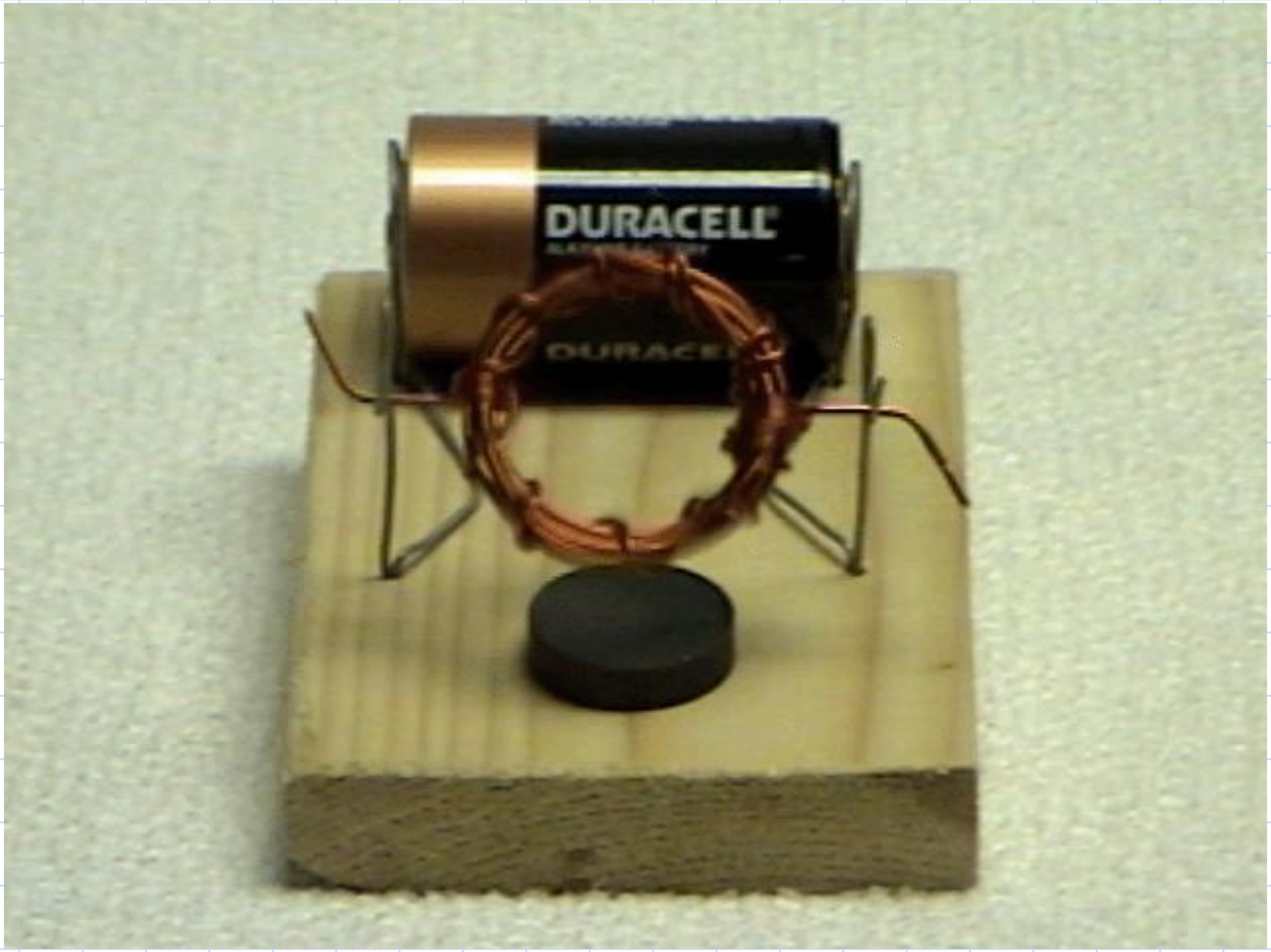
Magnetic pressure creates force between permanent magnets



Giphy

Magnetic Force

How to account for forces on currents?



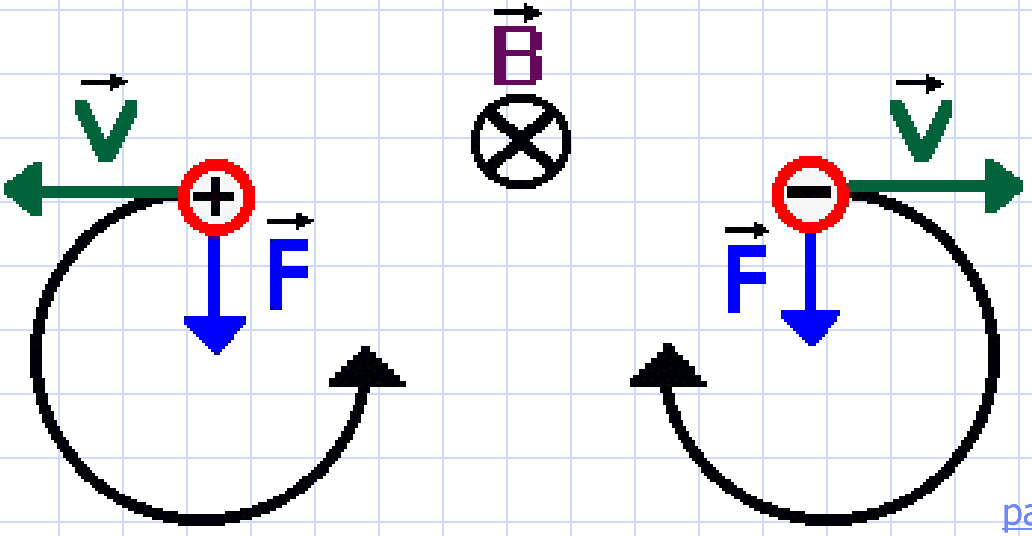
Giphy

Magnetic Force

Force on Currents

Force on a point charge: $\vec{F} = q \cdot (\vec{v} \times \vec{B})$

- For a constant B-field like the one below, F causes a change in v which in turn causes a change in F, creating a circular path.



panomics.pnnl.gov

Magnetic Force

The Large Hadron Collider

forbes.com

One of the LHC's superconducting dipole magnets (bnl.gov)

Magnetic Force

Force on Currents

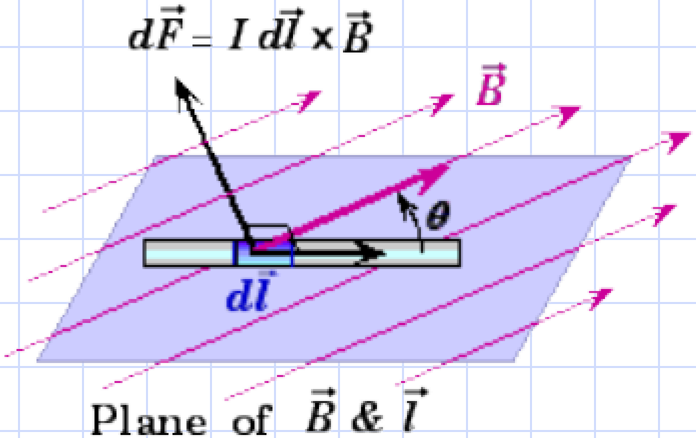
First approach - similar to that for individual particles

For one particle: $F = qE = q \cdot (v \times B)$

For many particles: $\frac{F}{volume} = \rho \cdot (\vec{v} \times \vec{B}) = \vec{j} \times \vec{B}$

For a wire in a magnetic field.

$$F = \int j \times B dv = \int I dl \times B$$

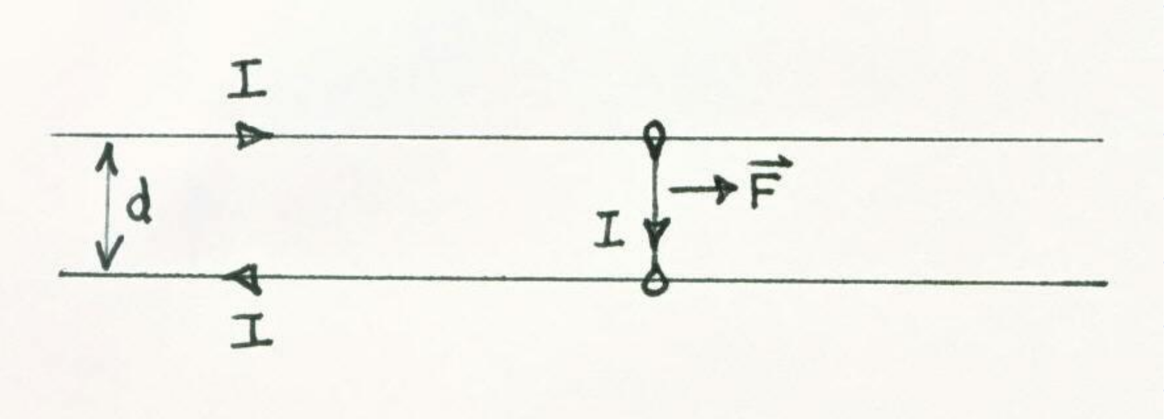


http://www.ac.wwu.edu/~vawter/PhysicsNet/Topics/MagneticField/MFOnWire.html

Magnetic Force

Rail Gun

If a sliding contact is placed across a two wire transmission line carrying a large current, a very large force can result on the contact. Assume that all the wires (including the slider) have a radius = *a* and that the transmission line wires are separated by a distance *d*.



Magnetic Force

Rail Gun

The external inductance of a two wire line of length *l* is given by (one of many forms:

$$L \approx l \frac{\mu_o}{4\pi} \cosh^{-1} \frac{d}{2a} \approx l \frac{\mu_o}{4\pi} \ln \frac{d}{a}$$

where we have used the fact that typically $d >> a$. The force on the sliding conductor will be:

$$\bar{F} = \nabla W_m = \frac{I^2}{2} \nabla L = \frac{I^2}{2} \frac{dL}{dl} \approx \frac{I^2}{2} \frac{\mu_o}{4\pi} \ln \frac{d}{a}$$

If $d/a = 5$ and $I = 10^5$ A, $F = 100$ Newtons

Magnetic Force

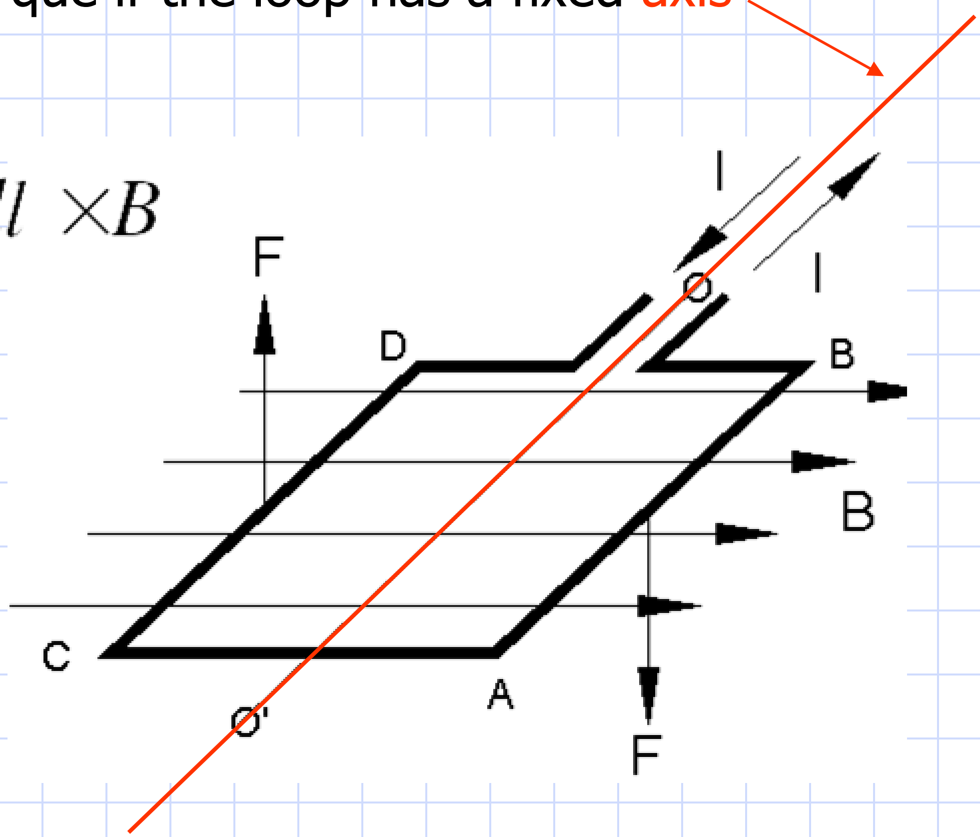
Rail Gun

Magnetic Force

Current Loop

The force on a current loop in a magnetic field can result in rotational torque if the loop has a fixed axis as shown.

$$F = \int j \times B dv = \int I dl \times B$$



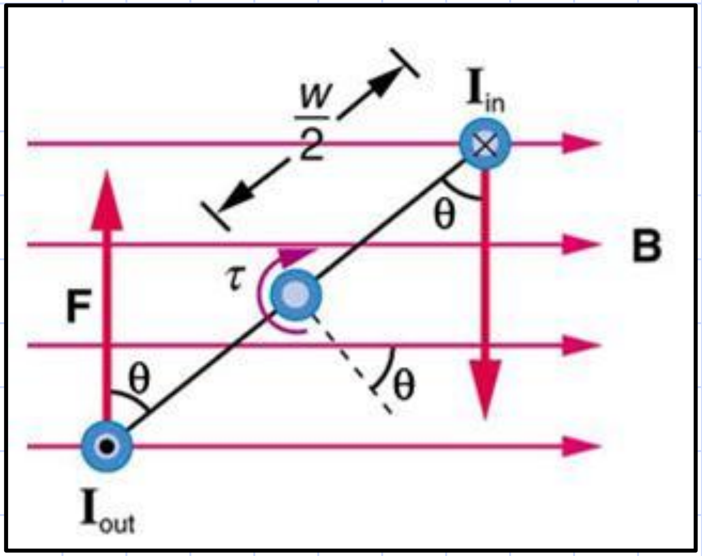
Magnetic Force

Current Loop

$$F = \int j \times B dv = \int I dl \times B$$

$$\tau = IAB sin\theta$$

B = field strength
θ = angle between the loop surface normal and direction of B field
A = area of loop



libretexts.org

Magnetic Force

Current Loop

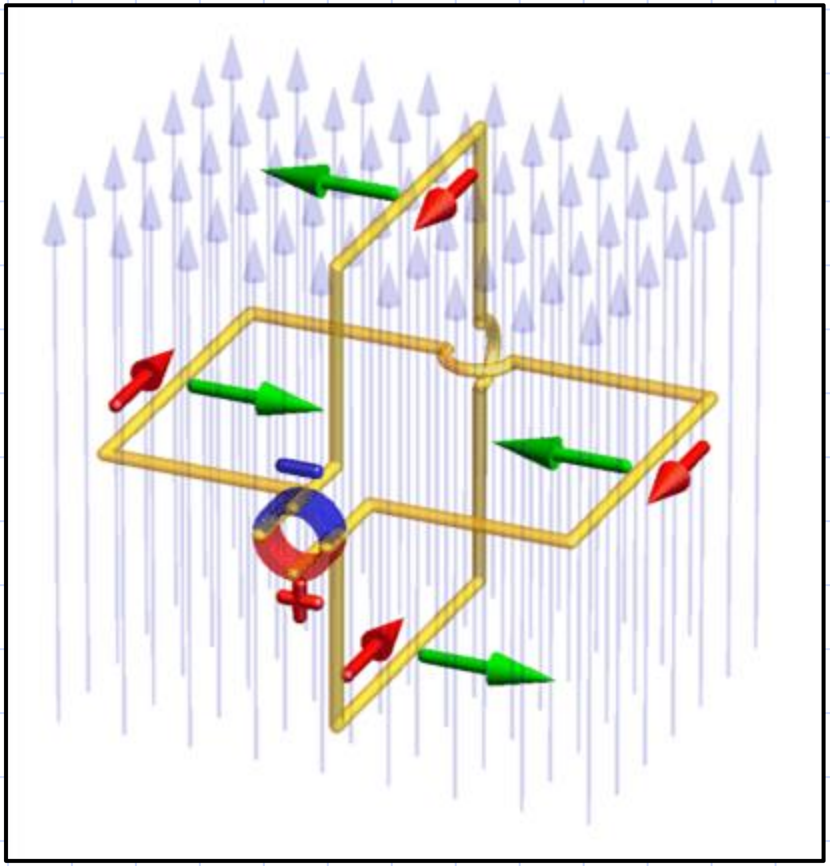
The presence of the rotational torque suggests that this loop could be used to make a DC motor. What happens when the is placed at an angle relative to the field, then allowed to rotate freely?

http://physics.bu.edu/~duffy/semester2/c13_torque.html

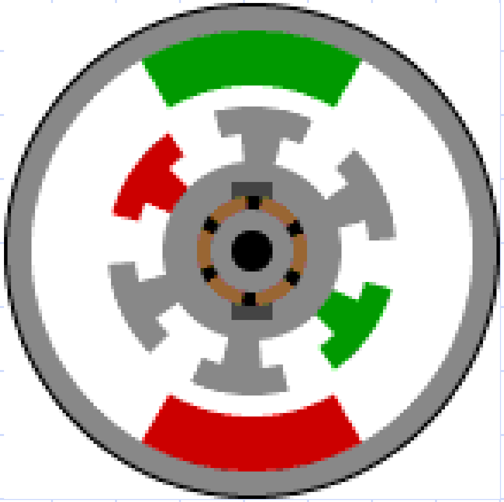
DC Motors

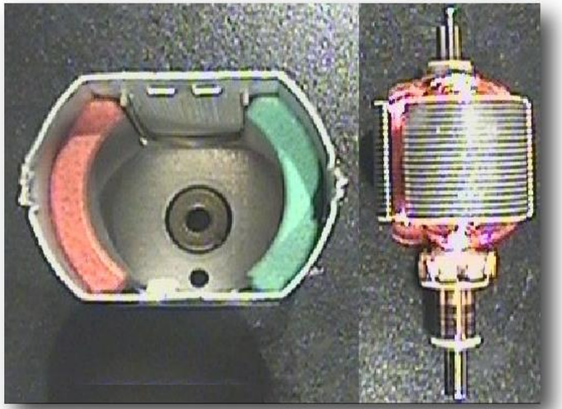
The Commutator

- A commutator allows a current loop to switch current direction at different stages of its rotation.
- As a result, the loop can achieve an average positive net torque through its rotation, and can rotate continuously while current is applied.



DC Motors

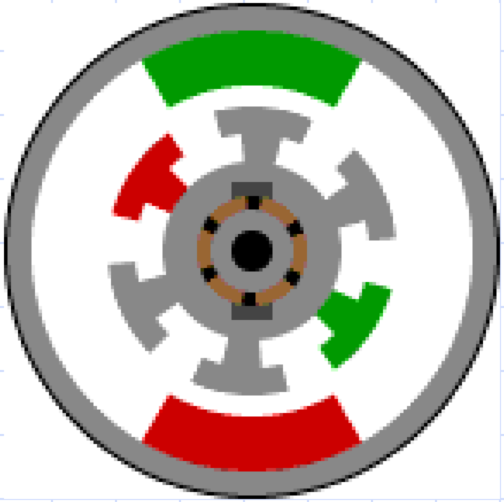




- The stator is the stationary outside part of a motor. The rotor is the inner part which rotates.
- In the motor animations, red represents a magnet or winding with a north polarization, while green represents a magnet or winding with a south polarization. Opposite, red and green, polarities attract.

http://www.freescale.com/files/microcontrollers/doc/train_ref_material/MOTORDCTUT.html

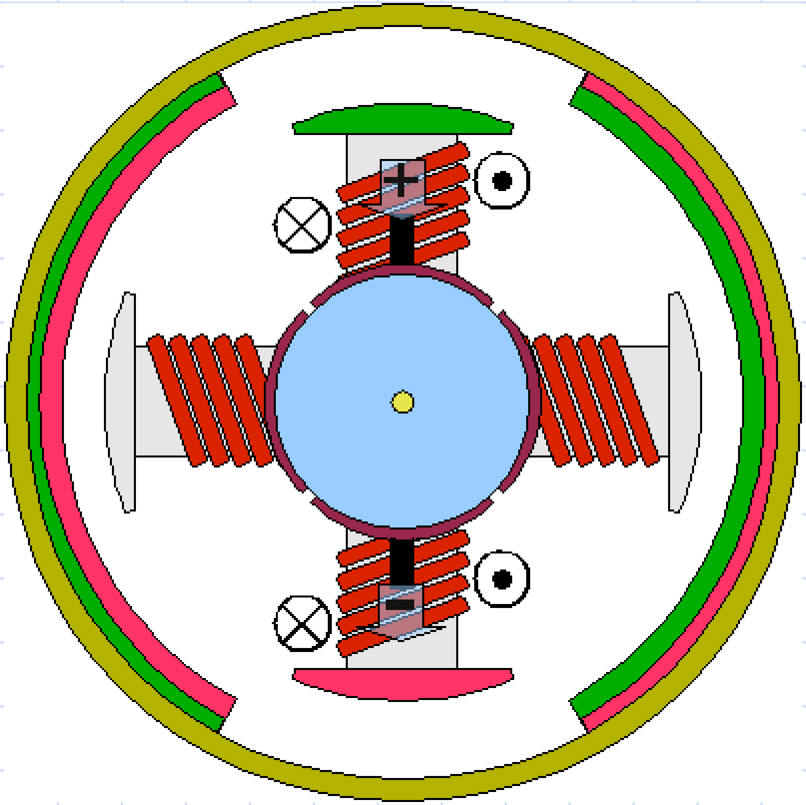
DC Motors



- Just as the rotor reaches alignment, the brushes move across the commutator contacts and energize the next winding.
- Above, the commutator contacts are brown and the brushes are dark grey.

http://www.freescale.com/files/microcontrollers/doc/train_ref_material/MOTORDCTUT.html

DC Motors



homofaciens.de

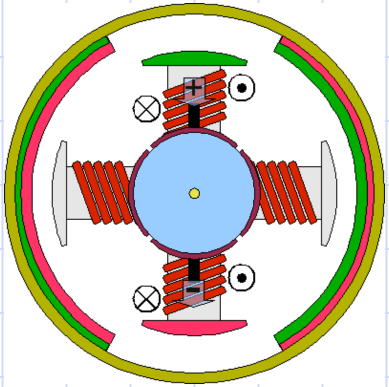
Another animated example

Motors vs Generators

What’s the difference between a motor and a generator?

Motors vs Generators

Mechanical Work ⟷ Electrical Work



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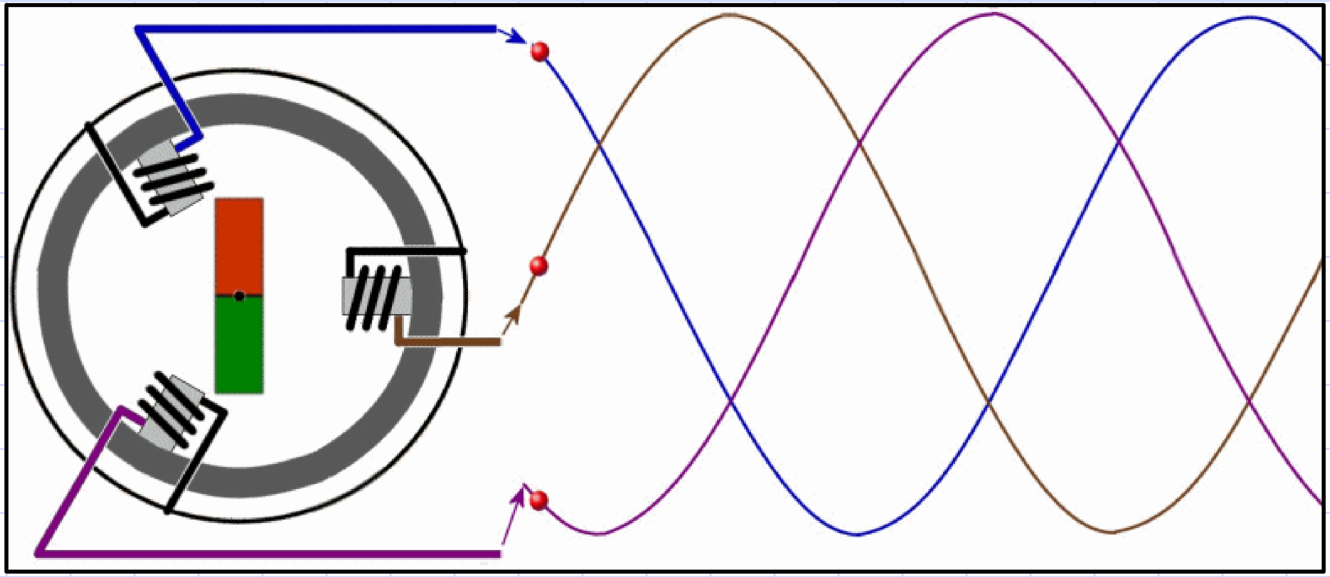
- In general, motors and generators can perform a two-way conversion of electrical and mechanical work (a motor can act as a generator and vice versa)
- You can think of motors and generators as a <u>single class</u> of electromechanical device with variations for specific applications

Motors vs Generators

AC Generator



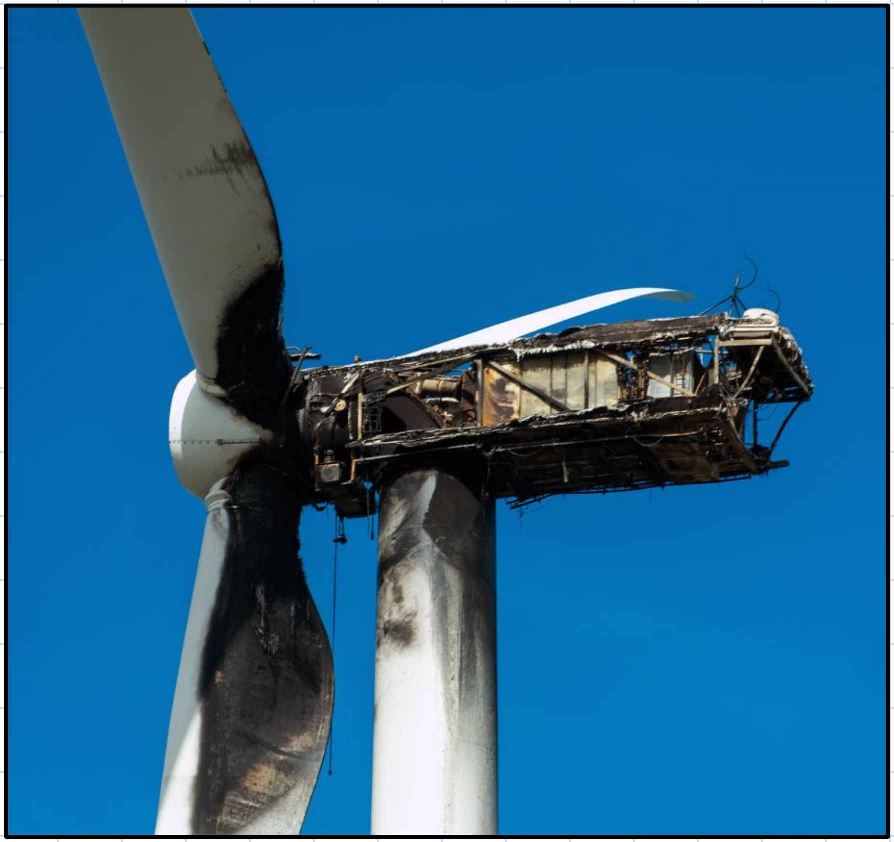
kfz-tech.de

- In an AC motor or generator, the coils are excited with an AC voltage
- This is the common method for power system applications

Motors vs Generators

Generator Motoring

- A generator can fail to provide the power demanded of it by a power system
- In this situation it begins to act like a motor, consuming power from the system until supply meets depend or a generator failure occurs



Motors vs Generators

Motors in Generator Mode

- Motors that are mechanically pushed beyond the speed at which they are being driven can enter generator mode, feeding power back into the system
- Electric cars can use this principle to recharge while going downhill

carmagazine.co.uk