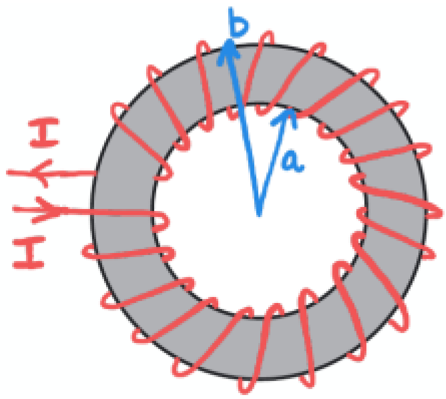
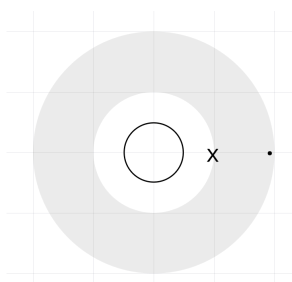
Hayden Fuller
Fields and Waves HW6

1. Ampere's Law:
   Magnetic Field of a Toroid Shown below is a toroid of inner radius a and outer radius b. The toroid lies parallel to the x-y plane and has a magnetic permeability of μ. The wire carries a current I and is wrapped N times around the toroid.



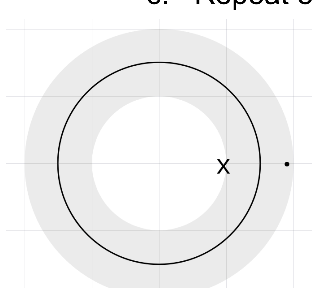
   a. Draw a diagram indicating the geometry and location of the Amperian loop you will use to solve for B for r < a.



   b. Using Ampere's Law, determine B for r < a. Justify any mathematical simplifications you made to the form of the B field using geometrical arguments (as we did in class).

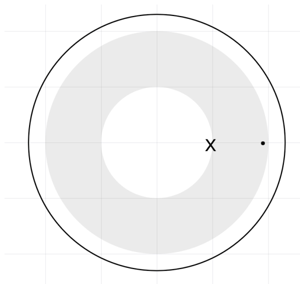
Ienc=0
B=0

   c. Repeat each of the steps in parts a and b to determine B for a < r < b.



B*2pi r = mu * Ienc
B=-mu_0 N I / (2 pi r)

   d. Repeat each of the steps in parts a and b to determine B for r > b.



Ienc=0
B=0

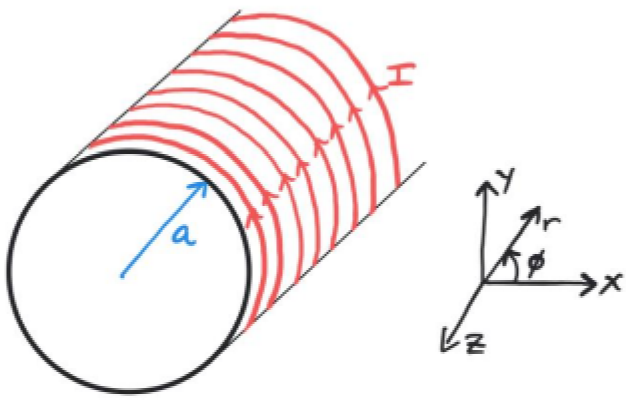
e. Verify that your solutions in parts b, c, and d satisfy $\vec{\nabla} \cdot \vec{B} = 0$.

\Nabla B = 1/r dB/dphi
B is not dependent on phi, so dB/dphi=0, so \Nabla B=0

2. Magnetic Vector Potential for a Solenoid
The solenoid below has a radius a and is oriented parallel to the z-axis. A current I flows in the φ direction and there are n wire windings per unit length of the solenoid. The magnetic vector potential is defined via the following two expressions:

$$\oint \vec{A} \cdot d\vec{l} = \oiint \vec{B} \cdot d\vec{S} \quad (1)$$

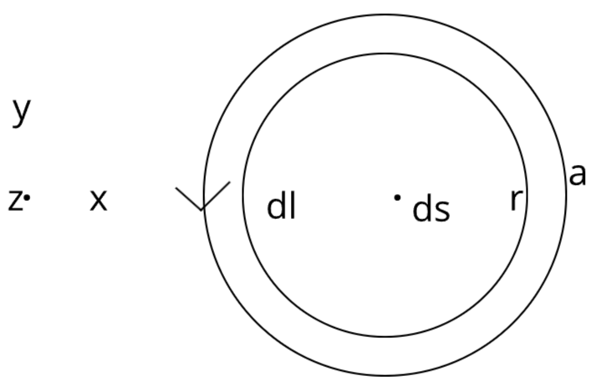
$$\vec{\nabla} \times \vec{A} = \vec{B} \quad (2)$$



a. Which components of A will be non-zero?

\hat A = A_phi \hat phi

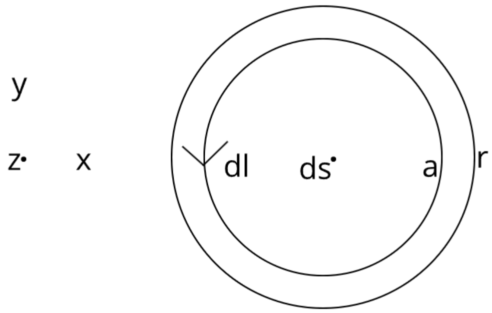
b. Draw a diagram of the loop and surface you will use to determine A for the region r < a, using equation (1) above.



c. Calculate A for r < a.

Int B dl = mu_0 Ienc
Bz l = mu 0 I n l
Bz= mu_0 I n \hat z
Int A dl = IntInt B ds
A 2pi r = mu_0 I n pi r^2
A=mu_0 I n r \hat phi /2

d. Draw a diagram of the loop and surface you will use to determine A for the region r > a.



e. Calculate A for r > a.

Int A dl = IntInt Bds
Aphi 2 pi r = mu_0 I n pi a^2
A=mu_0 I n a^2 \hat phi / 2r

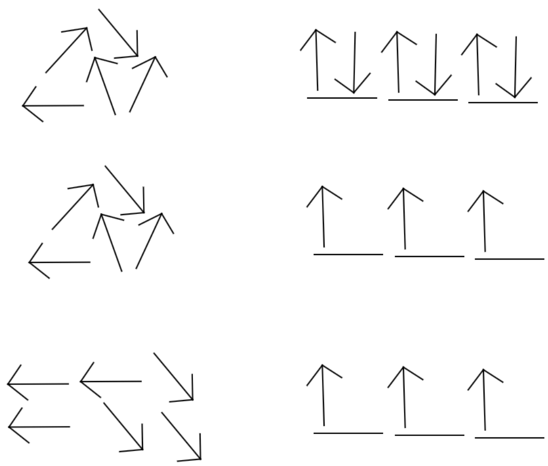
f. Verify that your solution satisfies the definition of the magnetic vector potential via equation (2) above in each of the regions.

\Nabla x A = B = 1/r d rA / dr \hat z = mu_0 I n \hat z
\Nabla x A = B = 1/r d rA / dr \hat z = 0

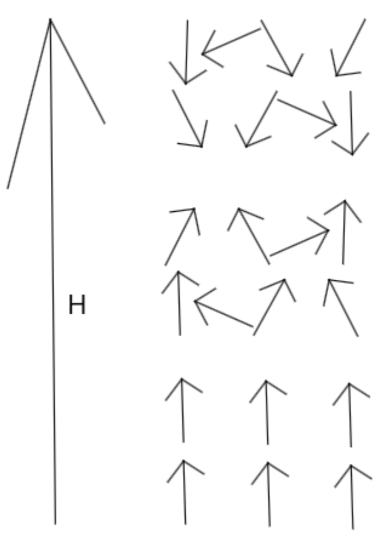
3. Magnetic Materials
    a. What are the three main classifications of materials in terms of their magnetization properties? By which physical property are materials categorized into these groups? How do they differ in terms of the internal magnetization field that is induced by an externally-applied magnetic field H?

Diamagnetic, xm<0, mu_r~=1, internal field opposes applied field
Paramagnetic xm>0, mu_r~=1, internal field aligns with applied field
Ferromagnetic xm>>1, mu_r>>1, internal field can shift to oppose or align with an applied field

    b. Sketch the typical orientation of the magnetic dipoles in each type of material in the absence of an externally applied magnetic field (i.e. H = 0).



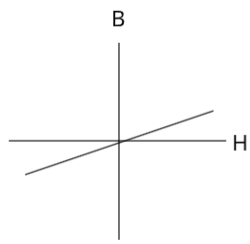
    c. Sketch the typical orientation of the magnetic dipoles in each type of material in the presence of an externally applied magnetic field (i.e. H ≠ 0).

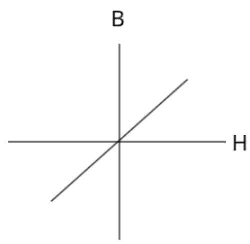


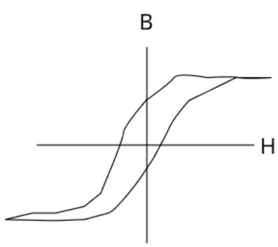
d. Assuming M points in the same direction as H (true for paramagnetic materials and can be true for ferromagnetic materials), explain why the B field is larger inside these materials where μ > μ0 than in free space where μ = μ0. Your reasoning should be based on your sketches in part c.

B field is larger because magnetic moments align with applied field and adds to it

e. Sketch a B-H magnetization diagram for each type of material.







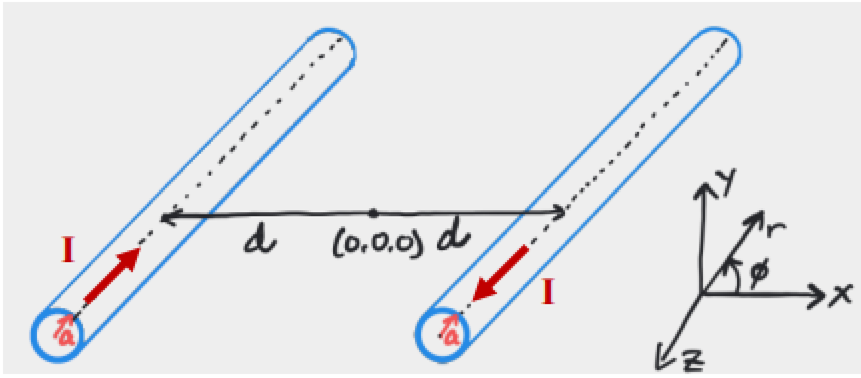
f. Ferromagnetic materials exhibit what is called "magnetic hysteresis". What does this mean in terms of the B-H magnetization curve for this type of material? What does this physically mean in terms of your sketches in part c?

The B field in the material has to change slowly, and will take slightly different paths on the up swing and down swing. Dipoles align and remain partially aligned after H field is removed.

g. Given a non-magnetized piece of a ferromagnetic material, how would you turn it into a permanent magnet?

Apply a strong H field

4. Inductance of a Parallel Wire Transmission Line and Faraday's Law

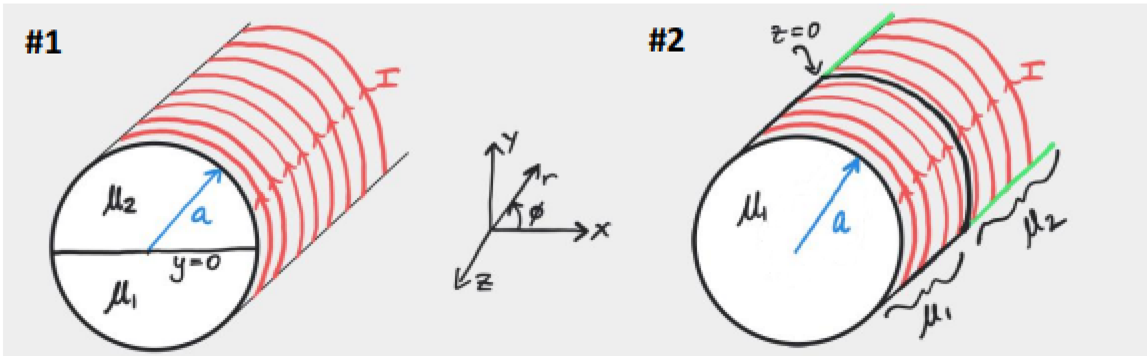


a. Calculate the total B field between the wires, in the x-z plane.

B=
B=-mu_0 I / 2pi * (1/|x+d|) + 1/|x-d|)

b. Calculate the magnetic flux Φ in a plane spanning the space between the wires (-d + a < x < d - a) in the x-z plane.
c. Calculate the inductance of the two-wire system via $L = \Lambda\ I$, where $\Lambda = N\Phi$ is the total flux linkage and N is the number of wire loops that the flux links.
d. Calculate a numerical value for the inductance L and inductance per unit length l if the length of the wires we're considering is 0.1m, a = 0.5mm, and d = 0.5cm.
e. Faraday's law states: $Vemf = -\ d\Phi\ dt$. If the current of the wires is now time-dependent, such that I = I(t), use your expression from part b to write $Vemf$ in the form $Vemf = -A\ d\mathrm{I}\ dt$, where A is a constant. What is this constant A? Does this expression for Vemf look familiar from circuit theory?
f. If $I(t) = I0\sin(2\pi ft)$, what is the EMF that would be generated across each of the wires? In which direction does the E field associated with the EMF point in each of the wires?

5. Magnetic Boundary Conditions
   Two solenoids are shown below: solenoid #1 consists of two materials with different magnetic permeabilities: μ1 for y < 0 and r < a and μ2 for y > 0 and r < a. Solenoid #2 also consists of two materials with different magnetic permeabilities: μ1 for z < 0 and r < a and μ2 for z > 0 and r < a.



   a. Calculate B and H for solenoid #1 in both regions for r < a.
   b. For solenoid #2:
      i. Calculate H and B in region #1 (where μ = μ1) using Ampere's Law (or simply use the expression for H of a solenoid from a previous problem).
      ii. Apply the normal boundary condition for B to find B in region #2 (where μ = μ2), then calculate H in region 2. You should find that your expression for H in region #2 is not what you would get by solving for it using Ampere's Law in region #2.
      iii. Find the current density at the surface of the solenoid in region #2 via the tangential boundary condition for H between region #2 and air. Is this different from the current density flowing at the surface of the solenoid in region #1?
   c. In electrostatics, we had the boundary conditions: { $D_{2n} - D_{1n} = \rho_s$ $E_{2t} - E_{1t} = 0$ which stated in mathematical terms that for:
      i. the normal component: the net electric flux density passing through the boundary is equal to the free surface charge density at that location (net flux is zero unless there's a source at the boundary: Gauss's Law says $\vec{\nabla} \cdot \vec{D} = \rho_s \rightarrow$ electric field lines originate from or end on free charges)
      ii. the tangential component: the electric field must be continuous across the boundary because the curl of the electric field is zero $\vec{\nabla} \times \vec{E} = 0$ (the E field has no rotation)
      Using the physical interpretations of Maxwell's equations for magnetostatics as your basis, explain why the difference in the normal B field across the boundary is always zero and the tangential magnetic field is equal to the surface current density at the boundary. { $\hat{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0$ $\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s$
   d. A surface current density can only exist at the boundary between certain types of materials. Which types of materials can have a surface current density at a boundary?