Fields and Waves I

Lecture 21

EM Waves

Polarization

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Course Admin

- Exam 3 is **Thursday, April 11 in class**
- Exam 3 review material + crib sheet is posted
- No class Monday. Remote review session lecture is posted
- Oral exams next week starting Tuesday (topics TBA)
- HW 6 solutions out (HW 5 solutions ASAP)

Plane Waves

Properties of a TEM

- Defines the connection between electric and magnetic fields of an EM wave
- Similar to the characteristic impedance (Z0) of a transmission line

Intrinsic impedance $$\eta = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} \quad [\Omega]$$

Phase velocity $$u_p = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} \quad [\text{m/s}]$$

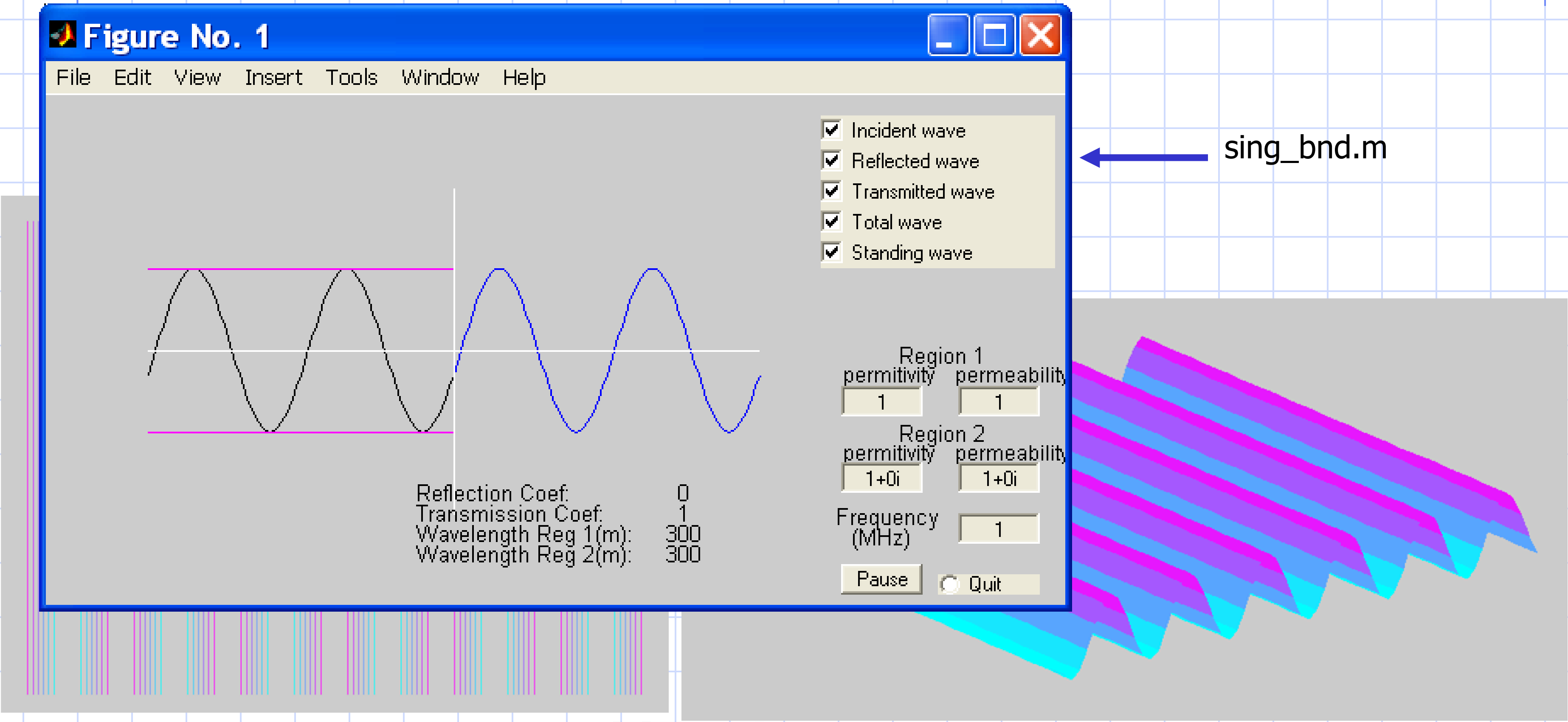
Wavelength $$\lambda = \frac{2\pi}{k} = \frac{u_p}{f} \quad [\text{m}]$$

If the medium is vacuum :

$u_p = 3x10^8$ [m/s], $\eta_0 = 377$ [Ω]

Plane Waves

For plane waves we will have only z-dependence.



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Plane Waves

The plane wave case allows us to treat wave propagation as a one-dimensional problem, just like transmission lines.

$$\frac{d^2 \tilde{E}_x}{dz^2} - \gamma^2 \tilde{E}_x = 0$$

$$\tilde{E}_x(z) = \tilde{E}_x^{+}(z) + \tilde{E}_x^{-}(z) = \underbrace{E_{x0}^{+} e^{-(\alpha + j\beta)z}}_{\text{forward traveling in +z direction}} + \underbrace{E_{x0}^{-} e^{(\alpha + j\beta)z}}_{\text{backward traveling in -z direction}}$$

Plane Waves

In general, a uniform plane wave traveling in the +z direction, may have x and y components

$$\begin{cases} \tilde{E}(z) = \tilde{E}_x^+(z)\hat{a}_x + \tilde{E}_y^+(z)\hat{a}_y \\ \tilde{H}(z) = \tilde{H}_x^+(z)\hat{a}_x + \tilde{H}_y^+(z)\hat{a}_y \end{cases}$$

The relationship between them

$$\boxed{\begin{aligned} \tilde{H} &= \frac{1}{\eta}\hat{a}_z \times \tilde{E} \\ \tilde{E} &= -\eta\hat{a}_z \times \tilde{H} \end{aligned}}$$

$$\eta = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}}$$

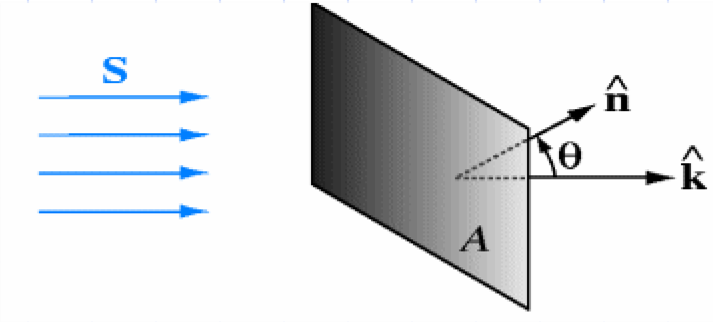
EM Wave Power Transmission

- Poynting Vector S , is defined

$$S = E \times H \qquad \text{[W/unit area]}$$

S is along the propagation direction of the wave

Ulaby



Total power

$$P = \int_A S \cdot \hat{a}_n \, dA \qquad \text{[W]}$$

OR $\quad P = |S| A \cos\theta \qquad$ [W]

Average power density of the wave [W/m²]

EM Wave Power Transmission

Average Power Density

$$\tilde{E}(z) = \tilde{E}_x^+(z)\hat{a}_x + \tilde{E}_y^+(z)\hat{a}_y$$

$$\tilde{E}(z) = (E_{x0}\hat{a}_x + E_{y0}\hat{a}_y)e^{-(\alpha+j\beta)z}$$

$$\tilde{H}(z) = \frac{1}{\eta_c}\hat{a}_Z \times \tilde{E} = \frac{1}{\eta_c}(-E_{y0}\hat{a}_x + E_{x0}\hat{a}_y)e^{-\alpha z}e^{-j\beta z}$$

Average power density

$$S_{av} = \frac{1}{2}\mathrm{Re}\left\{\tilde{E} \times \tilde{H}^*\right\} = \hat{a}_z \frac{1}{2}(|E_{x0}|^2 + |E_{y0}|^2)e^{-2\alpha z}\,\mathrm{Re}\left\{\frac{1}{\eta_c^*}\right\} \text{ [W/m}^2\text{]}$$

NOTE

EM Power Transmission

Plane wave in a Lossless Medium

$$\tilde{E}(z) = \tilde{E}_x^+(z)\hat{a}_x + \tilde{E}_y^+(z)\hat{a}_y$$

$$\tilde{E}(z) = (E_{x0}\hat{a}_x + E_{y0}\hat{a}_y)e^{-jkz}$$

$$\tilde{H}(z) = \frac{1}{\eta}\hat{a}_Z \times \tilde{E} = \frac{1}{\eta}(-E_{y0}\hat{a}_x + E_{x0}\hat{a}_y)e^{-jkz}$$

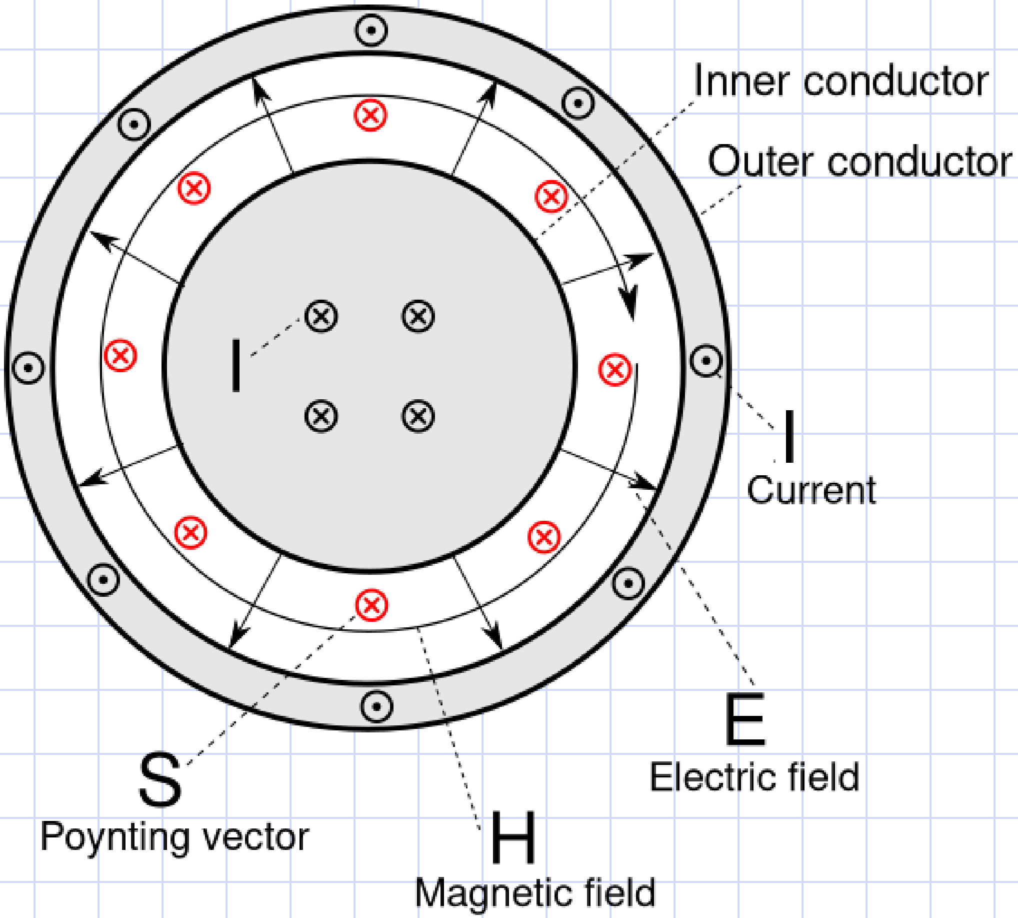
$$S_{av} = \hat{a}_z \frac{1}{2\eta}(|E_{x0}|^2 + |E_{y0}|^2)$$

$$\boxed{S_{av} = \hat{a}_z \frac{|\tilde{E}|^2}{2\eta}} \quad [\text{W/m}^2]$$

EM Wave Power Transmission

Coaxial Cable Poynting Vector

$$\vec{S} = \vec{E} \times \vec{H}$$

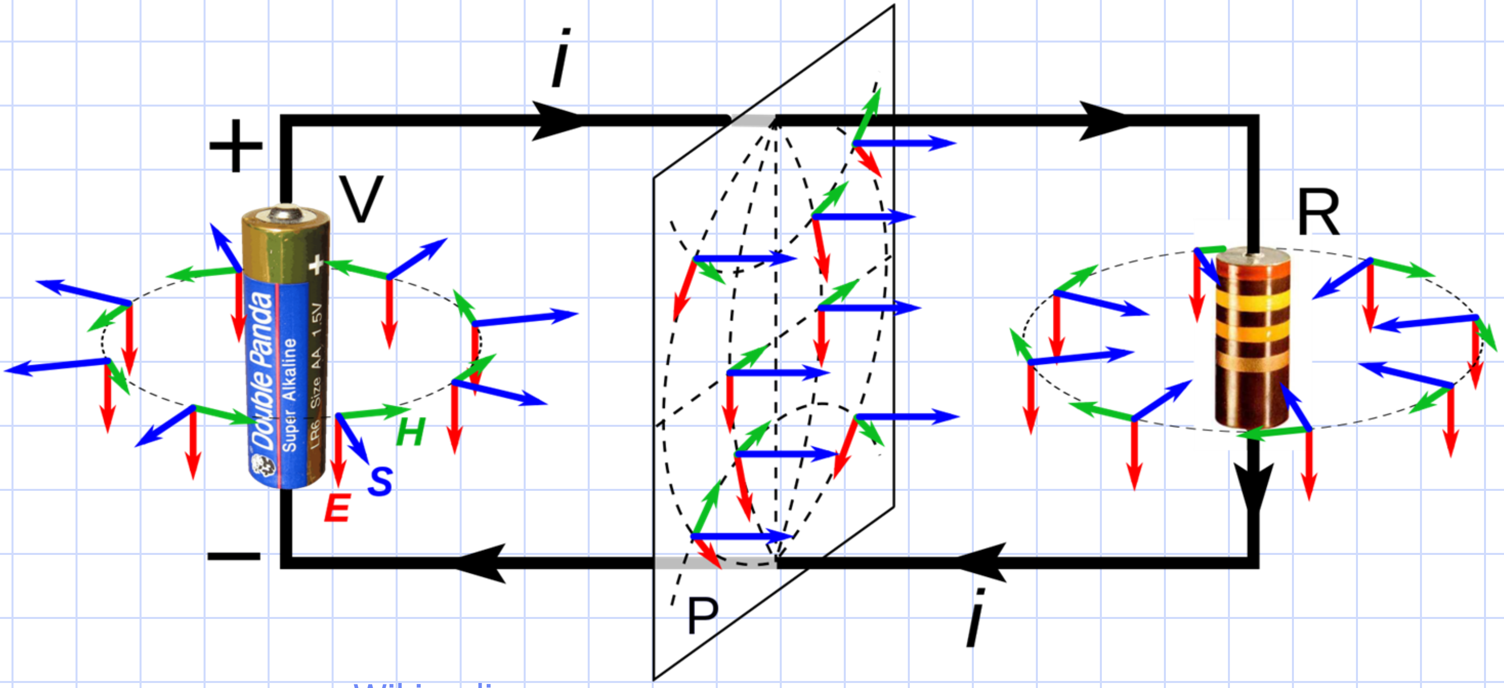


[Wikipedia]

EM Wave Power Transmission

Electric Circuit Poynting Vector

$$\vec{S} = \vec{E} \times \vec{H}$$



Wikipedia

EM Wave Power Transmission

Average Power Density

If η_c is written in polar form

$$\eta_c = |\eta_c| e^{j\theta_\eta}$$

Average power density

$$S_{av} = \hat{a}_z \frac{|E_0|^2}{2|\eta_c|} e^{-2\alpha z} \cos\theta_\eta \qquad [\mathrm{W/m^2}]$$

where

$$|E_0| = \left[|E_{x0}|^2 + |E_{y0}|^2 \right]^{1/2}$$

EM Wave Power Transmission

The transmitter is about 10 km from campus. What transmitter power is required to radiate the same power density into a sphere of radius 10 km?

EM Wave Power Transmission

The transmitter is about 10 km from campus. What transmitter power is required to radiate the same power density into a sphere of radius 10 km?

$$P_{total} = |\vec{S}_{av}| 4\pi R^2 = (8.5\times10^{-6})(4\pi)(10^4 m)^2 = \boxed{10.7\ kW}$$

In-Class Exercise

Do Lecture 21 Exercise 1 in groups of up to 4.

Review

- Electromagnetic waves (often shortened to EM waves) can travel through space and materials in 3 dimensions
- The media that EM waves travel will have frequency-dependent behavior

$$\nabla^2 \tilde{E} - \gamma^2 \tilde{E} = 0$$

$$\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu (\varepsilon' - j\varepsilon'')$$

$$\varepsilon' = \varepsilon \qquad \varepsilon'' = \frac{\sigma}{\omega}$$

Review

- Much like all transmission lines have a *characteristic impedance* Z_0, All materials have an *intrinsic impedance* η.

$$\eta = \frac{E(x,y,z)}{H(x,y,z)} \qquad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon'}}\left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$$

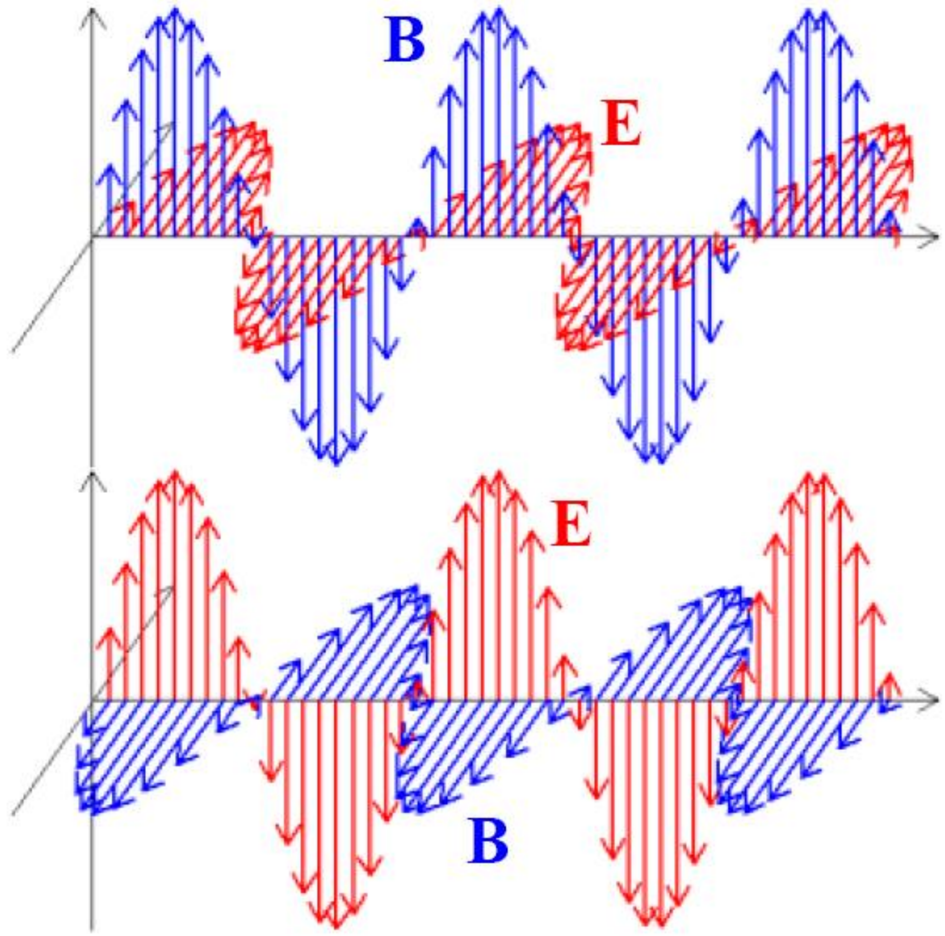
Review

- Materials that have a well-defined ε and σ will behave in one of three ways depending on the frequency:
  - **Good conductor ($\varepsilon''/\varepsilon'<10^{-2}$)**(EM wave mostly causes conduction current in medium as it propagates)
  - **Low-loss dielectric ($\varepsilon''/\varepsilon'>100$)**(EM wave mostly causes displacement current as it propagates)
  - Quasi-conductor (somewhere in between)

  - There is a general formula for calculating α, β and η and then simpler approximate formulas for good conductors and low-loss dielectrics

Wave Polarization

- For a +z-propagating wave, there are two possible directions for E (see right)
- A real wave can be a linear combination of these two possibilities
- In addition, it can be a combination of two components of different phase
- We use the direction of E to define a wave's polarization



Wave Polarization

a uniform plane wave traveling in the +z direction may have x and y components

$$\tilde{E}(z) = \tilde{E}_x^+(z)\hat{a}_x + \tilde{E}_y^+(z)\hat{a}_y$$

$$\tilde{E}(z) = (E_{x0}\hat{a}_x + E_{y0}\hat{a}_y)e^{-jkz}$$

Complex amplitudes

The phase difference between the complex amplitudes of x and y components of electric field can be defines with angle δ

$$E_{x0} = a_x$$

$$E_{y0} = a_y e^{j\delta}$$

a_x, a_y are the magnitudes of E_{x0} and E_{y0}

Wave Polarization

The phasor of electric field

$$\tilde{E}(z) = (a_x \hat{a}_x + a_y \hat{a}_y e^{j\delta}) e^{-jkz}$$

The corresponding instantaneous field

$$E(z,t) = \mathrm{Re}\left\{ \tilde{E}(z) e^{j\omega t} \right\}$$

$$E(z,t) = a_x \hat{a}_x \cos(\omega t - kz) + a_y \hat{a}_y \cos(\omega t - kz + \delta)$$

Wave Polarization

describes the shape and locus of tip of the E vector at a given point in space as a function of time

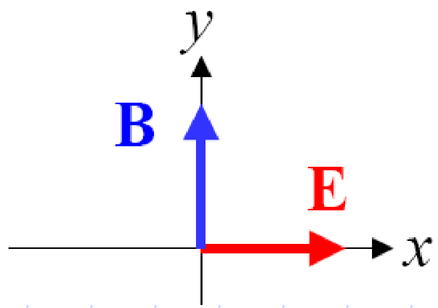
The locus of E, may have three different polarization states depends on conditions

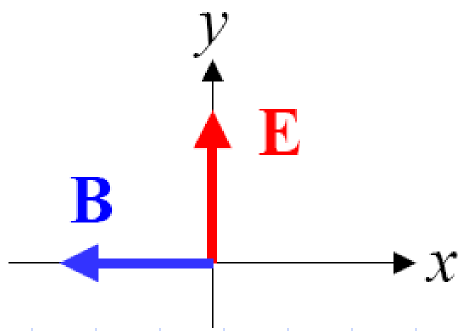
- Linear
- Circular
- Elliptical

Wave Polarization

Linear Polarization

Looking up from +z





<u>x-polarized or horizontal polarized</u>

$a_y=0$ $\psi=0°$ or $180°$

<u>y-polarized or vertical polarized</u>

$a_x=0$ $\psi=90°$ or $-90°$

Wave Polarization

Linear Polarization

A wave is said to be linearly polarized if $E_x(z,t)$ and $E_y(z,t)$

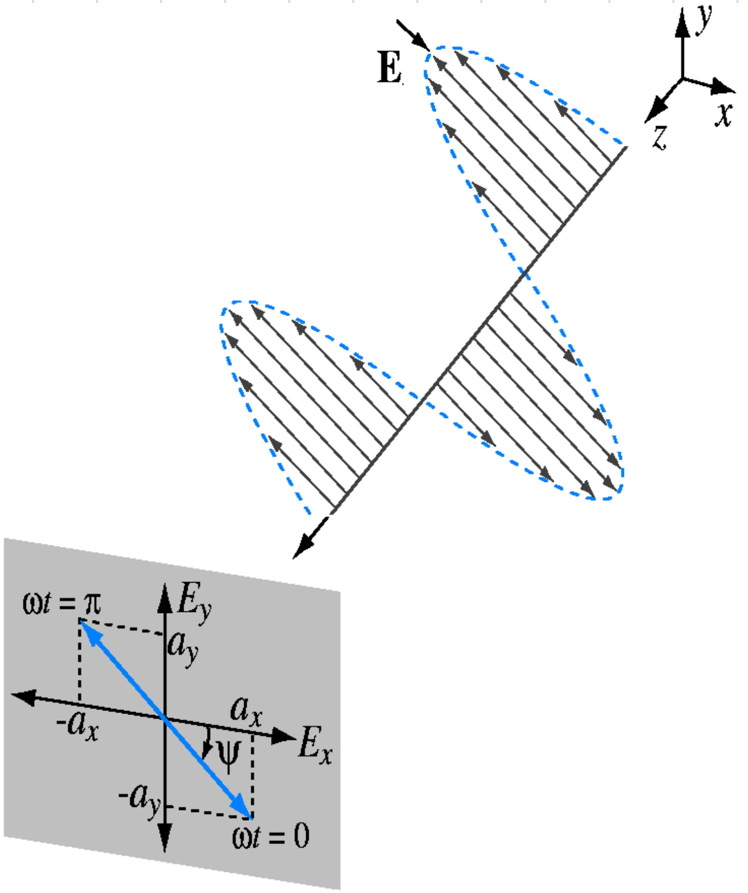
Are in phase ($\delta=0$) or out of phase ($\delta=\pi$)

$$\underset{\rightarrow}{E}(0,t) = (a_x \hat{a}_x + a_y \hat{a}_y)\cos \omega t \qquad \textit{In phase}$$

$$E(0,t) = (a_x \hat{a}_x - a_y \hat{a}_y)\cos \omega t \qquad \textit{Out of phase}$$

Wave Polarization

Linear Polarization



$$\left|E(0,t)\right| = \left[a_x^2 + a_y^2\right]^{1/2} \cos \omega t$$

$$\psi = \tan^{-1}\left(\frac{-a_y}{a_x}\right)$$

inclination angle

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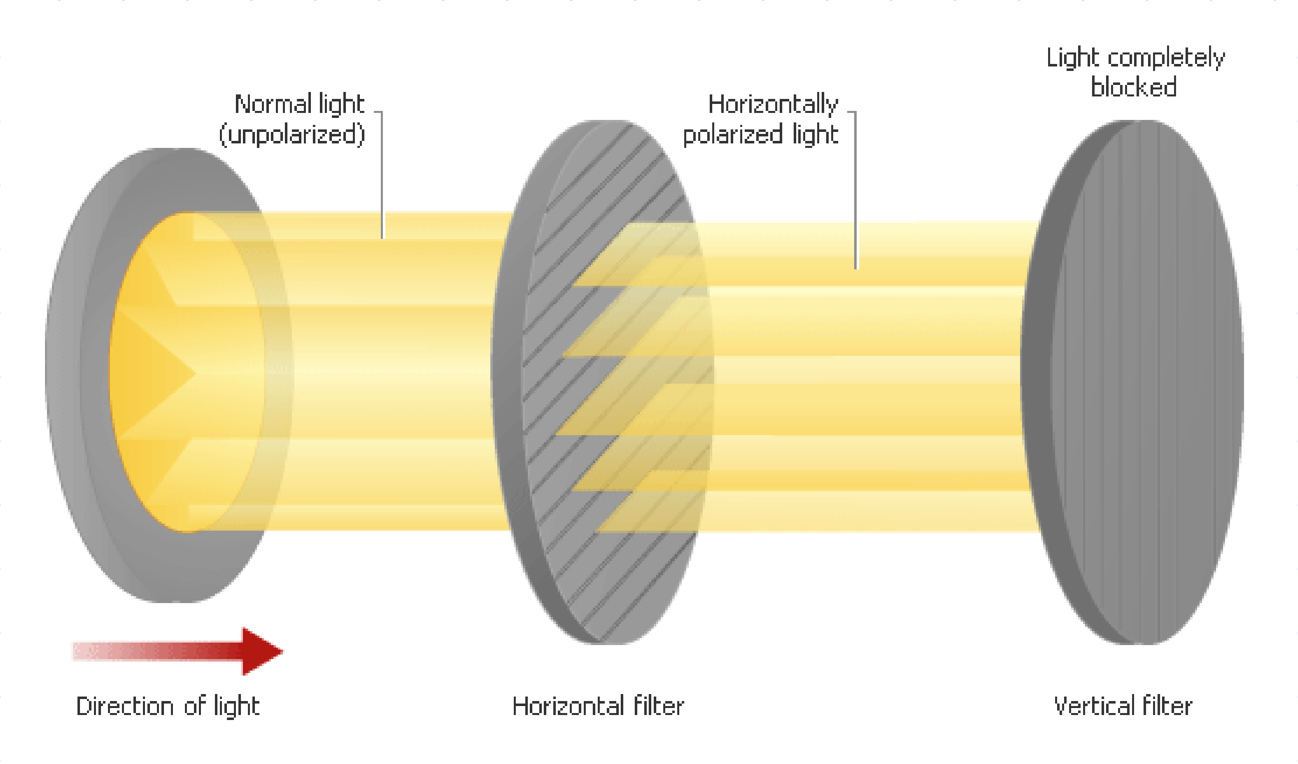
Wave Polarization



Encarta

Wave Polarization

Intensity and Inclination Angle

The intensity of

$$\left|E(z,t)\right| = \left[E_x^2(z,t) + E_y^2(z,t)\right]^{1/2}$$

$$= \left[a_x^{\;2}\cos^2(\omega t - kz) + a_y^{\;2}\cos^2(\omega t - kz + \delta)\right]^{1/2}$$

The inclination angle ψ

$$\psi(z,t) = \tan^{-1}\left(\frac{E_y(z,t)}{E_x(z,t)}\right)$$

generally they both are function of t and z

Wave Polarization

- Linear polarized light is separable as coherent superposition of two linearly polarized waves.
- A phase shift between the two linearly polarized components, changes linear into circular or elliptical polarization.

Wolfram applet

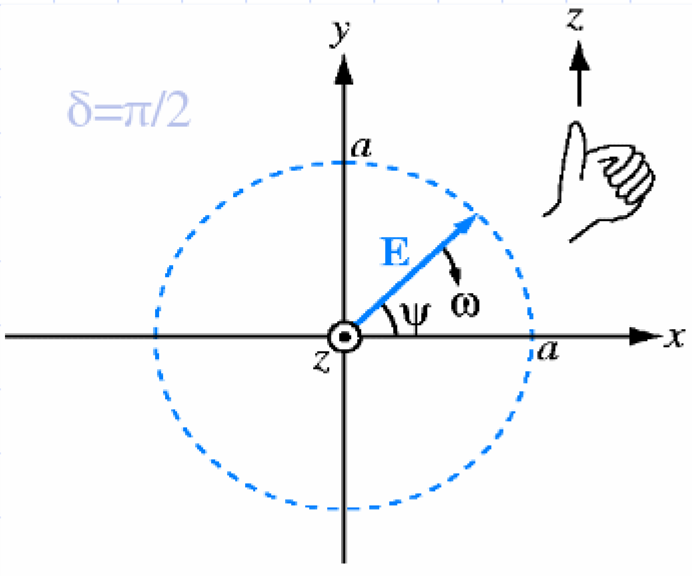
Also a very similar principle to Lissajous figures

Wave Polarization

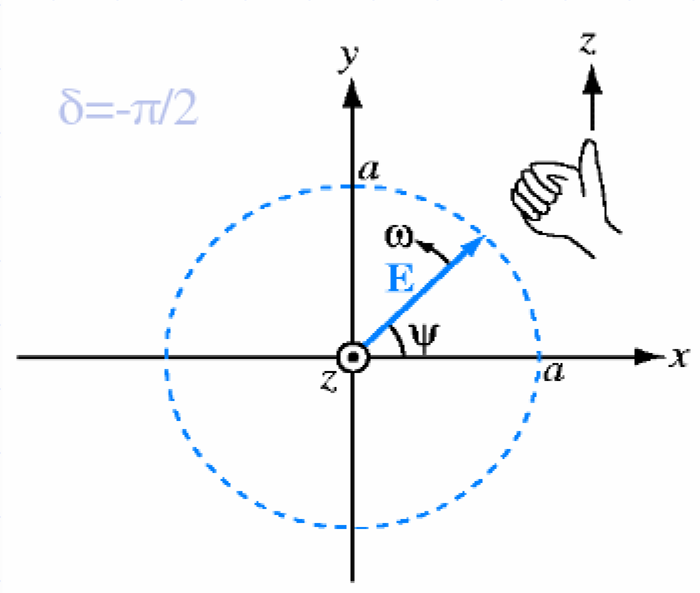
Circular Polarization

A wave is said to be circularly polarized if

- the magnitudes of $\tilde{E}_x(z)$ and $\tilde{E}_y(z)$ are equal and
- The phase difference is $\delta = \pm\pi/2$



LHC polarization



RHC polarization

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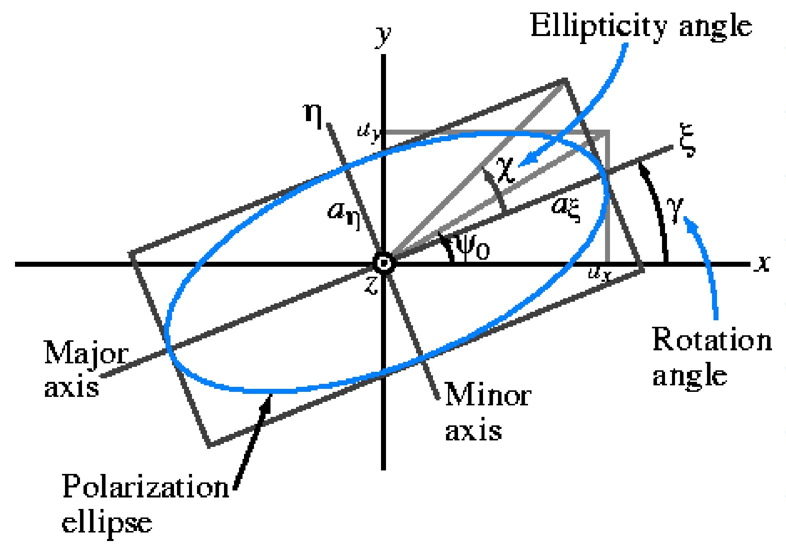
Wave Polarization

Elliptical Polarization

Generally $a_x \neq a_y \neq 0$ and $\delta \neq 0$. the tip of E traces an elliptical path in x-y plane



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<u>rotation angle, γ</u>

$-\pi/2 \leq \gamma \leq \pi/2$

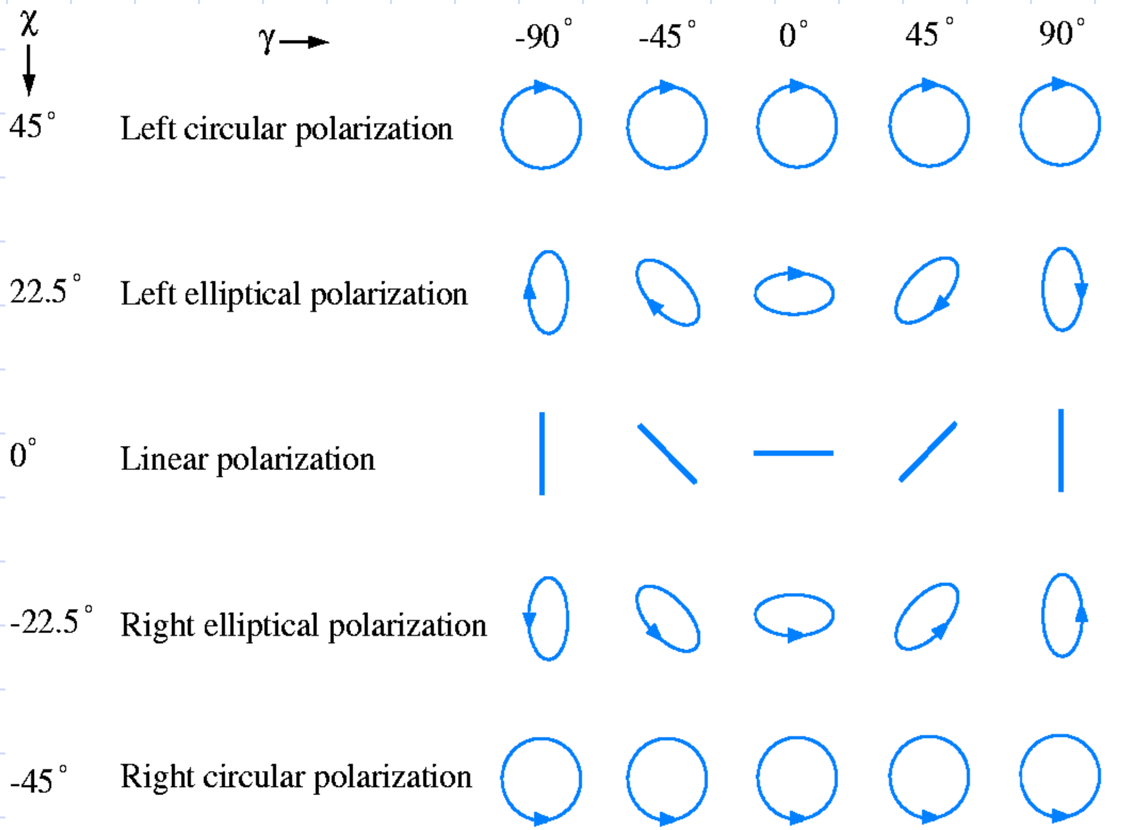
<u>Ellipticity angle, χ</u>

$$\tan \chi = \pm\left(\frac{-a_\eta}{a_\xi}\right) = \pm\frac{1}{R}$$

$-\pi/4 \leq \chi \leq \pi/4$

Wave Polarization

Polarization States



(the wave is traveling out of the slide)

Ulaby

Wave Polarization

Elliptical Polarization

$$\tan 2\gamma = (\tan 2\psi_0) \cos\delta \quad (-\pi/2 \leq \gamma \leq \pi/2), \qquad (7.59a)$$

$$\sin 2\chi = (\sin 2\psi_0) \sin\delta \quad (-\pi/4 \leq \chi \leq \pi/4), \qquad (7.59b)$$

where ψ_0 is an ***auxiliary angle*** defined by

$$\tan\psi_0 = \frac{a_y}{a_x} \qquad \left(0 \leq \psi_0 \leq \frac{\pi}{2}\right). \qquad (7.60)$$

Ulaby pg. 329

Wave Polarization

Example 1

Consider a wave travelling in the z direction whose electric field is given by $E(z,t) = 3cos(\omega t - \beta z)\hat{a}_x + Ccos(\omega t - \beta z + \phi)\hat{a}_y$

Describe the polarization (e.g. linear, right circular, etc.) and on an xy plot sketch the locus of $E(z,t)$ over a cycle for the following cases.

a) $C = 4\,V/m,\ \phi = 0^o$

b) $C = 3\,V/m,\ \phi = 45^o$

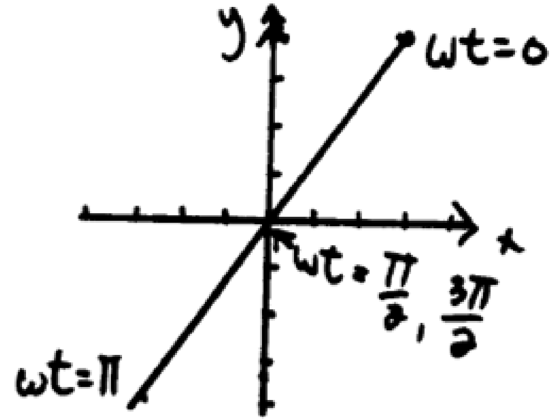
Wave Polarization

Example 1

a. $\vec{E} = 3\cos(\omega t - \beta z)\,\hat{a}_x + 4\cos(\omega t - \beta z)\,\hat{a}_y$

at $z = 0$ $\quad \vec{E}(0,t) = 3\cos\omega t\,\hat{a}_x + 4\cos\omega t\,\hat{a}_y$

$\boxed{\text{LINEAR POLARIZATION}}$



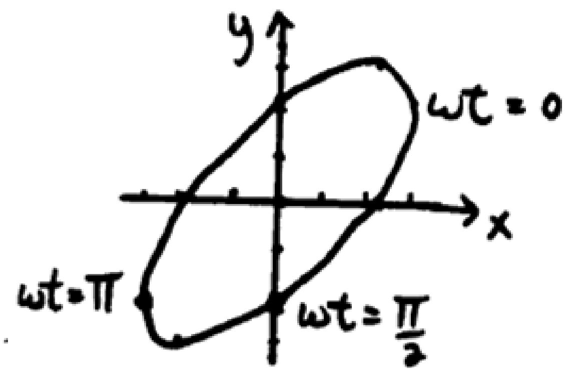
b. $\vec{E} = 3\cos(\omega t - \beta z)\,\hat{a}_x + 3\cos(\omega t - \beta z + \frac{\pi}{4})\,\hat{a}_y$

at $z=0$

$\vec{E}(0,t) = 3\cos(\omega t)\,\hat{a}_x + 3\cos(\omega t + \frac{\pi}{4})\,\hat{a}_y$

$\omega t = 0 \quad E = 3\hat{a}_x + \cancel{3\cos 45}\ \underbrace{3\cos\tfrac{\pi}{4}}_{2.1}\hat{a}_y$

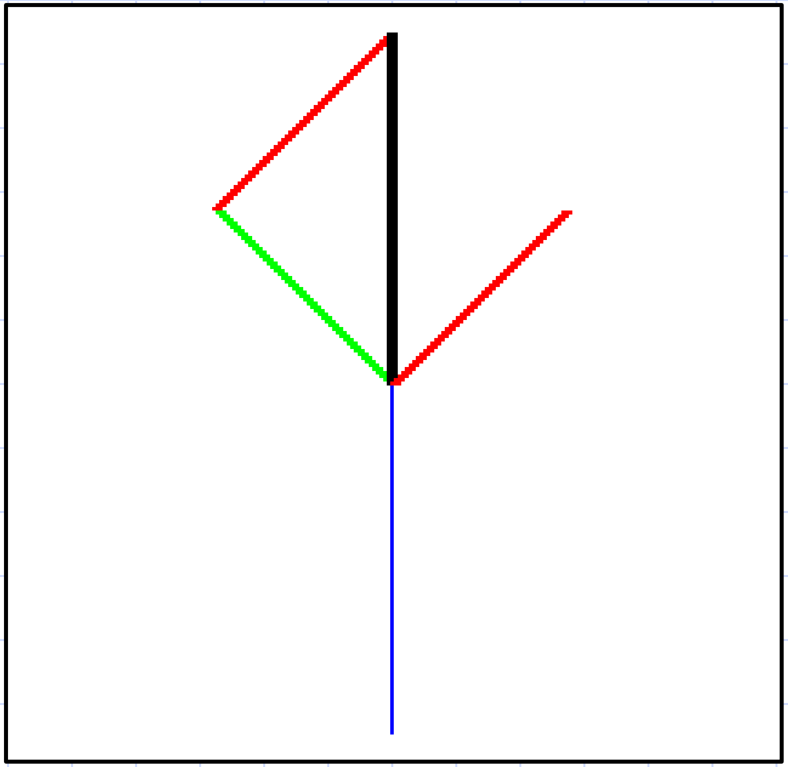
Similarly obtain other points



$\boxed{\text{ELLIPTICALLY POLARIZED}}$

Polarization and Materials

Linear polarized light is separable as coherent superposition of two linearly polarized waves. As shown in the figure to the right both waves (red and green amplitude) are polarized perpendicular with respect to each other and of identical amplitude.

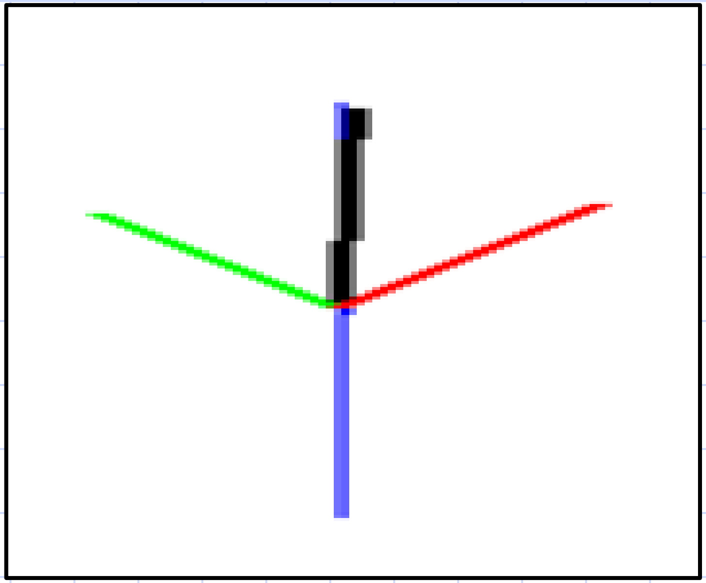


Alkwin Slenczka

Polarization and Materials

Birefringence, which causes a phase shift between the two linearly polarized components, changes linear into circular polarization.

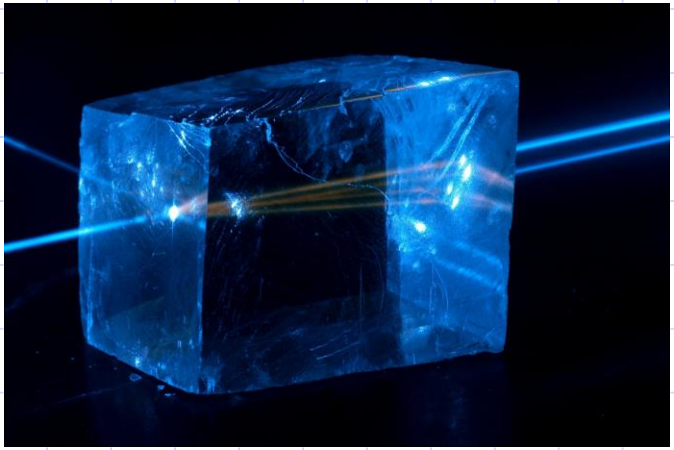
(or elliptical)



Alkwin Slenczka

Polarization and Materials

- Birefringence is the property by which the refraction index of material depends on the direction of the field hitting it
- Many crystals have this property
- Since the electric and magnetic fields of an EM wave have different directions, this can introduce a phase delay between them, turning linearly-polarized light into circularly or elliptically-polarized light

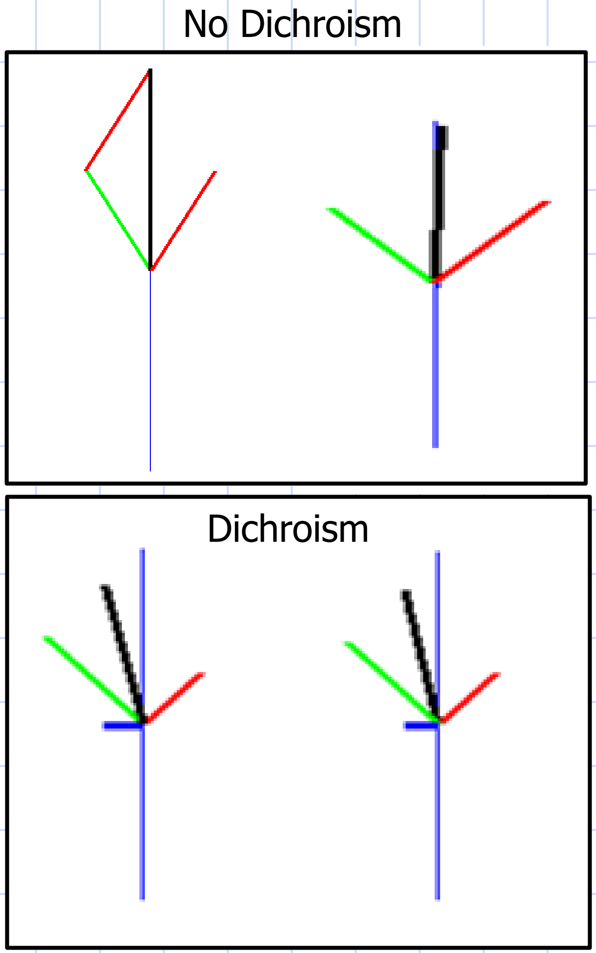


Wikipedia

Polarization and Materials

Linear dichroism, on the other hand, represents differing absorption of light depending on polarization direction (also a property of anisotropic materials.)

Linear dichroism produces a rotation in the plane of polarization.



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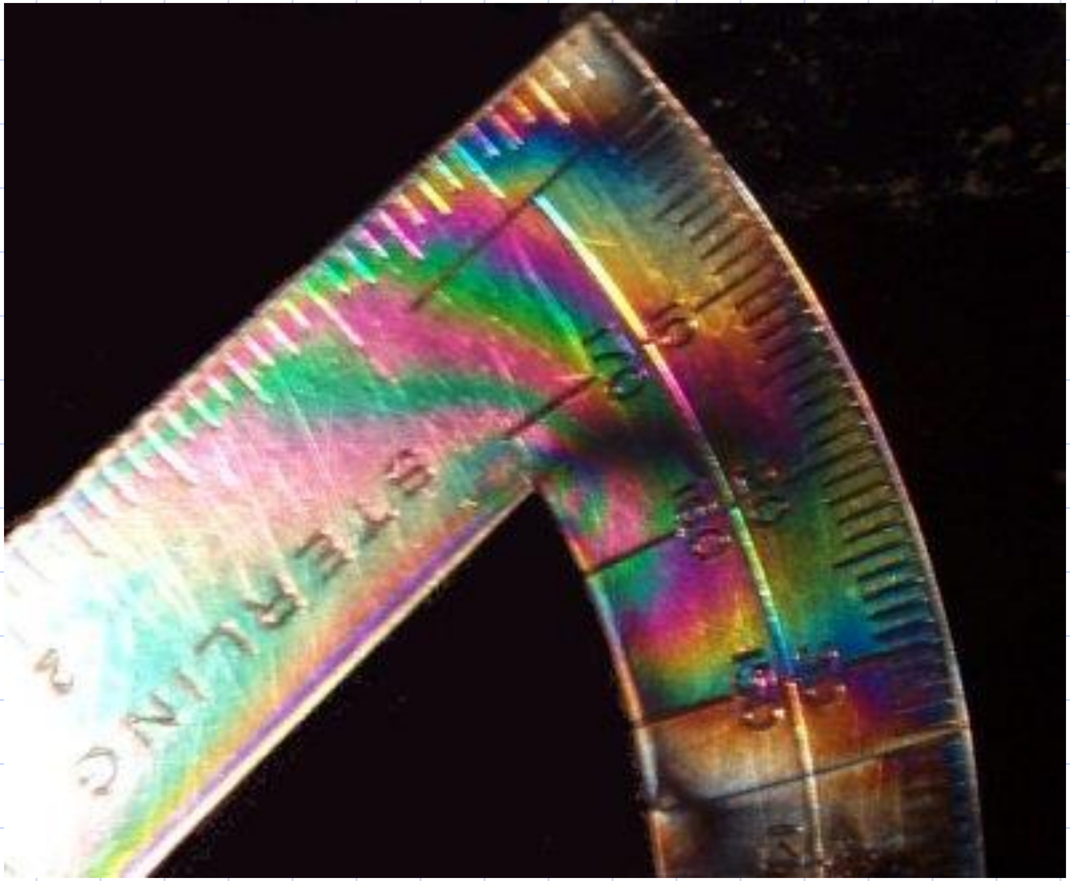
Polarization and Materials

Stress Birefringence



http://www.oberlin.edu/physics/catalog/demonstrations/optics/birefringence.html

Polarization and Materials

Birefringence on Plastic Film

http://www.engl.paraselene.de/html/birefringence_on_plastic_film.html

Polarization and Materials

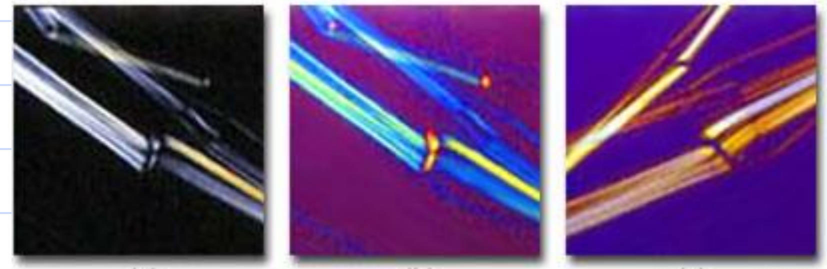
- Isotropic materials, which include gases, liquids, unstressed glasses and cubic crystals, demonstrate the same optical properties in all directions.
- They have only one refractive index and no restriction on the vibration direction of light passing through them.

Polarization and Materials

- Anisotropic materials, in contrast, which include 90 percent of all solid substances, have optical properties that vary with the orientation of incident light with the crystallographic axes.
- They demonstrate a range of refractive indices depending both on the propagation direction of light through the substance and on the vibrational plane coordinates
- More importantly, anisotropic materials act as beam splitters and divide light rays into two parts.
- The technique of polarizing microscopy exploits the interference of the split light rays, as they are re-united along the same optical path to extract information about these materials.

Polarization and Materials

Chrysotile Asbestos Fibers in Polarized Light

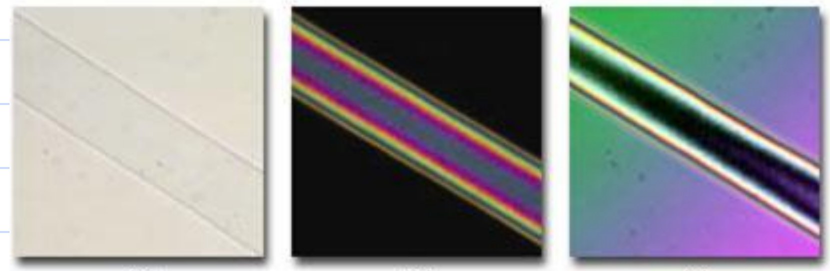


(a) (b) (c)

Natural and Synthetic Polymers in Polarized Light

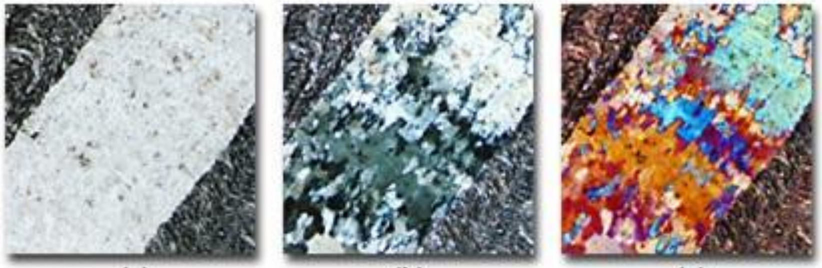
(a) (b) (c)

Nylon Fiber in Polarized Light



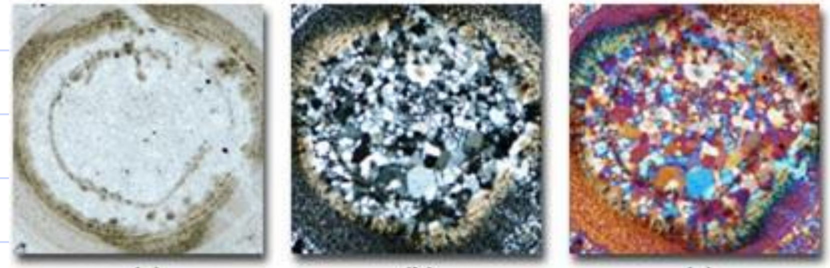
(a) (b) (c)

Phyllite Thin Section in Polarized Light



(a) (b) (c)

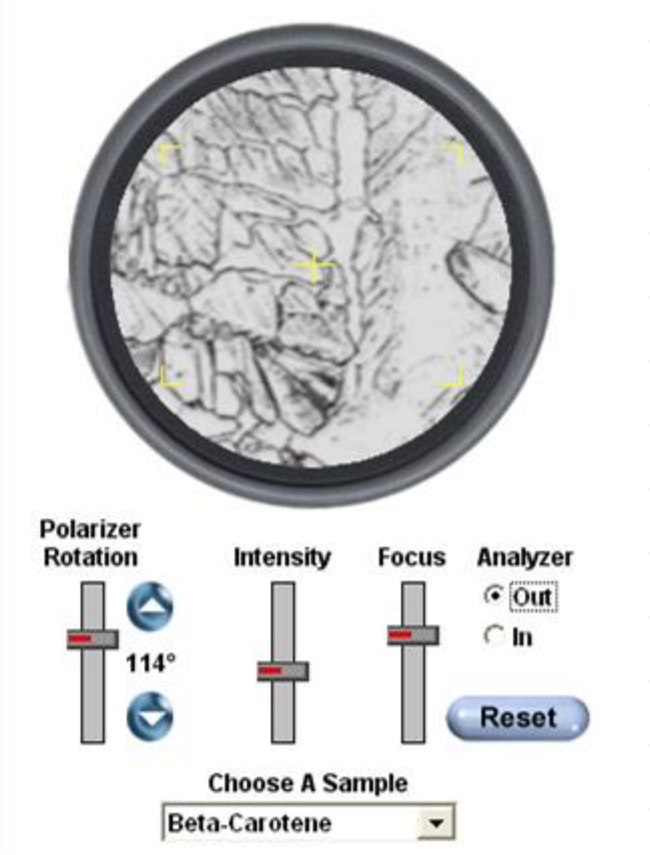
Oolite Thin Section in Polarized Light

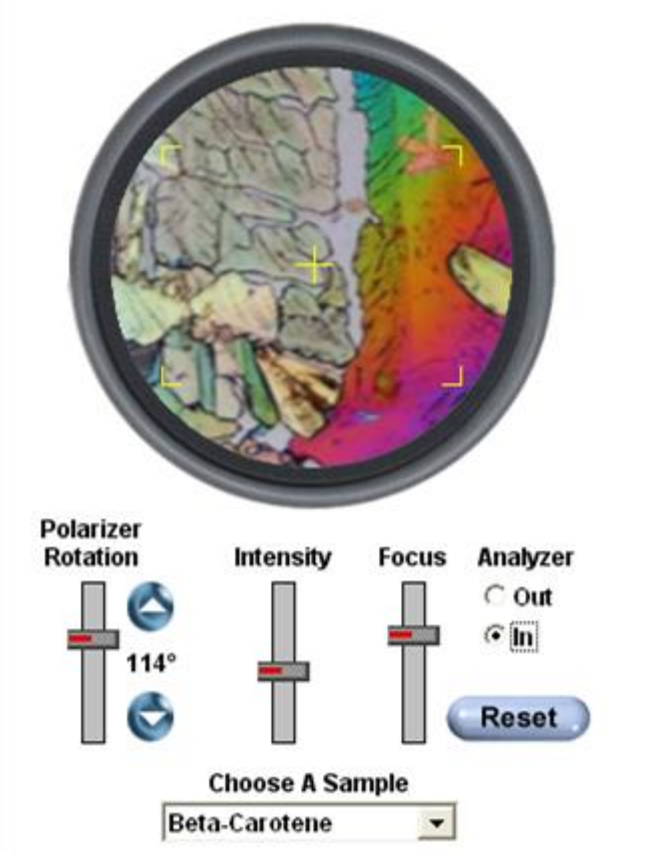


(a) (b) (c)

http://www.microscopyu.com/articles/polarized/polarizedintro.html

Polarization and Materials





http://www.microscopyu.com/tutorials/java/polarized/polarizerrotation/index.html

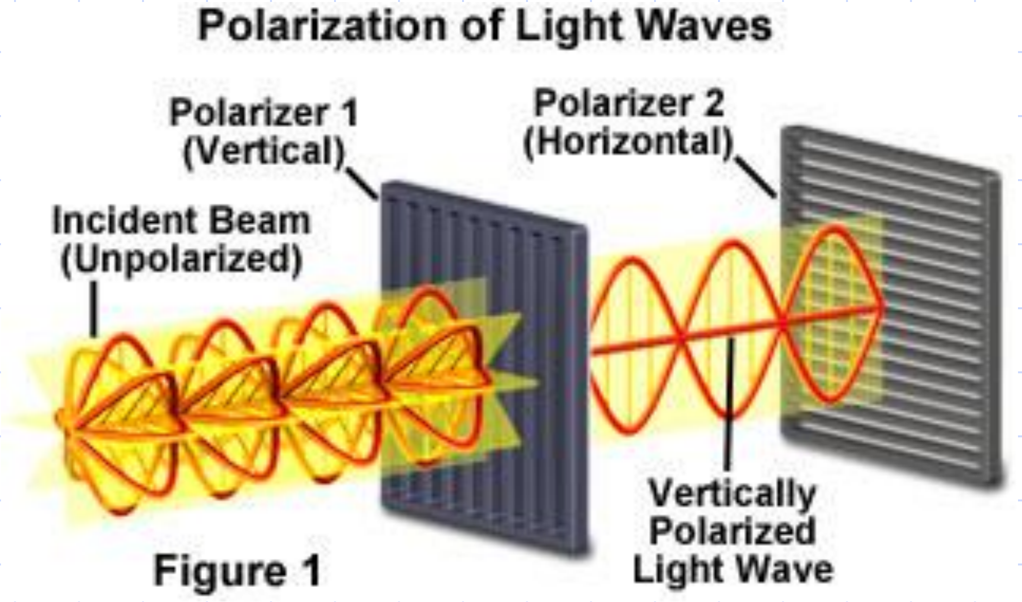
Polarization and Materials

- Light reflecting off of the surface of water is polarized, so polarizing lenses are used in photography to reduce glare

Wikipedia

Polarization and Materials

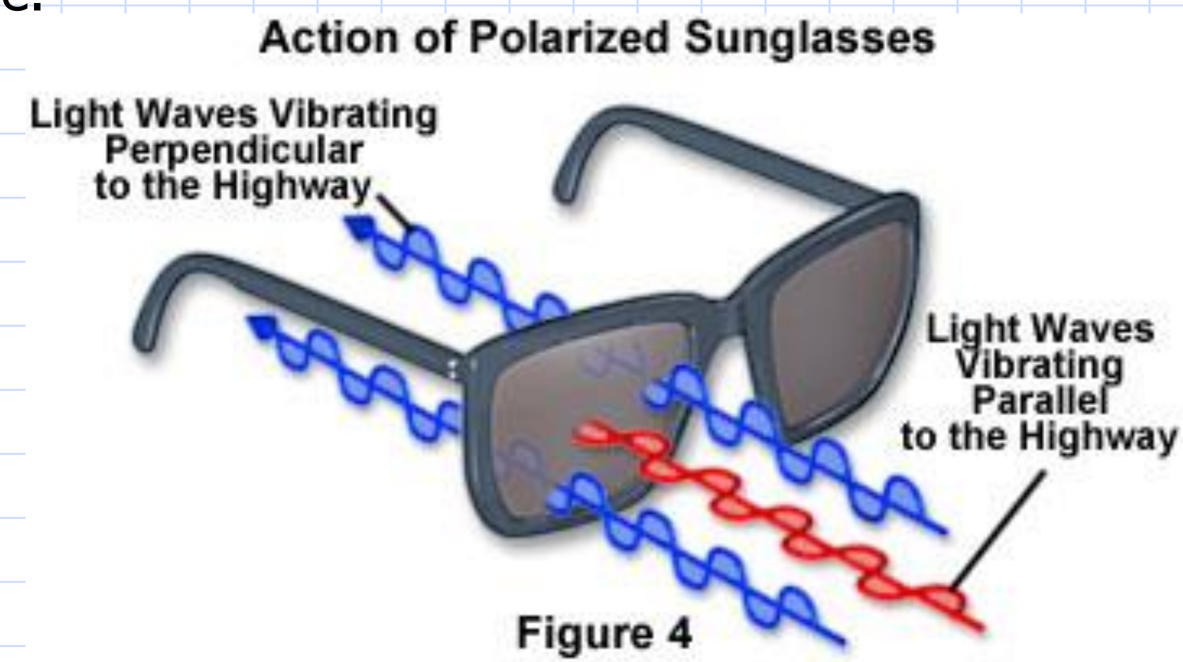
Naturally occurring light is randomly polarized. That is, it is equally probable for the electric field to be in any direction. A polarizing filter can select a particular polarization of light.



*http://www.mic-d.com/curriculum/lightandcolor/polarizedlight.html

Polarization and Materials

Sunglasses with polarizing lenses are made to block light that is reflected from highly reflective surfaces and, thus, can greatly reduce the effects of glare.



*http://www.mic-d.com/curriculum/lightandcolor/polarizedlight.html

Review

Do Lecture 21 Exercise 2 in groups of up to 4.