Problem Set #6

Innovation and Growth

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Due 4/14/23, 2PM

1. Dynamic inefficiency - quality ladders versus input varieties.

   (a) Argue that the equilibrium growth rate in the input varieties model is always inefficiently low by illustrating the social versus private value of an innovation in a graph of the intermediate input/innovation market.

   (b) Argue that the equilibrium growth rate in the quality ladder model may be either inefficiently low or high by illustrating the social versus private value of an innovation in a graph of the intermediate input/innovation market.

   (c) Which do you consider a more realistic interpretation? Why?

2. Consider the quality ladder model presented in class. However, instead of innovations increasing the productivity of the input category, assume that innovations reduce the marginal cost of producing the input. That is, each final goods producer has a production function of

$$Y_i = L_i^{1-\alpha} \sum_{j=1}^{N} X_{ij}^{\alpha}$$

   Where N is fixed, and the marginal productivity of inputs does not change with innovation. The marginal cost of producing one unit of X_j changes with innovation according to

$$c(k_j) = \frac{1}{q^{k_j}}, \qquad q > 1$$

   (a) Interpret this conceptualization of innovation. Do you think it is realistic? Provide a real world example of this type of innovation.

   (b) Set-up the final goods producer's maximization problem. Derive the aggregate demand of each input as a function of its price (first order condition).

   (c) Set-up the static maximization problem of the R&D firm with the monopoly right of sale to the current highest quality (lowest cost) in an arbitrary industry j. Try to derive the optimal monopoly price of inputs. It is OK if you can't get the math to work out exactly - do you expect the optimal price to be constant?

(d) Like we did in class, use a graph with $P(\$)$ on the vertical axis and quantity X on the horizontal axis to illustrate how the market for input x_j changes when innovation moves the quality (cost) from k to $k+1$. *Hint: which curve(s) shift?*