

Homework 1

Released: August 31st

Due: 11:59pm September 10th

1. Coaxial Cables

- The Ulaby textbook's companion website (<https://em8e.eecs.umich.edu/>) has many resources that we'll make use of during the course this semester. One of these resources is a set of applications that allow you to interact with fundamental concepts in electricity and magnetism. Navigate to the site and click on the Modules tab.
- Once on the Interactive Modules page, click on "Chapter 2: Transmission Lines", then "Coaxial Cable". Coaxial cables are one of the main types of transmission line, which you'll find in many different applications. It may take some time to load (and you may need to install Java).
- Use the drop-down menu next to "Select" to select "Impedance vs. Radius a".
- Let's consider the case of a coaxial cable with an inner diameter ($2a$) of 3mm, an outer diameter ($2b$) of 9mm, and a dielectric constant (ϵ_r) of 2.1. What is the characteristic impedance of this coaxial cable to the nearest ohm? Additionally, we will be considering a lossless coaxial cable here, so ignore imaginary components of the characteristic impedance Z_0 (and thus any frequency-dependence of Z_0).
- What is the velocity of the signal on this coaxial cable in m/s?
- If the wavelength of the signal being transmitted on this coaxial cable is 2m, what are the approximate phase constant β in rad/m and the frequency f in Hz?
- Now consider the case of a lossy transmission line, which the app calculates by default. How does Z_0 change as frequency increases? For a given frequency, what happens to Z_0 as the conductivity of the conductor (σ_c) decreases (as the line resistance increases)?

2. Phasor Notation

Note: Remember when you convert from the z - t domain to phasor notation, that you first have to convert to a cosine base, and that when converting from phasor notation back to the z - t domain, you obtain a cosine.

- Convert the following voltage from the time domain into phasor notation: $V(t) = 5j \sin(\omega t)$
- Convert the following current phasor into the time domain: $\tilde{I} = 0.10e^{-j(\beta z + \frac{\pi}{4})}$.
- Is the current wave in b) a traveling wave or standing wave? If it's a traveling wave, which direction is it traveling?

3. Traveling Waves and Standing Waves Part 1

The voltage and current on a transmission line are given by

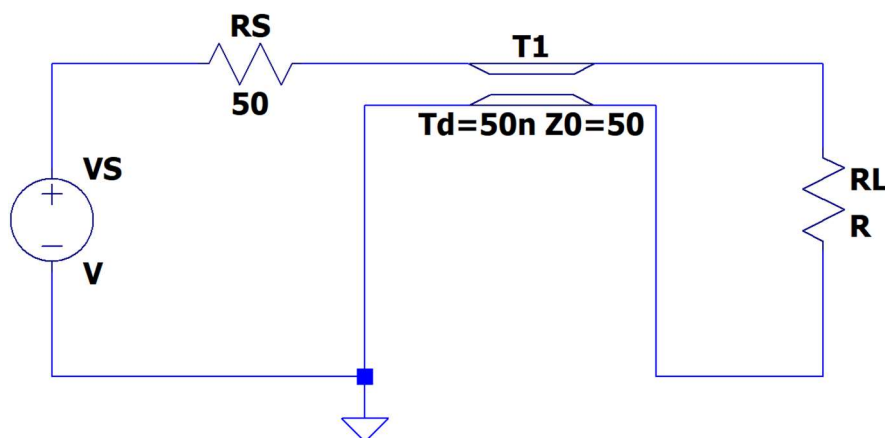
$$v(z, t) = 9 \cos(2\pi \times 10^7 t + 1.5\pi z)$$

$$i(z, t) = 0.09 \cos(2\pi \times 10^7 t + 1.5\pi z)$$

- Are the voltage and current waves traveling waves or standing waves or neither? If they are traveling waves, what is their frequency f in Hz and which direction are they traveling (+z or -z)?
- What are the period T and wavelength λ of the voltage and current waves?
- What is the characteristic impedance of the transmission line?
- What is the velocity of the signal on the transmission line?
- What are the capacitance and inductance per unit length of the transmission line?
- Express both the voltage and current waves in phasor form.

4. Traveling Waves and Standing Waves Part 2

Consider the following system consisting of a voltage source V_s , source resistance $R_s = 50\Omega$, transmission line with characteristic impedance $Z_0 = 50\Omega$, and load resistance R_L , as shown below. At time $t = 0$, the voltage source V_s is switched on and sends a voltage wave of the form $\tilde{V} = 2V^+ e^{-j\beta z}$ towards the transmission line.



- If $R_L = 50\Omega$, write the expression for \tilde{V}_L , the voltage across the load resistor R_L . How much of the incident wave \tilde{V} is reflected between the end of the transmission line and R_L ?
- For the rest of the problem, suppose instead that $R_L = 0$. What is the reflection coefficient Γ ? How much of the incident wave is reflected when the load is a short circuit?
- Write the expression for the total voltage on the transmission line, including both incident and reflected waves. Recall that the general solution for the Telegrapher's Equations in phasor form is $\tilde{V} = V^+ e^{-j\beta z} + V^- e^{+j\beta z}$ and that $V^- = \Gamma V^+$. Leave your answer in phasor form, in terms of V^+ .
- Convert your answer to the z - t domain. Your answer should be in the form of a sum of two cosines.
- Using the trigonometric identity $\cos(\theta) - \cos(\phi) = -2 \sin\left(\frac{\theta+\phi}{2}\right) \sin\left(\frac{\theta-\phi}{2}\right)$, convert your answer into a product of sine functions. What kind of wave is this?
- What is the reflection coefficient when the load resistance R_L is an open circuit ($R_L = \infty$)? Would you expect a standing wave to form in this case? Why or why not?