Small signal versus Large signal

Linear systems:

Output $\propto$ Input

Current $\propto$ Voltage

Effect $\propto$ Cause

Mathematics: $y = cx \Rightarrow y \propto x$

$F(ax) = aF(x)$

Non-linear systems

Effect not $\propto$ Cause

Linear circuit elements

R  $\Rightarrow I \propto V$

C  $\Rightarrow I \propto V$

L  $\Rightarrow I \propto V$

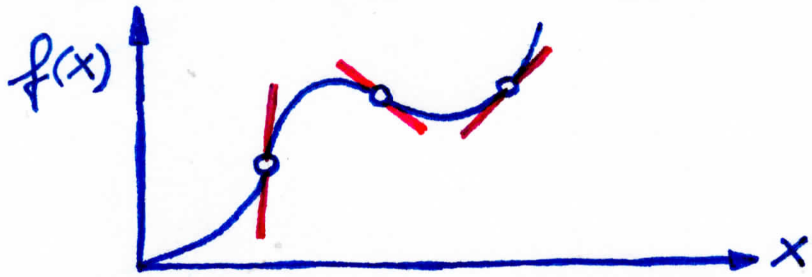
Non-linear circuit elements

Diode  $\Rightarrow I \propto \exp(V)$

BJT  $\Rightarrow I_c \propto \exp(V_{BE})$

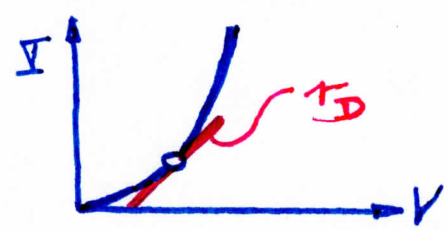
FET  $\Rightarrow I_D \propto g_m(V - V_{th})$

Mathematics: Any non-linear function $f(x)$ can be linearized in the vicinity of a point x.



Electronics: Any non-linear circuit can be linearized in the vicinity of its opeating point.

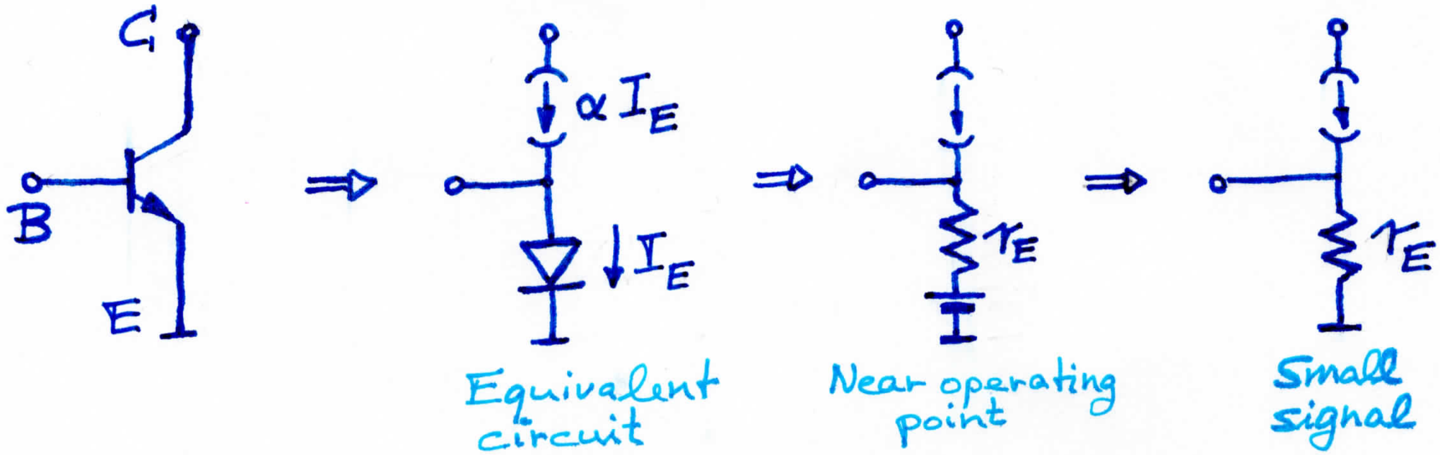
Example: Diode



$$r_D = \frac{dV}{dI} = \left(\frac{dI}{dV}\right)^{-1}$$

⇒ For small signals, near a certain operating point, the diode has the differential resistance $r_D = \left.\frac{dV}{dI}\right|_{\text{operating point}} = \left.\left(\frac{dI}{dV}\right)^{-1}\right|_{\text{oper. point}}$

Small-signal input impedance of BJT



Small-signal E resistance (input resistance)
↳ emitter

$$r_E = \frac{dV}{dI} = \left(\frac{dI}{dV}\right)^{-1} = \left(\frac{dI_E}{dV_{BE}}\right)^{-1} = \left(\frac{d}{dV_{BE}} I_E\right)^{-1}$$

$$= \left(\frac{d}{dV_{BE}} I_0 \left(e^{V_{BE}/V_t} - 1\right)\right)^{-1} \approx \left(\frac{d}{dV_{BE}} I_0\, e^{V_{BE}/V_t}\right)^{-1}$$

$$= \left(I_0\, e^{V_{BE}/V_t} \frac{1}{V_t}\right)^{-1} = \left(\frac{I_E}{V_t}\right)^{-1} = \frac{V_t}{I_E}$$

$$\Rightarrow \boxed{r_E = \frac{V_t}{I_E}}$$

Also $r_E = \dfrac{V_t}{I_E} = \dfrac{V_t}{I_B(\beta+1)} \approx \dfrac{V_t}{\beta I_B}$