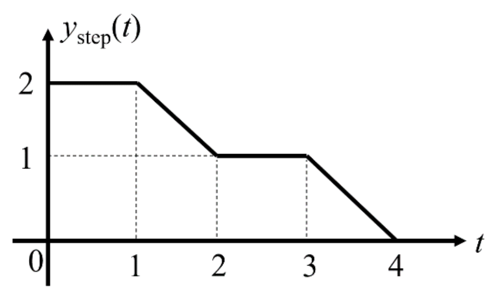
# Homework # 4

## Due: Monday, July 10th

**Problem 1.** (9 points) Let the impulse response function of an LTI system be $h(t) = 5e^{-t}u(t)$

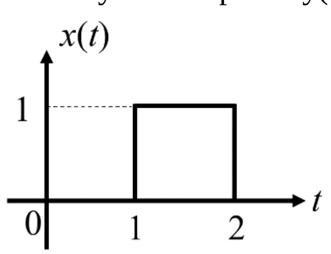
a) Determine the output of the system to x(t) = u(t) via the convolution integral.
b) Use the result of part (a) to find the output of the system to inputs $x_1(t) = u(t + 2)$ and $x_2(t) = u(t - 1)$.
c) Use the result of part (b) to find the output to the input $x(t) = 5(u(t + 2) - u(t - 1))$.

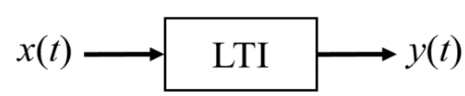
**Problem 2.** (15 points) If the step response ystep(t) of a LTI system is




a) Find the impulse response h(t) and sketch it.

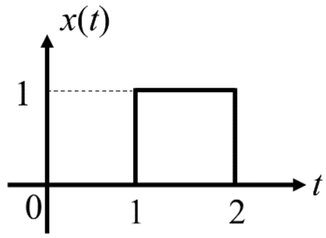
b) Sketch the system response y(t) when the input is



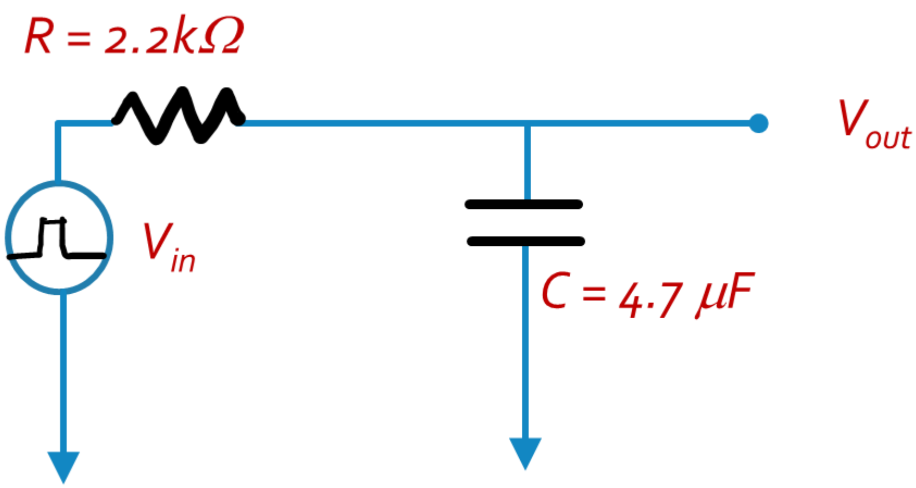

By convolving the input x(t) with the impulse response h(t) obtained in part a.

c) (10 points) Sketch the system response y(t) for the same input x(t) shown above by first expressing the input using step functions and then using superposition.





**Problem 3.** (36 points) Using the Discovery 2 board or M1K board, generate and plot an impulse response of an RC circuit shown below. Use an impulse of 1V amplitude, 1 second time period and 4% duty cycle. Also, by storing the impulse response digitally, convolute using Matlab to obtain the step response of the RC circuit. Experimentally verify the result on the Discovery 2 board or M1K board. Show all the results clearly along with an image of the circuit connected on the board.




**Problem 4**. (10 points) Express the following signal in terms of sinusoids.

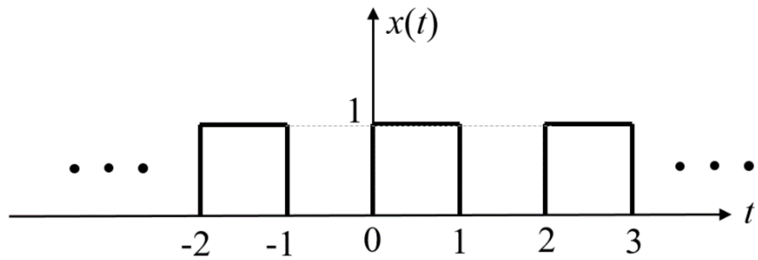
$$x(t) = je^{jt} - je^{-jt} + (1+j)e^{2jt} + (1-j)e^{-2jt}$$

That is, find real numbers $A_1$, $A_2$, $\phi_1$, $\phi_2$ in the sinusoidal representation below:

$$x(t) = A_1\cos(t - \phi_1) + A_2\cos(2t - \phi_2).$$

The amplitude $A_1$ and $A_2$ have to be positive.

**Problem 5.** (10 points) Consider the periodic signal $x(t)$ shown below. It can be expressed using the exponential Fourier series as




$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where $\omega_0 = {}^{2\pi}/_{T}$ with $T$ being the period of the signal. The Fourier Series coefficients $a_k$ are calculated by

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}\, dt.$$

Calculate the Fourier Series coefficients of $x(t)$. You must show your integral calculation (i.e., do not refer to table or textbook). Calculate the zeroth coefficient (*k=0*) separately if you encounter division by zero in your calculation.

**Problem 6**. (10 points) Consider the following signal

$$x(t) = 2\cos\left(300\pi t + \frac{\pi}{3}\right) + 5\sin(600\pi t) - 10\cos\left(900\pi t + \frac{\pi}{4}\right)$$

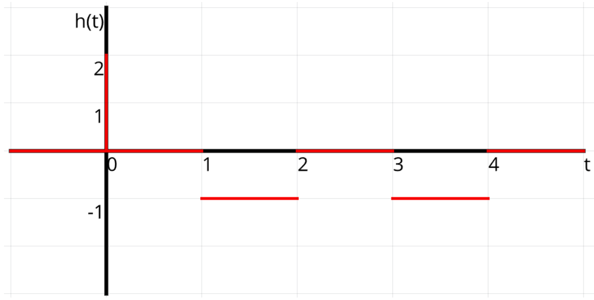
a) (10 points) Find the fundamental period of $x(t)$.
b) (20 points) Use Euler's formula to find the Fourier Series $x(t)$.

**Problem 7.** (10 points) Consider a complex signal $x(t)$ with Fourier coefficients $\{X_k\}$.

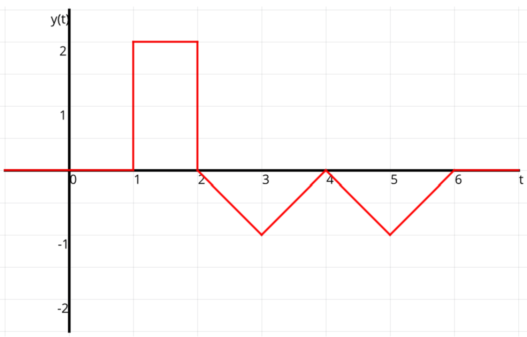
a) (15 points) Find the Fourier coefficients of Even$\{x(t)\}$ in terms of $\{X_k\}$.
b) (15 points) Find the Fourier coefficients of Odd$\{x(t)\}$ in terms of $\{X_k\}$.

1)
a)
h(t)*u(t)
=\int_-\inf^\inf h(T) \cdot u(t-T) dT
=5(1-e^-t)
b)
h(t)*x1(t)=5(1-e^-(t+2))
h(t)*x2(t)=5(1-e^-(t-1))
c)
h(t)*5((x1(t)-x2(t))
=25((1-e^-(t+2))-5(1-e^-(t-1)))

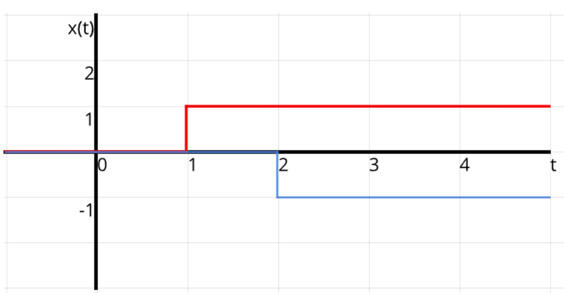
2)
a)
h(t)= { -1   1<t<2 , 3<t<4
      {   0      otherwise

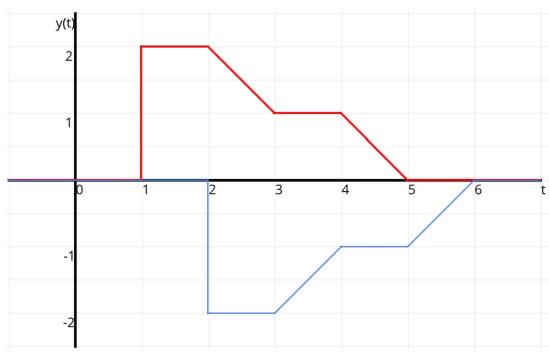


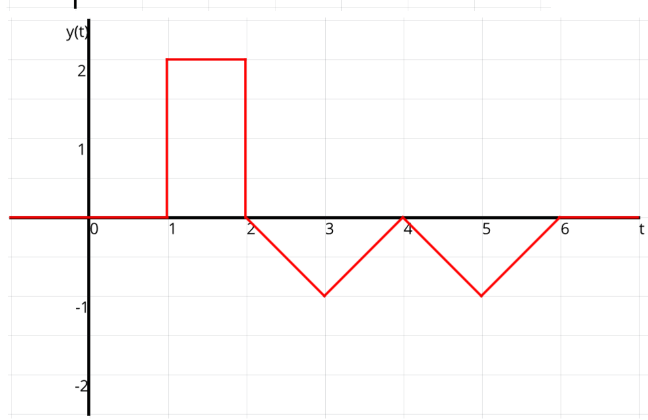

b)



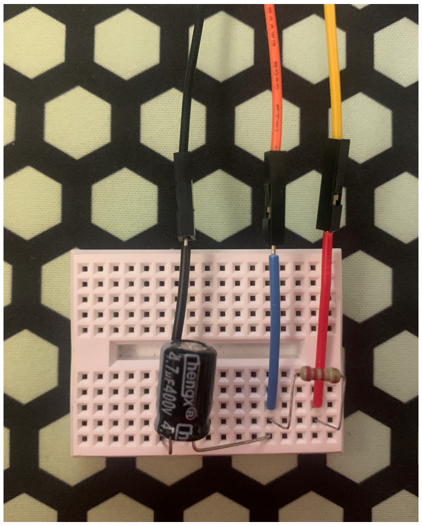

c)

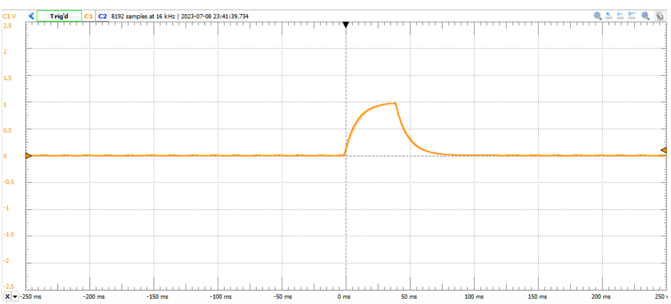


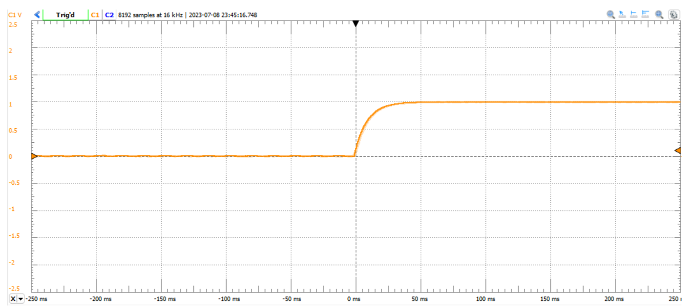



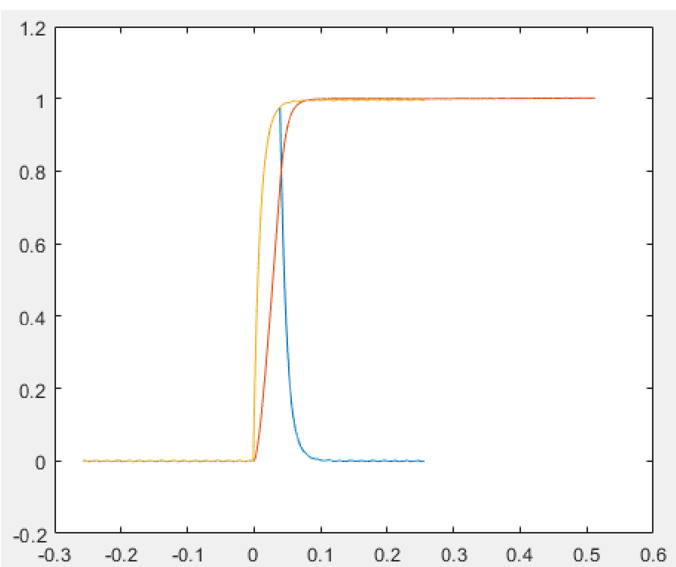



3)










The calculated step response (orange) has a slight delay compared to the experimental step response (yellow) because we could only approximate an impulse, giving us a slightly different transition, but after about 10ms they come to the same steady state.

4)
s=1
i(cos+isin)-i(cos-isin)
icos+i^2sin-(icos-i^2sin)
icos-sin-icos-sin
-2sin
2cos(x+pi/2)
A1=2
phi1=pi/2
s=2
(1+i)(cos+isin)+(1-i)(cos-isin)
1(cos+isin)+i(cos+isin)+1(cos-isin)-i(cos-isin)
cos+isin+icos-sin+cos-isin-icos-sin
2cos-2sin
2sqrt2 cos(2x+pi/4)
A2=2sqrt2
phi2=pi/4

A1 , A2 = 2 , 2sqrt2
phi1 , phi2 = pi/2 , pi/4

5)
T=2
ak=1/2 int_0^2 x(t) e^(-j k pi t) dt
ak=1/2 int_0^1 e^(-j k pi t) dt
ak=1/2 (sin(pi k) + j (cos(pi k)-1))/(pi k)
ak=sinc(pi k)/2 + j (cos(pi k)-1)/(2 pi k)
a0=.5+0j

a-3=j/3pi
a-2=0
a-1=j/pi
a0=.5
a1=-j/pi
a2=0
a3=-j/3pi

6)
a)
f0=GCD(150,300,450)=900Hz
T0=1/f0=1/900
b)
ak=900*int_0^1/900 x(t) e^(-j k 900 2pi t) dt

7)
a)
Even{x(t)}=1/2[x(t)+x(-t)]
evenXk=1/2[{Xk}+{X-k}]
b)
Odd{x(t)}=1/2[x(t)-x(-t)]
oddXk=1/2[{Xk}-{X-k}]