CSCI 2200 — Foundations of Computer Science (FoCS)
Problem Set 3 (document version 1.0)

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- ***Problem PS3.1.**

  a)

  we prove this with induction

  $n = 2$, $L_0 = 2$, $L_1 = 1$, $L_2 = 3$, $L_3 = 4$, $L_4 = 7$

  $L_0 + L_1 + L_2 = 2 + 1 + 3 = 6 = L_{2+2} - 1 = L_4 - 1 = 7 - 1 = 6$

  so $L_0 + L_1 + L_2 + ... + L_n = L_{n+2} - 1$ is true for $n = 2$

  assume $L_0 + L_1 + L_2 + ... + L_n = L_{n+2} - 1$ is true

  $L_0 + L_1 + L_2 + ... + L_n + L_{n+1} = L_{n+2+1} - 1 = L_{n+2+1-1} + L_{n+2+1-2} - 1 = L_{n+2} + L_{n+1} - 1$

  since $L_0 + L_1 + L_2 + ... + L_n = L_{n+2} - 1$, we can subtract from both sides which leaves us with $L_{n+1} = L_{n+1}$ which is true

  therefore $L_0 + L_1 + L_2 + ... + L_n = L_{n+2} - 1$ is true

  b)

  $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$

  $n = 2$, $L_2 = F_{2-1} + F_{2+1} = F_1 + F_3$, $3 = 1 + 2$

  so $L_n = F_{n-1} + F_{n+1}$ is true for $n = 2$

  $n = 3$, $L_3 = F_{3-1} + F_{3+1} = F_2 + F_4$, $4 = 1 + 3$

  so $L_n = F_{n-1} + F_{n+1}$ is true for $n = 3$

  assume $L_n = F_{n-1} + F_{n+1}$ and $L_{n+1} = F_{n+1-1} + F_{n+1+1} = F_n + F_{n+2}$ are true

  $L_{n+2} = F_{n+2-1} + F_{n+2+1} = F_{n+1} + F_{n+3}$

  $L_{n-1+2} + L_{n-2+2} = L_{n+1} + L_n = F_{n-1+1} + F_{n-2+1} + F_{n-1+3} + F_{n-2+3} = F_n + F_{n-1} + F_{n+2} + F_{n+1}$

  $L_{n+1} + L_n = F_n + F_{n-1} + F_{n+2} + F_{n+1}$

  $L_{n+1} = F_n + F_{n+2}$

  therefore $L_n = F_{n-1} + F_{n+1}$ is true

- ***Problem PS3.2.**

  we prove this with induction

  $2 + 6 + 12 + ... + (n^2 - n) = (n(n^2 - 1))/3$

  $n = 0$, $0^2 - 0 = 0 = (0(0^2 - 1))/3 = 0$

  so $2 + 6 + 12 + ... + (n^2 - n) = (n(n^2 - 1))/3$ is true for $n = 0$

  assume $2 + 6 + 12 + ... + (n^2 - n) = (n(n^2 - 1))/3$ is true

  $2 + 6 + 12 + ... + (n^2 - n) + ((n + 1)^2 - (n + 1)) = ((n + 1)((n + 1)^2 - 1))/3$

  $2 + 6 + 12 + ... + (n^2 - n) + ((n^2 + 2n + 1 - n - 1)) = ((n + 1)(n^2 + 2n + 1 - 1))/3$

  $(n(n^2 - 1))/3 + (n^2 + n) = ((n + 1)(n^2 + 2n))/3$

  $(n^3 - n)/3 + (n^2 + n) = (n^3 + 3n^2 + 2n)/3$

  $(n^3 - n) + (3n^2 + 3n) = (n^3 + 3n^2 + 2n)$

  $n^3 + 3n^2 + 2n = n^3 + 3n^2 + 2n$ is true

  therefore $2 + 6 + 12 + ... + (n^2 - n) = (n(n^2 - 1))/3$ is true

- ***Problem 6.21.** Prove that for $n \geq 1$, there is $k \geq 0$ and l odd such that $n = 2^k l$

  we prove this by strong induction

  if n is odd, $l = n$ and $k = 0$

  if n is even, it can be writen as $2j$, and $j < k$

if $n = 2^k l \wedge n = 2j \to j = 2^{k_2} l$: $k_2 = k + 1$