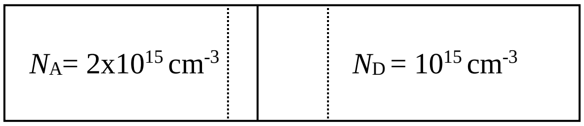
ECSE-2210 Microelectronics Technology
Homework 5 – Solution

1) a) The contact potential, V_{bi}

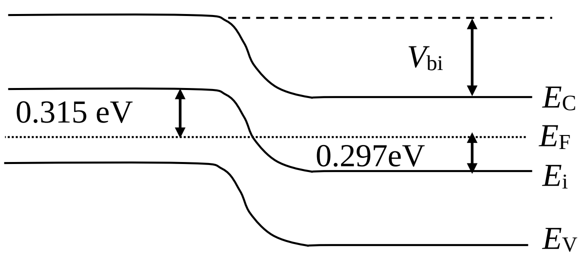


p-side: $E_i - E_F = kT \ln(\frac{p}{n_i}) = 0.0258 \text{ eV} \ln(\frac{2\times10^{15}}{10^{10}}) = 0.315 \text{ eV}$

n-side : $E_i - E_F = kT \ln(\frac{n}{n_i}) = 0.0258 \text{ eV} \ln(\frac{10^{15}}{10^{10}}) = 0.297 \text{ eV}$

$$V_{bi} = \frac{1}{q}[E_{i\,|\,\text{p-side}} - E_{i\,|\,\text{n-side}}] = \frac{1}{q}[E_F + 0.315 - (E_F - 0.297)] = 0.61 \text{ V}$$

Note: $V_{bi} = \frac{kT}{q} \ln(n_n p_p / n_i^2) = 0.0258 \text{ V} \ln(\frac{10^{15}\times 2x10^{15}}{10^{20}}) = 0.61 \text{ V}$



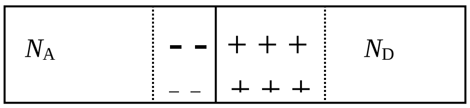
b) Total depletion layer width: $W = [\frac{2\varepsilon V_{bi}}{q}(\frac{1}{N_A} - \frac{1}{N_D})]^{1/2}$

$$W = [(2 \times 11.8 \times 8.85 \times 10^{-14} \text{ F/cm} \times 0.61 \text{ V}) / (1.6 \times 10^{-19} \text{ C}) \times$$
$$\times (\frac{1}{2\times10^{15}} - \frac{1}{10^{15}}) \text{ cm}^3]^{1/2} = 1.09 \times 10^{-4} \text{ cm} = 1.09\ \mu\text{m}$$

see page 214 in the textbook:

$x_p = W \times [N_D / (N_A + N_D)] = 0.09\ \mu m \times 10^{15}/(2\times10^{15} + 10^{15}) = 0.36\ \mu m$

$x_n = W \times [N_A / (N_A + N_D)] = 1.09\ \mu m \times 2\times10^{15}/(2\times10^{15} + 10^{15}) = 0.72\ \mu m$



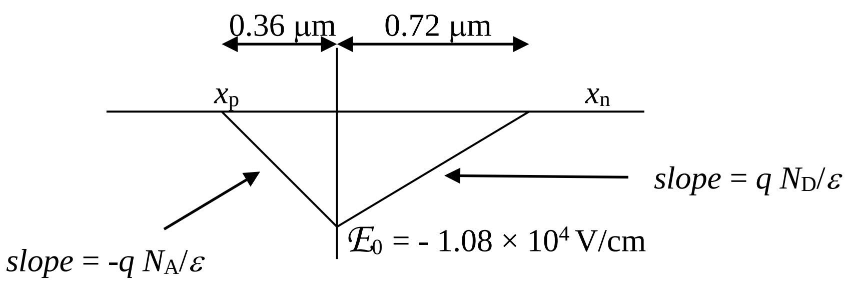
Note that, $N_A\, x_p = N_D\, x_n$

c) The maximum electric field occurs at the metallurgical junction.

$\mathcal{E}_0 = q/\varepsilon\, N_D\, x_n = -q/\varepsilon\, N_A\, x_p =$

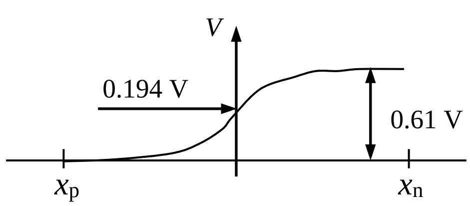
$= - [(1.6 \times10^{-19}\ C) / (11.8 \times 8\ 85 \times 10^{-14}\ F/cm)] \times 10^{15}\ cm^{-3} \times 0.72 \times 10^{-4}\ cm =$

$= - 1.08 \times 10^4\ V/cm$

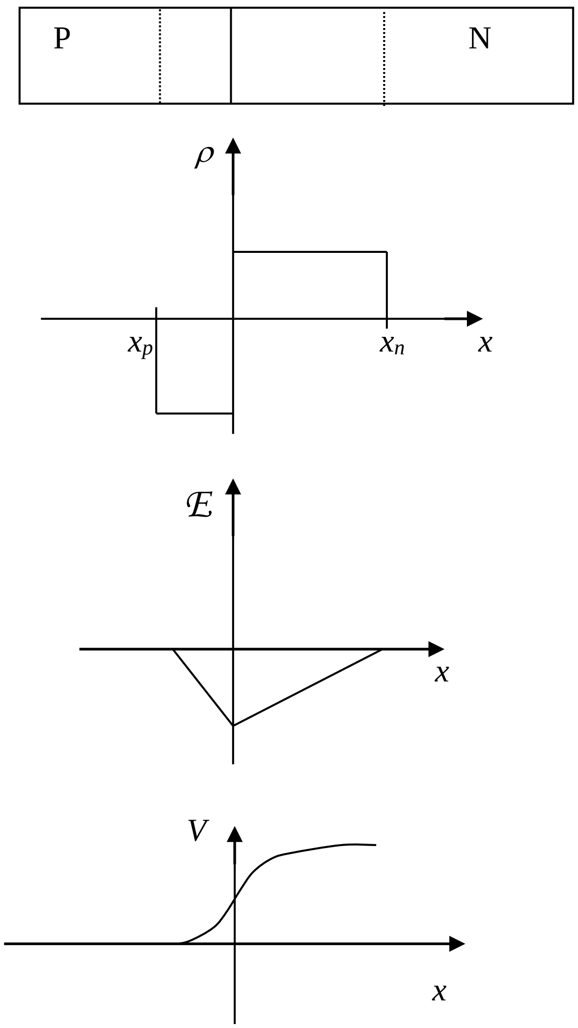


d) The potential at the metallurgical junction $V(0)$.

$V(0) = -\int \mathcal{E}\, dx = \mathcal{E}_0 \times 0.36\ \mu m \times 0.5 =$
$= 1.08 \times10^4\ V/cm \times 0.36 \times 10^4\ cm \quad = 0.194\ V$



e)



<u>Note:</u>
- Depletion layer extends further to lightly doped side.
- $\mathcal{E}$ field points in the negative x – direction everywhere.
- $\mathcal{E}$ – field varies linearly with x since doping constant everywhere: $d\mathcal{E}/dx = |\, q\, N_A / \mathcal{E}\, |$
- The potential varies as x^2 since $\mathcal{E}$-field varies linearly with x, $V = -\int \mathcal{E}\, dx$

2. a) $V_{bi} = (kT/q) \ln [(10^{17} \times 10^{15})/10^{20}] = 0.712$ V
   Larger than before. Follows directly from band diagram.

   b) $W = [(2\varepsilon V_{bi} / q)\, (1/N_A + 1/N_D)]^{1/2} = 0.95$ μm
   Narrower now, since the width on the p – side is very small.

   $x_p = 9.4 \times 10^{-3}$ μm

Much narrower depletion width on p-side since $N_A >>> N_D$

$x_n = 0.94\ \mu\text{m}$

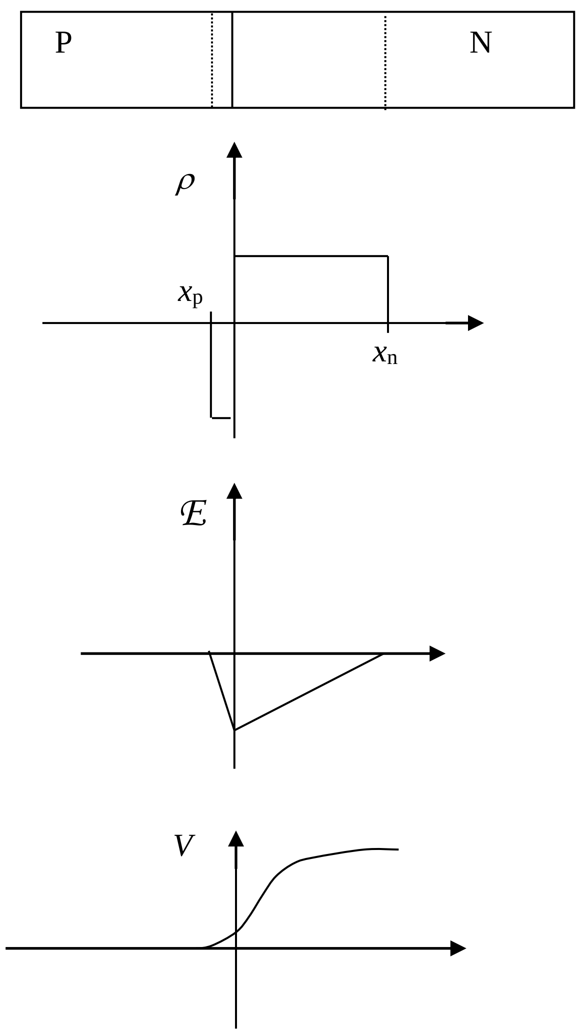
c) $\mathcal{E}_0 = q\, N_D \times_n =$

$= -(1.6 \times 10^{-19}\ \text{C}) / (11.8 \times 8.85 \times 10^{-14}\ \text{F/cm}) \times 10^{15}\ \text{cm}^{-3} \times 0.94 \times 10^{-4}\ \text{cm} =$

$= -1.44 \times 10^{4}\ \text{V/cm}$

d) $V(x = 0) = 1.44 \times 10^{4}\ \text{V/cm} \times 0.94 \times 10^{-4}\ \text{cm} \times 0.5 = 6.7 \times 10^{-3}\ \text{V}$

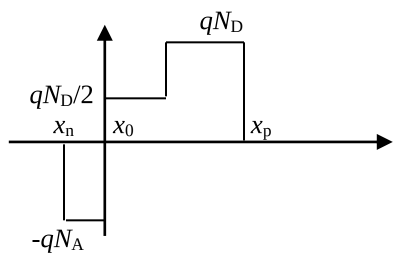
e)



3. a) Refer to Eq. 5.8: The built-in voltage depends only on the majority carrier concentration at the edges of the depletion region.
Note (Eq. 5.9a,b): $n(x_n) = N_D$ (since $x_n > x_0$) and $n(-x_p) = (n_i)^2/N_A$ (assuming the semiconductor is non-degenerately doped)

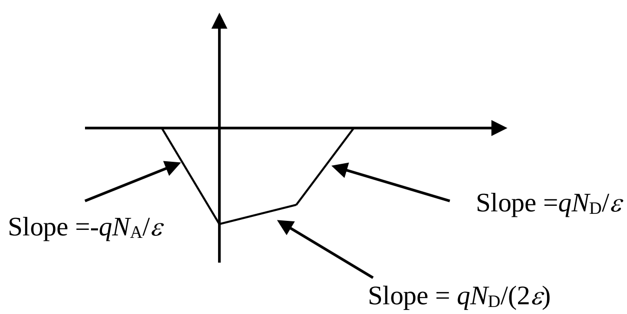
$\Rightarrow V_{bi} = (kT/q) \ln (N_A N_D / n_i^2)$

b) ρ (charge density) $= 0$ for $x < -x_p$ and $x_p > x_n$
$= -q N_A$ for $-x_p < x < 0$
$= q N_D /2$ for $0 < x < x_0$
$= q N_D$ for $x_0 < x < x_n$

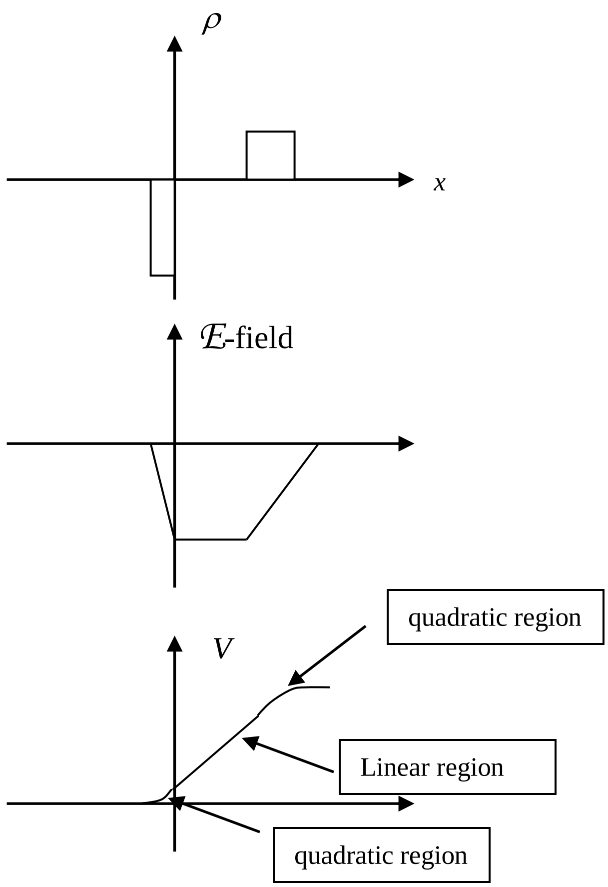


$d\mathcal{E}/dx = \rho / \varepsilon$ (Poisson's equation)

$d\mathcal{E}/dx = - qN_A/\varepsilon$ for $-x_p < x < 0$
$= qN_D/(2\varepsilon)$ for $0 < x < x_0$
$= qN_D / \varepsilon$ for $x_0 < x < x_n$
$= 0$ for $x < -x_p$, $x_p > x_n$



4. Charge density as a function of x:



Note: The banddiagram corresponds to the V-vs-x. curve plotted an upside-down

b. The built in voltage will be equal to kT/q ln $[N_A N_D / n_i^2]$ where N_A and N_D are the dopant concentrations at the depletion region edges. This is the same reasoning as in problem 3 except that there is an undoped region.