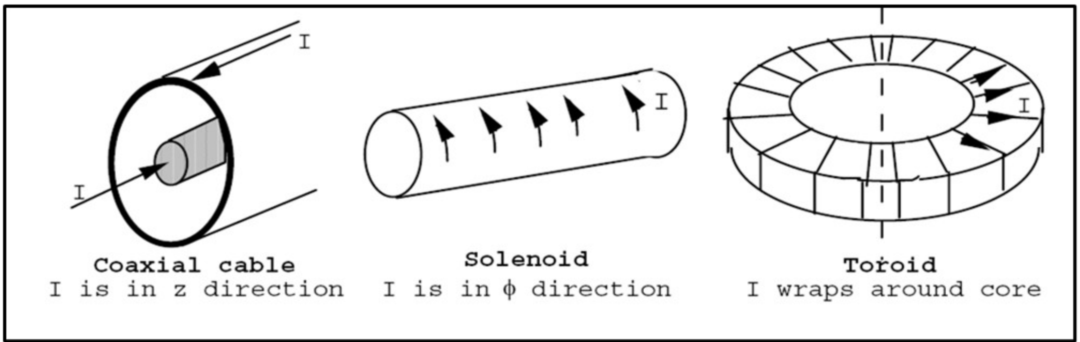
Exam 3 Crib Sheet

Magnetostatics / Ampere's Law

$\oint \vec{B} \cdot d\vec{S} = 0$	$\nabla \cdot \vec{B} = 0$
$\oint \vec{H} \cdot \vec{dl} = \int \vec{J} \cdot \vec{ds} = I_{enc}$	$\nabla \times \vec{H} = \vec{J}$
$\vec{B} = \mu \vec{H}$	$\mu_0 = 1.256 \times 10^{-6} H/m$



coax $\vec{B} = B_\varphi(r)\,\hat{a}_\varphi$ } ignoring end effects
solenoid $\vec{B} = B_z(r)\,\hat{a}_z$ }
torus $\vec{B} = B_\varphi(r,z)\,\hat{a}_\varphi$ ← assume tightly wound

Magnetic Vector Potential

$\vec{B} = \nabla \times \vec{A}$	$\psi = \oint \vec{A} \cdot \vec{dl}$

Flux, Flux Linkage and Inductance

Flux: $\psi = \int \vec{B} \cdot d\vec{S}$	**Flux linkage:** $\Lambda = N\psi$
$Emf = -L\dfrac{dI}{dt}$	$Emf = -\dfrac{d\Lambda}{dt}$
Faraday's Law 1: $\nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$	**Faraday's Law 2:** $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d}{dt}\int \vec{B} \cdot d\vec{s}$
$L = \dfrac{\Lambda}{I}$	

Magnetic Boundary Conditions*

$\vec{B}_{n1} = \vec{B}_{n2}$	$\vec{H}_{t1} - \vec{H}_{t2} = \vec{J}_s$

* Assuming that material 1 is on top and material 2 is on the bottom.

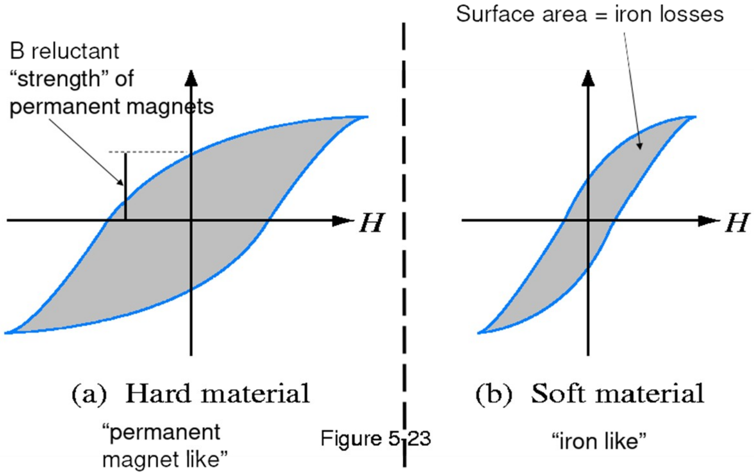
Magnetic Circuits

Magnetomotive force = NI	**Reluctance:** $\Re = \dfrac{l}{\mu A}$
$NI = \psi \cdot \Re$	

Magnetic Force and Energy

Energy Density:	$$w_m = \frac{1}{2}\frac{\lvert\vec{B}\rvert^2}{\mu} = \frac{1}{2}\mu\lvert\vec{H}\rvert^2$$
Total Energy:	$$W_m = \frac{1}{2}LI^2 = \int w_m dV$$
Force due to magnetic energy pressure: *(points across the normal of the boundary whose area is used)*	$$F = w_e \times Area$$
Force on a point charge:	$$\vec{F} = q(\vec{v} \times \vec{B})$$
Force on a current:	$$\vec{F} = \int (\vec{j} \times \vec{B})dV = \int I\vec{dl} \times \vec{B}$$

Magnetization Curves



Grad, Div, Curl and the Laplacian

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Cartesian Coordinates		$x = \rho\cos\varphi \quad y = \rho\sin\varphi \quad z = z$	$x = r\cos\varphi\sin\theta \quad y = r\sin\varphi\sin\theta$ $z = r\cos\theta$
Vector $\boldsymbol{A}$	$A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k}$	$A_\rho\widehat{\boldsymbol{\rho}} + A_\varphi\widehat{\boldsymbol{\varphi}} + A_z\widehat{\boldsymbol{z}}$	$A_r\widehat{\boldsymbol{r}} + A_\theta\widehat{\boldsymbol{\theta}} + A_\varphi\widehat{\boldsymbol{\varphi}}$
Gradient $\nabla\phi$	$\frac{\partial\phi}{\partial x}\boldsymbol{i} + \frac{\partial\phi}{\partial y}\boldsymbol{j} + \frac{\partial\phi}{\partial z}\boldsymbol{k}$	$\frac{\partial\phi}{\partial\rho}\widehat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial\phi}{\partial\varphi}\widehat{\boldsymbol{\varphi}} + \frac{\partial\phi}{\partial z}\widehat{\boldsymbol{z}}$	$\frac{\partial\phi}{\partial r}\widehat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\widehat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}\widehat{\boldsymbol{\varphi}}$
Divergence $\nabla\cdot\boldsymbol{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial A_\theta\sin\theta}{\partial\theta}$ $+\frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$
Curl $\nabla\times\boldsymbol{A}$	$\begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\begin{vmatrix} \frac{1}{\rho}\widehat{\boldsymbol{\rho}} & \widehat{\boldsymbol{\varphi}} & \frac{1}{\rho}\widehat{\boldsymbol{z}} \\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\varphi & A_z \end{vmatrix}$	$\begin{vmatrix} \frac{1}{r^2\sin\theta}\widehat{\boldsymbol{r}} & \frac{1}{r\sin\theta}\widehat{\boldsymbol{\theta}} & \frac{1}{r}\widehat{\boldsymbol{\varphi}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ A_r & rA_\theta & rA_\varphi\sin\theta \end{vmatrix}$
Laplacian $\nabla^2\phi$	$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2} + \frac{\partial^2\phi}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\phi}{\partial\theta}\right)$ $+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\phi}{\partial\varphi^2}$