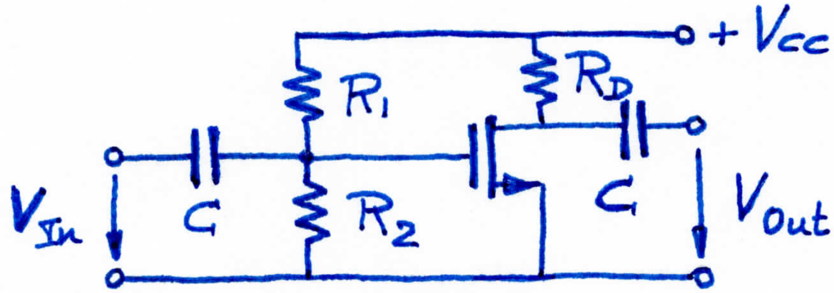
DC - biasing of FET

⇒ DC biasing circuit

⇒ Note: $\boxed{I_G = 0}$ Why? $\boxed{I_D = I_S}$ Why?

⇒ Mostly resistors. Some capacitors. Why?

1 FET biasing circuit



Assume that we know all circuit elements.

Quiescent-point determination

$$V_{GS} = \frac{R_2}{R_1 + R_2} V_{CC} \quad \text{(Voltage divider)} \qquad (1)$$

$$I_D = \frac{1}{2} k \left(V_{GS} - V_{TR}\right)^2 \quad \text{(Saturation)} \qquad (2)$$

⇒ 2 eqns. 2 unknowns (I_D and V_{GS})

⇒ Eqn. (1) yields V_{GS}. Eqn. 2 yields I_D

⇒ $V_{DS} = V_{CC} - R_D I_D$ ⇒ Verify that FET is in saturation. Saturation: $V_{DS} \geq V_{GS} - V_{TR}$

$\Rightarrow$ We solved the DC biasing problem

Numerical example: $V_{cc} = 10V$ $R_1 = 800k\Omega$ $R_2 = 200k\Omega$ $R_D = 500\Omega$ $k = 10\frac{mA}{V^2}$ $V_{th} = 1V$

Determine Q-point of FET

$$\Rightarrow V_{GS} = \frac{R_2}{R_1+R_2} V_{cc} = \frac{200k\Omega}{1000k\Omega} 10V = 2.0V$$

$$\Rightarrow I_D = \frac{1}{2}k\,(V_{GS} - V_{th})^2 = \frac{1}{2} 10\frac{mA}{V^2}\,(2V - 1V)^2$$

$$= \frac{1}{2}\, 10\, mA/_{V^2} \times (1V)^2 = 5.0\,mA$$

$$\Rightarrow V_{DS} = V_{cc} - R_D I_D = 10V - 500\Omega \times 5mA$$

$$= 10V - 2.5V = 7.5V$$

$$\Rightarrow \underbrace{V_{DS}}_{7.5V} \geq \underbrace{V_{GS} - V_{th}}_{1V} \Rightarrow \text{Yes} \Rightarrow \text{Saturation}$$

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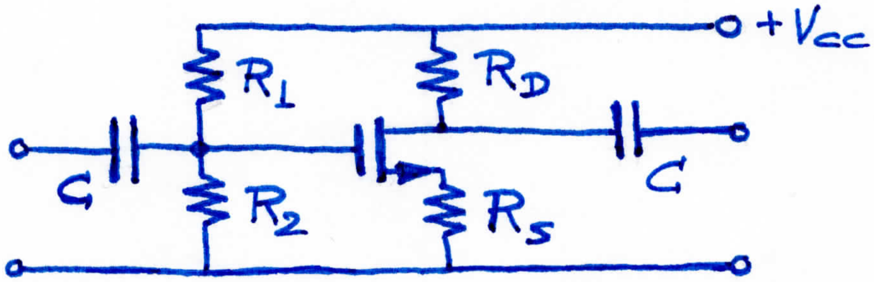
2 FET biasing circuit



$$V_G = \frac{R_2}{R_1+R_2} V_{cc}$$

$$V_S = R_S I_D$$

$$V_{GS} = V_G - V_S$$

$$V_{GS} = \frac{R_2}{R_1+R_2} V_{cc} - R_S I_D \qquad (1)$$

$$I_D = \frac{1}{2} k \left(V_{GS} - V_{th}\right)^2 \qquad \text{(Saturation)} \qquad (2)$$

$\Rightarrow$ 2 eqns 2 unknowns (V_{GS} and I_D)

Insert Eqn(2) into Eqn(1) $\Rightarrow$

$$V_{GS} = \frac{R_2}{R_1+R_2} V_{cc} - R_S \times \frac{1}{2} k \left(V_{GS} - V_{th}\right)^2$$

$\Rightarrow$ 1 eqn 1 unknown (V_{GS})

Rearrange eqn:

$$V_{GS} = \frac{R_2}{R_1+R_2} V_{cc} - \frac{1}{2} k R_S \left(V_{GS}^2 - 2 V_{GS} V_{th} + V_{th}^2\right)$$

$$\underbrace{\frac{1}{2} k R_S \underline{\underline{V_{GS}}}^2}_{a\underline{\underline{x}}^2} + \underbrace{\left(1 - k R_S V_{th}\right) \underline{\underline{V_{GS}}}}_{b\underline{\underline{x}}} + \underbrace{\frac{1}{2} k R_S V_{th}^2 - \frac{R_2}{R_1+R_2} V_{cc}}_{c} = 0$$

$$a\underline{\underline{x}}^2 + b\underline{\underline{x}} + c = 0$$

... this is a quadratic eqn.

Recall:

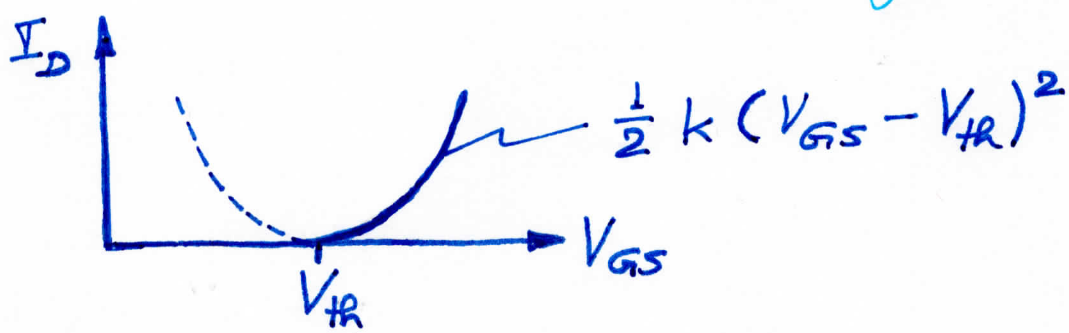
Quadratic eqn. $ax^2 + bx + c = 0$

Solution $x_{1/2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Q: Why does a quadratic eqn. have two solutions?

Q: Does it help writing the quadratic eqn as follows? $\underbrace{ax^2}_{\text{Parabola}} = \underbrace{-bx - c}_{\text{Straight line}}$

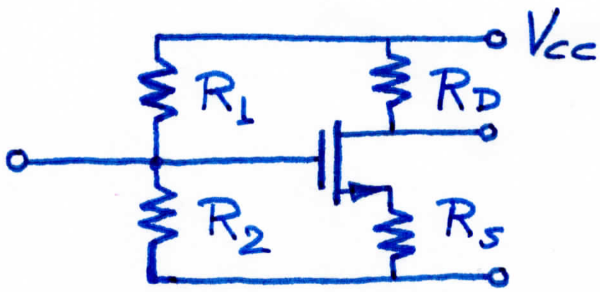
Recall



For the "right" solution, it is $V_{GS} > V_{th}$

Note: It may be prudent to determine the numerical values of a, b, and c and then solve the quadratic equation.

Numerical example



$V_{cc} = 10V$

$R_1 = R_2 = 100\,k\Omega$

$R_S = 100\,\Omega \quad R_D = 400\,\Omega$

$V_{th} = 2V$

$k = 5\,mA/V^2$

We use quadratic eqn derived above $aV_{GS}^2 + bV_{GS} + c = 0$

$$a = \frac{1}{2} k R_S = \frac{1}{2} 5 \frac{mA}{V^2} 100\Omega = 0.25 \frac{1}{V}$$

$$b = 1 - k R_S V_{th} = 1 - 5 \frac{mA}{V^2} 100\Omega\, 2V = 1 - 1 = 0$$

$$c = \frac{1}{2} k R_S V_{th}^2 - \frac{R_2}{R_1 + R_2} V_{cc} = \frac{1}{2} 5 \frac{mA}{V^2} 100\Omega\, 4V^2 - \frac{1}{2} 10V$$

$$= 1V - 5V = -4V$$

$\Rightarrow$ Quadratic eqn $\quad 0.25 \frac{1}{V} V_{GS}^2 - 4V = 0$

$$\Rightarrow V_{GS}^2 = 16V^2 \Rightarrow \underline{\underline{V_{GS} = 4V}}$$

$$\Rightarrow \underline{\underline{I_D}} = \frac{1}{2} k (V_{GS} - V_{th})^2 = \frac{1}{2} 5 \frac{mA}{V^2} (2V)^2 = \underline{\underline{10\,mA}}$$

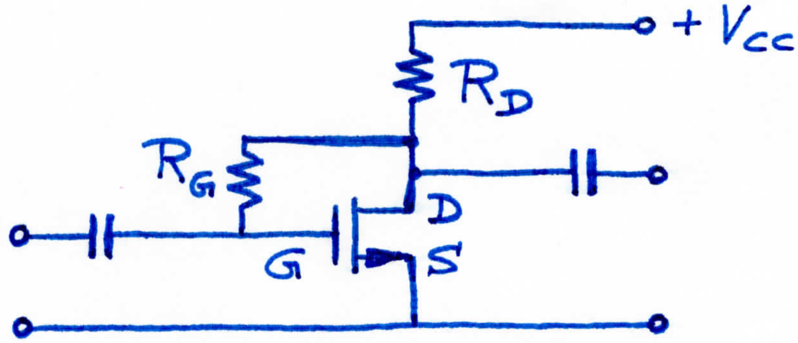
$$\Rightarrow \underline{\underline{V_{DS}}} = V_{cc} - R_D I_D - R_S I_D = 10V - 400\Omega\, 10mA - 100\Omega\, 10mA$$

$$= 10V - 4V - 1V = \underline{5V}$$

$$\underbrace{V_{DS}}_{5V} > \underbrace{V_{GS} - V_{th}}_{2V} \Rightarrow \text{FET is in saturation}$$

$\Rightarrow$ To conclude, we determined the Q-point

③ FET biasing circuit



Q: What is the value of the current flowing through R_G ? ⇒ Zero ⇒ Why ?

$$\underline{\underline{V_{GS}}} = V_D = V_{cc} - \underline{\underline{I_D}} R_D \qquad (1)$$

$$\underline{\underline{I_D}} = \frac{1}{2} k \left(\underline{\underline{V_{GS}}} - V_{th} \right)^2 \qquad (2)$$

Two eqns. and two unknowns. Insert Eqn.(2) into Eqn(1) yields

$$V_{GS} = V_{cc} - \frac{1}{2} k R_D \left(V_{GS}^2 - 2 V_{th} V_{GS} + V_{th}^2 \right)$$

Rearranging the eqn. yields

$$\underbrace{\frac{1}{2} k R_D V_{GS}^2}_{a V_{GS}^2} + \underbrace{(1 - k R_D V_{th}) V_{GS}}_{b V_{GS}} + \underbrace{\frac{1}{2} k R_D V_{th}^2 - V_{cc}}_{c} = 0$$

⇒ Quadratic eqn. ⇒ V_{GS} is unknown ⇒

We can solve the problem.

V_{GS} has two solutions. For the meaningful solution $V_{GS} > V_{th}$

⇒ Then calculate I_D

⇒ Then calculate $V_{DS} = V_{CC} - I_D R_D$

⇒ Verify that FET is in saturation (which requires $V_{DS} \geq V_{GS} - V_{th}$)