Rensselaer Polytechnic Institute
ECSE 2500: Engineering Probability
Spring 2023

Homework #1: due Sunday, Jan. 22$^{\text{th}}$
Show all work for full credit!
Submit your work as a single PDF on Gradescope, labeling each problem number with a page.

1. Good friends Raju and Bheem plan to spend 8 hours together, doing various activities. They can either go horseback/motorcycle riding (which takes exactly 4 hours), break into spontaneous dancing (which takes exactly 2 hours), or fight each other (which takes exactly 3 hours). They randomly choose an initial activity. After each activity is complete they immediately choose their next activity randomly, as long as it can fit entirely within their remaining time. If there's no time to complete a full activity in the remaining time, they go home early.

   (a) (6 points.) Determine the sample space corresponding to the set of possible outcomes (i.e., orders of activities). You can abbreviate the activities as **R** (riding), **D** (dancing), or **F** (fighting). How many outcomes are in the sample space?

   (b) (6 points.) Determine the set A corresponding to the event "The friends were dancing for the second activity".

   (c) (6 points.) Determine the set B corresponding to the event "The friends went horseback/motorcycle riding at least once".

   (d) (6 points.) Determine the set $A \cap B^c$.

   (e) (6 points.) A tiger attacks the friends at a randomly chosen time during the 8-hour period. Let X be the (continuous) random number corresponding to the cumulative length of time the friends have been fighting up to that point. Determine the sample space of X.

2. There are 10 kids fighting evil in Hawkins, Indiana. These include some older kids (Nancy, Jonathan, Robin, Steve) and some younger kids (Eleven, Mike, Dustin, Lucas, Will, Max). The members group into a 4-person team (heading to the Upside Down) and a 6-person team (staying in Indiana) to fight Vecna.

   (a) (10 points.) How many ways are there to form these teams? (Order within a team doesn't matter.)

   (b) (10 points.) After the teams are set, the kids on the 4-person team enter the Upside Down one at a time via an interdimensional portal. How many possibilities are there for the order in which they go through the portal?

   (c) (10 points.) Assuming each configuration in (a) is equally likely, what is the probability that Max and Lucas are both on the 6-person team?

   (d) (20 points.) What is the conditional probability that Max and Lucas are on the same team, given that Steve and Dustin are on the same team? (Think carefully about this one.)

More fun on the next page ⟶

3. A professor watches a mixture of TV shows on Netflix (40% of all shows), HBO Max (35% of all shows), and Disney+ (25% of all shows). 20% of the Netflix shows, 30% of the HBO Max shows, and 80% of the Disney+ shows involve a superhero.

   (a) (10 points.) Compute the overall probability that a show watched by the professor involves a superhero. What probability concept are you using?

   (b) (10 points.) If a randomly chosen show watched by the professor involves a superhero, what is the probability it is a Disney+ show? What probability concept are you using?