Hayden Fuller

Homework # 3

Due: Tuesday, June 20th

Problem 1. (20 points) Let the input-output relationship of a system be

$$y(t) = \frac{x(t) \cdot x(t+1). x(t+2)}{x^2(t-1)}$$

Where $x(t)$ denotes input and $y(t)$ denotes output. Determine whether the system satisfies the following properties. Justify your answer.

a) Linearity
b) Time invariance
c) System with memory
d) Causality
e) BIBO stability

Problem 2. (20 points) Convolve $x[n]$={2,-1, 5, 3, -1} and $x[n]$={6,-3, -7, 10, -11} using the array algorithm (include all steps). Also use Matlab to verify the answer. Include Matlab code as a part of your answer.

Problem 3. (20 points) Find the following convolutions using analytical approach, where we have defined

$$f[n] = \left(\frac{1}{3}\right)^n \cdot u[n] \quad \text{and} \quad g[n] = n$$

a) (5 pts) $f[n] * \delta[n]$
b) (5 pts) $f[n] * u[n]$
c) (10 pts) $f[n] * g[n]$

Problem 4. (20 points) Using Matlab, realize the complex exponential function $f(t) = \exp(\sigma + j\,\omega)t$ with f = 10Hz (where ω = 2*pi*f), and for three different values of $\sigma = 5, 0, \& -5$. Plot to show the un-damped, decaying and rising exponential complex function. Include plots as well as code used to generate the results as the part of the solution. Reasonably choose the number of data points to plot to show about 5 to 10 cycles of the exponential functions.

Problem 5. (20 points) Using the Discovery 2 board or M1K board, differentiate a square wave of fundamental frequency 1kHz, peak-to-peak amplitude 5V, to realize impulse at zero crossings. Also repeat the results by providing a delay of 0.25ms to the square wave. Results need to include, a photo of the protoboard, screenshots of scope and the circuit diagram.

1)
a) no, because the system is not additave. For example if x1(t)=t, x2(t)=t^2, and x3(t)=x1(t)+x2(t),
x1(t)*x1(t+1)*x1(t+2)/x1^2(t-1) + x2(t)*x2(t+1)*x2(t+2)/x2^2(t-1) /= x3(t)*x3(t+1)*x3(t+2)/x3^2(t-1)
(this is too much math to simplify and type out, but it can easily be confirmed on a calculator)
b) yes, this can be seen since all the x(t) functions only add to t, none scale it in any way, so a cahnge will affect them all equally and the output will shift the same as the input.
c) yes, y(0) depends on x(-1), x(0), x(1), and x(2).
d) no, y(0) depends on x(1) and x(2)
e) yes, if |x(t)| < B, then -B < x(t) < B, and B*B*B/B^2 < y(t) -B*-B*-B/-B^2, meaning y(t) has finite value

2)

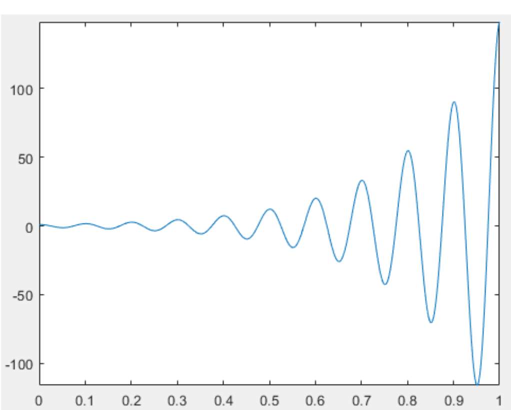
	2	-1	5	3	-1
6	12	-6	30	18	-6
-3	-6	3	-15	-9	3
-7	-14	7	-35	-21	7
10	20	-10	50	30	-10
-11	-22	11	-55	-33	11

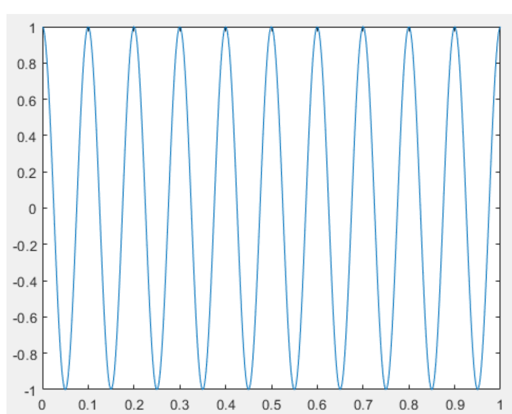
12	=12
-6-6	=-12
-14+3+30	=19
20+7-15+18	=30
-22-10-35-9-6	=-82
11+50-21+3	=43
-55+30+7	=-18
-33-10	=-43
11	=11

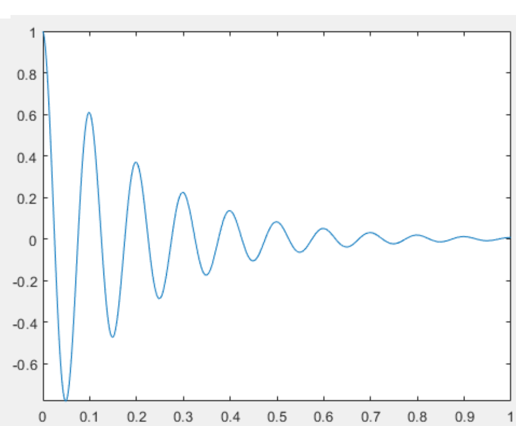
```
>> a=[2 -1 5 3 -1]
>> b=[6 -3 -7 10 -11]
>> c=conv(a,b)
c = 12   -12    19    30   -82    43   -18   -43
```

3)
a) (1/3)^n \cdot u[n] * \delta[n]
= \int_-inf^inf (1/3)^\tau \cdot u[\tau] \cdot \delta[n-\tau] d\tau
=(1/3)^n \cdot u[n]
b) (1/3)^n \cdot u[n] * u[n] = (1-(1/3)^n)\cdot u[n]/\ln3
= \int_-inf^inf (1/3)^\tau \cdot u[\tau] \cdot u[n-\tau] d\tau
= (1-(1/3)^n) \cdot u[n] / ln(3)
c) (1/3)^n \cdot u[n] * n
= \int_-inf^inf (1/3)^\tau \cdot u[\tau] \cdot n-\tau d\tau
= ((n \cdot ln(3))-1)/ln^2(3)

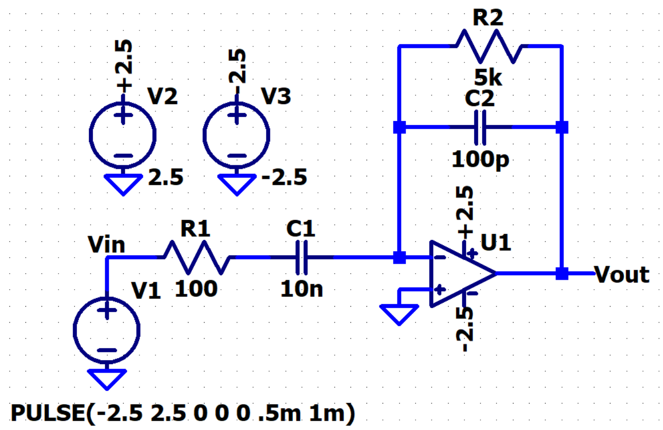
4)
```
>> w=2*pi*10>> s=5>> fplot(@(t)
real(exp((s+i*w)*t)),[0,1])
>> s=0
>> fplot(@(t) real(exp((s+i*w)*t)),[0,1])
>> s=-5
>> fplot(@(t) real(exp((s+i*w)*t)),[0,1])
```



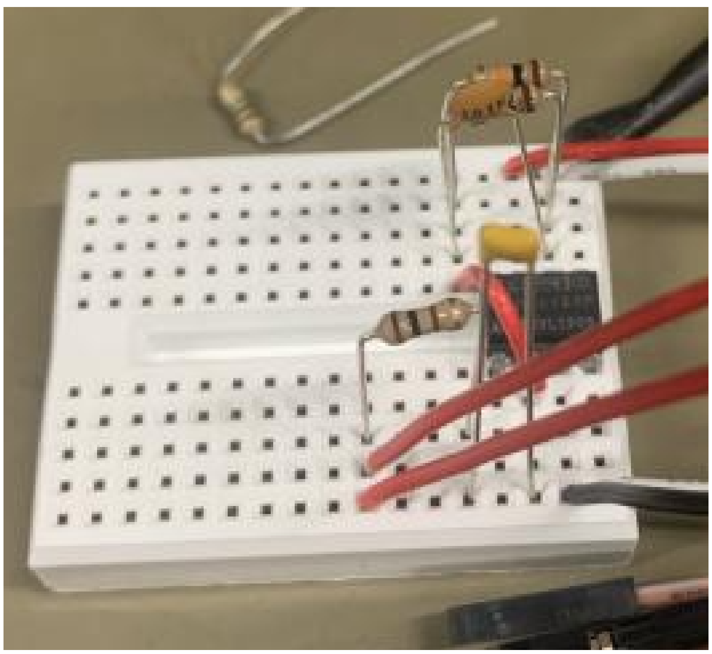


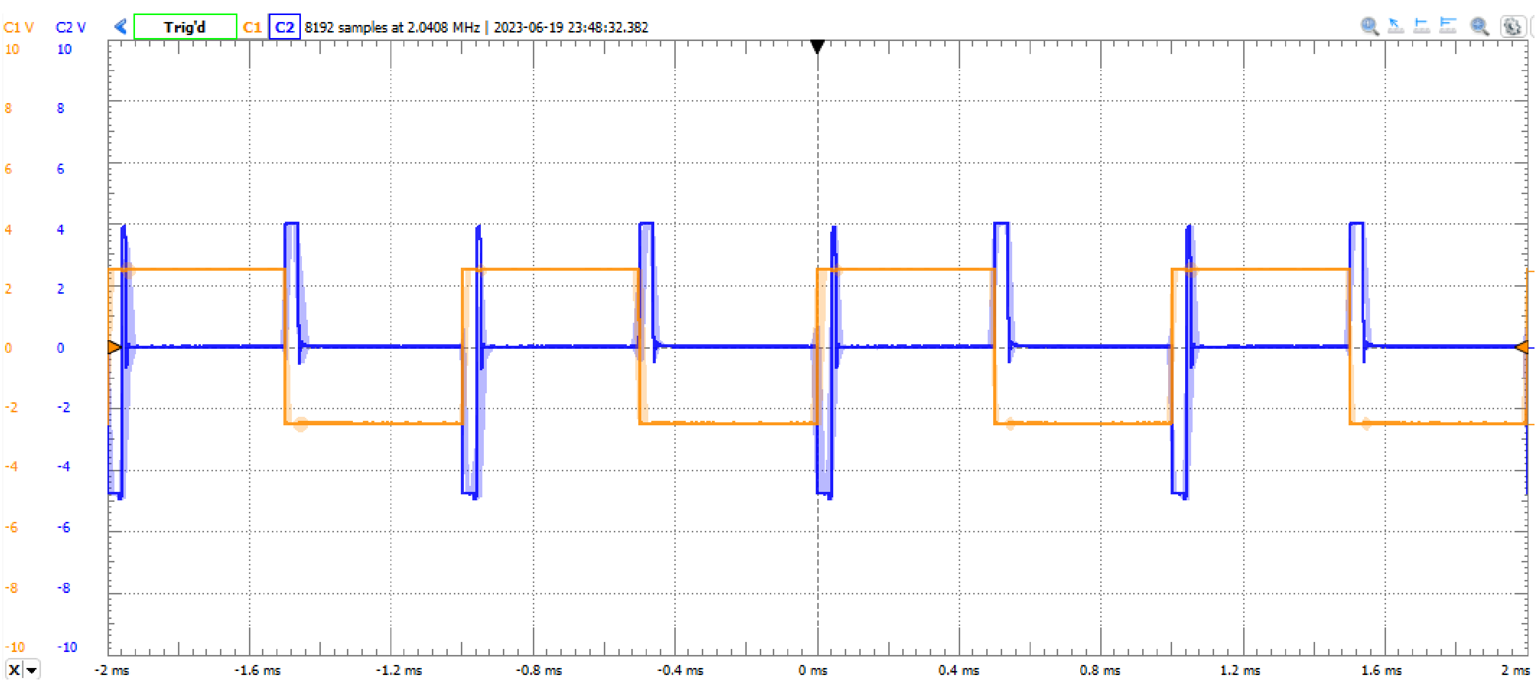


5)



R2
5k
C2
100p
+2.5
V2
2.5
-2.5
V3
-2.5
+2.5
U1
Vin
R1
100
C1
10n
V1
Vout
-2.5
PULSE(-2.5 2.5 0 0 0 .5m 1m)





C1 V
C2 V
Trig'd
C1
C2
8192 samples at 2.0408 MHz | 2023-06-19 23:48:32.382
-2 ms
-1.6 ms
-1.2 ms
-0.8 ms
-0.4 ms
0 ms
0.4 ms
0.8 ms
1.2 ms
1.6 ms
2 ms