Homework 3

Released: February 1st

Due: 11:59pm February 8th

1. Lossy Transmission Lines

You are given a length of transmission line with the following properties: $u_p = 0.66c$ and $Z_0 = 50\Omega$.

a) You measure the amplitude of the voltage at two positions on the transmission line and obtain the following measurements:

Distance (m)	Voltage (V)
0 (source)	5.00
900	4.97

What is the value of α, the attenuation factor for this line?

b) Assuming this is a low-loss transmission line, what is the value of r, the resistance per unit length of the line?

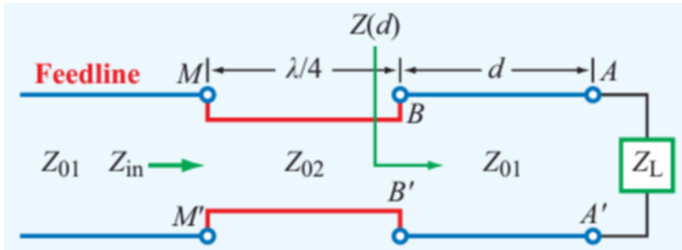
c) Knowing both u_p and Z_0 (assumed to be purely real in this case), you can calculate c (capacitance per unit length) and l (inductance per unit length) for the transmission line. What value of g (conductance per unit length) would you need to make this low-loss transmission line dispersionless?

d) What is the new value for α on this dispersionless transmission line?

e) If you were to measure the voltage at z = 900m on this new, dispersionless transmission line, what would it be? Has the addition of conductance noticeably affected the loss of this transmission line?

2. Impedance Matching with Smith Charts Part I

Given the circuit below, you are to use a transmission line of length d with characteristic impedance $Z_{01} = 50\Omega$ and a quarter-wave transformer of characteristic impedance Z_{02} to match the feedline impedance (which is also $Z_{01} = 50\Omega$) to the load impedance $Z_L = (40 + j35)\Omega$.



(Ulaby 2-33)

Finding the Length L of the Transmission Line Needed to Make $Z_{in,0}$ Real

a) Calculate the normalized impedance z.

b) Using the Smith chart on the next page, plot z and the VSWR circle.

c) Using the Smith chart, determine the magnitude and phase of Γ and the SWR. Explain how you found each of these values.

d) What length of transmission line (in terms of λ) is needed to transform z into a purely real impedance and what is the resulting impedance?

e) The VSWR circle you drew in part b) is a curve of constant $|\Gamma_L|$. Why must we remain on a curve of constant $|\Gamma_L|$ on the Smith chart when we are transforming impedances using transmission lines? (Hint: what parameters determine the magnitude of Γ_L?)

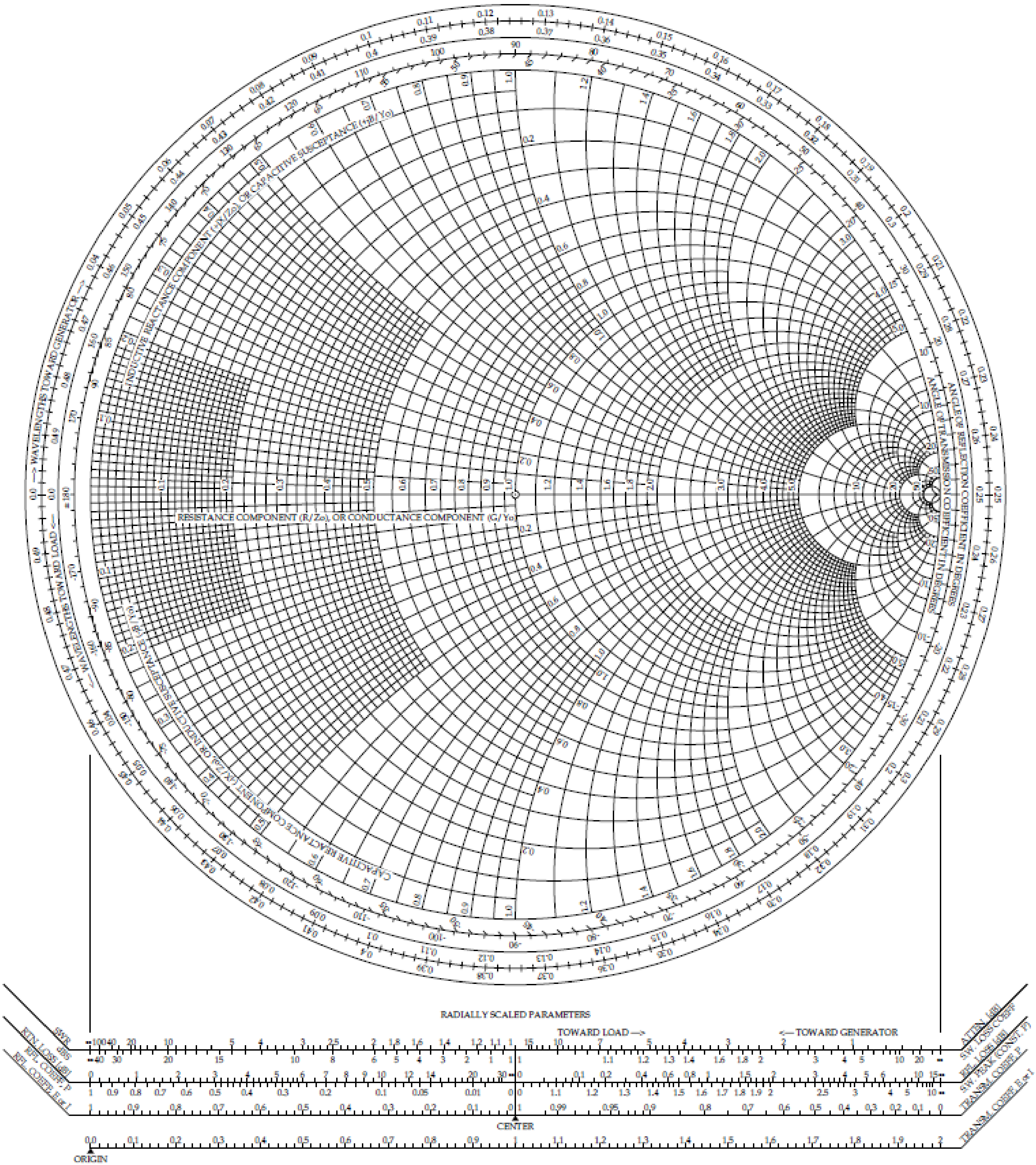
f) When you determined the length of the transmission line needed in part d), you traveled along the VSWR circle, on which the magnitude of Γ is constant, but the phase of Γ is changing. What does a change in the phase of Γ on a Smith chart physically correspond to? (Hint: Γ on the Smith chart can be thought of as the phase-shifted reflection coefficient Γ_d).

g) What do the impedances on the VSWR circle on the Smith chart represent? (Hint: recall the equation for wave impedance on the transmission line $Z(d) = \frac{1+\Gamma_L e^{-j2\beta d}}{1-\Gamma_L e^{-j2\beta d}}$, where θ_Γ is the phase of Γ_L at the load and d is the distance measured from the load in the direction of the generator).

Finding the Characteristic Impedance Z_{02} Needed to Match Z_{01} to Z_L

h) Determine the characteristic impedance Z_{02} of the quarter-wave transformer required to match the generator resistance to the transmission line you determined the length of in part d).

i) If the frequency of the signal is $f = 10$MHz and the velocity factor of both transmission lines is 0.66, how long is your transmission line from part d? How long is your quarter wave transformer from part g?


RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
CENTER
ORIGIN

3. Impedance Matching with Smith Charts Part II

In this question, you will verify you results from Problem 2 using the equations for the impedance of lossless transmission lines, instead of the Smith Chart.

a) Using the equation $\Gamma_L = \frac{z-1}{z+1}$, where z is the normalized impedance from 2a, calculate the magnitude and phase of the reflection coefficient at the load. Do they match the values you determined using the Smith chart in 2c?

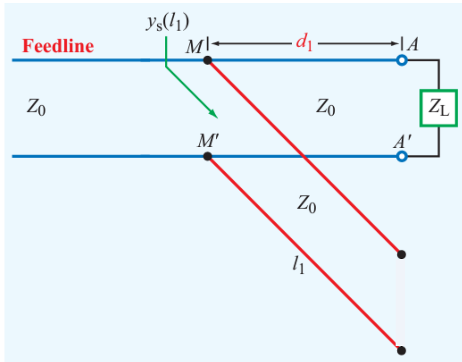
b) Insert the transmission line length L you determined in 2d into the equation for input impedance: $Z_{in}(L) = Z_0 \frac{z+j\tan(\beta L)}{1+jz\tan(\beta L)}$, where $Z_0 = 50\Omega$ as in problem 2. Does Z_{in} agree with the impedance you found on the Smith chart in 2d?

c) Assuming that your quarter-wave transformer was appropriately designed in 2h, what impedance does the generator circuit "see" looking to the right from R_g, into the transmission line?

d) Even though you've matched Z_L to R_g to achieve maximum power transfer to the load via a quarter-wave transformer (no reflections at the input to the transmission line in steady-state), the reflection coefficient at the load Γ_L (and at the generator Γ_g) is non-zero, meaning that reflections still occur on the λ/4 line. How is it possible that maximum power is still being transferred despite $\Gamma_L \neq 0$?

4. Impedance Matching with Stubs (also using Smith Charts)

You are to match a transmission line with a characteristic impedance of Z_0 =100Ω to a load impedance Z_L = (50 - j25) Ω using an open-circuited stub, as shown below:



(Adapted from Ulaby 2-33e)

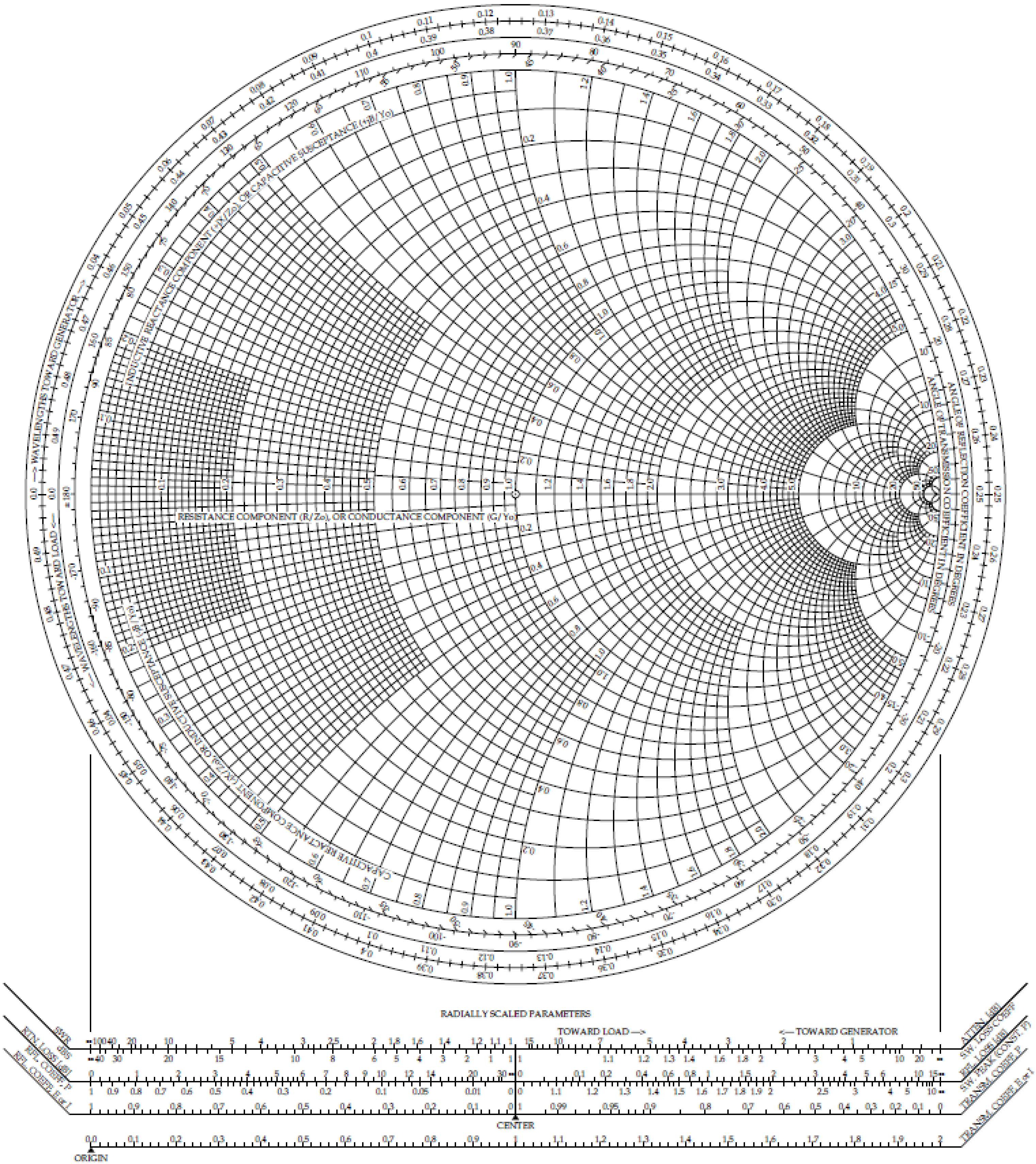
a) Find the normalized impedance of the load. Plot it on the Smith chart.

b) Using the Smith chart, find and plot the normalized admittance *y*.

c) What length of transmission line d_1 is required to transform the real part of *y* to be 1? What is *y* at this point? What is the significance of Re{*y*} = 1?

d) Plot the location of the open-circuit admittance and the susceptance that we will need for matching the load on the Smith chart. What length of stub l_1 do we need in order to provide this susceptance for matching?

e) Now we need to verify that our procedure on the Smith chart has actually matched the load impedance to the transmission line characteristic impedance of 100Ω. The input impedance seen by the transmission line is $Z_{in} = \frac{Z_{stub}^{oc} \cdot Z_{d1}}{Z_{stub}^{oc} + Z_{d1}}$, where Z_{stub}^{oc} is the input impedance of the open-circuited stub and Z_{d1} is the input impedance of the transmission line of length d_1.



Alex Patterson