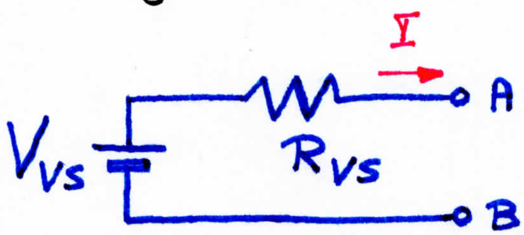
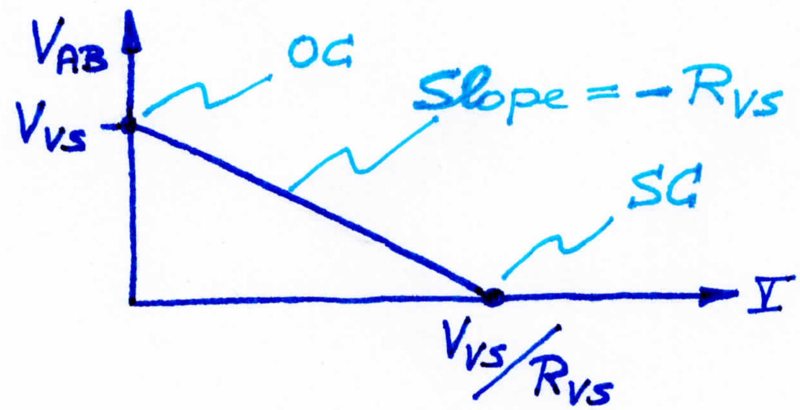
Equivalent sources (Method 1)

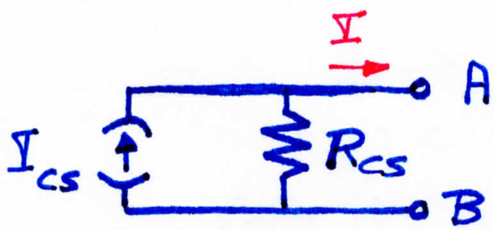
Voltage source (VS)



$$V_{AB} = V_{VS} - I R_{VS}$$

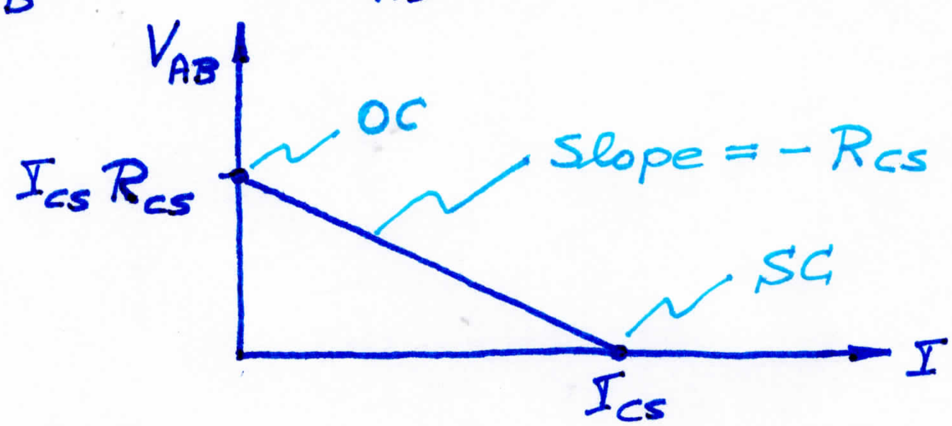


Current source



$$I = I_{CS} - V_{AB}/R_{CS}$$

$$\Rightarrow V_{AB} = (I_{CS} - I) R_{CS}$$



If the two sources are equivalent, then their OC voltages and SC currents must be the same.

$$\underbrace{V_{VS}}_{\text{OC voltages}} = \underbrace{I_{CS} R_{CS}}_{\text{OC voltages}} \Rightarrow I_{CS} = V_{VS} / R_{CS}$$

$$\underbrace{I_{CS}}_{\text{SC currents}} = \underbrace{V_{VS} / R_{VS}}_{\text{SC currents}}$$

Comparison of the last two eqns. yields

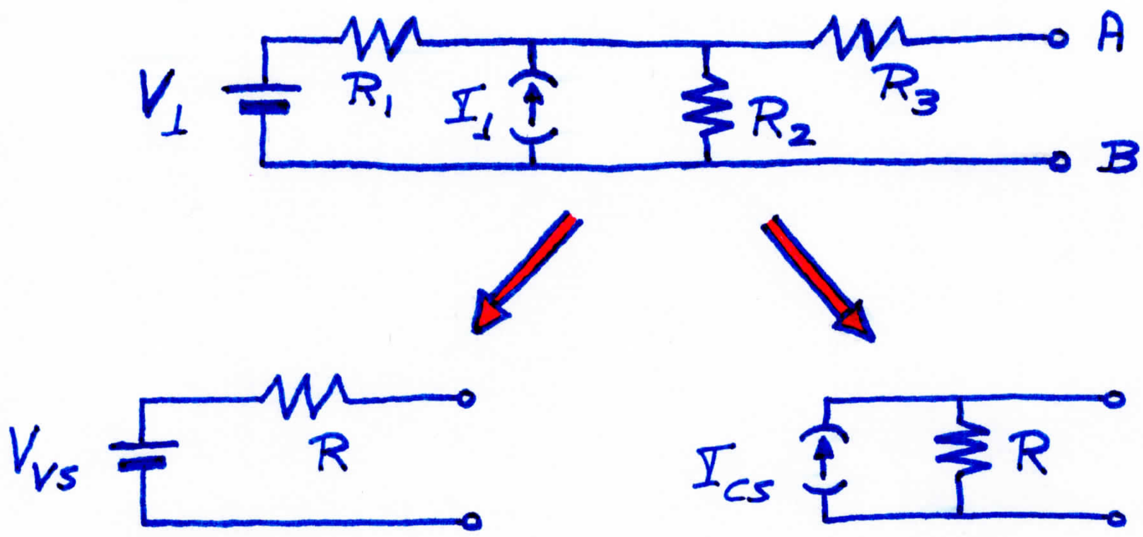
$$\boxed{R_{VS} = R_{CS} = R}$$

Internal resistance

"Looking into" the power sources, we "see" that they have the same internal resistance.

$\Rightarrow$ Two equivalent power sources have the same internal resistance.

Complex source



$\Rightarrow \quad R = R_3 + (R_1 \parallel R_2)$

$\quad$ ↳ Internal resistance

Equivalent V source

$$V_{VS} = V_{OC} \text{ of complex source}$$

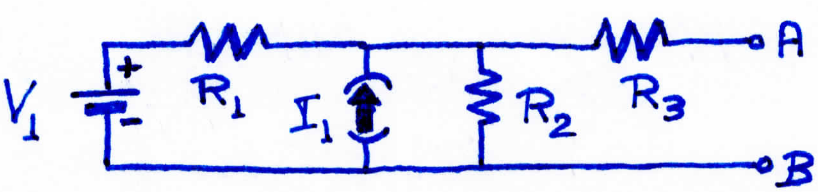
Equivalent I source

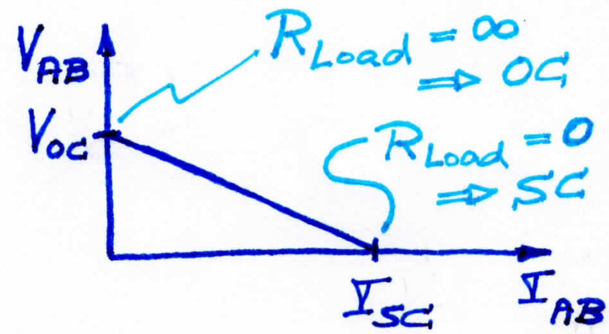
$$I_{CS} = I_{SC} \text{ of complex source}$$

Equivalent sources (Method 2)

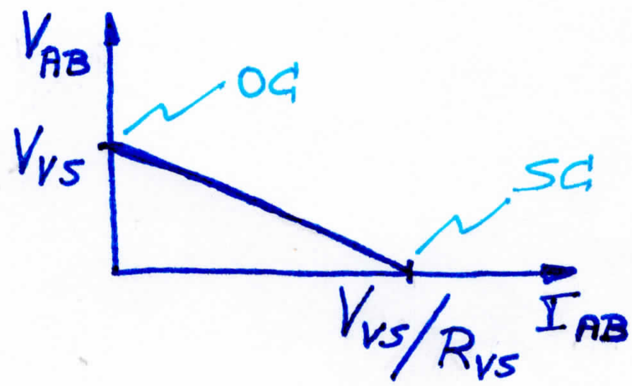
Any DC power source can be represented by a real voltage source (= ideal V source plus internal R) or a real current source (= ideal I source plus internal G)

Complex DC source

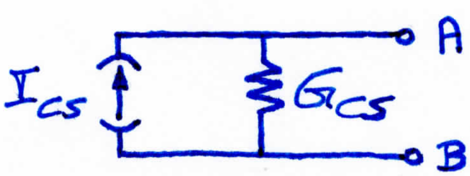


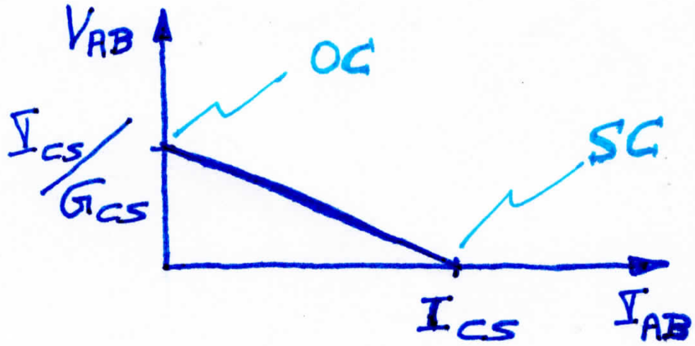


Equivalent V source



Equivalent I source





Voltage source (VS) is equivalent if...

$$\underline{V_{VS}} = V_{oc}$$

↳ of complex DC source

$$I_{SC} = V_{VS}/R_{VS} \Rightarrow \underline{R_{VS}} = V_{VS}/I_{SC}$$

↳ of complex DC source

Current source (CS) is equivalent if...

$$\underline{I_{CS}} = I_{SC}$$

↳ of complex DC source

$$V_{OC} = I_{CS}/G_{CS} \Rightarrow \underline{G_{CS}} = I_{CS}/V_{oc}$$

↳ of complex DC source

Determination of V_{oc} — of complex DC source

⇒ Superposition principle

$$\Rightarrow V_{OC} = \text{Effect of } V_1 + \text{Effect of } I_1$$

$$\text{Effect of } V_1 = V_1 \frac{R_2}{R_1+R_2}$$

$$\text{Effect of } I_1 = I_1 \,(R_1 \parallel R_2)$$

$$\Rightarrow \underline{V_{oc}} = V_1 \frac{R_2}{R_1+R_2} + I_1 \,(R_1 \parallel R_2)$$

Determination of I_{sc} ~ of complex DC source

⇒ Superposition principle

⇒ I_{sc} = Effect of V_1 + Effect of I_1

Effect of V_1

Ohm $V_1 = I_{R1}\left(R_1 + (R_2 \parallel R_3)\right)$

Also: $I_{R2} R_2 = I_{R3} R_3 \Rightarrow I_{R2} = I_{R3}\dfrac{R_3}{R_2}$

$$\Rightarrow V_1 = I_{R1}\left(R_1 + (R_2 \parallel R_3)\right)$$

$$= \left(I_{R3}\frac{R_3}{R_2} + I_{R3}\right)\left(R_1 + (R_2 \parallel R_3)\right)$$

$$= I_{R3}\left(\frac{R_3}{R_2} + 1\right)\left(R_1 + (R_2 \parallel R_3)\right)$$

$$\Rightarrow I_{R3} = \frac{V_1}{\left(R_3/R_2 + 1\right)\left(R_1 + (R_2 \parallel R_3)\right)}$$

Effect of I_1

~ $1/R_3$

Current divider $I_{R3} = I_1 \dfrac{G_3}{G_1 + G_2 + G_3}$

Total effect (due to V_1 and I_1) is the sum

$$\underline{\underline{I_{sc}}} = \frac{V_1}{\left(\frac{R_3}{R_2} + 1\right)\left(R_1 + (R_2 \parallel R_3)\right)} + I_1 \frac{G_3}{G_1 + G_2 + G_3}$$

We now determined V_{oc} and I_{sc} of complex DC source

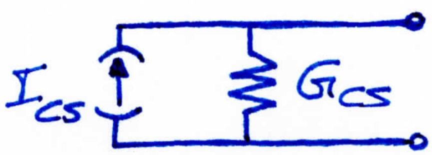
⇒ Equivalent V source (VS)

$$\Rightarrow \underline{\underline{V_{VS}}} = V_{oc}$$

$$\Rightarrow V_{VS}/R_{VS} = I_{sc}$$

$$\Rightarrow \underline{\underline{R_{VS}}} = V_{VS}/I_{sc}$$

⇒ Equivalent I source (CS)



$$\Rightarrow \underline{\underline{I_{CS}}} = I_{sc}$$

$$\Rightarrow V_{oc} = I_{CS}/G_{CS}$$

$$\Rightarrow \underline{\underline{G_{CS}}} = I_{CS}/V_{oc}$$