1a.)

$$\vec{B_{1n}} = \vec{B_{2n}} = 0.1\hat{y}$$

$$\vec{H_{2t}} - \vec{H_{1t}} = J_s = 0$$

$$\vec{H_{1t}} = \vec{H_{2t}}$$

$$\frac{\vec{B_{1t}}}{1000\mu_0} = \frac{\vec{B_{2t}}}{\mu_0}$$

$$\vec{B_{1t}} = 1000 \cdot (0.1y) = 100\hat{x}$$

$$\vec{B_1} = 100\hat{x} + 0.1\hat{y}$$

1b.)

$$\vec{B}_{1n} = \vec{B}_{2n} = 0.1\hat{y}$$

$$\vec{H}_{2t} - \vec{H}_{1t} = J_s$$

$$\frac{\vec{B}_{2t}}{1000\mu_0} - \frac{\vec{B}_{1t}}{\mu_0} = -1\ A/m$$ ← negative sign due to pointing away from us

$$\vec{B}_{1t} = \left(\mu_0 (1\ A/m) + \left(\frac{-0.2}{1000}\right)\right)\hat{x}$$

$$\vec{B}_{1t} = -0.0002\ \hat{x}$$

$$\vec{B}_1 = -0.0002x + 0.1\hat{y}$$

2a.)

$$\sigma = 5.9 \times 10^{7} \ S/m$$

$$R = \frac{l}{\sigma A} = \frac{1000m}{(5.9 \times 10^{7} \ S/m)(\pi (0.01)^2)} = 0.05395 \ \Omega$$

$$V = IR \qquad (0.1) = I\,(0.05395)$$

$$I = 1.854 \ A$$

$$I = J \cdot (\text{conductor area})$$

$$1.854 \ A = J \cdot \pi \cdot (0.01)^2$$

$$\vec{J} = 5901 \ A/m^2$$

2b.)

$$\oint \vec{H} \cdot \vec{dl} = I_{enc}$$

Inside the wire:

$$H \cdot 2\pi r = 1.854 \frac{r^2}{0.01^2}$$

$$\vec{H} = 2951 r \text{ A/m}$$

$$\vec{B} = \mu_0 \vec{H} = 3.71 r \text{ mT}$$ (in $\hat{\Phi}$ direction)

Outside the wire:

$$H \cdot 2\pi r = 1.854$$

$$H = \frac{0.295}{r} \text{ A/m}$$

$$\vec{B} = \mu_0 \vec{H} = \frac{0.371}{r} \mu T$$ (in $\hat{\Phi}$ direction)

c.)

$$\vec{B} = \nabla \times \vec{A}$$

$\vec{A}$ goes in the direction of current, so it only has a z component.

$$\vec{B} = \left(\cancelto{0}{\frac{1}{r}\frac{\partial A_z}{\partial \phi}} - \cancelto{0}{\frac{\partial A_\phi}{\partial z}}\right)\hat{r} + \left(\cancelto{0}{\frac{\partial A_r}{\partial z}} - \frac{\partial A_z}{\partial r}\right)\hat{\phi} + \frac{1}{r}\left(\cancelto{0}{\frac{\partial (r \cdot A_\phi)}{\partial r}} - \cancelto{0}{\frac{\partial A_r}{\partial \phi}}\right)\hat{z}$$

$$\vec{B} = -\frac{\partial}{\partial r} A_z \,\hat{\phi}$$

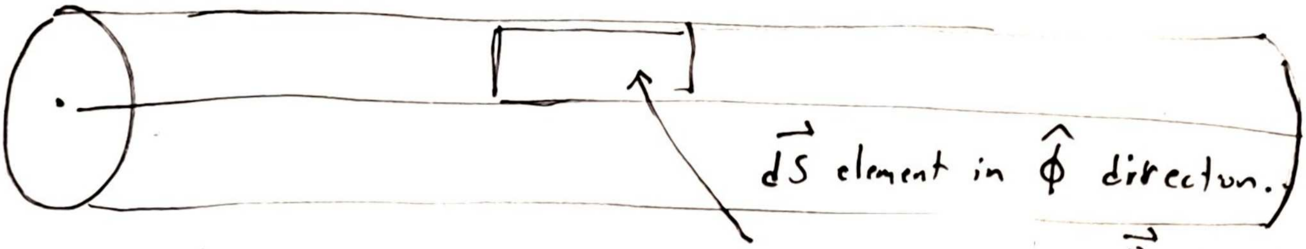
$$\frac{0.371}{r} = -\frac{\partial}{\partial r} A_z$$

$$A_z = -0.371 \ln(r) + C \quad (\text{in } \hat{\phi} \text{ direction})$$

d.) Since there is only one turn, flux is equal to flux linkage.

e.)

$$L = \frac{\Lambda}{I} = \frac{\psi}{I} \quad \text{(flux over current)}$$



$\vec{B}$ is pointed in $\hat{\phi}$ and will flow through it.

$$\Psi_m = \int \vec{B} \cdot \vec{ds} = \int_0^{1000\,m} dz \int_0^{0.01} (3.71 r)\, dr$$

$$= (1000) \int_0^{0.01} (3.71 r)\, dr$$

$$= (1000)(3.71)(0.01)^2 \quad /2$$

$$= 0.371 \text{ Wb} \quad /2$$

$$L = \frac{0.371 \text{ Wb}}{1.854 \text{ A}} = 200\,mH \quad /2$$

f.)

0 - 1000 m in z,

0.01 m - ∞ in r

g.) mag. energy density $= \frac{B^2}{2\mu_0}$

$$\doteq \frac{(3.71r \times 10^{-3})^2}{2\mu_0}$$

$$= 5.477r^2 \ \ J/m^3$$

3a.)

i.) air gap: $R = \dfrac{0.05}{\mu_0 (0.1)^2} = 3.98 \times 10^6$

metal loop: $R = \dfrac{1 - 0.05}{2500 \mu_0 (0.1)^2} = 3 \times 10^4$

$$R_{airgap} + R_{loop} \approx R_{air\ gap}$$

ii.) $\Psi = \dfrac{NI}{R} = \dfrac{1000}{3.98 \times 10^6} = 2.5 \times 10^{-4}$ Wb iii.) $L = \frac{\Lambda}{I} = \frac{1000 \cdot \Psi}{I} = 0.25 \text{ H}$

b.) $\Psi = \int \vec{B} \cdot \vec{ds}$

$2.5 \times 10^{-4} = B \cdot (0.1)^2$

$\vec{B} = 2.5 \times 10^{-2} \text{ T}$

Inside gap energy density $= \frac{B^2}{2\mu_0}$

$= 248 \text{ J/m}^3$

Inside metal energy density

$= \frac{B^2}{2 \cdot 2500 \mu_0}$

$= 0.099 \text{ J/m}^3$

$\frac{\text{Joules}}{m^3} = \frac{\text{Newtons}}{m^2}$

$(248 \text{ J/m}^3) \cdot (0.1 \text{ m})^2 = 2.48 \text{ N}$

4a.)

$$\mathcal{E} = -\frac{d\Psi}{dt}$$

$$\Psi = \int \vec{B} \cdot d\vec{S} = (0.1 \sin(200\pi t)) \cdot (0.05)^2$$

$$= (2.5 \times 10^{-4}) \sin(200\pi t) \text{ Wb}$$

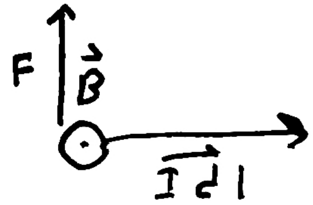
$$\mathcal{E} = -(2.5 \times 10^{-4})(200\pi) \cos(200\pi t)$$

$$= -0.157 \cos(200\pi t) \text{ V}$$

For 5 turns,

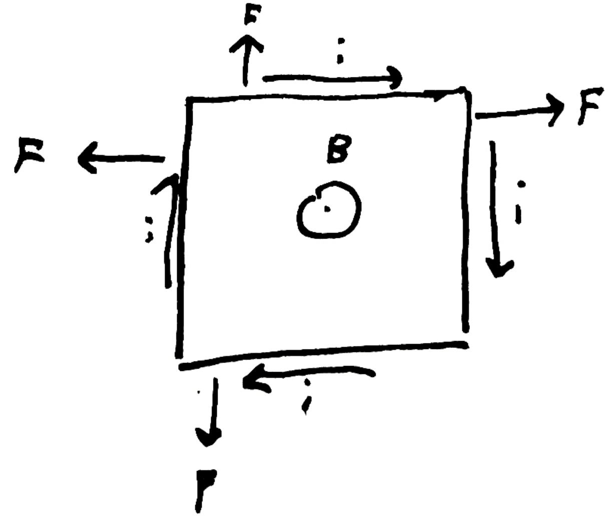
$$\mathcal{E} = -5 \frac{d\Psi}{dt} = -0.785 \cos(200\pi t)$$

b.) $\vec{F} = \int I\,dl \times B$



$\vec{F} = I l B$ pointing outward

$|\vec{F}| = (0.1)(0.1)(0.05) = 0.5\ mN$



The force pulls outward with equal magnitude on each section of the loop.

As a result, there is no net force or torque when the loop is in this position.

Problem 5

a.) Since, $B = \mu H$, the initial permeability will be determined by the slope of the dashed line near the origin of the graph.

b.) As H becomes strong, the material approaches total magnetization – that is, the atoms approach total alignment with the field. At this point, additional H field will not produce additional magnetization, so increase in the B field at this point will match that of free space. Hence, μ approaches μ_0 at very high H.

c.) Hard magnetic materials magnetize more strongly than soft magnetic materials. The advantage of hard magnetic materials is their ability to retain a strong magnetic field when the externally-applied H-field is removed, making them suitable materials for permanent magnets. Soft magnetic materials, on the other hand, have a smaller area within their hysteresis curve, which translates to less hysteresis energy loss when the externally-applied H-field changes. This makes soft magnetic materials more suitable for transformer and inductor cores.