

# 24A – Capacitance Concepts

A capacitor is a device in which equal and opposite charge is stored on two plates, resulting in the storage of energy in the form of electric field. The capacitance is a property of the geometry and of the plates and the material between the plates. The capacitor is usually charged by supplying electrical energy from a battery to separate positive and negative charges onto one plate and the other. The capacitance serves to relate the charge stored on a pair of plates to the potential difference required to store that charge.

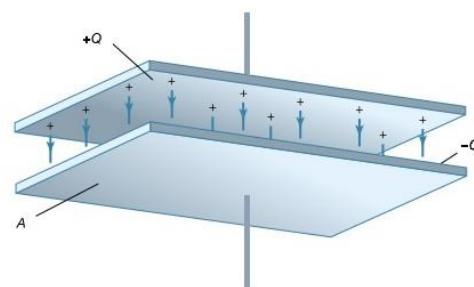
$$Q = C\Delta V \quad \text{Eq. 24a}$$

A capacitor is represented in circuits as two parallel lines placed close to one another. ( $\Rightarrow$ )



## Calculating capacitance from geometry

1. Consider two metal plates separated by a distance  $d$ . Shown schematically to the right. The plates have cross-sectional area  $A$ . The top plate is charged positively to  $+Q$  and the bottom to  $-Q$ . The area charge density on the capacitor plates is  $\sigma$ .

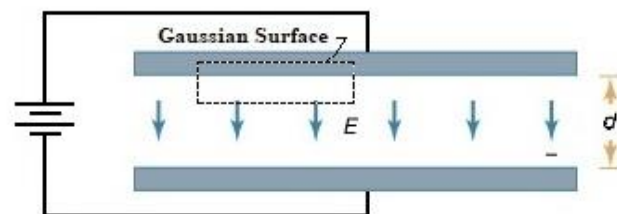


- a) Where do you expect the charge on the plate to be?

- ☒ A) Mostly on the inside (facing) surfaces,  
 B) Mostly on the outside surfaces.  
 C) Distributed throughout the metal.

- b) Explain your answer:

the opposite charges will



1a) A

- c) Is there a field inside the metal?

- A) Yes. ☒ B) No.

1c) B

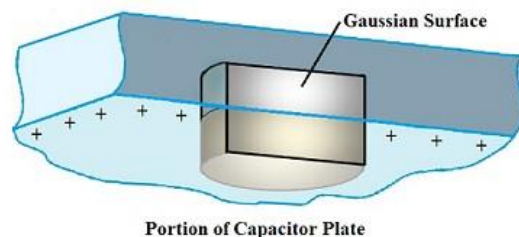
- d) Explain.

No, there can not be a field in the conductor because the particles will all move to the point of lowest potential until there is no more potential gradient

- e) What is the area charge density  $\sigma$  on the bottom surface of the top plate in terms of some of the given variables charge  $Q$ , plate area  $A$ , and plate separation  $d$ ?

e)  $Q/A$

- 2) Consider a disk-shaped Gaussian surface (right circular cylinder or “soup-can”) with the top surface inside the metal of the top plate and the bottom surface between the plates, as suggested by the sketch. Take the top surface area of the can to be  $A'$ .



- a) What is the charge  $Q'$  contained in the soup-can in terms of charge  $Q$ , plate area  $A$ , plate separation  $d$ , and can top area  $A'$ ?

a)  $QA'/A$

- b) Use Gauss' Law to find the field between the plates in terms of charge  $Q'$  and top area  $A'$ ?

b)  $Q'd/(A'e_0)$

- c) Now that you know the field, find the potential difference between the plates.

c)  $Q'd^2/(A'e_0)$

- d) What is the capacitance ( $Q/\Delta V$ ) in terms of  $d$ ,  $A$ , and physical constants?

d)  $e_0 A/d$

- e) What must the area of a certain capacitor be if it has plate separation of  $d = 2 \times 10^{-6}$  m and a voltage of  $\Delta V = 1$  V applied across the capacitor plates results in charge storage of  $Q = 10^{-12}$  C. Show your work.

$$Q/V = e_0 A/d$$

$$10^{-12}/1 = 8.85 \times 10^{-12} A/2 \times 10^{-6}$$

$$10^{-12} = 8.85 \times 10^{-6} A/2$$

$$10^{-6} = 8.85 A/2$$

$$2/8.85 \times 10^{-6} = A$$

$$A = 0.235 \times 10^{-6} \text{ m}^2$$

e)  $= 0.235 \text{ mm}^2$

- f) If the distance between the plates of the capacitor is halved ( $d/2$ ) while keeping the charge  $Q$  constant, does the potential difference change? (Yes or No)

f) Yes

- g) If so, what is the new potential difference?

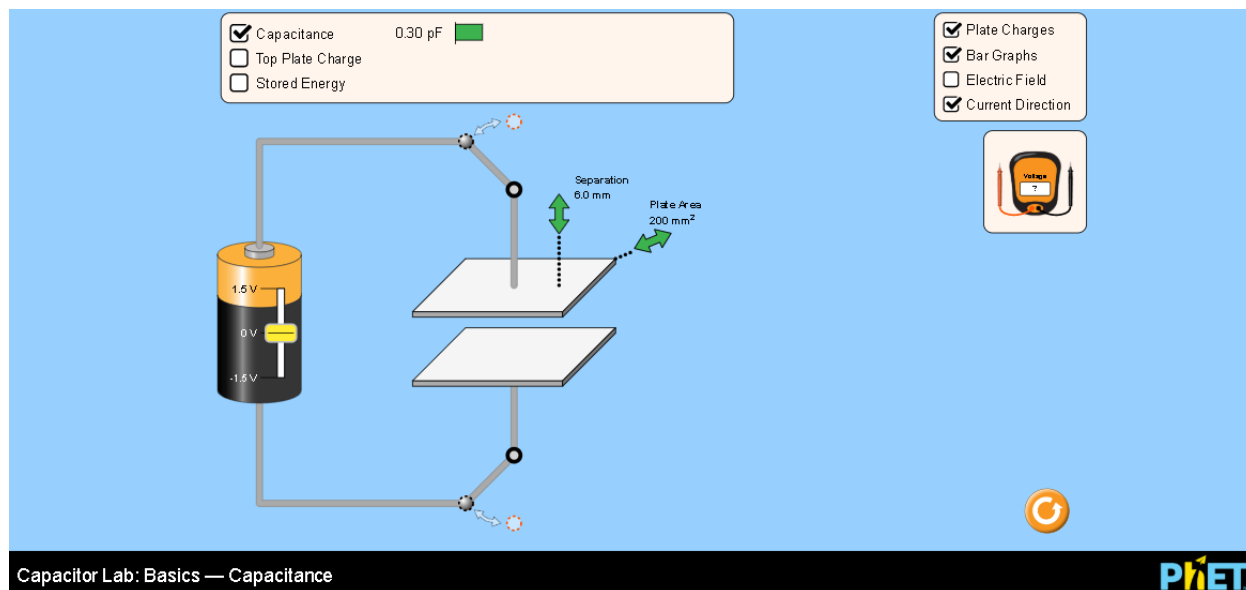
g)  $V/2$

## 24B – Simulation: Charge, Potential, Capacitance

In this module you will explore the behavior of a simulated capacitor and check to see if the creators of the simulation did it correctly.

This lab uses the simulation at the website: <https://phet.colorado.edu/en/simulation/capacitor-lab-basics>. Some of the activities here have been adapted from the “Capacitor Lab Basic” document by Trish Loeblein at the Colorado PhET site.

- Start the simulation and select the capacitance icon. The screen should look like this:



- Enable all the measurement capabilities of the simulation by selecting all of the boxes. Drag the voltmeter from its storage and touch the two leads to upper and lower capacitor plates.
1. Find the conditions (Battery Voltage, Plate Area, Plate Separation) that produce the largest capacitance. (Some of these parameters might not be relevant.)
    - a. Describe these parameters (e.g. – “largest plate spacing) in words.

maximize area, minimize distance, voltage not relevant

- b. Record the parameters for the largest and smallest capacitance. (Enter NA if the parameter doesn't affect the capacitance.)

	Largest	Smallest
Capacitance (pF)	1.77pf	0.09
Voltage	N/A	N/A
Area (mm <sup>2</sup> )	400	100
Plate Separation (mm)	2	10
Calculated Capacitance ( $\epsilon_0 A/d$ )	1.77pf	0.0885
Percent error $\frac{\text{Calculated} - \text{Given}}{\text{Given}} * 100$	0	-1.667 literally just a rounding error

- c. Apply the specified voltages to the largest and smallest capacitors and record the charge. Calculate the "experimental" capacitance using  $Q/V$  and compare it to the value given in the simulation.

#### LARGEST CAPACITOR

Voltage	Charge pC	$C_{\text{exp}}=Q/V$ pF	C(given) pF	Percent Error
0.150	.27	1.8	1.77	1.6949
0.300	.53	1.76667	1.77	-0.1883
0.500	.89	1.78	1.77	0.5649
1.000	1.77	1.77	1.77	0.000000
1.500	2.66	1.773333	1.77	0.18832

#### SMALLEST CAPACITOR

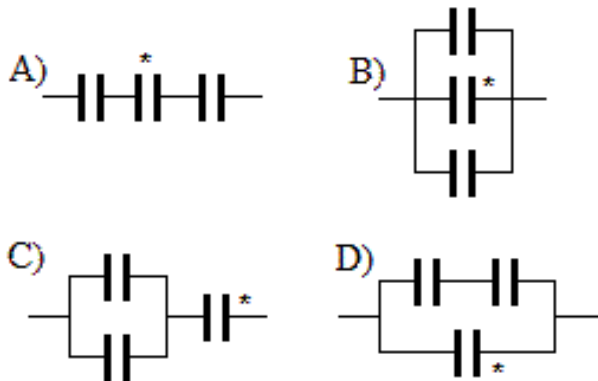
Voltage	Charge	$C_{\text{exp}}=Q/V$	C(given)	Percent Error
0.150	.01	.06667	.09	-25.9259
0.300	.03	.1	.09	11.1111
0.500	.04	.08	.09	-11.1111
1.000	.09	.09	.09	0.0000
1.500	.13	.086667	.09	-3.7003

Is the simulation perfect? Is it acceptable by the "Physics 2" standard of 2%?

not for the small cap at least, it's rounded very sharply and creates large errors

# 24C – Capacitors in Circuits

- Assume now that the capacitance for each capacitor in the circuits above is 2 microFarads. Find the equivalent capacitance of each circuit.



- A) 2/3mF      B) 6mF      C) 4/3mF      D) 3mF      C

- A potential of 10 V is applied across each circuit. Find the total energy stored in each circuit.

- A) 33.33mJ      B) 300mJ      C) 66.67mJ      D) 50mJ       $1/2 C V^2 = 50C$

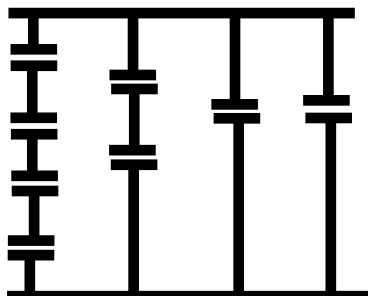
- A potential of 10 V is applied across each circuit. Find the charge stored on the equivalent capacitance of each circuit.

- A) 6.67mC      B) 60mC      C) 13.333mC      D) 30mC       $Q = C V = 10C$

- A potential of 10 V is applied across each circuit. Find the potential across the starred capacitor in each circuit.

- A) 6.67V      B) 10V      C) 6.67V      D) 10V

- You are given several 2.00-microFarad capacitors. Design and sketch a circuit that has a capacitance of 5.5 microFarads using fewer than ten capacitors.



$$(4 \cdot 2^{-1})^{-1} + (2 \cdot 2^{-1})^{-1} + 2 + 2 = 5.5$$

**24E – Experiment: Charge and Potential – Capacitance**

**Equipment:** 1 Black Capacitor Box; 1 Capacitance Board; 1 Small black power supply for the Capacitance Lab Board; 4 red and 3 black banana wires.

To measure the value of a capacitor you will use the Capacitance Board to apply a potential difference across a specific capacitor and then use the Capacitance Board to measure the charge stored on the capacitor. [You only need two wires to measure one capacitor. You will use the others to wire capacitors in series and parallel.]

1. Wire the Capacitance Board to Capacitor A. Dial in the potential difference you want to apply (10 V). Press the left button to charge the capacitor. Release when indicated. Press the right red button to measure the charge on the capacitor.

Potential Difference $\Delta V$ (V)	Charge $Q$ (nC)	Capacitance $C$ (Q/ $\Delta V$ )
10	1000nC	100nC

2. Perform the same process to measure capacitors B and C. Report results of these measurements in the next table.

Capacitor	Potential difference	Charge (nC)	Capacitance (nF)
B	10V	1000nC	100nF
C	10V	503.3nC	50.33nF

**24F – Experiment: Adding Capacitors**

**Equipment:** 1 Black Capacitor Box; 1 Capacitance Lab Board; 1 Small black power supply for the Capacitance Board; 4 red and 3 black banana wires.

- 1) What relationship would you use to compute the total capacitance if you were to wire two capacitors in series?

$$C = (C_1^{-1} + C_2^{-1})^{-1}$$

- 2) What relationship would you use to compute the total capacitance if you were to wire two capacitors in parallel?

$$C = C_1 + C_2$$

- 3) Fill out the table below with measured and computed capacitances for the following combinations. (Use your measured values from the previous section for computation.)

Series Parallel Combinations	Potential Difference $\Delta V$ (10V)	Measured Charge $Q$ (nC)	Capacitance $C$ Computed from $C = Q/\Delta V$ (nF)	Capacitance Computed from $C_A, C_B, C_C$ from <b>Module 24E</b> and capacitor addition rules
A,B in series	10V	544.9nC	54.49nF	50nF
A,B in parallel	10V	2000nC	200nF	200uF
A,C in series	10V	354.5nC	35.45nF	33.47967nF
A,C in parallel	10V	1500nC	150nF	150.33nF
A,B,C in series	10V	276.4nC	27.64nF	25.08nF
A,B,C in parallel	10V	2500nC	250nF	250.33nF

- 4) Are your parallel and series measurements consistent with the capacitance computed from the parallel and series rules? Discuss.

Yes, approximately. I think there was a slight deviation in one of our initial measurements leading to our final calculations deviating slightly from the experimental data.

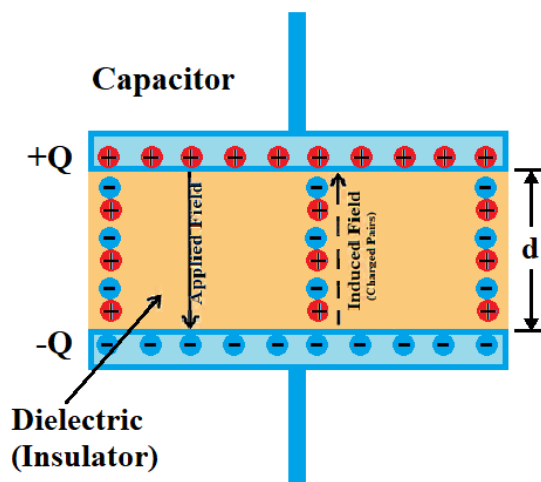
**24D – Dielectric Materials (Extra Credit, 2 pts)**

A dielectric is a non-conducting material. When a dielectric is placed in an electric field, the charges (e.g.- electrons and protons of the atoms) will shift slightly (by distances usually less than 1/100 of an atom width). This tiny shift can induce an internal electric field that opposes the applied field. The result is that the electric field inside of an insulator will be less than the applied field by a factor  $\kappa$ , known as the dielectric constant.

$$E_{net} = E_{applied}/\kappa.$$

The dielectric constant  $\kappa$  can be quite large, especially if the material is made up of polar molecules, such as water.

The applied field is due to the voltage  $V_{applied}$  between the plates, separated by distance  $d$ . Answer all questions below in terms of the applied voltage,  $d$ ,  $A$ ,  $\kappa$ , and constants.



- a. For a capacitor of area  $A$  and plate separation  $d$ , what would be the field between the plates for applied voltage  $V_{applied}$  if the space between the plates were under vacuum?

$$E_{applied} = \frac{V_{app}/d}{}$$

- b. What is the charge density  $\sigma_{vacuum}$  on the plates for the same conditions as in part a above?

$$\sigma_{vacuum} = \frac{V_{app} \cdot e_0/d}{}$$

- c. What is the total charge on the plates under the same conditions as in a?

$$Q = \sigma_v \cdot A = V_{app} \cdot e_0 \cdot A/d$$

- d. What is the Electric field  $E_{net}$  between the plates if the dielectric is inserted *without changing the charge  $Q$  on the plates*?

$$E_{net} = \sigma_v / (e_0 \cdot \kappa) = V_{app} / (d \cdot \kappa)$$

- e. What is the potential difference  $\Delta V_{dielectric}$  between the plates with the dielectric inserted (*without changing  $Q$  on the plates*)?

$$V_d = E_{net} \cdot d = V_{app} / \kappa$$

- f. Take the ratio of the charge on the plates to the potential between the plates to determine the capacitance when there is vacuum between the plates. (Your answer should be in terms of  $d$ ,  $A$ , and  $\epsilon_0$ .)

$$C = Q/V = (V_{app} \cdot e_0 \cdot A/d) / V_{app} = e_0 \cdot A/d$$



$$\underline{C = \epsilon_0 A/d}$$

- g. Take the ratio of the charge on the plates to the potential between the plates to determine the capacitance when there is dielectric between the plates. (Your answer should be in terms of  $d, A, \kappa$  and  $\epsilon_0$ .)

$$C = Q/V_d = (V_{app} \epsilon_0 A/d)/(V_{app}/\kappa) = \epsilon_0 A \kappa/d$$