1) Laplace transforms/Transfer functions

<u>Use Laplace transform tables!!!!</u>

1.1: Find the Laplace transform of

$f(t) = \left(\cos(2t) + e^{-4t}\right)\cdot u(t)$ (simplify into one ratio)

$$\frac{s}{s^2+4} + \frac{1}{s+4} = \frac{(s+4)s + \left(s^2+4\right)}{\left(s^2+4\right)\cdot(s+4)}$$

$$\frac{2s^2+4s+4}{\left(s^2+4\right)\cdot(s+4)}$$

1.2: Find the poles and zeros of the following functions. Indicate any repearted poles and complex conjugate poles. Expand the transforms using partial fraction expansion.

1. $F(s) = \dfrac{20}{(s+3)\cdot\left(s^2+8s+25\right)}$

$s^2 + 8s + 25 = 0$ $\begin{pmatrix} -4+3i \\ -4-3i \end{pmatrix}$

Zeros: 0
Poles: -3, -4 ±j3

2. $F(s) = \dfrac{2s^2+18s+12}{s^4+9\cdot s^3+34\cdot s^2+90\cdot s+100}$

Zeros: double -3
Poles:-2, -5, -1+j3, -1-j3

$$F(s) = \frac{2s^2+18s+12}{s^4+9\cdot s^3+34\cdot s^2+90\cdot s+100} = \frac{A_1}{s+2} + \frac{A_2}{s+5} + \frac{A_3}{s+1-j3} + \frac{A_3^*}{s+1+j3}$$

$s^4+9\cdot s^3+34\cdot s^2+90\cdot s+100 = 0$ $\begin{pmatrix} -2 \\ -5 \\ -1+3i \\ -1-3i \end{pmatrix}$

$(s+3)^2$

$(s+3)^2$

$$\frac{2(s + 9s + 6)}{(s+2)(s+5)(s+1-j3)(s+1+j3)}\cdot(s+2)$$ for s=-2

$$\frac{2\left[(-2)^2 + (9\times -2) + 6\right]}{(-2+5)(-2+1-3i)(-2+1+3i)} = -0.533$$

$$2(-2+3)^2 = 2$$

$$2\cdot(4 + -18 + 6) = -16$$

$$\frac{2(s + 9s + 6)}{(s+2)(s+5)(s+1-j3)(s+1+j3)}\cdot(s+5)$$ for s=-5

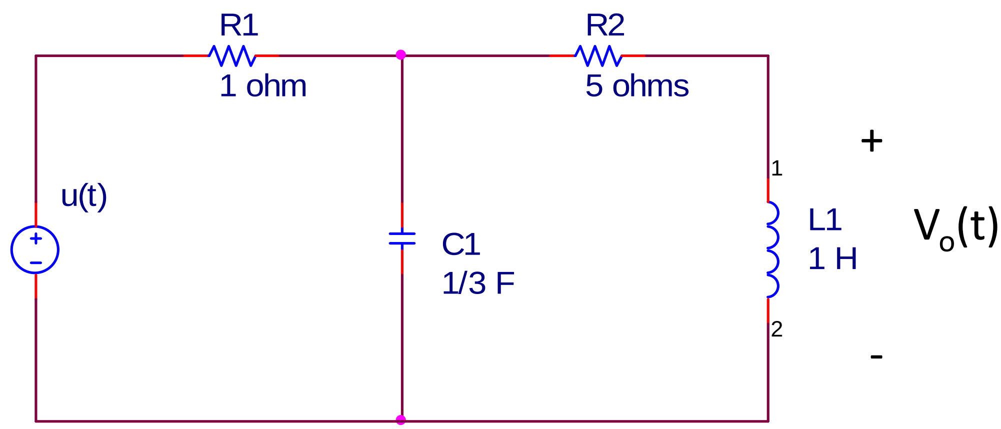
$$\frac{2\left[(-5)^2 + (9\times -5) + 6\right]}{(-5+2)(-5+1-3i)(-5+1+3i)} = 0.373$$

$$\frac{2(s + 9s + 6)}{(s+2)(s+5)(s+1-j3)(s+1+j3)}\cdot(s+1-j3)$$ for s=-1+j3

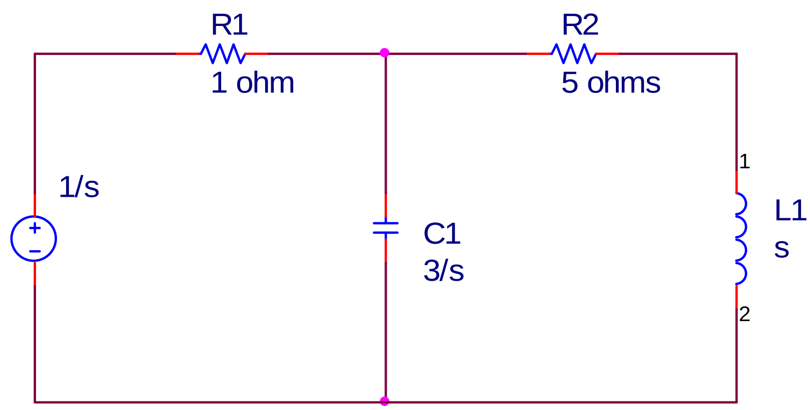
$$\frac{2\left[(-1+3i)^2 + \left[9(-1+3i)\right] + 6\right]}{(-1+3i+2)(-1+3i+5)(-1+3i+1+3i)} = 0.08 - 0.493i$$

$$F(s) = \frac{2s^2 + 18s + 12}{s^4 + 9\cdot s^3 + 34\cdot s^2 + 90\cdot s + 100} = \frac{-0.533}{s+2} + \frac{0.373}{s+5} + \frac{0.08 - 0.493i}{s+1-j3} + \frac{0.08 - 0.493i}{s+1+j3}$$

2) Circuits and Differential Equations



2.1: Draw the s-domain equivalent circuit. Assume all intial conditions are zero and the source is an arbitrary source.



2.2 Using impedances, determine the expression for Vo(t). Consider using mesh analysis then make one ratio.

Applying mesh analysis

mesh 1

$$\frac{1}{s} = \left(1 + \frac{3}{s}\right) i_1 - \frac{3}{s} \cdot i_2$$

mesh 2

$$0 = \frac{-3}{s} \cdot i_1 + \left(s + 5 + \frac{3}{s}\right) \cdot i_2 \qquad i_1 = \frac{1}{3} \cdot \left(s^2 + 5s + 3\right) \cdot i_2$$

substituting equation mesh 2 into mesh 1

$$\frac{1}{2} = \left(1 + \frac{3}{s}\right)\cdot\frac{1}{3}\cdot\left(s^2 + 5s + 3\right)\cdot i_2 - \frac{3}{s}\cdot i_2$$

multiply through by 3s

$$3 = \left(s^3 + 8s^2 + 18s\right)\cdot i_2 \qquad i_2 = \frac{3}{s^3 + 8s^2 + 18s}$$

$$v_o(s) = s\cdot i_2 = \frac{3}{s^2 + 8s + 18}$$

2.3 Find Vo(t) which is the VL(t) for t>0 using Vs=1 u(t).

Using Mesh analysis:

$$\frac{1}{s} = i_1\left(1 + \frac{3}{s}\right) - i_2\frac{3}{s}$$

$$0 = -i_1\cdot\left(\frac{3}{s}\right) + i_2\cdot\left(s + 5 + \frac{3}{s}\right)$$ solve for i1 $$i_1 = \frac{1}{3}\cdot\left(s^2 + 5s + 3\right)\cdot i_2$$

substitute in to find i2 which can be multiplied by s to find the voltage across L1

$$\frac{1}{s} = \frac{1}{3}\cdot\left(s^2 + 5s + 3\right)\cdot i_2\cdot\left(1 + \frac{3}{s}\right) - \frac{3}{s}\cdot i_2$$

multiply by 3s and solve for i2

$$i_2 = \frac{3}{s^3 + 8s^2 + 18s}$$

$$V_L = i_2\cdot s = \frac{3}{s^2 + 8s + 18}$$

Students will start with the end of 2.2 here...

$$s^2 + 8s + 18 = 0$$

$$\begin{pmatrix} -4 + \sqrt{2}\cdot i \\ -4 - \sqrt{2}\cdot i \end{pmatrix}$$

$$V_L = \frac{A_1}{s - (-4 - \sqrt{2}\,i)} + \frac{A_{1*}}{s - (-4 + \sqrt{2}\,i)}$$

$$\sqrt{2} = 1.414$$

$$\frac{3}{(s + 4 + \sqrt{2}\cdot i)\cdot(s + 4 - \sqrt{2}\cdot i)}\cdot[s - (-4 - \sqrt{2}\,i)] \qquad s = -4 - \sqrt{2}\,i$$

$$\frac{3}{(-4 - 1.414i) + 4 - 1.414i} = 1.061i$$

$$V_L = \frac{1.061i}{(s + 4 + \sqrt{2}\cdot i)} + \frac{-1.061i}{(s + 4 - \sqrt{2}\cdot i)}$$

$$V_L = 1.061i\cdot e^{\left(-4 - \sqrt{2}\,i\right)t} - 1.061i\cdot e^{\left(-4 + \sqrt{2}\,i\right)t}$$

not necessary but to check another way manipulate with laplace transform for e^at sin (bt)

$$\frac{3}{\sqrt{2}}\cdot\frac{\sqrt{2}}{(s + 4)^2 + (\sqrt{2})^2} = \frac{3}{\sqrt{2}}\cdot e^{-4t}\cdot\sin(\sqrt{2}\,t)$$

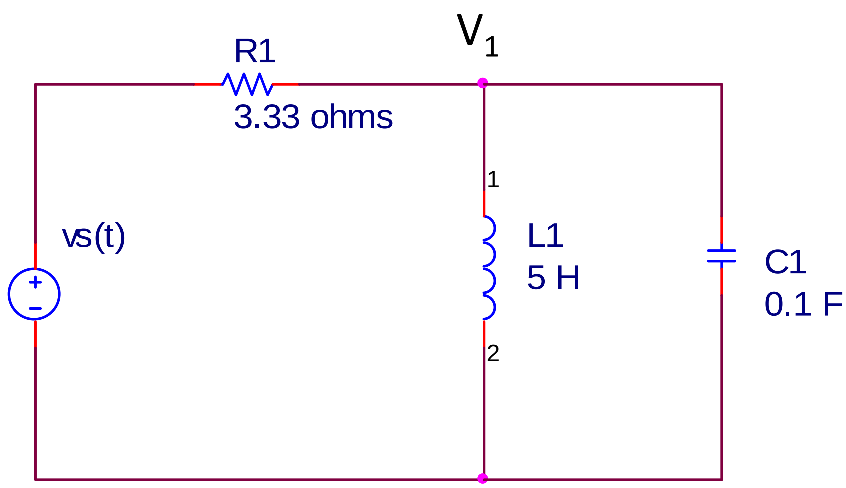
$$-2i\cdot 1.061i\cdot e^{-4t}\cdot\left[\frac{(-e)^{-\sqrt{2}\,i\,t} + e^{\sqrt{2}\,it}}{2i}\right]$$

Euler's formula $\quad \sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$

this should match with highlighted answer as well with some algebraic manipulation

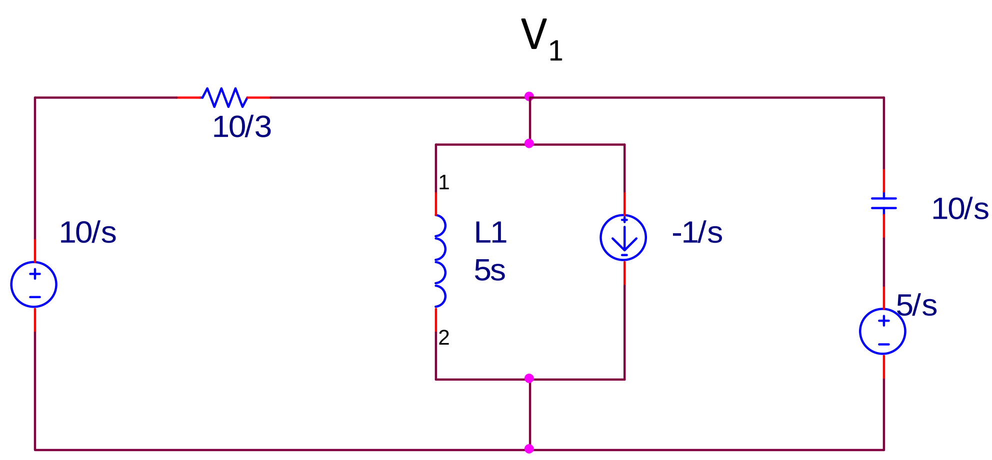
$$2.122e^{-4t}\cdot\sin(\sqrt{2}\,t) \qquad \frac{3}{\sqrt{2}} = 2.121$$

3) RLC and initial conditions



$v_S(t) = 10\,u(t)$ ***AND assume that -1A flows through the inductor and +5V is across the capacitor at t=0....i.e. vc(0)=5 and iL(0)= -1***

3.1: Draw the s-domain equivalent with initial conditions.



3.2: Find the value of the voltage across the capacitor, vc(t), using nodal analysis (at node V1) and laplace.

$$\frac{V_1 - \frac{10}{s}}{\frac{10}{3}} + \frac{V_1 - 0}{5s} - \frac{1}{s} + \frac{V_1 - \frac{5}{s}}{\frac{10}{s}} = 0$$

$$\frac{V_1}{\frac{10}{3}} - \frac{\frac{10}{s}}{\frac{10}{3}} + \frac{V_1}{5s} - \frac{1}{s} + \frac{V_1}{\frac{10}{s}} - \frac{\frac{5}{s}}{\frac{10}{s}} = 0$$

$$\frac{3V_1}{10} - \frac{3}{s} + \frac{V_1}{5s} - \frac{1}{s} + \frac{V_1 \cdot s}{10} - \frac{5}{10} = 0$$

group together V1 on left and other factors on right

$$V_1 \cdot \left(\frac{3}{10} + \frac{1}{5s} + \frac{s}{10} \right) = \frac{3}{s} + \frac{1}{s} + \frac{1}{2}$$

$$\frac{1}{10} \cdot \left(s + 3 + \frac{2}{s} \right) \cdot V_1 = \frac{3}{s} + \frac{1}{s} + 0.5$$

Take out 1/10 from right

$$\left(s + 3 + \frac{2}{s} \right) \cdot V_1 = \frac{30}{s} + \frac{10}{s} + 5$$

Multiply thorugh by 10

$$\left(s^2 + 3s + 2 \right) \cdot V_1 = 40 + 5s$$

multiply by s on both sides

$$V_1 = \frac{40 + 5s}{(s + 1)(s + 2)}$$

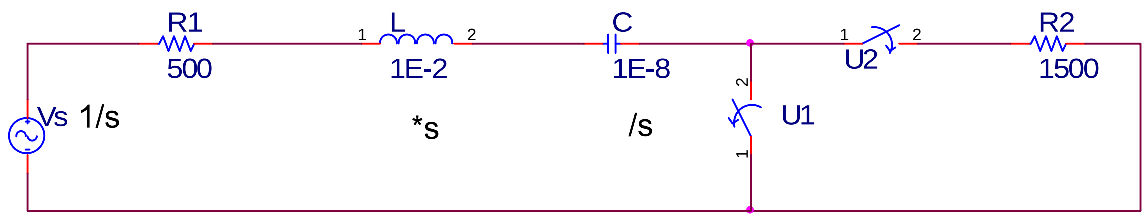
using partial fraction expansion

$$\frac{35}{s + 1} - \frac{30}{s + 2}$$

$$v_1(t) = v_c(t) = \left(35e^{-t} - 30e^{-2t} \right) \cdot u(t)$$

V

4) RLC parallel circuits



In the above circuit, the source turns on at t=0 with a voltage of 10V. Additionally, switch U1 is closed and switch U2 is open. At 15E-6 s switch U1 opens and switch U2 closes. The source also turns off at 15E-6 s.

a. Use Laplace analysis to determine the voltage across the capacitor as a function of time for 0<t<15E-6 (s)

$R_1 := 500\Omega \qquad L_1 := 1\cdot 10^{-2}\text{H} \qquad C_1 := 1\cdot 10^{-8}\text{F} \qquad R_2 := 1500\Omega$

$$V_C(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}\cdot s + \frac{1}{LC}}\cdot\left(V_s(s) + L\cdot I_L(0\text{-}) - \frac{V_C(0\text{-})}{s}\right) + \frac{V_C(0\text{-})}{s}$$

$$\frac{1}{L_1\cdot C_1} = 1\times 10^{10}\frac{1}{s^2} \qquad \frac{R_1}{L_1} = 5\times 10^4\frac{1}{s}$$

open circuit for cap and short for inductor at t = 0- and source is 0 so IL0- and VC0- are both 0

$$V_C(s) = \frac{1\cdot 10^{10}}{s^2 + 5\cdot 10^4\cdot s + 1\cdot 10^{10}}\cdot\left(\frac{10}{s}\right)$$

Use partial fraction expansion

$$s^2 + 5\cdot 10^4\cdot s + 1\cdot 10^{10} = 0$$

$$\begin{pmatrix} -25000 + 25000i\cdot\sqrt{15} \\ -25000 - 25000i\cdot\sqrt{15} \end{pmatrix}$$

$$V_C(s) = \frac{-5 + 1.29i}{s + 2.5\cdot 10^4 - 9.68\cdot 10^4 i} + \frac{-5 - 1.29i}{s + 2.5\cdot 10^4 - 9.68\cdot 10^4 i} + \frac{10}{s}$$

$$V_C(t) = \left[(-5 + 1.29i)\cdot\exp\left(-2.5\cdot 10^4 t + 9.68\cdot 10^4 i\, t\right) + (-5 - 1.29i)\cdot\exp\left(-2.5\cdot 10^4 t - 9.68\cdot 10^4 i\, t\right) + 10\right]u(t)$$

Note: The solution can be left in this form

b. Use Laplace analysis to determine the voltage across the capacitor as a function of time for t>15E-6 s

Use the final conditions of the first region as the initial conditions of the second region (at 15E-6).

$$V_C(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}\cdot s + \frac{1}{LC}}\cdot\left(V_s(s) + L\cdot I_L\left(15\cdot 10^{-6}-\right) - \frac{V_C\left(15\cdot 10^{-6}-\right)}{s}\right) + \frac{V_C\left(15\cdot 10^{-6}-\right)}{s}$$

INITIAL CONDITIONS:

Must find $V_C\left(15\cdot 10^{-6}-\right)$ and $L\cdot I_L\left(15\times 10^{-6}-\right)$

plug 15E-6 into equation from part a to get Vc(t at 15E-6)

Then use $I_C = C\cdot\frac{dVc}{dt}$ to find IL (which is the same as IC)

$$(-5 + 1.29i)\cdot\exp\left[-2.5\cdot 10^4\left(15\cdot 10^{-6}\right) + 9.68i\cdot 10^4\left(15\cdot 10^{-6}\right)\right] + (-5 - 1.29i)\cdot\exp\left[-2.5\cdot 10^4\left(15\cdot 10^{-6}\right) - 9.68i\cdot 10^4\left(15\cdot 10^{-6}\right)\right] + 10 =$$

$$V_C(0) = 7.425$$

Taking derivative of Vc(t) from previous solution.

$$(128.0 - 516250.0i)\cdot e^{-(25000.0-96800.0i)\cdot\left(15\cdot 10^{-6}\right)} + (128.0 + 516250.0i)\cdot e^{-(25000.0+96800.0i)\cdot\left(15\cdot 10^{-6}\right)} = 7.046\times 10^5$$

$$L\cdot C\cdot\frac{dVc}{dt} = 1\cdot 10^{-2}\cdot 1\cdot 10^{-8}\cdot 7.046\times 10^5 = 7.046\times 10^{-5}$$

$$1\cdot 10^{-2}\cdot 1\cdot 10^{-8}\cdot 7.046\times 10^5 = 7.046\times 10^{-5}$$

$$L\cdot I_L(0) = 7.046\cdot 10^{-5}$$

$Vs(s) = 0$ source turned off...

$$R_{12} := R_1 + R_2 = 2\times 10^3\,\Omega$$

$$\frac{R_{12}}{L_1} = 2\times 10^5\frac{1}{s}$$

$$\frac{1\cdot 10^{10}}{s^2 + 2\cdot 10^5\cdot s + 1\cdot 10^{10}}\cdot\left(7.046\cdot 10^{-5} - \frac{7.43}{s}\right) + \frac{7.43}{s}$$

$$\frac{s\cdot 7.046\cdot 10^5 - 7.43\cdot 10^{10}}{s\cdot\left(s^2 + 2\cdot 10^5 s + 1\cdot 10^{10}\right)} + \frac{7.43}{s}$$

$$s^2 + 2\cdot 10^5 s + 1\cdot 10^{10} = 0$$

$$\begin{pmatrix} -100000 \\ -100000 \end{pmatrix}$$

$$\frac{7.43}{s + 1\cdot 10^5} + \frac{1.447\cdot 10^6}{\left(s + 1\cdot 10^5\right)^2} + \frac{-7.43}{s} + \frac{7.43}{s}$$

PFE,
take inverse laplace and add time delay

$$\frac{7.43}{s + 1\cdot 10^5} + \frac{1.447\cdot 10^6}{\left(s + 1\cdot 10^5\right)^2}$$

$$V_{C2} = 7.43\cdot \exp\left[-10^5\left(t - 15\cdot 10^{-6}\right)\right] + 1.447\cdot 10^6\cdot\left(t - 15\cdot 10^{-6}\right)\cdot \exp\left[-10^5\left(t - 15\cdot 10^{-6}\right)\right]$$