39A – Matter Waves – Interference, Wavelength, and Momentum

You have learned that light sometimes presents itself as an electromagnetic wave and sometimes as a particle, depending on the physical probe that is used. Specifically, the relationship between wavelength and particle momentum *p* is given by:

$$p = \frac{h}{\lambda}$$

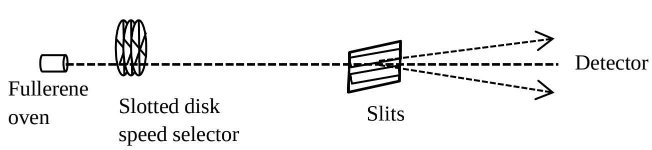
It was proposed by de Broglie in 1924 and has been experimentally verified many times that this relationship also applies to particles that have mass, including electrons, protons, and molecules.

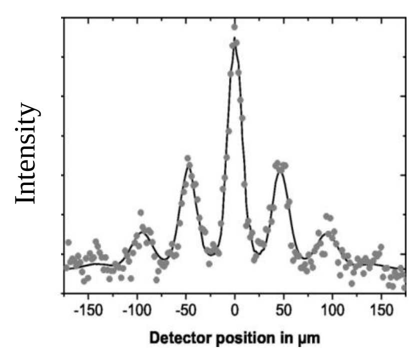
We will focus in this course on particles with mass that are moving at non-relativistic velocities although the de-Broglie relation applies to relativistic particles as well. A non-relativistic particle with mass *m* and velocity *v* has momentum $p = mv$, so $\lambda = \frac{h}{mv}$ and the classical kinetic energy is:

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

de Broglie Wavelength Calculations

One of the largest particles to demonstrate wavelike interference effects is the C60 fullerene molecule (consisting of 60 carbon atoms). A schematic of the experimental set-up used by Nairz et al (American Journal of Physics, 71 (2003)) is shown below.





1) The oven and slotted disk serve to provide molecules of a single speed. The molecules pass through collimators and slits and then travel 1.25 meters to a detector. The resulting interference pattern is shown to the right. The sharp peaks are due to interference between adjacent slits, with a spacing of 100 nm.

   a) Determine the wavelength for the C60 molecules in this experiment from the interference pattern and slit spacing. (Show logic.)

   lambda=d*y/D=100*10^-9*50*10^-6/1.25

   4*10^-12 m

b) The velocity of the C60 molecules was stated to be 120 m/s in this experiment. What wavelength do you predict from the de Broglie relation and the mass? Is this consistent with the number you calculated in part a? (Show logic.)

l=h/mv
m=720*10^-3/(6.022*10^23)

4.61*10^-12 m

c) About how big do you expect a C60 molecule to be? (Google™ it.) It should be several times the size of a carbon atom.) How does this compare with the de Broglie wavelength?

7A=0.7*10^-9m >> 4*10^-12

d) Assuming you could singly-ionize a C60 molecule moving at 120 m/s, how much electric potential would you have to apply to stop its motion? (Show logic.)

0.05378 V

e) You could increase the selected velocity of the C60 molecules by spinning the slotted disks faster. Would the angle (detector position) at which you observe the interference peaks increase or decrease if you increased the velocity? Explain your answer.

decrease, increased velocity decreases wavelength and decreases delta y and theta

2) Write a significant question that you and your team still have about matter wavelength.

what effect does temperture have when working with mater wavelength?
could something hot enough have high enough particle velocity to have noticable effects?
are these wavelengths affected by gravitational redshift

 Name Hayden Fuller Section 6

39C – Wavelength, Momentum, and Kinetic Energy

1) Calculate the wavelengths for an electron and for a proton, each moving at 10^5 m/s.

lambda=h/mv
electron: 7.27*10^-9 m
proton: 3.968*10^-12 m

Which has the greater wavelength?
electron

2) Calculate the wavelengths for an electron and for a proton, each moving with kinetic energy of 1×10^{-19} J.

K=h^2/(2m*lambda^2)
lambda=h/sqrt(2mK)

electron: 1.55*10^-9 m
proton: 3.625*10^-11 m

Which has the greater wavelength?

electron

3) Calculate the wavelengths for an electron and a proton, each moving with a classical momentum of 9.1×10^{-28} kg m/s.

lambda=h/sqrt(p^2/(2m))

electron: 7.29*10^-7 m
proton: 7.29*10^-7 m

Which has the greater wavelength?
they're the same

39D – Heisenberg Uncertainty Principle

The Heisenberg Uncertainty Principle states that it is not possible to simultaneously measure the position and momentum of a particle with absolute certainty. The mathematical statement of the principle in the textbook is:

$$\Delta x \Delta p = \sigma_x \sigma_p \geq h/4\pi$$

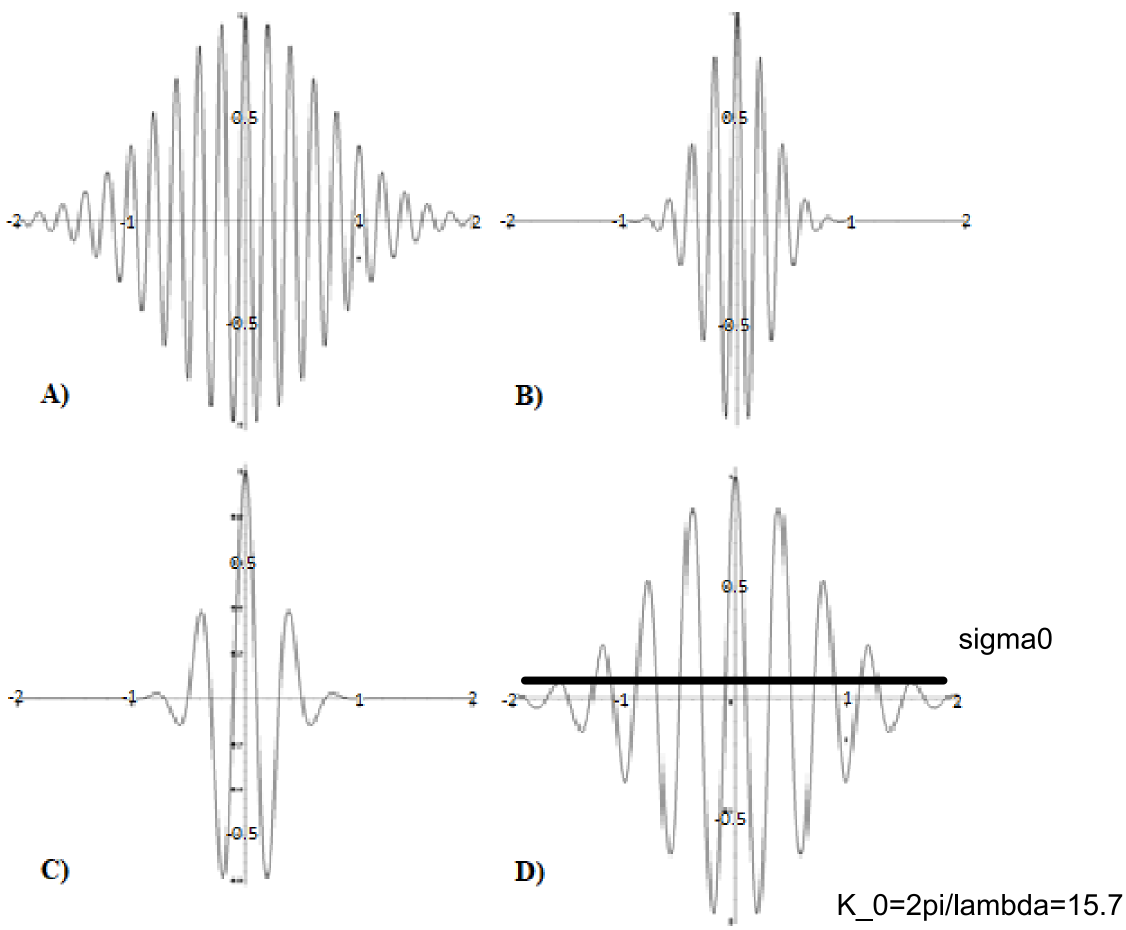
One of the approaches to understanding the uncertainty principle is to think about how to add waves of different wavelengths to one another to create a peaked waveform.

A solution to the wave equation for a photon moving in free space (U(x) = 0) is

$$\Psi(x,t) = Ae^{-\left\{\frac{(x-ct)}{2\sigma_x}\right\}^2} e^{ik_0(x-ct)}$$

where the average wave number $k_0 = p_0/\hbar$ and p_0 is the average momentum of the particle if it were measured many times.[1] This solution is called a "wavepacket". For the rest of this activity assume that k_0 >> $1/\sigma_x$.

1) Examples of various wavepackets are shown below. The horizontal scales are the same for all. Assume that the vertical scales are appropriate so that the wavepackets are properly normalized.



[1] The quantity "i" in the wavepacket equation is the unit imaginary number, $i = \sqrt{-1}$. Recall also Euler's relation from your calculus classes: $e^{iz} = \cos z + i \sin z$.

a. Assuming that the scale is the length in meters, estimate the wavelength of the wavepacket for each of the sketches above.

a .2 , b .2 , c .4 , d .4

b. Give a brief description of the following symbols given in the above wavepacket. What properties of the wavepacket does each represent?

i. σ_x.

variance

ii. k_0.

the wave number

iii. c.

speed of light

c. Label σ_x and k_0 on the figure for wavepacket D

d. Estimate σ_x for each of the wavepackets sketched above.

a 2, b 2, c 4, d 4

e. Rank the wavepackets above from the largest average momentum to the smallest. If two wavepackets have the same momentum, group them with parentheses.

(A,B),(C,D)

f. Rank the wavepackets above from the largest spatial uncertainty to the smallest. If two wavepackets have the same spatial uncertainty, group them with parentheses.

(A,D),(B,C)

g. Rank the wavepackets from the largest momentum uncertainty to the smallest. If two wavepackets have the same momentum uncertainty, group them with parentheses.

(B,C),(A,D)