32B – Electromagnetic Waves Background

From the text and lecture you now know that an oscillating electric field produces a magnetic field perpendicular to the electric field (Maxwell-Ampere Law), and an oscillating magnetic field produces an electric field (Faraday's Law). This mutual induced-field effect gives rise to travelling electromagnetic waves that transport information and energy. An electric field wave of frequency $f = \omega/2\pi$, wavelength λ, and speed $c = \lambda f$, *traveling in the direction* $\vec{r}$ can be written as:

$\vec{E} = \vec{E}_0 \cos(\vec{k} \cdot \vec{r} - \omega t + \phi)$.

Where the wave amplitude is $\vec{E}_0$ and the wave travels in the direction given by $\vec{k}$.

1) Here are some terms associated with a travelling wave. Describe the meaning of each one with a short sentence or phrase.
   a) $k = 2\pi/\lambda$ radians per meter

   b) $T = 1/f$ Time period is one over frequency.

2)
   a) Write down the relationship between the Poynting vector $\vec{S}$ and the electric -$\vec{E}$ and magnetic fields $\vec{B}$ in a traveling electromagnetic wave. Give the meaning of each term in your equation.

      S=1/\mu_0 ExB
      \mu_0 is the magnetic constant, S is the direction of propogation and energy per area,
      E is electric field, B is magnetic field

   b) Write down the relationship between the intensity (in W/m^2) and the electric and magnetic fields in an electromagnetic wave. Explain what each term means in a sentence.

I=c*.5*\epsilon_0*E^2=c*B^2/(2*\mu_0)
\epsilon_0 is electric constant, \mu_0 is magnetic constant, I is intensity (power per area),
c is the speed of light 299792458m/s, E is electric field, M is magnetic field

   c) Write down the relationship between the electric and magnetic fields and the pressure exerted on a mirror by a plane electromagnetic wave normally incident on the mirror.

      Power=I/c, Pressure=2*Power
      Pressure=\epsilon_0*E^2=B^2/\mu_0

3) The electric field for an electromagnetic wave is given by:

$$\vec{E}\,(x,y,z,t) = 1000\sin\left(\frac{20\pi}{1\text{m}}(y-ct)\right)\hat{\imath}\ \frac{V}{m}.$$

a) In what direction is this wave traveling (+x, +y, +z,-x,…)? +y

b) What is the wavelength of this wave? 1/10 m

c) What is the frequency of this wave? 10c Hz

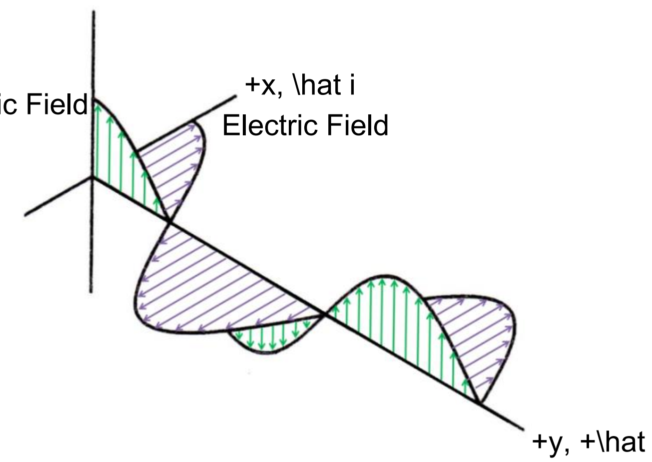
c=LF
k=20 pi c
F=10 c
L=c/F=1/10

d) Along what axis does the electric field point? \hat i

4) The electric field

$$\vec{E}\,(x,y,z,t) = 1000\sin\left(\frac{20\pi}{1\text{m}}(y-ct)\right)\hat{\imath}\ \frac{V}{m},$$

along with the corresponding magnetic field is shown in an illustration to the right at a time t = 0 s.



a) Label the axes and curves appropriately. (Be careful with signs.)

b) The energy density stored in an electric field is given by: $u_E = \frac{1}{2}\,\varepsilon_0 E^2$. Find the average energy density stored in the wave $\vec{E}\,(x,y,z,t) = 1000\sin\left(\frac{20\pi}{1\text{m}}(y-ct)\right)\frac{V}{m}\hat{\imath}$ by computing the integral $\bar{u}_E = \frac{1}{\lambda}\int_0^\lambda u_E(y)dy$. Show your work or make a logical argument to complete the integral.

\bar u_E=\epsilon_0/(2\lambda) \int_0^\lambda (1000\sin(20\pi y-20\pi ct))^2 dy
\bar u_E=5*10^6\epsilon_0 \int_0^\lambda (sin(20\pi y-20\pi ct))^2 dy
\bar u_E=10^6 \epsilon_0/(16\pi)(4\pi-2\sin(40\pi ct))
\bar u_E=10^6 \epsilon_0/(4\pi)

I think this is wrong?
\bar u_E=1/\lambda \int_0^\lambda*u_E(y) dy
\bar u_E=1/\lambda \int_0^\lambda*.5*\epsilon_0*E^2 dy
\bar u_E=1/\lambda \int_0^\lambda*.5*\epsilon_0*(1000*\sin(20*\pi*(y-ct))^2 dy
\bar u_E=1/\lambda \int_0^\lambda*.5*\epsilon_0*1000000*\sin^2(20*\pi*(y-ct)) dy, choose t=0
\bar u_E=5*10^5*\epsilon_0/\lambda \int_0^\lambda*(.5-.5*\cos(20*\pi*(y))) dy
\bar u_E=2.5*10^5*\epsilon_0/\lambda \int_0^\lambda*[1-\cos(20*\pi*y)] dy
\bar u_E=2.5*10^5*\epsilon_0/\lambda* [y-20*\pi*\sin(20*\pi*y)]|_0^\lambda dy
\bar u_E=2.5*10^5*\epsilon_0/\lambda* [[\lambda-20*\pi*\sin(20*\pi*\lambda)]-[0-20*\pi*\sin(20*\pi*0)]]
\bar u_E=2.5*10^5*\epsilon_0/\lambda* [\lambda-20*\pi*\sin(20*\pi*\lambda)]
\bar u_E=2.5*10^5*\epsilon_0/0.1* [0.1-20*\pi*\sin(20*\pi*0.1)]
\bar u_E=2.5*10^6*\epsilon_0* [0.1-20*\pi*\sin(2*\pi)]
\bar u_E=2.5*10^6*\epsilon_0* [0.1-20*\pi]

c) Write the equation for the magnetic field in terms of y, t, and constants for the wave in this problem.

B=1000/c * cos(20\pi(y-ct)) \hat z

d) Find the average energy density stored in the magnetic field for this wave.

\bar u_B=1/(2\mu_0) \int_0^\lambda B^2 dy

Since this wave moves at velocity *c*, the energy density moves at this speed as well. The time-averaged rate at which this energy density passes through a plane surface is known as the intensity and is given by: $I = \bar{S} = \frac{1}{2\mu_0}\left|\vec{E}_0 \times \vec{B}_0\right|$, where $\vec{E}_0$ ($\vec{B}_0$) is the amplitude of the electric (magnetic) wave.

e) Find the intensity of this wave.

I=1/(2\mu_0) (1000*1000/c)=10^6/(2\mu_0c)=1326.29 W/m^2

5) This summer if you were lucky you got to lay in the sun at the beach and you absorbed significant solar power and a lot of energy in an afternoon. This was quite noticeable, and you probably had to take breaks in the water to cool off. Assume that the intensity of sunlight is about 1000 W/m^2 and the area of your body is about 0.3 m^2. Assume also that the sunlight is completely absorbed by your body. Show your logical steps.

a) Estimate the power that your body absorbed.

P*A/c=1000*.3/(3*10^8)=10^-6

Power = 10^-6 W

b) Estimate the total energy that you absorbed over a three-hour sunning session.

Energy = 1.08*10^-2=0.0108

c) Estimate the force exerted by sunlight on your body during this time and compare it to the force of the Earth's gravitation on your body.

Sunlight force = 1.4*10^-6 N

Gravitational force = 9.81 N