Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 2500: Engineering Probability, Spring 2023

Homework #8: due Wednesday, April 26th, at 11:59PM.
Show all work for full credit!
Submit your work as a single PDF on Gradescope, labeling each problem number with a page.

1. (30 points) Let X and Y be defined by the joint PDF:

$$f_{XY}(x,y) = \begin{cases} \frac{6}{19}(x^2+y^3) & 0 \le x \le 2, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) (10 points) Compute $f_{X \mid Y}(x \mid y)$. (You need to compute the marginal $f_Y(y)$ along the way.)
   (b) (10 points) Compute $E(X \mid Y = y)$.
   (c) (10 points) Compute $E(X)$ using the law of iterated expectations.

2. (20 points) Suppose X is a uniform random variable on [0, 2] and Y is an exponential random variable with parameter $\lambda = 3$. X and Y are independent. Let $Z = X + Y$.

   (a) (5 points) Compute $E(Z)$.
   (b) (5 points) Compute $\text{Var}(Z)$.
   (c) (10 points) Compute $f_Z(z)$. Be sure to specify the PDF for all values of z.

3. (25 points) The weight of a Montana cow in tons has the PDF given by

$$f_X(x) = \begin{cases} \frac{3}{2}x - \frac{3}{4}x^2 & x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$$

   (a) (10 points) Use the Chebyshev inequality (the version in Equation 7.20 on p. 366) to find a lower bound on the probability that a collection of 100 independent cows weighs between 85 and 115 tons.
   (b) (10 points) Now use the Central Limit Theorem to estimate the probability that a collection of 100 independent cows weighs between 85 and 115 tons.
   (c) (5 points) How does this compare to the Chebyshev bound? What explains the difference? Which is more accurate?

More fun on the next page ⟶

4. (25 points) Rent The Runway (RTR) offers a service for customers to rent clothes through the mail. We call each package of clothes a Shipment. Suppose RTR's internal shipping and handling cost to process the exchange of Shipment i is X_i, a uniform random variable on [0.30, 1.20] dollars. Let Y_i be the (continuous) number of days a customer holds on to Shipment i, modeled as an exponential random variable with mean 5 days. For the purposes of this problem, assume the exchange of shipments is instantaneous, and that all the $\{X_i\}$ and $\{Y_i\}$ are mutually independent. Let N be the number of shipments a customer rents over the course of 30 days, and let Z be the internal monthly cost to RTR for shipping and handling these N shipments. Note that N is a discrete random variable (we can't ship half a package)!

   (a) (10 points) Determine the conditional expected value $E(Z \mid N)$.

   (b) (8 points) Determine the expected value of Z. Remember the connection between the exponential and Poisson random variables!

   (c) (7 points) How much should RTR charge for its monthly service to make an average of 5 dollars net profit per month from each customer?