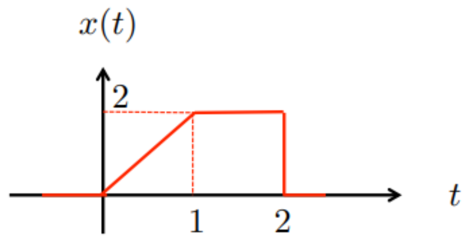
Homework # 6

Due: Tuesday, August 1st, 2023

Problem 1 (10 points each) Signal $x(t)$ is given in the following figure:

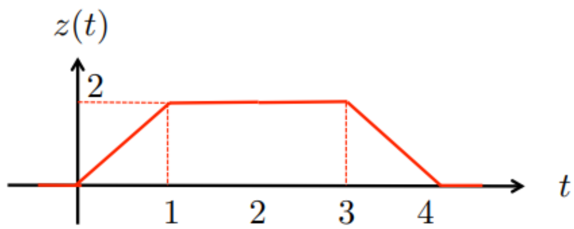


a) Find and sketch signal $y(t) = \frac{dx(t)}{d}$.

b) Find the Fourier transform of $y(t)$.

c) Use the result of part (b) to find the Fourier transform of $x(t)$.

d) Use the result of part (c) to find the Fourier transform of $z(t)$ given below.



Problem 2 (20 points) Compute the convolution $x(t)$ and $h(t)$ by using their Fourier transforms $X(\omega)$ and $H(j\omega)$.

$$x(t) = te^{3t}u(-t) \qquad \text{and} \qquad h(t) = e^{t}u(-t)$$

Problem 3 (20 points) The Fourier transform of signal $x(t)$ is given by

$$X(\omega) = \frac{2j\omega}{1+3j\omega}$$

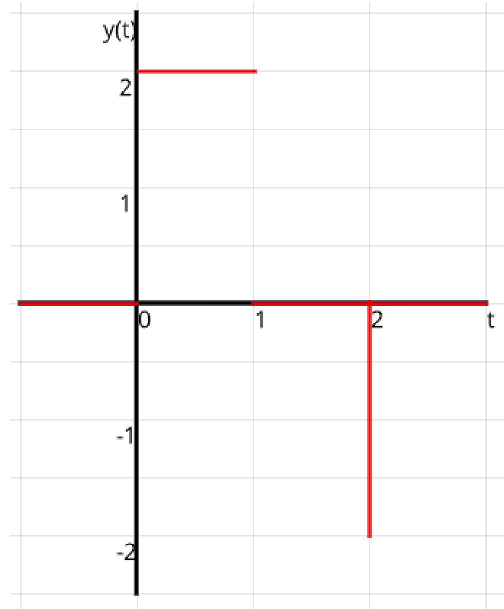
Find signal $x(t)$.

Problem 4 (20 points) Using the convolution theorem, find the inverse Fourier transform of signal $X(\omega)$ is given by

$$X(\omega) = \frac{1}{(a + j\omega)^2}$$

Find signal $x(t)$.

1)
a)



b)
y = 2pulse(t-1/2)-2delta(t-2)
pulse(t) == 2T sinc(wt) = sinc(w/2)
pulse(t-1/2) == e^-jw/2 sinc(w/2)
delta(t) == 1
delta(t-2) == e^-j2w
Y(w) = 2e^(-jw/2) sinc(w/2) - 2e^(-j2w)

c)
x(t) = int_-inf^t y(t) dt
X(w) = F{int_-inf^t y(t) dt = 1/jw Y(w) + pi Y(0) delta(w)
X(w) = [2e^(-jw/2) sinc(w/2) - 2e^(-j2w)]/jw + pi 0 delta(w)
X(w) = 2/jw (e^-jw/2 sinc(w/2) - e^-j2w)

d)
z(t) = x(t)+x(4-t)
x(t) time shift time reverse e^-j4w X(-w)
Z(w) = X(w) + e^-j4w X(-w)
Z(w) = 2/jw (e^-jw/2 sinc(w/2) - e^-j2w) + -2/jw e^-j4w (e^jw/2 sinc(w/2) - e^j2w)
Z(w) = 2/jw sinc(w/2) (e^-jw/2 - e^-j7w/2)

2)
X(w) = int_-inf^inf x(t) e^-jwt) dt
d/dw X(w) = int_-inf^inf -jt x(t) e^-jwt) dt
j d/dw X(w) = int_-inf^inf t x(t) e^-jwt) dt
F{t x(t)}=j d/dw X(w)
e^-at u(t) == 1/(a+jw)
e^at u(-t) == 1/(a-jw)
t e^at u(-t) == j d/dw 1/(a-jw) = -1/(a-jw)^2
X(w) = j d/dw 1/(3-jw) = -1/(3-jw)^2
H(w)=1/(1-jw)
Y(w)=X(w)H(w)=-1/((1-jw)(3-jw)^2)
Y(w)=-1/4/(1-jw)+1/4/(3-jw)+1/2/(3-jw)^2
y(t)=- 1/4e^t u(-t) + 1/4e^3t u(-t) - 1/2 te^3t u(-t)

3)
e^-at u(t) == 1/(a+jw)
d/dt e^-at u(t) == jw/(a+jw)
2 d/dt e^-at u(t) == 2jw/(a+jw)
F^-1{2jw/(1+3jw)} = F^-1{2/3jw/(1/3+jw)} == 2/3 d/dt e^(-1/3 t) u(t)
= 2/3 (-1/3 e^(-t/3) u(t) + delta(t))
=-2/9e^(-t/3) u(t) + 2/3 delta(t)

4)
X(w)=Y(w)^2
Y(w)=1/(a+jw)
y(t)=e^-at u(t)
x(t)=y(t)*y(t)=int_-inf^inf y(T)y(t-T) dT
x(t)=-ln(x/20)