Exam 3

● Graded

Student

Hayden Fuller

Total Points

34 / 60 pts

Question 1

Problem 1 **18** / 25 pts

1.1 **Skill 3g** **4** / 5 pts

- ✔ **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- **+ 1 pt** include unit
- ✔ **+ 0 pts** no unit included or incorrect units

1.2 **Skill 3i** **4** / 5 pts

- ✔ **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- **+ 1 pt** Units
- **+ 0 pts** No units

Needs more explanation

1.3 **Skill 3j** **3** / 5 pts

- **+ 4 pts** Complete mastery w/ calculations
- ✔ **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- **+ 1 pt** Correct units
- ✔ **+ 0 pts** Incorrect or incomplete units

Did not calculate inductance

1.4 **Skill 3k** **4 / 5 pts**

+ **4 pts** Complete mastery w/ calculations

✔ + **3 pts** Mastery w/ math errors

+ **2 pts** Partial mastery

+ **1 pt** Limited mastery

+ **0 pts** No mastery

✔ + **1 pt** include unit

+ **0 pts** no unit included or incorrect units

incomplete definition of force direction

1.5 **Skill 3f** **3 / 5 pts**

+ **4 pts** Complete mastery w/ calculations

+ **3 pts** Mastery w/ math errors

✔ + **2 pts** Partial mastery

+ **1 pt** Limited mastery

+ **0 pts** No mastery

✔ + **1 pt** Correct units

Question 2

Problem 2 **9** / 10 pts

2.1 **Skill 3d** **5** / 5 pts

- ✔ **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- ✔ **+ 1 pt** Correct units

2.2 **Skill 3l** **4** / 5 pts

- **+ 4 pts** Complete mastery w/ calculations
- ✔ **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** No mastery
- ✔ **+ 1 pt** Correct units
- **+ 0 pts** Incorrect units

Question 3

Problem 3 — **5** / 20 pts

3.1 **Skill 3a** — **0** / 5 pts

- **+ 4 pts** Complete mastery w/ calculations
- **+ 3 pts** Mastery w/ math errors
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- ✔ **+ 0 pts** No mastery
- **+ 1 pt** include unit

3.2 **Skill 3b** — **5** / 5 pts

- ✔ **+ 4 pts** Complete mastery
- **+ 3 pts** Mastery
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- **+ 0 pts** Blank / no mastery
- ✔ **+ 1 pt** Unit

3.3 **Skill 3c** — **0** / 5 pts

- **+ 4 pts** Complete mastery
- **+ 3 pts** Mastery
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- ✔ **+ 0 pts** Blank / no mastery
- **+ 1 pt** Direction

3.4 **Skill 3e** — **0** / 5 pts

- **+ 4 pts** Complete mastery
- **+ 3 pts** Mastery
- **+ 2 pts** Partial mastery
- **+ 1 pt** Limited mastery
- ✔ **+ 0 pts** Blank
- **+ 1 pt** Unit

Question 4

Problem 4 **2** / 5 pts

4.1 **Skill 3h** **2** / 5 pts

+ 0 pts Blank

✔ **+ 1 pt** Limited mastery

+ 2 pts Partial mastery

+ 3 pts Mastery

+ 4 pts Complete mastery

✔ **+ 1 pt** Unit

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Instructions

1.) Unless otherwise specified, you have one class period to complete the questions below.

2.) Read all directions carefully.

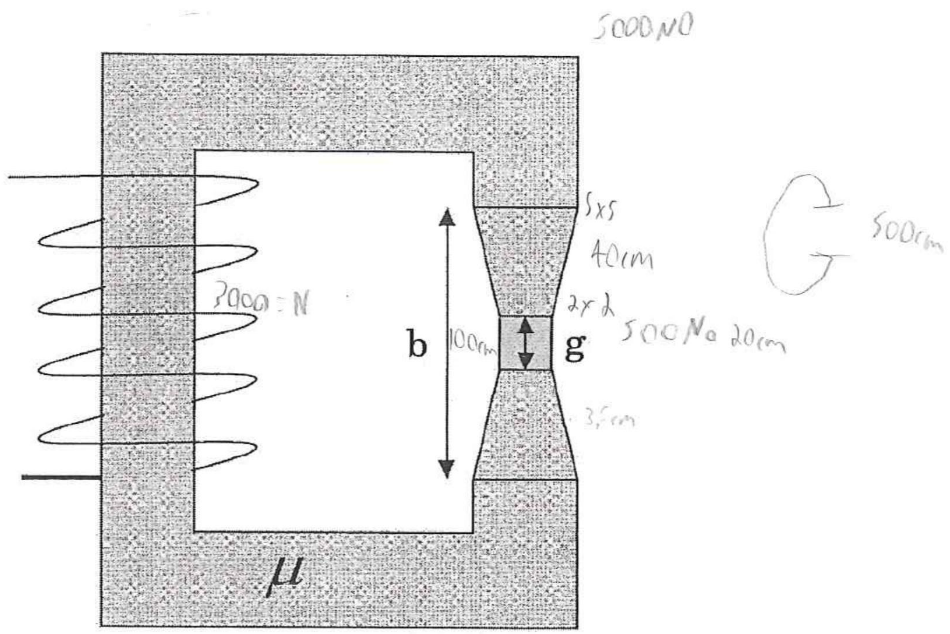
3.) Show your work in enough detail to allow the graders to completely follow your thought process.

4.) Make sure your calculator is set to perform trigonometric functions in radians & not degrees & use at least 2 significant digits.

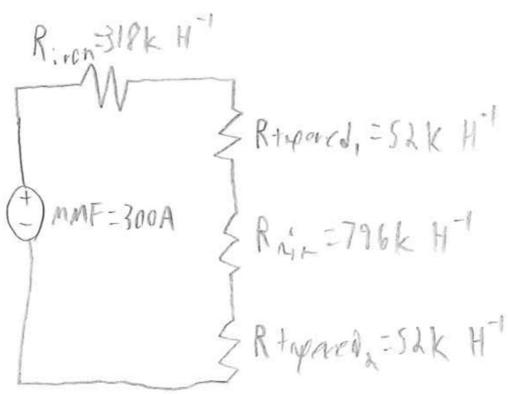
5.) Make sure to write your answers legibly. You can write on the back of the exam pages or ask for scratch paper

1. Magnetic Circuit

Consider the iron structure below. It is energized by a coil with 3000 turns. The permeability of the core is $5000\mu_0$. The perimeter of the core loop (measuring through its middle and not including the gap) is 500cm. The core's cross section is square and 5cm on each side. The region b is 100cm long and the gap region g is 20cm long, filled with a material with permeability $500\mu_0$. The two tapered pieces within the region are 5cm x 5cm on the wide end and 2cm x 2cm on the narrow end.



a.) Draw a magnetic circuit to represent the structure when the current through the coil is 100mA. Calculate and label the MMF (magnetomotive force) and show the reluctance of 4 different regions: the main iron core region, the two tapered regions, and the air gap. Also calculate the total magnetic flux through the core. (The tapered regions also have a square cross section. You may approximate them by using their average of 3.5cm as the diameter they have for their entire length.) $R = \frac{L}{\mu A}$



$R_{iron} = 5\,m / (.05 \cdot .05\, m^2 \cdot 5000 \cdot 1.256\times10^{-6}\, \frac{H}{m}) = 318{,}471\ H^{-1}$

$R_{tapered_1} = .4\,m / (.035 \cdot .035\, m^2 \cdot 5000 \cdot 1.256\times10^{-6}\, \frac{H}{m}) = 51{,}995\ H^{-1}$

$R_{air} = .2\,m / (.02 \cdot .02\, m^2 \cdot 500 \cdot 1.256\times10^{-7}\, \frac{H}{m}) = 796{,}178\ H^{-1}$

$R_{tapered_2} = R_{tapered_1} = 51995\ H^{-1}$

$MMF = NI = 3000 \cdot 100\,mA = 300\,A$

$R = 1218640\ H^{-1} = 1.218\ Meg\ H^{-1}$

b.) Calculate the flux linkage, and total inductance of the coil and magnetic circuit it is attached to. Also calculate the B-field at any point inside the circuit.

$\Lambda = N\Psi = 3000\,\Psi$

$MMF = \Psi R$

$\Psi = \frac{MMF}{R} = \frac{300A}{1.218\,Meg\,H^{-1}} = .000246 = 246\ \frac{\mu A}{H} = 246\ \mu T m^2$

c.) What is the energy density in the air gap? Are there net forces at the surfaces where the tapered cores meet the air gap? If so, what are their magnitudes and directions?

$w_m = \frac{1}{2}\frac{|B|^2}{\mu}$ $\quad \Psi = B \cdot A \quad B = \Psi/A = 246\mu A/H / (.02^2) = 0.61544\ T$

$w_m = \frac{1}{2}\frac{0.61544^2\ T^2}{500 \cdot 1.256\times10^{-6}\frac{H}{m}} = 301.56\ \frac{T^2 m}{H}$

yes, $F = w_e \times A = 301.56\ \frac{T^2 m}{H} \cdot (0.02^2)\ m^2 = 0.1206\ \frac{T^2 m^3}{H} = 0.1206\,N$

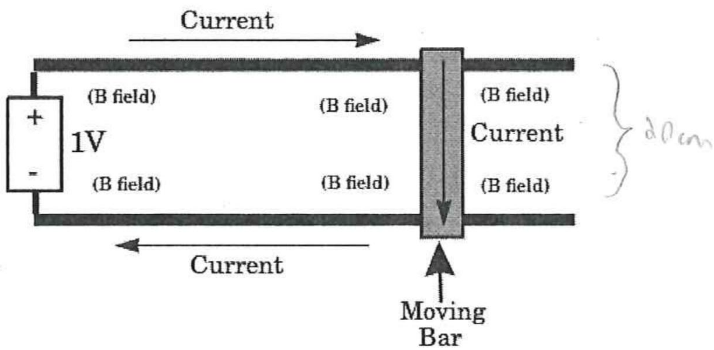
normal to the cross sectional area,

d.) If the current through the energizing coil is driven high enough that the core material starts to saturate, what effect will this have on the inductance?

decreased inductance

2. Rail gun

The figure below depicts a rail gun. A 1V voltage source is applied to the rails. The circuit is completed by a 20cm long metal bar which can also slide on the rails. A uniform magnetic field is present in the vicinity of the rail gun. Assume that all the current from the top rail passes into the bar rather than continuing on the top rail.



a.) You can apply a magnetic field of 0.2 T either up towards the sky or down towards the ground. This field will be uniform at any point that is between the two rails. Which direction do you apply the field in order to move the bar away from the voltage source?

Up

b.) Suppose that the rails have a resistance of 0.1 ohms per meter and the bar has negligible resistance. What will the force on the bar be (Give magnitude as well as direction.)

$I = \frac{V}{R} = \frac{1}{0.2x} = 5/x\ A$ $A = 0.2x\ m^2$ $B = 0.2T$ $\Psi = 0.2 \cdot 0.2x = 0.04x$

0.2N right

c.) As the bar begins to move to the right, the size of the loop that the current encloses will get larger, enclosing more magnetic flux. Therefore as long as the bar is moving, the loop's flux is growing with respect to time. This changing flux will also produce an electromotive force (or emf). Calculate how many volts this electromotive force will be as a function of the bar's velocity. Will the emf increase or decrease the force pushing the bar to the right? Does the bar have a top speed?

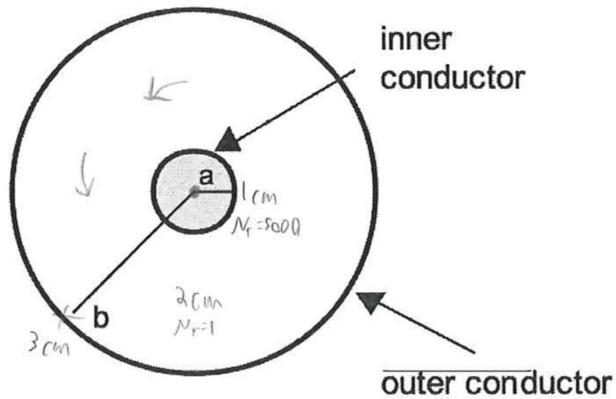
$$\psi = \int B \, ds = 0.04x$$

$$\frac{d\psi}{dt} = 0.04 \frac{dx}{dt}$$

decrease the force pushing the bar, yes, there is a top speed where this EMF cancels out the applied EMF, leading to no current, no force, no acceleration, and a constant velocity

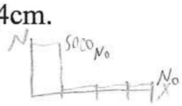
3. Coaxial Cable

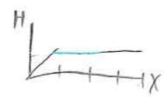
A coaxial cable has a cylindrical inner conductor with radius 1cm. This conductor is made of iron and has a relative permeability μ_r=5000. The outer conductor is a thin shell (treated as having no thickness for our purposes.) Between the two conductors is a 2cm thick dielectric layer that has the same permeability as free space.

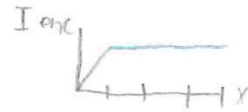


a.) A 50mA DC current flows in the inner conductor in the +z direction (out of the page). The current density is uniform inside the inner conductor. On the outer conductor, 50mA flows in the opposite direction (-z) as a uniform surface charge. Calculate the magnitude and direction of the H-field and B-field for 0cm≤r≤4cm.

$B = \mu H$







counter clockwise

$\int H \, dl = I_{enc}$

b.) Write the expression for magnetic vector potential for the interval 1cm≤ r≤ 3cm (the region between the conductors). If your expression contains a constant, set the constant to zero. c.) Determine the cable's inductance per unit length.

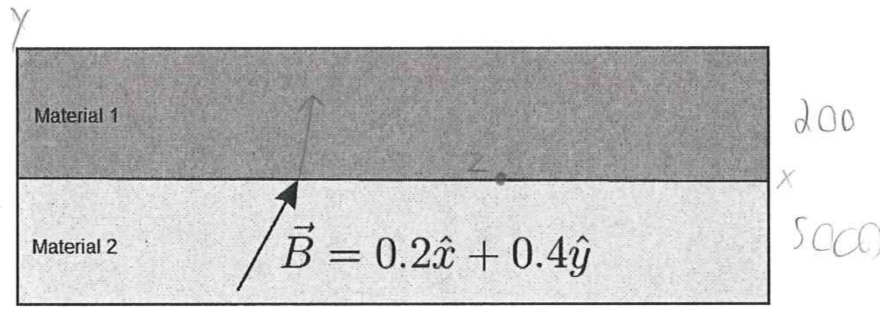
$L = \frac{\Lambda}{I}$

4. Magnetic Boundary

$B_{n1} = B_{n2}$

$H_{t1} - H_{t2} = J_s$

$H_{t1} = J_s + H_{t2}$



In the image above, a magnetic field is shown in the lower material. Material 2 has a relative permeability μr=5000 while Material 1 has μ_r=200.

a.) Calculate the x and y components of the field in Material 1. The x direction is parallel to the material boundary and the y direction is normal to it.

x: $0.2\hat{x}$

y: $0.4\hat{y} \frac{5000}{200} = 10\hat{y}$

b.) Suppose that a current begins to flow on the surface between the two materials. The current points in the +z direction (out of the page). The B-field in Material 2 stays the same. Will this affect the x component of the B-field in Material 1, the y component, neither, or both?

It will affect the y component