

Homework # 2**Due: Monday, June 12th**

Question 1. (30 points) Show that only 1-phase circuits have the double frequency component of power but not the balanced 3-phase circuits. Include detailed steps. Also implement this in Matlab environment and plot the relevant power components (include Matlab code as a part of answers).

Question 2. (30 points) A 440-V, 3-phase voltage feeds a balanced delta-connected 3-phase load with an impedance of $(25 + j10) \Omega/\text{phase}$

- (a) Calculate the line current and represent in phasor form
- (b) Calculate the power absorbed per phase and
- (c) Calculate the sum of all phase currents. Explain why this sum has such a value

Question 3. (40 points) A three-phase, star connected load with a resistance of 30Ω and a reactance of 25Ω per phase, is fed by a 230 V, three-phase, 60 Hz source. Calculate

- (a) all line currents
- (b) power consumed and
- (c) power factor.

If a delta connected, balanced 3-phase capacitor bank, is added to the same supply in parallel to the given load, with the 15 micro Farad capacitance/phase, then calculate the new overall power factor of the circuit.

1)

Single phase:

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$$P(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

since $\cos^2(x) = \frac{1}{2}[1 + \cos(2x)]$, this shows that $P(t)$ will have twice the frequency of $v(t)$ and $i(t)$

this is because we have positive power twice per period, one with positive voltage, and one with negative voltage.

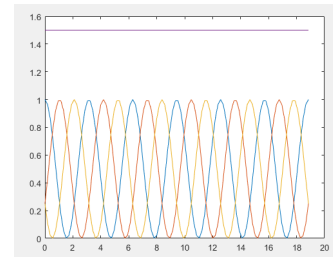
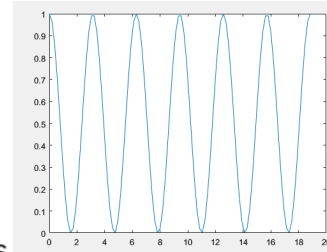
Three phase:

$$P(t) = v_a i_a + v_b i_b + v_c i_c$$

$$P(t) = V_m I_m \left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i + 0) + \cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 240) + \cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i + 240) \right]$$

$$P(t) = 3 V_m I_m \cos(\theta_v - \theta_i)$$

all the individual powers have twice the frequency still, but when you sum equally spaced cos's together, they all cancel out, giving us constant power.



2)

$$V_a = 440 \angle 0^\circ$$

$$V_b = 440 \angle -120^\circ$$

$$V_c = 440 \angle -240^\circ$$

a)

$$I_{ph-a} = V_{line} / Z_{ph} = (440 + j0) / (25 + j10) = 15.17 - j6.07 \text{ A}$$

$$I_{ph-a} = 16.34 \angle -21.79^\circ$$

$$I_{ph-b} = 16.34 \angle 98.21^\circ$$

$$I_{ph-c} = 16.34 \angle 218.21^\circ$$

$$I_{line} = I_{ph-a} - I_{ph-b} = 16.34 \angle -21.79^\circ - 16.34 \angle 98.21^\circ$$

$$I_{line} = 28.3 \angle -51.8^\circ$$

b)

$$S = V I^* = 440 (16.34 \angle 21.79^\circ) = 7190 \text{ VA}$$

$$P = 440 \cdot 16.34 \cos(21.79^\circ) = 7190 \text{ W}$$

c)

$$I = M(\cos(t) + \cos(t-120) + \cos(t+120)) = 0$$

Zero. Same as mentioned at the end of the previous question. Since it's a balanced system, they all cancel out, and are always at a net 0.

3)

$$V_{ph} = 230 / \sqrt{3} = 132.79 \text{ V}$$

$$Z_{ph} = 30 + j25 = 39 \angle 39.8^\circ \text{ ohms}$$

a)

$$I_{ph} = V_{ph} / Z_{ph} = 132.79 / (39 \angle 39.8^\circ) = 3.36 \angle -39.8^\circ \text{ A}$$

b)

$$\text{power consumed} = \sqrt{3} V_L I_L \cos(\phi) = 1028.3 \text{ W}$$

$$\text{reactive power} = \sqrt{3} V_L I_L \sin(\phi) = 856.8 \text{ VAR}$$

c)

$$\text{power factor} = \cos(39.8^\circ) = 0.77 \text{ lagging}$$

c.2)

$$3 V_L I_{ph} \sin(\phi) = 897 \text{ VAR}$$

close to 856.8, this will mostly cancel it out

$$856.8 - 1028.3 \tan(\phi_2) = 897$$

$$\tan(\phi_2) = (-40.3) / 1028 = -0.039$$

$$\cos(\phi_2) = 0.999$$

new power factor = 0.999, much better