

CSCI-2200 FOCS F 2022 Crib Sheet Exam 1 Tuesday, October 5, 2022 Hayden Fuller Notes:

NEGATION: $a \vee b \Rightarrow \neg a \wedge \neg b$; $a \wedge b \Rightarrow \neg a \vee \neg b$; if a , then $b = a \rightarrow b \Rightarrow a \wedge \neg b$

$\forall x, A(x) \Rightarrow \exists x : \neg A(x)$; $\exists x : A(x) \Rightarrow \forall x \neg A(x)$

$\forall x : (\exists y : 2x - y = 0) = T(\text{already know } x)$; $\exists y : (\forall x : 2x - y = 0) = F(\text{can't predict } x)$;

$\exists y : (\forall x : xy = 0) = T(x \text{ doesn't matter, } y=0)$

statement: $p \rightarrow q$ converse: $q \rightarrow p$ inverse: $\neg p \rightarrow \neg q$ contrapositive: $\neg q \rightarrow \neg p$

\cup union; \cap intersection; \subset proper subset; \subseteq subset(can be equal);

What type of proof is appropriate? Contradiction:

There is a prime number greater than ab

$2^{\frac{1}{p}}$ is irrational for any integer $p > 2$, given: $a, b, c, n \in \mathbb{N}, n > 2, a^n + b^n \neq c^n$

Contraposition:

Direct:

There is an even number greater than ab (either $ab + 1$ or $ab + 2$ is even)

if n and q are natural numbers, then there exists unique integers d and r satisfying $n = dq + r$, with $d \in 0 \cup \mathbb{N}$ and $0 \leq r < q$.

Leaping induction:

Induction:

$(a + b)^n \geq a^n + b^n$, when n is a natural number.

$n^2 \leq 2^n$ for all $n \geq 4$

5 divides $11^n - 6$ for all $n \geq 5$; show 5 divides $11^5 - 6$ and show for $n \geq 5$, if 5 divides $11^n - 6$, then 5 divides $11^{n+1} - 6$

Weak induction:

$11^n - 6$ is divisible by 5 if n is a natural number.