

$$\mathbf{A} = A + A^x \mathbf{e}_x + A^y \mathbf{e}_y + A^{xy} \mathbf{e}_x \wedge \mathbf{e}_y$$

$$\mathbf{B} = B + B^x \mathbf{e}_x + B^y \mathbf{e}_y + B^{xy} \mathbf{e}_x \wedge \mathbf{e}_y$$

$$\begin{aligned} \mathbf{AB} = & AB + A^x B^x - A^{xy} B^{xy} + A^y B^y \\ & + (AB^x + A^x B + A^{xy} B^y - A^y B^{xy}) \mathbf{e}_x \\ & + (AB^y + A^x B^{xy} - A^{xy} B^x + A^y B) \mathbf{e}_y \\ & + (AB^{xy} + A^x B^y + A^{xy} B - A^y B^x) \mathbf{e}_x \wedge \mathbf{e}_y \end{aligned}$$

$$\dot{\nabla} \mathbf{A} = (A^x + A \mathbf{e}_x + A^{xy} \mathbf{e}_y + A^y \mathbf{e}_x \wedge \mathbf{e}_y) \frac{\partial}{\partial x} + (A^y - A^{xy} \mathbf{e}_x + A \mathbf{e}_y - A^x \mathbf{e}_x \wedge \mathbf{e}_y) \frac{\partial}{\partial y}$$

$$\mathbf{A} \dot{\nabla} = (A^x + A \mathbf{e}_x - A^{xy} \mathbf{e}_y - A^y \mathbf{e}_x \wedge \mathbf{e}_y) \frac{\partial}{\partial x} + (A^y + A^{xy} \mathbf{e}_x + A \mathbf{e}_y + A^x \mathbf{e}_x \wedge \mathbf{e}_y) \frac{\partial}{\partial y}$$

$$\nabla (\mathbf{AB}) - (\nabla \mathbf{A}) \mathbf{B} - \left(\dot{\nabla} \mathbf{A} \right) \dot{\mathbf{B}} = A \partial_x B^x - A \partial_x B^x - A \partial_y B^y + A \partial_y B^y + A^x \partial_x B - A^x \partial_x B - A^x \partial_y B^{xy} + A^x \partial_y B^{xy} + A^{xy} \partial_y B^x - A^{xy} \partial_y B^x + A^{xy} \partial_x B^y - A^{xy} \partial_x B^y + A^y \partial_y B - A^y \partial_y B - A^y \partial_x B^{xy} + A^y \partial_x B^{xy} + (A \partial_x B - A \partial_x B + A \partial_y B^{xy} - A \partial_y B^{xy} + A^x \partial_x B^x - A^x \partial_x B^x + A^x \partial_y B^y - A^x \partial_y B^y - A^{xy} \partial_y B + A^{xy} \partial_y B - A^{xy} \partial_x B^{xy} + A^{xy} \partial_x B^{xy} + A^y \partial_y B^x - A^y \partial_y B^x + A^y \partial_x B^y - A^y \partial_x B^y) \mathbf{e}_x + (A \partial_y B - A \partial_y B + A \partial_x B^{xy} - A \partial_x B^{xy} + A^x \partial_y B^x - A^x \partial_y B^x + A^x \partial_x B^y - A^x \partial_x B^y + A^x \partial_y B^{xy} - A^x \partial_y B^{xy} + A^y \partial_y B^x - A^y \partial_y B^x + A^y \partial_x B^y - A^y \partial_x B^y) \mathbf{e}_y + (A \partial_y B - A \partial_y B + A \partial_x B^{xy} - A \partial_x B^{xy} + A^x \partial_y B^x - A^x \partial_y B^x + A^x \partial_x B^y - A^x \partial_x B^y + A^x \partial_y B^{xy} - A^x \partial_y B^{xy} + A^y \partial_y B^x - A^y \partial_y B^x + A^y \partial_x B^y - A^y \partial_x B^y) \mathbf{e}_x \wedge \mathbf{e}_y$$

$$\left(\dot{\mathbf{A}} \dot{\mathbf{B}} \right) \dot{\nabla} - \dot{\mathbf{A}} \left(\dot{\mathbf{B}} \dot{\nabla} \right) - \dot{\mathbf{A}} \left(\dot{\mathbf{B}} \dot{\nabla} \right) = A \partial_x B^x + A \partial_y B^y - A \partial_x B^x - A \partial_y B^y + A^x \partial_x B + A^x \partial_y B^{xy} - A^x \partial_x B - A^x \partial_y B^{xy} - A^{xy} \partial_y B^x + A^{xy} \partial_x B^y + A^{xy} \partial_y B^x - A^{xy} \partial_x B^y - A^y \partial_y B + A^y \partial_x B^{xy} + A^y \partial_y B - A^y \partial_x B^{xy} + B \partial_x A^x + B \partial_y A^y - B \partial_x A^x - B \partial_y A^y + B^x \partial_x A - B^x \partial_y A^{xy} - B^x \partial_x A + B^x \partial_y A^{xy} + B^{xy} \partial_y A^x - B^{xy} \partial_x A^y - B^{xy} \partial_x A^y - B^{xy} \partial_y A^x + B^{xy} \partial_x A^y + B^y \partial_y A + B^y \partial_x A^{xy} - B^y \partial_y A - B^y \partial_x A^{xy} + (A \partial_x B + A \partial_y B^{xy} - A \partial_x B - A \partial_y B^{xy} + A^x \partial_x B^x + A^x \partial_y B^y - A^x \partial_x B^x - A^x \partial_y B^y + A^{xy} \partial_y B^x - A^{xy} \partial_y B^x + A^{xy} \partial_x B^y - A^{xy} \partial_x B^y) \mathbf{e}_x + (A \partial_y B + A \partial_x B^{xy} - A \partial_y B - A \partial_x B^{xy} + A^x \partial_y B^x + A^x \partial_x B^y - A^x \partial_y B^x - A^x \partial_x B^y + A^x \partial_y B^{xy} - A^x \partial_y B^{xy} + A^y \partial_y B^x + A^y \partial_x B^y - A^y \partial_y B^x - A^y \partial_x B^y) \mathbf{e}_y + (A \partial_y B + A \partial_x B^{xy} - A \partial_y B - A \partial_x B^{xy} + A^x \partial_y B^x + A^x \partial_x B^y - A^x \partial_y B^x - A^x \partial_x B^y + A^x \partial_y B^{xy} - A^x \partial_y B^{xy} + A^y \partial_y B^x + A^y \partial_x B^y - A^y \partial_y B^x - A^y \partial_x B^y) \mathbf{e}_x \wedge \mathbf{e}_y$$

$$\mathbf{A} \wedge \mathbf{B} = AB + (AB^x + A^x B) \mathbf{e}_x + (AB^y + A^y B) \mathbf{e}_y + (AB^{xy} + A^x B^y + A^{xy} B - A^y B^x) \mathbf{e}_x \wedge \mathbf{e}_y$$

$$\nabla \wedge (\mathbf{A} \wedge \mathbf{B}) - (\nabla \wedge \mathbf{A}) \wedge \mathbf{B} - \left(\dot{\nabla} \wedge \mathbf{A} \right) \wedge \dot{\mathbf{B}} = -(-A \partial_x B + A \partial_x B) \mathbf{e}_x - (A \partial_y B - A \partial_y B) \mathbf{e}_y + (A \partial_y B^x - A \partial_y B^x + A \partial_x B^y - A \partial_x B^y - A^x \partial_y B + A^x \partial_y B + A^y \partial_x B - A^y \partial_x B) \mathbf{e}_x \wedge \mathbf{e}_y$$

$$\left(\mathbf{A} \dot{\wedge} \mathbf{B} \right) \wedge \dot{\nabla} - \mathbf{A} \wedge \left(\dot{\mathbf{B}} \wedge \dot{\nabla} \right) - \dot{\mathbf{A}} \wedge \left(\mathbf{B} \wedge \dot{\nabla} \right) = -(A \partial_x B - A \partial_x B + B \partial_x A - B \partial_x A) \mathbf{e}_x - (A \partial_y B - A \partial_y B + B \partial_y A - B \partial_y A) \mathbf{e}_y - (A \partial_y B^x - A \partial_x B^y - A \partial_y B^x + A \partial_x B^y + A^x \partial_y B - A^x \partial_y B - A^y \partial_x B + A^y \partial_x B - B \partial_y A^x + B \partial_x A^y + B \partial_y A^x - B \partial_x A^y - B^x \partial_y A + B^x \partial_y A + B^y \partial_x A - B^y \partial_x A) \mathbf{e}_x \wedge \mathbf{e}_y$$