3D Orthogonal Metric
Multvectors:

$$s = s$$

$$v = v^{x}\boldsymbol{e_{x}} + v^{y}\boldsymbol{e_{y}} + v^{z}\boldsymbol{e_{z}}$$

$$b = b^{xy}\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}} + b^{xz}\boldsymbol{e_{x}}\wedge \boldsymbol{e_{z}} + b^{yz}\boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

Products:

$$ss = s^{2}$$

$$s\wedge s = s^{2}$$

$$s\cdot s = 0$$

$$s\rfloor s = s^{2}$$

$$s\lfloor s = s^{2}$$

$$sv = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$s\wedge v = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$s\cdot v = 0$$

$$s\rfloor v = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$s\lfloor v = 0$$

$$sb = b^{xy}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

$$s\wedge b = b^{xy}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

$$s\cdot b = 0$$

$$s\rfloor b = b^{xy}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}}\wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}}\wedge \boldsymbol{e_{z}}$$

$$s\lfloor b = 0$$

$$vs = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$v\wedge s = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$v\cdot s = 0$$

$$v\rfloor s = 0$$

$$v\lfloor s = sv^{x}\boldsymbol{e_{x}} + sv^{y}\boldsymbol{e_{y}} + sv^{z}\boldsymbol{e_{z}}$$

$$vv = \left(v^{x}\right)^{2} + \left(v^{y}\right)^{2} + \left(v^{z}\right)^{2}$$

$$v\wedge v = 0$$

$$v\cdot v = \left(v^{x}\right)^{2} + \left(v^{y}\right)^{2} + \left(v^{z}\right)^{2}$$

$$v \rfloor v = (v^{x})^{2} + (v^{y})^{2} + (v^{z})^{2}$$

$$v \lfloor v = (v^{x})^{2} + (v^{y})^{2} + (v^{z})^{2}$$

$$vb = \left (- b^{xy} v^{y} - b^{xz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{x}} + \left (b^{xy} v^{x} - b^{yz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{y}} + \left (b^{xz} v^{x} + b^{yz} v^{y}\right) \boldsymbol{e}_{\boldsymbol{z}} + \left (b^{xy} v^{z} - b^{xz} v^{y} + b^{yz} v^{x}\right) \boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{y}}\wedge \boldsymbol{e}_{\boldsymbol{z}}$$

$$v \wedge b = \left (b^{xy} v^{z} - b^{xz} v^{y} + b^{yz} v^{x}\right) \boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{y}}\wedge \boldsymbol{e}_{\boldsymbol{z}}$$

$$v \cdot b = \left (- b^{xy} v^{y} - b^{xz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{x}} + \left (b^{xy} v^{x} - b^{yz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{y}} + \left (b^{xz} v^{x} + b^{yz} v^{y}\right) \boldsymbol{e}_{\boldsymbol{z}}$$

$$v \rfloor b = \left (- b^{xy} v^{y} - b^{xz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{x}} + \left (b^{xy} v^{x} - b^{yz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{y}} + \left (b^{xz} v^{x} + b^{yz} v^{y}\right) \boldsymbol{e}_{\boldsymbol{z}}$$

$$v \lfloor b = 0$$

$$bs = b^{xy} s\boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{y}} + b^{xz} s\boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{z}} + b^{yz} s\boldsymbol{e}_{\boldsymbol{y}}\wedge \boldsymbol{e}_{\boldsymbol{z}}$$

$$b \wedge s = b^{xy} s\boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{y}} + b^{xz} s\boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{z}} + b^{yz} s\boldsymbol{e}_{\boldsymbol{y}}\wedge \boldsymbol{e}_{\boldsymbol{z}}$$

$$b \cdot s = 0$$

$$b \rfloor s = 0$$

$$b \lfloor s = b^{xy} s\boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{y}} + b^{xz} s\boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{z}} + b^{yz} s\boldsymbol{e}_{\boldsymbol{y}}\wedge \boldsymbol{e}_{\boldsymbol{z}}$$

$$bv = \left (b^{xy} v^{y} + b^{xz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{x}} + \left (- b^{xy} v^{x} + b^{yz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{y}} + \left (- b^{xz} v^{x} - b^{yz} v^{y}\right) \boldsymbol{e}_{\boldsymbol{z}} + \left (b^{xy} v^{z} - b^{xz} v^{y} + b^{yz} v^{x}\right) \boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{y}}\wedge \boldsymbol{e}_{\boldsymbol{z}}$$

$$b \wedge v = \left (b^{xy} v^{z} - b^{xz} v^{y} + b^{yz} v^{x}\right) \boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{y}}\wedge \boldsymbol{e}_{\boldsymbol{z}}$$

$$b \cdot v = \left (b^{xy} v^{y} + b^{xz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{x}} + \left (- b^{xy} v^{x} + b^{yz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{y}} + \left (- b^{xz} v^{x} - b^{yz} v^{y}\right) \boldsymbol{e}_{\boldsymbol{z}}$$

$$b \rfloor v = 0$$

$$b \lfloor v = \left (b^{xy} v^{y} + b^{xz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{x}} + \left (- b^{xy} v^{x} + b^{yz} v^{z}\right) \boldsymbol{e}_{\boldsymbol{y}} + \left (- b^{xz} v^{x} - b^{yz} v^{y}\right) \boldsymbol{e}_{\boldsymbol{z}}$$

$$bb = - (b^{xy})^{2} - (b^{xz})^{2} - (b^{yz})^{2}$$

$$b \wedge b = 0$$

$$b \cdot b = - (b^{xy})^{2} - (b^{xz})^{2} - (b^{yz})^{2}$$

$$b \rfloor b = - (b^{xy})^{2} - (b^{xz})^{2} - (b^{yz})^{2}$$

$$b \lfloor b = - (b^{xy})^{2} - (b^{xz})^{2} - (b^{yz})^{2}$$

Multivector Functions:

$$s(X) = s$$

$$v(X) = v^{x} \boldsymbol{e}_{\boldsymbol{x}} + v^{y} \boldsymbol{e}_{\boldsymbol{y}} + v^{z} \boldsymbol{e}_{\boldsymbol{z}}$$

$$b(X) = b^{xy} \boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{y}} + b^{xz} \boldsymbol{e}_{\boldsymbol{x}}\wedge \boldsymbol{e}_{\boldsymbol{z}} + b^{yz} \boldsymbol{e}_{\boldsymbol{y}}\wedge \boldsymbol{e}_{\boldsymbol{z}}$$

Products:

$$\boldsymbol{\nabla} s = \partial_{x} s\boldsymbol{e}_{\boldsymbol{x}} + \partial_{y} s\boldsymbol{e}_{\boldsymbol{y}} + \partial_{z} s\boldsymbol{e}_{\boldsymbol{z}}$$

$$\boldsymbol{\nabla} \wedge s = \partial_{x} s\boldsymbol{e}_{\boldsymbol{x}} + \partial_{y} s\boldsymbol{e}_{\boldsymbol{y}} + \partial_{z} s\boldsymbol{e}_{\boldsymbol{z}}$$

$$\boldsymbol{\nabla} \cdot s = 0$$

$$\boldsymbol{\nabla} \rfloor s = 0$$

$$\boldsymbol{\nabla} \lfloor s = \partial_x s \boldsymbol{e_x} + \partial_y s \boldsymbol{e_y} + \partial_z s \boldsymbol{e_z}$$

$$\boldsymbol{\nabla} v = \partial_x v^x + \partial_y v^y + \partial_z v^z + \left(-\partial_y v^x + \partial_x v^y\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} + \left(-\partial_z v^x + \partial_x v^z\right) \boldsymbol{e_x} \wedge \boldsymbol{e_z} + \left(-\partial_z v^y + \partial_y v^z\right) \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$\boldsymbol{\nabla} \wedge v = \left(-\partial_y v^x + \partial_x v^y\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} + \left(-\partial_z v^x + \partial_x v^z\right) \boldsymbol{e_x} \wedge \boldsymbol{e_z} + \left(-\partial_z v^y + \partial_y v^z\right) \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$\boldsymbol{\nabla} \cdot v = \partial_x v^x + \partial_y v^y + \partial_z v^z$$

$$\boldsymbol{\nabla} \rfloor v = \partial_x v^x + \partial_y v^y + \partial_z v^z$$

$$\boldsymbol{\nabla} \lfloor v = \partial_x v^x + \partial_y v^y + \partial_z v^z$$

$$\boldsymbol{\nabla} b = \left(-\partial_y b^{xy} - \partial_z b^{xz}\right) \boldsymbol{e_x} + \left(\partial_x b^{xy} - \partial_z b^{yz}\right) \boldsymbol{e_y} + \left(\partial_x b^{xz} + \partial_y b^{yz}\right) \boldsymbol{e_z} + \left(\partial_z b^{xy} - \partial_y b^{xz} + \partial_x b^{yz}\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$\boldsymbol{\nabla} \wedge b = \left(\partial_z b^{xy} - \partial_y b^{xz} + \partial_x b^{yz}\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$\boldsymbol{\nabla} \cdot b = \left(-\partial_y b^{xy} - \partial_z b^{xz}\right) \boldsymbol{e_x} + \left(\partial_x b^{xy} - \partial_z b^{yz}\right) \boldsymbol{e_y} + \left(\partial_x b^{xz} + \partial_y b^{yz}\right) \boldsymbol{e_z}$$

$$\boldsymbol{\nabla} \rfloor b = \left(-\partial_y b^{xy} - \partial_z b^{xz}\right) \boldsymbol{e_x} + \left(\partial_x b^{xy} - \partial_z b^{yz}\right) \boldsymbol{e_y} + \left(\partial_x b^{xz} + \partial_y b^{yz}\right) \boldsymbol{e_z}$$

$$\boldsymbol{\nabla} \lfloor b = 0$$

$$s \boldsymbol{\nabla} = \partial_x s \boldsymbol{e_x} + \partial_y s \boldsymbol{e_y} + \partial_z s \boldsymbol{e_z}$$

$$s \wedge \boldsymbol{\nabla} = \partial_x s \boldsymbol{e_x} + \partial_y s \boldsymbol{e_y} + \partial_z s \boldsymbol{e_z}$$

$$s \cdot \boldsymbol{\nabla} = 0$$

$$s \rfloor \boldsymbol{\nabla} = \partial_x s \boldsymbol{e_x} + \partial_y s \boldsymbol{e_y} + \partial_z s \boldsymbol{e_z}$$

$$s \lfloor \boldsymbol{\nabla} = 0$$

$$ss = s^2$$

$$s \wedge s = s^2$$

$$s \cdot s = 0$$

$$s \rfloor s = s^2$$

$$s \lfloor s = s^2$$

$$sv = sv^x \boldsymbol{e_x} + sv^y \boldsymbol{e_y} + sv^z \boldsymbol{e_z}$$

$$s \wedge v = sv^x \boldsymbol{e_x} + sv^y \boldsymbol{e_y} + sv^z \boldsymbol{e_z}$$

$$s \cdot v = 0$$

$$s \rfloor v = sv^x \boldsymbol{e_x} + sv^y \boldsymbol{e_y} + sv^z \boldsymbol{e_z}$$

$$s \lfloor v = 0$$

$$sb = b^{xy} s \boldsymbol{e_x} \wedge \boldsymbol{e_y} + b^{xz} s \boldsymbol{e_x} \wedge \boldsymbol{e_z} + b^{yz} s \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$s \wedge b = b^{xy} s \boldsymbol{e_x} \wedge \boldsymbol{e_y} + b^{xz} s \boldsymbol{e_x} \wedge \boldsymbol{e_z} + b^{yz} s \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$s \cdot b = 0$$

$$s \rfloor b = b^{xy} s \boldsymbol{e_x} \wedge \boldsymbol{e_y} + b^{xz} s \boldsymbol{e_x} \wedge \boldsymbol{e_z} + b^{yz} s \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$s \lfloor b = 0$$

$$v\boldsymbol{\nabla} = \partial_x v^x + \partial_y v^y + \partial_z v^z + \left(\partial_y v^x - \partial_x v^y\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} + \left(\partial_z v^x - \partial_x v^z\right) \boldsymbol{e_x} \wedge \boldsymbol{e_z} + \left(\partial_z v^y - \partial_y v^z\right) \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$v \wedge \boldsymbol{\nabla} = \left(\partial_y v^x - \partial_x v^y\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} + \left(\partial_z v^x - \partial_x v^z\right) \boldsymbol{e_x} \wedge \boldsymbol{e_z} + \left(\partial_z v^y - \partial_y v^z\right) \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$v \cdot \boldsymbol{\nabla} = \partial_x v^x + \partial_y v^y + \partial_z v^z$$

$$v \rfloor \boldsymbol{\nabla} = \partial_x v^x + \partial_y v^y + \partial_z v^z$$

$$v \lfloor \boldsymbol{\nabla} = \partial_x v^x + \partial_y v^y + \partial_z v^z$$

$$vs = s v^x \boldsymbol{e_x} + s v^y \boldsymbol{e_y} + s v^z \boldsymbol{e_z}$$

$$v \wedge s = s v^x \boldsymbol{e_x} + s v^y \boldsymbol{e_y} + s v^z \boldsymbol{e_z}$$

$$v \cdot s = 0$$

$$v \rfloor s = 0$$

$$v \lfloor s = s v^x \boldsymbol{e_x} + s v^y \boldsymbol{e_y} + s v^z \boldsymbol{e_z}$$

$$vv = \left(v^x\right)^2 + \left(v^y\right)^2 + \left(v^z\right)^2$$

$$v \wedge v = 0$$

$$v \cdot v = \left(v^x\right)^2 + \left(v^y\right)^2 + \left(v^z\right)^2$$

$$v \rfloor v = \left(v^x\right)^2 + \left(v^y\right)^2 + \left(v^z\right)^2$$

$$v \lfloor v = \left(v^x\right)^2 + \left(v^y\right)^2 + \left(v^z\right)^2$$

$$vb = \left(-b^{xy} v^y - b^{xz} v^z\right) \boldsymbol{e_x} + \left(b^{xy} v^x - b^{yz} v^z\right) \boldsymbol{e_y} + \left(b^{xz} v^x + b^{yz} v^y\right) \boldsymbol{e_z} + \left(b^{xy} v^z - b^{xz} v^y + b^{yz} v^x\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$v \wedge b = \left(b^{xy} v^z - b^{xz} v^y + b^{yz} v^x\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$v \cdot b = \left(-b^{xy} v^y - b^{xz} v^z\right) \boldsymbol{e_x} + \left(b^{xy} v^x - b^{yz} v^z\right) \boldsymbol{e_y} + \left(b^{xz} v^x + b^{yz} v^y\right) \boldsymbol{e_z}$$

$$v \rfloor b = \left(-b^{xy} v^y - b^{xz} v^z\right) \boldsymbol{e_x} + \left(b^{xy} v^x - b^{yz} v^z\right) \boldsymbol{e_y} + \left(b^{xz} v^x + b^{yz} v^y\right) \boldsymbol{e_z}$$

$$v \lfloor b = 0$$

$$b\boldsymbol{\nabla} = \left(\partial_y b^{xy} + \partial_z b^{xz}\right) \boldsymbol{e_x} + \left(-\partial_x b^{xy} + \partial_z b^{yz}\right) \boldsymbol{e_y} + \left(-\partial_x b^{xz} - \partial_y b^{yz}\right) \boldsymbol{e_z} + \left(\partial_z b^{xy} - \partial_y b^{xz} + \partial_x b^{yz}\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$b \wedge \boldsymbol{\nabla} = \left(\partial_z b^{xy} - \partial_y b^{xz} + \partial_x b^{yz}\right) \boldsymbol{e_x} \wedge \boldsymbol{e_y} \wedge \boldsymbol{e_z}$$

$$b \cdot \boldsymbol{\nabla} = \left(\partial_y b^{xy} + \partial_z b^{xz}\right) \boldsymbol{e_x} + \left(-\partial_x b^{xy} + \partial_z b^{yz}\right) \boldsymbol{e_y} + \left(-\partial_x b^{xz} - \partial_y b^{yz}\right) \boldsymbol{e_z}$$

$$b \rfloor \boldsymbol{\nabla} = 0$$

$$b \lfloor \boldsymbol{\nabla} = \left(\partial_y b^{xy} + \partial_z b^{xz}\right) \boldsymbol{e_x} + \left(-\partial_x b^{xy} + \partial_z b^{yz}\right) \boldsymbol{e_y} + \left(-\partial_x b^{xz} - \partial_y b^{yz}\right) \boldsymbol{e_z}$$

$$bs = b^{xy}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$b \wedge s = b^{xy}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$b \cdot s = 0$$

$$b \rfloor s = 0$$

$$b \lfloor s = b^{xy}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} + b^{xz}s\boldsymbol{e_{x}} \wedge \boldsymbol{e_{z}} + b^{yz}s\boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$bv = \left(b^{xy}v^{y} + b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(-b^{xy}v^{x} + b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(-b^{xz}v^{x} - b^{yz}v^{y}\right)\boldsymbol{e_{z}} + \left(b^{xy}v^{z} - b^{xz}v^{y} + b^{yz}v^{x}\right)\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$b \wedge v = \left(b^{xy}v^{z} - b^{xz}v^{y} + b^{yz}v^{x}\right)\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}} \wedge \boldsymbol{e_{z}}$$

$$b \cdot v = \left(b^{xy}v^{y} + b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(-b^{xy}v^{x} + b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(-b^{xz}v^{x} - b^{yz}v^{y}\right)\boldsymbol{e_{z}}$$

$$b \rfloor v = 0$$

$$b \lfloor v = \left(b^{xy}v^{y} + b^{xz}v^{z}\right)\boldsymbol{e_{x}} + \left(-b^{xy}v^{x} + b^{yz}v^{z}\right)\boldsymbol{e_{y}} + \left(-b^{xz}v^{x} - b^{yz}v^{y}\right)\boldsymbol{e_{z}}$$

$$bb = -\left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

$$b \wedge b = 0$$

$$b \cdot b = -\left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

$$b \rfloor b = -\left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

$$b \lfloor b = -\left(b^{xy}\right)^{2} - \left(b^{xz}\right)^{2} - \left(b^{yz}\right)^{2}$$

General 2D Metric
Multivector Functions:

$$s(X) = s$$

$$v(X) = v^{x}\boldsymbol{e_{x}} + v^{y}\boldsymbol{e_{y}}$$

$$b(X) = v^{xy}\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}}$$

Products:

$$\boldsymbol{\nabla} s = \left(-\left(e_{x} \cdot e_{y}\right)\partial_{y} s + \left(e_{y} \cdot e_{y}\right)\partial_{x} s\right)\boldsymbol{e_{x}} + \left(\left(e_{x} \cdot e_{x}\right)\partial_{y} s - \left(e_{x} \cdot e_{y}\right)\partial_{x} s\right)\boldsymbol{e_{y}}$$

$$\boldsymbol{\nabla} \wedge s = \left(-\left(e_{x} \cdot e_{y}\right)\partial_{y} s + \left(e_{y} \cdot e_{y}\right)\partial_{x} s\right)\boldsymbol{e_{x}} + \left(\left(e_{x} \cdot e_{x}\right)\partial_{y} s - \left(e_{x} \cdot e_{y}\right)\partial_{x} s\right)\boldsymbol{e_{y}}$$

$$\boldsymbol{\nabla} \cdot s = 0$$

$$\boldsymbol{\nabla} \rfloor s = 0$$

$$\boldsymbol{\nabla} \lfloor s = \left(-\left(e_{x} \cdot e_{y}\right)\partial_{y} s + \left(e_{y} \cdot e_{y}\right)\partial_{x} s\right)\boldsymbol{e_{x}} + \left(\left(e_{x} \cdot e_{x}\right)\partial_{y} s - \left(e_{x} \cdot e_{y}\right)\partial_{x} s\right)\boldsymbol{e_{y}}$$

$$\boldsymbol{\nabla} v = \left(e_{x} \cdot e_{x}\right)\left(e_{y} \cdot e_{y}\right)\partial_{x} v^{x} + \left(e_{x} \cdot e_{x}\right)\left(e_{y} \cdot e_{y}\right)\partial_{y} v^{y} - \left(e_{x} \cdot e_{y}\right)^{2}\partial_{x} v^{x} - \left(e_{x} \cdot e_{y}\right)^{2}\partial_{y} v^{y} + \left(-\left(e_{x} \cdot e_{x}\right)\partial_{y} v^{x} + \left(e_{x} \cdot e_{y}\right)\right.$$

$$\boldsymbol{\nabla} \wedge v = \left(-\left(e_{x} \cdot e_{x}\right)\partial_{y} v^{x} + \left(e_{x} \cdot e_{y}\right)\partial_{x} v^{x} - \left(e_{x} \cdot e_{y}\right)\partial_{y} v^{y} + \left(e_{y} \cdot e_{y}\right)\partial_{x} v^{y}\right)\boldsymbol{e_{x}} \wedge \boldsymbol{e_{y}}$$

$$\boldsymbol{\nabla} \cdot v = \left(e_{x} \cdot e_{x}\right)\left(e_{y} \cdot e_{y}\right)\partial_{x} v^{x} + \left(e_{x} \cdot e_{x}\right)\left(e_{y} \cdot e_{y}\right)\partial_{y} v^{y} - \left(e_{x} \cdot e_{y}\right)^{2}\partial_{x} v^{x} - \left(e_{x} \cdot e_{y}\right)^{2}\partial_{y} v^{y}$$

$$\boldsymbol{\nabla} \rfloor v = \left(e_{x} \cdot e_{x}\right)\left(e_{y} \cdot e_{y}\right)\partial_{x} v^{x} + \left(e_{x} \cdot e_{x}\right)\left(e_{y} \cdot e_{y}\right)\partial_{y} v^{y} - \left(e_{x} \cdot e_{y}\right)^{2}\partial_{x} v^{x} - \left(e_{x} \cdot e_{y}\right)^{2}\partial_{y} v^{y}$$

$$\boldsymbol{\nabla} \lfloor v = \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_x v^x + \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_y v^y - \left(e_x \cdot e_y\right)^2 \partial_x v^x - \left(e_x \cdot e_y\right)^2 \partial_y v^y$$

$$s\boldsymbol{\nabla} = \left(-\left(e_x \cdot e_y\right)\partial_y s + \left(e_y \cdot e_y\right)\partial_x s\right)\boldsymbol{e_x} + \left(\left(e_x \cdot e_x\right)\partial_y s - \left(e_x \cdot e_y\right)\partial_x s\right)\boldsymbol{e_y}$$

$$s \wedge \boldsymbol{\nabla} = \left(-\left(e_x \cdot e_y\right)\partial_y s + \left(e_y \cdot e_y\right)\partial_x s\right)\boldsymbol{e_x} + \left(\left(e_x \cdot e_x\right)\partial_y s - \left(e_x \cdot e_y\right)\partial_x s\right)\boldsymbol{e_y}$$

$$s \cdot \boldsymbol{\nabla} = 0$$

$$s \rfloor \boldsymbol{\nabla} = \left(-\left(e_x \cdot e_y\right)\partial_y s + \left(e_y \cdot e_y\right)\partial_x s\right)\boldsymbol{e_x} + \left(\left(e_x \cdot e_x\right)\partial_y s - \left(e_x \cdot e_y\right)\partial_x s\right)\boldsymbol{e_y}$$

$$s \lfloor \boldsymbol{\nabla} = 0$$

$$ss = s^2$$

$$s \wedge s = s^2$$

$$s \cdot s = 0$$

$$s \rfloor s = s^2$$

$$s \lfloor s = s^2$$

$$sv = sv^x \boldsymbol{e_x} + sv^y \boldsymbol{e_y}$$

$$s \wedge v = sv^x \boldsymbol{e_x} + sv^y \boldsymbol{e_y}$$

$$s \cdot v = 0$$

$$s \rfloor v = sv^x \boldsymbol{e_x} + sv^y \boldsymbol{e_y}$$

$$s \lfloor v = 0$$

$$v\boldsymbol{\nabla} = \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_x v^x + \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_y v^y - \left(e_x \cdot e_y\right)^2 \partial_x v^x - \left(e_x \cdot e_y\right)^2 \partial_y v^y + \left(\left(e_x \cdot e_x\right)\partial_y v^x - \left(e_x \cdot e_y\right)\partial_x$$

$$v \wedge \boldsymbol{\nabla} = \left(\left(e_x \cdot e_x\right)\partial_y v^x - \left(e_x \cdot e_y\right)\partial_x v^x + \left(e_x \cdot e_y\right)\partial_y v^y - \left(e_y \cdot e_y\right)\partial_x v^y\right)\boldsymbol{e_x} \wedge \boldsymbol{e_y}$$

$$v \cdot \boldsymbol{\nabla} = \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_x v^x + \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_y v^y - \left(e_x \cdot e_y\right)^2 \partial_x v^x - \left(e_x \cdot e_y\right)^2 \partial_y v^y$$

$$v \rfloor \boldsymbol{\nabla} = \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_x v^x + \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_y v^y - \left(e_x \cdot e_y\right)^2 \partial_x v^x - \left(e_x \cdot e_y\right)^2 \partial_y v^y$$

$$v \lfloor \boldsymbol{\nabla} = \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_x v^x + \left(e_x \cdot e_x\right)\left(e_y \cdot e_y\right)\partial_y v^y - \left(e_x \cdot e_y\right)^2 \partial_x v^x - \left(e_x \cdot e_y\right)^2 \partial_y v^y$$

$$vs = sv^x \boldsymbol{e_x} + sv^y \boldsymbol{e_y}$$

$$v \wedge s = sv^x \boldsymbol{e_x} + sv^y \boldsymbol{e_y}$$

$$v \cdot s = 0$$

$$v \rfloor s = 0$$

$$v \lfloor s = sv^x \boldsymbol{e_x} + sv^y \boldsymbol{e_y}$$

$$vv = \left(e_x \cdot e_x\right)\left(v^x\right)^2 + 2\left(e_x \cdot e_y\right)v^x v^y + \left(e_y \cdot e_y\right)\left(v^y\right)^2$$

$$v \wedge v = 0$$

$$v \cdot v = \left(e_x \cdot e_x\right)\left(v^x\right)^2 + 2\left(e_x \cdot e_y\right)v^x v^y + \left(e_y \cdot e_y\right)\left(v^y\right)^2$$

$$v \rfloor v = \left(e_x \cdot e_x\right)\left(v^x\right)^2 + 2\left(e_x \cdot e_y\right)v^x v^y + \left(e_y \cdot e_y\right)\left(v^y\right)^2$$

$$v \lfloor v = \left(e_x \cdot e_x\right)\left(v^x\right)^2 + 2\left(e_x \cdot e_y\right)v^x v^y + \left(e_y \cdot e_y\right)\left(v^y\right)^2$$