Vector Manifold

$$\boldsymbol{X} = u\cos(v)\boldsymbol{e}_{\boldsymbol{x}} + u\sin(v)\boldsymbol{e}_{\boldsymbol{y}} + (u\cos(v) + w)\,\boldsymbol{e}_{\boldsymbol{z}}$$

Basis Vectors

$$\boldsymbol{e}_{u} = \cos(v)\boldsymbol{e}_{\boldsymbol{x}} + \sin(v)\boldsymbol{e}_{\boldsymbol{y}} + \cos(v)\boldsymbol{e}_{\boldsymbol{z}}$$

$$\boldsymbol{e}_{v} = -u\sin(v)\boldsymbol{e}_{\boldsymbol{x}} + u\cos(v)\boldsymbol{e}_{\boldsymbol{y}} - u\sin(v)\boldsymbol{e}_{\boldsymbol{z}}$$

$$\boldsymbol{e}_{w} = \boldsymbol{e}_{\boldsymbol{z}}$$

Reciprocal Basis Vectors

$$\boldsymbol{e}^{u} = \cos(v)\boldsymbol{e}_{\boldsymbol{x}} + \sin(v)\boldsymbol{e}_{\boldsymbol{y}}$$

$$\boldsymbol{e}^{v} = -\frac{\sin(v)}{u}\boldsymbol{e}_{\boldsymbol{x}} + \frac{\cos(v)}{u}\boldsymbol{e}_{\boldsymbol{y}}$$

$$\boldsymbol{e}^{w} = -\boldsymbol{e}_{\boldsymbol{x}} + \boldsymbol{e}_{\boldsymbol{z}}$$

Dot Products

$$\boldsymbol{e}_{u} \cdot \boldsymbol{e}^{u} = 1$$

$$\boldsymbol{e}_{u} \cdot \boldsymbol{e}^{v} = 0$$

$$\boldsymbol{e}_{u} \cdot \boldsymbol{e}^{w} = 0$$

$$\boldsymbol{e}_{v} \cdot \boldsymbol{e}^{u} = 0$$

$$\boldsymbol{e}_{v} \cdot \boldsymbol{e}^{v} = 1$$

$$\boldsymbol{e}_{v} \cdot \boldsymbol{e}^{w} = 0$$

$$\boldsymbol{e}_{w} \cdot \boldsymbol{e}^{u} = 0$$

$$\boldsymbol{e}_{w} \cdot \boldsymbol{e}^{v} = 0$$

$$\boldsymbol{e}_{w} \cdot \boldsymbol{e}^{w} = 1$$