$$Manifold = u\boldsymbol{e_x} + v\boldsymbol{e_y} + \left(u^2 + v^2\right)\boldsymbol{e_z}$$

$$\begin{aligned} dg = &\left(\frac{-4u^2 v\log(u) + 4v^3 + 4v^2 + v + 1}{u\left(4u^2 + 4v^2 + 1\right)}\right)\boldsymbol{e_x} \\ &+ \left(\frac{4u^2\log(u) - 4v^2 - 4v + \log(u)}{4u^2 + 4v^2 + 1}\right)\boldsymbol{e_y} \\ &+ \left(2\frac{v\log(u) + v + 1}{4u^2 + 4v^2 + 1}\right)\boldsymbol{e_z} \end{aligned}$$

$$dg(1,0) = \frac{1}{5}\boldsymbol{e_x} + \frac{2}{5}\boldsymbol{e_z}$$

$$G = v^2\boldsymbol{e_x} + u^2\boldsymbol{e_y} + \left(2uv\left(u+v\right)\right)\boldsymbol{e_z}$$

$$\begin{aligned} dG = &4\frac{uv\left(u+v\right)}{4u^2 + 4v^2 + 1} \\ &+ \left(2\frac{-4u^2 v + 4uv^2 + u - v}{4u^2 + 4v^2 + 1}\right)\boldsymbol{e_x}\wedge\boldsymbol{e_y} \\ &+ \left(2\frac{v\left(-4u^3 - 8u^2 v + 8uv^2 + 2u + 4v^3 - v\right)}{4u^2 + 4v^2 + 1}\right)\boldsymbol{e_x}\wedge\boldsymbol{e_z} \\ &+ \left(2\frac{u\left(4u^3 + 8u^2 v - 8uv^2 - u - 4v^3 + 2v\right)}{4u^2 + 4v^2 + 1}\right)\boldsymbol{e_y}\wedge\boldsymbol{e_z} \end{aligned}$$

$$dG(1,0) = \frac{2}{5}\boldsymbol{e_x}\wedge\boldsymbol{e_y} + \frac{6}{5}\boldsymbol{e_y}\wedge\boldsymbol{e_z}$$