# Limits for Stable Near-Circular Planetary or Satellite Orbits in the Restricted Three-Body Problem

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**In the vicinity of two massive bodies rotating in circles around their common center of mass there are regions in which near-circular orbits of a third body are stable, and other regions in which near-circular orbits are unstable. Expressions for calculating the limiting radii of the regions where stable near-circular orbits can exist are developed from Jacobi's integral to the equations of motion of a particle, and computed quantitative values for these radii are given in graphical and tabular form for mass ratios from $10^{-8}$ to 0.5. An example is given of the application of these limits to an idealized Earth-moon system (mass ratio, 0.012128563).**

A FUNDAMENTAL set of limits in the restricted three-body problem[2] may be established by exploring the question: In what regions of space in the vicinity of two massive bodies can a third body of infinitesimal mass remain in a stable near-circular orbit?

Since the computation of these limits is time consuming, and since the limits may be applied as approximations to a number of real problems in astronomy and orbit mechanics, we hope the results given here will be of general usefulness and interest.

The data, presented in tabular and graphical form, are solutions to a series of problems which are stated as follows. Determine the distance at which particles in circular orbits around either mass or around both masses together can exist, without exceeding the energy levels corresponding to certain critical values of the Jacobian constant $C$.

In the restricted three-body problem (reference coordinates rotating at the same angular velocity as the two massive bodies) it is well known that a set of zero relative velocity surfaces may be described around each of the individual bodies and around both bodies together. An example of the intersection of these surfaces with the $xy$ plane is given in Fig. 1. Sketches of the approximate shapes of these surfaces in the $xz$ plane and the $yz$ plane are given by Buchheim (1)[3] and Moulton (2). These zero velocity surfaces are approximately spheres in the immediate vicinity of each of the masses, and approximately cylinders enclosing both masses at a distance from the masses, and are associated with high numerical values of the Jacobian constant. As the value of $C$ is gradually reduced, the closed "spherical" surfaces around each of the massive bodies gradually expand, and the "cylindrical" surfaces around both bodies gradually contract. As $C$ is reduced to a certain value $C_1$ the expanding spheres around the two massive bodies impinge at a point $p_1$, the first Lagrangian libration point. As $C$ is reduced further, the closed contours around the two bodies coalesce into a closed shape resembling a budded yeast cell, while the cylindrical curtains continue to contract. At $C = C_2$, the expanding "yeast cell" and the contracting curtain impinge at $p_2$, the second Lagrangian point. Further reduction of $C$ results in coalescence and formation of a single surface. At $C = C_3$ another impingement of surfaces takes place at the third Lagrangian point $p_3$, and at $C = C_4$ the surfaces shrink down to two points and disappear at the fourth and fifth Lagrangian points $p_4$ and $p_5$, points in the $xy$ plane which form equilateral triangles with the two massive bodies.

These zero velocity surfaces have the following significance for the present problem: A real third body has associated with it a specific and conserved value of the Jacobian constant $C_n$ determined by its position and velocity. It cannot cross a surface of zero relative velocity corresponding to $C_n$, but is forever constrained to remain in regions of space where the zero velocity surfaces have a numerical value $C \geq C_n$. For example, if a particle has associated with it a value of $C = C_1$,

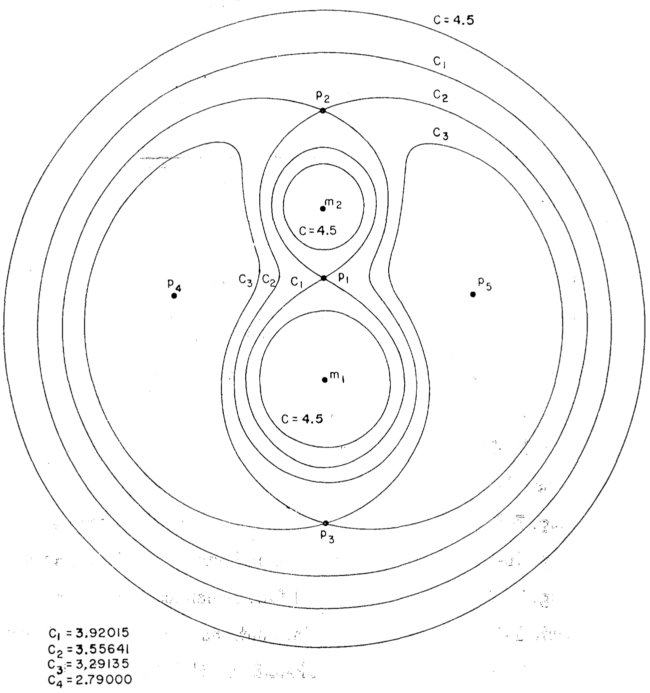


**Fig. 1 Zero velocity curves in $xy$ plane for mass ratio $\mu$ equal to 0.3**

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[2] Two massive bodies rotate in circles around their common center of mass. The third body, of infinitesimal mass, moves in the same plane as do the massive bodies.

[3] Numbers in parentheses indicate References at end of paper.

and is initially within a closed envelope surrounding the larger of the two massive bodies, it cannot escape from the region around $m_1$ delimited by the $C_1$ contour. If it is initially outside the outer $C_1$ contour, it cannot penetrate inside the space enclosed by this contour. On the other hand, if a particle has associated with it a value of $C \leq C_4$, it can potentially invade all parts of the space in the $xy$ plane, no region being excluded.

Now let us consider a particle within the closed $C_1$ contour around $m_1$ which has associated within it $C = C_1$. Since it cannot escape from this region, it will either orbit endlessly or eventually impact on $m_1$. Qualitatively the most stable orbit it can have will be a near-circular orbit around $m_1$. Even more stable would be near-circular orbits of particles having $C > C_1$.

If we now depart somewhat from the restricted three-body problem and assume that, as in the real world, small perturbing influences are present which can effectively increase or decrease the value of $C$, or change the velocity vector by a small amount, it may be seen that a high degree of stability is required in order to enable the third body to survive for long periods of time. Thus if a third body has a value of $C < C_1$, even if it is initially in a near-circular orbit around $m_1$, one would have doubts as to its long term stability, since it can potentially enter the regions surrounding both $m_1$ and $m_2$, and the probabilities of eventual collision with one of these bodies are much increased. Unfortunately, methods for assessing the long term stability of orbits in multi-body systems are not available. It may be said qualitatively, however, that near-circular orbits of particles with $C = C_1$ represent the outermost stable orbits which can exist in the closed space about $m_1$; this applies, also, for near-circular orbits around $m_2$. For particles in near-circular orbits around both bodies the associated value of $C$ must be related to the outer $C_2$ zero velocity curve such that $C \geq C_2$.

A graphical representation of the relationship between the Jacobian constant and particles moving at orbital velocity is given in Fig. 2. For an idealized Earth-moon case, values of $C$ were computed for particles moving at orbital velocity in the $xy$ plane and in a direction normal to the line joining the two masses. As may be seen from this figure, retrograde satellites of Earth have $C$ values greater than $C_1$, and may be regarded as stable out to a distance of 0.239 lunar units[4] (ca. 57,000 miles), but less than $C_1$ beyond this. Similarly, direct satellites of Earth are stable between Earth's surface and 0.635 lunar units (152,000 miles) and beyond 1.6056 lunar units (384,000 miles), but not between 0.635 and 1.6056 lunar units.

Direct satellites of the moon on near-circular orbits are stable out to 0.073 units (17,400 miles from the moon's center), whereas retrograde satellites are stable out to 0.043 units (10.300 miles from the center of the moon).

Using similar criteria it is possible to establish regions in which particles on near-circular orbits can exist in the neighborhood of two massive bodies as a function of the mass ratio $\mu$.

The expressions for computing the dimensions of these regions are developed from Jacobi's integral in the latter part of this paper.

For the example illustrated in Fig. 3 (where $\mu = 0.1$), stable direct near-circular orbits around $m_1$ can exist only inside region $A$, radius 0.44 units; stable retrograde near-circular orbits around $m_1$ can exist only inside region $B$, radius 0.21; stable direct near-circular orbits around $m_2$ can exist only inside region $D$, radius 0.14; stable retrograde near-circular orbits around $m_2$ can exist only inside region $E$, radius 0.086 units; stable direct near-circular orbits around both masses can exist only outside of region $F$, radius 2.24 units; apparently none of the near-circular retrograde orbits around both masses are stable.

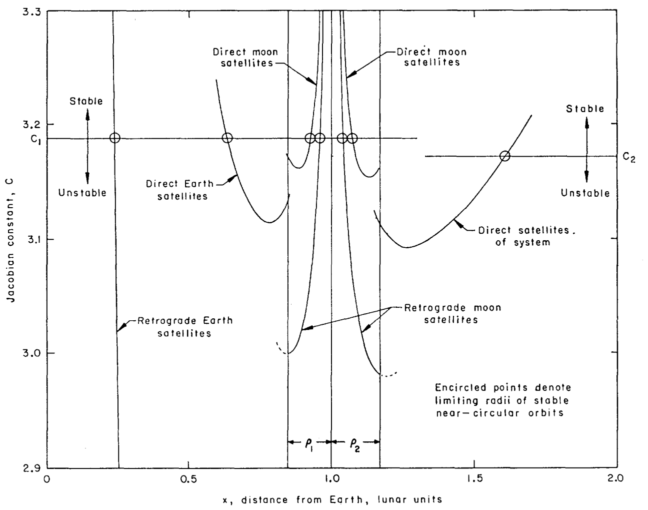


**Fig. 2 Values of Jacobian constant $C$ associated with particles crossing the $x$-axis perpendicular with orbital velocity, as a function of distance along $x$-axis. Mass ratio $\mu$ equal to 0.012128563**

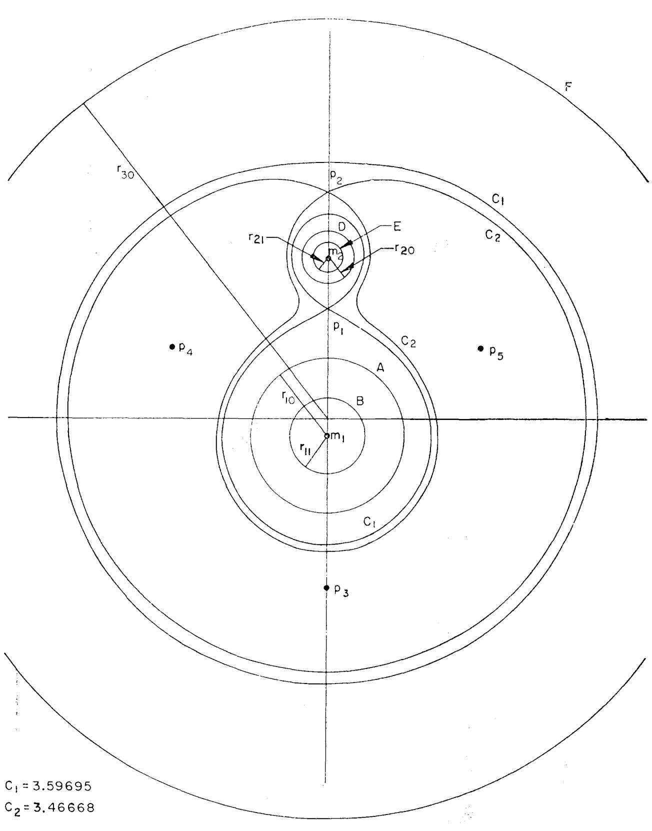


**Fig. 3 Zero velocity curves in $xy$ plane for mass ratio $\mu$ equal to 0.1. Extreme limits for stable near-circular satellite orbits indicated**

The regions inside the contours $A$, $B$, $D$ and $E$ and outside contour $F$ might be called "permitted regions" for direct and retrograde inferior planets, for direct and retrograde satellites, and for direct superior planets, respectively. All regions outside of those enclosed by $B$ and $E$ might be called "forbidden regions" for retrograde planets and satellites, whereas regions outside of $A$ and $D$ but inside $F$ might be called forbidden regions for direct planets and satellites. If we con-

[4] A lunar unit is taken to be 239,000 miles. Numerical agreement with 0.239 is coincidence.

**Table 1 Location of libration points and radii of limiting regions for various mass ratios**

| $\mu$ | $10^{-8}$ | $10^{-6}$ | $10^{-4}$ | $10^{-3}$ | 0.01 | 1/82.45[a] | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\rho_1$ | $1.493 \times 10^{-3}$ | $6.918 \times 10^{-3}$ | 0.03183 | 0.06771 | 0.14192 | 0.15085 | 0.29096 | 0.36192 | 0.41387 | 0.45838 | 0.50000 |
| $\rho_2$ | $1.495 \times 10^{-3}$ | $6.950 \times 10^{-3}$ | 0.03253 | 0.07092 | 0.15677 | 0.16773 | 0.35970 | 0.47105 | 0.55673 | 0.63081 | 0.69841 |
| $\rho_3$ | $5.833 \times 10^{-9}$ | $5.833 \times 10^{-7}$ | $5.833 \times 10^{-5}$ | $5.833 \times 10^{-4}$ | $5.833 \times 10^{-3}$ | $7.075 \times 10^{-3}$ | 0.05839 | 0.11716 | 0.17679 | 0.23795 | 0.30159 |
| $C_1 - 3$[b] | $2.009 \times 10^{-5}$ | $4.294 \times 10^{-4}$ | $8.989 \times 10^{-3}$ | 0.03995 | 0.16754 | 0.18814 | 0.59695 | ... | 0.92015 | ... | 1.00000 |
| $C_2 - 3$[b] | $2.005 \times 10^{-5}$ | $4.280 \times 10^{-4}$ | $8.856 \times 10^{-3}$ | 0.03861 | 0.15422 | 0.17199 | 0.46668 | 0.55239 | 0.55641 | 0.48196 | 0.45680 |
| $C_3 - 3$[b] | $1.000 \times 10^{-8}$ | $1.000 \times 10^{-6}$ | $1.000 \times 10^{-4}$ | $1.000 \times 10^{-3}$ | $9.998 \times 10^{-3}$ | 0.01213 | 0.09958 | 0.19732 | 0.29135 | 0.37908 | 0.45680 |
| $3 - C_4$[b] | $1.000 \times 10^{-8}$ | $1.000 \times 10^{-6}$ | $9.999 \times 10^{-5}$ | $9.990 \times 10^{-4}$ | $9.900 \times 10^{-3}$ | 0.00198 | 0.09000 | 0.16000 | 0.21000 | 0.24000 | 0.25000 |
| 1 $(D)(r_{20})$ | $7.311 \times 10^{-4}$ | $3.410 \times 10^{-3}$ | 0.01565 | 0.03316 | 0.06910 | 0.073[c] | 0.14140 | ... | 0.20564 | ... | 0.25725 |
| 2 | $7.327 \times 10^{-4}$ | $3.407 \times 10^{-3}$ | 0.01559 | 0.03292 | 0.06822 | ... | 0.13869 | ... | 0.20110 | ... | 0.25115 |
| 3 $(E)(r_{21})$ | $4.187 \times 10^{-4}$ | $1.946 \times 10^{-3}$ | $8.991 \times 10^{-3}$ | 0.01924 | 0.04091 | 0.043[c] | 0.08635 | ... | 0.12537 | ... | 0.15267 |
| 4 | $4.188 \times 10^{-4}$ | $1.946 \times 10^{-3}$ | $8.988 \times 10^{-3}$ | 0.01923 | 0.04087 | ... | 0.08618 | ... | 0.12510 | ... | 0.15235 |
| 5 $(A)(r_{10})$ | 0.99543 | 0.97903 | 0.90729 | 0.81396 | 0.65436 | 0.625[c] | 0.43514 | ... | 0.31878 | ... | 0.25725 |
| 6 | 0.99484 | 0.97643 | 0.89764 | 0.79882 | 0.63693 | ... | 0.42296 | ... | 0.31065 | ... | 0.25115 |
| 7 $(B)(r_{11})$ | 0.25000 | 0.24998 | 0.24949 | 0.24772 | 0.24022 | 0.239[c] | 0.21182 | ... | 0.17883 | ... | 0.15267 |
| 8 | 0.25000 | 0.24998 | 0.24949 | 0.24772 | 0.24019 | ... | 0.21164 | ... | 0.17853 | ... | 0.15235 |
| 9 $(F)(r_{30})$ | 1.00459 | 1.02153 | 1.10448 | 1.23847 | 1.56291 | 1.60556 | 2.24305 | 2.41239 | 2.40770 | 2.22160 | 2.16517 |
| 10 | 1.00519 | 1.02420 | 1.11534 | 1.25773 | 1.59084 | 1.63396 | 2.27921 | 2.44086 | 2.43289 | 2.24289 | 2.16517 |

[a] Mass ratio for idealized Earth-moon system, $\mu = 0.012128563$.
[b] Values for $C_1 - 3$, $C_2 - 3$, $C_3 - 3$ and $3 - C_4$ are tabulated, since $C_1$, $C_2$, $C_3$ and $C_4$ all approach 3 very closely for small values of $\mu$.
[c] Approximate values (interpolated).

sider only direct planets of $m_1$, the larger of the two massive bodies, it may be seen that $m_2$ has created a broad band around $m_1$ in which no planets on near-circular orbits could be expected to exist. Such planets would be expected only inside contour $A$ and outside contour $F$.

Dimensions of permitted regions in the restricted three-body problem for the range of $\mu$ from $10^{-8}$ to 0.5 are given in Table 1, and some illustrations drawn to scale of these regions for various mass ratios are given in Fig. 4.

## Analysis

Jacobi's integral to the equations of motion of a particle in the vicinity of two massive bodies that are rotating in circles around their common center of mass may be written as given by Buchheim (1)

$$v^2 = \omega^2(x^2 + y^2) + 2K(1 - \mu)/r_1 + 2K\mu/r_2 - c$$

Fig. 5 illustrates the nomenclature used in this paper.

If we normalize by letting the distance between the two masses equal unity, and the sum of the masses equal unity, for circular motion of the two massive bodies

$$\omega^2 = K$$

by setting $c/\omega^2 = C$ and $v/\omega = V$, the integral may be rewritten

$$C = x^2 + y^2 + 2(1 - \mu)/r_1 + 2\mu/r_2 - V^2$$

Now consider a particle in circular orbit around the smaller of the two masses in the $xy$ plane and with motion in the direct sense ($CCW$). At time $t = 0$, it is assumed to be on the $x$-axis ($y = 0$, $\dot{x} = 0$) between the two masses, and its velocity in intertial space is

$$-\sqrt{K\mu/r_2} = -\omega\sqrt{\mu/r_2}$$

However its velocity in rotating space

$$v = -\omega\sqrt{\mu/r_2} + \omega r_2$$

and

$$V^2 = \mu/r_2 - 2\sqrt{r_2\mu} + r_2^2$$

also

$$x = 1 - \mu - r_2$$

and

$$r_1 = 1 - r_2$$

and Jacobi's integral becomes

$$C = (1 - \mu - r_2)^2 + 2(1 - \mu)/(1 - r_2) + \mu/r_2 + 2\sqrt{r_2\mu} - r_2^2$$

If $C$ is set equal to $C_1$ and the equation solved for $r_2$, we obtain the maximum distance ($r_{20}$) from the smaller mass $\mu$ that a particle can be moving $CCW$ at orbital velocity, without having so much energy that it could potentially (if perturbed slightly) cross the $C_1$ zero relative velocity surface and escape from the neighborhood of the smaller mass.

Direct, between

$$C_1 = (1 - \mu - r_{20})^2 + \frac{2(1 - \mu)}{1 - r_{20}} + \frac{\mu}{r_{20}} + 2\sqrt{r_{20}\mu} - r_{20}^2 \quad [1]$$

A similar expression which considers a particle in direct motion around $\mu$, but on a point on the $x$-axis on the side away from the larger mass, yields closely similar results.

Direct, beyond

$$C_1 = (1 - \mu + r_{20})^2 + \frac{2(1 - \mu)}{1 + r_{20}} + \frac{\mu}{r_{20}} + 2\sqrt{r_{20}\mu} - r_{20}^2 \quad [2]$$

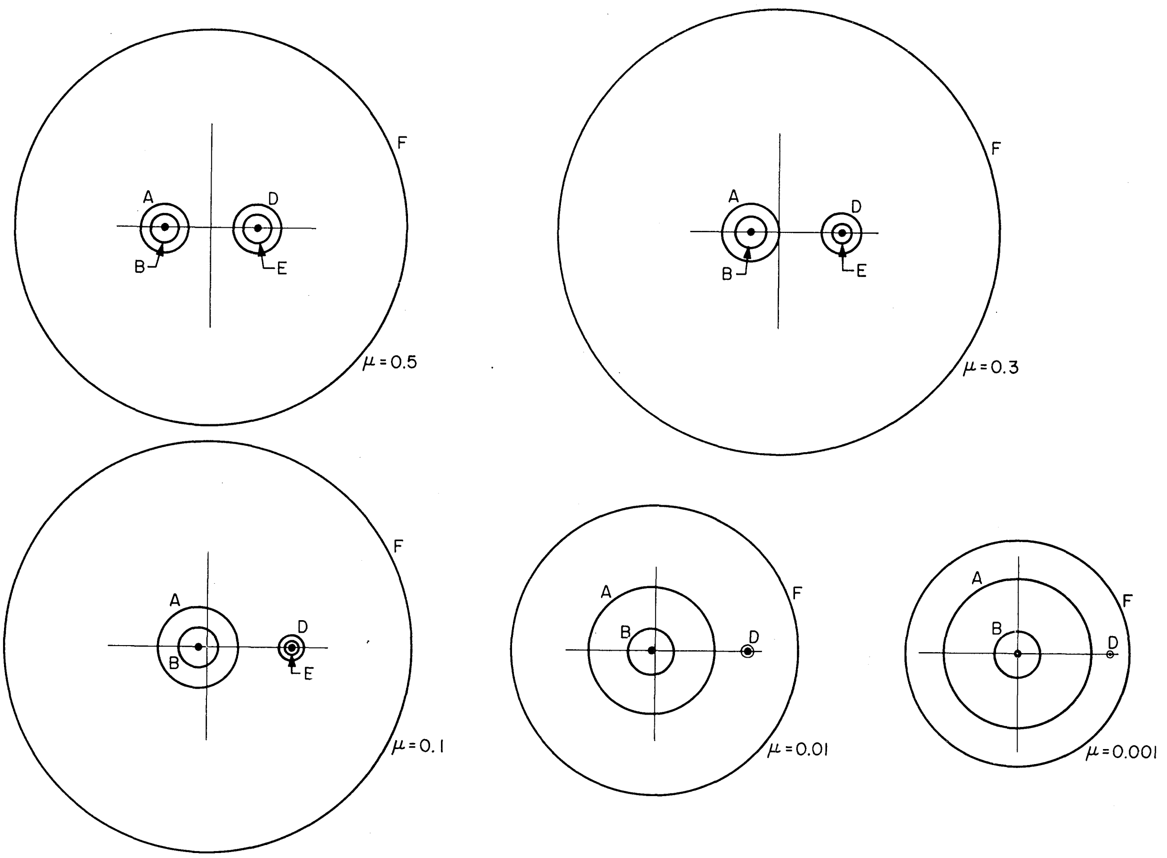


**Fig. 4 Extreme limits for stable near-circular planetary and satellite orbits for various mass ratios**

For retrograde motion around the smaller mass, analogous expressions follow:

Retrograde, between

$$C_1 = (1 - \mu - r_{21})^2 + \frac{2(1-\mu)}{1 - r_{21}} + \frac{\mu}{r_{21}} - 2\sqrt{r_{21}\mu} - r_{21}^2 \quad [3]$$

Retrograde, beyond

$$C_1 = (1 - \mu + r_{21})^2 + \frac{2(1-\mu)}{1 + r_{21}} + \frac{\mu}{r_{21}} - 2\sqrt{r_{21}\mu} - r_{21}^2 \quad [4]$$

The maximum distance of particles in near-circular orbits around the larger mass $(1 - \mu)$, where velocities and distances correspond to $C = C_1$, is given in the following equations:

Direct, between

$$C_1 = (r_{10} - \mu)^2 + \frac{1-\mu}{r_{10}} + \frac{2\mu}{1 - r_{10}} + 2\sqrt{r_{10}(1-\mu)} - r_{10}^2 \quad [5]$$

Direct, behind

$$C_1 = (r_{10} + \mu)^2 + \frac{1-\mu}{r_{10}} + \frac{2\mu}{1 + r_{10}} + 2\sqrt{r_{10}(1-\mu)} - r_{10}^2 \quad [6]$$

Retrograde, between

$$C_1 = (r_{11} - \mu)^2 + \frac{1-\mu}{r_{11}} + \frac{2\mu}{1 - r_{11}} - 2\sqrt{r_{11}(1-\mu)} - r_{11}^2 \quad [7]$$

Retrograde, behind

$$C_1 = (r_{11} + \mu)^2 + \frac{1-\mu}{r_{11}} + \frac{2\mu}{1 + r_{11}} - 2\sqrt{r_{11}(1-\mu)} - r_{11}^2 \quad [8]$$

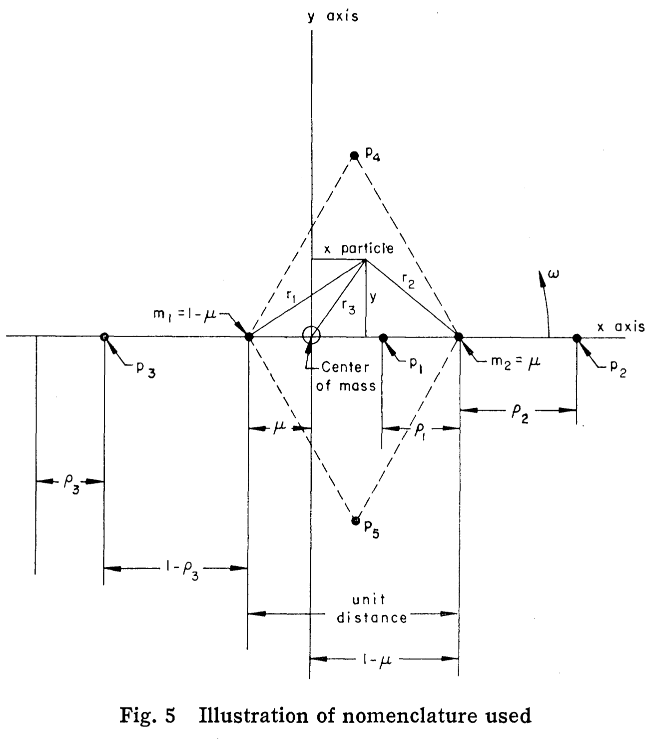


**Fig. 5 Illustration of nomenclature used**

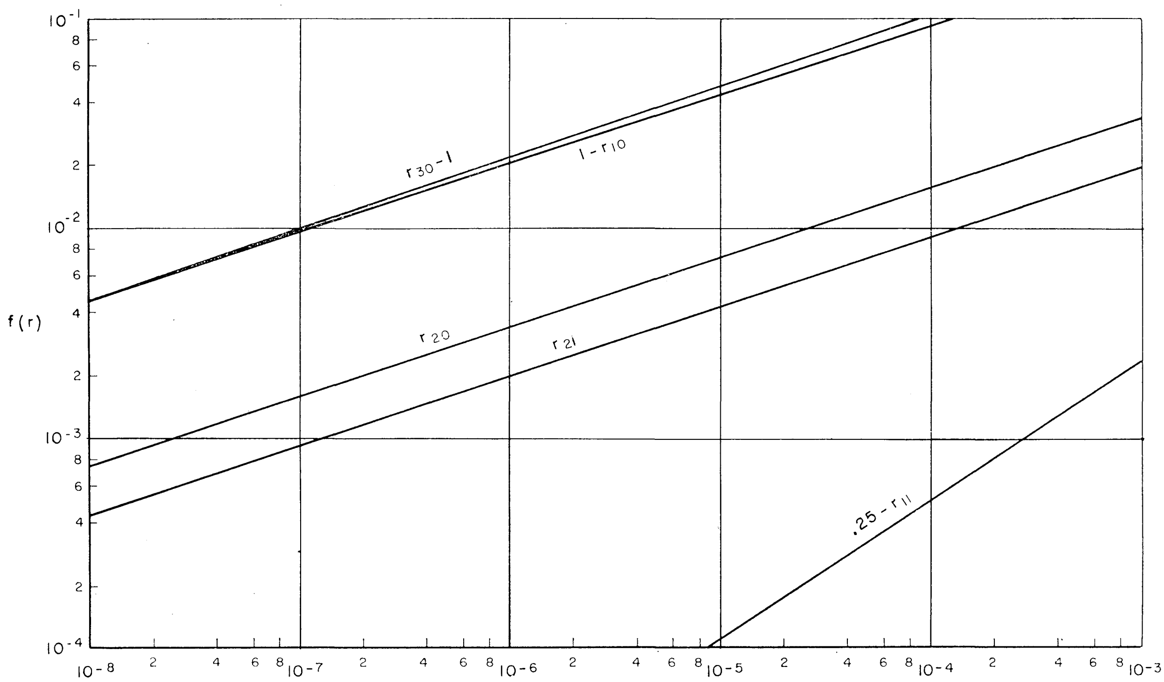


**Fig. 6 Limiting radii of stable near-circular planetary and satellite orbits in restricted three-body problem as a function of mass ratio $\mu$**



Following is the equation for the minimum distance of particles in near-circular orbit around both masses, outside the outer $C_2$ contour where velocities and distances correspond to $C_2$ and where $r_{30}$ is the distance from the common center of mass to the particle.

"Direct," beyond

$$C_2 = 2\sqrt{r_{30}} - \frac{1}{r_{30}} + \frac{2(1-\mu)}{r_{30}+\mu} + \frac{2\mu}{r_{30}-(1-\mu)} \quad [9]$$

"Direct," behind

$$C_2 = 2\sqrt{r_{30}} - \frac{1}{r_{31}} + \frac{2(1-\mu)}{r_{30}-\mu} + \frac{2\mu}{r_{31}+(1-\mu)} \quad [10]$$

"Retrograde," beyond

$$C_2 = -2\sqrt{r_{31}} - \frac{1}{r_{31}} + \frac{2(1-\mu)}{r_{31}+\mu} + \frac{2\mu}{r_{31}-(1-\mu)} \quad [11]$$

"Retrograde," behind

$$C_2 = -2\sqrt{r_{31}} - \frac{1}{r_{31}} + \frac{2(1-\mu)}{r_{31}-\mu} + \frac{2\mu}{r_{31}+(1-\mu)} \quad [12]$$

Results of machine computations of the roots of Equations [1 through 10] are given in Table 1. Equations [11 and 12] have no roots for values of $r_{31} > 1 - \mu + \rho_2$, hence "retrograde" motion of particles at orbital velocity corresponding to $C \geq C_2$ apparently is not possible, and no stable "retrograde" near-circular orbits around both masses can exist. (Particles moving at orbital velocity around both masses will always have retrograde motion with respect to the rotating coordinate system. The terms "direct" and "retrograde" refer to motion with respect to inertial space.)

Results are shown graphically in Figs. 6 and 7. The functions $(1 - r_{10})$, $(0.25 - r_{11})$ and $(r_{30} - 1)$ are plotted against $\mu$, since $r_{10}$ and $r_{30}$ both approach 1.0 asymptotically, and $r_{11}$ approaches 0.25 for small values of $\mu$.

## Nomenclature

$A$ = region around larger mass $m_1$ within which stable direct near-circular inferior planetary orbits can exist, radius $= r_{10}$

$B$ = region around larger mass $m_1$ within which stable retrograde near-circular inferior planetary orbits can exist, radius $= r_{11}$

$c$ = Jacobian constant, absolute units

$C$ = Jacobian constant, canonical units, $C = c/\omega^2$

$C_1$ = value of Jacobian constant associated with zero relative velocity surface passing through first Lagrangian libration point $p_1$
$= (1 - \rho_1 - \mu)^2 + 2(1 - \mu)/(1 - \rho_1) + 2\mu/\rho_1$

$C_2$ = value of Jacobian constant associated with zero relative velocity surface passing through second Lagrangian libration point $p_2$
$= (1 + \rho_2 - \mu)^2 + 2(1 - \mu)/(1 + \rho_2) + 2\mu/\rho_2$

$C_3$ = value of Jacobian constant associated with the zero relative velocity surface passing through third Lagrangian libration point $p_3$
$= (1 - \rho_3 + \mu)^2 + 2(1 - \mu)/(1 - \rho_3) + 2\mu/(2 - \rho_3)$

$C_4$ = value of Jacobian constant at fourth and fifth Lagrangian libration points (equilateral points) $p_4$ and $p_5$
$= 3 - \mu(1 - \mu)$

$D$ = region around smaller mass $m_2$ within which stable direct near-circular satellite orbits can exist, radius $= r_{20}$

$E$ = region around smaller mass $m_2$ within which stable retrograde near-circular satellite orbits can exist, radius $= r_{21}$

$F$ = region around both masses outside of which stable direct near-circular superior planetary orbits can exist, radius $= r_{30}$

$G$ = gravitational constant

$K$ = product of gravitational constant $G$ and total mass $m_0$ of system

$m_0$ = total mass of system, $m_0 = m_1 + m_2$

$m_1$ = mass of larger body

$m_2$ = mass of smaller body

$p_1$ = first Lagrangian libration point, in-line point between $m_1$ and $m_2$

$p_2$ = second Lagrangian libration point, in-line point between $m_2$ and $+\infty$

$p_3$ = third Lagrangian libration, in-line point between $m_1$ and $-\infty$

$p_4, p_5$ = fourth and fifth Lagrangian libration points, "equilateral points"

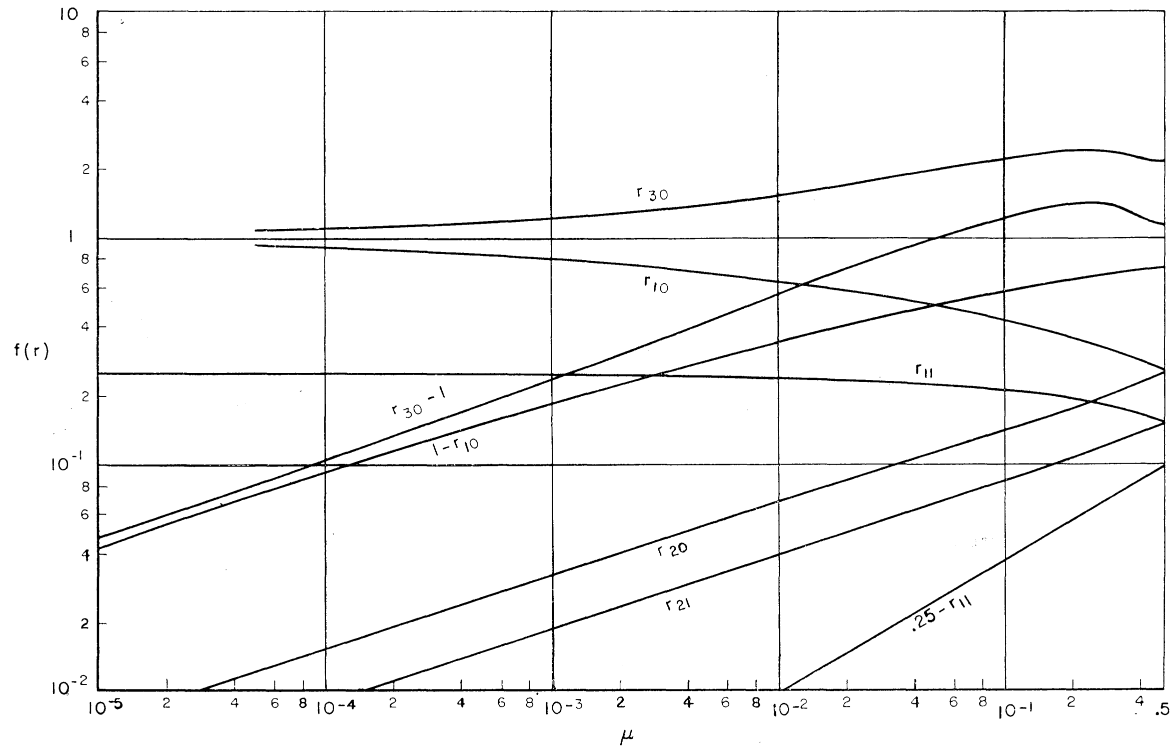


**Fig. 7 Limiting radii of stable near-circular planetary and satellite orbits in restricted three-body problem as a function of mass ratio $\mu$**

$r_1$ = radial distance from larger mass to particle
$r_2$ = radial distance from smaller mass to particle
$r_3$ = radial distance from center of mass to particle
$r_{10}$ = radius of outermost stable direct near-circular inferior planetary orbit of larger mass $m_1$
$r_{11}$ = radius of outermost stable retrograde near-circular inferior planetary orbit of larger mass $m_1$
$r_{20}$ = radius of outermost stable direct near-circular satellite orbit of smaller mass $m_2$
$r_{21}$ = radius of outermost stable retrograde near-circular satellite orbit of smaller mass $m_2$
$r_{30}$ = radius of innermost stable direct near-circular superior planetary orbit around center of mass
$t$ = time
$v$ = magnitude of velocity of a particle in rotating space
$V$ = particle velocity in rotating space, canonical units $V = v/\omega$
$x$ = component of distance between particle and center of mass of system, measured parallel to line joining the two massive bodies (rotating coordinate system)
$y$ = component of distance between particle and center of mass of system, measured perpendicular to line joining the two massive bodies (rotating coordinate system)
$\mu$ = ratio of smaller of the two masses to total mass of system
= $m_2/m_1 + m_2 = m_2/m_0$
for $m_0 = 1$, $\mu = m_2$ and $1 - \mu = m_1$
$\rho_1$ = distance from $m_2$ to $p_1$
$\rho_2$ = distance from $m_2$ to $p_2$
$\rho_3$ = one minus distance from $m_1$ to $\rho_3$
$\omega$ = angular velocity of the two massive bodies

$\rho_1$, $\rho_2$ and $\rho_3$ were computed from the following quintic expressions derived by Moulton (2)

$$\rho_1^5 - (3 - \mu)\rho_1^4 + (3 - 2\mu)\rho_1^3 - \mu\rho_1^2 + 2\mu\rho_1 - \mu = 0$$
$$\rho_2^5 + (3 - \mu)\rho_2^4 + (3 - 2\mu)\rho_2^3 - \mu\rho_2^2 - 2\mu\rho_2 - \mu = 0$$
$$\rho_3^5 - (7 + \mu)\rho_3^4 + (19 + 6\mu)\rho_3^3 - (24 + 13\mu)\rho_3^2 + (12 + 14\mu)\rho_3 - 7\mu = 0$$

## References


1 Buchheim, R. W., "Motion of a Small Body in Earth-Moon Space," The Rand Corp., Research Memo. RM-1726, June 4, 1956.
2 Moulton, F. R., "An Introduction to Celestial Mechanics." The Macmillan Co., N. Y., 1959, 2nd revised edition, chap. VIII.