Example Linear Algebra Competency Test Solutions

The 40 questions below are a combination of True or False, multiple choice, fill in the blank, and computations involving matrices and vectors. In the latter case, some operations may not be feasible. Where that is the case, for example, where two vectors or matrices cannot be added together please indicate so by writing "N/A."

1. Compute the sum of $[2 \quad 9 \quad -1]$ and $[1 \quad 5]$.

   N/A vectors and matrices must be of the same dimensions for addition to be defined.

2. Compute the sum of $[2 \quad 9 \quad -1]$ and $[1 \quad 5 \quad 3]$.

   $[\mathbf{3} \quad \mathbf{14} \quad \mathbf{2}]$

3. Compute the sum of A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 5 \\ 4 & 5 & 2 \end{bmatrix}$.

   $\begin{bmatrix} \mathbf{3} & \mathbf{5} & \mathbf{7} \\ \mathbf{5} & \mathbf{3} & \mathbf{9} \\ \mathbf{7} & \mathbf{9} & \mathbf{3} \end{bmatrix}$

4. The two matrices A and B in the question above are:
   a. Transposes of one another
   b. Asymptotic
   c. Parenthetical
   d. Symmetric

5. Compute the sum of A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 4 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

   N/A vectors and matrices must be of the same dimensions for addition to be defined.

6. Compute the dot product of $[1 \quad 5 \quad 2]$ and $[1 \quad -1 \quad 3]$.

   2

7. Compute the transpose of this matrix: $\begin{bmatrix} 1 & 3 & 8 \\ 3 & 1 & 4 \\ 8 & 4 & 1 \end{bmatrix}$.

   $\begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{8} \\ \mathbf{3} & \mathbf{1} & \mathbf{4} \\ \mathbf{8} & \mathbf{4} & \mathbf{1} \end{bmatrix}$

8. Compute the dot product of $[1 \quad 0 \quad -1]$ and $[0 \quad 1]$.

   N/A vectors must be of the same dimensions to compute a dot product.

9. Compute the product of the two matrices A = $\begin{bmatrix} 1 & 4 & 3 \\ 1 & -1 & 1 \\ 3.5 & 1 & -2.5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & 1.75 \\ 8 & -2 & 0.5 \\ 6 & 2 & -1.25 \end{bmatrix}$.

   $\begin{bmatrix} \mathbf{52} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{6} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{9.75} \end{bmatrix}$

10. The rows of the matrix A = $\begin{bmatrix} 1 & 4 & 3 \\ 1 & -1 & 1 \\ 3.5 & 1 & -2.5 \end{bmatrix}$ are:
    a. Orthogonal
    b. Orthonormal
    c. Parenthetical
    d. Peculiar
11. (T/F) Where the dimension of matrices is defined as *rows*×*columns*, you can multiply a 3×3 matrices on the left by a 2×3 matrix on the right.
    a. True
    b. False
12. (T/F) Where the dimension of matrices is defined as *rows*×*columns*, you can multiply a 3×2 matrices on the left by a 2×3 matrix on the right?
    a. True
    b. False
13. (Fill in the blank) Find the solution to this system of equations:

$$2x + 3y - 4z = 10$$
$$5x + 2y + z = 20$$
$$x - 6y + 3z = 12$$

    a. x = **_≈5.018181818______**
    b. y = **_≈-1.85454545______**
    c. z = **_≈-1.38181818______**

14. Denote the matrix of coefficients for the left-hand side of the system of equations above as A = $\begin{bmatrix} 2 & 3 & -4 \\ 5 & 2 & 1 \\ 1 & -6 & 3 \end{bmatrix}$. Denote the matrix equivalent with the row echelon form (REF) of this matrix as R.
What is the matrix P that accomplishes the equivalent operations in the row reduction steps that you performed for the previous questions? In other words, what is the matrix P that satisfies PA = R given A and R?

$$\mathbf{P} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{-2.5} & \mathbf{1} & \mathbf{0} \\ \mathbf{2.9091} & \mathbf{-1.3636} & \mathbf{1} \end{bmatrix}$$

15. (Fill in the blank) Denote the right-hand side of the system of equations in Problem 13 as b. what is the product of the matrix P specified in the question above multiplied by b, that is, what is Pb? Write the components of Pb relating to each of the 3 equations in the same order as the equations are presented.
    a. Equation 1: ___**10**_______________
    b. Equation 2: ___**-5**_______________
    c. Equation 3: ___**13.818182**_______________

16. (T/F) This matrix, $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ 5 & 4 & 1 \end{bmatrix}$, has an inverse.
    a. True
    b. False

17. Compute the inverse of this matrix: $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$.

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \end{bmatrix}$$

18. Compute the inverse of this matrix, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

19. Compute the inverse of this matrix, $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

$$\begin{bmatrix} \mathbf{0.75} & \mathbf{-0.5} & \mathbf{0.25} \\ \mathbf{-0.5} & \mathbf{1} & \mathbf{-0.5} \\ \mathbf{0.25} & \mathbf{-0.5} & \mathbf{0.75} \end{bmatrix}$$

20. (Fill in the Blank) The magnitude of this vector $[12 \quad 16]$, using the Euclidean distance as the definition of magnitude, is: ____**20**_______________
21. (Fill in the Blank) The magnitude of the projection of this vector $[5 \quad -2 \quad 3 \quad 6]$ onto this vector, $[1 \quad 2 \quad 7 \quad 3]$, is: __**3.58**_________
22. (T/F) These two vectors are orthogonal: $[1, \quad 2, \quad 1]$ and $[-5, \quad 4, \quad -3]$.
    - **a. True**
    - b. False
23. (T/F) These two vectors are orthogonal: $[1, \quad 1, \quad 1]$ and $[1, \quad 0, \quad 1]$.
    - a. True
    - **b. False**
24. What is the rank of this matrix: $\begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$?

    1

25. (T/F) An $m \times n$ matrix of rank n-2 has a non-empty (right) null space.
    - **a. True**
    - b. False
26. (Fill in the blank) The maximum rank of an $m \times n$ matrix that has fewer rows than columns is: ____***m***_____
27. (T/F) An $m \times n$ matrix with $m < n$ can be symmetric.
    - a. True
    - **b. False**
28. (T/F) A symmetric matrix A has the same (right) null space as A^T.
    - **a. True**
    - b. False
29. (T/F) This matrix provides a valid basis for 3-dimensional space: $\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.4 & 0.1 & 0.5 \\ 0.5 & 0.4 & 0.1 \end{bmatrix}$.
    - **a. True**
    - b. False
30. Compute the LU decomposition (factorization) of this matrix: $\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.4 & 0.1 & 0.5 \\ 0.5 & 0.4 & 0.1 \end{bmatrix}$.

    $$\boldsymbol{\begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 4.00 & 1.00 & 0.00 \\ 5.00 & 1.11 & 1.00 \end{bmatrix} \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.0 & -1.9 & -1.1 \\ 0.0 & 0.0 & -0.684 \end{bmatrix}}$$

31. Compute the eigenvalues for this matrix: $\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$.

    2, 1

32. Compute the eigenvectors for this matrix: $\begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$.

$\begin{bmatrix} \mathbf{0.7071} \\ \mathbf{0.7071} \end{bmatrix}$ **and** $\begin{bmatrix} \mathbf{0.4472} \\ \mathbf{0.8944} \end{bmatrix}$

33. Compute the magnitude of the projection of this vector, $[2, -1,5]$, onto this vector, $[1,4,1]$.

1.58

34. Write the permutation matrix P that multiplies the matrix A on the left side, as in PA, which exchanges the 2nd and 4th rows of a 5 × 5 matrix A.

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

35. Write the permutation matrix P that multiplies the matrix A on the left side, as in PA, which exchanges the 2nd and 4th rows of a 5 × 5 matrix A while multiplying the 3rd row by 3.

$$\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{3} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

36. (T/F) These two vectors are parallel: $[2, \quad 3\,], [8, \quad 12\,]$
    - **a. True**
    - b. False
37. (T/F) These two vectors are parallel: $[1, \quad 1\,], [1, \quad -1\,]$
    - a. True
    - **b. False**
38. (T/F) A singular value decomposition exists only for $m \times n$ matrices where $m = n$.
    - a. True
    - **b. False**

39. Suppose that the matrix A is 3×3, and its columns are orthonormal. That is, it is an orthogonal matrix. What is the singular value decomposition of A^TA in the form A^TA = UΣV? Give the values of the matrix elements denoted by the letters below.

$$U\Sigma V = \begin{bmatrix} a & b & * \\ * & c & * \\ * & * & d \end{bmatrix} \begin{bmatrix} e & f & * \\ * & g & * \\ * & * & h \end{bmatrix} \begin{bmatrix} i & j & * \\ * & k & * \\ * & * & l \end{bmatrix}$$

a. a = __1___
b. b = __0____
c. c = __1___
d. d = __1____
e. e = __1____
f. f = __0____
g. g = __1____
h. h = __1____
i. i = ___1___
j. j = ___0___
k. k = ___1___
l. l = ___1___

40. (Fill in the blank) The singular value decomposition of this matrix, $\begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$, is:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f & g \\ h & i & j \end{bmatrix} \begin{bmatrix} k & l & m \\ n & o & p \\ q & r & s \end{bmatrix}$$

a. a = -0.70710678
b. b = -0.70710678
c. c = -0.70710678
d. d = 0.70710678
e. e = 3.46410162
f. f = 0
g. g = 0
h. h = 0
i. i = 3.16227766
j. j = 0
k. k = -0.4085
l. l = -0.8165
m. m = -0.4082
n. n = -0.8944
o. o = 0.4472
p. p = 0
q. q = -0.1826
r. r = -0.3651
s. s = 0.91287