Chapter 3

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Combinatorics is the study of the number of ways a set of objects can be arranged, combined, or chosen; or the number of ways a succession of events can occur.

- Each result is called an **outcome.**
- An **event** is a subset of outcomes.
- When several events occur together, we have a **compound event.**

The **Fundamental Counting Principle** states that the total number of a ways a compound event may occur is $n_1 . n_2 . n_3 . n_4 \ldots . n_i$ where n_1 represents the number of ways the first event may occur, n_2 represents the number of ways the second event may occur, and so on.

Example 1

How many ways can you create a pizza choosing a meat, a vegetable and a type of cheese if you have 3 choices of meats, 4 choices for vegetables and 2 cheese choices?

Answer = 24

- A permutation of a set of *n* objects is an ordered arrangement of the objects.

$$_nP_r = n\,.(n-1).(n-2)\ldots 3.2.1 = n!$$

- When arranging! of then objects, the formula is

$$_nP_r = \frac{n!}{(n-r)!}$$

Examples 2 & 3

2. In how many ways can 6 people be seated in a row?

Answer = 720

3. In how many ways can 3 of the 6 symbols, &^%$#@ be arranged?

Answer = 120

- When we allow repeated values, The number of orderings of *n* objects taken *r* at a time, with repetition is *nr.*

- **Example 4:**

- In how many ways can you write 4 letters on a tag using each of the letters C O U G A R with repetition?

Answer = 6 . 6 . 6 . 6 = 1296

- The number of permutations, P, of *n* objects taken n at a time with *r* objects alike, *s* of another kind alike, and *t* of another kind alike is

$$P = \frac{n!}{r!\, s!\, t!}$$

How many different words (they do not have to be real words) can be formed from the letters in the word MISSISSIPPI?

Answer = 34’650

The number of circular permutations of *n* objects is *(n* -1)!

Example 6:

In how many ways can 12 people be seated around a circular table?

Answer = Approximately 39 Million (11!)

- A combination gives the number of ways of picking *r* unordered outcomes from *n* possibilities. The number of combinations of a set of *n* objects taken *r at* a time is

$$ {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} $$

- Note that with a combination, order does not matter.

Example 7

In how many ways can a committee of 5 be chosen from a group of 12 people?

Answer = 792

12! / 5! (12-5)!