# Random Variables

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- A **random variable** is a variable whose value is a numerical outcome of a random phenomenon. It assigns one and only one numerical value to each point in the sample space for a random experiment.
  - A discrete random variable is one that can assume a countable number of possible values.
  - A continuous random variable can assume any value in an interval on the number line.
  - A probability distribution table of X consists of all possible values of a discrete random variable with their corresponding probabilities.

Suppose a family has 3 children.

# Example 1

- A. Show all gender combinations with no regard to birth order.
  - {GGG, BBB, BBG, GGB}
- B. Determine the number of different birth order arrangements that are possible.
  - {GGG, GGB, GBG, BGG, BBB, BGB, BBG, GBB}
- C. Create a probability distribution for the number of girls in the family using the birth order arrangements

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X=G) | 1/8 | 3/8 | 3/8 | 1/8 |

- D. Find P(X >= 2)
  - P(X=2) + P (X=3) = 3/8 + 1/8 = 1/2

- The mean, or **expected value** of a random variable X is found using the formula

$$\mu_x = E[X] = x_1p_1 + x_2p_2 + \ldots + x_np_n$$

- The variance of a random variable X can be found using the formula:

$$\sigma_x^2 = Var[X] = (x_1 - \mu_x)^2p_1 + (x_2 - \mu_x)^2p_2 + \ldots + (x_n - \mu_x)^2p_n$$

*or using*

$$\sigma_x^2 = Var[X] = E[X^2] - (E[X])^2$$

- **Using the probability distribution from example 1, find the expected number of girls in the family and the standard deviation for the number of girls in the family.**

| X | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| P(X=G) | 1/8 | 3/8 | 3/8 | 1/8 |

**A. Draw the Probability Distribution Table**

**B. Find the Mean using the Expectation Function**

- **E[X] = 0*1/8 + 1*3/8 + 2*3/8 + 3*1/8 = 1.5**

**C. Calculate the Variance as $E[X^2] – E[X]^2$**

- **$E[X^2]$ = 0(1/8) + 1(3/8) + 4(3/8) + 9(1/8) = 3**
- **Var[X] = 3 – $(1.5)^2$**

**D. Find StD as the root of Variance.**

# Example 3

- Use the probability distribution to calculate what follows:

| X | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $P(X=x_i)$ | 0.15 | 0.05 | 0.1 | | 0.1 | 0.15 | 0.15 |

- $P(X=4)$
- $P(X<2)$
- $P(2<X<=5)$
- $P(X>=3)$
- $E[X]$
- $Var[X]$
- $Std[X]$

A: 0.3, 0.15, 0.5, 0.8, 4.2, 3.66, 1.91

- Suppose X is a random variable and we define *W* as a new random variable such that *W= aX+b* where *a* and *b* are real numbers. The mean and variance of W are:

$$\mu_W = E[W] = E[aX + b] = aE[X] + b$$

$$\sigma^2_W = Var[W] = Var[aX + b] = a^2 Var[X]$$

- Likewise, we have a formula for random variables that are combinations of two or more independent random variables.
  - Let X and *Y* be independent random variables.
  - The means and variances of the combinations of X and Y are:

$$\mu_{x+y} = E[X + Y] = E[X] + E[Y]$$

$$\sigma_{x+y}^2 = Var[X + Y] = Var[X] + Var[Y]$$

*and*

$$\mu_{x-y} = E[X - Y] = E[X] - E[Y]$$

$$\sigma_{x-y}^2 = Var[X - Y] = Var[X] - Var[Y]$$

Suppose you have a distribution, X, with mean = 22 and standard deviation= 3.

Define a new random variable *Y* = 3X + 1.

- a. Find the variance of X
  - Var[X] = $3^2$ = 9
- b. Find the mean of Y.
  - E[Y] = E[3X+1] = 3E[X] + 1 = 3(22) + 1 = 67
- c. Find the variance of *Y.*
  - *Var[Y] = Var[3X+1] =* $3^2$*Var[X] = 9(9) = 81*
- d. Find the standard deviation of *Y.*
  - *Std[Y] = SQRT(Var[Y]) = 9*

- *Note that:* $\sigma_{aX+b} = a.\sigma_X$