Normal Distribution

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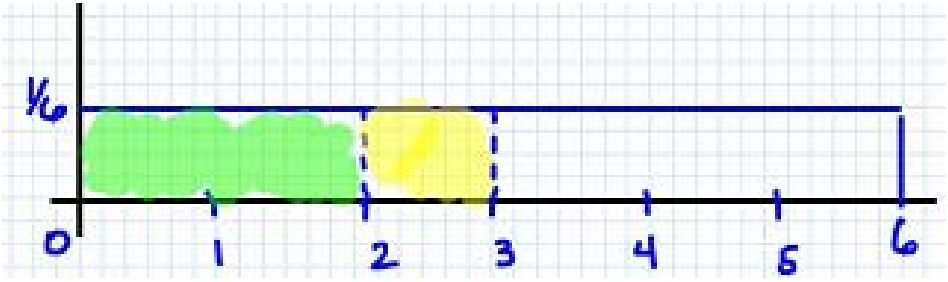
Density Curves

- A density curve is a graph whose area between it and the x-axis is equal to one.
  - These graphs come is a variety of shapes but the most familiar "normal" graph is bell shaped.
  - The area under the curve in a range of values indicates the proportion of values in that range.

Example

- Consider a uniform density curve defined from x = 0 to X = 6
- Sketch

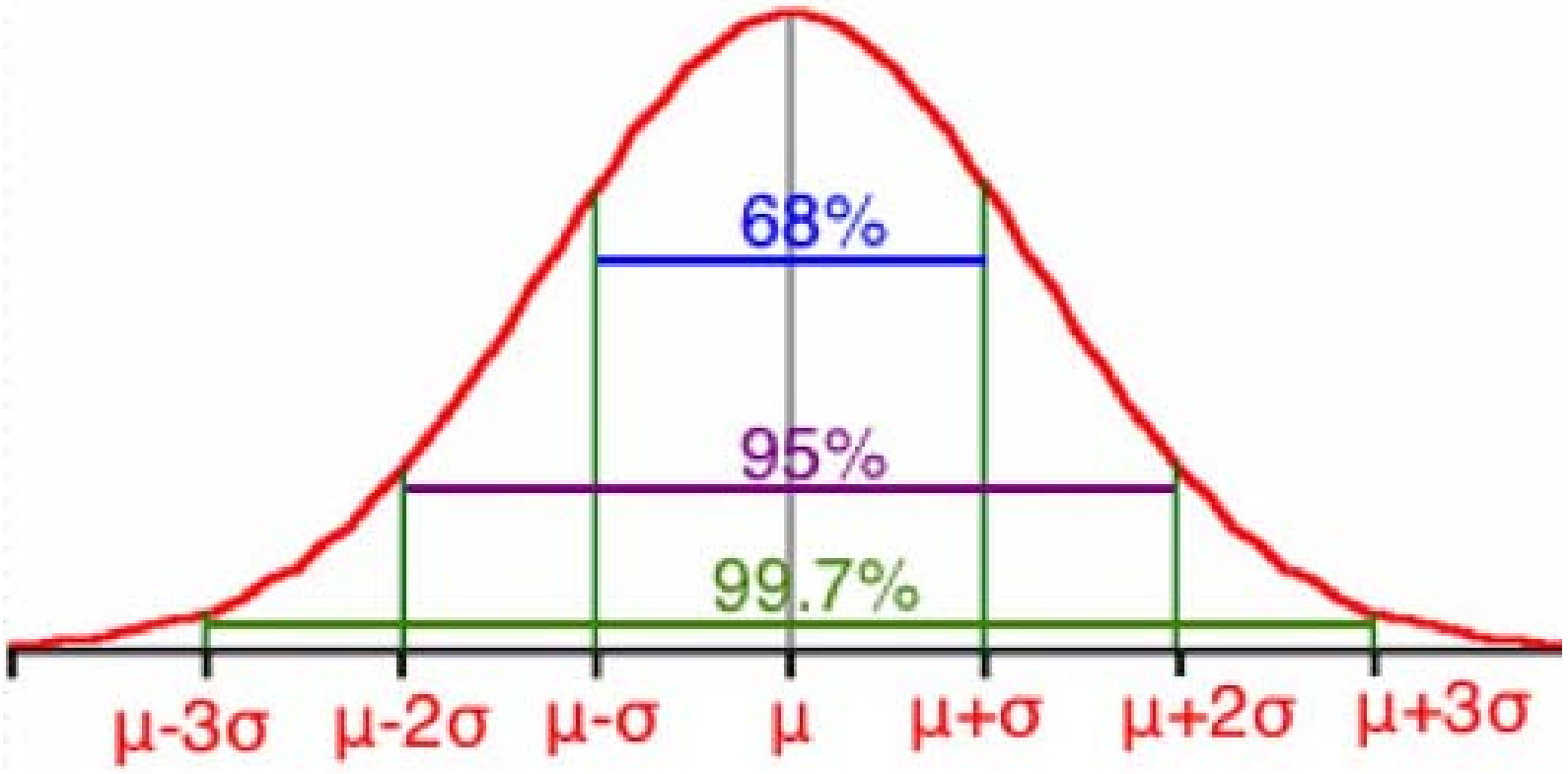


- What percent of observations fall below 2? P(X<2)
- What percent of observations lie between 2 and 3? P(2<X<3)
- Find the median.

- A density curve that is symmetric, single peaked and bell shaped is called a normal distribution.
- The normal distribution with mean μ and standard deviation σ is represented by $N(\mu, \sigma)$.
- **The Empirical Rule:**

The Empirical Rule states if a distribution has a normal distribution,

- Approximately 68% of all observations fall within one standard deviation of the mean.
- Approximately 95% of all observations fall within two standard deviations of the mean.
- Approximately 99.7% of all observations fall within three standard deviations of the mean.


68%
95%
99.7%
μ-3σ
μ-2σ
μ-σ
μ
μ+σ
μ+2σ
μ+3σ

In MATLAB

- If we know the value and want to find the probability of x such that P(X < x), we use the following command:
- normcdf(x, mean, std)

- If we know the probability and want to find the value of x such that P(X < x), we use the following command:
- norminv(p, mean, std)

- The length of time needed to complete a certain test is normally distributed with mean 60 minutes and standard deviation 10 minutes.
  - Sketch the distribution and shade in the area in question.
  - What is the probability that someone will take between 40 and 80 minutes to complete the test? 95%
  - Find the interval that contains the middle 68% of completion times for all people taking the test. (50, 70)
  - What percent of people take more than 80 minutes to complete the test? 2.5%
  - What is the probability that someone will take less than 45 minutes to complete the test? $P(x<45)$
  - What is the probability that someone will take more than 30 minutes to complete the test? $P(X > 30)$
  - What is the probability that someone will take between 30 and 45 minutes to complete the test? $P(30<X<45)$
  - How long would it takes someone to finish a test if they are in the top 10% of the times? $P(X<x) = 0.9$

Standard Normal Calculations

- As suggested in the previous section, all normal distributions share many common properties.
- In fact, if change the units to σ and center the graph at μ = 0, all normal distributions would be exactly the same.
  - This is called standardizing.
- If x is an observation from a normal distribution with mean μ. and standard deviation σ, the standardized value of x is called the *z-score* and is computed with the formula below.

$$\text{Z-score:} \quad z = \frac{x - \mu}{\sigma}$$

- A z-score tells us how many standard deviations the observed value falls from the mean.
- We can use z-scores to “standardize” values that are on different scales to compare them.

- **Example:**

Bon took the ACT and scored 31. Craig took the SAT and scored (CR+M) 1390. If both tests are normally distributed, who did better? The ACT has a mean of 21.1 and a standard deviation of 4.7. The SAT has a mean of 1010 and a standard deviation of 174.5.