

Basic Probability Models

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- The **probability** of any outcome of a random phenomenon is the proportion of times the outcome would occur in a very long series of repetitions.
 - The **sample space** of a random phenomenon is the set of all possible outcomes.
 - An **event** is an outcome or a set of outcomes of a random phenomenon. It is a subset of the sample space. A **simple event** is an event consisting of exactly one outcome.
 - To compute the probability of some event E occurring, divide the number of ways that E can occur by the number of possible outcomes the sample space, S , can occur:

$$P(E) = \frac{n(E)}{n(S)}$$

Basic Rules of Probability

1. All events have a probability between zero and one. $0 \leq P(E) \leq 1$
2. All possible outcomes together must have a probability of one. $P(S) = 1$
3. Complement Rule: For any event E ,
$$P(E^c) = 1 - P(E)$$
4. Addition Rule: If E and F are disjoint events (mutually exclusive),
then $P(E \cup F) = P(E) + P(F)$
5. If E and F are **any** events of an experiment,
then $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

Example 1

- Suppose we draw a single card from a deck of 52 fair playing cards.

$$n(S) = 52$$

- What is the probability of drawing a heart?

$$n(H) = 13 ,$$

$$P(H) = 13 / 52 = 1 / 4$$

- What is the probability of drawing a queen?

$$n(Q) = 4 ,$$

$$P(Q) = 4 / 52 = 1 / 13$$

Example 2

- If **5 marbles** are drawn at random **all at once** from a bag containing 8 white and 6 black marbles, **what is the probability that 2 will be white and 3 will be black ?**

1. $n(S) = {}_{14}C_5 = 2002 \text{ (} 14!/9!.5! \text{)}$
2. $n(2 \text{ W \& } 3 \text{ B}) = {}_8C_2 \cdot {}_6C_3 = 560 \text{ (} 8!/6!.2! \cdot 6!/3!.3! \text{)}$
3. $P(2 \text{ W \& } 3 \text{ B}) = 560 / 2002 = .28$

Example 3

- The qualified applicant **pool for six management trainee positions** consists of **seven women and five men**. ($n(S) = {}_{12}C_6 = 924$)
 - What is the probability that a randomly selected trainee class will **consist entirely of women**?

$$n(\text{all women}) = {}_7C_6 = 7$$

$$P(\text{all women}) = 7 / 924$$

- What is the probability that a randomly selected trainee class will **consist of an equal number of men and women**?

$$n(3 \text{ men \& 3 women}) = {}_5C_3 \cdot {}_7C_3 = 350$$

$$P(3 \text{ men \& 3 women}) = 350/924 = 0.379$$

Example 4

A sports survey taken at UH shows that 48% of the respondents liked soccer, 66% liked basketball, and 38% liked hockey. Also, 30% liked soccer and basketball, 22% liked basketball and hockey, and 28% liked soccer and hockey. Finally, 12% liked all three sports:

- What is the probability that a randomly selected student likes basketball or hockey? Solve this by also using an appropriate formula.

$$\begin{aligned} P(\text{Basketball} \cup \text{Hockey}) &= P(\text{Basketball}) + P(\text{Hockey}) - P(\text{Basketball} \cap \text{Hockey}) \\ &= .66 + .38 - .22 = .82 \end{aligned}$$

- What is the probability that a randomly selected student does not like any of these sports?
 - Draw and then you can see !