Binomial Distributions

Dr. Bashar Al-Shboul

The University of Jordan

- A **Bernoulli Trial** is a random. Experiment with the following features:
  - The outcome can be classified as either a success or a failure (only two options and each is mutually exclusive).
  - The probability of success is p and probability of failure is $q = 1 - p$.
- A **Bernoulli random variable** is a variable assigned to represent the successes in a Bernoulli trial.
- If we wish to keep track of the <u>number</u> of successes that occur in repeated Bernoulli trials, we use a **binomial random variable**.
- A **binomial experiment** occurs when the following conditions are met:
  - Each trial can result in one of only two mutually exclusive outcomes (success or failure).
  - There are a fixed number of trials.
  - Outcomes of different trials are independent.
  - The probability that a trial results in success is the same for all trials.

- The random variable X = number of successes of a binomial experiment is a binomial distribution with parameters *p* and *n* where *p* represents the probability of a success and *n* is the number of trials.
- • The possible values of X are whole numbers that range from. 0 to *n.* As an abbreviation, we say

X~B(n, *p).*

- • Binomial probabilities are calculated with the following formula:

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}={}_nC_kp^k(1-p)^{n-k}$$

Binomial Distribution using MATLAB

- Binopdf (X,N,P) – Binomial Probability Density Function
- Computes the binomial pdf at each of the values of X using the corresponding number if trials in N and probability of success for each trial in P.
- N must be positive integers
- P falls in the interval [0, 1]

- The mean and variance of a binomial distribution are computed using the following formulas :
- $\mu_x = E[X] = np$
- $\sigma_x^2 = np(1-p)$

A fair coin is flipped 30 times. N=30, p = 1/2

- a. What is the probability that the coin comes up heads exactly 12 times?
  - $p\,(X{=}\ 12) = {}_{30}C_{12}\,(1/2)^{12}\,(1-1/2)^{(30-12)} = 0.0806$
- b. What is the probability the coin comes up heads less than 12 times ?
  - P (X < 12) = P *(X* <= 11) = *0.0509*
- c. What is the probability the coin comes up heads more than 12 times?
  - P(X>12) = 1-P(X<=12) = 1-Binomial(30, 0.5, 12) = 0.9194
- d. What is the expected number of heads?
  - E[X] = np = 30 (1/2) = 15

Suppose it is known that 80% of the people exposed to the flu virus will contract the flu. Out of a family of five exposed to the virus, what is the probability that:

- No one will contract the flu ?
  - $P(X=0) = {}_5C_0\ (0.8)^0\ (1-0.8)^{(5-0)} = 0.00032$
- All will contract the flu ?
  - $P(X=5) = {}_5C_5(0.8)^5(1-0.8)^{(5-5)} = 0.3277$
- Exactly two will get the flu ?
  - $P(X=2) = {}_5C_2\ (0.8)^2\ (1-0.8)^{(5-2)} = 0.0512$
- At least two will get the flu ?
  - $P(X>=2) = 1 – P(X<=1) = 0.9936$

- Using the binomial distribution in example 2,
- a. Let X = number of family members contracting the flu. Create the probability distribution table of X.

X	0	1	2	3	4	5
P(X)	0.00032	0.0064	0.0572	0.2048	0.4096	0.32768

- b. Find the mean and variance of this distribution.
  - E[X] = 0(0.00032) + 1(0.0064) + 2(0.0572) + 3(0.2048) + 4(0.4096) + 5(0.32768)

    = np = 5 (0.8) = 4

  σ_x^2 = np(1-p) = 5 (0.8)(0.2) = 0.8