

01. Sphere [1]

$$\sum_{i=1}^D x_i^2$$

$$-100 \leq x_i \leq 100$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \dots \quad 0]$$

02. Rastrigin [1]

$$\sum_{i=1}^D [x_i^2 - 10 \cos(2\pi x_i) + 10]$$

$$-5.12 \leq x_i \leq 5.12$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \dots \quad 0]$$

03. Ackley [1]

$$-20 \exp \left(-0.2 \sqrt{\frac{1}{D} \sum_{i=1}^D x_i^2} \right) - \exp \left(\frac{1}{D} \sum_{i=1}^D \cos(2\pi x_i) \right) + 20 + e$$

$$-32 \leq x_i \leq 32$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \dots \quad 0]$$

04. Griewank [1]

$$\frac{1}{4000} \sum_{i=1}^D x_i^2 - \prod_{i=1}^D \left[\cos \left(\frac{x_i}{\sqrt{i}} \right) \right] + 1$$

$$-600 \leq x_i \leq 600$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \dots \quad 0]$$

05. Schwefel P2.22 [1]

$$\sum_{i=1}^D |x_i| + \prod_{i=1}^D |x_i|$$

$$\begin{aligned}
-10 &\leq x_i \leq 10 \\
F^* &= 0 \\
X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}
\end{aligned}$$

06. Rosenbrock [1]

$$\sum_{i=1}^{D-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right]$$

$$\begin{aligned}
-30 &\leq x_i \leq 30 \\
F^* &= 0 \\
X^* &= \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}
\end{aligned}$$

07. Schwefel P2.21 [1]

$$\max_{1 \leq i \leq D} |x_i|$$

$$\begin{aligned}
-100 &\leq x_i \leq 100 \\
F^* &= 0 \\
X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}
\end{aligned}$$

08. Quartic (De-Jong) [1]

$$\sum_{i=1}^D (ix_i^4) + \text{random}[0, 1)$$

$$\begin{aligned}
-1.28 &\leq x_i \leq 1.28 \\
F^* &= 0 \\
X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}
\end{aligned}$$

09. Schwefel P1.2 [1]

$$\sum_{i=1}^D \left(\sum_{j=1}^i x_j \right)^2$$

$$\begin{aligned}
-100 &\leq x_i \leq 100 \\
F^* &= 0 \\
X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}
\end{aligned}$$

10. Penalized 1 [1]

$$\frac{\pi}{D}\left\{10\sin^2(\pi y_1)+\sum_{i=1}^{D-1}(y_i-1)^2\left[1+10\sin^2(\pi y_{i+1})\right]+(y_D-1)^2\right\}+\sum_{i=1}^Du\left(x_i,10,100,4\right)$$

$$y_i=1+\frac{1}{4}(x_i+1), u\left(x_i,a,k,m\right)=\left\{\begin{array}{ll}k(x_i-a)^m&x_i>a\\0&-a\leqslant x_i\leqslant a\\k(-x_i-a)^m&x_i<-a\end{array}\right.$$

$$\begin{array}{c} -50\leqslant x_i\leqslant 50\\ F^*=0\\ X^*=\left[\begin{array}{cccc} -1 & -1 & \cdots & -1 \end{array}\right] \end{array}$$

11. Penalized 2 [\[1\]](#)

$$0.1\left\{\sin^2(3\pi x_1)+\sum_{i=1}^{D-1}(x_i-1)^2\left[1+\sin^2(3\pi x_{i+1})\right]+(x_D-1)^2\left[1+\sin^2(2\pi x_D)\right]\right\}+$$

$$\begin{array}{c} \sum_{i=1}^Du\left(x_i,5,100,4\right)\\ u\left(x_i,a,k,m\right)=\left\{\begin{array}{ll}k(x_i-a)^m&x_i>a\\0&-a\leqslant x_i\leqslant a\\k(-x_i-a)^m&x_i<-a\end{array}\right.\end{array}$$

$$\begin{array}{c} -50\leqslant x_i\leqslant 50\\ F^*=0\\ X^*=\left[\begin{array}{cccc} 1 & 1 & \cdots & 1 \end{array}\right] \end{array}$$

12. Schwefel P2.26 [\[1\]](#)

$$-\sum_{i=1}^Dx_i\sin\Big(\sqrt{|x_i|}\Big)$$

$$\begin{array}{c} -500\leqslant x_i\leqslant 500\\ F^*=-418.982887272433799807913601398D\\ X^*=\left[\begin{array}{cccc} 420.968746 & 420.968746 & \cdots & 420.968746 \end{array}\right] \end{array}$$

13. Step [\[1\]](#)

$$\sum_{i=1}^D\left(\lfloor |x_i+0.5|\rfloor\right)^2$$

$$\begin{array}{c} -100\leqslant x_i\leqslant 100\\ F^*=0\\ X^*\in\left[-0.5,0.5\right) \end{array}$$

14. Kowalik [\[3\]](#)

$$\sum_{i=11}^{11}\left[a_i-\frac{x_1\left(b_i^2+b_ix_2\right)}{b_i^2+b_ix_3+x_4}\right]^2$$

$$b=\left[\begin{array}{cccccccccccc}4&2&1&\frac{1}{2}&\frac{1}{4}&\frac{1}{6}&\frac{1}{8}&\frac{1}{10}&\frac{1}{12}&\frac{1}{14}&\frac{1}{16}\end{array}\right]$$

$$a=\left[\begin{array}{cccccccccccc}0.1957&0.1947&0.1735&0.1600&0.0844&0.0627&0.0456&0.0342&0.0323&0.0235&0.0246\end{array}\right]$$

$$-5\leqslant x_i\leqslant 5,D=4$$

$$F^*=0.00030748610$$

$$X^*=\left[\begin{array}{cccc}0.192833&0.190836&0.123117&0.135766\end{array}\right]$$

15. Shekel's Foxholes [\[1\]](#)

$$\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^2\left(x_i-a_{ij}\right)^6}\right)^{-1}$$

$$a_{ij}=\left[\begin{array}{ccccccccccccccccccccccc} -32 & -16 & 0 & 16 & 32 & -32 & -16 & 0 & 16 & 32 & -32 & -16 & 0 & 16 & \cdots & 32 \\ -32 & -32 & -32 & -32 & -32 & -16 & -16 & -16 & -16 & -16 & 0 & 0 & 0 & 0 & \cdots & 32 \end{array}\right]$$

$$-65.536\leqslant x_i\leqslant 65.536,D=2$$

$$F^*=0.998003837794449325873406851315$$

$$X^*=\left[\begin{array}{cc}-31.97833&-31.97833\end{array}\right]$$

16. Goldstein-Price [\[1\]](#)

$$\left[1+\left(x_1+x_2+1\right)^2\left(19-14 x_1+3 x_1^2-14 x_2+6 x_1 x_2+3 x_2^2\right)\right]$$

$$\times\left[30+\left(2 x_1-3 x_2\right)^2\left(18-32 x_1+12 x_1^2+48 x_2-36 x_1 x_2+27 x_2^2\right)\right]$$

$$-2\leqslant x_i\leqslant 2$$

$$F^*=3$$

$$X^*=\left[\begin{array}{cc}0&-1\end{array}\right]$$

17. Shekel 5 [\[3\]](#)

$$-\sum_{i=1}^m\frac{1}{c_i+\sum_j^D\left(x_j-a_{ij}\right)^2},m=5,a=\left[\begin{array}{cccc}4.0&4.0&4.0&4.0\\1.0&1.0&1.0&1.0\\8.0&8.0&8.0&8.0\\6.0&6.0&6.0&6.0\\3.0&7.0&3.0&7.0\end{array}\right],c=\left[\begin{array}{ccccc}0.1&0.2&0.2&0.4&0.6\end{array}\right]$$

$$0\leqslant x_i\leqslant 10,D=4$$

$$F^*=-10.1527$$

$$X^*=\left[\begin{array}{cccc}4&4&4&4\end{array}\right]$$

18. Branin [1]

$$\left(x_2 - \frac{5.1x_1^2}{4\pi^2} + \frac{5x_1}{\pi} - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$$

$$-5 \leq x_1 \leq 10, 0 \leq x_2 \leq 15, D = 2$$

$$F^* = 0.39788735772973816$$

$$X^* = [-\pi \quad 12.275], [\pi \quad 2.275], [9.42478 \quad 2.475]$$

19. Hartmann 3 [1]

$$-\sum_{i=1}^m c_i \exp\left(-\sum_{j=1}^n a_{ij}(x_j - p_{ij})^2\right), m = 4, n = 3, a = \begin{bmatrix} 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \\ 3.0 & 10 & 30 \\ 0.1 & 10 & 35 \end{bmatrix}, c = [1.0 \quad 1.2 \quad 3.0 \quad 3.2],$$

$$p = \begin{bmatrix} 0.36890 & 0.1170 & 0.2673 \\ 0.46990 & 0.4387 & 0.7470 \\ 0.10910 & 0.8732 & 0.5547 \\ 0.03815 & 0.5743 & 0.8828 \end{bmatrix}$$

$$0 \leq X \leq 1, D = 3$$

$$F^* = -3.86278214782076$$

$$X^* = [0.1 \quad 0.55592003 \quad 0.85218259]$$

20. Shekel 7 [3]

$$-\sum_{i=1}^m \frac{1}{c_i + \sum_j^D (x_j - a_{ij})^2}, m = 7, a = \begin{bmatrix} 4.0 & 4.0 & 4.0 & 4.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 8.0 & 8.0 & 8.0 & 8.0 \\ 6.0 & 6.0 & 6.0 & 6.0 \\ 3.0 & 7.0 & 3.0 & 7.0 \\ 2.0 & 9.0 & 2.0 & 9.0 \\ 5.0 & 5.0 & 3.0 & 3.0 \end{bmatrix}, c$$

$$= [0.1 \quad 0.2 \quad 0.2 \quad 0.4 \quad 0.4 \quad 0.6 \quad 0.3]$$

$$0 \leq X \leq 10, D = 4$$

$$F^* = -10.3999$$

$$X^* = [4 \quad 4 \quad 4 \quad 4]$$

21. Shekel 10 [3]

$$-\sum_{i=1}^m \frac{1}{c_i + \sum_j^D (x_j - a_{ij})^2}, m = 10, a = \begin{bmatrix} 4.0 & 4.0 & 4.0 & 4.0 \\ 1.0 & 1.0 & 1.0 & 1.0 \\ 8.0 & 8.0 & 8.0 & 8.0 \\ 6.0 & 6.0 & 6.0 & 6.0 \\ 3.0 & 7.0 & 3.0 & 7.0 \\ 2.0 & 9.0 & 2.0 & 9.0 \\ 5.0 & 5.0 & 3.0 & 3.0 \\ 8.0 & 1.0 & 8.0 & 1.0 \\ 6.0 & 2.0 & 6.0 & 2.0 \\ 7.0 & 3.6 & 7.0 & 3.6 \end{bmatrix}, c = [0.1 \quad 0.2 \quad 0.2 \quad 0.4 \quad 0.4 \quad 0.6 \quad 0.3 \quad 0.7 \quad 0.5 \quad 0.5]$$

$$0 \leq X \leq 10, D = 4$$

$$F^* = -10.5319$$

$$X^* = [4 \quad 4 \quad 4 \quad 4]$$

22. Six-Hump Camel-Back [\[1\]](#)

$$4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$$

$$-5 \leq X \leq 5, D = 2$$

$$F^* = -1.031628453489877$$

$$X^* = \begin{bmatrix} -0.08984201368301331 & 0.7126564032704135 \\ 0.08984201368301331 & -0.7126564032704135 \end{bmatrix}$$

23. Hartmann 6 [\[1\]](#)

$$-\sum_{i=1}^m c_i \exp\left(-\sum_{j=1}^n a_{ij}(x_j - p_{ij})^2\right), m = 4, n = 6, c = [1.0 \quad 1.2 \quad 3.0 \quad 3.2]$$

$$a = \begin{bmatrix} 10.0 & 3.00 & 17.0 & 3.50 & 1.70 & 8.00 \\ 0.05 & 10.0 & 17.0 & 0.10 & 8.00 & 14.0 \\ 3.00 & 3.50 & 1.70 & 10.0 & 7.0 & 8.00 \\ 17.0 & 8.00 & 0.05 & 10.0 & 0.10 & 14.0 \end{bmatrix}$$

$$p = \begin{bmatrix} 0.1312 & 0.1696 & 0.5569 & 0.0124 & 0.8283 & 0.5886 \\ 0.2329 & 0.4135 & 0.8307 & 0.3736 & 0.1004 & 0.9991 \\ 0.2348 & 0.1415 & 0.3522 & 0.2883 & 0.3047 & 0.6650 \\ 0.4047 & 0.8828 & 0.8732 & 0.5743 & 0.1091 & 0.0381 \end{bmatrix}$$

$$0 \leq X \leq 1, D = 6$$

$$F^* = -3.32236801141551$$

$$X^* = [0.20168952 \quad 0.15001069 \quad 0.47687398 \quad 0.27533243 \quad 0.31165162 \quad 0.65730054]$$

24. Zakharov [\[1\]](#)

$$\sum_{i=1}^D x_i^2 + \left(\sum_{i=1}^D 0.5 i x_i \right)^2 + \left(\sum_{i=1}^D 0.5 i x_i \right)^4$$

$$-5 \leq X \leq 10$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \dots \quad 0]$$

25. Sum Squares (Axis Parallel Hyper Ellipsoid) [\[2\]](#)

$$\sum_{i=1}^D i x_i^2$$

$$-10 \leq x_i \leq 10$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \dots \quad 0]$$

26. Alpine [\[1\]](#)

$$\sum_{i=1}^D |x_i \sin(x_i) + 0.1 x_i|$$

$$-10 \leq X \leq 10$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \dots \quad 0]$$

27. Michalewicz [\[1\]](#)[\[11\]](#)

$$-\sum_{i=1}^D \sin(x_i) \sin^{2m} \left(\frac{i x_i^2}{\pi} \right), m = 10$$

$$0 \leq x_i \leq \pi$$

$$D = 1 : F^* = -0.801303410098552549, X^* = [2.20290552017261]$$

$$D = 2 : F^* = -1.80130341009855321, X^* = [2.20290552014618 \quad 1.57079632677565]$$

$$D = 5 : F^* = -4.687658$$

$$D = 10 : F^* = -9.66015$$

28. Exponential [\[1\]](#)

$$\exp \left(-0.5 \sum_{i=1}^D x_i^2 \right)$$

$$\begin{aligned} -1 &\leq X \leq 1 \\ F^* &= -1 \\ X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix} \end{aligned}$$

29. Schaffer [\[1\]](#)

$$0.5 + \frac{\sin^2\left(\sqrt{x_1^2 + x_2^2}\right) - 0.5}{\left[1 + 0.001\left(x_1^2 + x_2^2\right)\right]^2}$$

$$\begin{aligned} -100 &\leq X \leq 100, D = 2 \\ F^* &= 0 \\ X^* &= \begin{bmatrix} 0 & 0 \end{bmatrix} \end{aligned}$$

30. Bent Cigar [\[1\]](#)

$$x_1^2 + 10^6 \sum_{i=2}^D x_i^2$$

$$\begin{aligned} -100 &\leq X \leq 100 \\ F^* &= 0 \\ X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix} \end{aligned}$$

31. Bohachevsky 1 [\[1\]](#)

$$x_1^2 + 2x_2^2 - 0.3 \cos(3\pi x_1) - 0.4 \cos(4\pi x_2) + 0.7$$

$$\begin{aligned} -50 &\leq X \leq 50, D = 2 \\ F^* &= 0 \\ X^* &= \begin{bmatrix} 0 & 0 \end{bmatrix} \end{aligned}$$

32. Elliptic (Ellipsoid) [\[4\]](#)

$$\sum_{i=1}^D (10^6)^{\frac{i-1}{D-1}} x_i^2$$

$$\begin{aligned} -100 &\leq x_i \leq 100 \\ F^* &= 0 \\ X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix} \end{aligned}$$

33. Drop Wave [\[1\]](#)

$$-\frac{1+\cos\left(12\sqrt{x_1^2+x_2^2}\right)}{0.5\left(x_1^2+x_2^2\right)+2}$$

$$\begin{array}{l} -5.12 \leqslant X \leqslant 5.12, D=2 \\ F^*=-1 \\ X^*=\left[\begin{array}{cc} 0 & 0 \end{array}\right] \end{array}$$

34. Cosine Mixture [\[1\]](#)

$$0.1\sum_{i=1}^D\cos(5\pi x_i)-\sum_{i=1}^Dx_i^2$$

$$\begin{array}{l} -1 \leqslant X \leqslant 1 \\ F^*=-0.1D \\ X^*=\left[\begin{array}{cccc} 0 & 0 & \cdots & 0 \end{array}\right] \end{array}$$

35. Ellipsoidal [\[5\]](#)

$$\sum_{i=1}^D\left(x_i-i\right)^2$$

$$\begin{array}{l} -D \leqslant x_i \leqslant D \\ F^*=0 \\ X^*=\left[\begin{array}{cccc} 1 & 2 & \cdots & D \end{array}\right] \end{array}$$

36. Levy and Montalvo 1 [\[6\]](#)

$$\frac{\pi}{D}\left[10\sin^2(\pi y_1)+\sum_{i=1}^{D-1}\left(y_i-1\right)^2\left[1+10\sin^2(\pi y_{i+1})\right]+\left(y_D-1\right)^2\right],y_i=1+\frac{1}{4}\left(x_i+1\right)$$

$$\begin{array}{l} -10 \leqslant X \leqslant 10 \\ F^*=0 \\ X^*=\left[\begin{array}{cccc} -1 & -1 & \cdots & -1 \end{array}\right] \end{array}$$

37. Easom [\[1\]](#)

$$-\cos(x_1)\cos(x_2)\exp\left[-(x_1-\pi)^2-(x_2-\pi)^2\right]$$

$$\begin{array}{l} -10 \leqslant X \leqslant 10, D=2 \\ F^*=-1 \\ X^*=\left[\begin{array}{cc} \pi & \pi \end{array}\right] \end{array}$$

38. Sum of Different Power (Powell Sum) [7]

$$\sum_{i=1}^D |x_i|^{i+1}$$

$$-1 \leq x_i \leq 1$$

$$F^* = 0$$

$$X^* = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}$$

39. Levy and Montalvo 2 [6]

$$0.1 \left\{ \sin^2(3\pi x_1) + \sum_{i=1}^D (x_i - 1)^2 [1 + \sin^2(3\pi x_i + 1)] + (x_D - 1)^2 [1 + \sin^2(2\pi x_D)] \right\}$$

$$-5 \leq X \leq 5$$

$$F^* = 0$$

$$X^* = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}$$

40. Holzman [1]

$$\sum_{i=1}^D i x_i^4$$

$$-10 \leq X \leq 10$$

$$F^* = 0$$

$$X^* = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}$$

41. Xin-She Yang 1 [1]

$$\left[\exp \left(- \sum_{i=1}^D \left(\frac{x_i}{\beta} \right)^{2m} \right) - 2 \exp \left(- \sum_{i=1}^D (x_i - c)^2 \right) \right] \prod_{i=1}^D \cos^2(x_i), m = 5, \beta = 15, c = 0$$

$$-20 \leq X \leq 20$$

$$F^* = -1$$

$$X^* = \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}$$

42. Xin-She Yang 6 [1]

$$\left[\sum_{i=1}^D \sin^2(x_i) - \exp \left(- \sum_{i=1}^D x_i^2 \right) \right] \exp \left(- \sum_{i=1}^D \sin^2 \sqrt{|x_i|} \right)$$

$$\begin{aligned}
-10 &\leq X \leq 10 \\
F^* &= -1 \\
X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}
\end{aligned}$$

43. Beale [1]

$$(1.5 - x_1 + x_1 x_2)^2 + (2.25 - x_1 + x_1 x_2^2)^2 + (2.625 - x_1 + x_1 x_2^3)^2$$

$$\begin{aligned}
-4.5 &\leq X \leq 4.5, D = 2 \\
F^* &= 0 \\
X^* &= \begin{bmatrix} 3 & 0.5 \end{bmatrix}
\end{aligned}$$

44. Shubert [3]

$$\left[\sum_{i=1}^5 i \cos(i+1)x_1 + i \right] \left[\sum_{i=1}^5 i \cos(i+1)x_2 + i \right]$$

$$\begin{aligned}
-10 &\leq X \leq 10, D = 2 \\
F^* &= -186.7309 \\
X^* &= \begin{bmatrix} -7.0835 & 4.8580 \end{bmatrix}, \begin{bmatrix} -7.0835 & -7.7083 \end{bmatrix}, \begin{bmatrix} -7.0835 & -1.4251 \end{bmatrix}, \\
&\begin{bmatrix} -7.7083 & -7.0835 \end{bmatrix}, \begin{bmatrix} -7.7083 & 5.4828 \end{bmatrix}, \begin{bmatrix} -7.7083 & -0.8003 \end{bmatrix}, \\
&\begin{bmatrix} -1.4251 & -7.0835 \end{bmatrix}, \begin{bmatrix} -1.4251 & -0.8003 \end{bmatrix}, \begin{bmatrix} -1.4251 & 5.4828 \end{bmatrix}, \\
&\begin{bmatrix} 4.8580 & -7.0835 \end{bmatrix}, \begin{bmatrix} 4.8580 & 5.4828 \end{bmatrix}, \begin{bmatrix} 4.8580 & -0.8003 \end{bmatrix}, \\
&\begin{bmatrix} 5.4828 & 4.8580 \end{bmatrix}, \begin{bmatrix} 5.4828 & -7.7083 \end{bmatrix}, \begin{bmatrix} 5.4828 & -1.4251 \end{bmatrix}, \\
&\begin{bmatrix} -0.8003 & -7.7083 \end{bmatrix}, \begin{bmatrix} -0.8003 & -1.4251 \end{bmatrix}, \begin{bmatrix} -0.8003 & 4.8580 \end{bmatrix}
\end{aligned}$$

45. Inverted Cosine Mixture [8]

$$0.1D - 0.1 \sum_{i=1}^D \cos(5\pi x_i) - \sum_{i=1}^D x_i^2$$

$$\begin{aligned}
-1 &\leq x_i \leq 1 \\
F^* &= 0 \\
X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}
\end{aligned}$$

46. Salomon [1]

$$1 - \cos(2\pi \|x\|) + 0.1 \|x\|, \|x\| = \sqrt{\sum_{i=1}^D x_i^2}$$

$$\begin{aligned}
-100 &\leq X \leq 100 \\
F^* &= 0 \\
X^* &= \begin{bmatrix} 0 & 0 & \cdots & 0 \end{bmatrix}
\end{aligned}$$

47. Matyas [1]

$$0.26\left(x_1^2+x_2^2\right)-0.48 x_1 x_2$$

$$-10 \leqslant X \leqslant 10, D=2$$

$$F^*=0$$

$$X^*=\left[\begin{array}{cc} 0 & 0 \end{array}\right]$$

48. Leon [1]

$$100\left(x_2-x_1^3\right)^2+\left(x_1-1\right)^2$$

$$-1.2 \leqslant X \leqslant 1.2, D=2$$

$$F^*=0$$

$$X^*=\left[\begin{array}{cc} 1 & 1 \end{array}\right]$$

49. Paviani [2]

$$\sum_{i=1}^D\left[\ln ^2\left(x_i-2\right)+\ln ^2\left(10-x_i\right)\right]-\left(\prod_{i=1}^D x_i\right)^{0.2}$$

$$2.001 \leqslant x_i \leqslant 9.999, D=10$$

$$F^*=-45.7784684040686$$

$$X^*=\left[\begin{array}{cccc} 9.350266 & 9.350266 & \cdots & 9.350266 \end{array}\right]$$

50. Sinusoidal [12]

$$-\left\{2.5 \prod_{i=1}^D \sin \left(x_i-\frac{\pi}{6}\right)+\prod_{i=1}^D \sin \left[5\left(x_i-\frac{\pi}{6}\right)\right]\right\}$$

$$0 \leqslant x_i \leqslant \pi$$

$$F^*=-3.5$$

$$X^*=\left[\begin{array}{cccc} \frac{2 \pi}{3} & \frac{2 \pi}{3} & \cdots & \frac{2 \pi}{3} \end{array}\right]$$

51. k-tablet [9]

$$\sum_{i=1}^K x_i^2+\sum_{i=K+1}^D\left(100 x_i\right)^2, K=int\left(\frac{D}{4}\right)$$

$$-5.12 \leq x_i \leq 5.12$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \cdots \quad 0]$$

52. Noncontinuous Rastrigin [\[4\]](#)

$$\sum_{i=1}^D [y_i^2 - 10 \cos(2\pi y_i) + 10], y_i = \begin{cases} x_i & |x_i| < 0.5 \\ \frac{\text{round}(2x_i)}{2} & |x_i| \geq 0.5 \end{cases}$$

$$-5.12 \leq x_i \leq 5.12$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \cdots \quad 0]$$

53. Fletcher [\[10\]](#)

54. Levy [\[9\]](#)

$$\sin^2(\pi w_1) + \sum_{i=1}^{D-1} (w_i - 1)^2 [1 + 10 \sin^2(\pi w_i + 1)] + (w_D - 1)^2 [1 + \sin^2(2\pi w_D)], w_i = 1 + \frac{x_i - 1}{4}$$

$$-10 \leq x_i \leq 10$$

$$F^* = 0$$

$$X^* = [1 \quad 1 \quad \cdots \quad 1]$$

55. Davis [\[1\]](#)

$$(x_1^2 + x_2^2)^{0.25} \left\{ \sin^2 \left[50(3x_1^2 + x_2^2)^{0.1} \right] + 1 \right\}$$

$$-100 \leq X \leq 100, D = 2$$

$$F^* = 0$$

$$X^* = [0 \quad 0]$$

56. Pathological [\[1\]](#)

$$\sum_{i=1}^{D-1} 0.5 + \frac{\sin^2 \left(\sqrt{100x_i^2 + x_{i+1}^2} \right) - 0.5}{1 + 0.001(x_i^2 - 2x_i x_{i+1} + x_{i+1}^2)^2}$$

$$-100 \leq X \leq 100$$

$$F^* = 0$$

$$X^* = [0 \quad 0 \quad \cdots \quad 0]$$

57. Schwefel P2.20 (SumPower) [3]

$$\sum_{i=1}^D |x_i|$$

$$\begin{aligned} -100 &\leq X \leq 100 \\ F^* &= 0 \\ X^* &= [0 \quad 0 \quad \cdots \quad 0] \end{aligned}$$

58. Booth [2]

$$(x_1 + 2x_2 - 7)^2 + (2x_1 + x_2 - 5)^2$$

$$\begin{aligned} -10 &\leq X \leq 10, D = 2 \\ F^* &= 0 \\ X^* &= [1 \quad 3] \end{aligned}$$

59. Zettl [1]

$$(x_1^2 + x_2^2 - 2x_1)^2 + 0.25x_1$$

$$\begin{aligned} -1 &\leq X \leq 5, D = 2 \\ F^* &= -0.003791237220468656 \\ X^* &= [-0.02989597760285287 \quad 0] \end{aligned}$$

60. Powell's Quartic [1]

$$(x_1 + 10x_2)^2 + 5(x_3 + x_4)^2 + (x_2 + 2x_3)^4 + 10(x_1 + 10x_4)^4$$

$$\begin{aligned} -10 &\leq X \leq 10, D = 4 \\ F^* &= 0 \\ X^* &= [0 \quad 0 \quad 0 \quad 0] \end{aligned}$$

61. Tablet [13]

$$10^6 x_1^2 + \sum_{i=2}^D x_i^6$$

$$\begin{aligned} -1 &\leq x_i \leq 1 \\ F^* &= 0 \\ X^* &= [0 \quad 0 \quad \cdots \quad 0] \end{aligned}$$

62. Brown [1]

$$\sum_{i=1}^{D-1} (x_i^2)^{(x_{i+1}^2+1)} + (x_{i+1}^2)^{(x_i^2+1)}$$

$$\begin{aligned} -1 &\leq X \leq 4 \\ F^* &= 0 \\ X^* &= [0 \quad 0 \quad \cdots \quad 0] \end{aligned}$$

63. Chung Reynolds [1]

$$\left(\sum_{i=1}^D (x_i^2) \right)^2$$

$$\begin{aligned} -100 &\leq X \leq 100 \\ F^* &= 0 \\ X^* &= [0 \quad 0 \quad \cdots \quad 0] \end{aligned}$$

64. Csendes [1]

$$\begin{cases} \sum_{i=1}^D x_i^6 \left(2 + \sin \frac{1}{x_i} \right) & \text{if } \prod_{i=1}^D x_i \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} -1 &\leq X \leq 1 \\ F^* &= 0 \\ X^* &= [0 \quad 0 \quad \cdots \quad 0] \end{aligned}$$

65. Bohachevsky 2 [1]

$$x_1^2 + 2x_2^2 - 0.3 \cos(3\pi x_1) \cos(4\pi x_2) + 0.3$$

$$\begin{aligned} -50 &\leq X \leq 50, D = 2 \\ F^* &= 0 \\ X^* &= [0 \quad 0] \end{aligned}$$

66. Bohachevsky 3 [1]

$$x_1^2 + 2x_2^2 - 0.3 \cos(3\pi x_1 + 4\pi x_2) \cos(4\pi x_2) + 0.3$$

$$\begin{aligned} -50 &\leq X \leq 50, D = 2 \\ F^* &= 0 \\ X^* &= [0 \quad 0] \end{aligned}$$

67. Colville [1]

$$\begin{aligned} &[100\left(x_1-x_2\right)]^2+\left(1-x_1\right)^2+90\left(x_4-x_3^2\right)^2+\left(1-x_3\right)^2+10.1\left[\left(x_2-1\right)^2+\left(x_4-1\right)^2\right] \\ &+19.8\left(x_2-1\right)\left(x_4-1\right) \\ &-10 \leq X \leq 10, D=4 \\ &F^*=0 \\ &X^*=\left[\begin{array}{cccc} 1 & 1 & 1 & 1 \end{array}\right] \end{aligned}$$

68. Bartels Conn [1]

$$\begin{aligned} &\left|x_1^2+x_2^2+x_1 x_2\right|+\left|\sin \left(x_1\right)\right|+\left|\cos \left(x_2\right)\right| \\ &-500 \leq X \leq 500, D=2 \\ &F^*=1 \\ &X^*=\left[\begin{array}{cc} 0 & 0 \end{array}\right] \end{aligned}$$

69. Bird [1]

$$\begin{aligned} &\left(x_1-x_2\right)^2+\sin \left(x_1\right) \exp \left(\left[1-\cos \left(x_2\right)\right]^2\right)+\cos \left(x_2\right) \exp \left(\left[1-\sin \left(x_1\right)\right]^2\right) \\ &-2 \pi \leq X \leq 2 \pi, D=2 \\ &F^*=-106.7645367198034 \\ &X^*=\left[\begin{array}{cc} 4.701055751981055 & 3.152946019601391 \end{array}\right],\left[\begin{array}{cc} -1.582142172055011 & -3.130246799635430 \end{array}\right] \end{aligned}$$

In []: