

```
In[ ]:= SetDirectory@NotebookDirectory[];
        |设置目录      |当前笔记本的目录
Import["QLanczos_package.m"];
        |导入
```

Parameters

```
In[ ]:= th = 10; (*multiple of noise*)
        d = 5;
        Id = IdentityMatrix[d];
            |单位矩阵
        κ = 0.1;
        η = 10.-15; (*machine precision*)

In[ ]:= MList = Table[10.j, {j, 4, 20, 1}];
            |表格

        (*the measurement number for a real matrix element is M*)
        rep = 100; (*replications*)
```

Model

```
In[ ]:= Ham = HeisenbergHam;
```

Spectrum

```
In[ ]:= {Λ, U} = funSpectrum[Ham];
        HamNorm = Max[Abs[Λ]]
            |... |绝对值

        Λ = Λ / HamNorm;
        Eg = Λ[[1]]

Out[ ]:= 17.0321

Out[ ]:= -1.

In[ ]:= htot = 27. / HamNorm

Out[ ]:= 1.58524
```

Reference state

```
In[ ]:= φ = φHeisenberg;
        φ = Flatten[Conjugate[U].φ];
            |压平      |共轭
        probφ = Abs[φ]^2;
            |绝对值
```

```

In[ ]:= pg = probφ[[1]] (*pg>10^-3*)
      ER = Total[probφ * Δ];
           |总计
      eR = ER - Eg
Out[ ]:= 0.682614

Out[ ]:= 0.119312

```

Power

```

In[ ]:= E0 = Eg + 1.;
      {Hmat, Smat} = funMatP[Δ, E0, d, probφ];

In[ ]:= EB = Hmat[[d, d]] / Smat[[d, d]];
      eB = EB - Eg (*used to identify τ for GP, ITE and F*)
Out[ ]:= 0.0072981

In[ ]:= {EK, cn} = funSubDiag[Hmat + η * Id, Smat + η * Id];
      eK = EK - Eg (*10^-9<eK<10^-2*)
Out[ ]:= 0.000390209

In[ ]:= costH = 1.;
      costS = 1.;
      ePList = funThrPracGP[MList, rep, Hmat, Smat, d, costH, costS, th, Eg];

```

Chebyshev Polynomial

```

In[ ]:= E0 = 0.;
      {Hmat, Smat} = funMatCP[Δ, E0, d, probφ, htot];

In[ ]:= costH = 1.;
      costS = 1.;
      eCPList = funThrPracF[MList, rep, Hmat, Smat, d, costH, costS, th, Eg];

```

Gaussian-Power

```

In[ ]:=  $\tau$ MIN = 0;
 $\tau$ MAX = 64;
While[True,
  |While... |真
     $\tau$  = ( $\tau$ MIN +  $\tau$ MAX) / 2.;
    {Hmat, Smat} = funMatGP[ $\Lambda$ , Eg,  $\tau$ , 1, prob $\phi$ , htot];
    EK = Hmat[[1, 1]] / Smat[[1, 1]];
    err = EK - Eg;
    If[err >  $\epsilon$ B,  $\tau$ MIN =  $\tau$ ];
    |如果
    If[err <  $\epsilon$ B,  $\tau$ MAX =  $\tau$ ];
    |如果
    (*Print[{err,  $\tau$ MIN,  $\tau$ MAX}];*)
    |打印
    If[Abs[err -  $\epsilon$ B] < 10-10, Break[]];
    |... |绝对值 |跳出循环
  ];
 $\tau$ 

```

Out[]:= 6.34638

```

In[ ]:= E0 = Eg;
{Hmat, Smat} = funMatGP[ $\Lambda$ , E0,  $\tau$ , d, prob $\phi$ , htot];

In[ ]:= costH = htot;
costS = 1.;
eGPList = funThrPracF[MList, rep, Hmat, Smat, d, costH, costS, th, Eg]; (*rescaled*)

```

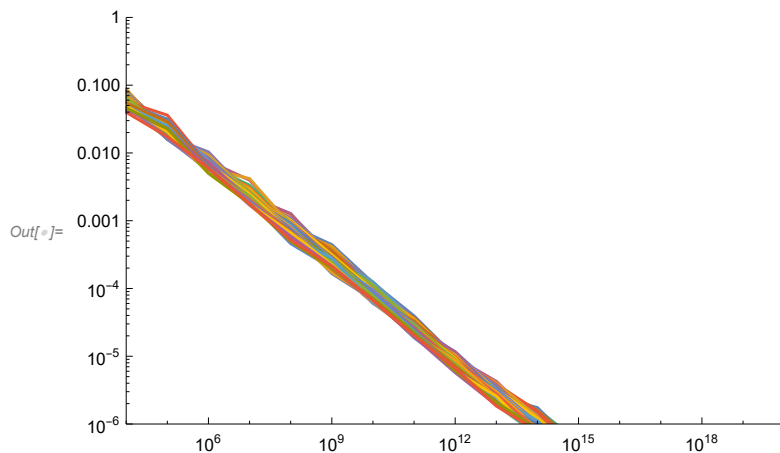
Gaussian-Power with different E0

```

In[ ]:=  $\delta$ List = Table[0.002 * j, {j, -50, 50}];
|表格
 $\delta$ Curves = {};
Do[
  |Do循环
     $\delta$  =  $\delta$ List[[j]];
    E0 = Eg +  $\delta$ ;
    {Hmat, Smat} = funMatGP[ $\Lambda$ , E0,  $\tau$ , d, prob $\phi$ , htot];
    eGPList = funThrPracF[MList, rep, Hmat, Smat, d, costH, costS, th, Eg];
    eGPList $\kappa$  = funExtract[eGPList, MList, rep,  $\kappa$ ];
    AppendTo[ $\delta$ Curves, Transpose[{MList, eGPList $\kappa$ }]];
    |附加 |转置
  , {j, 1, Length[ $\delta$ List]}];
  |长度

```

```
In[ ]:= PR = {{104, 1020}, {10-6, 100}};
ListLogLogPlot[δCurves, PlotRange → PR, Joined → True]
|点集的双对数图 |绘制范围 |连接点 |真
```



Inverse Power

```
In[ ]:= E0 = Eg - 1.;
{Hmat, Smat} = funMatIP[Δ, E0, d, probφ];

In[ ]:= costH = 1.;
costS = 1.;
εIPList = funThrPracGP[MList, rep, Hmat, Smat, d, costH, costS, th, Eg];
```

Imaginary-time evolution

```
In[ ]:= τMIN = 0;
τMAX = 64;
While[True,
|While... |真
  τ = (τMIN + τMAX) / 2.;
  {Hmat, Smat} = funMatITE[Δ, Eg, τ, d, probφ];
  EK = Hmat[[d, d]] / Smat[[d, d]];
  err = EK - Eg;
  If[err > εB, τMIN = τ];
  |如果
  If[err < εB, τMAX = τ];
  |如果
  (*Print[{err, τMIN, τMAX}];*)
  |打印
  If[Abs[err - εB] < 10-10, Break[]];
  |... |绝对值 |跳出循环
];
τ
Out[ ]:= 1.1768
```

```

In[ ]:= E0 = Eg;
        {Hmat, Smat} = funMatITE[Δ, E0, τ, d, probφ];

In[ ]:= costH = 1.;
        costS = 1.;
        eITEList = funThrPracGP[MList, rep, Hmat, Smat, d, costH, costS, th, Eg];

```

Real-time evolution

```

In[ ]:= ΔtList = Table[ $\frac{2. * \pi}{100} j$ , {j, 1, 100}];
           |表格

        eKList = ConstantArray[0, Length[ΔtList]];
           |常量数组 |长度

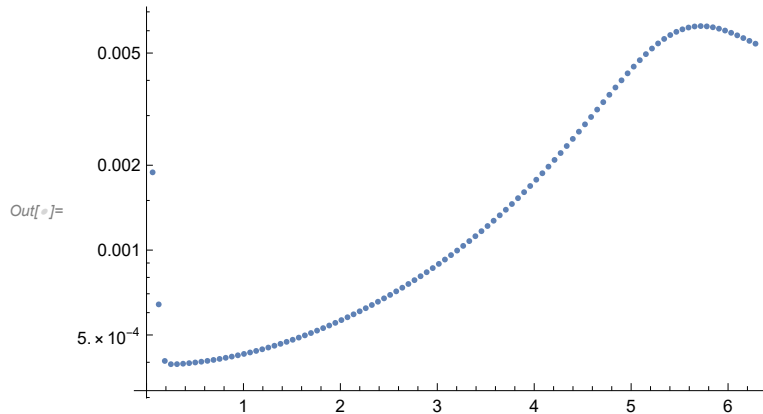
        Do[
           |Do循环
            Δt = ΔtList[[j]];
            {Hmat, Smat} = funMatRTE[Δ, Eg, Δt, d, probφ];
            {EK, cn} = funSubDiag[Hmat + η * Id, Smat + η * Id];
            err = EK - Eg;
            eKList[[j]] = err;
            , {j, 1, Length[ΔtList]}]
           |长度

        Δt = ΔtList[[Position[eKList, Min[eKList]][1, 1]]];
           |位置 |最小值

        ListLogPlot[Transpose[{ΔtList, eKList}], PlotRange → Full]
           |点集的对数图 |转置 |绘制范围 |全范围

Out[ ]:= 0.251327

```



```

In[ ]:= E0 = Eg;
        {Hmat, Smat} = funMatRTE[Δ, E0, Δt, d, probφ];

In[ ]:= costH = 1.;
        costS = 1.;
        eRTEList = funThrPracRTE[MList, rep, Hmat, Smat, d, costH, costS, th, Eg];

```

Filter

```

In[ ]:=  $\tau$ MIN = 0;
 $\tau$ MAX = 64;
While[True,
  |While... |真
     $\tau$  = ( $\tau$ MIN +  $\tau$ MAX) / 2.;
    {Hmat, Smat} = funMatF[ $\Delta$ , Eg, 0,  $\tau$ , 1, prob $\phi$ ];
    EK = Hmat[[1, 1]] / Smat[[1, 1]];
    err = EK - Eg;
    If[err >  $\epsilon$ B,  $\tau$ MIN =  $\tau$ ];
    |如果
    If[err <  $\epsilon$ B,  $\tau$ MAX =  $\tau$ ];
    |如果
    (*Print[{err,  $\tau$ MIN,  $\tau$ MAX}];*)
    |打印
    If[Abs[err -  $\epsilon$ B] < 10-10, Break[]];
    |... |绝对值 |跳出循环
  ];
 $\tau$ 

```

Out[]:= 11.2447

```

In[ ]:= ΔEList = Table[ $\frac{2.}{d * 100} * j$ , {j, 1, 100}];
           [表格]

εKList = ConstantArray[0, Length[ΔEList]];
           [常量数组]           [长度]

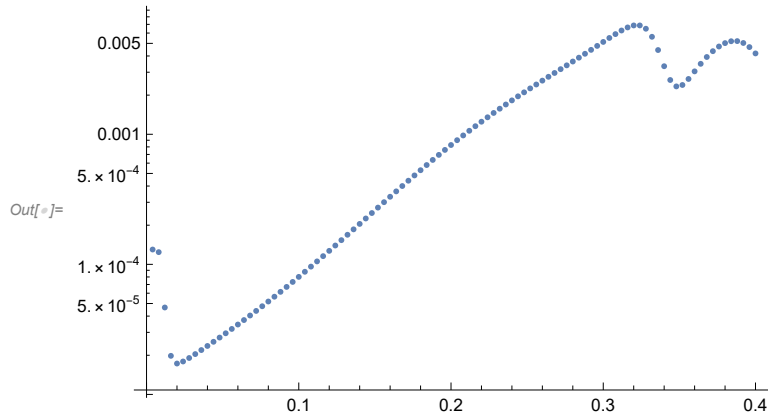
Do[ (
  [Do循环]
    ΔE = ΔEList[[j]];
    {Hmat, Smat} = funMatF[Δ, Eg, ΔE, τ, d, probφ];
    {EK, cn} = funSubDiag[Hmat + η * Id, Smat + η * Id];
    err = EK - Eg;
    εKList[[j]] = err;
  ), {j, 1, Length[ΔEList]}]
           [长度]

ΔE = ΔEList[[Position[εKList, Min[εKList]]][1, 1]]
           [位置]           [最小值]

ListLogPlot[Transpose[{ΔEList, εKList}], PlotRange → Full]
           [点集的对数图] [转置]           [绘制范围] [全范围]

```

Out[]:= 0.02



```

In[ ]:= E0 = Eg;
        {Hmat, Smat} = funMatF[Δ, E0, ΔE, τ, d, probφ];

In[ ]:= costH = 1.;
        costS = 1.;
        εFList = funThrPracF[MList, rep, Hmat, Smat, d, costH, costS, th, Eg];

```

Plot

```

In[ ]:= εPListκ = funExtract[εPList, MList, rep, κ];
        εCPListκ = funExtract[εCPList, MList, rep, κ];
        εGPListκ = funExtract[εGPList, MList, rep, κ];
        εIPListκ = funExtract[εIPList, MList, rep, κ];
        εITEListκ = funExtract[εITEList, MList, rep, κ];
        εRTEListκ = funExtract[εRTEList, MList, rep, κ];
        εFListκ = funExtract[εFList, MList, rep, κ];

```

```

In[ ]:= PR = {{104, 1020}, {10-6, 100}};

plot1 = ListLogLogPlot[ $\delta$ Curves, PlotRange  $\rightarrow$  PR,
  |点集的双对数图 |绘制范围

  Joined  $\rightarrow$  True, PlotStyle  $\rightarrow$  Table[LightBlue, Length[ $\delta$ Curves]],
  |连接点 |真 |绘图样式 |表格 |浅蓝色 |长度

  Frame  $\rightarrow$  True, FrameStyle  $\rightarrow$  Directive[Black, Thickness[0.002]],
  |边框 |真 |边框样式 |指令 |黑色 |粗细

  FrameTicksStyle  $\rightarrow$  Directive[Black, Thickness[0.002]], FrameLabel  $\rightarrow$  {"M", "€"}];
  |边框刻度样式 |指令 |黑色 |粗细 |边框标签

plot2 = ListLogLogPlot[{Transpose[{MList, 0. * MList + 1. *  $\epsilon$ K}],
  |点集的双对数图 |转置

  Transpose[{MList, 0. * MList + 2. *  $\epsilon$ K}], Transpose[{MList,  $\epsilon$ PList $\kappa$ ]},
  |转置 |转置

  Transpose[{MList,  $\epsilon$ CList $\kappa$ ]}, Transpose[{MList,  $\epsilon$ GList $\kappa$ ]},
  |转置 |转置

  Transpose[{MList,  $\epsilon$ IList $\kappa$ ]}, Transpose[{MList,  $\epsilon$ ITList $\kappa$ ]},
  |转置 |转置

  Transpose[{MList,  $\epsilon$ RTEList $\kappa$ ]}, Transpose[{MList,  $\epsilon$ FList $\kappa$ ]}],
  |转置 |转置

  PlotRange  $\rightarrow$  PR, Joined  $\rightarrow$  True, PlotStyle  $\rightarrow$  {{Thickness[0.004], Cyan},
  |绘制范围 |连接点 |真 |绘图样式 |粗细 |蓝绿色

  {Thickness[0.004], Brown}, {Thickness[0.004], Red, Dashed},
  |粗细 |棕色 |粗细 |红色 |虚线

  {Thickness[0.004], Yellow, Dashed}, {Thickness[0.004], Blue, Dashed},
  |粗细 |黄色 |虚线 |粗细 |蓝色 |虚线

  {Thickness[0.004], Green, Dashed}, {Thickness[0.004], Orange, Dashed},
  |粗细 |绿色 |虚线 |粗细 |橙色 |虚线

  {Thickness[0.004], Purple, Dashed}, {Thickness[0.004], Magenta, Dashed}},
  |粗细 |紫色 |虚线 |粗细 |品红色 |虚线

  Frame  $\rightarrow$  True, FrameStyle  $\rightarrow$  Directive[Black, Thickness[0.002]],
  |边框 |真 |边框样式 |指令 |黑色 |粗细

  FrameTicksStyle  $\rightarrow$  Directive[Black, Thickness[0.002]], FrameLabel  $\rightarrow$  {"M", "€"},
  |边框刻度样式 |指令 |黑色 |粗细 |边框标签

  LabelStyle  $\rightarrow$  {FontSize  $\rightarrow$  18, FontFamily  $\rightarrow$  "Arial"}, ImageSize  $\rightarrow$  500, PlotLegends  $\rightarrow$ 
  |标签样式 |字体大小 |字体系列 |图像尺寸 |绘图的图例

  Placed[LineLegend[{" $\epsilon_K$ ", "2 $\epsilon_K$ ", "P", "CP", "GP", "IP", "ITE", "RTE", "F"},
  |放置 |线的图例

    LegendFunction  $\rightarrow$  (Framed[#, FrameStyle  $\rightarrow$  LightGray] &), LegendMarkerSize  $\rightarrow$ 
    |图例函数 |加边框 |边框样式 |浅灰色 |图例标记尺寸

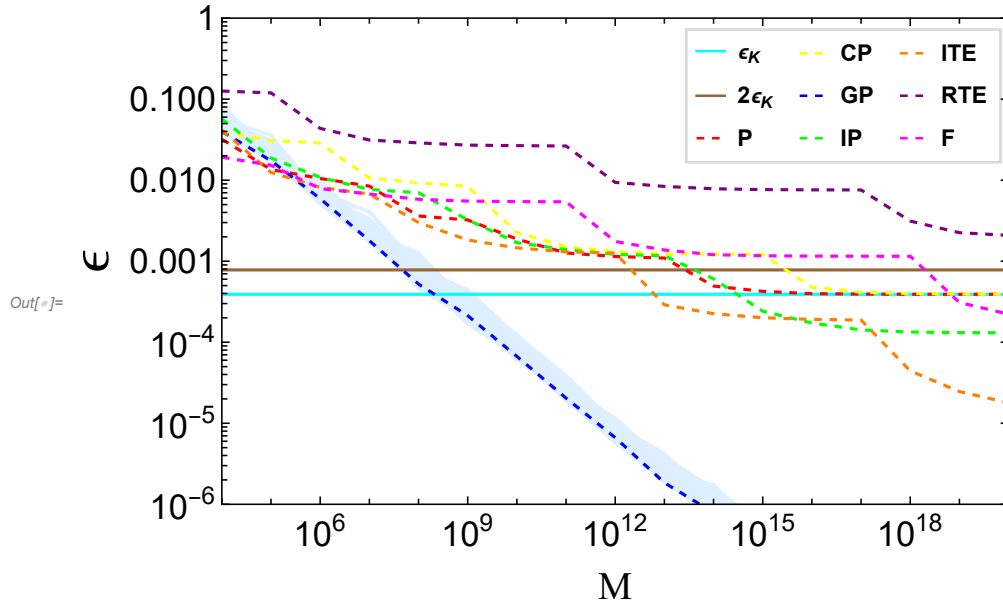
    {16, 8}, LabelStyle  $\rightarrow$  {Black, Bold, FontSize  $\rightarrow$  12, FontFamily  $\rightarrow$  "Arial"},
    |标签样式 |黑色 |粗体 |字体大小 |字体系列

    LegendMargins  $\rightarrow$  0, LegendLayout  $\rightarrow$  {"Column", 3}], {0.79, 0.84}]]];
    |图例边幅 |图例布局 |列

Fig = Show[plot1, plot2,
  |显示

  LabelStyle  $\rightarrow$  {FontSize  $\rightarrow$  18, FontFamily  $\rightarrow$  "Arial"}, ImageSize  $\rightarrow$  500]
  |标签样式 |字体大小 |字体系列 |图像尺寸

```

```

In[ ]:= l1leg1 = LineLegend[{Cyan}, {" $\epsilon_K$ "}, LegendMarkerSize → {14, 7},
  |线的图例 |蓝绿色 |图例标记尺寸
  LabelStyle → {Black, Bold, FontSize → 12, FontFamily → "Arial"}, LegendMargins → 0];
  |标签样式 |黑色 |粗体 |字体大小 |字体系列 |图例边幅
l1leg2 = LineLegend[{Brown}, {" $2\epsilon_K$ "}, LegendMarkerSize → {14, 7},
  |线的图例 |棕色 |图例标记尺寸
  LabelStyle → {Black, Bold, FontSize → 12, FontFamily → "Arial"}, LegendMargins → 0];
  |标签样式 |黑色 |粗体 |字体大小 |字体系列 |图例边幅
p1leg1 = PointLegend[{Red}, {"P"}, LegendMarkerSize → {14, 7},
  |点的图例 |红色 |图例标记尺寸
  LabelStyle → {Black, Bold, FontSize → 12, FontFamily → "Arial"}, LegendMargins → 0];
  |标签样式 |黑色 |粗体 |字体大小 |字体系列 |图例边幅
p1leg2 = PointLegend[{Yellow}, {"CP"}, LegendMarkerSize → {14, 7},
  |点的图例 |黄色 |图例标记尺寸
  LabelStyle → {Black, Bold, FontSize → 12, FontFamily → "Arial"}, LegendMargins → 0];
  |标签样式 |黑色 |粗体 |字体大小 |字体系列 |图例边幅
p1leg3 = PointLegend[{Blue}, {"GP"}, LegendMarkerSize → {14, 7},
  |点的图例 |蓝色 |图例标记尺寸
  LabelStyle → {Black, Bold, FontSize → 12, FontFamily → "Arial"}, LegendMargins → 0];
  |标签样式 |黑色 |粗体 |字体大小 |字体系列 |图例边幅
p1leg4 = PointLegend[{Green}, {"IP"}, LegendMarkerSize → {14, 7},
  |点的图例 |绿色 |图例标记尺寸
  LabelStyle → {Black, Bold, FontSize → 12, FontFamily → "Arial"}, LegendMargins → 0];
  |标签样式 |黑色 |粗体 |字体大小 |字体系列 |图例边幅
p1leg5 = PointLegend[{Orange}, {"ITE"}, LegendMarkerSize → {14, 7},
  |点的图例 |橙色 |图例标记尺寸
  LabelStyle → {Black, Bold, FontSize → 12, FontFamily → "Arial"}, LegendMargins → 0];
  |标签样式 |黑色 |粗体 |字体大小 |字体系列 |图例边幅
p1leg6 = PointLegend[{Purple}, {"RTE"}, LegendMarkerSize → {14, 7},
  |点的图例 |紫色 |图例标记尺寸
  LabelStyle → {Black, Bold, FontSize → 12, FontFamily → "Arial"}, LegendMargins → 0];
  |标签样式 |黑色 |粗体 |字体大小 |字体系列 |图例边幅
p1leg7 = PointLegend[{Magenta}, {"F"}, LegendMarkerSize → {14, 7},
  |点的图例 |品红色 |图例标记尺寸
  LabelStyle → {Black, Bold, FontSize → 12, FontFamily → "Arial"}, LegendMargins → 0];
  |标签样式 |黑色 |粗体 |字体大小 |字体系列 |图例边幅

```

Fig2 =

```

Legended[Show[Table[ListLogLogPlot[{Transpose[{MList, 0. * MList + 1. *  $\epsilon_K$ }],
图例 显示 表格 点集的双对数图 转置

Transpose[{MList, 0. * MList + 2. *  $\epsilon_K$ }], Transpose[{MList,  $\epsilon_{PList}[i]$ }],
转置 转置

Transpose[{MList,  $\epsilon_{CList}[i]$ }], Transpose[{MList,  $\epsilon_{GPList}[i]$ }],
转置 转置

Transpose[{MList,  $\epsilon_{IPList}[i]$ }], Transpose[{MList,  $\epsilon_{ITEList}[i]$ }],
转置 转置

Transpose[{MList,  $\epsilon_{RTEList}[i]$ }], Transpose[{MList,  $\epsilon_{FList}[i]$ }], PlotRange  $\rightarrow$  PR,
转置 转置 绘制范围

Joined  $\rightarrow$  {True, True, False, False, False, False, False, False, False},
连接点 真 真 假 假 假 假 假 假 假

Frame  $\rightarrow$  True, FrameStyle  $\rightarrow$  Directive[Black, Thickness[0.002]], FrameTicksStyle  $\rightarrow$ 
边框 真 边框样式 指令 黑色 粗细 边框刻度样式

Directive[Black, Thickness[0.002], 18], FrameLabel  $\rightarrow$  {"M", " $\epsilon$ "},
指令 黑色 粗细 边框标签

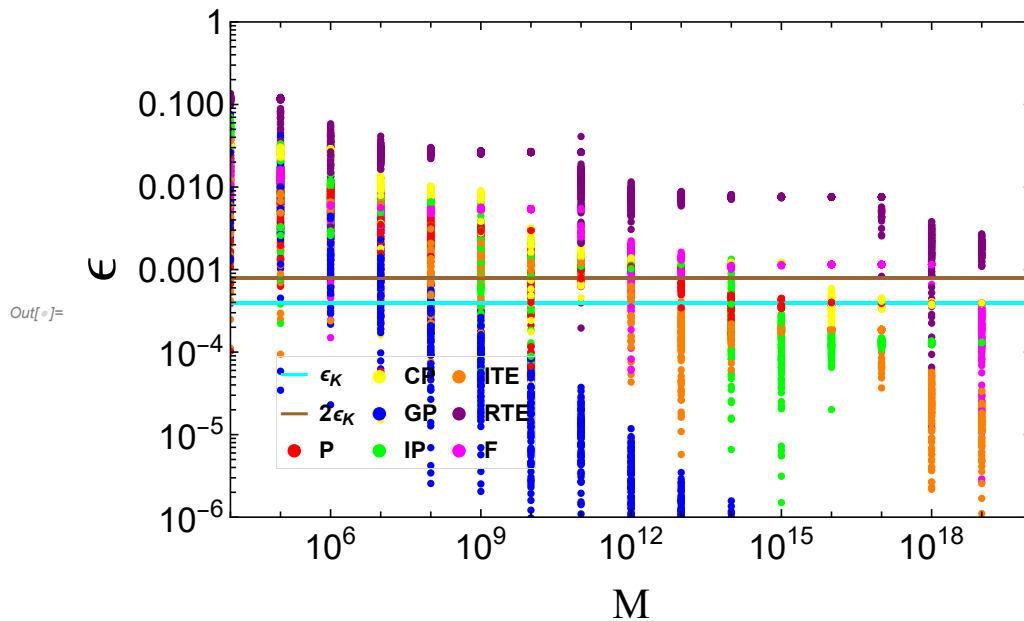
LabelStyle  $\rightarrow$  {FontSize  $\rightarrow$  22, FontFamily  $\rightarrow$  "Arial"}, ImageSize  $\rightarrow$  500, PlotStyle  $\rightarrow$ 
标签样式 字体大小 字体系列 图像尺寸 绘图样式

{Cyan, Brown, Red, Yellow, Blue, Green, Orange, Purple, Magenta}], {i, 1, rep}]],
蓝绿色 棕色 红色 黄色 蓝色 绿色 橙色 紫色 品红色

Placed[Grid[{{lleg1, pleg2, pleg5}, {lleg2, pleg3, pleg6}, {pleg1, pleg4, pleg7}},
放置 格子

Alignment  $\rightarrow$  Left, Frame  $\rightarrow$  True, FrameStyle  $\rightarrow$  LightGray], {{0.22, 0.45}, {0.5, 1.5}}]]
对齐 左 边框 真 边框样式 浅灰色

```



```

In[ ]:= Export["C:\\Users\\John\\Desktop\\Revise\\Second
Revision\\Thresholding comparison th=10.pdf", Fig];
导出 常量

```