HW1

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1 Problem 1

1.1 a

Since this truth table has three entries which compute F, we construct four conjuncts $\psi_i (1 \le i \le 3)$. We read the ψ_i off that table by listing the disjunction of all atoms, where we negate those atoms which are true in those lines

$$\psi_1 = (\neg p \vee \neg q), \ \psi_2 = (p \vee \neg q) \text{ and } \psi_3 = (\neg p \vee q)$$

The resulting ϕ in CNF is therefore,

$$\phi_1 = (\neg p \vee \neg q) \wedge (p \vee \neg q) \wedge (\neg p \vee q)$$

1.2 b

$$\phi_2 = (\neg p \lor \neg q \lor r) \land (\neg p \lor q \lor \neg r) \land (\neg p \lor q \lor r) \land (p \lor \neg q \lor r) \land (p \lor q \lor r)$$

1.3 c

$$\phi_3 = (\neg r \vee \neg s \vee \neg q) \wedge (\neg r \vee s \vee \neg q) \wedge (r \vee \neg s \vee \neg q) \wedge (r \vee \neg s \vee q) \wedge (r \vee s \vee \neg q)$$

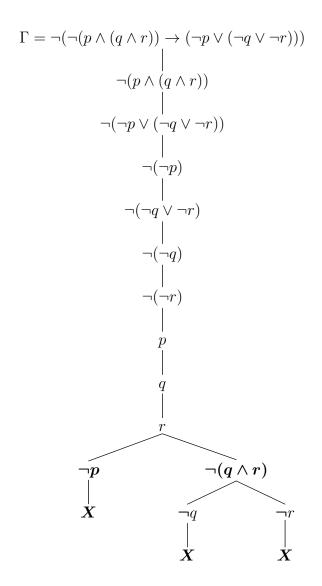
2 Problem 2

2.1 (1)

2.	1.	$\neg(p \land (q \land r))$	assumption
4. $ \neg p \lor (\neg q \lor \neg r) $ $\lor I 3$ \bot $\neg 2, 5$ 6. $ \neg \neg p $ $\neg I 3 - 5$ 7. $ \neg q $ assumption 8. $ \neg q \lor \neg r $ $\lor I 7$ $ \neg p \lor (\neg q \lor \neg r) $ $\lor I 8$ 10. $ \bot$ $ \neg q $ $\neg I 7 - 10$ 12. $ \neg r $ assumption 13. $ \bot$ $ \neg q \lor \neg r $ $\lor I 12$ $ \neg p \lor (\neg q \lor \neg r) $ $\lor I 13$ $ \bot$ $ \neg p \lor (\neg q \lor \neg r) $ $\lor I 13$ 15. $ \bot$ $ \neg r $ $ \neg I 12 - 15$ 17. $ p $ $ \neg \neg 6$ 18. $ q $ $ \neg \neg I 1$ 19. $ r $ $ \neg \neg 16$ 20. $ p \land (q \land r) $ $ \land I 17 - 19$ 21. $ \bot$ $ \neg 1, 20$ $ \neg \neg (\neg p \lor (\neg q \lor \neg r)) $ $ \neg I 2 - 21$	2.	$\neg(\neg p \lor (\neg q \lor \neg r))$	assumption
5. 6. $\neg \neg p$ $\neg I 3 - 5$ 7. $\neg q$ assumption 8. $\neg q \lor \neg r$ $\lor I 7$ 9. $\neg p \lor (\neg q \lor \neg r)$ $\lor I 8$ 10. \bot $\neg q$ $\neg I 7 - 10$ 12. $\neg r$ assumption 13. \bot $\neg q \lor \neg r$ $\lor I 12$ 14. \bot $\neg p \lor (\neg q \lor \neg r)$ $\lor I 13$ 15. \bot $\neg r$ $\neg I 12 - 15$ 17. p $\neg \neg 6$ 18. q $\neg \neg 11$ 19. r $\neg \neg 16$ 20. $p \land (q \land r)$ $\land I 17 - 19$ 21. \bot $\neg 1, 20$ 22. $\neg \neg (\neg p \lor (\neg q \lor \neg r))$ $\neg I 2 - 21$	3.	$\neg p$	assumption
6. $\neg p$ $\neg I 3 - 5$ 7. q assumption 8. $q \lor \neg r$ $\lor I 7$ 9. $1 . 1 . 1 . 1 . 1 . 1 . 1 . 1 . 1 . 1$	4.		∨I 3
7.	5.		$\neg 2, 5$
8.	6.	$\neg \neg p$	¬I 3 – 5
9.	7.	$\neg q$	assumption
10. $\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$	8.	$ \mid \mid \mid \neg q \lor \neg r $	∨I 7
11. $\neg \neg q$ $\neg I \ 7 - 10$ 12. $\neg r$ assumption 13. $\neg q \lor \neg r$ $\lor I \ 12$ 14. $\neg p \lor (\neg q \lor \neg r)$ $\lor I \ 13$ 15. \bot $\neg T$ $\neg I \ 12 - 15$ 17. p $\neg \neg 6$ 18. q $\neg \neg 11$ 19. r $\neg \neg 16$ 20. $p \land (q \land r)$ $\land I \ 17 - 19$ 21. \bot $\neg \neg (\neg p \lor (\neg q \lor \neg r))$ $\neg I \ 2 - 21$	9.		∨I 8
12. $\neg r$ assumption 13. $\neg q \lor \neg r$ $\lor I 12$ 14. $\neg p \lor (\neg q \lor \neg r)$ $\lor I 13$ 15. \bot $\neg T$ $\neg I 12 - 15$ 17. p $\neg T$ $\neg G$ 18. q $\neg T$ $\neg G$ 19. r $\neg G$ 20. $p \land (q \land r)$ $\land I 17 - 19$ 21. \bot $\neg G$ 22. $\neg G$	10.		¬ 2, 9
13.	11.	$\neg \neg q$	¬I 7 – 10
14. $ \neg p \lor (\neg q \lor \neg r) $ \lor I 13 15. $ \bot $ $\neg 2$, 14 16. $ \neg \neg r $ \neg I 12 - 15 17. $ p $ $\neg \neg 6$ 18. $ q $ $\neg \neg 11$ 19. $ r $ $\neg \neg 16$ 20. $ p \land (q \land r) $ \land I 17 - 19 21. $ \bot $ $\neg 1$, 20 22. $ \neg \neg (\neg p \lor (\neg q \lor \neg r)) $ \neg I 2 - 21	12.	$\neg r$	assumption
15. \bot $\neg 2, 14$ 16. $\neg \neg r$ $\neg I 12 - 15$ 17. p $\neg \neg 6$ 18. q $\neg \neg 11$ 19. r $\neg \neg 16$ 20. $p \land (q \land r)$ $\land I 17 - 19$ 21. \bot $\neg 1, 20$ 22. $\neg \neg (\neg p \lor (\neg q \lor \neg r))$ $\neg I 2 - 21$	13.		∨I 12
16. $\neg \neg r$ $\neg I \ 12 - 15$ 17. p $\neg \neg 6$ 18. q $\neg \neg 11$ 19. r $\neg \neg 16$ 20. $p \land (q \land r)$ $\land I \ 17 - 19$ 21. \bot $\neg 1, 20$ 22. $\neg \neg (\neg p \lor (\neg q \lor \neg r))$ $\neg I \ 2 - 21$	14.		∨I 13
17. p $\neg\neg 6$ 18. q $\neg\neg 11$ 19. r $\neg\neg 16$ 20. $p \land (q \land r)$ $\land I \ 17 - 19$ 21. \bot $\neg 1, 20$ 22. $\neg\neg (\neg p \lor (\neg q \lor \neg r))$ $\neg I \ 2 - 21$	15.		$\neg 2, 14$
18. q $\neg\neg 11$ 19. r $\neg\neg 16$ 20. $p \land (q \land r)$ $\land I \ 17 - 19$ 21. \bot $\neg 1, 20$ 22. $\neg\neg (\neg p \lor (\neg q \lor \neg r))$ $\neg I \ 2 - 21$	16.	$\neg \neg r$	¬I 12 – 15
19. r $\neg \neg 16$ 20. $p \land (q \land r)$ $\land I \ 17 - 19$ 21. \bot $\neg \neg (\neg p \lor (\neg q \lor \neg r))$ $\neg I \ 2 - 21$	17.	p	¬¬ 6
20. $p \wedge (q \wedge r) \qquad \wedge I 17 - 19$ 21. $\perp \qquad \neg 1, 20$ 22. $\neg \neg (\neg p \vee (\neg q \vee \neg r)) \neg I 2 - 21$	18.	q	¬¬ 11
21.	19.	r	¬¬ 16
22.	20.	$p \wedge (q \wedge r)$	∧I 17 – 19
(1 (1 //)	21.		¬ 1, 20
$23. \qquad (\neg p \lor (\neg q \lor \neg r)) \qquad \neg \neg 22$	22.	$\neg\neg(\neg p \lor (\neg q \lor \neg r))$	\neg I 2 $-$ 21
	23.	$(\neg p \vee (\neg q \vee \neg r))$	$\neg \neg 22$

2.2 (2)

We show $(\neg(p \land (q \land r)) \rightarrow (\neg p \lor (\neg q \lor \neg r)))$ is valid (a tautology) by showing its negation is a contradiction (unsatisfiable) :



3 Problem 3

consider event t_{j_1,j_2} is T when $j_1=j_2$

3.1 (a)

$$\bigwedge\{r_{i,j}|j\in\{1,...,n\}\} \land (r_{i,j_1} \land r_{i,j_2} \land t_{j_1,j_2})$$

On the left of \wedge we ensure at least one queen at row i.

On the right of the \land we ensure at most one queen at row i. i.e. if there are 2 queens at row i, they are the same queen at the same location.

3.2 (b)

 $\bigwedge\{r_{i,j}|i\in\{1,...,n\}\}\ \land (r_{i_1,j}\land r_{i_2,j}\land t_{i_1,i_2})$ On the left of \land we ensure at least one queen at column j.

On the right of the \land we ensure at most one queen at column j. i.e. if there are 2 queens at column j, they are the same queen at the same location.

3.3 (c)

 $r_{x_1,y_1} \wedge r_{x_2,y_2} \wedge t_{(x_2-x_1),(y_2-y_1)} \to t_{x_1,x_2} \wedge t_{y_1,y_2}$

If there are 2 queens in one diagonal, they are the same queen at the same location.

3.4 (d)

 $r_{x_1,y_1} \wedge r_{x_2,y_2} \wedge t_{(x_1-x_2),(y_2-y_1)} \to t_{x_1,x_2} \wedge t_{y_1,y_2}$

If there are 2 queens in one anti-diagonal, they are the same queen at the same location.

4 Problem 4

The condition that "The set Γ is finitely satisfiable" seems not true. Now we have an infinite board.

5 Problem 5

https://github.com/Zongshun96/CS511/blob/master/hw2/parity.smt2

6 Problem 6

https://github.com/Zongshun96/CS511/blob/master/hw2/parity.py