HW1

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1 Problem 1

1.1 a

Since this truth table has three entries which compute F, we construct four conjuncts $\psi_i (1 \leq i \leq 3)$. We read the ψ_i off that table by listing the disjunction of all atoms, where we negate those atoms which are true in those lines

$\psi_1 = (\neg p \vee \neg q)$, $\psi_2 = (p \vee \neg q)$ and $\psi_3 = (\neg p \vee q)$

The resulting ϕ in CNF is therefore,

$\phi_1 = (\neg p \vee \neg q) \wedge (p \vee \neg q) \wedge (\neg p \vee q)$

1.2 b

$\phi_2 = (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r)$

1.3 c

$\phi_3 = (\neg r \vee \neg s \vee \neg q) \wedge (\neg r \vee s \vee \neg q) \wedge (r \vee \neg s \vee \neg q) \wedge (r \vee \neg s \vee q) \wedge (r \vee s \vee \neg q)$

2 Problem 2

2.1 (1)

1.	$\neg(p \wedge (q \wedge r))$	assumption
2.	$\neg(\neg p \vee (\neg q \vee \neg r))$	assumption
3.	$\neg p$	assumption
4.	$\neg p \vee (\neg q \vee \neg r)$	$\vee$I 3
5.	$\bot$	$\neg$ 2, 5
6.	$\neg\neg p$	$\neg$I 3 – 5
7.	$\neg q$	assumption
8.	$\neg q \vee \neg r$	$\vee$I 7
9.	$\neg p \vee (\neg q \vee \neg r)$	$\vee$I 8
10.	$\bot$	$\neg$ 2, 9
11.	$\neg\neg q$	$\neg$I 7 – 10
12.	$\neg r$	assumption
13.	$\neg q \vee \neg r$	$\vee$I 12
14.	$\neg p \vee (\neg q \vee \neg r)$	$\vee$I 13
15.	$\bot$	$\neg$ 2, 14
16.	$\neg\neg r$	$\neg$I 12 – 15
17.	p	$\neg\neg$ 6
18.	q	$\neg\neg$ 11
19.	r	$\neg\neg$ 16
20.	$p \wedge (q \wedge r)$	$\wedge$I 17 – 19
21.	$\bot$	$\neg$ 1, 20
22.	$\neg\neg(\neg p \vee (\neg q \vee \neg r))$	$\neg$I 2 – 21
23.	$(\neg p \vee (\neg q \vee \neg r))$	$\neg\neg$ 22

2.2 (2)

We show $(\neg(p \wedge (q \wedge r)) \rightarrow (\neg p \vee (\neg q \vee \neg r)))$ is valid (a tautology) by showing its negation is a contradiction (unsatisfiable) :

$$\Gamma = \neg(\neg(p \wedge (q \wedge r)) \to (\neg p \vee (\neg q \vee \neg r)))$$

$$\neg(p \wedge (q \wedge r))$$

$$\neg(\neg p \vee (\neg q \vee \neg r))$$

$$\neg(\neg p)$$

$$\neg(\neg q \vee \neg r)$$

$$\neg(\neg q)$$

$$\neg(\neg r)$$

$$p$$

$$q$$

$$r$$

$$\boldsymbol{\neg p} \qquad \boldsymbol{\neg(q \wedge r)}$$

$$\boldsymbol{X} \qquad \neg q \qquad \neg r$$

$$\boldsymbol{X} \qquad \boldsymbol{X}$$

3 Problem 3

consider event t_{j_1,j_2} is T when $j_1 = j_2$

3.1 (a)

$\bigwedge\{r_{i,j} | j \in \{1, ..., n\}\} \wedge (r_{i,j_1} \wedge r_{i,j_2} \wedge t_{j_1,j_2})$
On the left of $\wedge$ we ensure at least one queen at row i.
On the right of the $\wedge$ we ensure at most one queen at row i. i.e. if there are 2 queens at row i, they are the same queen at the same location.

3.2 (b)

$\bigwedge\{r_{i,j}|i \in \{1,...,n\}\} \wedge (r_{i_1,j} \wedge r_{i_2,j} \wedge t_{i_1,i_2})$ On the left of $\wedge$ we ensure at least one queen at column j.
On the right of the $\wedge$ we ensure at most one queen at column j. i.e. if there are 2 queens at column j, they are the same queen at the same location.

3.3 (c)

$r_{x_1,y_1} \wedge r_{x_2,y_2} \wedge t_{(x_2-x_1),(y_2-y_1)} \rightarrow t_{x_1,x_2} \wedge t_{y_1,y_2}$
If there are 2 queens in one diagonal, they are the same queen at the same location.

3.4 (d)

$r_{x_1,y_1} \wedge r_{x_2,y_2} \wedge t_{(x_1-x_2),(y_2-y_1)} \rightarrow t_{x_1,x_2} \wedge t_{y_1,y_2}$
If there are 2 queens in one anti-diagonal, they are the same queen at the same location.

4 Problem 4

The condition that "The set Γ is finitely satisfiable" seems not true. Now we have an infinite board.

5 Problem 5

`https://github.com/Zongshun96/CS511/blob/master/hw2/parity.smt2`

6 Problem 6

`https://github.com/Zongshun96/CS511/blob/master/hw2/parity.py`