

Package ‘FlexBayesMed’

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Type Package

Title Modeling Skewed and Heavy-Tailed Errors in Bayesian Mediation Analysis

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Description Flexible Bayesian mediation models, including simulation from the centered two-piece t-distribution (CTPT). Provides methods for skewed and heavy-tailed error modeling in a mediation framework.

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Imports rstan,
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dctpt	<i>Density of the Centered Two-Piece Student t-Distribution (CTPT)</i>
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Description

Density of the Centered Two-Piece Student t -Distribution (CTPT)

Usage

```
dctpt(x, gamma, nu, log = FALSE)
```

Arguments

x	Numeric vector. Quantiles at which to compute the density.
gamma	Positive numeric scalar. Skewness parameter.
nu	Numeric scalar $\nu > 1$ (degrees of freedom) or "normal".
log	Logical. Return log-density? Default: FALSE.

Details

The CTPT distribution is defined by shifting the original two-piece Student t -distribution (with skewness parameter γ and degrees of freedom ν) by its mean, ensuring the resulting distribution has zero mean. The density of the CTPT distribution is given by:

$$p(\varepsilon|\gamma, \nu) = \frac{2}{\gamma + \frac{1}{\gamma}} \left[f_{\nu} \left(\frac{\varepsilon + m(\gamma, \nu)}{\gamma} \right) I_{[-m(\gamma, \nu), +\infty)}(\varepsilon) + f_{\nu}(\gamma(\varepsilon + m(\gamma, \nu))) I_{(-\infty, -m(\gamma, \nu))}(\varepsilon) \right], \quad \gamma \in \mathbb{R}_+, \nu > 1,$$

where:

- f_{ν} is the Student t -density with ν degrees of freedom.
- $m(\gamma, \nu)$ is the mean of the unadjusted two-piece distribution, computed by [mean_two_piece_student](#).
- $I_A(\varepsilon)$ denotes the indicator function for set A .

Value

Numeric vector of densities.

Examples

```
x <- seq(-5, 5, length.out = 100)
density <- dctpt(x, gamma = 2, nu = 5)
plot(x, density, type = "l")
```

flex_med

*Mediation Analysis with Centred Two-Piece Student t (CTPT) Errors***Description**Mediation Analysis with Centred Two-Piece Student t (CTPT) Errors**Usage**

```
flex_med(
  X,
  M,
  Y,
  model = "full",
  q00 = 1/3,
  q01 = 1/3,
  alpha_gamma = 2,
  beta_gamma = 2,
  gamma_lower = 0.05,
  gamma_upper = 20,
  d_nu = 0.01,
  nu_lower = 1.01,
  iter = 30000,
  warmup = floor(iter/2),
  chains = 1,
  seed = 123,
  verbose = FALSE,
  refresh = 2500,
  repetitions = 1,
  method = "normal",
  bridgesampling_cores = 1,
  use_neff = TRUE,
  maxiter = 1000,
  silent = FALSE,
  ...
)
```

Arguments

X	A numeric vector of length n (the independent variable).
M	A numeric vector of length n (the mediator).
Y	A numeric vector of length n (the response).
model	A string: which model to use ("full", "gamma-only", "nu-only", "normal")
q00	A non-negative numeric value giving the conditional probability $p(\alpha = 0, \beta = 0 \mid \alpha\beta = 0)$. Defaults to 1/3.
q01	A non-negative numeric value giving the conditional probability $p(\alpha = 0, \beta \neq 0 \mid \alpha\beta = 0)$. Defaults to 1/3. Here, $q00 + q01 + q10 = 1$, where $q10 = p(\alpha \neq 0, \beta = 0 \mid \alpha\beta = 0)$ is computed internally as $1 - q00 - q01$. These three probabilities correspond to

the disjoint ways in which the mediation effect can be zero (i.e., $\alpha\beta = 0$). They must be non-negative and sum to 1.

alpha_gamma, beta_gamma Shape and rate for the Gamma prior on skewness parameter γ .

gamma_lower, gamma_upper Truncation bounds for γ ($0 < \gamma_{\text{lower}} \leq \gamma \leq \gamma_{\text{upper}}$).

d_nu Rate for the exponential prior on degrees-of-freedom ν .

nu_lower Lower truncation bound for ν (must satisfy $\nu > 1$).

iter, warmup, chains, seed, verbose, refresh, ... Additional arguments passed to `rstan::stan()`.

repetitions, method, bridgesampling_cores, use_neff, maxiter, silent Additional arguments passed to `bridgesampling::bridge_sampler()`.

Value

A list with the following elements:

X2M_with_alpha A stanfit object for the mediator model with $\alpha \neq 0$.

XM2Y_with_beta A stanfit object for the outcome model with $\beta \neq 0$.

X2M_without_alpha A stanfit object for the mediator model with $\alpha = 0$.

XM2Y_without_beta A stanfit object for the outcome model with $\beta = 0$.

samples_med A numeric vector of posterior draws for $\alpha\beta$.

bf_med A numeric value giving the Bayes factor for the mediation effect.

flex_mr	<i>Fit a Linear Regression with Centred Two-Piece Student t (CTPT) Errors</i>
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Description

Fit a Linear Regression with Centred Two-Piece Student t (CTPT) Errors

Usage

```
flex_mr(
  X,
  Y,
  model = "full",
  alpha_gamma = 2,
  beta_gamma = 2,
  gamma_lower = 0.05,
  gamma_upper = 20,
  d_nu = 0.01,
  nu_lower = 1.01,
  iter = 30000,
  warmup = floor(iter/2),
  chains = 1,
  seed = 123,
  verbose = TRUE,
  refresh = 2500,
  ...
)
```

Arguments

X	A numeric matrix of predictors ($n \times k$).
Y	A numeric vector of length n (the response).
model	A string: which model to use ("full", "gamma-only", "nu-only", "normal")
alpha_gamma, beta_gamma	Shape and rate for the Gamma prior on skewness parameter γ .
gamma_lower, gamma_upper	Truncation bounds for γ ($0 < \gamma_{\text{lower}} \leq \gamma \leq \gamma_{\text{upper}}$).
d_nu	Rate for the exponential prior on degrees-of-freedom ν .
nu_lower	Lower truncation bound for ν (must satisfy $\nu > 1$).
iter, warmup, chains, seed, verbose, refresh, ...	Additional arguments passed to <code>rstan::stan()</code> .

Value

An object of class `stanfit`.

mean_two_piece_student

Mean of the Uncentered Two-Piece Student t (or Normal) Distribution

Description

Computes the theoretical mean $m(\gamma, \nu)$ of the two-piece Student t_ν distribution with skewness parameter γ . If `nu = "normal"` the function returns the two-piece *normal* limit $m^{\text{normal}}(\gamma) = \sqrt{2/\pi} (\gamma - 1/\gamma)$ as $\nu \rightarrow \infty$. This corresponds to Eq. (1) in Fernández & Steel (1998).

Usage

```
mean_two_piece_student(gamma, nu)
```

Arguments

gamma	Positive numeric scalar. Skewness parameter $\gamma > 0$.
nu	Either a numeric scalar $\nu > 1$ (degrees of freedom) <i>or</i> the literal string "normal" for the $\nu \rightarrow \infty$ limit.

Details

The closed-form expression for finite ν is

$$m(\gamma, \nu) = \frac{2\nu\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi\nu}(\nu-1)\Gamma(\frac{\nu}{2})} \left(\gamma - \frac{1}{\gamma} \right).$$

For the normal limit (`nu = "normal"`) the mean simplifies to $\sqrt{2/\pi} (\gamma - 1/\gamma)$.

Value

A numeric scalar giving the mean $m(\gamma, \nu)$.

References

Fernández, C. & Steel, M. F. J. (1998). “On Bayesian Modeling of Fat Tails and Skewness.” *Journal of the American Statistical Association*, **93**(441), 359–371.

Examples

```
mean_two_piece_student(gamma = 2, nu = 5)          # finite-nu case, approximately 1.424
mean_two_piece_student(gamma = 3, nu = "normal") # normal limit, approximately 2.128
```

mode_ctpt	<i>Mode of the Centered Two-Piece Student t (or Normal) Distribution</i>
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Description

Computes the theoretical mode $mode(\gamma, \nu)$ of the centred two-piece Student t_ν distribution with skewness parameter γ . If `nu = "normal"` the function returns the two-piece *normal* limit $mode^{\text{normal}}(\gamma) = -\sqrt{2/\pi} (\gamma - 1/\gamma)$ as $\nu \rightarrow \infty$.

Usage

```
mode_ctpt(gamma, nu)
```

Arguments

<code>gamma</code>	Positive numeric scalar. Skewness parameter $\gamma > 0$.
<code>nu</code>	Either a numeric scalar $\nu > 1$ (degrees of freedom) <i>or</i> the literal string "normal" for the $\nu \rightarrow \infty$ limit.

Details

The closed-form expression for finite ν is

$$mode(\gamma, \nu) = -\frac{2\nu\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi\nu}(\nu-1)\Gamma(\frac{\nu}{2})} \left(\gamma - \frac{1}{\gamma} \right).$$

For the normal limit (`nu = "normal"`) the mean simplifies to $-\sqrt{2/\pi} (\gamma - 1/\gamma)$.

Value

A numeric scalar giving the mean $mode(\gamma, \nu)$.

Examples

```
mode_ctpt(gamma = 2, nu = 5)          # finite-nu case, approximately -1.424
mode_ctpt(gamma = 3, nu = "normal") # normal limit, approximately -2.128
```

pctpt	<i>Cumulative Distribution Function (CDF) of the Centred Two-Piece Student t-Distribution (CTPT)</i>
-------	--

Description

Cumulative Distribution Function (CDF) of the Centred Two-Piece Student *t*-Distribution (CTPT)

Usage

```
pctpt(q, gamma, nu, lower.tail = TRUE, log.p = FALSE)
```

Arguments

q	Numeric vector of quantiles.
gamma	Positive scalar. Skewness parameter ($\gamma > 0$).
nu	Degrees of freedom. Either numeric >1 or "normal" for the Gaussian limit.
lower.tail	Logical; if TRUE (default), probabilities are $P(X \leq q)$.
log.p	Logical; if TRUE, returns log probabilities.

Details

The CDF is piecewise-defined with split point at $\varepsilon = -m(\gamma, \nu)$:

For $\varepsilon < -m(\gamma, \nu)$:

$$F(\varepsilon \mid \gamma, \nu) = \frac{2}{1 + \gamma^2} F_\nu[\gamma(\varepsilon + m(\gamma, \nu))]$$

For $\varepsilon \geq -m(\gamma, \nu)$:

$$F(\varepsilon \mid \gamma, \nu) = \frac{2\gamma^2}{1 + \gamma^2} F_\nu\left[\frac{\varepsilon + m(\gamma, \nu)}{\gamma}\right] + \frac{1 - \gamma^2}{1 + \gamma^2}$$

Where:

- $m(\gamma, \nu)$ is the mean-adjustment term (see Proposition 1 in manuscript)
- F_ν = CDF of Student *t* (ν df) or standard normal ($\nu = \text{"normal"}$)
- Gaussian mean: $m(\gamma) = \sqrt{\frac{2}{\pi}} \left(\gamma - \frac{1}{\gamma}\right)$

References

TBA

See Also

[mean_two_piece_student\(\)](#) for the mean-adjustment computation

Examples

```
# Symmetric case (gamma=1) matches standard t
pctpt(q = 0, gamma = 1, nu = 5) # Returns 0.5
pt(0, df = 5)                   # Returns 0.5

# Skewed case
pctpt(q = 0, gamma = 2, nu = 5)

# Normal limit
pctpt(q = 0, gamma = 1.5, nu = "normal")
```

qctpt	<i>Quantile Function of the Centred Two-Piece Student t-Distribution (CTPT)</i>
-------	---

Description

Inverts the CDF of the CTPT distribution to find ε such that $P(X \leq \varepsilon) = p$.

Usage

```
qctpt(p, gamma, nu, lower.tail = TRUE, log.p = FALSE)
```

Arguments

p	Numeric vector of probabilities in $[0, 1]$.
gamma	Positive scalar ($\gamma > 0$). The skewness parameter.
nu	Either a numeric > 1 (degrees of freedom) or the string "normal".
lower.tail	Logical; if TRUE (default), probabilities are $P(X \leq q)$.
log.p	Logical; if TRUE, p is treated as $\log(p)$.

Details

The distribution is defined by a piecewise CDF:

$$F(\varepsilon|\gamma, \nu) = \begin{cases} \frac{2}{1+\gamma^2} F_\nu[\gamma(\varepsilon+m)], & \varepsilon < -m, \\ \frac{2\gamma^2}{1+\gamma^2} F_\nu\left(\frac{\varepsilon+m}{\gamma}\right) + \frac{1-\gamma^2}{1+\gamma^2}, & \varepsilon \geq -m, \end{cases}$$

where $m = m(\gamma, \nu)$ is the mean-adjustment term, and F_ν is the standard Student t (if $\nu > 1$) or standard normal (if $\text{nu} = \text{"normal"}$) CDF.

Value

Numeric vector of quantiles corresponding to the given probabilities.

See Also

[pctpt](#) for the CDF and [dctpt](#) for the PDF.

Examples

```
qctpt(p = 0.975, gamma = 1, nu = "normal") # Returns 1.96
qctpt(p = 0.025, gamma = 2, nu = 2) # Returns -3.40
```

rctpt	<i>Generate Samples from the Centred Two-Piece Student t-Distribution (CTPT)</i>
-------	--

Description

Produces independent and identically distributed (i.i.d.) samples from the CTPT distribution, which is a zero-mean adjusted version of the two-piece Student t -distribution (or normal). The adjustment ensures that the error terms satisfy the zero-mean assumption, which is needed in cases such as mean regression or mediation analysis.

Usage

```
rctpt(N, gamma, nu)
```

Arguments

N	Positive integer. The number of samples to generate.
gamma	Positive numeric scalar. Skewness parameter ($\gamma > 0$).
nu	Either a numeric scalar $\nu > 1$ (degrees of freedom) or the string "normal" for the normal limit ($\nu \rightarrow \infty$).

Details

The CTPT distribution is defined by shifting the original two-piece Student t -distribution (with skewness parameter γ and degrees of freedom ν) by its mean, ensuring the resulting distribution has zero mean. The density of the CTPT distribution is given by:

$$p(\varepsilon|\gamma, \nu) = \frac{2}{\gamma + \frac{1}{\gamma}} \left[f_{\nu} \left(\frac{\varepsilon + m(\gamma, \nu)}{\gamma} \right) I_{[-m(\gamma, \nu), +\infty)}(\varepsilon) + f_{\nu}(\gamma(\varepsilon + m(\gamma, \nu))) I_{(-\infty, -m(\gamma, \nu))}(\varepsilon) \right], \quad \gamma \in \mathbb{R}_+, \nu > 1,$$

where:

- f_{ν} is the Student t -density with ν degrees of freedom.
- $m(\gamma, \nu)$ is the mean of the unadjusted two-piece distribution, computed by [mean_two_piece_student](#).
- $I_A(\varepsilon)$ denotes the indicator function for set A .

Value

A numeric vector of length N containing the samples.

References

TBA

See Also

[mean_two_piece_student](#) for computing the mean of the unadjusted distribution.

Examples

```
# Generate samples from CTPT with gamma=2, nu=5 (Student t case)
samples_t <- rctpt(N = 1000, gamma = 2, nu = 5)
hist(samples_t, breaks = 30, main = "CTPT (nu=5)")

# Generate samples from CTPT with gamma=2 (normal case)
samples_norm <- rctpt(N = 1000, gamma = 2, nu = "normal")
hist(samples_norm, breaks = 30, main = "CTPT (normal)")
```

skewness_ctpt

Calculate the Fisher's Moment Coefficient of Skewness for the Centred Two-Piece Student t (CTPT) Distribution

Description

This function calculates the skewness of the centred two-piece Student t (CTPT) distribution for a given skewness parameter $\gamma > 0$ and degrees of freedom $\nu > 3$. It also handles the limiting normal case if $\nu = \text{"normal"}$.

Usage

```
skewness_ctpt(gamma, nu = "normal")
```

Arguments

- | | |
|-------|---|
| gamma | A positive numeric value giving the skewness parameter. |
| nu | Either: <ul style="list-style-type: none"> • A numeric value > 3, giving the degrees of freedom for the Student t (required for a finite skewness). • The character string "normal" to indicate the limiting two-piece normal case. |

Details

The formula for finite ν follows Proposition 3 in your manuscript. When $\nu = \text{"normal"}$, the skewness is computed under the limiting two-piece normal case.

Value

A numeric value giving the Fisher's moment coefficient of skewness.

Examples

```
# Skewness for gamma=2, nu=5
skewness_ctpt(gamma = 2, nu = 5)

# Skewness in the normal limit for gamma=2
skewness_ctpt(gamma = 2, nu = "normal")
```

var_ctpt	<i>Variance of the Centred Two-Piece Student t (or Normal) Distribution</i>
----------	---

Description

This function calculates the variance of the centred two-piece Student t distribution. It also handles the limiting "normal" case (i.e., $\nu \rightarrow \infty$).

Usage

```
var_ctpt(gamma, nu)
```

Arguments

- | | |
|-------|---|
| gamma | A positive numeric value giving the skewness parameter (must be $\gamma > 0$). |
| nu | Either: <ul style="list-style-type: none"> • A numeric value > 2, giving the degrees of freedom ν for the Student t (required for a finite variance). • The character string "normal" to indicate the limiting two-piece normal case (i.e., $\nu = \infty$). |

Details

When nu is a finite numeric value > 2 , the variance formula is:

$$\text{Var}(\varepsilon \mid \gamma, \nu) = \frac{\nu}{\nu - 2} \left(\gamma^2 - 1 + \frac{1}{\gamma^2} \right) - m(\gamma, \nu)^2,$$

where

$$m(\gamma, \nu) = \frac{2\nu \Gamma((\nu + 1)/2)}{\sqrt{\pi} \nu (\nu - 1) \Gamma(\nu/2)} \left(\gamma - \frac{1}{\gamma} \right).$$

When nu = "normal", the variance is that of the centred two-piece normal distribution:

$$\text{Var}(\varepsilon \mid \gamma, \nu = \infty) = \left(\gamma^2 - 1 + \frac{1}{\gamma^2} \right) - \left(\sqrt{\frac{2}{\pi}} \left(\gamma - \frac{1}{\gamma} \right) \right)^2.$$

Value

A numeric value giving the variance of the centred two-piece distribution.

Examples

```
# Variance for gamma=2, nu=5
var_ctpt(2, 5)

# Variance in the limiting normal case for gamma=2
var_ctpt(2, "normal")
```

var_two_piece_student	<i>Variance of the Uncentered Two-Piece Student t (or Normal) Distribution</i>
-----------------------	--

Description

This function calculates the variance of the uncentred two-piece Student t distribution (see Proposition 2 in ...) for $\gamma > 0$ and $\nu > 2$. It also handles the limiting "normal" case (i.e., $\nu \rightarrow \infty$).

Usage

```
var_two_piece_student(gamma, nu)
```

Arguments

gamma	A positive numeric value giving the skewness parameter (must be $\gamma > 0$).
nu	Either: <ul style="list-style-type: none"> • A numeric value > 2, giving the degrees of freedom ν for the Student t (required for a finite variance). • The character string "normal" to indicate the limiting two-piece normal case (i.e., $\nu = \infty$).

Details

When nu is a finite numeric value > 2 , the variance formula is:

$$\text{Var}(\varepsilon \mid \gamma, \nu) = \frac{\nu}{\nu - 2} \left(\gamma^2 - 1 + \frac{1}{\gamma^2} \right) - m(\gamma, \nu)^2,$$

where

$$m(\gamma, \nu) = \frac{2 \nu \Gamma((\nu + 1)/2)}{\sqrt{\pi} \nu (\nu - 1) \Gamma(\nu/2)} \left(\gamma - \frac{1}{\gamma} \right).$$

When nu = "normal", the variance is that of the uncentred two-piece normal distribution:

$$\text{Var}(\varepsilon \mid \gamma, \nu = \infty) = \left(\gamma^2 - 1 + \frac{1}{\gamma^2} \right) - \left(\sqrt{\frac{2}{\pi}} \left(\gamma - \frac{1}{\gamma} \right) \right)^2.$$

Value

A numeric value giving the variance of the uncentred two-piece distribution.

Examples

```
# Variance for gamma=2, nu=5
var_two_piece_student(2, 5)

# Variance in the limiting normal case for gamma=2
var_two_piece_student(2, "normal")
```

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