Package ‘FlexBayesMed’

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Type Package

Title Modeling Skewed and Heavy-Tailed Errors in Bayesian Mediation Analysis

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Description Flexible Bayesian mediation models, including simulation from
the centered two-piece t-distribution (CTPT). Provides methods
for skewed and heavy-tailed error modeling in a mediation framework.

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Imports rstan,
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bridgesampling

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Contents

`dctpt` *Density of the Centered Two-Piece Student's t-Distribution (CTPT)*

Description

Density of the Centered Two-Piece Student's t-Distribution (CTPT)

Usage

```
dctpt(x, gamma, nu, log = FALSE)
```

Arguments

`x`	Numeric vector. Quantiles at which to compute the density.
`gamma`	Positive numeric scalar. Skewness parameter.
`nu`	Numeric scalar $\nu > 1$ (degrees of freedom) or "normal".
`log`	Logical. Return log-density? Default: `FALSE`.

Details

The CTPT distribution is defined by shifting the original two-piece Student's t-distribution (with skewness parameter γ and degrees of freedom ν) by its mean, ensuring the resulting distribution has zero mean. The density of the CTPT distribution is given by:

$$p(\varepsilon|\gamma,\nu) = \frac{2}{\gamma + \frac{1}{\gamma}} \left[f_\nu\left(\frac{\varepsilon + m(\gamma,\nu)}{\gamma}\right) I_{[-m(\gamma,\nu),+\infty)}(\varepsilon) + \right.$$
$$\left. f_\nu\left(\gamma\big(\varepsilon + m(\gamma,\nu)\big)\right) I_{(-\infty,-m(\gamma,\nu))}(\varepsilon) \right], \quad \gamma \in \mathbb{R}_+,\ \nu > 1,$$

where:

- f_ν is the Student's t-density with ν degrees of freedom.
- $m(\gamma,\nu)$ is the mean of the unadjusted two-piece distribution, computed by `mean_two_piece_student`.
- $I_A(\varepsilon)$ denotes the indicator function for set A.

Value

Numeric vector of densities.

Examples

```
x <- seq(-5, 5, length.out = 100)
density <- dctpt(x, gamma = 2, nu = 5)
plot(x, density, type = "l")
```

`flex_med` *Mediation analysis with mean-adjusted two-piece Student's t (CTPT) errors*

Description

Mediation analysis with mean-adjusted two-piece Student's t (CTPT) errors

Usage

```
flex_med(
  X,
  M,
  Y,
  model = "full",
  q00 = 1/3,
  q01 = 1/3,
  alpha_gamma = 2,
  beta_gamma = 2,
  gamma_lower = 0.05,
  gamma_upper = 20,
  d_nu = 0.01,
  nu_lower = 1.01,
  iter = 30000,
  warmup = floor(iter/2),
  chains = 1,
  seed = 123,
  verbose = FALSE,
  refresh = 2500,
  repetitions = 1,
  method = "normal",
  bridgesampling_cores = 1,
  use_neff = TRUE,
  maxiter = 1000,
  silent = FALSE,
  ...
)
```

Arguments

`X`	A numeric vector of length n (the independent variable).
`M`	A numeric vector of length n (the mediator).
`Y`	A numeric vector of length n (the response).
`model`	A string: which model to use ("full", "gamma-only", "nu-only", "normal")
`q00`	A non-negative numeric value giving the conditional probability $p(\alpha = 0, \beta = 0 \mid \alpha\beta = 0)$. Defaults to `1/3`.
`q01`	A non-negative numeric value giving the conditional probability $p(\alpha = 0, \beta \neq 0 \mid \alpha\beta = 0)$. Defaults to `1/3`. Here, $q00 + q01 + q10 = 1$, where $q10 = p(\alpha \neq 0, \beta = 0 \mid \alpha\beta = 0)$ is computed internally as `1 - q00 - q01`. These three probabilities correspond to

the disjoint ways in which the mediation effect can be zero (i.e., $\alpha\beta = 0$). They must be non-negative and sum to 1.

`alpha_gamma`, `beta_gamma`
: Shape and rate for the Gamma prior on skewness parameter γ.

`gamma_lower`, `gamma_upper`
: Truncation bounds for γ ($0 < \gamma_{\text{lower}} \leq \gamma \leq \gamma_{\text{upper}}$).

`d_nu` Rate for the exponential prior on degrees-of-freedom ν.

`nu_lower` Lower truncation bound for ν (must satisfy $\nu > 1$).

`iter`, `warmup`, `chains`, `seed`, `verbose`, `refresh`, `...`
: Additional arguments passed to `rstan::stan()`.

`repetitions`, `method`, `bridgesampling_cores`, `use_neff`, `maxiter`, `silent`
: Additional arguments passed to `bridgesampling::bridge_sampler()`.

Value

A list with the following elements:

`X2M_with_alpha` A `stanfit` object for the mediator model with $\alpha \neq 0$.

`XM2Y_with_beta` A `stanfit` object for the outcome model with $\beta \neq 0$.

`X2M_without_alpha` A `stanfit` object for the mediator model with $\alpha = 0$.

`XM2Y_without_beta` A `stanfit` object for the outcome model with $\beta = 0$.

`samples_med` A numeric vector of posterior draws for $\alpha\beta$.

`bf_med` A numeric value giving the Bayes factor for the mediation effect.

`flex_mr` *Fit a linear mean regression with mean-adjusted two-piece Student's t (CTPT) errors*

Description

Fit a linear mean regression with mean-adjusted two-piece Student's t (CTPT) errors

Usage

```
flex_mr(
  X,
  Y,
  model = "full",
  alpha_gamma = 2,
  beta_gamma = 2,
  gamma_lower = 0.05,
  gamma_upper = 20,
  d_nu = 0.01,
  nu_lower = 1.01,
  iter = 30000,
  warmup = floor(iter/2),
  chains = 1,
  seed = 123,
  verbose = TRUE,
  refresh = 2500,
  ...
)
```

Arguments

- `X` A numeric matrix of predictors ($n \times k$).
- `Y` A numeric vector of length n (the response).
- `model` A string: which model to use ("full", "gamma-only", "nu-only", "normal")
- `alpha_gamma`, `beta_gamma`
  Shape and rate for the Gamma prior on skewness parameter γ.
- `gamma_lower`, `gamma_upper`
  Truncation bounds for γ ($0 < \gamma_{\text{lower}} \le \gamma \le \gamma_{\text{upper}}$).
- `d_nu` Rate for the exponential prior on degrees-of-freedom ν.
- `nu_lower` Lower truncation bound for ν (must satisfy $\nu > 1$).
- `iter`, `warmup`, `chains`, `seed`, `verbose`, `refresh`, `...`
  Additional arguments passed to `rstan::stan()`.

Value

An object of class `stanfit`.

`mean_two_piece_student` *Mean of the uncentered two-piece Student* t *(or normal) distribution*

Description

Computes the theoretical mean $m(\gamma, \nu)$ of the two-piece Student's t_ν distribution with skewness parameter γ. If `nu = "normal"` the function returns the two-piece *normal* limit $m^{\text{normal}}(\gamma) = \sqrt{2/\pi}\,(\gamma - 1/\gamma)$ as $\nu \to \infty$. This corresponds to Eq. (1) in Fernández & Steel (1998).

Usage

```
mean_two_piece_student(gamma, nu)
```

Arguments

- `gamma` Positive numeric scalar. Skewness parameter $\gamma > 0$.
- `nu` Either a numeric scalar $\nu > 1$ (degrees of freedom) *or* the literal string `"normal"` for the $\nu \to \infty$ limit.

Details

The closed-form expression for finite ν is

$$m(\gamma, \nu) = \frac{2\nu\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\,(\nu-1)\Gamma\left(\frac{\nu}{2}\right)}\left(\gamma - \frac{1}{\gamma}\right).$$

For the normal limit (`nu = "normal"`) the mean simplifies to $\sqrt{2/\pi}\,(\gamma - 1/\gamma)$.

Value

A numeric scalar giving the mean $m(\gamma, \nu)$.

References

Fernández, C. & Steel, M. F. J. (1998). "On Bayesian Modeling of Fat Tails and Skewness." *Journal of the American Statistical Association*, **93**(441), 359–371.

Examples

```
mean_two_piece_student(gamma = 2, nu = 5)        # finite-nu case, approximately 1.424
mean_two_piece_student(gamma = 3, nu = "normal") # normal limit, approximately 2.128
```

`mode_ctpt` *Mode of the centered two-piece Student* t *(or normal) distribution*

Description

Computes the theoretical mode $mode(\gamma, \nu)$ of the centred two-piece Student's t_ν distribution with skewness parameter γ. If `nu = "normal"` the function returns the two-piece *normal* limit $mode^{\text{normal}}(\gamma) = -\sqrt{2/\pi}\,(\gamma - 1/\gamma)$ as $\nu \to \infty$.

Usage

```
mode_ctpt(gamma, nu)
```

Arguments

`gamma`	Positive numeric scalar. Skewness parameter $\gamma > 0$.
`nu`	Either a numeric scalar $\nu > 1$ (degrees of freedom) *or* the literal string `"normal"` for the $\nu \to \infty$ limit.

Details

The closed-form expression for finite ν is

$$mode(\gamma, \nu) = -\frac{2\nu\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\,(\nu-1)\Gamma\left(\frac{\nu}{2}\right)}\left(\gamma - \frac{1}{\gamma}\right).$$

For the normal limit (`nu = "normal"`) the mean simplifies to $-\sqrt{2/\pi}\,(\gamma - 1/\gamma)$.

Value

A numeric scalar giving the mean $mode(\gamma, \nu)$.

Examples

```
mode_ctpt(gamma = 2, nu = 5)        # finite-nu case, approximately -1.424
mode_ctpt(gamma = 3, nu = "normal") # normal limit, approximately -2.128
```

`pctpt` *Cumulative Distribution Function (CDF) of the Centred Two-Piece Student's t-Distribution (CTPT)*

Description

Computes the CDF for the Mean-Adjusted Two-Piece Student's t-Distribution (CTPT), a skewed heavy-tailed distribution with zero mean used in robust mediation analysis.

Usage

```
pctpt(q, gamma, nu, lower.tail = TRUE, log.p = FALSE)
```

Arguments

`q`	Numeric vector of quantiles.
`gamma`	Positive scalar. Skewness parameter ($\gamma > 0$).
`nu`	Degrees of freedom. Either numeric `>1` or `"normal"` for the Gaussian limit.
`lower.tail`	Logical; if `TRUE` (default), probabilities are $P(X \leq q)$.
`log.p`	Logical; if `TRUE`, returns log probabilities.

Details

The CDF is piecewise-defined with split point at $\varepsilon = -m(\gamma, \nu)$:

For $\varepsilon < -m(\gamma, \nu)$:

$$F(\varepsilon \mid \gamma, \nu) = \frac{2}{1 + \gamma^2} F_\nu[\gamma (\varepsilon + m(\gamma, \nu))]$$

For $\varepsilon \geq -m(\gamma, \nu)$:

$$F(\varepsilon \mid \gamma, \nu) = \frac{2\gamma^2}{1 + \gamma^2} F_\nu[\tfrac{\varepsilon + m(\gamma,\nu)}{\gamma}] \; + \; \frac{1 - \gamma^2}{1 + \gamma^2}$$

Where:

- $m(\gamma, \nu)$ is the mean-adjustment term (see Proposition 1 in manuscript)
- F_ν = CDF of Student's t (ν df) or standard normal (`nu = "normal"`)
- Gaussian mean: $m(\gamma) = \sqrt{\frac{2}{\pi}} \left(\gamma - \frac{1}{\gamma}\right)$

References

TBA

See Also

`mean_two_piece_student()` for the mean-adjustment computation

Examples

```
# Symmetric case (gamma=1) matches standard t
pctpt(q = 0, gamma = 1, nu = 5)  # Returns 0.5
pt(0, df = 5)                      # Returns 0.5

# Skewed case
pctpt(q = 0, gamma = 2, nu = 5)

# Normal limit
pctpt(q = 0, gamma = 1.5, nu = "normal")
```

---

qctpt *Quantile Function of the Centred Two-Piece Student's t-Distribution (CTPT)*

Description

Inverts the CDF of the CTPT distribution to find ε such that $P(X \le \varepsilon) = p$.

Usage

```
qctpt(p, gamma, nu, lower.tail = TRUE, log.p = FALSE)
```

Arguments

`p`	Numeric vector of probabilities in $[0, 1]$.
`gamma`	Positive scalar ($\gamma > 0$). The skewness parameter.
`nu`	Either a numeric > 1 (degrees of freedom) or the string `"normal"`.
`lower.tail`	Logical; if `TRUE` (default), probabilities are $P(X \le q)$.
`log.p`	Logical; if `TRUE`, `p` is treated as $\log(p)$.

Details

The distribution is defined by a piecewise CDF:

$$F(\varepsilon|\gamma,\nu) = \begin{cases} \dfrac{2}{1+\gamma^2}\, F_\nu\big[\gamma\,(\varepsilon+m)\big], & \varepsilon < -m, \\ \dfrac{2\,\gamma^2}{1+\gamma^2}\, F_\nu\Big(\frac{\varepsilon+m}{\gamma}\Big) \;+\; \dfrac{1-\gamma^2}{1+\gamma^2}, & \varepsilon \ge -m, \end{cases}$$

where $m = m(\gamma, \nu)$ is the mean-adjustment term, and F_ν is the standard Student's t (if $\nu > 1$) or standard normal (if `nu="normal"`) CDF.

Value

Numeric vector of quantiles corresponding to the given probabilities.

See Also

`pctpt` for the CDF and `dctpt` for the PDF.

Examples

```
qctpt(p = 0.975, gamma = 1, nu = "normal") # Returns 1.96
qctpt(p = 0.025, gamma = 2, nu = 2) # Returns -3.40
```

`rctpt` *Generate Samples from the Centred Two-Piece Student's t-Distribution (CTPT)*

Description

Produces independent and identically distributed (i.i.d.) samples from the CTPT distribution, which is a zero-mean adjusted version of the two-piece Student's t-distribution (or normal). The adjustment ensures that the error terms satisfy the zero-mean assumption, which is needed in cases such as mean regression or mediation analysis.

Usage

```
rctpt(N, gamma, nu)
```

Arguments

`N`	Positive integer. The number of samples to generate.
`gamma`	Positive numeric scalar. Skewness parameter ($\gamma > 0$).
`nu`	Either a numeric scalar $\nu > 1$ (degrees of freedom) or the string "normal" for the normal limit ($\nu \to \infty$).

Details

The CTPT distribution is defined by shifting the original two-piece Student's t-distribution (with skewness parameter γ and degrees of freedom ν) by its mean, ensuring the resulting distribution has zero mean. The density of the CTPT distribution is given by:

$$p(\varepsilon|\gamma, \nu) = \frac{2}{\gamma + \frac{1}{\gamma}} \left[f_\nu \left(\frac{\varepsilon + m(\gamma, \nu)}{\gamma} \right) I_{[-m(\gamma,\nu),+\infty)}(\varepsilon) + \right.$$
$$\left. f_\nu \left(\gamma\big(\varepsilon + m(\gamma, \nu)\big)\right) I_{(-\infty,-m(\gamma,\nu))}(\varepsilon) \right], \quad \gamma \in \mathbb{R}_+, \ \nu > 1,$$

where:

- f_ν is the Student's t-density with ν degrees of freedom.
- $m(\gamma, \nu)$ is the mean of the unadjusted two-piece distribution, computed by `mean_two_piece_student`.
- $I_A(\varepsilon)$ denotes the indicator function for set A.

Value

A numeric vector of length N containing the samples.

References

TBA

See Also

mean_two_piece_student for computing the mean of the unadjusted distribution.

Examples

```
# Generate samples from CTPT with gamma=2, nu=5 (Student's t case)
samples_t <- rctpt(N = 1000, gamma = 2, nu = 5)
hist(samples_t, breaks = 30, main = "CTPT (nu=5)")

# Generate samples from CTPT with gamma=2 (normal case)
samples_norm <- rctpt(N = 1000, gamma = 2, nu = "normal")
hist(samples_norm, breaks = 30, main = "CTPT (normal)")
```

---

skewness_ctpt — *Calculate the Fisher's moment coefficient of skewness for CTPT*

Description

This function calculates the skewness of the Mean-Adjusted Two-Piece Student's t (CTPT) distribution for a given skewness parameter gamma > 0 and degrees of freedom nu > 3. It also handles the limiting normal case if nu = "normal".

Usage

```
skewness_ctpt(gamma, nu = "normal")
```

Arguments

gamma	A positive numeric value giving the skewness parameter.
nu	Either: • A numeric value > 3, giving the degrees of freedom for the Student's t (required for a finite skewness). • The character string "normal" to indicate the limiting two-piece normal case.

Details

The formula for finite nu follows Proposition 3 in your manuscript. When nu = "normal", the skewness is computed under the limiting two-piece normal case.

Value

A numeric value giving the Fisher's moment coefficient of skewness.

Examples

```
# Skewness for gamma=2, nu=5
skewness_ctpt(gamma = 2, nu = 5)

# Skewness in the normal limit for gamma=2
skewness_ctpt(gamma = 2, nu = "normal")
```

var_ctpt *Variance of the centred two-piece student's t (or normal) distribution*

Description

This function calculates the variance of the centred two-piece Student's t distribution It also handles the limiting "normal" case (i.e., $\nu \to \infty$).

Usage

```
var_ctpt(gamma, nu)
```

Arguments

gamma	A positive numeric value giving the skewness parameter (must be $\gamma > 0$).
nu	Either: • A numeric value > 2, giving the degrees of freedom ν for the Student's t (required for a finite variance). • The character string "normal" to indicate the limiting two-piece normal case (i.e., $\nu = \infty$).

Details

When nu is a finite numeric value > 2, the variance formula is:

$$\mathrm{Var}(\varepsilon \mid \gamma, \nu) = \frac{\nu}{\nu - 2}\Big(\gamma^2 - 1 + \tfrac{1}{\gamma^2}\Big) \; - \; m(\gamma, \nu)^2,$$

where

$$m(\gamma, \nu) = \frac{2\,\nu\,\Gamma\big((\nu + 1)/2\big)}{\sqrt{\pi\,\nu}\,(\nu - 1)\,\Gamma\big(\nu/2\big)}\,\big(\gamma - \tfrac{1}{\gamma}\big).$$

When nu = "normal", the variance is that of the centred two-piece normal distribution:

$$\mathrm{Var}(\varepsilon \mid \gamma, \nu = \infty) = \big(\gamma^2 - 1 + \tfrac{1}{\gamma^2}\big) \; - \; \Big(\sqrt{\tfrac{2}{\pi}}\big(\gamma - \tfrac{1}{\gamma}\big)\Big)^2.$$

Value

A numeric value giving the variance of the centred two-piece distribution.

Examples

```
# Variance for gamma=2, nu=5
var_ctpt(2, 5)

# Variance in the limiting normal case for gamma=2
var_ctpt(2, "normal")
```

`var_two_piece_student` *Variance of the Uncentered Two-Piece Student's t (or Normal) Distribution*

Description

This function calculates the variance of the uncentred two-piece Student's t distribution (see Proposition 2 in ...) for $\gamma > 0$ and $\nu > 2$. It also handles the limiting "normal" case (i.e., $\nu \to \infty$).

Usage

```
var_two_piece_student(gamma, nu)
```

Arguments

`gamma`	A positive numeric value giving the skewness parameter (must be $\gamma > 0$).
`nu`	Either: • A numeric value > 2, giving the degrees of freedom ν for the Student's t (required for a finite variance). • The character string `"normal"` to indicate the limiting two-piece normal case (i.e., $\nu = \infty$).

Details

When `nu` is a finite numeric value > 2, the variance formula is:

$$\mathrm{Var}(\varepsilon \mid \gamma, \nu) = \frac{\nu}{\nu - 2}\Big(\gamma^2 - 1 + \tfrac{1}{\gamma^2}\Big) \; - \; m(\gamma, \nu)^2,$$

where

$$m(\gamma, \nu) = \frac{2\,\nu\,\Gamma\big((\nu+1)/2\big)}{\sqrt{\pi\,\nu}\,(\nu - 1)\,\Gamma\big(\nu/2\big)}\,\big(\gamma - \tfrac{1}{\gamma}\big).$$

When `nu = "normal"`, the variance is that of the uncentred two-piece normal distribution:

$$\mathrm{Var}(\varepsilon \mid \gamma, \nu = \infty) = \big(\gamma^2 - 1 + \tfrac{1}{\gamma^2}\big) \; - \; \Big(\sqrt{\tfrac{2}{\pi}}\big(\gamma - \tfrac{1}{\gamma}\big)\Big)^2.$$

Value

A numeric value giving the variance of the uncentred two-piece distribution.

Examples

```
# Variance for gamma=2, nu=5
var_two_piece_student(2, 5)

# Variance in the limiting normal case for gamma=2
var_two_piece_student(2, "normal")
```

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