Predicting Stock Volatility with Time-Series Analysis

Spring 2019

Springboard Capstone Project 2

Final project

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Problem Description

The purpose of the following project is to attempt to create a model that makes useful predictions about the movement of Microsoft stock. In particular the focus will be on using time-series analysis in an attempt to predict the 10 day historical volatility of the stock, which is a measurement of how much the stock tends to move over a period of 10 days. From a business perspective this could be useful in a number of ways. If volatility can be predicted, trading strategies can be structured around those predictions. In particular buying or selling options would be ways to profit from periods of predicted high or low volatility respectively. Another different but related use of this information would be for portfolio construction. Portfolio managers could use the predictions to increase or decrease market exposure based on expected volatility. Additionally, while this paper will focus specifically on Microsoft stock, the techniques used here could be applied to any individual stock or even to indexes or futures, and therefore could be used to choose stocks when constructing a diverse portfolio.

A Brief Description of Volatility

In order to understand the focus of this project it is important to have at least a cursory understanding of volatility and how it is measured. Within this paper two different types of volatility will be mentioned, historical, and implied. Historical volatility is simply a measure of how much a stock moves and is measured in percentage terms. For example if a stock has a historical volatility of 17% then it is expected to have a standard deviation of 1%, meaning it should move 1% or less about two-thirds of the time. Implied volatility is a measure of expected historical volatility that comes from the price of the stock's options. Implied volatility is usually slightly higher than historical volatility, meaning that the price of the option is "too high" based on how much the option is currently moving. This can be thought of in a fashion similar to insurance premiums. If you were to insure a car the insurer might predict that you had a 10% per year chance of an accident. The insurer might price the policy as if you had a 15% chance of having an accident as a way to profit or at least protect against miscalculation or unforeseen circumstances. Option premiums are also affected by the date of maturity of the option. For instance an option that expires in 30 days is more expensive that an option that expires in 10 days because there is more opportunity for the price of the underlying stock to move in the additional 10 days, in much the same way that a 1 year insurance policy would be less expensive than a 2 year policy. This report will make specific mention of 10-day historical volatility, which is an average of the historical volatility over the last 10 trading days.

Data Acquisition and Wrangling

The first step in preparing for the analysis of volatility in Microsoft stock was to find appropriate data. Since statistical analysis of stocks is an area of high interest, there are a wealth of resources on the internet detailing the movement of stock prices. While it would have been possible to calculate historical volatility by using the closing prices of the stock, there were some other factors I wanted to examine in relation to historical volatility, specifically the "implied volatility" of the stock based on the price of the stock's options. Implied volatility is a measure of the expected movement of the stock.

I was able to find a source of data that provided not only pricing data about Microsoft stock that included closing price, high, low, daily volume and several others, but also provided some data on options. Quandl.com contains free data on many stocks traded on exchanges spanning the globe. They also provide a premium service that provides a good deal of information on the options of many stocks. Option data on Microsoft stock was provided for free as an example of the premiums service and therefore should be available to anyone who like to reproduce any of the research provided herein.

The first step in working with the Quandl data was to determine how to use their API. Quandl provides documentation and an ID for anyone requesting it, and the unique identifier is entered from the Python command line (and thus not included in the Jupyter notebook). Once this step was completed the stock data could be pulled into a notebook by way of a csv file. After converting the csv into a Pandas dataframe, I checked the newly created dataframe for NaN values. After a quick check revealed no NaN values I was ready to move on to the option data.

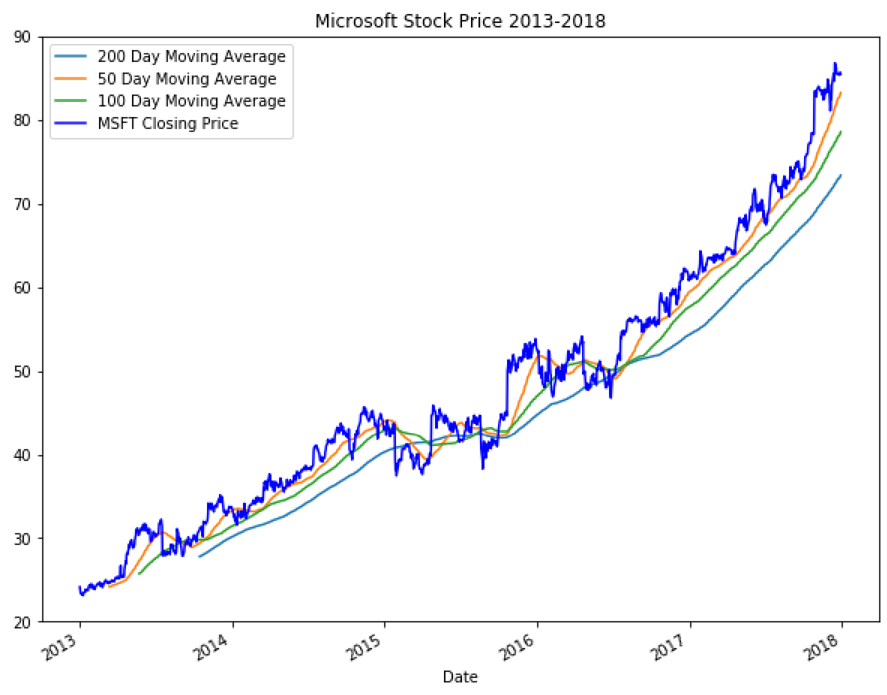
In working with the option data it quickly became obvious that there were a number of NaN values and a decision had to be made about how to handle them. Since I knew I planned to focus on shorter dated measures of volatility, I immediately dropped some of the longer dated measures (over 60 days) to see if this relieved the issue. Since after deleting older data there was still quite a bit of invalid data, I next attempted to drop all the columns containing NaN's, but since this would have left only 6 remaining columns from the original 55, I deemed this solution unacceptable. I conducted a little more exploration of the dataset, which revealed that 3 records (rows) were causing all the NaN values. Since I was already looking for a way to limit my data set which ranged from 1986 through 2018, and the records in question were all from before 2006, I just chose to limit my exploration to the years of 2013 through 2017. This eliminated all of the NaN related issues.

I also noticed a small issue when plotting the price of the stock. The closing price did not include stock splits, so when graphed, it appeared that the stock had a number of huge drops in price. After spending a few minutes looking over the split dates and how they were handled, I realized that the data set included an “Adj Close” column that already adjusted for the split price (as well as high low and volume issues). I resolved to just use the “Adj” columns going forward. I then used a relatively simple join to create a single Dataframe that included both stock and option data. I pickled that dataframe for later use, and was ready to move forward to the next phase of the project.

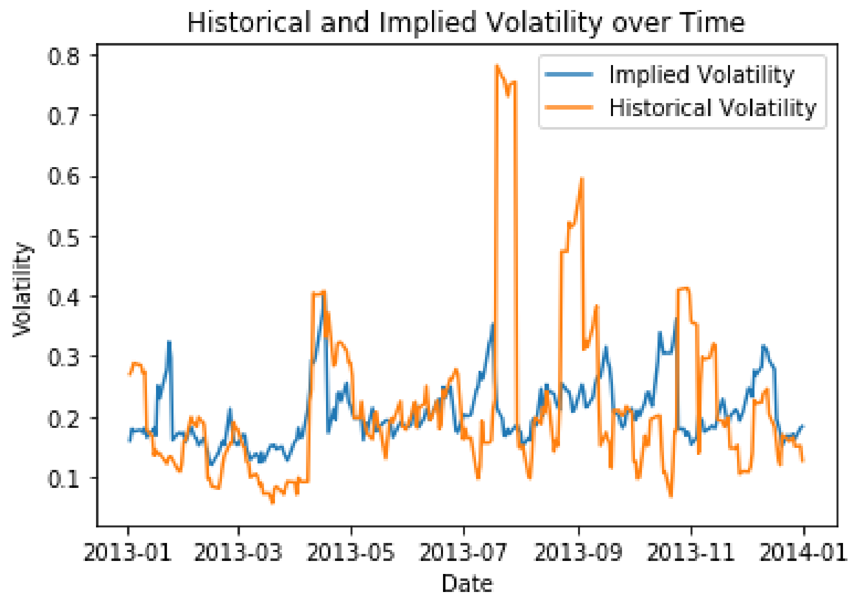
Data Exploration and Storytelling

The next step upon completion of the data cleaning and wrangling was to begin to explore the dataset through visualisations. I hoped to find noticeable patterns in the historical volatility of Microsoft stock that could be concisely visualized. I also hoped to find out more about the relationship of historical volatility to other features within the dataset.

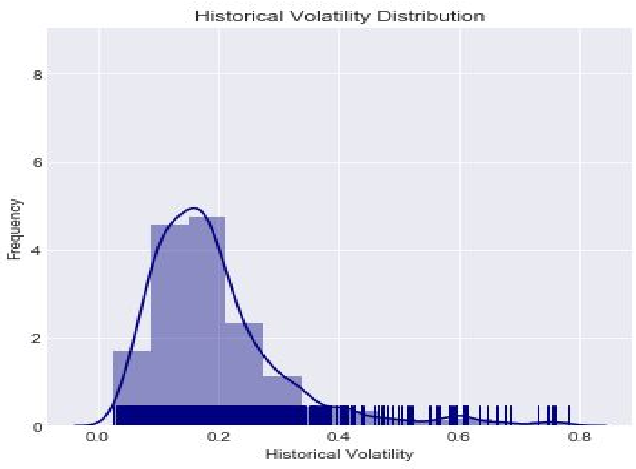
I began this journey by unpacking the previously pickled data, and making sure it appeared to be in order. After this step was completed I began by plotting the stock’s closing price as well as its range for a fixed period, from March 2016, to the end of the dataset, to get and idea of the daily ranges of the stock. When this was completed, I moved on to building a chart of Microsoft’s daily closing price for the period being examined. I also included the 50-day, 100-day and 200-day moving averages of the stock price, knowing that these ‘lag variables’ are often used in technical analysis and can be used to identify patterns in price. In particular crossing moving averages often indicate a change in the stocks current trend (see the chart below).

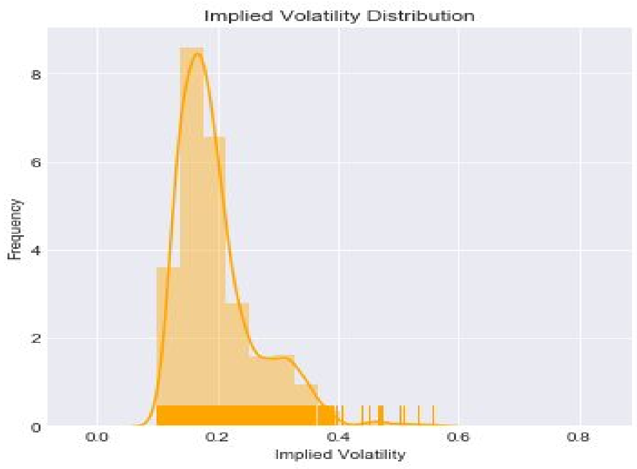


After examining the chart of the stocks price, I decided that an exploration of the Microsoft's historical volatility, specifically in respect to the implied volatility of the options was in order. After initially charting the entire period I decided to focus on a more condensed period. The next figure shows both the historical and implied volatility in the year 2013. It could be seen that at several times throughout the year historical volatility spiked well above the implied volatility.

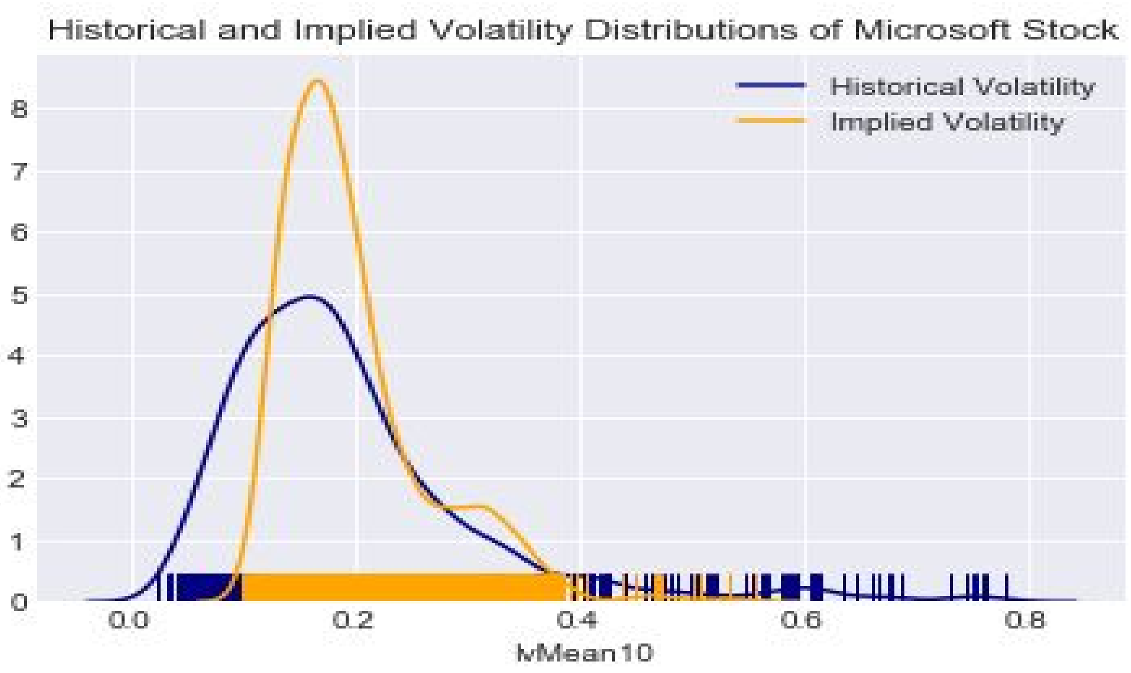


This pattern of historical volatility jumping to levels well above those indicated by option prices deserved additional analysis. The next step was to plot the distributions of each type of volatility via histogram.





In this diagram it becomes obvious that historical volatility tends to spike above the implied volatility in tail events. It also shows that more often than not historical volatility is slightly lower than implied volatility. While each type of volatility has a similar mean, the shape of the distribution curves are very different. When drawn on top of one another this relationship becomes even more obvious.



These visualizations helped to provide insight into patterns in Microsoft stock. Both trends and seasonal patterns were discovered in the closing prices of the stocks. Also a number of observations about the relationship between historical and implied volatility were made, as seen above in the visualizations from the applicable Jupyter notebook. With these realizations in mind it was time to begin a deeper dive into these statistical patterns.

Applications of Inferential Statistics

Once the data had been explored visually, I began to perform statistical analysis on historical volatility of the stock, as well as how the historical volatility related to the other features contained in the data set. I was interested in knowing how each of the features might relate to one another, so I used a heatmap to visualize the Pearson's correlation coefficients of each one (pictured on next page). Unsurprisingly the historical volatilities showed the most correlation to historical volatilities of other lengths, and to Parkinson volatilities (another measure of historical volatility that uses highs and lows instead of just closing prices). There was also a positive correlation to volume (more of the stock traded in volatile periods) and skew (a measure of increased implied volatility on out of the money options). Higher prices, on the other hand seemed to indicate lower volatility.

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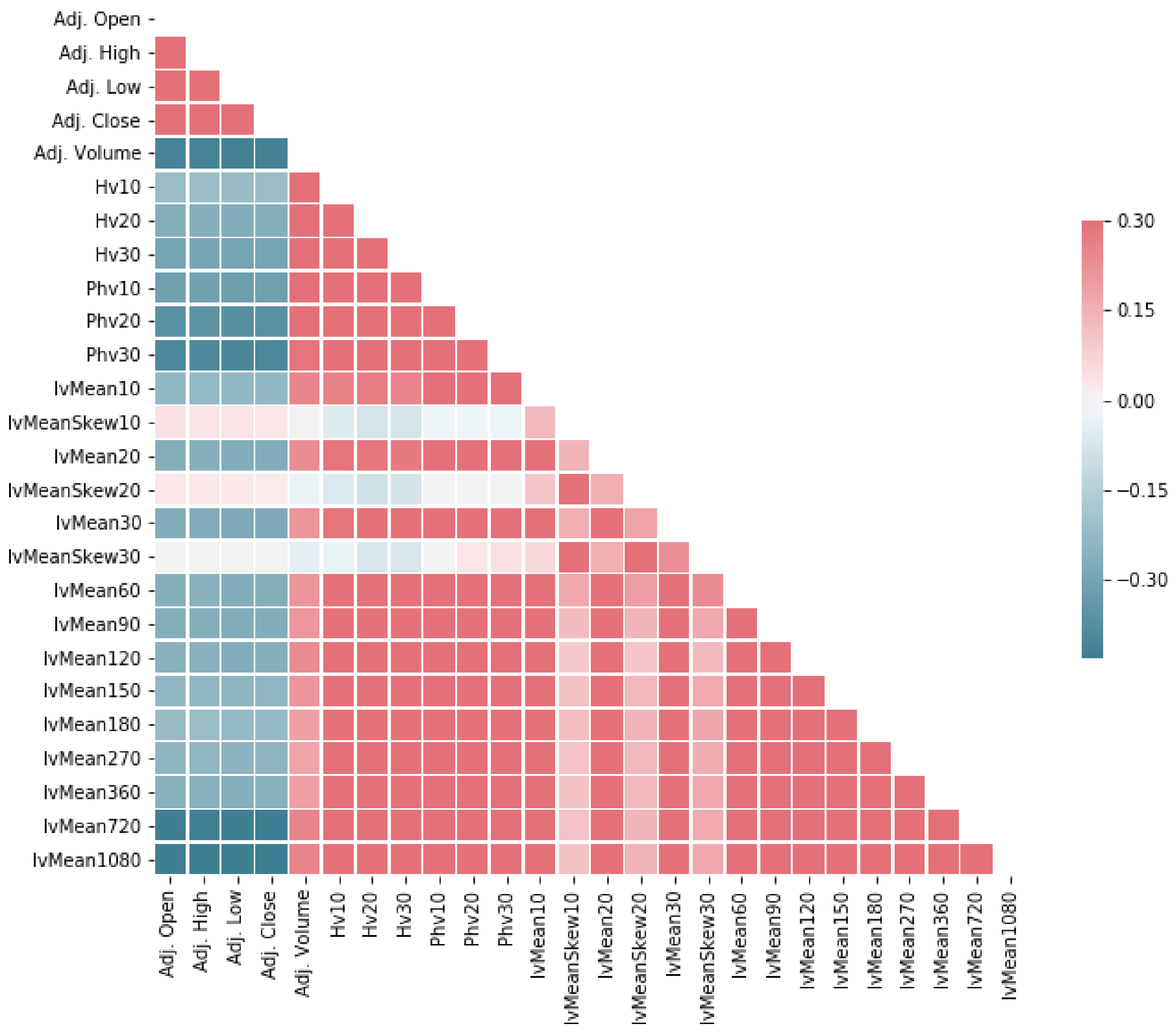
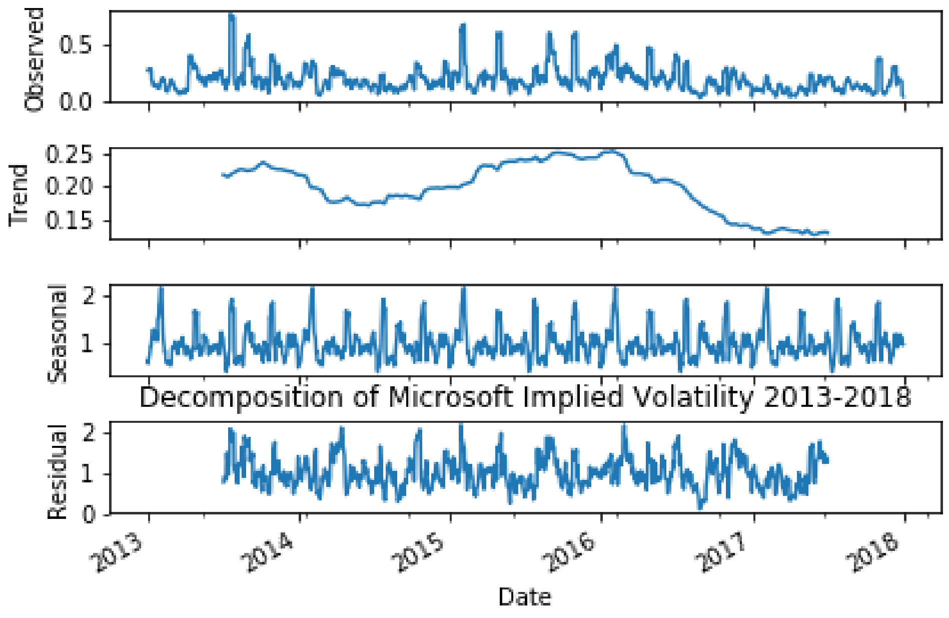
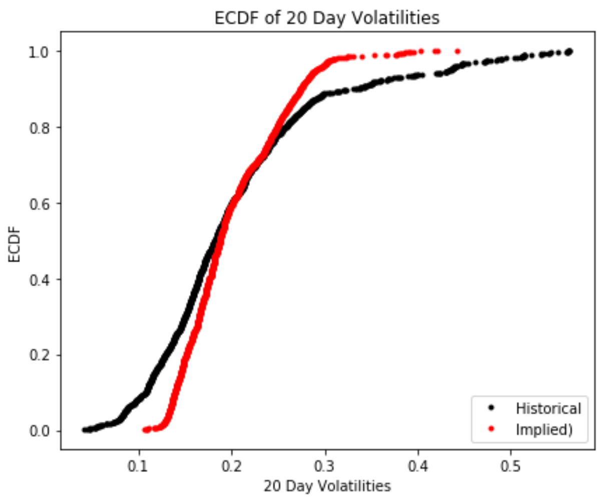
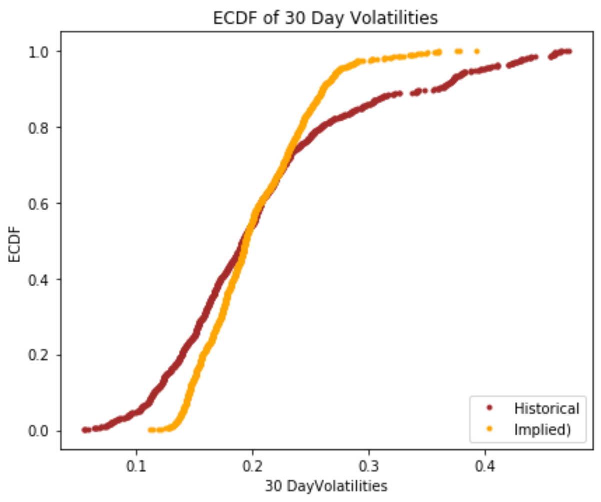


In the data visualization notebook I used the statsmodel seasonal decomposition function to examine Microsoft stock prices (not pictured). I once again applied this function, but this time to historical volatility. I found that the seasonal trend was very much inverse to that of the stock price. Low volatility could be found late in the year, a time when the stock traditionally rallies, and volatility increased in January, a period when prices often moved lower. While the decomposition also identified some semblance of a trend in volatility it was nowhere near as clear as the trend in the stock's price.

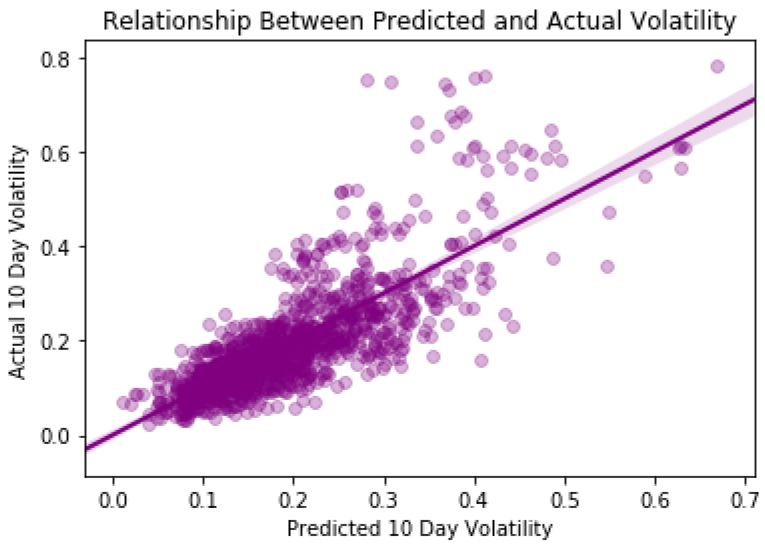


In an an attempt to further study the relationship between implied and historical volatility I ran empirical cumulative distribution functions (ECDF) on both.The results showed differences between implied and historical volatilities increased when looking at longer periods of time. This implies that the option markets assume historical volatility will revert to its mean over time.





I then used statsmodel’s ols function to further breakdown the relationship between implied and historical volatility. I found an adjusted r-squared score of 68 % meaning that a reasonable amount of variance in historical volatility of Microsoft stock could be explained by a model using implied volatility of at-the-money options as a predictor. The t-score (P>|t|) of zero (likely just a very small number) indicated that there was a statistically significant relationship between the two variables. I also used a regression plot better visualize the relationship between those two variables.



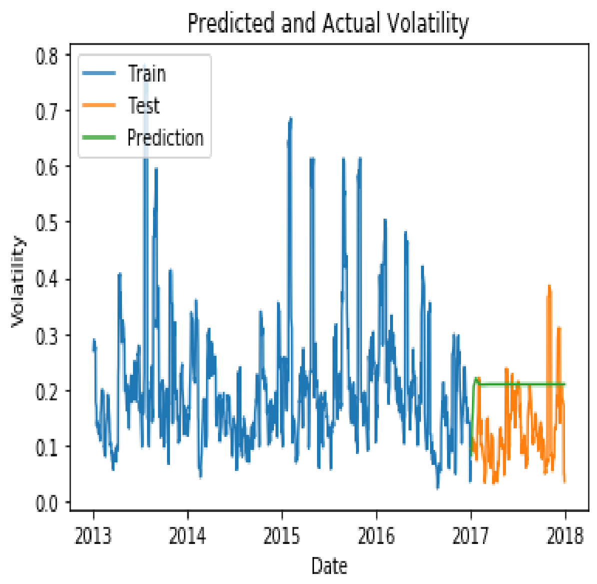
The regression plot showed that when actual volatility went above 40% the model had difficulty predicting the actual volatility. Conversely there are are a reasonable amount of predictions in the 30-40% range that are higher than the actual volatility.

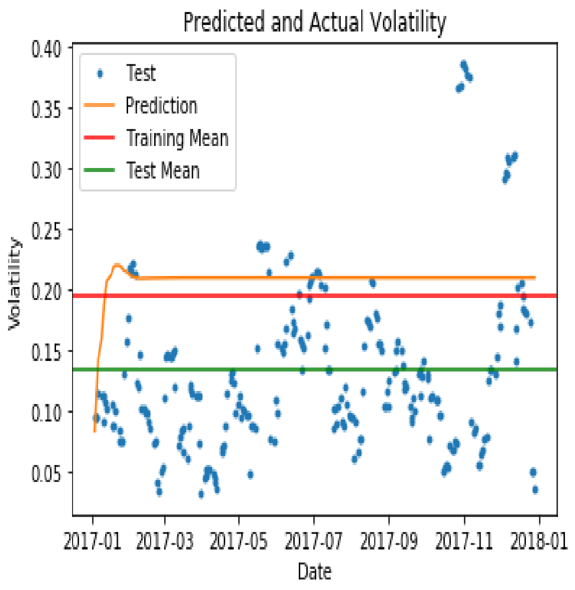
Applications of Machine Learning to the Problem

The next phase of the project was to move on to model-building and actually attempting to predict historical volatility. Since I had already determined that there were seasonal and trend components to the movement of the stock I decided to focus first on ARIMA (Autoregressive Integrated Moving Average), and then on Prophet--a model built by Facebook and recently made available to the public--both of which are capable of utilizing these factors. Specifically, I would use auto-ARIMA, a model with built-in parameter tuning, which could automatically find the best parameters for the model using a methodology similar to grid search.

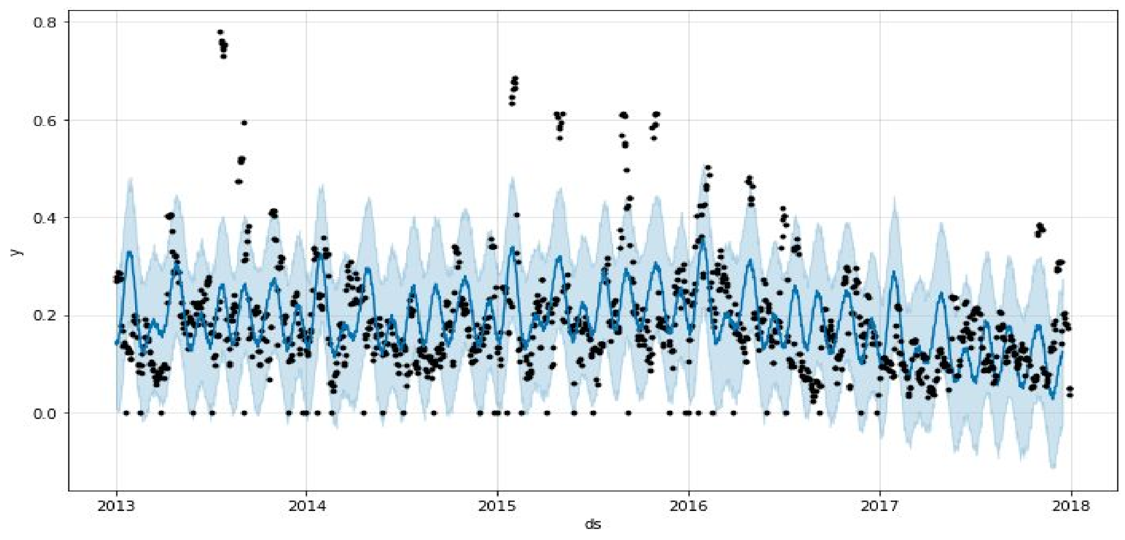
I began by running two ordinary least squares models using the statsmodel package. One model focussed solely on historical volatility, while the other include several other features (such as implied volatility, volume and skew). The model with additional features outperformed the more basic model (in terms of AIC, or Akaike Information Criteria), which led me to believe, that additional features might improve forecasts going forward.

In order to begin the search for the best predictive model I had to start with a baseline. I chose an auto-ARIMA model as the starting point for attempting to predict Microsoft stock’s 10-day historical volatility. This initial model seemed to start off very close to the actual volatility but then quickly gravitated to a spot slightly above the mean of the training set (2013-2016) where it stayed through most of 2017. The straight line drew my attention to an issue that would be present in several of the models. A forecast built in this fashion was attempting to predict a full year's data based on data for the previous 4 years, rather than taking into account the most recent measure of the stock's volatility. For this reason the model performed managed to track volatility reasonably well for a few days, but quickly degraded.

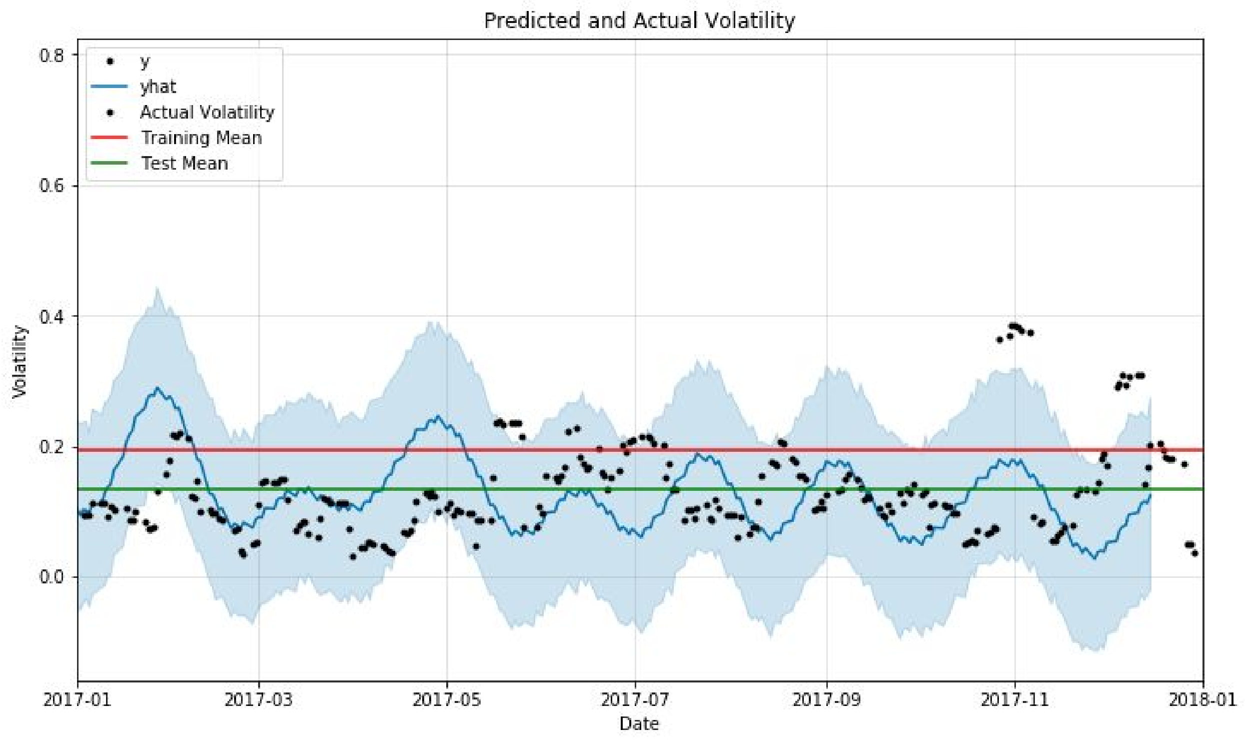




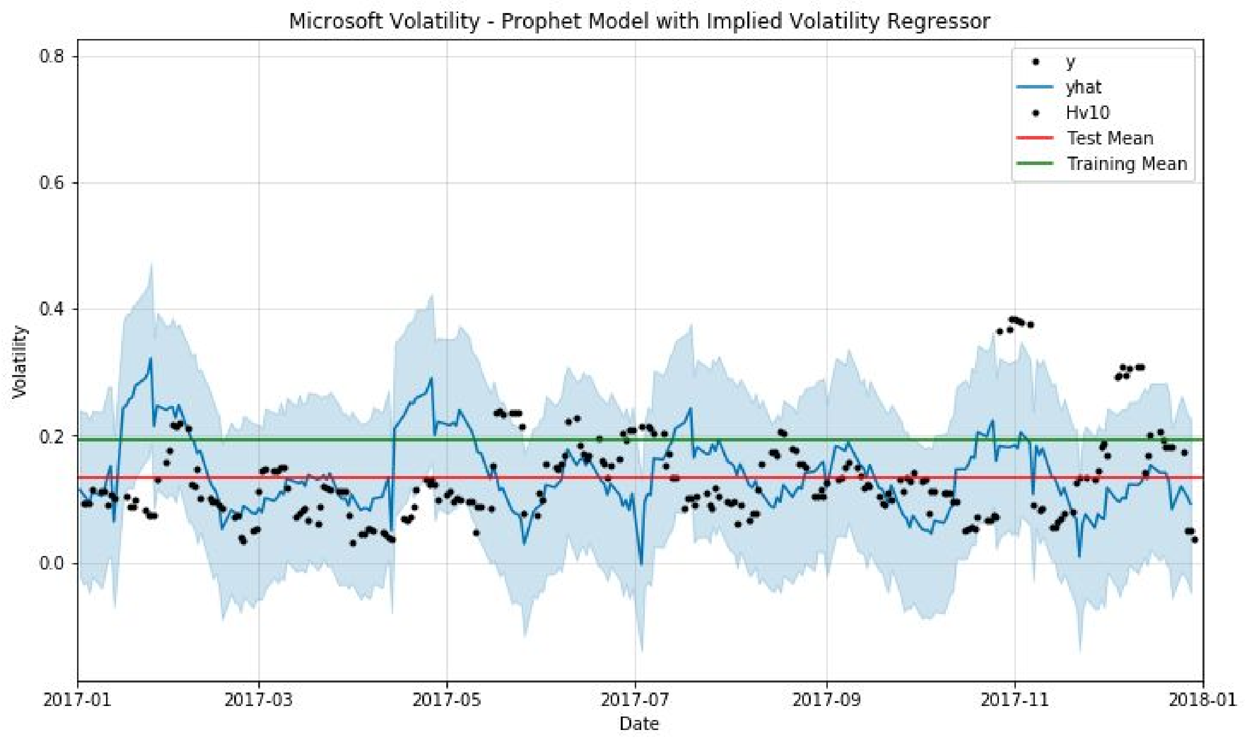
The next foray into modeling utilized Facebook Prophet (FBProphet) for Python. This model claims to outperm ARIMA, and I wanted to put that claim to the test. The first look at the model showed immediate improvement.



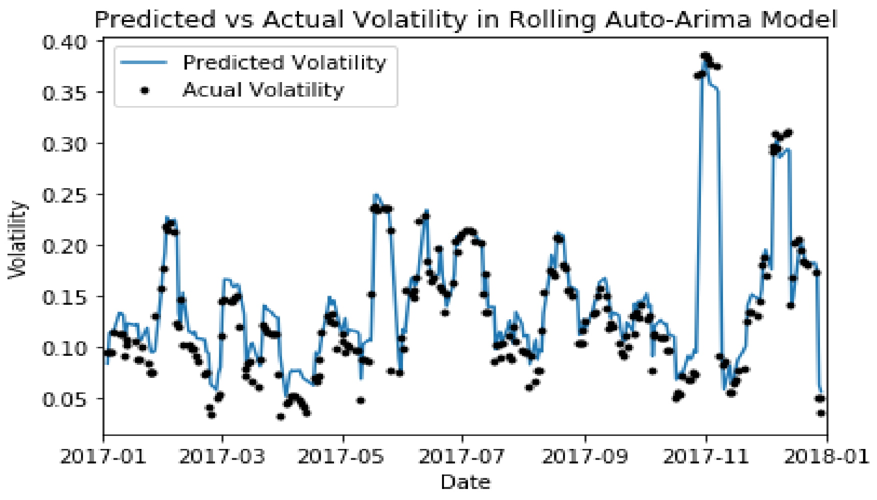
At the very least, the model above shows an attempt at modeling based on current volatility as well as seasonal patterns and trends. Close examination of 2017 shows that the model captured the market's move towards decreased volatility over the period (a general downtrend in 2017 predictions). A closer look at the 2017 forecast shows it to be significantly better than the baseline model. In addition to what can be seen on the chart, this model showed improvement in both root mean-squared error (rmse) and r-squared when compared with the baseline model.



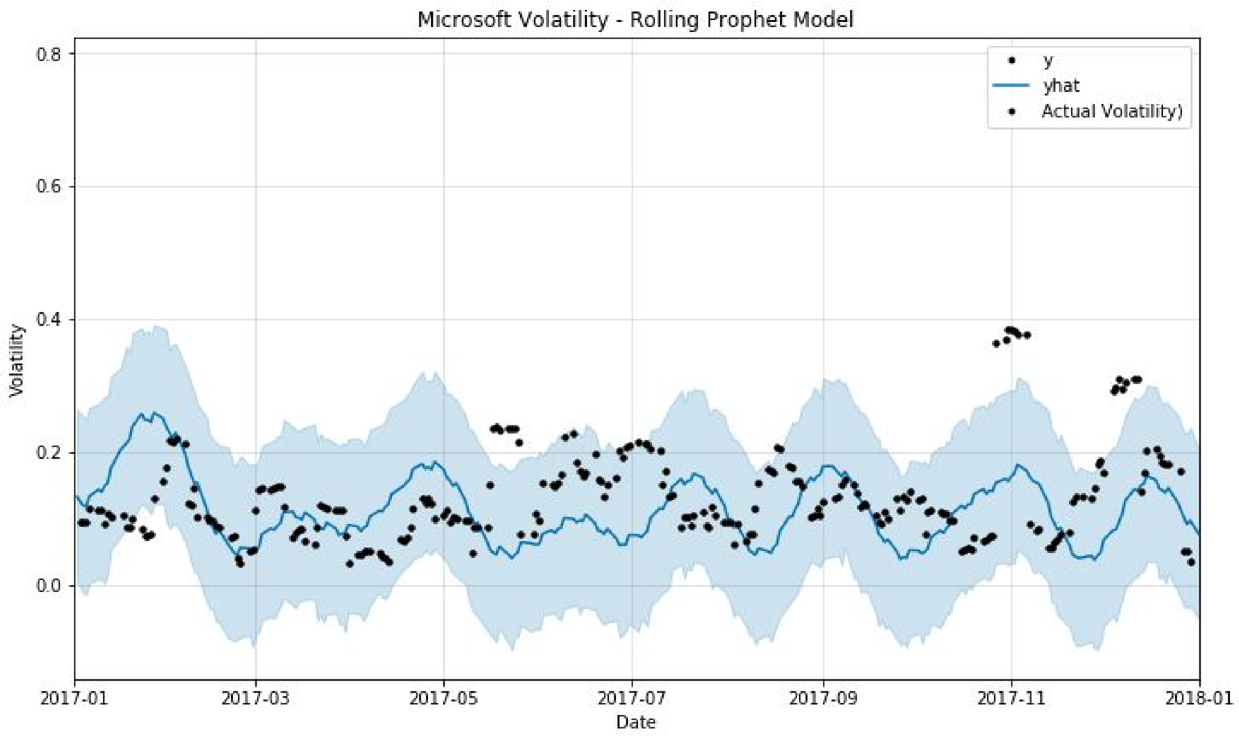
After proving that the initial Prophet model was more effective than the original auto-ARIMA baseline, I wanted to see if the Prophet model could be improved even further. I thought that by adding additional features, the forecasting accuracy of the this model might be improved. Prophet allows added regressors to be built into the model. Since the heatmap presented earlier showed a reasonable correlation between implied volatility and historical volatility I chose implied volatility as the feature to add. This attempt showed marked improvement over each of the first two in terms of R-squared and RMSE, meaning it was the most accurate model constructed so far. Once again though. R-squared value remained negative.



One thing I noticed about the models is that they seemed to build-in seasonal and trend patterns but not apply as much weight to recent values as I thought would be appropriate. After contemplating this issue I realized that this was a function of the way the previous models had been structured. The models essentially used data from 2013 through 2016 to build a forecast for the entirety of 2017. Only the regressor model had any 2017 data to work with (daily implied volatility) when making prediction. I speculated that fitting the model on all the data up to the day of the prediction might remedy this. To accomplish this I built a loop that would instantiate a new auto-ARIMA model for each day and output the result to a new dataframe.

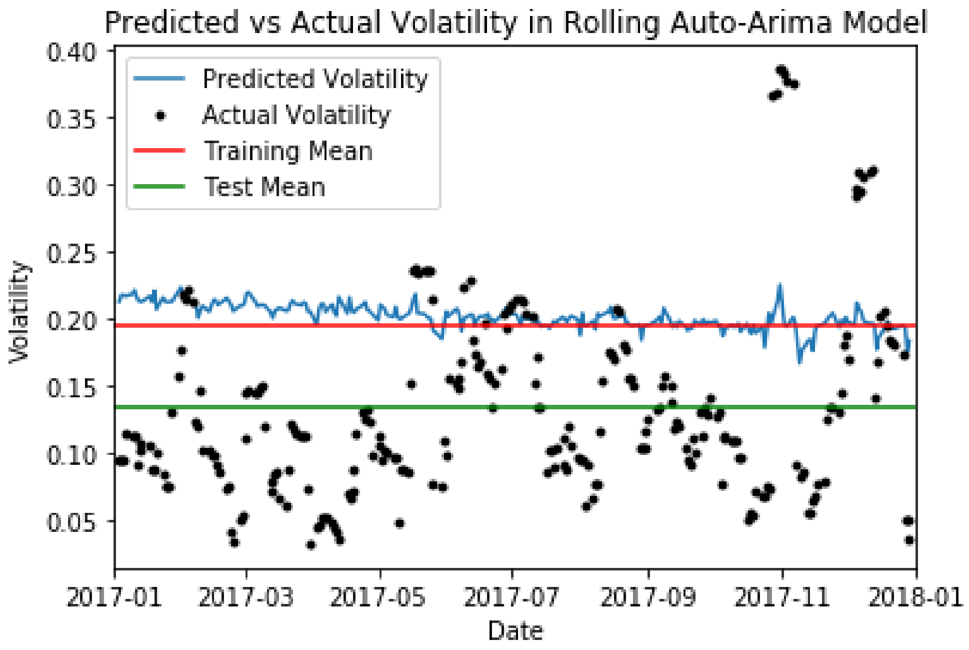


As seen above, the new rolling model was wildly successful in comparison to the previous three. It showed great improvement in RMSE, in addition to raising the R-squared value to .718 from a a value of -.688 in the next closest model. I then went ahead and created a new version of the prophet model to see how it would function if built in a similar, rolling, fashion.

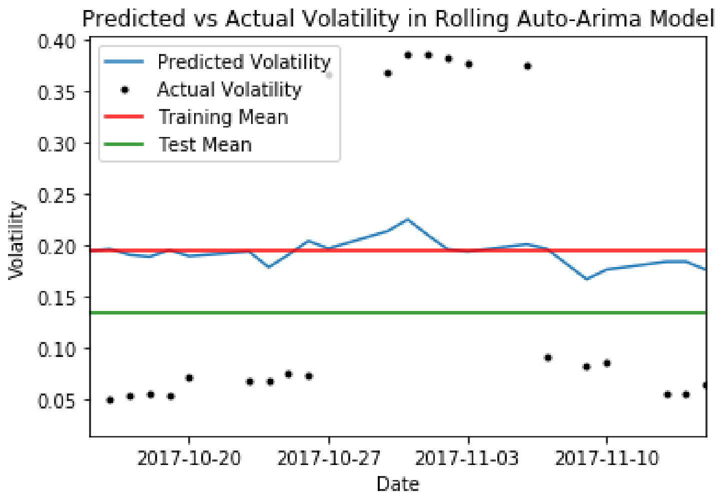


This Prophet model was actually less effective at forecasting than the rolling auto-ARIMA model. A glance at the plot above shows that this model seems to apply more weight to the trend and seasonality and less weight on the previous day's volatility than the Auto-ARIMA model.

The amount of improvement from the first auto-ARIMA model to the rolling one was a bit startling. I began this project wondering if I would be able to predict volatility in a way that would generate a positive R-squared value at all. To see it jump from -0.6 to .71 in one iteration of the model was definitely unexpected. After spending some time pondering the reasons for the dramatic turnaround, I uncovered a flaw in the premise of the rolling auto-ARIMA. The model was attempting to predict a 10-day average of historical volatility. By fitting the model to the last day before the prediction, it was given access to 9 of the 10 days included in that average. The model was essentially able to fit itself to 90% of the test set before making a prediction. To remedy this situation I built a new version that fit a model and then predicted 10 days out each time.

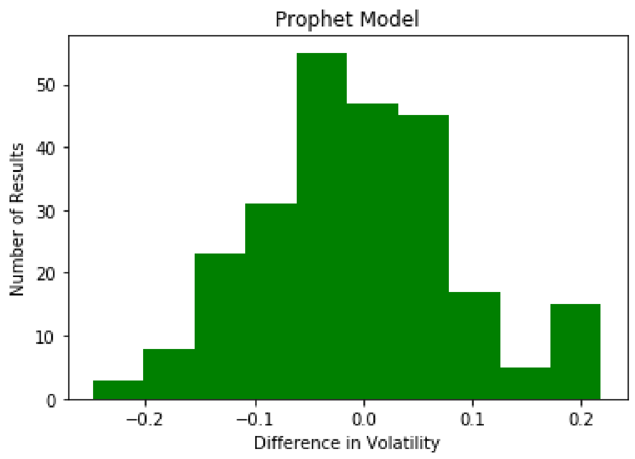


The resulting model is pictured above. This model is a large step back in terms of R-squared (-.948 the second worst model) but it actually has many positive aspects to it. First off, the jagged line shows that this model is actually taking the daily data into account and using it in the short term prediction. Secondly the overall downtrend of the prediction eventually moves the forecast below mean of the training set. In fact, in exploring the data for the last quarter of 2017, the model actually outperforms the mean of the training set and returns a positive R-squared for the period. Essentially the model is applying too much early weight to the previous period of higher volatility, but 'learning' to predict better as the model moves forward. There are also noticeable instances where the model makes accurate predictions about the direction of volatility, but is unable to accurately predict the scale of those moves, as in the diagram below.



Extended Analysis

While this final auto-ARIMA model is far from perfect, I believe it represents a major step in the right direction. Although I was unable to improve the metrics in the way I had hoped, the model does provide a positive R-squared in the final quarter of 2017. It also shows a number of instances of accurately predicting the direction of volatility if not the scale of those moves. The chart below shows the various models graded by how they performed in terms of RMSE and R-Squared. It also includes the upper and lower bounds for 90% density, meaning only 5 % of predictions fell below the lower bound of the test set and 5% fell above the upper bound. Prior to that is a histogram of the residuals of the best fitting (Prophet with regressor) model.



Model Performance

Model	RMSE	R-Squared	Lower Bound	Upper Bound
Auto-Arima (baseline model)	0.103	-1.143	-0.16	0.084
Auto Arima rolling (10 day window)	0.098	-0.948	-0.154	0.087
Prophet (Base)	0.097	-0.88	-0.122	0.198
Prophet with Implied Volatility Regressor	0.091	-0.688	-0.147	0.183
Prophet rolling	0.087	-0.501	-0.083	0.191
Auto Arima rolling (1 day window)	0.038	0.718	-0.053	0.028

While the above table shows the Prophet models to have significant statistical advantages, I do believe that the final auto-ARIMA model showed improved flexibility in adapting to market conditions. I believe that both models could have useful applications in the further examination of this data set.

Conclusions

The obvious takeaway from this exploration is that forecasting stock volatility is not a simple task. While I was unable to obtain the degree of forecasting accuracy I had initially hoped for, many improvements were made from the initial baseline model. Shortening the forecast period from a full year to 10 days was a major improvement in the quality of the model. Adding regressors was also helpful and is something that could potentially be expanded on.

Future Work

While I am uncertain as to whether 10-day historical volatility, specifically, can ever be accurately predicted, I believe these initial models to be an excellent jumping off point for attempting to predict moves in stock volatility. I believe there are a number of explorations that could be made that might significantly improve the forecasting accuracy of these initial models.

1. **Regressors**: Adding the implied volatility of the 10-day options to the prophet model provided significant improvement over the previous Prophet model. I believe that adding additional regressors could improve the predictive accuracy of these forecasts going forward. By adding additional regressors the model is directed to focus more on current market conditions than on longer term data which I suspect improves forecast quality.

2. **Shorter Term Training Set**: The final auto-ARIMA model was obviously hampered by the 4-year training set that took place in an era of higher volatility. This led to a model that performed significantly better at the end of the test period than at the beginning of it. The four-year test period was appropriate when an entire year was modeled at once. A shorter training period of say 40 days (for an 80/20 split) might be more appropriate when only predicting 10 days. The downside of that approach is that some seasonality in the model may be lost with a training period that short.

3. **Shorter Term Prediction**:- A general comment that can be made about predicting future events is that the further in the future they are, the harder they are to predict. In this project the length of the predictive period may have too large a hurdle. While 10-day historical volatility was a nice “out of the box” metric provided by Quandl, I’m not sure predicting what is essentially two full weeks of market activity is the easiest or most appropriate time period for the model. I believe that a shorter time period, perhaps 3 days, might be easier to forecast, while providing just as much, if not more, useful information from a business perspective.

Customer Recommendations

The goal of this exploration was to provide a ten day historical volatility prediction that would be actionable in the trading arena for a client. While I believe the project fell short in that regard, the modeling did provide some useful insight into the market. I was able to identify yearly and weekly patterns as well as overall trends in volatility that could provide guidance for a client.

Also, while the final auto-ARIMA model did not provide the accuracy I had hoped for, I do believe that it made useful predictions about the direction of market volatility, and that these predictions might be of use when making decisions. To be useful, the model need not necessarily predict the scale of moves.

Even a model that could predict the direction of volatility at a better success rate than a coin flip would be useful, and if it were to approach 60% accuracy, it could definitely be implemented into profitable trading strategies. Conversely, a model that could predict large moves without predicting direction could also be considered a success, and would be very useful for providing ‘smoother’ returns, something money managers are often measured by.