Programming Practice

2018-11-22

Week 12

Notice

Final Project

- We are going to start final project from this week.
- Final project is long-term project.
- You have to do both homework and final project.
- More details will be introduced later.

Practice Lecture

Numerical Analysis

- Calculate approximate value rather than exact answer, using computer.
  - ex) find the root of $\ln(x^4+1) * \ln(x^2+1) = 7$
  - hard to solve mathematically

Newton-Raphson Method

- Method for finding successively better approximations to the root of $f(x) = 0$.

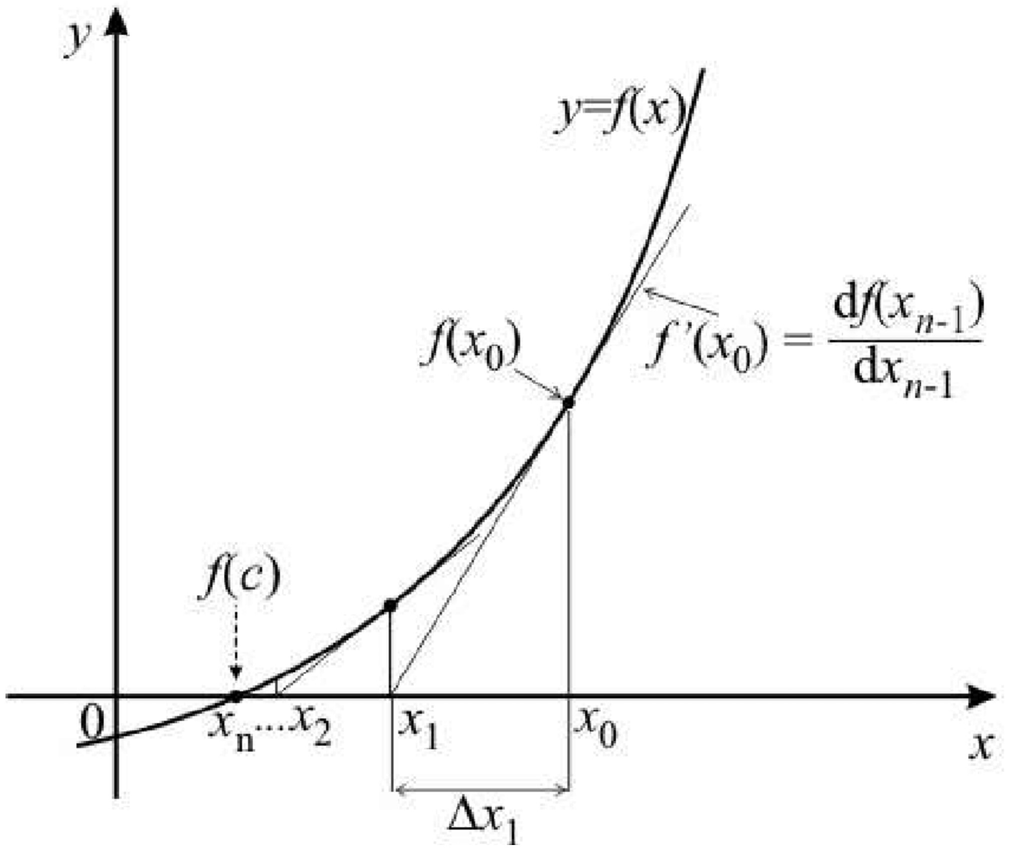
- Root of $f(x) = 0$ equals to the point where $y = f(x)$ and x-axis intersect.

reference: https://en.wikipedia.org/wiki/Newton%27s_method

Newton-Raphson Method

- Start with initial value x_0.
- Draw a tangent line at $(x_0, f(x_0))$.
  - tangent line: a line with slope $f'(x_0)$ and passes through $(x_0, f(x_0))$.
- new value x_1 equals to the point where the tangent line and x-axis intersect.
- x_1 is closer to the root c.

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$



Newton-Raphson Method

- Repeatedly, you can calculate x_{i+1} from x_i in a same way.
- x_{i+1} is better approximation. Use $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$
- Do until your approximation is sufficiently close to the root.
- How to know it is sufficient?
  - $|x_i - x_{i-1}| \leq 10^{-6}$
- This method is not always available – related to the shape of function & initial value (Poor initial value problem)

Newton-Raphson Method

- Example: $f(x) = 8x^3 - 12x^2 + 6x - 1 = 0$, $x_0 = 0.8$
  - $f'(x) = 24x^2 - 24x + 6$
  - absolute answer is 0.5

```
start: 0.800000
 -> 0.700000 -> 0.633333 -> 0.588889 -> 0.559259 -> 0.539506
 -> 0.526337 -> 0.517558 -> 0.511706 -> 0.507804 -> 0.505202
 -> 0.503468 -> 0.502312 -> 0.501541 -> 0.501028 -> 0.500685
 -> 0.500457 -> 0.500304 -> 0.500203 -> 0.500135 -> 0.500090
 -> 0.500060 -> 0.500040 -> 0.500027 -> 0.500018 -> 0.500012
 -> 0.500008 -> 0.500005 -> 0.500004 -> 0.500002 -> 0.500002
Process returned 0 (0x0)   execution time : 0.180 s
Press any key to continue.
```

Summary

- Initial value x_0 and equation $f(x) = 0$ is given.
- Start with x_0, calculate $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$ repeatedly.
- Stop when $|x_{i+1} - x_i| \leq 10^{-6}$. That x_{i+1} is the answer.

Homework Problems

1. Newton-Raphson Method

Problem. 1

Newton-Raphson Method

Description

- Your only job is to implement newtonMethod function which calculates and returns(do not print) the approximate root of $f(x) = 0$ for arbitrary $f(x)$, $f'(x)$, and initial value x_0.
- Provided skeleton code newton.c, implement newtonMethod and submit whole of it.
- Do not modify other functions : main(), f1(), df1(), f2(), df2().
- You can assume that the function will only handle the available situation.
- Absolute error is allowed up to 10^-4.

Function Prototype :

```
double newtonMethod(double x0, double (*fp)(double), double (*dfp)(double))
```

x0: initial value x_0
fp: function pointer of $f(x)$
dfp: function pointer of $f'(x)$

Skeleton Code

Function Name	Description
`main()`	Reads information of f1(x), f2(x), and initial value of those. It will call **`newtonMethod(x01,f1,df1)`** and **`newtonMethod(x02,f2,df2)`**, and print out the results.
`f1(x)`	Prepared function to test **newtonMethod**.
`df1(x)`	First derivative of f1(x).
`f2(x)`	Another prepared function to test **newtonMethod**.
`df2(x)`	First derivative of f2(x).
`newtonMethod(x0,f,df)`	TODO :: implement **newtonMethod** function

main()

- how does main call newtonMethod function
- f1, df1, f2, df2 are function pointers

```
int main() {
    double x01, x02;
    double result1, result2;
    scanf("%d %d %d %d", &a, &b, &c, &d);
    scanf("%lf %lf", &x01, &x02);
    result1 = newtonMethod(x01, f1, df1);
    result2 = newtonMethod(x02, f2, df2);
    printf("%lf %lf\n", result1, result2);
    return 0;
}
```

How to Test

- First line of input contains 4 integers a, b, c and d. f1 and f2 will be
  - $f_1(x) = ax^3 + bx^2 + cx + d$
  - $f_2(x) = bx^3 + ax^2 + cx + d$
- Second line contains $(x_0)_1$ and$(x_0)_2$.
  - $(x_0)_1$ is initial value for $f_1(x)$
  - $(x_0)_2$ is initial value for $f_2(x)$
- Each line of output is the approximate root of $f_1(x) = 0$ and $f_2(x) = 0$.
- Try various inputs and check your output with tools such as wolframalpha.

www.wolframalpha.com

Wolframalpha

-x^3+3x^2+5x+7=0

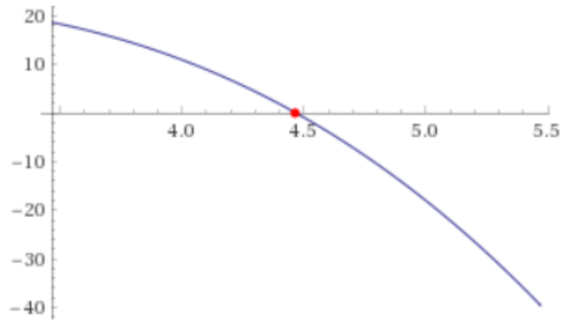
Browse Examples   Surprise Me

Input:

$$-x^3+3x^2+5x+7=0$$

Open code

Root plot:



Alternate forms:

$$x(5-(x-3)x)+7=0$$

$$-x^3+3x^2+5x=-7$$

$$-(x-1)^3+8(x-1)+14=0$$

Real solution:

More digits   Exact form   Step-by-step solution

$$x \approx 4.4692$$

Sample Testcases

[input]	[output]
4 3 2 1	-0.605830
-1.5 -1.4	-1.000000

[input]	[output]
-1 3 5 7	4.469220
5.5 -2.3	-0.863895