# Fertility, Migration and Capital Flows in China

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**Abstract**

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## 2.1 Setup

Consider the fertility choice with saving in the OLG model in which each agent live three periods, characterized by young(y), middle-aged(w), old(o). Only the middle-aged can work and the labor supply of each worker is normalized to one. Individual born on time t cannot decide his consumption in the young period, which is assumed to be zero for simplicity. Parents make the transfer to the children by feeding their children with education and consumption which is assumed to be proportional to half of the parents' total income which is after-tax and after-transfer. This setting is similar to Coeurdacier et al.(2013) but simplified into three periods.

The children's human capital accumulated with the amount of the education resources subject to the law of diminishing returns. We assume the individual productivity is proportional to the human capital and production only require labor with constant return to scale. The adult worker endowed with the human capital fed by his parents in the last period has the wage

$$w_{w,t} = A_t E_{w,t}^{1-\alpha} \tag{1}$$

Where $0 < \alpha < 1$, and $A_t$ is the economy-wide productivity. Note that this wage setting is deterministic. In fact, it could be the case that the agent with higher human capital earns

less wage because of the labor market friction. (need literature to support). However, the wage distribution in the searching model makes the return to the human capital uncertain and create too much individual heterogeneity which is not relevant to the main purpose of this paper.

Children and parents sign the contract which is articulated that when children grow up into workers, they have to give the proportion $\delta$ of his after-tax wage to his parents as the return to the education the parents spend on their children. We assume the inter-generational transfers given by each child are decreasing in the number of offspring due to the free-rider problem. Coeurdacier et al.(2013) give the evidence that the income elasticity of transfers is close to 1 and the elasticity of transfers with respect to the number of offspring is about 0.65.

One has to notice that either introducing free-rider problem in the transfer or the children's consumption which cannot increase the transfer they give to their parents is important. Think about the model where the consumption is not taken into consideration. Without the children's intergenerational transfer relevant to the number of siblings they have, the parents in terms of the program represented by utility function and budget constraints would decide to have more and more children until they meet the natural credit constraint, which is deduced by the fact that their consumption cannot be negative. The constant return to the number of children and the diminishing return to the education both generate this result. One may think that the free-rider problem acts as the decreasing return to the number of children could solve this problem. Then, More children each with higher marginal return to the education give lower transfers to the old parents. This can be valid in the theory. In fact, Mankiw et al.(1992) gives the human capital elasticity to the education is about 0.37, which is quite smaller compared to the transfer elasticity to the number of siblings. Therefore, we need to add the children's consumption as the friction of having children as the assets to avoid the corner solution.

While the rate of return to capital has indeed been very high in China documented by Bai, Hsieh, and Qian (2006), there is evidence that average households may not have access to assets with high returns, (see, for example, Song, Storesletten, Wang, and Zilibotti

(2014)). Song et al.(2011) build the model where the decreasing share of the State-owned enterprises depresses the supply of the asset supply and the bank allocates increasing share of household savings to the foreign assets. In this paper, we focus on the demand side of the assets. Therefore, we assume the extreme case that there is no physical capital in the model which means the absolute scarcity of asset, except later we introduce the housing production with capital. However, the average individual cannot get access to the housing production which is monopolized by the government. (need literature and evidence) Thus, The households save in the bank which in turn save on the foreign assets.

**Preferences and Budget constraints** Each household is consisted of two workers. After marriage, they together make the choice of the number of children, the education fed to their children and the assets positions. The household maximizes its life-time utility which includes the utility from consumption in two periods and benefits from raising children.

$$U_t = u(c_{w,t}) + vu(n_{y,t}) + \beta(1-x)u(c_{o,t+1}) \tag{2}$$

Where $\beta$ is the subjective discount rate. $1-x$ is the probability of the middle-aged getting into the old period. Lower x means higher life expectancy. v reflects the preference for the children. The budget constraints for the household are

$$c_{w,t} + a_{w,t} = 2w_{w,t}(1-\delta n_{w,t}^{\theta-1})(1-\frac{1}{2}\phi_0 n_{y,t}) - n_{y,t}E_{y,t} \tag{3}$$

$$c_{o,t+1} = \delta w_{w,t+1} n_{w,t+1}^{\theta} + Ra_{w,t} \tag{4}$$

Where $n_{w,t+1} = n_{y,t}$ and $E_{w,t+1} = E_{y,t}$. R is the interest rate of the foreign assets.

## 2.2 Household decisions

The Chinese government began to impose the population planning by making the limits on the number of children per couple to two nationwide in 1978, which is followed by the one-child policy enforced in the urban areas after 1980. The policy implementation in

the rural areas is not as strict as in the cities. For example, household with two daughters are permitted to have another children. Because in the rural areas, households need males to do manual labor in the agriculture; also, ...(need some cases and evidence). In contrast, in the city, individuals who have two kids cannot apply for the legal residence for their second child, which is called "Hukou" in China. Without the citizen registration, the agent does not have any right to education and legal job contract according to the constitution law in China. Therefore, it is not reasonable for people to have the second child in the city as long as they have more children mainly for the intergenerational transfer from them after retirement, since the child has no education and right to work will not have the income when he grows up. Therefore, the strict one-child policy is only applied to the urban areas and the rural areas are given enough discretion to make their fertility choices. Base on the data from the China Bureau of Population Statistics, the population is divided equally into two areas and the fertility rate is 1.6. With the fertility rate in the urban areas known to be 0.5, such rate in the rural area should be 2.7, which is greatly larger than 2. In this paper, we assume the extreme case that households in the rural areas are free to have the number of children they want and households in the urban areas are strictly subject to the one-child policy.

The one-child policy is imposed almost at the same time as the "Open Policy" led by Deng Xiaoping as one important tool in the package of the China's great economic reform.(we need a figure that show the gdp and fertility over time on the one graph) As Deng pointed out in his famous conversation, "Let small groups of people rich first and let them bring others to be rich then", the Chinese government focus on the economic development in the urban areas by building infrastructure, attracting FDI and giving subsidies to the local high-tech firms. In contrast, fewer resources are allocated to the rural areas. As a consequence, the technology level in the urban areas is largely greater than the counterpart in the rural areas. (need literature to support) Therefore, we assume that the worker can only gain the wage which is inferior to the worker in the city. The

after-tax wages are given by

$$w^u_{w,t} = (1-\tau^u)A_t {E^u_{w,t}}^{1-\alpha} \tag{5}$$

$$w^r_{w,t} = (1-\tau^r)\lambda A_t {E^r_{w,t}}^{1-\alpha} \tag{6}$$

Where $\tau^u$ and $\tau^r$ are the tax rates for the urban and rural residents at time t.

The financial services in the rural areas in China keep underdeveloped. Yao (2006) points out that financial organizations are in bad finance position. Rural Credit Co-operatives, which is the official financial intermediaries in the rural areas in China, keep monopolization in the rural financial market, which continue weakening the financial services. (need more literature) We simply assume that the workers in the rural areas are credit constrained. Since they have zero labor income when they are old, the borrowing constraint can be written as

$$a^r_{w,t} \geq 0 \tag{7}$$

We further assume that the productivity level in the rural areas is sufficiently low that the borrowing constraint is always binding. In brief, for the households in the urban areas, they meet the birth constraint while the rural counterparts are subject to the credit constraint. Also, In this economy, the absence of bequests means that the only individuals that optimize their saving are the middle-aged. With the assumption of the log utility, we solve the optimal fertility choice, education and saving for two types of households.
For the households in the rural areas,

$$n^r_{w,t+1} = \frac{2}{c_3\phi_0} \tag{8}$$

$$E^r_{w,t+1} = \frac{\beta(1-x)(1-\alpha)\phi_0 w^r_{w,t}(1-\delta {n^r_{w,t}}^{\theta-1})}{v-\beta(1-x)(1-\alpha)vc_1} \tag{9}$$

$$a^r_{w,t} = 0 \tag{10}$$

where $c_1 = \frac{1-\alpha-\theta}{(1-\alpha)v}$ and $c_3 = 1 + \frac{1}{v}\frac{\beta(1-x)(1-\alpha)+1}{1-\beta(1-x)(1-\alpha)c_1}$. For the households in the urban areas,

$$n^u_{w,t+1} = 1 \tag{11}$$

$$E^u_{w,t+1} = (\frac{A_t}{c_2})^{\frac{1}{\alpha}} \tag{12}$$

$$\begin{aligned} a^u_{w,t} = \frac{1}{[\beta(1-x_t)+1]R}[&\beta R(1-x_t)(1-\phi_0)(1-\delta)2w^u_{w,t} \\ &- \beta R(1-x_t)(\frac{A_t}{c_2})^{\frac{1}{\alpha}} - \delta A_{t+1}(\frac{A_t}{c_2})^{\frac{1-\alpha}{\alpha}}] \end{aligned} \tag{13}$$

Where $c_2 = \frac{R}{\delta(1+z)(1-\alpha)}$.

**Discussion** The basic trade-off in this model is between the decreasing return to education and increasing "financial friction" with more children, which is the children's consumption, accompanied with decreasing return to the number of children in terms of transfer. The latter factors dominate for the rural households. More children as the assets means increasing "financial friction" which is proportional to the parents' wage. Since the credit constraint for the rural household is binding, they can only have the certain number of children even with the increasing wage. As shown in the Eq. 8, The fertility rate in the rural areas at any time is a constant which is not relevant to the wage and productivity. As a result, all the increase in the wage is allocated to the children's education with the fertility rate remaining the same. Also, the fertility rate decreases with life expectancy. Yakita (2001) gives the similar result that the life expectancy is negatively related to the fertility rate.

**Assumption 1** The difference between human capital elasticity and transfer elasticity has to be small, which means

$$c_1 < \frac{1}{\beta(1-x)(1-\alpha)} \tag{14}$$

Actually, with the typical value shown in Coeurdacier et al. (2013), $c_1$ is negative, which satisfies our assumption under which the education level and fertility choice are strictly positive.

The urban households have been subject to the direct birth control since 1980. The

one-child policy distorts the education choice by increasing the education per child with greater growth rate than the productivity growth rate. From the wages determined in both area before, the gap between the real wage in both areas is broadening due to the more human capital over-accumulation in the urban area. (need some evidence: education gap and real wage gap) This is also corresponding to the quality-quantity trade-off in the theory literature starting with Becker and Lewis (1973). Urban parents devote overwhelming education to their only child. However, because of the decreasing return to the education, parents cannot perfectly compensate for quantity with quality. Therefore, they decrease the education spending that they would spend on the children in the case where there is no birth control. Also, the decrease in the number of children saves the parents financial friction to raise children as assets. Both are the two components of saving on the foreign reserve. Notice that less friction and more human capital per capita do not mean the birth control is beneficial in the sense that the marginal return to have an additional child is much higher than just save on the foreign assets. (need evidence on the increasing education and savings)

**Steady state** The steady state is characterized by the constant productivity growth, $1+z$, and constant state variables $n_{w,t} = n_{ss}$, $E_{w,t}/A_t^{\frac{1}{\alpha}} = E_{ss}$, when time t tends to infinity. We define $E_{w,t}/A_t^{\frac{1}{\alpha}}$ as the effective education spending.

For the rural households, the Eqs. 8 and 9 in the long run give the steady state fertility choice and education spending per child when $T \to \infty$.

$$E_{ss}^r = [\frac{\lambda\beta(1-\tau^r)(1-x)(1-\alpha)\phi_0(1-\delta {n_{ss}^r}^{\theta-1})}{[v-\beta(1-x)(1-\alpha)vc_1](1+z)^{\frac{1}{\alpha}}}]^{\frac{1}{\alpha}} \tag{15}$$

$$n_{ss}^r = \frac{2}{c_3\phi_0} \tag{16}$$

The effective education spending per child in the rural areas tend to be positively related to the discount factor on the rural productivity, $\lambda$, which means the higher the gap between the rural and urban areas, the smaller effective education spending will be devoted to the rural children, which further depresses their future income despite the low productivity. (need some literature on cross-country evidence) It also increases with

the children's consumption because it decreases the number of children rural households can have and make them devote more education to each child. As indicated in Eqs. 15 and 16, the 1% increase in the friction multiplier decreases the fertility rate by 1%, but raises the education spending per child by $\frac{1}{\alpha}\%$ due to the diminishing return to education. Also, with longer life expectancy, more patient rural individuals are willing to save on more foreign assets with lower interest rate but without financial friction. As a result, they tend to have a lower fertility rate and higher education per child because of the quantity-quality trade-off.

For the urban households, eq. 12 shows the steady state education spending per child and eq. 13 gives the foreign assets per capita in the urban area in the long run.

$$E^u_{ss} = [\frac{\delta(1-\alpha)}{R}]^{\frac{1}{\alpha}} \tag{17}$$

$$\begin{aligned} \frac{a^u_{w,t}}{E^u_{w,t}} = \{&\frac{1}{\beta(1-x)+1}[2\beta(1-x)(1-\phi_0)(1-\delta)(1-\tau^u_t)- \\ &\beta(1-x)(1+z)^{\frac{1-\alpha}{\alpha}} - \frac{1}{1-\alpha}(1+z)^{\frac{1-\alpha}{\alpha}}]\} \end{aligned} \tag{18}$$

**Proposition 1** The education level of the urban children is higher than the level of the rural counterparts in the steady state, when the the financial friction of having children as assets satisfies:

**Proof:** see Appendix A.

The human capital of urban individuals is indeed statistically higher than their counterparts in the rural areas, where they typically have no less than two children. (need evidence) This important fact tells the truth that the indirect birth control implemented by the limited financial resources supply is weaker than the direct one-child policy, which makes the urban human capital over-accumulation severer.

Another important implication from eq. 17 is that the foreign assets substitute the investment in the child's education in the sense that the increase in the return of the foreign assets depresses the education spending on each child. However, for the total saving on the foreign assets, there are both substitution effect and income effect. The income

effect comes from the lower human capital level led by the less education investment from their parents in the steady state. The income effect dominates which can be seen from the striking result shown in eq. 18 that The foreign assets per urban household is proportional to their education level. We assume that this proportion is positive to avoid the negative value of the saving, which would lead to the contradiction with the presumption that urban households are not credit constrained. With the contract that regulates more transfer from children to parents after their retirement, which means the marginal return of education investment rises, the household makes more education spending but less saving. One last thing to mention about is that the longer life expectancy plays no role in the education choice but increases the saving. In other words, it only affects the trade-off between the consumption today and tomorrow of the parents without any impact on the their choices on the children. So the intuition is simple which is that the longer one live, the more she saves to smooth the consumption.

**Proposition 2** With binding fertility constraint, the saving rate is non-decreasing with the productivity over a generation.

**Proof:** see Appendix B.

With the typical human capital elasticity to the education, the saving rate is increasing with the productivity.

With the urban households has the saving rate that is increasing with the productivity under the typical human capital elasticity to the education and also the rising productivity level after 2001, can we conclude that there will be more and more capital outflows from China? No, absolutely, because we have not taken into account the law of motion for the population in the urban areas. Actually, with the total separation between rural and urban areas, thanks to the one-child policy, the population in the urban areas become one-half of its original size after one generation. The growth rate of the foreign asset position during the same period is $(1+z)^{\frac{1}{\alpha}}$. Therefore, the growth rate of the total foreign reserve would be $\frac{1}{2}(1+z)^{\frac{1}{\alpha}}$. The productivity growth rate is typically 1% per year(need literature or evidence) after 2001. If we assume eighteen years are just one generation, then the eighteen years' productivity growth is about 20%, which implies that

the foreign asset positions should decrease almost 33%. However, the China's foreign reserve indeed balloons over the past 20 years and becomes the largest foreign claimant on US obligations. Therefore, we need to consider the migration from rural areas to the urban. These rural-born individuals choose to work in cities and then settle down there. Zhao (1999) documents that the migration of rural labor to urban areas in China since the mid-1980's has created the largest labor flow in world history. Seemingly, the rural individuals are fully responding to the relaxation of long-standing controls over rural-to-urban migration. In fact, this kind of cases becomes much more common after 2001 when China becomes the member of the WTO.(need literature and evidence from the Railway department, Chunyun) The access to trade and participation in the global supply chain, in which case people call China "the world industry", booms the manufacture in the urban areas and factories in turn recruit the rural individuals to do the simple and routine work, which does not need much education and has lower pay.(need evidence about chinese manufactural goods export share is rising and rural-born workers wage is lower) However, such low-pay jobs are still more profitable than the rural jobs because of the higher technology level in the urban areas, which attracts thousands of millions of rural middle-aged work there and some of them choose to settle down. With this great transition of the population in both regions and the reallocation of the labor resources across urban and rural areas, we generate the exact dynamics of the foreign reserve and provide possible explanations for the recent decline and strong recovery of it.

One last but important thing is that our theory is also consistent with the foreign reserve positions before 2000 after the implementation of the one-child policy in 1980. The average TFP growth rate during that period is 2%, the twenty years' productivity growth is about 50%, which implies that the total foreign reserve decreases 8% over the twenty years if we assume two regions are totally separated and workers are not allowed to move across regions. Indeed, as shown in table (chunyun), the increase in the labor flows are not enough to compensate the huge decrease in the population in towns and cities, or these new rural-born citizens do not have enough saving to trigger the positive increase in the total saving, which we analyze in Section 2.3. Therefore, since there is zero foreign reserve at the beginning of the one-child policy, such decrease in proportion

still makes the foreign reserve remain zero.

## 2.3 Migration and marriage decisions

The "open policy" package includes the relaxation of the migration control together with the birth control and market economy and participation to the international trade. This policy generates the dramatically rising labor flow from rural to the cities and towns. Alan de Brauw et al.(2002) document that from around 15% in 1981 by 1995, 32% of the rural labor force found some employment off-farm. They estimate that off-farm rural employment in China rose from less than 40 million in 1981 to more than 150 million in 1995. In 2002, one year after China entered the WTO, there was a sharp increase in the labor flows and the agricultural labor share declines steadily thereafter.(need evidence) However, as Zhao (1999) points out, still some rural individuals did not move. Such unwillingness to move can be explained by several factors. Most of existing empirical studies have found significant effects of individual, household, and community characteristics upon the migration decision (Hein Mallee, 1999; Xin Meng, 1999; Zhao, 1999); Banerjee et al. (2012) show that the government infrastructure investment, especially the transportation networks, significantly lower down the mobility cost for the rural labor, which booms the labor flow after the relaxation of the migration restriction; Zhang and Song (2003) estimate that the interprovince migrants were encouraged by the rural–urban income gap and discouraged by their geographic distances to destinations.

The rural middle-aged are assumed to incur a fixed cost after they move and work in the urban areas, which is consisted by two parts. The first part is common to all the agents which is the house rent in the urban areas; another part is the fixed labor cost, which is heterogeneous across agents and randomly drawn from a distribution. This distribution is a public information known both to individuals in the urban and rural areas and its cdf is $G(\cdot)$. The second part includes all the specific individual factors that may enter the decision of migration. The fixed cost is written as

$$f_t^i = \beta_1 w_{w,t}^u p_t + w_{w,t}^r b_i \tag{19}$$

Where $\beta_1$ is a coefficient, which will be clear later and $b_i$ is the fixed labor cost of individual i. $p_t$ is the housing price in terms of urban wage in urban area at time t. The incentive to work in the city is the rural-urban income gap which is arised from the discount factor in the rural productivity level. Naturally, the individuals endowed with huge fixed cost which cannot be covered by the income gap prefer to stay in the village. Then the cutoff fixed cost is defined as equal to the income gap, which in turn determines the fixed labor cost threshold.

$$b_t^* = (\frac{w_{w,t}^{ru}}{w_{w,t}^r} - 1)(1 - \delta n_{w,t}^{\theta-1}) - \beta_1 p_t \frac{w_{w,t}^{ru}}{w_{w,t}^r} \tag{20}$$

We consider a continuum of individuals in both regions, of which the measure is $N_t^u$, $N_t^r$. By the law of large number, the number of the rural workers choose to migrate is $G(b_t^*)N_t^r$, and the remaining choose to stay in the rural areas. Once they work in the urban area, they earn the wage from the combination of the urban technology and their rural human capital. Therefore, the after-tax wage of rural-born urban worker is

$$w_{w,t}^{ru} = (1 - \tau^u) A_t {E_{w,t}^r}^{1-\alpha} \tag{21}$$

Then, $\frac{w_{w,t}^{ru}}{w_{w,t}^r} = \frac{1-\tau^u}{\lambda(1-\tau^r)}$, which we define as $c_4$, a constant. For the rural-born urban workers, they can take the advantage of the advanced technology level in the urban areas and also they raise their children in their hometown where the fertility choice is not subject to the one-child policy, as long as they are married with rural workers no matter where they work. The only drawback is that they have the lower human capital compared to the urban workers and then earn less. We assume that even though the they can earn more than the counterparts who choose to stay, this income gap subtracting the fixed cost which is assumed to account for a large share of the income gap is not big enough to get the individual out of the situation where they are credit constrained. (need some evidence that nong mingong basically has no saving.)

The decision of migration is followed by the decision of marriage, which is the key mechanism that makes the urban population expand. There are three possibilities on who

the rural-born urban workers (rbu) can marry. One may marry rural-born rural workers (rbr); rural-born urban workers (rbu); urban-born workers (ubu). The chance of whom they may marry is proportional to the population of these three types of the middle-aged, which are

$$P_{w,t}^{(rbu,rbr)} = \frac{(1 - G(b_t^*))N_{w,t}^r}{N_{w,t}^r + N_{w,t}^u} \tag{22}$$

$$P_{w,t}^{(rbu,rbu)} = \frac{G(b_t^*)N_{w,t}^r}{N_{w,t}^r + N_{w,t}^u} \tag{23}$$

$$P_{w,t}^{(rbu,ubu)} = \frac{N_{w,t}^u}{N_{w,t}^r + N_{w,t}^u} \tag{24}$$

For the households where one of the parents work in the city, It is not profitable to bring the children with them to the urban areas for two reasons. First, the children also incur the fixed cost when they move to the urban school to get education; Another reason is that rural-born children are prohibited to be educated in the middle school in the urban areas. (need evidence) which means they have to transfer to their hometown school and again incur some fixed cost including adjustment cost. Table (here we need a table showing children are educated in their hometown) shows that most of the rural-born urban workers raise their children in the rural areas who are taken care of by their spouse who chooses to stay. In fact, children care and paying transfer to their parents which may includes not only the money transfer but the healthcare is the main reason that increases the rural middle-aged fixed cost so much that make them stay. Therefore, we drop the case that rural workers bring their children to the cities. On the contrary, the rural-born urban workers who marry urban-born workers have to follow the one-child policy under which they can apply the permanent residence permit which is called "Hukou" in China for their child and then, he/she can get educated in the urban areas. To give birth to the second child is illegal even one of the parents in the household is from rural area. As for the family that both of the parents work in the city, the children have to be brought with them which, we assume for simplicity, only incurs a certain amount of fixed cost for each child, which is denoted by $\frac{1}{2}[2w_{w,t}^{ru}(1 - \delta n_{w,t}^{\theta-1}) - b_{w,t}^i - b_{w,t}^j]\phi_1$. Therefore, this type of household has lower fertility rate than the households with one rbr and another rbu or

both rbrs. (need evidence comes from tan xiaoqing)

The rural-born rural workers have no chance to get married with the urban-born workers as they cannot meet with each other since they stay in the separating regions, not to mention love and then, marriage. Then, there are five types of households which are rbr-rbr, rbu-rbr, rbu-rbu, rbu-ubu, ubu-ubu. The number of each type of the households in this economy are

$$F_{w,t}^{(rbr,rbr)} = \frac{1}{2}[(1 - G(b_t^*))N_{w,t}^r - P_{w,t}^{(rbu,rbr)}G(b_t^*)N_{w,t}^r] \tag{25}$$

$$F_{w,t}^{(rbu,rbr)} = P_{w,t}^{(rbu,rbr)}G(b_t^*)N_{w,t}^r \tag{26}$$

$$F_{w,t}^{(rbu,rbu)} = \frac{1}{2}[G(b_t^*)N_t^r - (P_{w,t}^{(rbu,rbr)} + P_{w,t}^{(rbu,ubu)})G(b_t^*)N_{w,t}^r] \tag{27}$$

$$F_{w,t}^{(rbu,ubu)} = P_{w,t}^{(rbu,ubu)}G(b_t^*)N_{w,t}^r \tag{28}$$

$$F_{w,t}^{(ubu,ubu)} = \frac{1}{2}[N_{w,t}^u - P_{w,t}^{(rbu,ubu)}G(b_t^*)N_{w,t}^r] \tag{29}$$

The first three types of households are rural families. They are free to make the fertility choice, but still credit constrained since even the rural-born urban workers are not willing to save as we assumed. In other words, these households consume all their wage after giving transfer to their parents and feeding their children with education and some loss because of the financial friction. Therefore, in the case where the two regions are totally separated, their fertility, education and saving choices are subject to Eqs. 8, 9 and 10. However, with the relaxation of the migration restriction, parents should have in mind that their children have the possibility of migration to the city, earn more wage, and give more transfer. Since the representative household has the measure of $n_{w,t+1}$ children, $G(b_{t+1}^*)n_{w,t+1}$ will work in the city and the remaining stay in the village. Then, the program is rewritten as the same objective function Eq. 2 and budget constraint Eq. 3, but with another different constraint,

$$c_{o,t+1} = [(c_4 - 1)G(b_{t+1}^*) + 1]\delta w_{w,t+1}n_{w,t+1}^{\theta} + Ra_{w,t} \tag{30}$$

The basic trade-off in this problem is that with more children, each child would pay less transfer due to the free-rider problem among siblings and there would be more financial

friction. However, it can also make the work in the urban areas more profitable because the after-transfer income gap becomes larger which increases the cutoff fixed cost at the next period and then the proportion of rural-born urban workers in all rural workers. Also, having more children can increase the number of siblings working in the urban areas who give higher transfers. For simplicity, we assume the distribution of fixed cost is a uniform distribution of which the density function is $\frac{1}{b_{max}-b_{min}}$.

The fertility rate and the education level per child are now denoted by

$$\begin{aligned}\frac{\phi_0 n_{w,t+1}}{2-\phi_0 n_{w,t+1}} = [\beta(1-x)(1-\alpha) - \theta\beta(1-x) - v] - \\ \frac{\beta(1-x)(c_4-1)^2\delta(1-\theta)}{(c_4-1)G(b^*_{t+1})+1}\frac{1}{b_{max}-b_{min}} n_{w,t+1}^{\theta-1}\end{aligned} \tag{31}$$

$$E_{w,t+1} = \frac{\beta(1-x)(1-\alpha)}{1+\beta(1-x)(1-\alpha)} 2w^r_{w,t}(1-\delta n_{w,t}^{\theta-1})(\frac{1}{n_{w,t}} - \frac{1}{2}\phi_0) \tag{32}$$

**Steady state** The steady state is characterized by the constant productivity growth, $1+z$, and constant state variables $n_{w,t} = n_{ss}$, $E_{w,t}/A_t^{\frac{1}{\alpha}} = E_{ss}$, when time t tends to infinity.

$$\begin{aligned}\frac{\phi_0 n^r_{ss}}{2-\phi_0 n^r_{ss}} = [\beta(1-x)(1-\alpha) - \theta\beta(1-x) - v] - \\ \frac{\beta(1-x)(c_4-1)^2\delta(1-\theta)}{(c_4-1)G(b^*_{ss,t+1})+1}\frac{1}{b_{max}-b_{min}} {n^r_{ss}}^{\theta-1}\end{aligned} \tag{33}$$

$$E^r_{ss} = [\frac{\beta(1-x)(1-\alpha)}{1+\beta(1-x)(1-\alpha)}\frac{2(1-\tau^r)(1-\delta {n^r_{ss}}^{\theta-1})}{(1+z)^{\frac{1}{\alpha}}}(\frac{1}{n^r_{ss}} - \frac{1}{2}\phi_0)]^{\frac{1}{\alpha}} \tag{34}$$

**Proposition 2** The relaxation of the migration stimulates the fertility rate and lowers the education investment in each child in the steady state.

**Proof:** see Appendix B. (need evidence to prove that at the beginning of migration, the fertility rate increases)

Suppose initially the economy is in the steady state where the migration is prohibited. Now, with the shock of the relaxation on the migration, some rural workers move and work in the city and enjoy higher wages. The fertility rate suddenly jump up to a new

level which is denoted by Eq. 33 and the education investment per child jump down to the level denoted by Eq. 34. In other words, such migration from rural to urban areas mitigates the bad effect which is that the marginal return of raising another child is higher than the return of the foreign assets. This is caused by indirect restriction on the fertility choice imposed by the strict credit constraint.

The last two types are urban families. They are fertility constrained but give their child more education and also save which help them to smooth the consumption after retirement together with the transfer from their children. Thus, their fertility, education and saving choices are subject to Eqs. 11, 12 and 13.

However, since each rural-born urban worker draws different fixed cost randomly from a public-known distribution, different households have various incomes as a whole within type 2, 3, 4. The problem is that the huge heterogeneity in the household income leads to different education choices in type 2 and 3, which, fortunately, does not affect the fertility choice even it is not in the steady state. For the type 3, the fertility rate is lower because of the additional financial friction.

$$\frac{(\phi_0+\phi_1)n_{ss}^{r_3}}{2-(\phi_0+\phi_1)n_{ss}^{r_3}} = [\beta(1-x)(1-\alpha)-\theta\beta(1-x)-v]- \tag{35}$$

$$\frac{\beta(1-x)(c_4-1)^2\delta(1-\theta)}{(c_4-1)G(b_{ss,t+1}^*)+1}\frac{1}{b_{max}-b_{min}}n_{ss}^{r_3\theta-1} \tag{36}$$

However, for the rural types 1, 2 and 3, the education choices are totally heterogeneous across agents with type 3 spend the most on the education investment. Note that even children in type 3 households have higher education level, they have to go back to the rural areas to continue middle high school thanks to the "Hukou" institution. These children with higher human capital earn more wage when they grow up and in turn devote more resources to their children's education. Therefore, this virtuous cycle makes the human capital per capita averagely higher than the case where the migration is not allowed. Though the average education level could be higher, it is still quite lower compared to the level in the urban area, which, we assume for simplicity, makes them still credit constraint. Actually, in the general setting, they can have some savings after the education level is

higher enough. However, this saving is negligible compared to the saving of the urban workers with the strict birth control. Moreover, it is impossible to track the households generations by generations to know when they are no longer credit constrained.

**Proposition 3** The law of motion for the average education level in the rural areas is denoted by

$$\begin{aligned}\overline{E^r_{w,t+1}} = & \frac{\beta(1-x)(1-\alpha)}{1+\beta(1-x)(1-\alpha)} 2(1-\delta n_{w,t}^{\theta-1})(\frac{1}{n_{w,t}} - \frac{1}{2}\phi_0)\frac{1}{N^r_{w,t}} \\ & [(1-\tau^r)\lambda A_t \overline{E^r_{w,t}}^{1-\alpha}\phi_0 n_{w,t+1}^{r_1} F_{w,t}^{(rbr,rbr)} + \\ & \frac{1}{2}[(1-\tau^r)\lambda A_t \overline{E^r_{w,t}}^{1-\alpha} + (1-\tau^u)A_t \overline{E^r_{w,t}}^{1-\alpha} - \overline{b_t}]\phi_0 n_{w,t+1}^{r_2} F_{w,t}^{(rbu,rbr)} + \\ & ((1-\tau^u)A_t \overline{E^r_{w,t}}^{1-\alpha} - \overline{b_t})(\phi_0+\phi_1) n_{w,t+1}^{r_3} F_{w,t}^{(rbu,rbu)}]\end{aligned} \tag{37}$$

Where $\overline{b_t} = \int_{b_t^*}^{b_{max}} bg(b)db$. The growth rate of the average education level is higher than $(1+z)^{\frac{1}{\alpha}}$.

**Proof:** see Appendix C.

**Timeline** It is useful to summarize what we have discussed about and arrange them in the timeline. In the rural areas, after children have been educated, at the beginning of the period of middle age, they first randomly draw a fixed cost from a distribution which is assumed to be public information, based upon which they decide whether to move and work in the urban areas. After the decision of migration, they make the decision of the marriage at the same time. There are three possibilities for the rural-born urban workers. Once getting married, they make the fertility, education, and saving choices depending on where they settle down. Then, they work and earn the wage during the whole period to give the transfer to their parents and invest in the children's human capital. After retirement, there could be possibility that they die. Otherwise, they receive transfer from their children. The story is almost the same for the urban individuals except they do not make the migration decision.

With the sequence of the events in mind, we now give the law of motion for the

population in both areas.

$$N^{r}_{w,t+1} = n^{r_1}_{w,t+1} F^{(rbr,rbr)}_{w,t} + n^{r_2}_{w,t+1} F^{(rbu,rbr)}_{w,t} + n^{r_1}_{w,t+1} F^{(rbu,rbu)}_{w,t} \tag{38}$$

$$N^{u}_{w,t+1} = F^{(rbu,ubu)}_{w,t} + F^{(ubu,ubu)}_{w,t} \tag{39}$$

Apparently, the law of motion for the population depends on the evolution of the housing price, which in turn determines the foreign reserve growth rate. Next section we will focus on how the housing price is determined and how this interacts with rural individual's migration and marriage decisions.

## 2.4 Housing price