Q1

In the provided source code, there is a list of 16 normally distributed random values. Treat these 16 values as if they were a vector. What is the 2-norm of that vector? What is the average value of those 16 values? For this question, you should try five different seeds for the random number generator to get five different vectors of random values. Record your seeds together with your results, and let your seeds be six-to-eight digits in length, randomly selected by you so as to ensure your submission is unique from all other students.

Discuss why one may be significantly smaller than the other.

Q2

In Question 1, you were asked to look at taking the average of sixteen random numbers. When you look at many of the graphics showing the number of new cases of COVID-19, they often provide, for example, a 10-day average. Thus, the number for today is the average of today and the last nine days. The number for yesterday the average of yesterday and the nine days prior to that.

Explain why this gives a better trend than just showing a graph of the new cases per day. Explain one source of noise that may affect the number of new cases. One that you may not use, but is given as an example, is the weather. If the weather is miserable one day, fewer people may go in to get tested, and thus, the number of new cases may similarly go down.

Q3

Moving on to the sine and cosine functions, you are given 17 different vectors. The first is a vector of all 1s. The rest are vectors where the entries of the form $\cos(2\pi fi+1/216)$ or $\sin(2\pi fi+1/216)$ where i takes on values from 0 to 15 and where the frequency f is an integer from 1 to 8. Together, these define seventeen 16-dimensional vectors.

Exactly one of these has no useful information. Which one, and why?

Q4

What is the 2-norm of each of the remaining sixteen 16-dimensional vectors? Normalize these vectors by dividing these vectors by their 2-norm. You may use Python math.sqrt() function or you can use a minimum of 10 digits in approximating the 2-norm using a calculator or some other application and simply hard-code the divisor.

Give the 2-norm of each of the sixteen vectors in your answer. In subsequent questions, you will be required to work with these normalized vectors.

Q5

Now that you normalized your sixteen 16-dimensional vectors, what is the inner product of each pair of these vectors?

Record your answers but comment on those that are not exactly zero.

Q6

Now that you have normalized these sixteen 16-dimensional vectors, you will be picking a number of random seeds and generate corresponding 16-dimensional vectors of normally-distributed random variables (white noise). Record your seeds in your answer.

Try calculating the coefficient for each of the sixteen normalized vectors you created in the previous question. Try it with five different seeds and see if any of the coefficients are always significantly larger in magnitude than the other. Is any coefficient consistently significantly larger? Does this match your expectation?

Q7

Create a 16-dimensional vector where the entries are of the form Asin(2πfi+1/216+ϕ) where you pick a frequency from 1 to 8 and where A is the amplitude and ϕ (pronounced "fee") is a phase shift. Pick an amplitude that is not that close to ±1 (but not larger than 100 in absolute value) and a phase shift that is not very close to a multiple of π2, but it should be unique, so a number like ±0.9197057831 or ±2.129583835 or ±5.30813508.

Calculate the coefficients of your new vector for each of the 16 normalized vectors from above. What is the relationship between the coefficients and the 2-norm of your vector? (Yes, there is one, and you must find it.)

Q8

Now, pick a frequency that is not an integer, and preferably, not too close to an integer, say 4.2. Do the coefficients give that much information about your original 16-dimensional vector? Why or why not?

Q9

The following is a signal that contains white noise and two frequencies. This may be, for example, a signal taken from listening to the heartbeat of a fetus where there will be interference from the mother's heartbeat and general gastrointestinal noise (white noise).

-1.90, -6.05, -2.48, 2.82, -.324, 3.99, 5.22, -2.90, -.599, -1.64, -3.76, -2.97, 3.39, 4.23, 3.26, 3.35

What are your best estimation as to the amplitude and the frequency of the fetus's heartbeat and that of the mother? Of course, here we are going to assume that the heartbeat is a sinusoidal pulse, which is, of course, not correct, but this is sufficient for a first-year course. You are welcome to state any assumptions you make.

Q10

Finally, suppose you sampled a signal with a higher frequency, such as $\cos(2\pi 13i+1/216+2)$ where i goes from 0 to 15. This would give the points

1. -0.1591651345
2. -0.9730117519
3. -0.5855458235
4. 0.5248543817
5. 0.9872519740
6. 0.2307555723
7. -0.8106393076,
8. -0.8511920292
9. 0.1591651223
10. 0.9730117520
11. 0.5855458262
12. -0.5248543789
13. -0.9872519761
14. -0.2307555755
15. 0.8106393115
16. 0.8511920414

What are the sixteen coefficients if you took the inner products with the sixteen vectors you normalized above?

What frequency does this appear to suggest the data comes from?

Suppose you were sampling a sensor at 16 times per minute, but the sensor was reacting to an activity that was cycling 13 times per minute. This would result in the above situation. Give one idea as to how you could fix this so that you could accurately find the activity with a cycle of 13 per minute.

Q11

Write down the system of linear equations to describe the process of burning a mixture of methane, sugar, carbon and hydrogen together with oxygen to produce water and carbon dioxide. The equation is:

$aCH_4+bC_6H_{12}O_6+cC+dH_2+eO_2\rightarrow fH_2O+gCO_2$.

Before you start, how many free variables do you expect?

Next, solve this system and describe what would be the most appropriate choice of free variables, as opposed to the choice of free variables we used in class?

Finally, suppose you perform this combustion in an overabundance of oxygen. Consequently, there may be some oxygen left over. In this case, the equation is now

$aCH_4+bC_6H_{12}O_6+cC+dH_2+eO_2 \rightarrow fH_2O+gCO_2+hO_2$.

Without solving this, once again, try to deduce how many additional free variables this will introduce, and what would any additional free variables mean?